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Abstract: The article presents a survey of the principal quantitative tools adopted by the major financial institutions in the credit market, pointing out their limits and new directions.

The valuation approach is essentially based on quantitative models that focus on the implied probability of default, as well as on the implied correlations and on the implied time to default. The basic idea is to extract the parameters of the credit risk models directly from the market value. The two most commonly used approaches to model credit risk are structural models and reduced form models. The structural model was developed by Merton(1973) and Black, Scholes(1973). While, the reduced form approach was devised by statisticians and can be found in Duffie and Singleton (1997), Jarrow and Turnbull (1995), and Lando (1998). Due to their easy adaptation to market values, the reduced form models are the most used. Indeed, as we will see, they do not require too many parameters except for the recovery rate. The expected value of a credit with a face value 1 can be expressed as follows:

$$1 - (1 - R)q(t) \qquad 1.1$$

Where R denotes the recovery rate and q(t) is the probability of default density function between times t and t + Δt as seen at time zero. Some problems can arise with this formulation. The recovery rate is stochastic and can be correlated to the probability of the default density function. In the structural model, this is determined implicitly by the expected value of the loss that is a Put option written on the assets value with strike price equal to the face value of the credit. If we equal the value of the Put option to the credit risk spread by calibrating the implied volatility, we can get the implied recovery rate by using formula 1.1 where the probability of default is determined on the base of the Put option formula. This procedure is rather complicated and does not allow us to price quickly. Hence, in practice, is used the average of the recovery rate. The probability of the default density function q(t) can be written as:

$$q(t) = h(t)\, exp - \int_0^t h(\tau)\, d\tau$$

Where the *hazard rate* function h(t) is the instantaneous probability of default.

On the other hand, the survival probability p(t) is given by:

$$p(t) = exp - \int_0^t h(\tau)\, d\tau$$

The two formulations are linked by the following relation:

$$\int_0^T q(t)\, dt = 1 - p(T)$$

Formulation 1.1 can be rewritten as:

$$exp - (1 - R)q(t)\, \Delta t \qquad 1.2$$

This is an expected value in t + Δt. Thus, we have to use a risk free discount factor D(t) = *exp* – *r*(t) Δt to obtain its present value:

$$exp - [r(t) + (1 - R)q(t)]\, \Delta t \qquad 1.3$$

Where *r*(t) denotes the continuously compounded risk free interest rate and (1 – R)q(t) the credit risk spread. Note it is possible to calibrate formula 1.3 to the market value and obtain the implied default probability of the credit, in order make decisions on this base. This parameter expresses the liquidity and systemic risk as well. Obviously the pricing formula in general becomes:

$$exp - \int_0^T [r(t) + (1 - R)q(t)]\, dt \qquad 1.4$$

Some problems can arise in the computation of the integral. The interest rate and the credit risk spread can be stochastic and correlated. If we assume that they are independent for the interest rate, we can use a default free zero coupon bond as forward measure so that the integral of the credit risk spread can be extracted from the market value. Along the same lines, we obtain the following formula for the credit default swap spread:

$$s(0,T) = \frac{(1 - R) \int_0^T q(t)D(t)\, dt}{\Sigma^n_{j=1} D(T_j)p(T_j)}$$

We can note that the credit default swap spread is directly determined by the credit risk spread priced in the market to avoid arbitrage opportunity. We also see an inversion, where the market for the credit default swap determines the value of the credit risk market. This is a formula for a single name credit default swap, but we can have a multi name credit default swap. In this case, the problem is to determine the joint probability of default. The solution was given by copula approaches where the univariate marginal distributions are used to determine the joint probability given the correlations between the different names. Copula functions provide a unifying and flexible way to study multivariate distributions. The most common multivariate distributions are the Frank copula function, Clayton copula function, t-student copula function and Gaussian copula functions. For a bivariate normal distribution we have:

$$C(q_1,q_2) = N_2\, [N^{-1}(q_{1(T)}), N^{-1}(q_{2(T)}), \rho]$$

Where N_2 is the cumulative distribution of a bivariate normal distribution with correlation coefficient ρ . N^{-1} is the inverse of the cumulative univariate normal distribution. Now we can see how the implied default correlation of a multi name credit default swap can be extracted given the probability of default of each single name from the market value of the credit default swap. This parameter expresses the systematic risk associated with a portfolio of credits, i.e. the possibility of multiple defaults. This is an important parameter for the assessment of a multi names credit default swap and it is used for the decisions on credit market positions. We have to note that a multi name credit default swap is diversified because the main risk is the systemic risk because we can suffer just some loss in normal condition, The real problem is to estimate the joint probability of default but we can use a simple and immediate approach that is in line with the probabilistic solution. For a bivariate normal distribution we have:

$$N(q_{1(T)})\, N(q_{2(T)})\, (1 + \rho)$$

Now it is simple to extend the analysis to the multi name credit default swap. Another tool in the credit risk market is the time to default implied in the market value of the credits. We propose this valuation model:

$$q(T)R\; exp - \int_0^{\tau} r(t)\, dt \;+\; p(T)\; exp - \int_0^{T} r(t)\, dt$$

By equaling this formula to the value of the credit, we can derive a result for the implied time to default τ . This procedure allows us to compare different credits by choosing those with a greater implied time to default. In this case we can obtain more information with respect to the implied probability of default just if we use the recovery rate for the specific Rating class. The real problem is when we don't have a market for the credit, in this case we can compute the probability of default by using the Poisson distribution along Credit Risk^{+}. The Poisson distribution determines how many time an event will occur in an interval of time where the average and the volatility is given respectively by $\lambda(t,\tau)$ and $\sqrt{\lambda(t,\tau)}$:

$$\frac{\lambda^{n}\, e^{-\lambda}}{n!}$$

$\lambda(t,\tau)$ is the expected value of the number of default, or the basic probability of default that can be collected from the Rating agency. The parameter n permits to determine the probability that the event will occur n -times. At this point by putting $\lambda(t,\tau) = n$ we obtain the probability of default that we have to transform in the probability of default density function. We have to note that *n!*, for $n<1$, is equal to n itself. In the case we use the Poisson distribution to compute the probability of default of a portfolio of credits n is equal to the number of credits.

Conclusion

These approaches fails to capture the effective risk embedded in the credit market due to other factors that the market does not predict. A sound method should combine a qualitative and a quantitative approach essentially through microeconomic and macroeconomic analysis in order to avoid that a financial crisis jeopardizes the equity value.

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