How to arrange a Singles Party

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In memory of Andrey Malishevski

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Abstract. The study addresses important question regarding the computational aspect of coalition formation. Almost as well known to find payoffs (imputations) belonging to a core, is prohibitively difficult, NP-hard task even for modern super-computers. In addition to the difficulty, the task becomes uncertain as it is unknown whether the core is non-empty. Following Shapley (1971), our *Singles Party Game* is convex, thus the presence of non-empty core is fully guaranteed. The article introduces a concept of coalitions, which are called nebulouses, adequate to critical coalitions, Mullat (1979). Nebulouses represent coalitions minimal by inclusion among all coalitions assembled into a semi-lattice of sets or kernels of "Monotone System" Mullat, (1971,1976,1995), Kuznetsov et al. (1982). An equivalent property to convexity, i.e., the monotonicity of the *singles game* allowed creating an effective procedure for finding the core by polynomial algorithm, a version of P-NP problem. Results are illustrated by MS Excel spreadsheet.

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1. **Introduction**

Leo Võhandu in his article published in "Teacher's Paper" (February, 2010) was concerned about students in Estonian universities and colleges stopping schooling after a half-year they start their studies; only 20% continue. According to Võhandu the root of problem rest on the fact that for many education programs are not mutually acceptable both for the students, and students themselves in training under particular program are not acceptable for the universities and colleges. Indeed, so-called mutual mismatch of priorities was the reason that students drop out of schools wasting in vain their time and "*the entitlement to government financial support.*" Therefore the problem arises how to match students and education programs best. Berge (1958, p.104, Russian translation) mentioned this type of matching problem in his book "The Theory of Graphs." Leo Võhandu proposed a way to solve the problem by introducing a *total regret* as the sum of pair-wise regrets selected within two directions of priorities: h*orizontal priorities* of a student with respect to programs, and *vertical priorities* of an institution with respect to students specializing in one program. The best solution, following LV, is where the total regret reaches its minimum. It is the lowest sum (amount) of *mutual regrets* found over all thinkable matching assignments of horizontal and vertical priorities of students and programs available for schooling in the institution. However, finding the best solution is a difficult task. Instead, LV proposed a greedy type workaround, Carmen et al. (2001), which in his words will approximate (be close enough to) the best solution matching students and programs by moving along increasing order of their *mutual regrets*.

It seems to us that LV's proposal to the solution in this wide area of research, is a typical approach in the spirit of classical utilitarianism, when the sum of utilities has to be maximized or minimized, Bentham, *The Principals of Morals and Legislation* (1789), Sidgwick, *The Methods of Ethics*, London 1907,… However, the weak point of utilitarian approach noted by Rawls in his "Theory of Justice" is the risk that, when the **max** or **min** is reached, there will still be members left in the society at very low utility levels say at very low levels of *regret compensations*, for example, transfer payments to the poor. Arguing for the ***maximorum minmorum*** principal, which was called the "Second Principal of Justice," Rawls suggests an alternative to the utilitarian approach. Along the lines of monotonic game, Mullat (1979), the motive determining this study was to address by example an alternative to LV, as well as all other solutions, that *the lowest regret compensation* should be maximized. The reader studying matchings can also find useful information about the issues, including the application of the idea of monotone systems, where a number of ways of constructing an optimal matching have been described, Tarmo Veskioja Dr. thesis on informatics and system engineering (2005, p.50).

Learning by example is of high value because the conventional core solution in a coalition game and well established *NM-solution* (Neuman-Morgenstrn) connected to the core of convex games cannot be well explained unless the scholars make themselves familiar with *utopian reality*, which does not exists. Thus, a rigorous set up of the game in simple terms will help to understand the otherwise complicated mathematics. By this note we hope to shed lights on what we call "The *Singles Party Game*." It should be emphasized once and for all that the game primitives actually represent an independent mathematical object in a completely different context despite we have used the concept of regret proposed by Võhandu and his scholars. We used the scale of regret for the transition to scale of mismatch compensations as well as to scale of incitements to dating.

The rest of the paper is organized as follows. We start with the preliminaries where the game primitives are explained. Next, the core concept of stability is introduced and illustrated but in connection to *singles game*. In Section 4, the reader will come across unconventional concept of nebulouses, coalitions minimal by inclusion, in accordance with our formal scheme. Afterwards we continue explaining our techniques and procedures of how to locate a core – collection of payoffs to all players. The results are illustrated by example in MS Excel spreadsheet where 20 single women and 20 single men attend a party. Section 7 ends the study with a conclusion.

2. **Preliminaries.**

Imagine a situation when 5 single women and 5 single men are willing to participate in Singles Party in order to find a suitable partner. The entrance fee is 50€[1] to cover refreshments, beverages, orchestra and arrangements expenses. The cashier disposes at most over the sum of 500€. Just before the orchestra was ready to play the last tango all gests have been kindly asked to fulfil a scheme of priorities: women asked to express their priorities about men, and visa versa, for each man about each woman. Those who agree to fulfil the scheme have been promised to receive a *chocolate*, which costs 10€, otherwise chocolates are not available if the scheme will not be disclosed. By definition, these players are *blind* and do not participate in the game. We also suppose that there was a plenty of time, before the last tango, making these priorities (preferences) clear to everyone. Below we continue in elaborating the rules of compensations to those who disclosed the schemes; we call them *participants*. By the rules *participants* will be compensated regardless of whether someone decides to date with suitable or unsuitable partner the end of the party or not.

[1] Note that red colour point at negative number.

We already supposed that during the party all 5 women, $\{1,...i,...5\}$ and 5 men $\{1,...j,...5\}$ got to know their mutual identities and priorities (preferences). Further on we use index i for a woman notification, and an index j for a man. Each woman i, $i=\overline{1,5}$, expressed her straight priorities w_i towards men, and each man j, $j=\overline{1,5}$, towards women as priorities m_j. Therefore, the priorities may be arranged into two 5×5 tables: $W=w_{i,j}$ and $M=m_{i,j}$, where the rows in table W are some permutations w_i of numbers $\langle 1,2,3,4,5\rangle$; some other permutations m_j stand in columns of the table M. In Table 1 below priorities w_{ij} (numbers $\langle \overline{1,5}=1,2,3,4,5\rangle$) might repeat themselves in columns of table W; repetitions may also happen in rows of the table M, i.e., at identical priority level a woman may be chosen by more than one man, and a man by many women. Mutual regrets $r_{i,j}=w_{i,j}+m_{i,j}$ occupy the cells in table R.

Table 1 $W = $ (Women Priorities) $+M=$ (Men Priorities) $=R=$ (Regret compensation)

Women Priorities (W)

	M₁	M₂	M₃	M₄	M₅
W₁	1	5	3	2	4
W₂	5	4	1	2	3
W₃	3	5	4	2	1
W₄	2	5	3	1	4
W₅	4	3	1	2	5

Men Priorities (M)

	M₁	M₂	M₃	M₄	M₅
W₁	3	4	2	1	2
W₂	1	3	4	2	4
W₃	5	2	3	4	3
W₄	4	5	1	3	1
W₅	2	1	5	5	5

Regret compensation (R)

	M₁	M₂	M₃	M₄	M₅
W₁	4	9	5	3	6
W₂	6	7	5	4	7
W₃	8	7	7	6	4
W₄	6	10	4	4	5
W₅	6	4	6	7	10

Assume that some participants will claim mismatch compensation since only unsuitable partners remained. Suitable were already taken, they went to dating. Others, those fortunate to go to dating will receive in advance an incitement payment, a prepaid ticket to happening, restaurant, concert, etc. Suppose that no one has found a suitable partner. Let for a moment, the mismatch compensation equals $c_{i,j}=\frac{1}{2}r_{i,j}\cdot 10$€. However, to compensate the participants in proportion to their level of regret is not just and fair, because there is a danger of misrepresentation of priorities due to profit motive, cheating, hiding, etc. Doing so, for example, couple $(5,5)$ may raise their compensation up to 50€! In Table 1, the **lowest regret** among all participants is $r_{1,4}=3$. Therefore arrangers of the party are ready to follow Rawls principle of justice as fairness – the principal of ***minimorum***, which equals to $\frac{1}{2}r_{1,4}\cdot 10=15$ € what we suppose they do regardless of it is a bad or good decision, right or wrong, etc. From the cashier point of view, it is also a reasonable compensation. Thus, the balance of payments for **all** participants, inclusive the cost of chocolate, will be negative: 50€+15€+10€ = 25€, i.e., -50€ as entrance fee, 15€ received as mismatch compensation, and 10€ as a chocolate.

What happens if couple $(1,4)$ decides to date after the party ends? Assume that each of them will receive an incitement for a date. Let, for simplicity the incitement equals to a doubled value of mismatch compensation or higher. Nothing prevents us to apply any other rule. Notice that the entire table R should be reorganized to reflect the fact that the rest, i.e., the women $\{2,3,4,5\}$ and men $\{1,2,3,5\}$, no longer be able to count on $(1,4)$ as potential partners. The priorities will fall; the scale $\langle 1,2,3,4,5 \rangle$ packs together or condenses into $\langle 1,2,3,4 \rangle$. Table 1 transforms into: [2]

Table 2 $W =$

	M1	M2	M3	M4	M5
W1	0	0	0	0	0
W2	4	3	1	0	2
W3	2	4	3	0	1
W4	1	4	2	0	3
W5	3	2	1	0	4

Women Priorities

$+M =$

	M1	M2	M3	M4	M5
W1	0	0	0	0	0
W2	1	3	3	0	3
W3	4	2	2	0	2
W4	3	4	1	0	1
W5	2	1	4	0	4

Men Priorities

$= R =$

	M1	M2	M3	M4	M5
W1	0	0	0	0	0
W2	5	6	4	0	5
W3	6	6	5	0	3
W4	4	8	3	0	4
W5	5	3	5	0	8

Regret compensation

The ***minmorum*** mismatch compensation did not change and equals to $c_{3,5} = 15€$. However, couple's $(1,4)$ potential balance $-50€ + 10€ + 2 \cdot 15€ = 10€$ of payments improves. N.B., W1 and M4 each receive an incitement to date of $30€$ in accord with rule that the incitement is always higher than a doubled mismatch compensation! For the rest, the balance remains negative (in deficit) and equals 15€. The balance may only improve monotonically, in case the couple $(3,5)$ decides to date as well. The important thing is that given the value of the balance one can determine: who is who, i.e., who will go on dating and who would not, receiving the mismatch compensation.

Orchestra starts playing the last tango. Party is over. Decisions have been made and passed in writing to the organizers of the party. What should represent the best collective decision in the *Singles Party Game* based on the principle of ***maximorum*** of ***minimorum***?

3. Concept of stability. [3]

Our motive here and now will be to illustrate the well-established solution in many persons' games, called the core, which is a conventional concept of stability. It is helpful first to focus on our model of *singles game* without any warranty of stability.

[2] This is the property and the only one property fostering the birth of MS, the "monotone system." Otherwise, the MS vocabulary remain sterile if used in any types of serialization methods applied for data analysis like regret scales or compensations, or what ever scales we use, etc., to bring analysis results in order for observations.

[3] Terminology, which we shall use below, is conventional. We use a payment, payoff(s), imputation, reward, etc., instead of the term compensation.

Let us turn to a more general study of the situation. The gests attending the party are: n single women $\Omega_w = \{1,...i,...n\}$ and m single men $\Omega_m = \{1,...j,...m\}$. All arrange a union $\Omega_w \cup \Omega_m$, $|\Omega_w \cup \Omega_m| = n + m$. Some of the guests expressed their willingness to participate in the game by disclosing their priorities. Those who refused have been called *blind players*; see above. Those who agreed arrange the grand coalition $\mathbf{P} \subseteq \Omega_w \cup \Omega_m$. Further on we label them by $i, j, \alpha,...,\sigma \in \mathbf{P}$. Recall that all an only the members of $\mathbf{P}$ are the *participants*. In contrast to i, j, labels $\alpha,...,\sigma$ emphasize no difference of sexes. For the reason to make notifications short we refer when needed to $\alpha,...,\sigma$ as an eventual matching or couple.

In many-persons games, by a coalition is understood a subset H of participants/members of $\mathbf{P}$, $H \subseteq \mathbf{P}$. Coalition H in *singles game* represents participants who did not yet decided with whom they wish to date. The situation when all did, $H = \varnothing$, or, in contrast, no one did, $H = \mathbf{P}$, is thinkable, blind guests $\alpha \notin \mathbf{P}$. Quite reasonable to think that he/she might feel sorry to date despite disclosing the scheme and will be hereby labeled as a *dummy* but not as a blind player. Among all coalitions $H \in 2^{\mathbf{P}}$ we usually distinguish rational coalitions – a member α in such coalition extracts from the interaction in the coalition a benefit, which is satisfactory for $\alpha \in H$. Sometimes it is further stipulated that extraction of this benefit is ensured independently of the actions of the players entering the anti-coalition.[4] Acting rationally a subset of members in a coalition H can improve their individual positions joining some other coalition. Such action is an example of instability.

In his work "Cores of Convex games" Shapley (1971) studied convex games, where this type of instability could be eliminated, so-called games with a nonempty core. The core is a convex set of end-points (imputations) of a multidimensional octahedron, i.e., the payoffs to all players. Using membership to the coalition H in the *singles game* we can always construct payoffs to all players $\mathbf{P}$. The inverse is also true that any payoff (imputation) clearly states who belongs to the coalition and who is the member of anti-coalition. Recall that the members of the anti-coalition $\overline{H}$ receive an incitement to date, which is equal or is higher than doubled mismatch compensation. Therefore, in the *singles game* a one to one correspondence exists between the core and a set of special coalitions *generated by the core*. We call them nebulouses. Members of nebulouses are not able to improve their positions by participating in any other coalition belonging to a nebulous, say internally stable.[5]

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[4] This situation leads to 2-person game, the coalition H against anti-coalition $\overline{H}$, $\mathbf{P} = H \cup \overline{H}$. It was proved that 2-person game always has a solution. In particular, for the coalition game it establishes the Shapley value.

[5] Our vocabulary is somewhat unconventional in this connection, but convenient.

In contrast to individual payoffs, improving or worsening the positions of members, when playing a coalition game, a payoff to a coalition H as a whole is called the characteristic function $v(H) > 0$. In classical cooperative game theory payoff $v(H)$ to coalition H is known with certainty. The convex property reflects a kind of synergy effect when two coalitions S and T join together subject to $v(S) + v(T) \le v(S \cup T) + v(S \cap T)$, called *super-modularity* condition. In case a couple or participant σ joins or leaves the coalition H, $\pi(\sigma, H) = v(H) - v(H \setminus \{\sigma\})$ defines marginal utility $\pi(\sigma, H)$ of $\sigma \in H$. Marginal utility expresses an increase (decrease) in utilities $\pi(\alpha, H \setminus \{\sigma\}) \le \pi(\alpha, H)$ of the membership for $\alpha \ne \sigma$, $\alpha \in H \setminus \{\sigma\}$ and is analogous to *returns to scale*, associated with convex production functions in economics, etc. Monotonic condition $\pi(\alpha, H \setminus \{\sigma\}) \le \pi(\alpha, H)$ is equivalent to super-modularity of characteristic function $v(H)$, which provides a full guarantee of the existence of a nonempty core. In general such a warranty cannot be given. This property, but inverse, was used to find solutions of many combinatorial problems, Petrov and Cherenin (1948), Edmons (1970), Nemhauser et al. (1978). In current study we analyze marginal utilities independently of super-modularity as a self-object.

Below we define, the payoffs for all members $\alpha \in H$ using pair-wise minimum of regrets in table R, see above. Recall that we eliminated rows and columns in tables W and M for the members of anti-coalition $\overline{H}$. The pair-wise priorities in H will decrease yielding to recalculation of regrets. The elimination result is labeled by $r_{i,j}(H)$, $i, j \in H$. Aligning the ***minimorum*** principle with the principle of justice as fairness, the reward or payoff (mismatch compensation) to any member $\alpha \in H$ is defined and equals to $F(H) = ½\, min_{(i,j) \in H}\, r_{i,j}(H) \cdot 10$ €; all receive an equal reward $F(H)$. Payoffs to dating couples as incitements to members of anti-coalition H, $i, j \in \overline{H}$ will be doubled. Thus, the value $v(H) = |H| \cdot F(H)$ defines the characteristic function of *singles game* [6]. Monotonic property $\pi(\alpha, H \setminus \{\sigma\}) \le \pi(\alpha, H)$ holds, because participant σ, joining $H \leftarrow H \cup \{\sigma\}$, expands the priority scale for members in H. A member $\sigma \in H$ leaving H, $H \leftarrow H \setminus \{\sigma\}$, in contrast, condense the scale. Thus, the following proposition is valid.

Proposion 1. *Singles Party Game* is a convex and there exists a non-empty core.

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[6] Check that $v(\mathbf{P})$, for $\mathbf{P} = \Omega_w \cup \Omega_m$, $|\mathbf{P}| = 10$, see the Table 1, equals to 150€.

4. **Concept of a nebulous.**

In view of "monotone system," Mullat (1971), exactly as in Shapley's convex games, the basic requirement of our model validity lies in monotonic property $\pi(\alpha,H\setminus\{\sigma\})\le\pi(\alpha,H)$. In other words, this means that eliminating an element α from H the utilities (weights) on the rest will fall or remain the same. To our knowledge there exist two kinds of monotone system, which we think is important to distinguish. The first class of monotone systems, called **p**-monotone: the order of utilities (weights) $\langle\pi(\alpha,H)\rangle$ on each subset H follows the initial order $\langle\pi(\alpha,\mathbf{W})\rangle$ on the grand set $\mathbf{W}$, or the grand coalition $\mathbf{P}$. The fall of utilities on **p**-monotone system does not change the weights ordering on any subset H, Kuznetsov et al. (1985). The second class: the grand order does not necessarily hold on some subset $H\subset\mathbf{W}$.

Clearly, any serialization (greedy) method on the grand set $\mathbf{W}$ immediately catches the structure of any subset $H\subset\mathbf{W}$ of **p**-monotone system. One may argue that **p**-monotone systems are quite sterile and do not bring much on the surface comparing to any serialization method. Economists, according to Narens and Luce (1983), will say that subsets H of **p**-monotone systems *perform* on interpersonally compatible scales. The *singles game* is twisted enough. The levels of regret in contrast to their initial ordering may fall in the table C in inverse order despite that priorities themselves of participants about each other remain in tact on any coalition $H\subset\mathbf{P}$. One will say then that levels of regret, in general, are not necessarily interpersonally compatible. Indeed, the order of two different pair-wise levels of regret for two eventual couples of participants can move when falling in the opposite direction compared to their original order in grand coalition $\mathbf{P}$.

Finally, we are ready to formulate some propositions.

Definition 1. *By kernel of singles game we call a coalition* $H^*=\arg\max_{H\subseteq\mathbf{P}}F(H)$; $\{H^*\}$ *is the set of all kernels.*

Proposition 2. *The set of all kernels* $\{H^*\}$ *arrange an upper semi-lattice of sets, i.e., the union* $H_1^*\cup H_2^*\in\{H^*\}$. *The set* $\{H^*\}$ *is closed under union operation of sets.*

Definition 2. *A kernel* $\mathbf{n}\in\{H^*\}$ *minimal by inclusion is called a nebulous – it does not include any proper kernel* $H^*\subset\mathbf{n}$: $H^*\not\subset\mathbf{n}$ *is true for all* $H^*\neq\mathbf{n}$.

The final step will require the concept of imputation, Owen (1968). A simple variant of imputation definition connected to *singles game* is a $|\mathbf{P}|$-vector of payoffs to all participants. Each coalition H induces an imputation $s(H)$, cf. Tables 3,4.

Proposition 3. *The set* $\{s(\mathbf{n})\}$ *of imputations, induced by nebulouses, arranges a core of the singles game. Imputations in* $\{s(\mathbf{n})\}$ *are non-dominant by any other imputation* $s(\mathbf{n}')\in\{s(\mathbf{n})\}$, $s(\mathbf{n}')\prec s(\mathbf{n})$ *or* $s(\mathbf{n}')\succ s(\mathbf{n})$ *not true.*

5. Finding the core.

Finding the core is a NP-hard task. The number of "operations" increases with exponent speed depending on the number of participants. However, we claim that the core of *singles game* may be found by polynomial algorithm. The basic idea lies in "Monotone System" technique by constructing so-called defining sequences of pair-wise matches first in increasing order of their pair-wise regrets (peakedness), and then decreasing. There are several variants of the same procedure, which do not coincide. Apparently there is no need to cover all the details. However, the algorithm below is given in a more convenient form to combine the procedure with a *singles game*.

Following algorithm finds maximal by inclusion kernel $\mathbf{H}$, i.e., *for* $\forall H^* \in \{H^*\} \rightarrow H^* \subset \mathbf{H}$.

1. **Input.** Build the regret table R, $R = W + M$, simple operation in MS Excel spreadsheet.
2. **Output**. Highlight by H_t those participants who have not yet decided to date after the party ends**;** $t \leftarrow 0$**,** $H_t \leftarrow \mathbf{P}$; define F as maximizer of F_t; pointer $t_p = t$ points to H_t accompanying the maximizer F. Recall that $\mathbf{P}$ is the coalition of participants of the game.

While $\mathbf{P}$ is not empty

$F_t = min_{(i,j)\in H_t} ½\pi((i,j),H_t)$€; $\alpha_t \leftarrow arg\,min_{(i,j)\in H_t} ½\pi((i,j),H_t)$€,

$H_{t+1} \leftarrow H_t \setminus \{\alpha_t\}$ highlights the row and column of indicated by couple α_t in tables W, M and R as deleted, $\mathbf{P} \leftarrow \mathbf{P} \setminus \{\alpha_t\} : t \leftarrow t+1$. Rearrange priorities in tables W and M counting on participants from $\mathbf{P}$ as potential partners for matchings, then recalculate remaining levels of regret in table R, those not yet highlighted as deleted.

End While Returns the set accompanying the maximizer coalition $\mathbf{H} \leftarrow H_{t_p}$

Definition 3. *A sequence of participants* $\langle \alpha_t \rangle = \langle \alpha_1,...,\alpha_{|\mathbf{P}|} \rangle$ *ordering participants of* $\mathbf{P}$ *is called the defining sequence of the singles game if in the sequence of coalitions* $\langle \mathbf{P} = H_1,...,H_{|\mathbf{P}|} \rangle$, $H_{t+1} = H_t \setminus \alpha_t$, $t = \overline{1,|\mathbf{P}|}$, *there exists a subsequence* $\langle \mathbf{P} = H_1 = \Gamma_{t_1}, \Gamma_{t_2},...,\Gamma_{t_p} \rangle$ *such that:*

a) *for any matching* $\alpha_t \in \Gamma_{t_k} \setminus \Gamma_{t_{k+1}}$ *of the sequence* $\langle \alpha_t \rangle$ *the pair-vise regret* $\pi(\alpha_t; H_t) < F_{t_k}$ $(k = 1,...,p-1)$;

b) *in coalition* Γ_{t_p} *no sub-coalitions exist at the level of regret higher than* F_{t_p}.

Definition 4. *Coalition* $\mathbf{D}$ *is called defining when* $\mathbf{D} = \Gamma_p$ *for certain defining sequence* $\langle \alpha_t \rangle$.

Proposition 4. The algorithm returns the largest kernel $\mathbf{H}$ of the *singles game*, $\mathbf{H} = \mathbf{D} = \Gamma_{t_p}$.

Before we finish this section a comment is in place. Technically minded person may notice that coalitions H_t are of two types. The *first* when operation $\alpha_t \leftarrow arg\,min_{(i,j)\in H_t}$ ½$\pi((i,j),H_t)$€ produces only one $\alpha_t \in H_t$. The *second* type when many α_t are available. In general, independently of $\exists H_t \big| t \geq t_p$ such that $F = F_t$ holds, α_t might be of *first* or *second* type. Let H_{t_R} is the last coalition to the right in the sequence $\langle \alpha_t \rangle$ maximizing F, thus the inequalities $F > F_t$ are true for the rest of $t > t_R$, where $t_p < t_R \leq |\mathbf{P}|$. Clearly, in case the coalition H_{t_R} is of *first* type it represents only one coalition, which is the only nebulous $\mathbf{n} = H_{t_R}$ of the game, and hereby the only one imputation $s(H_{t_R})$ belonging to the core. However, when H_{t_R} is of *second* type the situation is different. In fact, a difficulty arrives: In which order to select couples α_t to facilitate constricting the *defining sequence* $\langle \alpha_t \rangle$? We can solve the problem by backtracking technique. Explanation of backtracking is out of the scope of current investigation. However, one may find useful guidelines, Dumbadze (1989), who investigated different algorithms that effectively solve the problem. We intend to clarify the essence of this difficulty by experiment, that is the subject of the next section.

6. Experiments.

Recall that in the *singles game* the input to the algorithm above contains two tables: $W = w_{i,j}$ – priorities w_i of women about men in the form of permutations of $\overline{1,n}$ in rows, and the table $M = m_{i,j}$ – priorities m_j of men about women in the form of permutations of $\overline{1,m}$ in columns. Tables are well-suited objects in MS Excel spreadsheet that features calculation, graphing tools, pivot tables and a macro programming language called VBA (Visual Basic for Applications).

A spreadsheet was developed in order to illustrate our idea in search for nebulouses of the *singles game*, i.e., stable coalitions with imputations belonging to the core induced by these coalitions. Spreadsheet is available for downloading from http://www.datalaundering.com/download/singles-game.xls, or from http://www.datalaundering.com by (contact) request, as attached to E-mail. It takes for granted a "state of arts" of MS Excel uses. We first provide the user with the list of macros written in VBA, and then, after the macro has performed the calculus, we supply by comments the tables, extracted from the spreadsheet. We also hope that the spreadsheet exercise will be useful facilitating or at least illustrating programming technology of backtracking.

List of *singles game* macro-programming routines.

- **Dummy.** Ctrl+d — Removing (blind) gests from the list of participants who wish not play the game, or who feel sorry for dating. We call them DUMMY players.[7]
- **Match.** Ctrl+a — Constructing *defining sequence* by performing ***arg min*** operation, see the algorithm above.
- **Perform.** Ctrl+p — Re-establishes payoffs of dummy players counting that she/he once again is available for dating.
- **RandM.** Ctrl+m — Randomly rearranges columns constructing random permutations, priorities, in men's priority table ***M***.
- **RandW.** Ctrl+w — Randomly rearranges rows constructing random permutations, priorities, in women's priority table ***W***.
- **TrackB.** Ctrl+b — Restores the status of *Women*-***W*** and *Men*-***M*** priorities saved by **TrackF** macro: Performing Backtracking.
- **TrackF.** Ctrl+f — Saves the status of *Women*-W and *Men*-M priorities to backtrack by **TrackB** macro: Preparing Backtracking.

Spreadsheet layout. There are 20 single women and 20 single men attending the party, $n, m = 20$. Three tables will present themselves: **the Pink table *W***, women's priorities, **the Blue table *M***, men's priorities, and the **Yellow table *R*** – the mutual regrets table. The column to the right of the table R shows $min_{j=\overline{1,20}} r_{i,j}$ level of regret of couples (i, j) listing all women $i = \overline{1,20}$. The row down of the bottom of table R shows $min_{i=\overline{1,20}} r_{i,j}$ level of regret of couples (i, j) listing all men $j = \overline{1,20}$. In the corner-cell, down-to-the right stands the lowest $min_{i=\overline{1,20}, j=\overline{1,20}} r_{i,j}$ level of regret over the whole table R. Notice that the **green cells** in the table R visualize the effect of $arg\,min_{i=\overline{1,20}, j=\overline{1,20}} r_{i,j}$ operation. In the areas **V24:AO25** and **V26:AO26**, will be implemented the construction process of the defining sequence accordingly, together with the levels of regret accompanying this sequence; the players' balance of payments occupies the area **V31:AO32**. Some cells reflecting the *state of finances* of cashier are located below the area **V31:AO32**. Cells in row-1 and column-A contain the names of the guests. We use names of guests in macros **Dummy** and **Perform**.

Functional test. Users of the spreadsheet are invited first to perform a functional test. Calculations in MS Excel can be done in two modes, ***automatic*** and ***manual***. Choose properties and set the calculus in manual mode. Manual mode significantly speeds up performance of our macros. The purpose of the test is to be familiar with the effects of **ctrl-keys** attached to macros and what to do when something goes wrong. Recommended order of actions performing the test follows below.

[7] Actually it is irrelevant who is who, blind or dummy players.

- **TrackB.** Ctrl+b — When something goes wrong use this macro. However, it can also be used when backtracking is useful, see above.
- **RandM.** Ctrl+m — Make the cell A1 active! Perform the macro by Ctrl+m. Notice the effect upon men's priority table ***M***.
- **RandW.** Ctrl+w — Make the cell A1 active! Perform the macro by Ctrl+w. Notice the effect upon women's priority table ***W***.
- **Dummy.** Ctrl+d — Select an active cell in the row-1 or column-A to mark a man or women as a DUMMY player. Perform the macro by Ctrl+d.
- **Perform.** Ctrl+p — Warning. This macro can be used only when all dummy or blind players of the game have been selected in row-1 or column-A, and the macro **Dummy** performed for each of selected dummy player accordingly. Otherwise, that could lead to inconsistencies of calculations. Select an active cell in row-1 or column-A. Perform by Ctrl+p, as needed. Notice the effect in the area **V24:O26**.
- **Match.** Ctrl+a — We recommend first making the cell A1 active, however, it is not mandatory. Perform the macro, for example, twice: Ctrl+a, Ctrl+a. Notice the effect in the area **V24:O26**.
- **TrackF.** Ctrl+f — The effect of this macro is invisible. It can be used whenever it is appropriate to save the current status of all tables and areas necessary to restore the status by **TrackB** macro.

Extracting nebulouses of the game. We came closer to the goal of our experiment, where we visually demonstrate the main features of the theoretical model of the game by example. So-called *defining sequence* construction constitutes the skeleton of the theory. *Defining sequence* constriction goes on by steps. At each step in the construction sequence, we add to the right to already built sequence on previous steps, a matching with the lowest level of regret, i.e., a couple that decided to date after the party ends. Doing so until all participants are matched finishes the construction. On can easily check that at the beginning the levels of regrets increase, while to the end the levels will fall. The peakedness of levels of regret in row 3 of Table 3, rests on the monotonic property $\pi(\alpha,H\setminus\{\sigma\})\leq\pi(\alpha,H)$, see above. Recall that after a couple was matched we always recalculate levels of regret: the priority scales will "condense" or "pack together" upon all not yet matched participants are matched together. Let us try to build up a *defining sequence* using the **Match** macro 20 times: press Ctrl+a 20 times. Following data will occupy the area **V4:O28**:

Table 3.

		Nr.1	Nr.2	Nr.3	Nr.4	Nr.5	Nr.6	Nr.7	Nr.8	Nr.9	Nr.10	Nr.11	Nr.12	Nr.13	Nr.14	Nr.15	Nr.16	Nr.17	Nr.18	Nr.19	Nr.20
Row 1	**Women matched**	2	5	7	10	20	1	4	16	6	9	11	15	8	13	18	3	14	12	19	17
Row 2	**Men matched**	12	2	9	1	18	8	14	6	10	3	17	11	19	13	7	16	20	15	5	4
Row 3	**Levels of regret**	4	5	5	4	4	5	6	5	5	5	5	4	4	4	3	3	3	3	2	2
Row 4	**W-payoffs**	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €
Row 5	**M-payoffs**	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €	10 €

As agreed we label a couple (i,j) by α, thus the notation $\langle\alpha_t\rangle$ of a *defining sequence,* $t=\overline{1,20}$, is appropriate. Rows 1,2 correspond to the sequence $\langle\alpha_t\rangle$ together with levels of regret in Row 3; mismatch compensations incitements for dating are not payable, only the costs of chocolates of 10€'s occupy rows 4,5. Observe, that in accordance with monotonic property, the ***minimorum*** levels of regret first increase starting at 4, reaching 6 and then are falling down to 2. In column nr.2 regrets jump from 4 to 5, and in column nr.7, from 5 to 6. There are two nested coalitions accompanying the jumps: the coalition Γ_2 starting at column nr.2, and Γ_7 starting at nr.7. Notifying by Γ_0 the initial coalition $\mathbf{P}$ (the grand coalition) of participants, we conclude that $\mathbf{P}=\Gamma_0\supset\Gamma_2\supset\Gamma_7$. We also claim that the coalition Γ_7 is the maximal coalition $\mathbf{H}=\mathbf{D}$ by inclusion. Therefore $\forall\mathbf{n}\in\{\mathbf{n}\}|\ \mathbf{n}\subset\Gamma_7$, $\Gamma_7=\mathbf{D}$: each nebulous of the game lies inside the coalition $\mathbf{D}$ say the cloud of nebulouses. The cloud $\mathbf{D}$ was found by the algorithm but how to find a nebulous?

Let us backtrack to the status, when only 6 matches (dates) accomplished, i.e., all columns to right including the Nr.7 are empty. The status is tracked back by Ctrl+b, and 6 times by Ctrl+a. However, one can check that the coalition Γ_7 is not a nebulous because it is not minimal by inclusion. Pay attention to green cells at the level of regret **6**. In fact, a nebulous can be found in the following way. Perform exactly as follows. Make **m5** active and press Ctrl+d. Make **w19** active and press Ctrl+d. Activate **m5** and Ctrl+p. Activate **w19** and Ctrl+p. Column 7 instead of couple $(4,14)$ contains now $(19,5)$, a new matching – a dating couple nr.7. The nebulous, visualized below by Table 4, is found (manually) as the complement: a) of the set of original matchings, couples nr.1-nr to nr.6 in Table 3; b) together with couple $(19,5)$, to the set of all participants $\mathbf{P}$. Pink and Blue colors mark those who decided to date after the party ends, **Yellow** – those who did not yet taken decisions. Hereby, **Yellow** participants mark the members of a nebulous coalition inducing an imputation occupying rows 4-5 belonging to the core, i.e., a payoffs to all 40 participants. N.B., the green cells.

Table 4.

		Nr.1	Nr.2	Nr.3	Nr.4	Nr.5	Nr.6	Nr.7	Nr.8	Nr.9	Nr.10	Nr.11	Nr.12	Nr.13	Nr.14	Nr.15	Nr.16	Nr.17	Nr.18	Nr.19	Nr.20
Row 1	**Women matched**	2	5	7	10	20	1	19													
Row 2	**Men matched**	12	2	9	1	18	8	5													
Row 3	**Levels of regret**	4	5	5	4	4	5	6													
Row 4	**W-payoffs**	70 €	70 €	40 €	40 €	70 €	40 €	70 €	40 €	40 €	70 €	40 €	40 €	40 €	40 €	40 €	40 €	40 €	40 €	70 €	70 €
Row 5	**M-payoffs**	70 €	70 €	40 €	40 €	70 €	40 €	40 €	70 €	70 €	40 €	40 €	70 €	40 €	40 €	40 €	40 €	40 €	70 €	40 €	40 €

7. Conclusion.

One of the drawbacks in the application of formal schemes of monotone systems is its weakness or absence of appropriate interpretation of the results of the analysis. When the process of extracting nebulouses with the basic procedures ends, our nebulouses itself challenge the researcher's further advance. Usually, trying to interpret the results, the most in that what the researcher can expect is common sense. Regardless of the mathematical complexity, cleverly twisted rules, compensations, incitements etc., the *singles game* still has a point how to arrange a singles party. However, this is not enough in the social sciences, particularly in the economy, when it comes to reality that does not exist, e.g., the difference in political views, positions, political affiliation, etc. Monotone system's scheme does not allow coming closer in answering the question what is right or wrong, what is good and what is bad. Therefore, applying well-known and well-understood concepts and categories that have been successfully applied in the past, we can move forward in the right direction. Our advantage is that such a relationship was found and came out in the form of a well-known concept of the core in the theory of coalition games say in the theory of collective behavior.

In the theory of coalition games we usually do not indicate clearly what kind of coalition will be able to enforce or accomplish one or another imputation in the list belonging to the imputations in the core. Given imputation(s) of *singles game* (the game is so arranged) one can always specify by which coalition it was accomplished. Therefore, going through all of the coalitions, we are going over all of the imputations, there are simply no others. Consequently, setting the core, one can uniquely specify by which coalitions the core was accomplished. This allowed determining both by theoretical means and by experimental simulation that the computational aspect of coalitions formation in *singles game* is a version of P-NP problem.

We have not yet investigated the connection between the core induced by our nebulouses and NM-solution. Typically, the NM-solution must coincide with the core. Apparently it would be easy to give a constructive proof of this fact. All necessary facilities are available as it is possible to position all participants of any but not nebulous coalition in the order of the corresponding defining sequence. Now, it is easy to figure out that payments to all players grow, as mentioned earlier, while the construction of the sequence proceeds reaching a certain maximum point, then the payoffs decrease. Therefore, "*moving*" the players "*to the right or to the left*" along the defining sequence, formed in this way, the coalition would be either inside or cover some nebulous coalition. Now, we only need to add some players into the "emerging coalition" expanding the coalition in question by these players to form a coalition, or to exclude players from the "emerging coalition, " in order to coincide exactly with some nebulous coalition. The verification of the validity of our conjecture is quite possible. To do this, one who doubts or wish to confirm the conjecture just made, we recommend performing an experiment using the MS-spreadsheet available by request.

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