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The Reality and Masquerade behind Bargaining over Welfare Pie Sizing, Delivery and Slicing [a]

September 4, 2010

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Abstract:

The present analysis addresses the apparently critical issue of circulation of wealth in society. Three actors play the game of welfare-related taxation. The first actor, in the role of Negotiator No.1, stands up for citizens' legal and moral rights to primary needs. The second actor, in the role of Negotiator No.2, proceeds in response to public will for the provision and delivery of public goods. Quite the opposite, the third actor, hereinafter named the Voter, who represents the taxpayers, prefers personal consumption to moral understanding and public activity. In fact, backed by electoral maneuvering, the Voter emanates a risk to break down negotiations. The result of the simulation provides an evidence for the claim that a 50% median income is close enough to be considered a realistic choice of poverty line within the variety or rules of the alternating-offers bargaining game and conditions for unanimous consent of voter-citizens.

JEL: C78, H21
Key words: bargaining, policy, public goods, simulation, taxation, voting

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1. Introduction

A welfare state presupposes the existence of both a market economy and a democratic political system. The hallmark of it is that the government intervenes in the distribution of goods and services to ensure a more equitable allocation than the free market can readily provide. The more welfare in the form of more and wider transfer payments, is it better? It depends, at least, who we look at and how the benefits are provided. The term welfare is used deliberately in this context since emphases are put on the government role. It is used, closely in the debates, which devote attention to what the State "ought to" or "should" not deliver. The high social security level lies in a problem area, because it might encourage certain behavior to low savings when economic security is guaranteed, to high wage demands because the state ensures the event of unemployment, to low social and geographical mobility, because the benefits are high, for black labor market as taxes are high, "moonlighting," – people working multiple jobs, and finally to work "too little" because it simply does not pay to "run longer and faster." The result is that human capital does not develop quickly and well enough. Our primary goal is to highlight the negative effects, called welfare hazard, i.e., an inverse working incentive of transfer payments, and long-term consequences of this using a theoretical model of flimsy or fictitious governmental institutions. Welfare hazard in this framework is closely related to delivery of benefits and subsidies.

Imagine a scenario of *real life masquerade* or *realistic utopia* when two flimsy or fictitious governmental institutions play a bargaining game. *Public agencies* want to maximize welfare pie as much as possible – more to share in the long term. They tend to focus on regulations of private business, limit government's scope of using its powers, to combat criminal violence and commercial fraud. *Social agencies* tend to see the source of evil in disproportion of consumption, unjust redistribution of wealth and income, the profit motive and private property. Therefore, social agencies prefer an additional equitable sharing of the available stock of goods and services here and now, which market forces are not suitable to provide, and which necessitates government intervention. Consider the choice of poverty line through social institutions that are aware of citizens tax sacrifices to finance services and subsidies to the needy. Whatever it is, the greater the sacrifice, the less private budget for consumption of all citizens, and so larger the quantity of public goods to be delivered. Social services and public goods are always the basic expectations that contribute to political debates about tax policies towards consumers who may see the effects of tax sacrifices on private consumption opportunities as voters represented by the social and public agencies.

Our brief remark hopefully clarified some goals of the State by which we might conclude that welfare policy in a representative democracy always faces conflicting interests of institutions. This paper tries to shed light on how a consensus is reached among government institutions and whether the consensus reflects tax policy criteria to be the greatest benefit for all members of society. To address this issue, we use two flimsy or fictitious actors, named negotiators. Our strategy masquerades negotiators with a political mandate to initiate proposals that either support or oppose expectations of citizens for more or better welfare services. A balance of transfer payments for benefits and services claimed by citizens lays the constraint on delivery of public goods bringing under control *the scene* of negotiations. The negotiators acting on the scene within limits of the constraint in the roles of social and public agencies try to get funding for the proposals, which is the classic problem of public finance allocation set up by the alternating-offers bargaining game on how to *share* the tax revenue, also called *the welfare pie.* The third actor behind the scene, the representative voter, is left with no options other than of voting for or against higher taxes.

We believe that the size of the welfare pie increases progressively depending on the poverty line increase. Despite the size increase, poverty line in our scheme will shape the expectations of negotiators in a different way: single $\cap$-peaked for social agencies, Fig. 1; all but not subsidies represent expectations of public agencies in accord with the size of the pie, Fig. 2. Notice, elevated single $\cap$-peaked curve **S** on Fig. 1 corresponds, to the lower but progressively increasing, concave curve **S** of expectations on Fig. 2, and other work around for **P** to **P**; x-axis indicate poverty line, y-axis expectations. In support to our belief, the expectations, shaped this way, emerge from income distributions inside two-man economy endowed by agents' income abilities marginalized on income level, i.e., agents with low incomes below the poverty line receive subsidies, agents with higher incomes, above the line, do not.

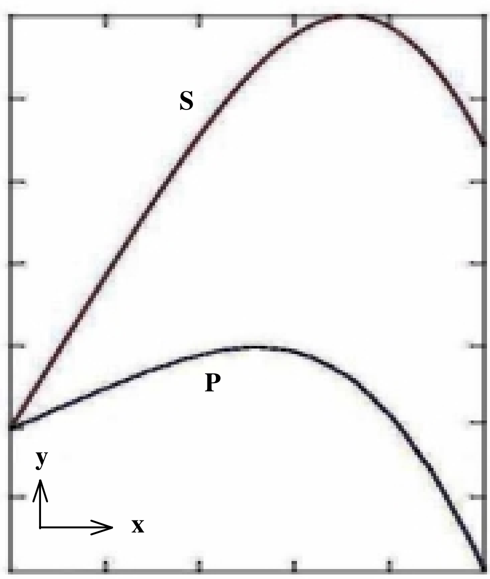

Figure 1, social agencies expectations

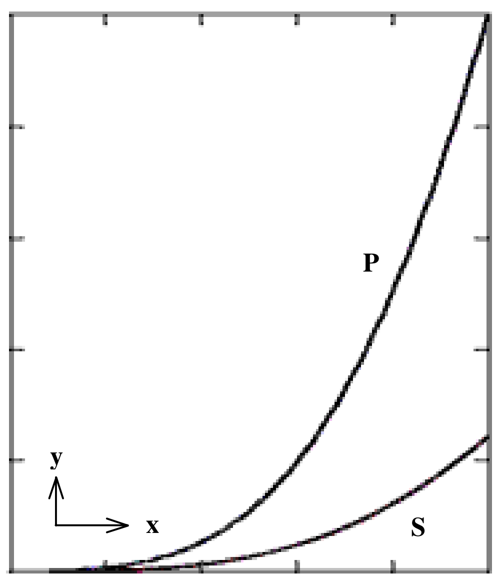

Figure 2, public agencies expectations

In respect to these so-called non-conforming expectations, when an offer is made (irrelevant who makes the offer), social agencies tend to control the poverty line parameter ξ independently. Varying the parameter, social agencies are supposed to reach the peak of their expectations. In making those suppositions, we hope together with Rawls [1971: 304] precepts of justice that "*The sum of transfers and benefits…*" (the subsidies $B(\xi)$, see below)… "*from essential public goods should be arranged so as to enhance the expectations of the least favored consistent with the required saving and the maintenance of equal liberties*."

Single peakedness plays a major role in collective decision-making, when the decision is arrived by a vote, originated from Black, (1948: 27). The expectation of social minimum to be single peaked is the result of an assumption that tax sacrifice taken upon income equal to the poverty line parameter is a progressively increasing function of the parameter itself provided negotiators commit to the slice of the pie in eventual agreement, observation 2.

Arranged traditionally, the bargaining problem consists of sharing some monetary resources in rational or just way. Literally speaking, the slicing of the pie is the main problem, what the players try to solve during the negotiations. Given that the expectations of players are non-conforming, i.e., single peaked for the first in contrast to the other, the traditional procedure of how to slice the pie may be put differently, which suites better, as we see it, to meet some circumstances of "real life controversies," – not any more as a division of a resource. The main idea of our message is the most important point that slices bargain procedure can be *rescheduled*, then, into poverty lines parametric space in a way that negotiations upon slices occur instead within some interval of the parameter, which turns to be the *scope of negotiations*. The other way around is true as well; in fact, Cardona and Ponsattí (2007: 628) also noticed that "*the bargaining problem is not radically different from negotiations to split a private surplus,*" when all in the bargain have conforming $\cap$-expectations. The situation holds true even the expectations of the second player are principally non-conforming, not single peaked but concave. What should be pointed out here concerns individual rationality, Roth (1977), known as a set of negotiations or bargaining set or "*well defined bargaining problem*," which allows dropping the axiom of "Pareto Efficiency." Well-defined bargaining set is rescheduled then to one-dimensional space, and the *status quo* or *breakdown* might be viewed as inefficient policy with notably low poverty line, outside the scope of negotiations.

To negotiate on slices or poverty lines, both procedures, we can say, are two sides of the same bargain portfolio. Therefore, are the players bargaining on slices of pie or trying to agree on poverty lines makes no difference. The main advantage of parametric procedure comes trough: it brings about a number of different patterns of interpretations of outcomes in the game, binding, for example, poverty lines to the average amount of taxable income, called the wealth, tax returns or, to the lowest tax rate, … etc., all as indictors of the most desirable sacrifice by taxpayers. Moreover, in consideration of alternatives ways to set the poverty line, which includes possibilities to describe the outcomes of collective bargaining in form of voting, or voting in form of bargaining, in any voting scheme the scope of negotiations brings the voting and bargaining under one roof. In this way we hope to enrich the range of interpretations for both – the bargaining and voting schemes.

Our welfare game comprises as an indicator the bargaining power of social agencies. To deal properly with the indicator, the breakdown point, however, cannot be given exogenously. To overcome the obstacle, one can supply the game with a point of breakdown extracted endogenously on condition, called *equity of breakdown*.

Beyond the perception of how to negotiate an expected slice of the welfare pie, it is also reasonable to believe that income distribution is, perhaps, the only target for control and an exclusive source to assess welfare policy. The welfare game is neither justified based on empirical income distribution nor does it provide the data and empirical support from the field. Even as this key weakness is understood, the poverty line, drawn under the rules of the game across typical income distribution, is close enough to be considered a realistic match (Table 1) with Fuchs point (50% of median income) by which terms Rawls (1971: 98) came out with an alternative to the second principle of justice with no reference to social position.

No part of the literature has dealt with parametric bargaining while *slicing the welfare pie* among institutions of "welfare state" and directly linking it to (a) welfare policy, (b) alternating-offers game, and (c) wealth redistribution. We are looking forward by the scheme to enroll such a *tripod-linkage* in the calculus of bargaining game solutions since it may be useful for institutional economics. Richter (2005: 387), in a review on "*Handbook of New Institutional Economics,*" indeed, pointed out "*that the sociological analysis…and large institutional structures in economic life is still at an early stage…game theory, and computer simulation could help to further develop the new institutional approach…game theory might be a defendable heuristic device of NIE.*"

To discern the root cause of the results and to find solutions for the welfare game, we try to move in all three directions of the *linkage* scheme that aims to bring on the surface the economic content in a rigorous mathematical form and to lay the way for the presentation of results along the following modules:

Stabilization	*Once balanced, the portion of the welfare pie for funding subsidies, throughout and in spite of volatility in the economy, must remain balanced;*
Bargaining	*The negotiations between social and public agencies of how to slice of the welfare pie comply with the rules and regulations in the alternating-offers bargaining game;*
Unanimous consent	*Bringing a motion to a vote is necessary to meet consumer perception against high taxes and excessive public spending. Whether it is good or bad or whether it ought to be acknowledged or not, or rejected or accepted, this motion must be carried out by the unanimous consent of voter-citizens.*

The tripod-linkage scheme is nothing but requirements to be met by rules and regulations of the welfare game, which we are going to discuss later, c.f. "Rational man" deliberation, Rubinstein (1998: 7). We hope to show that the scheme will enable us to state the view under which conduct these modules like a cascade are embedded into welfare policy of the state. To perform poverty and income distribution analysis this cascade is accessible for observation and could be subject to computer simulations in evaluating wealth redistribution policies, appendix C. Our initiative by itself could also be of certain value for unifying the theoretical structure of economic analysis of institutions, for passing judgment on social and public organizations, or for systematic inquiry into impacts of governmental decisions and actions.

In the next section, we discuss the relevant trends and issues of welfare policy encircled by the three modules mentioned above. In Section 3, we invite the reader to play *a sugar pie* game in a way that illustrates the standard of how to adjust the bargaining power of negotiators to make sure the agreement if a specific outcome of negotiations is desirable. We also hope that, before advancing any further, the reader will try to solve our *sweet exercise* at the end of the section. In Section 4, we discuss the assumptions to be made a priori on the functions and primitives involved and then we go along the cascade of tripod-linkage scheme. In the first module, we disclose a volatility constraint, under which restraint a balanced portion of the welfare pie for funding subsidies in amalgamation with *stability* constraint holds down inverse working incentives effect, which is called welfare hazard. In the second module, we make an effort to embrace the ambivalence and multifaceted welfare perception of citizens from the angle of the alternating-offers game. Our work associates the policy on poverty with bargaining related to monetary expectations of two actors – the social and public agencies. In principal, given arbitrary income distribution, it would be possible, within the scope of negotiations, to obtain an exact analytical solution of the game (find the proof in the appendix B).

The actors on the opposite sides of the bargaining table might haggle over the terms of outcomes and delay the decision on consolidating a draft to the agreement. The draft might not necessarily be the best outcome, and citizens may vote against the draft, emanating a risk to break down the negotiations. Thus, in lines with our bargaining procedure, only in the third module do we reveal an appropriately settled bargaining problem for the game that would probably enable voter-citizens to accept a proposal by unanimous consent. In Section 5, we discuss an equity condition and the possibilities of setting up a breakdown point of the game endogenously. Section 6 summarizes and ends the study with a postscript of rules and regulations, associated with the welfare game.

2. Relevant trends and issues

"*The interface between economics and politics is still in a primitive state in our theories but its development is essential if we are to implement policies consistent with intentions,*" as pointed out by North (2005: 29). Feldstein (2008: 132) also noticed: "*Unfortunately, there is no reason to be pleased about the analysis in policy discussions of the efficiency effects…of the welfare consequences of proposed tax changes.*" In what follows, we examine the publications on economic behavior, which deal with the sociological effects of welfare using public finance, which is the best starting point to go into our *tripod-linkage* scheme of the welfare game.

Public finance focuses on the revenue side of tax policy. It deals, in particular, with tax-induced efficiency effects (c.f. Formby and Medema [1995]) and implications on equity principles of welfare, whereas for agents with low-income levels, the welfare policy is a different matter. It is worth considering on the grounds of legal and moral rights of citizens. Empirical evidence consistent with legal obligations can be found in the literature of welfare policy: "*...Henderson poverty line. The line was initially set (in 1966) equal to the level of the minimum wage plus family benefits for one-earner couple with two children,*" Saunders (1997: 29). Hypothesis consistent with moral obligations can be found in the literature of economic politics (c.f. Eichenberger and Oberholzer-Gee [1996], Feld and Frey [2002]).

Musgrave (1959) examined two main approaches to taxation: the "*benefit approach*" and "*ability-to-pay approach*," which put taxation accordingly into efficiency and equity context. We intend to augment the existing principles of welfare policy by the benefit approach, as we see it, allocating a guaranteed minimum or level of basic goods for the lowest taxes. Moreover, to keep taxes *fairly levied*, we think the best tax for all citizens injects optimal equity into the tax system according to the ability-to-pay principle of "*proportional sacrifice*."

Stabilization. The purchase, production, and delivery of social services and public goods give rise to public spending. Often referred to in common parlance as welfare expenses, a portion of expenditure is meant to reimburse agents, those who had a misfortune, through subsidies. To be specific, subsidies are benefits for agents with low incomes and limited assets, providing an adequate chance to improve their disposable income. Agents who are eligible for benefits do not have many assets; they are not flexible in the labor market; and their income lies below the poverty line, driving them into social exclusion. Therefore, beneficiaries who decide to claim benefits would be better off under social administration. At other extreme, because of implemented declines in welfare and services, the administration revokes the benefits by submitting a request to some permanent clients who might find themselves worse off and to decide to be flexible once again in the labor market. That is to say, the emphasis on welfare implementation may manifest itself in hidden ambiguity as a result of economic growth, decline or stagnation, demographic shift, pit or migration, political change and change in scarcity of resources, property rights, level skills and education of labor force, etc., by disturbing the rules and regulations of social services. Now, while the services either improve or decay, the agents whose disposable income is in proximity of the poverty line may cause a feedback, a hazard (h-factor) effect on subsidies and benefits, which may have an impact on the tax compliance of all citizens.

A principal source to pay for subsidies and benefits is taxation. The social agencies have to solve the problem of stability of public spending in cooperation with other fiscal institutions, as it cannot be solved by market mechanism alone. Thus, the first module in welfare policy cascade discloses the stability paradigm in welfare policy.

According to the ability-to-pay principle in public finance to make the distortion of tax policies stable, the known terms of warranty rely on exogenous taxes enforced on the productivity of agents. A variant of the classic public finance and the like (Berliant and Page Jr. [1996]) this concept applies when an agent with given productivity does not shift his/her labor supply after all adjustments to the tax formula have been implemented; optimal taxation enforces optimal labor supply.

Yet, another "treatment of policies," closely related to institutional stability, entails equity of pre- and post-tax positions of taxpayers. Such a view to demarcate between agents attracts the attention of economists and tax policymakers. Credit-tax-scheme analysis opposes the income-tested program in the rich-and-the-poor, two-man economy (Kesselman and Garfinkel (1978). Fuchs (1965) was among the few who first observed properties of the relative poverty margin at one-half of median family income as arbitrary but reasonable. Sen (1976), Atkinson (1987), and Hunter et al. (2002) succeed on poverty measurements. Horizontal inequalities seem to occupy a place in Stewart's (2000) paper, which reviews the connection linking income distribution and economic growth. Ebert (2008) developed a principle of concentration for the redistribution of income. Peñalosa and Wen (2004) investigate redistribution, which operates as a form of social insurance.

We continue to rely on postulation that, throughout and in spite of volatility in the economy, welfare policy remains stable. By postulating this, we end up in stabilization for public spending in an attempt to control the circulation of wealth through social and public agencies and explain how can the wealth redistribution reach all in society.

***Bargaining*.** Entering the bargaining module of the cascade while pushing along the policy, which is stable to move ahead, it is realistic to imagine a scene where our actors play the alternating-offers "bargaining drama" in the roles of social and public agencies. "*These flimsy structures, however, are used by individuals to allocate resource flows to participants according to rules that have been devised in tough constitutional and collective-choice bargaining situations over time,*" Ostrom (2005: 823) stated. Bargaining, after all, can be risky, because if the terms of voter-citizens are not met, the voter may vote behind the scene against the draft to the agreement. Therefore, by Osborn and Rubinstein's (1990: 71) variant of the bargaining game with exogenous risk of breakdown, we reveal an analytical solution setting up the outcome of the game as poverty line instead of traditional solutions like slices dividing the welfare pie. Despite social and public agencies are pursuing their own causes, we demonstrate that social and public agencies might end up being able to agree upon compatible portions of the welfare pie for funding the primary or basic goods (needs), and all non-primary, called public goods. Bargaining has been a theme of a wide range of publications (Roth [1985]).

Unanimous consent. The interaction of taxpayers in terms of the scheme to reject by a single representative vote the draft to the agreement between social and public agencies is in reality an ultimate threat to achieve the goal of taxpayers (citizens) – the less tax, the better. The private well-being of citizens is the reason why the message is taken into consideration at the third module. At this module, it is rational to go and to maintain the momentum of citizens by focusing on moderate portions of the welfare pie for the provision and delivery of primary or basic (the subsidies and benefits) and non-primary (the public goods) in order to improve the perception and behavior of consumers against high taxes. Otherwise, disoriented consumers or disagreeing social and public agencies at previous modules could further block the attempts in making favorable policies for citizens.

Buchanan (1967: 71) underlined "*…that of earmarking, the individual 'votes for' designated taxes to finance specific public outlay. In the other, general-fund financing, he 'votes for' the same taxes to finance, not a single service, but a budgetary bundle of several services.*" In contrast, we put an extra emphasis on the design and analysis of a debating and voting platform for the welfare policy on poverty (Table 1). However, we neither design nor analyze any voting system or scheme by which voter-citizens express their expectation as taxpayers. As noticed by Roberts (1977: 329), "*The point is not whether choices in the public domain are made through a voting mechanism but whether choice procedures mirror some voting mechanism,*" and thus, we adhere to all voting guidelines where each tax proposal comes with or includes its own tax (c.f. Mueller [2003: 67]). We see no reason to depart from this and wish to stress that welfare policy would not be approved by the unanimous consent as long as the goal of minimizing taxes (observation 6) is desirable.

3. The sugar pie game

We are correct, as it seems to us, in believing that those who stay behind baking a welfare pie for public consumption do not realize that citizens' demands of welfare and public goods to a greater extent might sometimes worsen the quality of cooking the pie in the welfare policy oven. This is how we perceive it and hope that the process of finding the solution in a sugar pie game is the best way to understand what happens. We invite the reader to play the game, which explains the situation with welfare pie in simple terms.

The game may be connected to how a piece of sugar pie is fairly sliced between two people: HE, a soft negotiator, not very keen on sweets but with emphasis on quality; and SHE, a tough negotiator, likes sweets, whatever they are. The question about the size of the pie we leave temporarily aside for the present.

The axiomatic bargaining theory finds the asymmetric Nash solution by maximizing the product of players' expectations above the disagreement point $d=\langle d_1,d_2\rangle$:

$$\arg\max_{0\le x+y\le 1} f(x,y,\alpha)=(u(x)-d_1)^{\alpha}\cdot(g(y)-d_2)^{1-\alpha},$$

the asymmetric variant (Kalai [1977]).

Although the answer may be known to game theory purists, the questions often asked by many include the following: "*What are* x*,* y*,* α*,* $u(x)$ *and* $g(y)$*? What does the point* $\langle d_1,d_2\rangle$ *mean? How is the* $\arg\max$ *formula used?*" The answer looks like this:

x	is HIS slicing the pie, and α is HIS bargaining power, $0\le x\le 1$, $0\le\alpha\le 1$;
$u(x)$	is HIS expectation, for example $u(x)\equiv x$, of HIS x slicing the pie;
y	is HER slicing the pie, and $1-\alpha$ is HER bargaining power, $0\le y\le 1$;
$g(y)$	is HER expectation, for example $g(y)\equiv\sqrt{y}$, of HER y slicing the pie.

In widely accepted vocabulary, we call $s=\langle u(x),g(y)\rangle$ the utility pair. The disagreement point $d=\langle d_1,d_2\rangle$ is what HE and SHE collect if they disagree how to slice the pie. The sugar pie disagreement point is $d=\langle d_1,d_2\rangle=\langle 0,0\rangle$; disagreeing players collect nothing. Further, we believe that expectations from the pie are more valuable for HER indicating HER desire $g(½)=\sqrt{½}=0.707$, which is greater than HIS desire $u(½)=0.5$.

Now considering the $\arg\max$ formula of $f(x,y,\alpha)$ one may ask a new question: "*What standard will HE, the sugar pie negotiator, base HIS decision on to obtain an equal half of the pie?*" That is to ask, what standard will facilitate HIS negotiating power α to obtain the half of the pie if SHE may only accept or reject the proposal. A technical person can shed light on the solution. First, replace $u(x)$ with x, put $y=1-x$, replace $g(y)$ with $\sqrt{1-x}$, and take the derivative of the result $f(x,1-x,\alpha)$ with respect to the variable x by evaluating $f'_x(x,1-x,\alpha)$. Later, replace $x=½$, and finally solve the equation $f'_x(½,½,\alpha)=0$ for α; the equation $f'_x(½,½,\alpha)=0$ resolves for $\alpha=1/3$.

In general, one might feel comfort in the following judgment. "*Even in the face of the fact that SHE is twice as tough a negotiator,* [1] *to count on the half of the pie is a realistic attitude towards HIS position of negotiations. Surely, rather sooner than later, since HE revealed that SHE likes sweets, HE would have HER to agree to a concession.*" This attitude might well be the standard if a half of the pie is desirable as a specific outcome of negotiations.

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[1] Let us say, she pays her solicitor twice as much as he does.

Exercise. In our sugar pie slicing game, rational players have a reason to align operating procedure of any eventual agreement. The negotiators suppose to play the bargaining drama of alternative offers resulting in some commitments of how to slice the pie. When agreement on slice of pie is reached a new *real life problem* arise: Who should order the pie in the bakery, to select the size, and to ensure a safe delivery of the pie to its end destination. Usually players negotiate on issues when there are equal preconditions in place that guarantee equal rights in bargaining. We go in opposite direction, pointing out our believes in a normative manner, correct or not, what so ever they are, because our approach expects to deal with skills shortage of the second player. The decision-making on size of the pie is an exclusive privilege of the first player. However, the second player will preserve its advisory rights regarding the size of the pie.

Indeed, assume that only HE has all *relevant skills of cooking*, whereas SHE does not; HE possesses some hidden knowledge, which can be used in situations where HE can enforce the *bakery to cook properly*, or effectively retaliate for breaches if the quality of cooking does not meet its goal whereas SHE cannot. Moral-Principal problem may also arise, because we believe in addition that HE acts on HER behalf. Having fewer skills about cooking SHE cannot completely monitor HIS actions or intentions; HE may have an incentive to act inappropriately (from HER viewpoint) if the interests of both are not aligned in eventual agreement. We believe that SHE lacks such abilities and knowledge and might show willingness to agree or, at least, not to resist HIS privileges to make an order and to decide upon the size of the pie. On the other hand, the bakery has limits of its own within realistic utopia, e.g., the size of the pie might be too large to fit into the oven, or deficit of finance may occur forcing the bakery to close its activities, etc., i.e., a risk of bankruptcy (fiscal inconsistency) event is pending. Bakery utopian limitations (in real life the tax system inconsistency or volatility of economic resources) are a common knowledge, as we believe they are; both players know all these circumstances, which must be taken into account. Therefore the risk of breakdown may be the driving force for both players to reach the agreement in order to bake the pie reasonably.

Suppose now that in the background of HIS judgment the quality of the pie first increases when the size is small, but reaching the peak point it starts to decline: the quality, one will say, is $\cap$-single peaked upon the size. For HER the pie is desirable whatever it is. Let us try to play the sugar pie game in a different way when HE alone *placing an order for the delivery to the bakery* prescribes the size z of the pie. In the example SHE *just recommends* the size z that HE is not committed to accept, but HE is committed to the slice x aligned by the agreement. Let the utility pair $\langle u, g\rangle$ of HIS and HER expectations is given by:

$$u(\mathrm{z},x)=\mathrm{z}\cdot[(1+x/2)-\mathrm{z}]\,,\ g(\mathrm{z},y)=\mathrm{z}\cdot\sqrt{y}\,,\ \mathrm{z}\in[0,1],\ x,y\in[0,1].$$

Define binding slices to the size z as a curve $x(\mathrm{z})$, which resolve $\mathrm{u}'_{\mathrm{z}}(\mathrm{z},x)=0$ for x. Evaluating x from $\mathrm{u}'_{\mathrm{z}}(\mathrm{z},x)=0$ and then replacing $x(\mathrm{z})$ into $u(\mathrm{z},x)$ and $g(\mathrm{z},x)$ we get $u(\mathrm{z})=\mathrm{z}^2$ and $g(\mathrm{z})=\mathrm{z}\cdot\sqrt{3-4\cdot\mathrm{z}}$. Hereby, the bargaining problem $\langle S,d\rangle$ transfers into parametric space $S_{\mathrm{b}}=\langle u(\mathrm{z}),g(\mathrm{z})\rangle$ of the size parameter $\mathrm{z}\in[\frac{1}{2},\frac{3}{4}]\subset[0,1]$. I call the interval $[\frac{1}{2},\frac{3}{4}]$ by the scope of negotiations: $\frac{1}{2}$ and $\frac{3}{4}$ resolve $\mathrm{u}'_{\mathrm{z}}(\frac{1}{2},0)=0$ and $\mathrm{u}'_{\mathrm{z}}(\frac{3}{4},1)=0$ for z accordingly. In HIS view the pie must fit to size, since outside the interval $[\frac{1}{2},\frac{3}{4}]$ it is too small or has a low quality of baking but useful. Therefore, the disagreement occurs at $d=\langle u(\frac{1}{2}),g(\frac{3}{4})\rangle=\langle\frac{1}{4},0\rangle$. When we impose a constraint that the size of the welfare pie remains fixed (*stable*) during the delivery to its end destinations, the Nash symmetric solution to the game is found at $\mathrm{z}=0.69$, $x=0.74$. However, HIS asymmetric power 0.212 is sufficient to negotiate with HER about the half of the pie provided the size $\mathrm{z}=0.62$ is suitable for cooking in the oven.

4. The welfare pie game

Welfare theory is based on that the optimal redistribution of goods in a society is achieved through optimal allocation of society's resources. Fundamentally, this is about redistribution of wealth. If we identify a basis for welfare policy, we must find the point for optimal provision and delivery of basic and primary goods. Because of moral and political grounds basic goods are unrelated in principle to the other goods, e.g., national defense, public safety, environment protection, education and health services, roads and highway systems, etc. Conventionally, national defense, … etc., on the list, are all the primary goods. Naming these goods in the list, in contrast to basic goods, as "non-primary" but public goods, suites better to our purposes. However, when basic and public goods are on delivery to its end destinations we preserve the traditional notification for both goods as public goods.

Preliminaries. We believe that splitting up the public goods into basic and other wants is acceptable, and, then, it is, more or less, reasonable to follow the same pattern like "playing the sugar pie game." Indeed, the situation looks more like a bargaining game between social agencies negotiating favorable portion, i.e., the slice x, $0 \le x \le 1$, of the pie with public agencies. Following the traditional rules of alternative offers game of how to slice the welfare pie, when the pie is desirable for both parties, the negotiators (bargainers) changing roles commit to offers (x, y), $x + y = 1$. Setting up the game, the rules and regulations of social agencies, valid for financing a desirable level of subsidies, require, in contrast, an additional control by a poverty line parameter ξ. Such a requirement is a separate matter connected to the size of the pie. The way of putting the matter is to suppose that higher values of the poverty line need an increased taxation $\tau(\xi, x)$ to finance subsidies through tax channels with excessive burden of tax rate $\tau(\xi, x)$ upon the poverty line ξ increase, $\tau'_\xi(\xi, x) > 0$, $\tau''_\xi(\xi, x) > 0$, $x = const$.

The subsidy $s(\xi, \sigma) > 0$ compensates for the unfair subsistence of the poor agent $\sigma < \xi$ and is a supplement for the poor to compose the eligible "poverty basket" for food, clothing and shelter, fuel and lights, …etc. In lines with Rawls [1971: 92] that "*primary goods are things which it is supposed a rational man wants whatever he wants,*" we define the expectation of social agencies by level $u(\xi, x)$, called also the level of basic goods, the guaranteed social minimum or cost of living. All but not expenses on subsidies represent the expectation $g(\xi, x)$ of public agencies, what we already named above as "public goods." Provided the negotiators commit to the slice x, here and further on, we suppose that level u is a single $\cap$-peaked upon ξ increase. Expectation g of public agencies are decreasing with x but increasing with ξ. Expectations $\langle u, g \rangle$ in detail, c.f. Fig. 1-2, are considered to be analytic functions $u(\xi, x)$, $g(\xi, x)$ as follows. Given an interval $[\xi_1 \le \xi \le \xi_2]$, called later on the scope of negotiations, u supposed to be single peaked, $u''_\xi < 0$ upon ξ increase, $u'_\xi(\xi_1, x) > 0$, $u'_\xi(\xi_2, x) < 0$. Upon x increase utilities u are convex, $u''_x > 0$, $u'_x > 0$. Upon ξ increase utilities g are concave with $g'_\xi > 0$, $g''_\xi > 0$. In contrast, utilities g decrease upon x increase either with $g''_x > 0$ or $g''_x < 0$.

Stabilization module: Stable policies of social agencies. The main responsibility of the provision and delivery of welfare goods lies in the public sector, which counteracts negative contingency, if it occurs. It is universal in the sense that the entire population is covered by the public sector intervention, regardless of one's situation before or after the contingency. The benefits are of high quality, enough to support social minimum, Greve (2008: 58); poverty is not allowed. This course provides a relatively high level of public spending, high level of taxes and governments' budget deficit in balance with expenses on welfare, i.e., a fiscal inconsistency or misbalance with tax returns. In addition to the level of welfare measured largely independently of market forces this might have an adverse effect of the market economy, which should not be borne by the individual alone since the State has duty to help.

Let us try to give the situation a sugar pie game analogy. When HE and SHE have signed an agreement, the size of the pie has been prescribed by HIM and remained unchanged (stable) under the delivery to its end destinations. We intend to go through similar situation: first party to prescribe the size and to propose a slice of pie, the other accepts or rejects. The game continues: the other to propose a slice but has only one authority to recommend the size what the first party may not obligated to accept. The first party, under any such agreement, has a commitment to the slice without a commitment to the size. In this way within the scope of negotiations the agreement is reached. In case the pie is changing too rapidly, both players know that under the delivery the size might change. Is it true that the size instability might confuse negotiators in making decisions? The difficulty may be that such instability may drive the fair bargain to unknown destiny. In fact, subsidies could smooth out the balance of payments! The welfare pie is not stable, and thus may break into the behavior of our rational negotiators. Therefore, we need a method of splitting the pie of tax returns not only fairly but also steadily. Otherwise, in any agreement, the rules and regulations of the game are not living up to their claims unless the welfare policies of social agencies do not enforce balance. We need a criterion of how to detect the stability.

In our further discussions, an issue that justifies the set-up involves quantification. We take a quantum of average income for the measurement of all variables and functions over income distributions family $P(\sigma,\xi)$ parameterized by poverty line ξ. Hence, the average income per capita establishes the ratio scale. [2]

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[2] Monetary scale satisfies an interpersonal comparability of utilities (c.f. Narens and Luce [1983: 249]).

It is helpful to focus first on welfare policy without any warranty of stability. The policy might be set by the poverty line ξ to decide who is living in poverty and to transfer wealth from the rich to the poor: the variable σ to *the left from* the line ξ, $0 < \sigma \leq \xi$, allocates the income of the poor; the income of the rich *to the right* is σ, $\xi < \sigma < \infty$. An agent $\sigma < \xi$ claims and receives subsidy or benefits $s(\xi,\sigma)$ in a way that the agent's disposable income $\sigma + s(\xi,\sigma) = \xi$. The agent $\sigma = \xi$ receives a zero subsidy besides all agents $\sigma > \xi$.

In what follows, we study a specific scheme emphasizing the readiness of the society for funding public spending. We suppose that averages, in other words subsidies B per capita and taxable income W, depend on poverty line parameter ξ; $B \equiv B(\xi)$, $W \equiv W(\xi)$. Next, suppose that negotiators in the welfare game prefer to commit to the slice x and will agree to hold the balanced way of financing subsidies B, i.e., $B(\xi) = \tau \cdot x \cdot W(\xi)$, where $\tau \cdot W(\xi)$ is the size z of the pie. Based on the perception of income distributions family $P(\sigma,\xi)$, the product $\tau \cdot W(\xi)$ estimates the tax revenue, i.e., the size z of the welfare pie; $W(\xi)$ is the average taxable income; we call $W(\xi)$ – the wealth. Let the average of the cost of subsidies is $B(\xi)$, and the average cost of public goods is $g(\xi)$. Public spending equals the size of the pie $z(\xi) = \tau \cdot W(\xi)$ whereas $\tau \cdot W(\xi) = B(\xi) + g(\xi)$, what means that the revenue is spent, and the delivery of public, i.e., basic and public goods, has reached its end.

Let the social agencies are ready to finance subsidies, i.e., to refund $B(\xi)$ via tax revenue $\tau \cdot W(\xi)$. The agencies commit to keep the balance $B(\xi) = x \cdot \tau \cdot W(\xi)$ of payments between credits $B(\xi)$ and debts $x \cdot \tau \cdot W(\xi)$ as a portion x of revenue $\tau \cdot W(\xi)$. The balance specifies an effect of policy ξ, poverty line ξ, welfare policy, welfare reform, pact, program, etc. Although the balance is valid, it might break ahead of the beginning – policy ξ bids fair of being unstable towards adjustment in ξ. As far as the balanced way of payments for funding subsidies is required, only a few would question the balance; however, almost every one, perhaps for different reasons, prefers a stable way to implement the balance. Our focus next is on a constraint that embodies the stability of welfare policy ξ; in other words, embodies a criterion for a safe delivery of the pie to its end destinations.

The problem arises: the size of the pie could vary too fast, what may require frequent adjustments of taxes. Rules and regulation for the delivery of basic (primary) and public (non-primary) goods do not provide an adequate funding of the expenses, i.e., do not match the rules of taxation, nor prevent numerous changes and adjustments. The study emphasizes that the pie being on delivery must be controlled and adjusted steadily. We proclaim a simple rule for the decision to be stable: We demand, if the first decision is implemented, the second one taken by the same protocol must coincide with the first. Such schemes $C(C(X)) = C(X)$ known as idempotence decision rules $C(X)$ originate from social choice mechanisms. In our vocabulary, appendix A, this means that multiple adjustments of rules and regulations do not change the *machinery* of how the subsidies are legally paid out, and, in particular, implemented twice give the same result. Here we came upon a sequence $\ldots,\xi',\xi'',\ldots$ of poverty lines multiple adjustments. Such an understanding requires, in particular, that stable poverty lines, coming in pairs amid the sequence, will coincide at some point.

One can say, then, that the economy is immune against volatility $B(\xi) \neq x \cdot \tau \cdot W(\xi)$ of the balance for subsidies. That is to say that the immunity holds down the welfare hazard in the environment of rules and regulation of basic goods on delivery. Likewise, implementing the policy ξ under *immune* or *stabilized composition* $[B(\xi), W(\xi)]$ is like saying that, to make sure the balance, it requires implementing the rules and regulations of the policy only once. For this reason, we assume that the stability of the balance has been secured.

In this mode, the rules and regulations reflect the policy ξ that outlines the stabilization of public spending by which the policy might be brought to conclusion. We therefore conclude that the account of expenses $x \cdot \tau \cdot W(\xi)$ meant for social spending must be in balance not only for funding subsidies $B(\xi)$, when the particular policy ξ takes effect, but also in the whole spectrum of current and future events; the policy ξ enforces the stability in future.

The balance $B(\xi) = x \cdot \tau \cdot W(\xi)$ is a static relationship leading to functional dependency $\tau \equiv \tau(\xi, x) = \dfrac{B(\xi)}{x \cdot W(\xi)}$ binding ξ and x variables. Now, the tax τ becomes a function of ξ and x, $\tau \equiv \tau(\xi, x)$. Agent $\sigma = \xi$ after-tax position is $\pi(\xi, \tau) = (1 - \tau) \cdot (\xi - \phi) + \phi$, where φ is the personal allowance establishing a single tax bracket $[\phi, \infty)$ (c.f. Malcomson [1986: 266]). According to rules and regulations valid for tax schedules in the moment, the dependency $\tau \equiv \tau(\xi, x)$ transforms $\pi(\xi, \tau)$ into fiscally realistic social minimum $\pi(\xi, \tau(\xi, x))$. Let the level u of basic goods (the minimal cost of living) highlights the expectation of social agencies. However, the cost u of living does not necessarily match the fiscal level $\pi(\xi, \tau(\xi, x))$! Therefore, to avoid this undesirable incident, if one prefers realistic and stable rules, an equation for stable poverty line ξ should be resolved as necessary condition for stability in the future, as follows.

Observation 1. *Constraint* $u = \pi(\xi, \tau(\xi, x))$ *is the necessary condition to uphold a dynamic stability of the static balance* $B(\xi) = x \cdot \tau \cdot W(\xi)$.

All proofs are in the appendix B.

Corollary. *Adjustments* ξ', ξ'',... *are unnecessary if* $u = \pi(\xi, \tau(\xi, x))$ *resolves for* ξ. *The only chance, remaining for agents with incomes* $\sigma < \xi$ *or* $\sigma > \xi$, *to change their social positions is irrational, and thus the root* ξ *allows to prevent the welfare hazard effect.*

Stable policy ξ induces a stabilized composition $[B(\xi), W(\xi)]$, which is the basis for the welfare game solutions. A reasonable question now emerges: "*Which policy* ξ *represents a stable averages for subsidies* $B(\xi)$ *and for taxable income* $W(\xi)$ *respectively?*" [3] The answer is in the following three constraints:

Delivery constraint, all taxes are spent, i.e., the delivery of basic and public goods reached its end; this form of constraint makes sense only for proportional, or flat taxes. The case will later substantially simplify the method of function minimization with constraints.

$$\tau \cdot W(\xi) = B(\xi) + g \tag{1}$$

The balance for funding subsidies with the portion x of the welfare pie (tax revenue) credited to and deposited (debited) in social agencies' account; $B(\xi)$ is the average of subsidies shifted by the policy ξ.

$$B(\xi) = x \cdot \tau \cdot W(\xi) \tag{2}$$

The constraint verifying safe delivery of subsidies, i.e., the dynamic stability of (2). In contrast to $(\sigma, \tau) \in \Re^2$, we distinguish levels $u = \pi(\xi, \tau)$ as indifference curves $(\xi, \tau) \in \Re \subset \Re^2$!

$$u = (1 - \tau) \cdot (\xi - \phi) + \phi \tag{3}$$

Taking the expression $\tau(\xi, x) \equiv \dfrac{B(\xi)}{x \cdot W(\xi)}$ out from constraint (2) and replacing $\dfrac{B(\xi)}{x \cdot W(\xi)}$ into $u = \pi(\xi, \tau(\xi, x))$, constraint (3) must resolve but for stable policy ξ:

$$L(\xi, x, u) := (\xi - \phi) \cdot B(\xi) - x \cdot (\xi - u) \cdot W(\xi) = 0. \tag{4}$$

Constraint (4), also called volatility constraint, sets up the stabilization module. It holds down the welfare hazard effect by balance (2) in amalgamation with constraint (3).

[3] Below we continue to call the average taxable income – the wealth.

Summary. An outcome $\phi,\xi \Rightarrow z,x,\alpha,\tau,\langle u,g\rangle$ constitutes the citizens' bargaining shield for wealth circulation that relates to a bundle of variables or constants. Controls, variables ϕ,ξ ; the status is set to z,x,α,τ ; $\langle u,g\rangle$ embodies competing expectations (proposals):

ϕ – the personal allowance establishing single tax bracket $[\phi,\infty)$;
it is an ex-ante, a control (tuning) variable, $0<\phi=const<\xi$;

ξ – the poverty line; a policy to decide who is living in poverty,
the choice or the control parameter as well;

z – the size of welfare pie $z=\tau\cdot W(\xi)$, the tax revenue z equals
public spending; the case of proportional taxes;

x – the slice of welfare pie z ; a portion x of z to be deposited in
favor of social agencies for funding subsidies, $0\le x\le 1$;

α – the negotiating power of social agencies, $0\le\alpha\le 1$;

τ – the marginal tax rate, the wealth tax function $\tau(\xi,x)$ is set up by (1);

u – the guaranteed social minimum, the level of basic goods, monetary
expectation function $u(\xi,x)$ of social agencies is set up by (2) and (3);

g – the monetary expectation function $g(\xi,x)$ of public agencies is set up
by (1) and (2); all but not expenses on subsidies, amount of public goods.

- The slice x and the marginal tax rate τ, due to constraints (1-3), became functions of variables ξ,g: $x\equiv x(\xi,g)$ and $\tau\equiv\tau(\xi,x(\xi,g))$. This form of dependence appears later at the module of *welfare pie bargaining*.

- In order to perform simulations, the formulas for average cost $B(\xi)$ of subsidies and average taxable income $W(\xi)$ may be additionally parameterized by ex-ante parameter θ given over distribution families $P(\sigma,\theta+h\cdot\xi)$ in a more specific form,

$$B(\xi)=\int_0^{\xi} s(\xi,\sigma)\cdot P(\sigma,\theta+h\cdot\xi)\cdot d\sigma\ ,\ s(\xi,\sigma)\text{ is the subsidy function;}$$

$$W(\xi)=\int_0^{\xi}(\sigma+s(\xi,\sigma)-\phi)\cdot P(\sigma,\theta+h\cdot\xi)\cdot d\sigma+\int_{\xi}^{\infty}(\sigma-\phi)\cdot P(\sigma,\theta+h\cdot\xi)\cdot d\sigma\ ,$$

where the h-factor reveals the inverse working incentives – the feedback of social clients.

- In our welfare pie game, the policy ξ in general is also an issue of the average income $a(\theta+h\cdot\xi)$ maintenance to uphold the restraint $a(\theta+h\cdot\xi)>W(\xi)$ by proper choice of the personal allowance constant $\phi>0$, the tax bracket $[\phi,\infty)$.

- The income density function $P(\sigma,\theta)$ reflects the initial position when the circulation of wealth through tax channels just starts. If triggered by the push $\xi'>\xi$ or pull $\xi'<\xi$ on policy ξ, the distribution $P(\sigma,\theta)$ embodies the factor $h=0$ hiding the *rate of change* $Hz(\xi)=h\cdot\dot{a}(\theta+h\cdot\xi)<0$ of inverse working incentives, $\dot{a}(\theta+h\cdot\xi)>0$, $h<0$. Policy ξ' brings the distribution $P(\sigma,\theta)$ into an undisclosed position $P(\sigma,\theta+h\cdot\xi')$.

Bargaining module: The procedure and policies. In the welfare state there will always be disagreement about sharing the pie of tax returns and whether it should be sliced at all – policies that will be an impossible task, a wrangling between conflicting groups. Consider two flimsy (fictitious) institutions, social agencies acting in the role of negotiators over basic (primary) goods and public agencies over public goods. Like in sugar pie game, the expectations of negotiators' solely depend on efficient policies of social agencies within the framework of how to set the poverty line parameter as precondition for the agreement. Indeed, an efficient poverty line sets up a one-to-one corresponding to "binding slices." Accepting the precondition, public agencies will only propose binding slices for obvious reason that all but not the lines corresponding to efficient slices will be rejected for sure. We exclude, however, the situation when public agencies by negligence, mistake or some other reason, or, whatever is done to make the recommendation inefficient, happen to suggest an <u>inefficient recommendation</u>, and the other side, prescribing its own efficient line, disregards the recommendation but accepts the proposed slicing anyway. Also note, in contrast, that even if social agencies will have an intention to disregard an <u>efficient recommendation</u> they will not do it because ultimately in any agreement they will be committed to a binding slice by a proposition. Therefore, we believe that recommendations on poverty lines provide a rational argument what social agencies cannot resist and must accept, or reject making a new proposal in standard way. Such an account explains that the outcome of the bargaining game might be a desirable poverty line ξ^o for both parties, instead of an agreement upon slices. Within an interval $[\xi_1, \xi_2]$, which we call the scope of negotiations, public agencies would as well either accept or reject proposals of efficient poverty lines. Therefore, only the scope $[\xi_1, \xi_2]$ bids proposals, which do not need to be accompanied any more by binding slices x and, then, the bargaining drama performs exclusively within the interval $[\xi_1, \xi_2]$. Expectations on the interval arrange so-called bargaining frontier, Fig.4-5. This is our unconventional idea of how the bargain portfolio is assembled. The portfolio has changed its *color*; negotiators may be conceived as themselves making poverty line proposals. If rejected the roles of actors' change and a new proposal is submitted. The game continues in a traditional way of alternating offers.

So understood, social agencies propose a slice x of the welfare pie they commit to, and prescribe the maximum level ξ^o of the poverty line, which is fiscally possible to hold. Public agencies accept or reject the proposal. If rejected, public agencies, as well, make an alternative proposal y of a slice they commit to but only recommend the level ξ for poverty line that social agencies are not committed to accept. However, within the scope of negotiations $[\xi_1, \xi_2]$ the recommendation ξ looks like a proposal since rejecting social agencies would violate the commitment x to the slice they committed to by the agreement. The game continues this way; each actor takes turn in making a proposal, x or y about the slice and prescription ξ^o or recommendation ξ about the poverty line in opposition of the other. When a rejection occurs, the momentary phase of the game consolidates a draft.

Despite one can secure that the rules are stable, the game itself contains a new challenge. Increased poverty lines does not necessarily increase the level u because we believed that the welfare hazard (h-effect) keeps coming up: increased number of claims may have a declining effect on social minimum; too many lower income agents may claim subsidies. As a result, increased poverty lines will decrease the disposable level of basic goods for the poor despite required unavoidable increase of taxes. Provided social agencies commit to slice x and public agencies to $y = 1 - x$, the *space of stable social minimums* corresponds to points, which arrange a single $\cap$-peaked curve depending on poverty line as a parameter [4]. The peak ξ^o of such a $\cap$-curve (if the maximum is reachable, what we believe it is) represents an efficient welfare policy ξ^o of social agencies. Thus, for any fixed slice x^o, the bargain portfolio will contain an efficient policy ξ^o as a function of x^o, $\xi^o = \arg\max_\xi u(\xi, x^o)$. Given the other way around, for any efficient policy $\xi^o \in [\xi_1, \xi_2]$, which corresponds to a top value u^o, a unique slice x^o resolves $u(\xi^o, x) = u^o$ for x; $g(\xi^o, x^o) = g^o$ represents the non-conforming expectation of public agencies. We call the slice x^o a binding slice to ξ^o. Depicted in various projections (on expectations $\langle u^o, g^o \rangle$ at Fig. 4, and on the wealth W contra τ at Fig. 5), efficient peaks ξ^o, which correspond to binding slices x^o, arrange the geometry what we call the *bargaining frontier*. It highlights top values u^o – efficient policies of social agencies at peaks ξ^o. One, perhaps, would recognize here the vocabulary of the Laffer curve, but for different domain, similar to: first, "*guaranteed social minimum being proposed in the normal range of poverty line parameter.*" Next, "*by passing through the top point, the poverty proposals will be assessed and reviewed in the range of prohibited values.*"

We have looked above at average subsidies $B(\xi)$ and at average taxable income $W(\xi)$. Now we are ready to see how in our two-man economy the stabilized composition $[B(\xi), W(\xi)]$ contributes to the rules and regulations of the welfare game. The composition, at the end of the subsection leads to an appropriately settled bargaining problem, which associates the risk of a breakdown with the third party (the voter). We have already dealt with the rules of the game but now we make two rigorous suppositions. Let us first specify the welfare game expectations of all parties involved:

[4] We already emphasized the worsening quality of cooking the welfare pie in the welfare policy oven.

Expectations of the Negotiators 1 and 2, and the Voter:

Negotiator No.1, u	–	level of basic goods, cost of living or a guaranteed social minimum to fulfil basic necessities;
Negotiator No.2, g	–	all but not expenses on subsidies, amount of public goods, expectation that benefits all in the society;
The Voter, q	–	risk of higher taxes $\tau(\xi, x)$ emanating from electoral maneuvering of citizens to break down negotiations.

Suppose that the rules and regulations to slice the welfare pie include the volatility constraint (4), which certifies the stabilized composition $[B(\xi), W(\xi)]$ for policy ξ. This assumption is pending on the conclusions of the analysis undertaken above because funding the stabilized composition in the welfare game could not be implemented unless the volatility constraint $L(\xi, x, u) = 0$ (observation 1).

Suppose next that varying ξ under their own rules and regulations social agencies propose a stable policy ξ^o, which for each slice $x = x^o$ they commit to, ensures an effective social minimum $u(\xi^o, x^o) = \max_\xi u(\xi, x^o)$ (observation 2) binding x^o to ξ^o; no matter who is making proposals x^o or y^o. Clearly, for each slice y^o proposed by public agencies, social agencies reject for sure ineffective recommendation $\xi \neq \xi^o$; effective policy $\xi = \xi^o$ must occurs at $\langle u(\xi^o, x^o), g(\xi^o, x^o)\rangle$ amid binding slices x^o as ongoing precondition for the agreement; *the procedure* was already discussed. Thus, social agencies have no reason to reject efficient recommendation ξ^o otherwise they cannot keep to eventual commitment x^o.

Observation 3. *The curve* $S_b = \langle u(\xi, x), g(\xi, x)\rangle$ *of effective policies* ξ *of social agencies, which certify the stabilized compositions* $[B(\xi), W(\xi)]$, *must satisfy the constraint*

$$D(\xi, x, u) := \frac{\partial}{\partial \xi} L(\xi, x, u) = \frac{\partial}{\partial \xi}\left[(\xi - \phi)\cdot B(\xi) - x\cdot(\xi - u)\cdot W(\xi)\right] = 0. \tag{5}$$

We call the curve S_b a bargaining frontier. We provide the proof of necessary and sufficient conditions for constraint (5) in the appendix B.

It is evident that level u of basic goods and amount g of public goods at the frontier S_b depend but exclusively on policies ξ, $\langle u(\xi), g(\xi)\rangle \in S_b$. Recall that parties are conceived then as themselves making proposals over policies ξ, instead of slice-proposals (x, y). Frontier $S_b = u(g)$ in Fig. 4 illustrates the expectations. Particularly, the constraints

$$\left.\begin{array}{ll} Q(\xi,\tau,g)=0 & \text{– Delivery (1);} \\ L(\xi,x,u)=0 & \text{– Volatility (4);} \\ D(\xi,x,u)=0 & \text{– Frontier (5);} \end{array}\right\} \begin{array}{l}\text{as rules and regulations} \\ \text{for the welfare game between} \\ \text{institutions of the welfare state,}\end{array}$$

in case of proportional (flat) taxes, following a succession of collections of sub-expressions and simplifications, imply an analytical solution, which turns to be without constraints: [5]

$$u(\xi)=\xi-\tau(\xi)\cdot(\xi-\phi),\ \tau(\xi)=\frac{z(\xi)}{W(\xi)},\ z(\xi)=B(\xi)+g(\xi);$$
$$g(\xi)=\frac{W(\xi)}{v(\xi)}-B(\xi),\ \text{where}\ v(\xi)=1+(\xi-\phi)\cdot\left(\frac{B'(\xi)}{B(\xi)}-\frac{W'(\xi)}{W(\xi)}\right);$$

$\pm$ rates $W'(\xi)\leq 0$, $W'(\xi)\geq 0$ of changes in $W(\xi)$ are essential for the analysis, whereas function $B(\xi)$ is valid only with $B'(\xi)>0$, and $0<\phi<\xi$.

Now, so understood, the bargain portfolio contains a scope $[\xi_1,\xi_2]$ for negotiations that parties will follow in traditional way. Social agencies propose $\xi\in[\xi_1,\xi_2]$; public agencies either accept or reject. If rejected the roles of actors' change and public agencies make an effective recommendation – a proposal ξ to social agencies. The game continues changing roles until a proposal ξ is accepted. The policy ξ on poverty, or the poverty line, is a control parameter of policy rules and regulations. From now on, we refer to efficient policy ξ as to a proposal of policy on poverty when the bargaining over slices (x,y) of the welfare pie in eventual agreement is under negotiations.

Before we go any further, let us recollect the phenomenon of risk the voter creates for negotiators. Recall that rejecting proposals negotiators consolidate a draft, see above. It is a risky moment to continue the bargain. The voter may vote against the draft, emanating a risk of breakdown when negotiators try to continue bargaining over too costly proposals without fulfilling the terms of the voter, and the game ends with disagreement or the breakdown. In lines with Osborn and Rubinstein, "*We can interpret a breakdown as the result of the intervention of a third party, who exploits the mutual gains. A breakdown can be interpreted also as the event that a threat made by one of the parties to halt the negotiations is actually realized. This possibility is especially relevant when a bargainer is a team (e.g. government), the leaders of which may find themselves unavoidably trapped by their own threats.*" (1990: 72).

5 *Mathematical derivation available on request*

Suppose that negotiators bargain over all stable policies ξ within the scope of negotiations $[\xi_1,\xi_2]$, $\xi \in [\xi_1,\xi_2]$. Below we follow the alternating-offers game $\Gamma(q)$ with an exogenous risk q, $0<q<1$, of breakdown (Osborne and Rubinstein [1990: 71-76]). Each time the proposal ξ is rejected by one of the negotiators, the momentary phase of the game consolidates a draft. Negotiators do not feel safe to continue bargaining upon the draft if the terms of voter are not met since voter-citizens (taxpayers) may vote against the draft emanating a risk q of breakdown. The worst-case scenario $\{\langle u_1 = u(\xi_1), g_1 = g(\xi_1)\rangle, \langle u_2 = u(\xi_2), g_2 = g(\xi_2)\rangle\}$ extracted from the endpoints $\xi_1 < \xi_2$ of bargaining frontier S_b naturalizes the risk $q > 0$ of breakdown.

A closer look at what is known the "*well-defined bargaining problem,*" or individual rationality, associated with the traditional definition $\langle S, d\rangle$ of the bargaining scheme, seems to be instructive. Indeed, inequalities $g_1 > g_2$, $u_1 < u_2$ hold for the pair $d = \langle d_1 = u_1, d_2 = g_2\rangle$ composed from endpoints $\langle u_1, g_1\rangle$ and $\langle u_2, g_2\rangle$ of the bargaining interval $[\xi_1,\xi_2]$. The point d, introduced here, naturalizes the disagreement point of the problem $\langle S_b, d\rangle$, $S_b \subset \Re^1$. Then, compared to traditional approach of compact convex set $S \subset \Re^2$, inequalities $s > d$ are also true for any pair $s \in S_b$. Therefore, the pair $\langle S_b, d\rangle$ for the bargaining frontier S_b becomes a well-defined bargaining problem. However, it is not immediately apparent, whether the point d is a stable. Following observation rules out such a doubt.

Observation 4. *To testify whether the point* $d = \langle d_1, d_2\rangle = \langle u_1, g_2\rangle$ *becomes a stable outcome of the welfare pie game, it is necessary and sufficient that there exists a policy* δ *on poverty, which resolves the equation:*

$$(\delta - \phi)\cdot(B(\delta) + d_2) - (\delta - d_1)\cdot W(\delta) = 0; \qquad (6)$$

condition $\delta \notin [\xi_1,\xi_2]$ *is necessarily required.*

Notice that in the worst-case δ the average amount of wealth circulating in society is $W(\delta)$; the average cost for funding subsidies is $B(\delta)$. Proposal δ depends on endpoints of the interval $[\xi_1,\xi_2]$. This dependence, if equation (6) can be resolved for δ, provides the basis for validation of *equity condition of breakdown,* next section.

Finally, what follows is the proper choice of expectations in the alternating-offers game with a risk q of breakdown, Osborn and Rubinstein, (1990: 75), encapsulating the bargaining power α of negotiators for an appropriately settled bargaining problem.

Observation 5. *In the alternating-offers game* $\Gamma(q)$ *with the risk* $q > 0$ *(probability) of breakdown* $\langle d_1, d_2 \rangle$ *the functions* $(u(\xi) - d_1)^{\alpha}$ *and* $(g(\xi) - d_2)^{1-\alpha}$ *imply the expectations of social and public agencies accordingly. The solution* λ *of the well-defined bargaining problem* $\langle S_{\mathrm{b}}, d \rangle$ *is close to the pair* (λ_1, λ_2), $\lambda_1 \leq \lambda \leq \lambda_2$, *resolving equations* $(1-q)\cdot(u(\lambda_1) - d_1)^{\alpha} = (u(\lambda_2) - d_1)^{\alpha}$ *and* $(1-q)\cdot(g(\lambda_2) - d_2)^{1-\alpha} = (g(\lambda_1) - d_2)^{1-\alpha}$ *for variables* λ_1, λ_2; without proof.

Unanimous consent module: Electoral maneuvering. Only the voting results can reveal what are the true incentives of citizens that give the democracy its final judgment, so it is through the democratic process the voters step up in the roles of actors to whom the opportunity is granted to govern institutions and who intend to act in people's interest to allocate optimally the social and public resources. It is voters' inequalities, life plans, social class and origin, native endowments, political capital, etc., which determine what bulletin to collect at the voting table. As consequences of this, voters' reasonable disagreements or interpretations of reality that advance voters' own choice would affect electoral maneuvering and thus the voting result. The results are not totally predictable because there are always unpredictable deviations in voters' political views and opinions of how the wealth redistribution ought to be organized. Trouble is that welfare proposals, which benefit all, sometimes require higher taxes but our negotiators (social and public agencies) would be confronted with taxpayers' selfish attitude towards lower taxes. Such an attitude deserves, perhaps, critical examination – emphasis that will personalize taxpayers acting as well in the role of a representative voter. Given these points our question is: Why the negotiators should care about lower taxes?

The situation is well suited to introduce electoral maneuvering of taxpayers bringing about an external threat q, $0 < q < 1$, of breakdown by the bargaining game $\Gamma(q)$ (Osborn and Rubinstein, 1990: 72). Indeed, sufficient to finance the expectations of our rational negotiators for different taxes τ, the Fig. 5 depicts the curve of efficient welfare policies (poverty lines) into the space of taxable income (the wealth) W and the income tax τ. The pair $\langle u, g \rangle$ is embedded in each point but not visible on the graph. These invisible pairs in the upper part of the graph symbolize lower basic (primary) but higher public (non-primary) goods, the lower part symbolize lower public but higher basic goods. As we believe, all views are represented, then, the expectations $\langle u, g \rangle$ of negotiators for each tax τ (in voters' view) are better off for some coalitions of voters against others. Herby, efficient poverty lines resulting from eventual agreements between negotiators are two-folded. Some voters (now in negotiators view) will accept higher, others prefer lower taxes. Therefore, the "upper coalitions" of voters will always disagree with "lower coalitions" unless the tax is too high. For lower taxes, disagreements may drop off and vanish totally at the tax minimum. Thus, the taxes higher than minimum, emanate a risk of breakdown that some voters will reject the draft to the agreement reached in the momentary phase of the game, see above. The lowest tax, then, is the one among many necessary conditions for the draft to pass by unanimous consent. Given the probability q about voters' dissatisfaction of the draft, the risk $q > 0$ is, however, a rough characteristic of the situation.

We have not yet passed by all the troubles to avoid, if possible, the risk of breakdown. A number of efficient welfare policies wait to be put into the bargain portfolio but only the minimum tax is desirable (see Table 1, $q = 0$). Indeed, negotiators may overlook the choice of voters because the minimum tax is not necessarily a desirable welfare policy in negotiators view. It may happen against voters' free will, that the power of social or public agencies, as negotiators, is strong enough in making *selfish* decisions favorable either for social or public agencies alone. Here the bargaining power indicator α, $0 < \alpha < 1$, of social agencies comes into consideration; $1 - \alpha$ signifies the bargaining power of public agencies. To be sure that the power α must be adjusted satisfactorily towards desirable outcome, i.e., towards the minimum tax (as it was in our sugar pie game with the desired half of the pie when the whole pie was suitable to be put in the oven) we adjust the power indicator α in a way that the minimum tax inherits the balance of power between social and public agencies. This means that the rules and regulations of agencies activities have to be adjusted accordingly.

Hereby, we arrived at an idea of the resemblance between the sugar pie game, when two highly pragmatic people try to slice a sugar pie, and the welfare pie game, when social and public agencies try to slice the welfare pie. The voter, who represents taxpayers refunding the pie, votes only for or against the draft accepting or not the momentary phase of the negotiations. The policy λ, which minimizes the wealth tax, is what voters are seeking. We adjust the bargaining power α of social agencies to make sure that the slice of welfare pie would be also set so that it safeguards the outcome of negotiations to be the standard bargaining solution λ. In doing so the outcome of alternating-offers game $\Gamma(q)$ also safeguards the same solution λ when $q = 0$, which we are going for to avoid the risk $q > 0$ of breakdown associated with electoral maneuvering of taxpayers as voters.

Marginal tax minimization, i.e., the wealth-tax minimum at the bargaining frontier, is the voters' preference desirable by all members in the society. Following our analytical solution without constraints and extracting the τ expression from (1), the frontier $S_{\mathrm{b}} = u(g)$ is a curve $\langle u(\xi), g(\xi) \rangle$ by which shape the voters' preference turns into problem:

$$\min_{\xi \in [\xi_1, \xi_2]} \tau(\xi) \equiv \frac{B(\xi) + g(\xi)}{W(\xi)}.$$

Observation 6. *Condition* $\lambda = \arg\min_{\xi \in [\xi_1, \xi_2]} \tau(\xi)$ *is necessary to put forward a poverty proposal* λ *before voter-citizens by unanimous consent. At the bargaining frontier* S_b, *proposal* λ *outlines a unique outcome* $\phi, \xi \Rightarrow z, x, \alpha, \tau(\lambda), \langle u(\lambda), g(\lambda) \rangle \in S_b$.

Provided $\tau(\xi)$ is smooth enough, the solution of the voter problem is the root λ of the equation $\tau'(\xi) = 0$. The root λ allows settling the negotiation power α of social agencies in their negotiations with public agencies in a way that rules and regulations of α are sufficient to persuade public agencies to agree upon social minimum $u(\lambda)$.

Indeed, putting a new spin on the old idea of the sugar pie game of how to adjust the bargaining power appears to be clear. The *old standard* affecting HIS negotiating power α can be the *new standard* for social agencies to affect their negotiating power α in the welfare game. Let $f(\xi, \alpha) = (u(\xi) - d_1)^{\alpha} \cdot (g(\xi) - d_2)^{1-\alpha}$. Recall that we must first take the derivative of $f(\xi, \alpha)$ with respect to ξ evaluating $f'_{\xi}(\xi, \alpha)$, and then we must replace ξ by λ (do not replace $\xi = ½$ this time like in the case of the sugar pie game). Now, the negotiating power α of social agencies must solve the equation $f'_{\xi}(\xi|_{\xi=\lambda}, \alpha) = 0$ for α.

5. Equity condition of breakdown

The breakdown in the game $\Gamma(q)$ is an initial environment upon which the players of the game do not have any influence. In our case the initial environment, when the negotiations start, takes account of taxes τ and the wealth W. The product $\tau(\xi) \cdot W(\xi)$ identifies the size z of the pie within an interval $[\xi_1, \xi_2]$ named the scope of negotiations. The scope $[\xi_1, \xi_2]$ establishes the boundary for negotiators, i.e., their initial, most favorable proposal ξ_1 for public agencies, which is the most negative for social agencies; proposal ξ_2 offers exactly the opposite. It is like saying what is the most favorable for one party is the most negative for the opposite, and other work around.

Since the adjustment of power indicator α of negotiators comprises the breakdown point $d = \langle d_1, d_2 \rangle = \langle u_1, g_2 \rangle$ allegedly given exogenously, it seems suitable to continue the analysis. In trying to extract the point, which is conditionally encoded but endogenously into the income distributions family $P(\sigma, \xi)$ [6], we are working here upon specific form for equity normalizing the breakdown under the description valid in the alternating-offers game $\Gamma(q)$.

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[6] Note that the breakdown is a pair of pairs $\{\langle u_1, g_1 \rangle, \langle u_2, g_2 \rangle\}$.

Traditionally, the disagreement (breakdown) in the alternative offers game are the payoffs when one player gets all and the other gets nothing and visa versa. In accordance with players' non-conforming expectations, like in sugar pie game, the first player in our scheme acts on behalf of the second. Therefore, it is important for the second player to arrange the terms of the contract in a way that the first player does not exploit or misuse its skills and abilities to win some advantages for nothing. In the bad end the quality and the size of the pie should preferably be equal for both players' because they will eventually posses the whole pie (the first player or the other) independently of circumstances when the breakdown happens. The standard bad end corresponds to a *pair of pairs* $\{\langle 1,0\rangle,\langle 0,1\rangle\}$ of utilities. In this form the breakdown is usually found using normalization by an ex-ante linier transformation. The standard breakdown, in particular, is an equity representing an exogenous environment, in which the players cannot and will not make binding agreements. However, in our case the breakdown environment includes additional parameters – the tax τ and taxable income W upon which the players can agree a priori in order to equalize (normalize) the quality and the size of the pie in case the bad end of negotiations is a reality. Without a priori knowledge or warranty of equity existence, such normalization is unrealistic. Therefore, in view of taxpayers electoral maneuvering, it is certainly better, to set the breakdown according to some rational principle. Below, and finally, we contribute to, if possible, implementing typical cost-benefit rationality of how to set the breakdown endogenously.

Consider a situation of expectations driving the welfare policy in the context of cost-benefit analysis. The differences in the amounts of wealth and taxes for funding low-cost welfare policy ξ_1 against an expensive policy ξ_2, $\xi_1 < \xi_2$, i.e., funding expectations $\langle u_1, g_2\rangle$ for ξ_1 against $\langle u_2, g_2\rangle$ for ξ_2, $u_1 < u_2$, $g_1 > g_2$, could amplify misunderstandings among negotiators and contribute to delays. Indeed, we already know that public spending $z(\xi_1)$, $z(\xi_2)$ or the sizes of the welfare pie at the endpoints of the scope of negotiations $[\xi_1, \xi_2]$ would require delivery of wealth $W(\xi_1)$ and $W(\xi_2)$ amounts for taxes $\tau(\xi_1)$ and $\tau(\xi_2)$. These amounts and taxes have been settled prior to the start of the game: In mundane terms, the wealth and prices for the delivery of needed *supplies of commodities* to its end destinations, which safeguard the expectations $s_1 = \langle u_1, g_1\rangle$ and $s_2 = \langle u_2, g_2\rangle$. This interpretation clearly emerges from the definition of the size of the pie, which equals to $\tau \cdot W$. Such an interpretation, however, is only thinkable for flat (proportional) taxes. In order to bring the negotiators, if possible, into just and equal positions prior to negotiations, it might be rational to equalize wealth amounts W and taxes τ at ξ_1 and ξ_2. Therefore, the condition $\tau(\xi_1) = \tau(\xi_2)$, $W(\xi_1) = W(\xi_2)$, highlights the equity in the positions of negotiators prior to negotiations starts. In particular, such a consideration will equalize prior to negotiations fiscally realistic demands for public spending, i.e., the size of welfare pie $z(\xi_1) = z(\xi_2)$. We do not claim that the equity can be reached in all circumstances, but we found a number of examples where the validity of the condition was detected.

Let us turn to the Fig. 5 once again to highlight the situation. In the following lines of reasoning, we label the condition an *equity of breakdown* associated with electoral maneuvering of taxpayers. Thus, the exercise would be to fix an interval $[\xi_1, \xi_2]$ solving a system of two non-linear equations: $W(\xi_1) = W(\xi_2)$ and $\tau(\xi_1) = \tau(\xi_2)$ for two variables ξ_1 and ξ_2, i.e., to find a point (W^*, τ^*) where the bargaining frontier crosses its own contour on the plain with W, τ as *XY*-axis coordinates. Although this not a hard exercise to quantify the cross point (ξ_1^*, ξ_2^*), we did not explained away the problem of the existence of breakdown point $\langle d_1, d_2 \rangle = \langle u(\xi_1^*), g(\xi_2^*) \rangle$. Nevertheless, we found this point in a number of cases.

Finally, a conjecture would be observed true by example or simulations like one below.

Conjecture. *Under the condition of equity of breakdown, the policy η on poverty with equal power of social and public agencies, as negotiators, minimizes the wealth $W(\xi)$ (average taxable income) channeled via the tax system:* $\eta \approx \arg\min_{\xi \in [\xi_1, \xi_2]} W(\xi)$.

6. Concluding remarks

It is time to review the knowledge that the welfare pie game brought to us. We followed the welfare policy pushing along the edge of poverty line to reimburse the needy, and to treat them fairly, deciding who needs subsidies, which we considered from the point of view of the cost of living. Then we revised a way of how to estimate the expenses on basic (primary) and public (non-primary) goods. Together, these estimates become functions of poverty line as a parameter. Elevated poverty line gave rise to inverse working incentives, called welfare hazard (feedback effect) or h-factor, which unbalanced the budget on delivery of public goods. For this reason, the budget under the scheme of stable subsidies becomes crucial to coordinate the welfare game.

Setting up the game, we used incomes to serve the self-interest of agents. We addressed, then, the public finance problem of wealth redistribution among agents with lower incomes against those with higher incomes. The root of the problem was to find a solution in the alternating-offers game with two actors having conflicting interests and representing the public institutions pursuing their own causes. Social agencies acted their role by arguing to put in all efforts to increase the poverty line. The public agencies' expectation, in response to the public will, was to meet the need for public goods. Electoral maneuvering against higher taxes emanated risk to break down negotiations, and the threat of breakdown was the only driving force in favor of taxpayers. In doing so, we gave credit to the tax system to guarantee reasonably high social minimum disposable for the poor. It is now an elementary exercise to find that arguments, demanding an increase in the poverty line, are weak since too costly proposals require a raise of funding to subsidize an increased number of low-income agents claiming benefits. Increased number of claims may have a declining effect on the quality of social services guaranteed for the poor. Hereby, if the decline is undesirable, we cannot any more demand an increase of tax returns, and must restrict the scope of negotiations to a reasonable interval of poverty lines related to our bargaining procedure.

Bearing all these circumstances in mind, our message was a pretext for the analysis of the domain and the extent of bargain portfolio of institutions in the welfare state. Building on the prerogatives of the so-called tripod-scheme embedded into the welfare policy of the state, we embodied from the scheme the actors of an alternating-offers game. Actors were supposed to inherit non-conforming expectations and represent institutions. We provided a guide to how the decisions ought to be analyzed and interpreted within the scope of poverty lines at the bargaining frontier, instead of decisions on portions of tax returns. An adjusted bargaining power of actors (institutions) was used in compliance with citizens' desirable vision and ambitions as a solution of the bargaining problem.

We initially thought that due to the uncertainty of how to select the breakdown point, the model could treat the power indicator as a variable given only exogenously. However, fortunately enough, we found a condition, at least true in valuable examples, to encode the power indicator endogenously, and called it "*the equity of breakdown*." Last, but not least, despite one experiment not making a trend, we presented an evidence for the claim that the recognized poverty line, defined as 50% of the median income, is close enough to be considered as matching the poverty line minimizing the wealth tax.

Postscript. This paper encapsulated our long-term vision and believes of a three-divided mind of voter-citizens arguing like three individuals, about the level of basic goods, amount of (non-primary) public goods, and taxes. We did not provide empirical and conceptual support for the key elements of the scheme. We presumed, however, a fundamental difference between social security system and public sector providing vital public services but did not get the difference from a more primitive utility specification of monetary functions. How the income of households is assembled and how they use it, for example, to buy private health insurance or services of nursing housing, and why cannot the provision of equivalently valued public services be a perfect substitute? All these and similar questions are left apart. In short, we did not merit a debate on what was good or bad and what was right or wrong in the economic or political environment involving the two agents and a voter. Nevertheless, an addendum setting up the list of rules and regulations of how to coordinate the game might be informative since these rules led to a well-defined bargaining problem. We solve the problem analytically. Brought in as outcomes of the game, these solutions, put differently, assign a number of appealing interpretations within the space of poverty line parameter establishing the main result of the paper.

Welfare policy rules and regulations of institutions in the welfare game

1. The administration of social agencies knew the true and exact incomes of social clients, and thus it was required to implement an appropriate auditing regulation.
2. The behavioral pattern of agents remained endogenous. Agents demarcated themselves as *rich or poor* in compliance with current rules and regulations related to whether to compensate for the unfair subsistence of the poor and the needy with subsidies.
3. The regulation and maintenance of the balance between debts and credits for funding subsidies was crucial. Debts and credits remained balanced throughout and in spite of volatility in the economy.
4. Subsidies eligible for claims may become progressively attractive for the needy or moderately attractive for the permanent clients, which was likely to be the result of the inverse working incentives of agents towards shifting the subsidy budget out of balance. In this context, official rules and regulations were necessary to keep the balance, i.e., to neutralize so-called welfare hazard (h-factor) effect. Adoption of these rules and regulations on delivery of public goods predict and enforce a stable policy of public spending.
5. Rules and regulations of taxation: (a) exclusively proportional (flat) tax, (b) enforced tax schedules equaled taxable income, and (c) the tax revenue accumulated via tax schedules was spent entirely on public needs; i.e., the delivery of basic and public goods has reached its end.
6. Social agencies' conduct over poverty lines was self-dependent in a sense that, once negotiators were committed to the agreement of how to slice the welfare pie, internal rules and regulations of social agencies allowed them to guarantee an efficient level of basic goods. Public agencies conduct over the line was only in advisory authority.
7. In any variety of rules and regulations of how to extrapolate and assess tax revenue, income distribution was considered the only legal repository for tax return information.

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Appendix A: Concept of Stability.

In many-persons games, by a coalition is understood a subset H of agents Ω or participants in H. Among all coalitions $H \in 2^{\Omega}$ we usually single out rational coalitions – a participant in such coalition extracts from the interaction in the coalition a benefit, which is satisfactory for the agent $\sigma \in H$. Sometimes it is further stipulated that extraction of this benefit is ensured independently of the actions of the players not entering into the coalition. It is thinkable that a subset or participants in a coalition can improve their individual positions joining some other coalition. Such action is an example of instability. In his work "Cores of Convex Games" Shapley (1971: 11) investigated a class of n-person's games, $|\Omega| = n$ with special convex property, which guarantees an existence of coalitions, participants of which cannot improve their positions when joining any other coalition, hereby making such coalitions (collection, say the core) stable.

In contrast to individual payoffs improving or worsening the positions of agents, when playing a coalition game, a payoff to a coalition H as a whole is called the characteristic function $v(H) > 0$. In classical cooperative game theory payoffs $v(H)$ to coalitions H are known with certainty. The convex property reflects a kind of synergy effect when two coalitions S and T join together subject to $v(S) + v(T) \le v(S \cup T) + v(S \cap T)$, called *super-modularity* condition; compare super-modularity with Cherenin (1962). In case agent σ joins or leaves the coalition H, the increment or decrement $\pi(\sigma, H) = v(H) - v(H \setminus \{\sigma\})$

in the total payoff $v(H)$ defines the marginal utility $\pi(\sigma,H)$ of an agent $\sigma \in H$. This expresses an increase/decrease in marginal utility $\pi(\alpha,H \setminus \{\sigma\}) \le \pi(\alpha,H)$ of membership for all $\alpha \ne \sigma$, $\alpha \in H \setminus \{\sigma\}$ and is analogous to property of returns to scale' associated with convex production functions in economics. Monotonic condition $\pi(\alpha,H \setminus \{\sigma\}) \le \pi(\alpha,H)$ is equivalent to convex property of characteristic functions in Convex Game. In current study we analyze marginal utilities independently of super-modularity as self-object. This property, but inverse, was introduced for computational optimization problems, Nemhauser et al. (1978: 269). Therefore, nothing is wrong to look at marginal utilities as individual payoffs from the angle of agents post tax position $\pi(\sigma,H)$. It will allow investigating coalitions formation analytically.

Convex property guarantees a relationship between marginal utilities – incentives $\pi(\sigma,H)$ of participants to join the coalition H, which are necessarily monotonically increasing as the coalition grows, so that one might expect a 'snowballing' or 'band-wagon' effect when the game is played cooperatively. Below we investigate coalitions formation using marginal utilities in the form of after tax incomes $\pi(\sigma,H)$ of agents σ looking at the coalition H in view of a tax base H consisting of active taxpayers. The monotonic property is guaranteed provided the tax burden is an inverse function of the tax base H when the base is shrinking or growing, observation 2.

Our story of tax base stability lies in the State intervention; however, it is still similar to the core notion. If not the State, due to snowballing effect, the grand coalition formation (no benefits or subsidies are paid out) is immanent. As soon as the State wishes to guarantee a social minimum u to cover cost of living by providing basic goods the situation change. Now, one can discover that in terms of coalition game we deal with formation (marginalization) of two non-intersecting coalitions $\overline{H} \cup H = \Omega$: those $\overline{H}$ who are below the margin ξ of poverty line and those H who stay above. Thus, in contrast to formation of all eventual coalitions $H \in 2^{\Omega}$, only two (feasible) coalitions $\overline{H} \cup H = \Omega$ are under investigation; cost of living $u = \pi(\xi,H)$ is defined as post tax position of an agent $\sigma = \xi$ having an income equal to the poverty line ξ. All agents σ above poverty line ξ, $\sigma > \xi$ receive payment $\pi(\sigma,H)$, all $\sigma \le \xi$ receive the payment equal to social minimum u.

Trying to implement the coalition game theory to just described scheme of payoffs, the post tax position $\pi(\sigma,H)$ of an agent is a personal payoff, which represents a marginal utility with monotone property – the pie of tax returns increases when a new taxpayer joins the coalition H of active taxpayers and visa versa. Coalitions of taxpayers in ordinary vocabulary are already called the tax base H. Indeed, joining a tax base of active taxpayers improve the position of all citizens, leaving the base H results in additional transfer payments to the needy $\sigma \notin H$ increasing the tax burden upon those who still stay in the coalition (active taxpayers) by decreasing their post-tax position. We already dealt with the marginalization of income level. Therefore, the welfare system intervention in the form of transfer payments will interact into incentives of agents in two directions. In the first, those who find them better off due to monotone property of their post-tax position will try to join the coalition of active taxpayers, and those in opposite direction will try to join to participants who receive transfer payments, the needy. Now, the problem to answer the question of stability emerges, e.g., when this process stops – no one can and intend to claim subsidies legally and no one wishes to leave the tax base H. Provided the marginalization occurs at poverty level ξ, the situation becomes stable and the delivery of basic goods reaches its destination safely. Such a situation may be understood, also, as rules and regulation of welfare system, which do not change in the long run of how the transfer payments to be payout.

To make this concept of stability rigorous we introduce first the notion a rational coalition. Let the margin u be the cost of living. We define coalition H of agents σ as rational if for all participants $\sigma \in H$, the inequality $\pi(\sigma,H) \geq u$ holds. Upon the set of feasible coalitions $\overline{H} \cup H = \Omega$, rational or not, define the mapping $C(H) = \{\sigma \in H | \pi(\sigma,H) \geq u\}$. In doing so, we extract from a coalition H those agents $\sigma \in H \setminus C(H)$, $\pi(\sigma,H) < u$, who need support, and who can legally claim subsides, say the needy. As soon as all needy leave the coalition H and became the members of coalition $\overline{H} = \Omega \setminus H$, they will find themselves better off under the administration of welfare system bringing instability into the tax system: the database is now shrinking to the set of agents $C(H) \subset H$ because of inequalities $\pi(\sigma,C(H)) < \pi(\sigma,H)$, $\sigma \in C(H)$. There are, however, no guarantees that for a coalition $H^{u} = C(C(...C(H)))$, which is the result of shrinking process, the chain H^{u} would stabilize unless it stops at some point H^{u}: we get $H^{u} = C(H^{u})$. Our goal, which we try to achieve, sounds like the following. Given the population of all agents Ω and implementing the chain of mappings $C(C(...C(\Omega)))$ unless the process stops we can marginalize in the end the agents in Ω into two coalitions $\overline{H}^{u} \cup H^{u} = \Omega$, where $H^{u} = C(H^{u})$; H^{u}

becomes a fixed point of the mapping C. Given the marginalization at the level ξ of an income equal to poverty line when the State interacts into the process of coalition formation, the marginalization $\overline{H}^{\mathrm{u}} \cup H^{\mathrm{u}} = \Omega$ represents a core consisting of two coalitions: the needy $\overline{H}^{\mathrm{u}}$ and active taxpayers H^{u}. In other words, the solution, which guarantees the safe delivery of basic goods, complies with both, a constraint as fixed point of the mapping C, and the core notion. Thus in the end, when the chain stops, the agents have neither intention nor reason to leave the coalitions $\overline{H}^{\mathrm{u}}$, H^{u}; collection of two sets $\{\overline{H}^{\mathrm{u}}, H^{\mathrm{u}}\}$ arrange a core in classical sense. It was proved, Mullat (1980: 1471), [7] that there exists a unique rational coalition H^{u} and the chain $C(C(...C(\Omega)))$ highlights the procedure finding the coalition H^{u} at the moment the chain stops. Marginalization in this way represents two-man economy: the needy $\overline{H}^{\mathrm{u}}$ and active taxpayers H^{u}. Moreover, there exist a unique coalition H^{*} of active taxpayers, called kernel, where the cost of living $u_{\max}$ represents the last fiscal opportunity or balanced situation for the State to pay for subsidies; no fiscal solutions $u > u_{max}$ are available.

Only one, issue remains, in view of the basic cost of living, to explain the relationship between single peakedness of expectations u of social agencies and the coalition game. We emphasized previously a moral-principal that social agencies act on the behalf of public agencies. However, the agencies, i.e., the negotiators in bargaining game, have to reach some kind of understanding what role they play. For this reason, a slice x of the welfare pie, which is in reality a portion of tax returns, is at least one of items at the agenda upon which both negotiators must agree in the form of commitment to the agreement of how to divide the welfare pie. A portion x of the pie under the agreement must go to social agencies and the other, equal to $1-x$, to public agencies. Given the negotiators will commit to the slice x in eventual agreement, such a consideration guarantees the monotonic property of marginal utilities $\pi(\sigma, H)$ when σ joins or leaves the coalition of active taxpayers H. Thus, carrying on along the curve of excessive burden of taxation, secures the monotonic property of marginal utilities, observation 2, as well as the single peakedness of expectations u of social agencies. In case the commitment does not hold it will be unclear in which direction the slice is going to move, *upstream* or *downstream*, with the welfare pie expansion or shrinking, allowing social agencies, despite the tax decrease/increase, to move in *opposite direction* to increase/decrease the subsidies. The situation goes out of control and rational coalitions do not exist in the sense described above; the coalition game will not be convex any more.

[7] Mullat, J.E. (1979). Устойчивые коалиции в монотонных играх, Автоматика и Телемеханика, №10, стр. 84-95, (1980). Automation and Remote Control, Plenum Publishing Corporation, 1469-1478.

Appendix B: Verification

Proof of observation 1. On the contrary, suppose that $u > u' = \pi(\xi, \tau(\xi, x))$. Let social agencies attempt *to improve* after-tax income u' of poverty line ξ. The agencies attempt to implement $u > u'$ by initiating a regulation for policy $\xi' > \xi$. At once, for some highly pragmatic agents σ an option becomes visible *to claim for subsidies* to be *better off* because of inequalities $u \geq \pi(\sigma, \tau(\xi, x)) > u'$. These pragmatic agents σ would *increase* the cost $B(\xi') > B(\xi)$ for subsidies and would shift the balance $B(\xi) = x \cdot \tau(\xi, x) \cdot W(\xi)$ onto *deficit* $B(\xi') > x \cdot \tau(\xi, x) \cdot W(\xi)$; the balance was valid in the past, whereas in the past $\tau(\xi, x) \equiv \dfrac{B(\xi)}{x \cdot W(\xi)}$. Since social agencies must stay committed to x (x is fixed by agreement), the only option remains to keep the balance adjusting $\tau(\xi, x)$ to $\tau(\xi, \xi', x) = \dfrac{B(\xi')}{x \cdot W(\xi)} > \tau(\xi, x)$. Otherwise, keeping the old policy ξ in tact, the agencies could, but cannot, eliminate the *deficit* through a *decrease* in x. If $u > \pi(\xi', \tau(\xi', \xi, x))$ the agencies must continue the adjustment policy of taxes by $\tau(\xi', \xi'', x) > \tau(\xi, \xi', x)$ but adjusting now upon the welfare policy ξ' and proposing $\xi'' > \xi'$ in order to eliminate a new *deficit* $B(\xi'') > x \cdot \tau(\xi, \xi', x) \cdot W(\xi')$. These *improvements* $u > u'' > u'$ initiate a sequence of policies $..., \xi'' > \xi', ...$ and after-tax positions $..., u'' > u', ...$. The limit $u = u''$ with $\xi' = \xi''$, however, contradicts the assumption that the equation $u = \pi(\xi, \tau(\xi, x))$ cannot be resolved for ξ; the sequence $..., \xi'' > \xi', ...$ continues forever. ■

The chain of reasoning with $u' < u$ is similar. Just follow the instructions below.

Replace	*to implement an improved*	by	*to make a decline in*
–	*better off*	–	*worse off*
–	*improve* *improvement*	–	*decline* *deterioration*
–	*to claim for subsidies*	–	*that the subsidies have been revoked*
–	*deficit*	–	*surplus*
–	$\geq, >$	–	$\leq, <$
Transpose:	*an increase*	with	*a decrease*

In what follows, we investigate stable expectations $\langle u,g\rangle \in S_{\mathrm{b}}$ of social and public agencies. The bargaining agreement occurs at outcomes $\phi,\xi \Rightarrow \mathrm{z},x,\alpha,\tau,\langle u,g\rangle$ under the constraint that the variation of policy ξ does not improve the position of social agencies – the point turns up at the bargaining frontier $S_{\mathrm{b}} = u(g)$.

For stable outcomes, the variables of income level u, slice x, policy ξ, and tax rate τ depend on each other. The slice $x = x^o$, if settled as a possible agreement, redirects the level $u = \pi(\xi,\tau(\xi,x^o)$ to become a function $u = u(\xi,x^\circ)$. So, the peak point of level u with regard to the best welfare policy looks like

$$\xi^\circ = \arg\max_\xi u(\xi,x^o) \tag{B.1}$$

Observation 2. *Assume that social agencies do not shift from the slice* $x = x^o$. *Let the volatility constraint* (4) *solve for two different policies* $\xi_1 < \xi_2$. *Let the tax sacrifice* $t(\xi,x^o) = \tau(\xi,x^o)\cdot(\xi - \phi)$ *be a differentiable function of* ξ *progressively increasing with* ξ *increase within closed interval* $[\xi_1,\xi_2]$, *i.e., the derivatives*

$$\left.\frac{\partial}{\partial\xi}t(\xi,x^o)\right|_{\xi=\xi_1} > 0,\ \left.\frac{\partial}{\partial\xi}t(\xi,x^o)\right|_{\xi=\xi_2} < 0 \text{ and } \frac{\partial^2}{\partial\xi^2}t(\xi,x^o) > 0,$$

then the social minimum level $u(\xi,x^o) = \xi - t(\xi,x^o)$ *is single* $\cap$*-peaked function of* ξ.

Corollary. *There exists a unique interior policy* ξ^o *maximizing* u *at* $\left.\frac{\partial}{\partial\xi}u(\xi,x^o)\right|_{\xi=\xi^\circ} = 0$.

The standpoint coming next concerns the necessary and sufficient conditions for the stable policy ξ to occur at the bargaining frontier.

Observation 3. *Assume that the volatility constraint* (4) *is differentiable of its variables. The income level* $u = u(\xi,x^o)$ *is differentiable and strictly convex with respect to the policy* ξ *within some closed interval* $[\xi_1,\xi_2]$. *For a stable outcome* $\phi,\xi^o \Rightarrow \mathrm{z}^o,x^o,\alpha,\tau^o,\langle u^o,g^o\rangle$ *to occur on the bargaining frontier* $S_{\mathrm{b}} = u(g)$, *it is necessary and sufficient that the policy* ξ° *resolves the equation:*

(i) $\left.\frac{\partial}{\partial\xi}L(\xi,x^o,u^o)\right|_{\xi=\xi^\circ} = 0$, where $u^o = u(\xi^o,x^o)$ provided that

(ii) $\left.\frac{\partial}{\partial u}L(\xi^o,x^o,u)\right|_{u=u^\circ} \neq 0$.

Proof. Necessity. Let the stable outcome $\phi,\xi^o \Rightarrow z^o,x^o,\alpha,\tau^o,\langle u^o,g^o\rangle$ on the bargaining frontier $S_b = u(g)$ maximizes (B.1) at $u^o = u(\xi^o,\tau(\xi^o,x^o))$. Varying the policy ξ around ξ^o of the outcome $\phi,\xi^o \Rightarrow z^o,x^o,\alpha,\tau^o,\langle u^o,g^o\rangle$ and substituting $u = u(\xi,\tau(\xi,x^o))$ into the volatility constraint (4), we obtain an identity $L(\xi,x^o,\pi(\xi,\tau(\xi,x^o))) \equiv 0$. Within the proximity of (ξ^o,u^o), we exhibit for variables ξ,u:

$$\frac{\partial}{\partial \xi}L(\xi,x^o,u^o) + \frac{\partial}{\partial u}L(\xi^o,x^o,u)\cdot\frac{\partial}{\partial \xi}\pi(\xi,\tau(\xi,x^o)) = 0, \tag{B.2}$$

from which we deduce the necessity statement for $\xi = \xi^o$ and $u = u^o$.

Sufficiency. Suppose the condition **(ii)** holds. Let **(i)** resolves for ξ^o at the stable outcome $\phi,\xi^o \Rightarrow z^o,x^o,\alpha,\tau^o,\langle u^o,g^o\rangle$. Combining **(i)** and (B.2), we conclude that

$$\left.\frac{\partial}{\partial \xi}\pi(\xi,\tau(\xi,x^o))\right|_{\xi=\xi^o} = 0.$$

The sufficiency statement (B.1) holds since $u = u(\xi,x^o)$ is a strictly convex function of ξ .■

Proof of observation 4. The statement is correct as soon as we find a stable policy δ for implementation of the pair $\langle d_1,d_2\rangle$. First, replace the variable g by d_2 in the formula for constraint (1). Then, take out the expression for $\tau = \dfrac{B(\delta)+d_2}{W(\delta)}$ from (1) and substitute it into $(1-\tau)...$ of constraint (3), where u should be replaced by d_1 in advance. By simplifying, we end up with the statement of the observation.■

Sketch of the proof (observation 6). Looking at the wealth tax value $\tau > \tau_{min}$, for any outcome $...,\tau,\langle u,g\rangle \in S_b$, one may, indeed, prefer a counter outcome as a motion $...,\tau,\langle u',g'\rangle$, which outlines $...,\tau,\langle u'>u,g'<g\rangle$ or $...,\tau,\langle u'<u,g'>g\rangle$. However, since the frontier $S_b = u(g)$ is a curve of efficient preferences $\langle u,g\rangle$ for social minimum $u(g)$, no one could put forward a motion $u'>u^o$ or $g'>g^o$ against an outcome $...,\tau_{min},\langle u^o,g^o\rangle$ at $\tau = \tau_{min}$. We argue that the only way to fulfil the expectations and requests of taxpayers and to carry out the motion is by unanimous consent $...,\tau_{min} = \tau(\lambda),\langle u^o = u(\lambda), g^o = g(\lambda)\rangle$.■

Appendix C: Income Distribution Analysis
Bargaining game point of view

We recommend performing the analysis of income density $P(\sigma)$ as follows.

1. Parameterization of $P(\sigma)$ by θ in the form $P(\sigma,\theta)$. Parameter θ must keep to the original shape $P(\sigma)$ of the distribution, but *crash* it to the right when θ increases and *protrude* to the left when it decreases. Below we also use a constant m to make the distribution more equal/flat when m increases, and a more unequal/peaked like when m decreases. Equal/unequal parameterization of the original $P(\sigma)$ is not mandatory.

2. Here, the h-factor inserts the adverse working incentives of agents. Replace the parameter θ by $\theta + h\cdot\xi$, where ξ represents the policy on poverty and choose $h < 0$. In doing so, the average income $a(\theta + h\cdot\xi)$ of the distribution $P(\sigma,\theta + h\cdot\xi)$, as an indicator on the provision side of all goods (primary and non-primary, i.e., basic + public), inherits the feedback effect of the welfare hazard. To prevent this effect, the social administration must know the true incomes of social clients. The administration accepts all eligible claims and revokes all ineligible claims. Once the policy ξ takes effect, an agent with *income σ below ξ claims and receives the benefits, $\sigma < \xi$. No benefits at all if $\sigma \geq \xi$ or the benefits are revoked even if agents fall under ξ in the past but are now above ξ. The income densities $P(\sigma,\theta)$, as well as the adverse working incentive, hidden in its undisclosed position $P(\sigma,\theta + h\cdot\xi)$, both have a characteristic "tail" to the right, which is typical for societies sharply divided into very rich and very poor people (Fig. 3).

3. Selection of the subsidy function $s(\xi,\sigma)$. We emphasize that subsidies eligible for claims are paid out as benefits: an agent's $\sigma < \xi$ disposable income $\sigma + s(\xi,\sigma) = \xi$. A more comprehensive assistance rules could be easily incorporated.

4. We have already pointed out at the personal allowance ϕ-constant, $\xi > \phi > 0$. The choice of the constant is of crucial importance to restrain the amount of wealth $W(\xi) < a(\theta + h\cdot\xi)$. Without borrowing or printing money to maintain the wealth $W(\xi)$ as average taxable income, greater than the average income $a(\theta + h\cdot\xi)$, is impossible. For $\phi \approx 0$ we obtain *too sterile*, a solution $\tau_{min} \approx 0$. In contrast, the condition $\phi \approx \xi$, if the delivery constraint (1) is obligatory, could lead to excessive public spending.

5. Given $P(\sigma,\theta + h\cdot\xi)$, evaluate the stabilized composition $[B(\xi),W(\xi)]$. Transform the composition using analytical solution into expectations $\langle u(\xi),g(\xi)\rangle$ of social and public agencies accordingly. In doing so we reassign the negotiations to the bargaining frontier of poverty lines, instead of slices upon the welfare pie in hope to show the way of how to implement the bargaining procedure upon arbitrary income distribution.

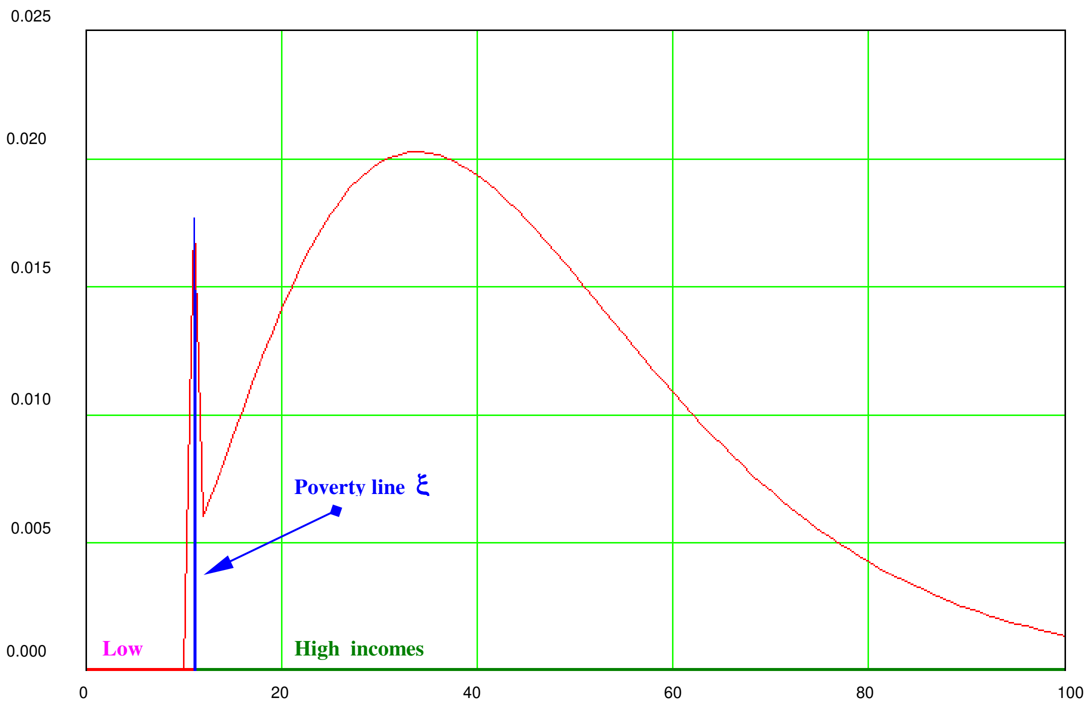
6. It is valuable to get hold of the principle associated with equity of breakdown. Otherwise, we run into the shortcomings of the bargaining scheme about the disagreement point $d = \langle d_1,d_2\rangle$. A proper choice of breakdown allows extracting the interval or the scope of negotiations $[\xi_1,\xi_2]$, $\xi_1 < \xi_2$, internally encoded in distributions $P(\sigma,\theta + h\cdot\xi)$.

Example. We proceed with a specific position of welfare state encapsulating an income distribution density similar to an exponential function:

$$P(\sigma,\theta+h\cdot\xi)=\frac{1}{(\theta+h\cdot\xi)\cdot\Gamma(m)}\left(\frac{\sigma}{\theta+h\cdot\xi}\right)^{m-1}\cdot exp\left(-\frac{\sigma}{\theta+h\cdot\xi}\right), \text{ where}$$

$m=4.1$, $\theta=11.9$, $\Gamma(m)=6.81$ and $h=-0.09$. The average income of this σ-density equals $a(\theta+h\cdot\xi)=\int_0^\infty \sigma\cdot P(\sigma,\theta+h\cdot\xi)\cdot d\sigma$; $\Gamma(m)$ is an extension of $(m-1)!$ to real variables. The subsidy $s(\xi,\sigma)=\xi-\sigma>0$ supplements the income of the poor up to the poverty line. Incorporation of more comprehensive rules of assistance is possible.

Figure 3. Income distribution density with characteristic "*bifurcation phenomenon*" around the poverty line



It is worth the efforts to validate that a disagreement policy δ under the primacy of equity principle of breakdown might be an outcome of the game. There is no reason why the equation $(\delta-\phi)\cdot(B(\delta)+d_2)-(\delta-d_1)\cdot W(\delta)=0$, in accordance with observation 4, has a solution in general. However, for the particular example of income distribution families $P(\sigma,\theta+h\cdot\xi)$ (see above), the equation could be resolved. One can check, to be sure, that for the wealth $W^*=41.32$ and the tax $\tau^*=26.38\%$, if we use the monetary expectations $\langle u,g\rangle$ at the endpoints $\langle u_1=4.54, g_1=10.9\rangle$, $\langle u_2=29.45, g_2=1.29\rangle$ of the scope $[\xi_1=5.32, \xi_2=39.16]$ of negotiations, the pair $d=\langle d_1,d_2\rangle=\langle u_1=4.54, g_2=1.29\rangle$, $u_1<u_2$, $d_1=u_1$, $d_2=g_2$, $g_1>g_2$, consolidates the breakdown policy $\delta=4.61\notin[5.32, 39.16]$.

Simulation. Recall already known proposals for incomes η, λ_1, λ, λ_2, δ (bear in mind that δ is outside of the scope of negotiations $\delta \notin [\xi_1, \xi_2]$); and a poverty proposal $\frac{1}{2}\mu$, as follows:

η	the policy on poverty with equal power of negotiators; the social and public agencies are in symmetric positions or equal roles;
λ_1	an alternating-offer of social agencies, what the public agencies accept;
λ	the poverty line, a policy minimizing public spending; minimizing the welfare pie instead of the wealth tax;
$\frac{1}{2}\mu$	50% of the median income; μ such that half the population have income above μ, and half have income below that, c.f. Bowman (1973);
λ_2	an alternating-offer of public agencies, what the social agencies accept;
δ	the most negative outcome: the policy of breakdown or disagreement; the result of electoral maneuvering of taxpayers when the draft to the agreement about how to set the slice for the welfare pie was rejected by voter-citizens.

After a quick glance at the table below, we are going to summarize and judge the "*eventualities of the burden of taxation*" by the magnitude and dimension of poverty proposals ought to be debated or implemented.

Table 1. Numerical experiment behind the bargaining game of welfare policy-making and delivery; [8] **SA – Social Agencies, PA – Public Agencies**

Obtained by means of income distribution density (Fig. 3); personal allowance $\phi = 2.34$, $\theta = 11.9$, $h = -0.09$, $m = 4.1$		**Policy of equal, symmetric power of negotiators** η	***SA proposal accepted by PA*** λ_1, $q = 5\%$	***Proposal minimizing wealth tax*** λ, $q \approx 0\%$	**Poverty line, 50% of median income** $\frac{1}{2}\mu$	***PA proposal accepted by SA*** λ_2, $q = 5\%$	**Policy of disagree-ment, the breakdown** δ
Poverty line– welfare policy	$\xi =$	25.45	*18.9*	*17.36*	16.76	*15.78*	4.61
Poverty rate: percentage of agents below the poverty line		25.65%	*10.63%*	*8.1%*	7.2%	*5.9%*	0.06%
Negotiating power of social agencies	$\alpha(\xi)$	0.5	*0.24*	*0.2*	0.19	*0.17*	Not defined
Guaranteed social minimum	$u(\xi)$	20.62	*15.57*	*14.35*	13.86	*13.08*	4.54
Average of public goods	$g(\xi)$	6.1	*7.23*	*7.43*	7.5	*7.62*	1.29
Average taxable income, the wealth	$W(\xi)$	37.94	*38.5*	*38.75*	38.86	*39.05*	41.47
Wealth-tax, marginal tax rate	$\tau(\xi)$	20.9%	*20.09%*	*20.05%*	20.06%	*20.09%*	3.1%
Average public spending, the size of the welfare pie	$z(\xi)$	7.93	*7.73*	*7.77*	7.80	*7.84*	1.29
Average income, provision indicator	$a(\theta + h \cdot \xi)$	38.43	*40.3*	*40.71*	40.87	*41.13*	43.74

[8] The table suggests a proposal laid out before voter-citizens to vote on.

Judgment. A chance behind the welfare game to masquerade economic reality might be the answer to the following question: "*Is it, nonetheless, true that funding subsidies and keeping the budget stable is going to be difficult to match together in case the tax burden for all citizens is decreasing?*" On the surface, it seems that, at some point, fairness and equity may disappear because the rich simply gets richer and the poor gets poorer. The effect of "tax relief for the rich" seems to drop down the welfare. In the face of these controversies, no one can guess what fallout may result from such eventualities, but we judge that tax relief actually guarantees a fair level of wealth. This consideration in the value judgment below deserves, perhaps, some emphasis on following four features.

First, let us suppose that, playing the game, social agencies reached an agreement with public agencies. Will the rules and regulations of the game stand any chance of a just and fair solution? In Table 1, we present the percentage of agents below the poverty line establishing the poverty rate. Taken separately, it is, however, a deeply flawed measurement of justice. In fact, when breakdown of negotiations occurs, the solution given by rate 0.06% is allegedly the most just and fair!?

Second, the welfare pie redistribution compensates for the inequalities of agents' incomes up to the poverty line. The poverty line is set by the national government to decide who is living in poverty. The official number is adjusted annually according to social minimum to take into account increases in the cost of living. Our major assumption was that the variety of rules and regulation to prescribe the poverty line in the welfare game are under the social agencies mandate, but public agencies set up an advisory authority. Thus, in order to reach an efficient policy of guaranteed social minimum, we assumed, then, that social agencies are in privileged position allowing them to prescribe the poverty line independently.

Next to the poverty line, the power indicator α highlights the amount of resources, skills and competence of social agencies, etc., to maintain their duties under the principles as to how the state ought to behave when trying to fulfil its welfare mission. We see no value in a separate judgment of this interpretation. However, due to public agencies central position of purchasing and delivering vital public services, to impose a higher grade $1-\alpha$, $0 \le \alpha \le 1$, to public agencies, but lower grade α to social agencies, is appropriate. Therefore, adjusting the power indicator α specifically as a desirable outcome, we imbedded the welfare pie to suit into fiscally realistic welfare policy aiming to settle the rules and regulations of the game closer to legal responsibilities and moral obligations of citizens, what benefits all in society.

The last feature is the unanimous consent; a situation in which no one can find a reason to object a motion to the agreement. To reach consensual agreement is difficult enterprise and time-consuming process. In the welfare game with a risk of higher taxes, the consent on condition of minimizing taxes brought the problem into focus. In view of agents receiving subsidies, a higher tax rate might be the subject of the debates and the most favorable and just solution. In contrast, the minimum tax policy for the consumer is out of the question, as we assume it is, not least that it is also a just and fair redistribution of wealth without a single objection. Therefore, if we agree in the debates upon the rules and regulations of the game, the outcome, which minimizes taxes, offers a vision of what policy should entail. To reach such an outcome is worth the time and efforts even if the vision is a realistic utopia.

Now so understood, it goes without saying that entering the realm of obvious utopia, the policy $\eta = 25.45$ (see the Table 1) with equal power of negotiators is less just and less fair than the policy $\lambda = 17.36$, where the minimum of taxes is reached; only the policy λ on poverty (Fig. 5) has a chance for a vote by unanimous consent. Indeed, in the variety of welfare game regulations, when engaged in an interaction to implement equal policy *25.45* (like HE and SHE engaged to obtain a piece of sugar pie), the equal power $\alpha = 0.5$ of social agencies' negotiators was stronger than *0.2*. Nevertheless, the incident with weakened power *0.2* is yet to be determined and the aim of customers can still be reached on policy *17.36* for the tax rate *20.05% < 20.9%*. Thus, regardless of the reduced obligations of taxpayers, social agencies, even with their weakened bargaining position, will be able to come to a desirable agreement with the public agencies to maintain a fair level of wealth**.**

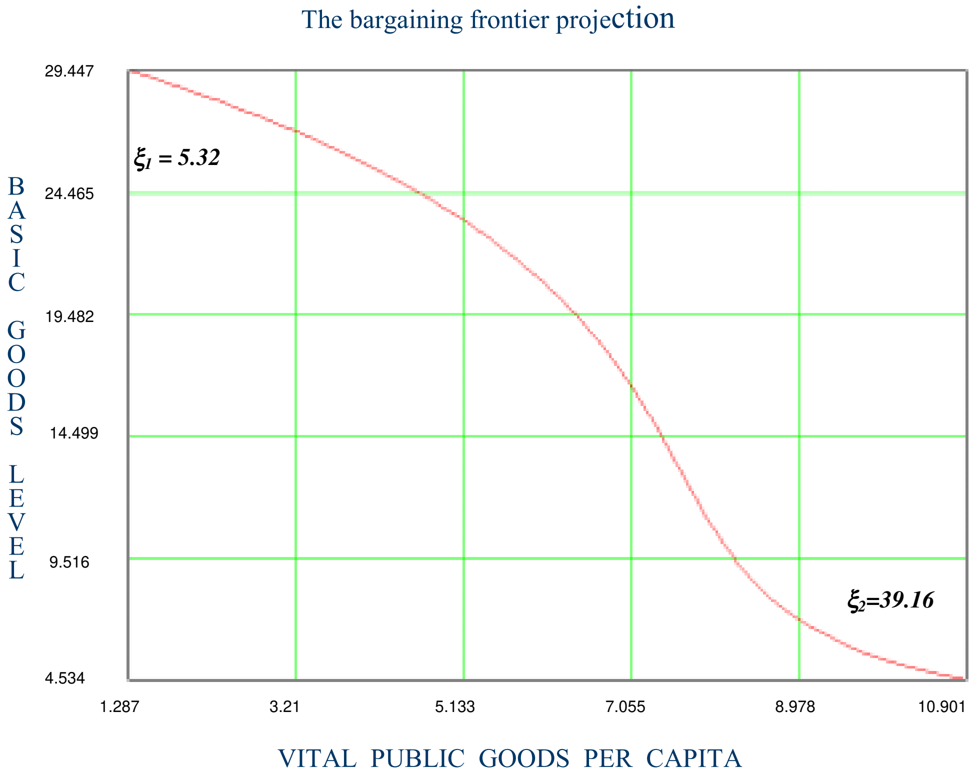


Figure 4. The monetary expectations of social and public agencies are depicted on the vertical and the horizontal axes, respectively. The graph represents the bargaining frontier $S_b = u(g)$ of guaranteed social minimum (level of basic goods) sloping down from the left-top ξ_1 towards right-bottom ξ_2. It is the projection found by resolving the frontier constraint (5).

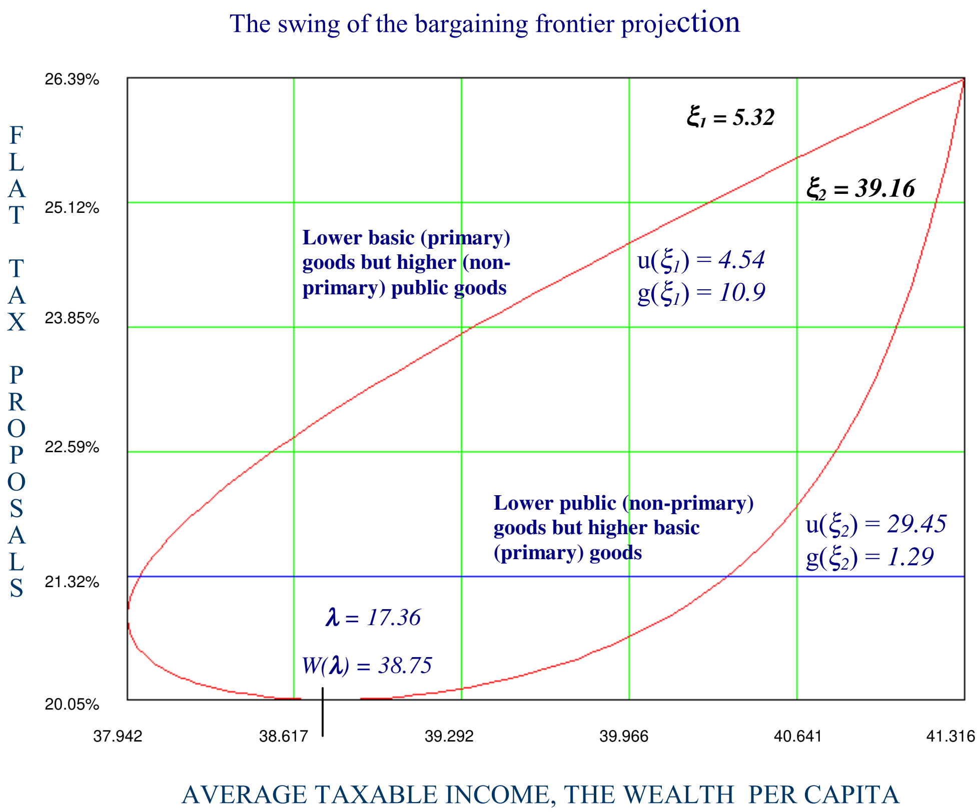


Figure 5. As it follows from the graph, presenting a motion for a vote on the amount W=*38.75* of wealth for the least tax $\tau = 20.05\%$ is a realistic proposal that may pass by unanimous consent (observation 6). In contrast, marked by '——', the higher tax *21.32% > 20.05%* may course a discontent because the same tax level on wealth refunds both: the higher (lower) level of basic goods but lower (higher) public goods, (Table 1).

Mathematical derivation

(1) $\tau \cdot W(\xi) = B(\xi) + g$ Delivery constraint: the tax returns = welfare pie = sum of primary (subsidies) and non-primary (public) goods g

(2) $B(\xi) = x \cdot \tau \cdot W(\xi)$ Balance of subsidies with the slice x of welfare pie

(3) $u = (1 - \tau) \cdot (\xi - \phi) + \phi$ Stability constraint to remove the welfare hazard effect

$u = \xi - \tau \cdot (\xi - \phi)$ Cost of living, social minimum or basic goods: Poverty line income ξ minus tax sacrifice $\tau \cdot (\xi - \phi)$

Replacing $\tau = \dfrac{B(\xi) + g}{W(\xi)}$ from (1) into $u = \xi - \tau \cdot (\xi - \phi)$ leads to $u = \xi - \dfrac{B(\xi) + g}{W(\xi)} \cdot (\xi - \phi)$

(3[1]) Doing differently, replacing $\tau = \dfrac{B(\xi)}{x \cdot W(\xi)}$ from (2) into (3) we get (c.f., at p. 17)

(4) $L(\xi, x, u) = (\xi - \phi) \cdot B(\xi) - x \cdot (\xi - u) \cdot W(\xi) = 0$ called volatility constraint, which amalgamates (2) and (3), p.22

(6) $D(\xi, x, u) = \dfrac{d}{d\xi} L(\xi, x, u) = \dfrac{d}{d\xi} ((\xi - \phi) \cdot B(\xi) - x \cdot (\xi - u) \cdot W(\xi)) = 0$ Bargaining frontier constraint

(6[1]) $\dfrac{d}{d\xi} ((\xi - \phi) \cdot B(\xi) - x \cdot (\xi - u) \cdot W(\xi)) = \blacksquare$

$$\blacksquare = B(\xi) + (\xi - \phi) \cdot \frac{d}{d\xi} B(\xi) - x \cdot W(\xi) - x \cdot (\xi - u) \cdot \frac{d}{d\xi} W(\xi) = 0$$

$F(\xi) = \dfrac{d}{d\xi} B(\xi)$ $E(\xi) = \dfrac{d}{d\xi} W(\xi)$ Now, renaming derivatives F=B', E=W' at p.22 in (6 [1])

yields at last to $B(\xi) + (\xi - \phi) \cdot F(\xi) - x \cdot W(\xi) - x \cdot (\xi - u) \cdot E(\xi) = 0$

(6²) $$D(\xi, x, u) = B(\xi) + (\xi - \phi)\cdot F(\xi) - x\cdot(W(\xi) + (\xi - u)\cdot E(\xi)) = 0$$

Extracting then x from (4) we substitute variable $x = \dfrac{(\xi - \phi)\cdot B(\xi)}{(\xi - u)\cdot W(\xi)}$ into (6²)

(6³) $$B(\xi) + (\xi - \phi)\cdot F(\xi) - \frac{(\xi - \phi)\cdot B(\xi)}{(\xi - u)\cdot W(\xi)}\cdot(W(\xi) + (\xi - u)\cdot E(\xi)) = 0$$ what results in

(6⁴) $$\frac{(B(\xi) + (\xi - \phi)\cdot F(\xi))\cdot(\xi - u)\cdot W(\xi)}{(\xi - u)\cdot W(\xi)} - \frac{(\xi - \phi)\cdot B(\xi)}{(\xi - u)\cdot W(\xi)}\cdot(W(\xi) + (\xi - u)\cdot E(\xi)) = 0$$

(6⁵) $$(B(\xi) + (\xi - \phi)\cdot F(\xi))\cdot(\xi - u)\cdot W(\xi) - (\xi - \phi)\cdot B(\xi)\cdot(W(\xi) + (\xi - u)\cdot E(\xi)) = 0$$

$A = (\xi - u)$ Collect (6⁵) on subexpression $A = (\xi - u)$

$$(B(\xi) + (\xi - \phi)\cdot F(\xi))\cdot A\cdot W(\xi) - (\xi - \phi)\cdot B(\xi)\cdot(W(\xi) + A\cdot E(\xi)) = 0$$

in the form of (6⁶) below

(6⁶) $$((B(\xi) + (\xi - \phi)\cdot F(\xi))\cdot W(\xi) - (\xi - \phi)\cdot B(\xi)\cdot E(\xi))\cdot A - (\xi - \phi)\cdot B(\xi)\cdot W(\xi) = 0$$

(6⁷) $$A = \frac{(\xi - \phi)\cdot B(\xi)\cdot W(\xi)}{((B(\xi) + (\xi - \phi)\cdot F(\xi))\cdot W(\xi) - (\xi - \phi)\cdot B(\xi)\cdot E(\xi))}$$ $A = (\xi - u)$

$$\xi - u = \frac{(\xi - \phi)\cdot B(\xi)\cdot W(\xi)}{((B(\xi) + (\xi - \phi)\cdot F(\xi))\cdot W(\xi) - (\xi - \phi)\cdot B(\xi)\cdot E(\xi))}$$

(6⁸) $$u = \xi - \frac{(\xi - \phi)\cdot B(\xi)\cdot W(\xi)}{((B(\xi) + (\xi - \phi)\cdot F(\xi))\cdot W(\xi) - (\xi - \phi)\cdot B(\xi)\cdot E(\xi))}$$

Substitute now $u = \xi - \dfrac{B(\xi) + g}{W(\xi)} \cdot (\xi - \phi)$ from (3^1) into (6^8) what yields to

$$(6^9) \qquad \xi - \frac{B(\xi) + g}{W(\xi)} \cdot (\xi - \phi) = \xi - \frac{(\xi - \phi) \cdot B(\xi) \cdot W(\xi)}{(B(\xi) + (\xi - \phi) \cdot F(\xi)) \cdot W(\xi) - (\xi - \phi) \cdot B(\xi) \cdot E(\xi)}$$

$$(6^{10}) \qquad \frac{B(\xi) + g}{W(\xi)} \cdot (\xi - \phi) = \frac{B(\xi) \cdot W(\xi) \cdot (\xi - \phi)}{(B(\xi) + (\xi - \phi) \cdot F(\xi)) \cdot W(\xi) - (\xi - \phi) \cdot B(\xi) \cdot E(\xi)}$$

$$(B(\xi) + g) \cdot (\xi - \phi) = \frac{B(\xi) \cdot W(\xi) \cdot W(\xi) \cdot (\xi - \phi)}{(B(\xi) + (\xi - \phi) \cdot F(\xi)) \cdot W(\xi) - (\xi - \phi) \cdot B(\xi) \cdot E(\xi)}$$

$$(6^{11}) \qquad B(\xi) + g = \frac{B(\xi) \cdot W(\xi) \cdot W(\xi)}{(B(\xi) + (\xi - \phi) \cdot F(\xi)) \cdot W(\xi) - (\xi - \phi) \cdot B(\xi) \cdot E(\xi)}$$

$$(6^{12}) \qquad g = \frac{B(\xi) \cdot W(\xi) \cdot W(\xi)}{(B(\xi) + (\xi - \phi) \cdot F(\xi)) \cdot W(\xi) - (\xi - \phi) \cdot B(\xi) \cdot E(\xi)} - B(\xi) \qquad C = (\xi - \phi)$$

Collect the denominator $(B(\xi) + (\xi - \phi) \cdot F(\xi)) \cdot W(\xi) - (\xi - \phi) \cdot B(\xi) \cdot E(\xi)$

in (6^{12}) on subexpression $C = (\xi - \phi)$

$$(B(\xi) + C \cdot F(\xi)) \cdot W(\xi) - C \cdot B(\xi) \cdot E(\xi) = (F(\xi) \cdot W(\xi) - B(\xi) \cdot E(\xi)) \cdot C + B(\xi) \cdot W(\xi)$$

$B(\xi) \cdot W(\xi) + (\xi - \phi) \cdot (F(\xi) \cdot W(\xi) - B(\xi) \cdot E(\xi))$ Denominator in (6^{12})

(6^{13}) $$g = \frac{B(\xi)\cdot W(\xi)\cdot W(\xi)}{B(\xi)\cdot W(\xi) + (\xi - \phi)\cdot(F(\xi)\cdot W(\xi) - B(\xi)\cdot E(\xi))} - B(\xi)$$

(6^{14}) $$B(\xi) = \frac{B(\xi)\cdot(B(\xi)\cdot W(\xi) + (\xi - \phi)\cdot(F(\xi)\cdot W(\xi) - B(\xi)\cdot E(\xi)))}{B(\xi)\cdot W(\xi) + (\xi - \phi)\cdot(F(\xi)\cdot W(\xi) - B(\xi)\cdot E(\xi))}$$

(6^{15}) $$g = \frac{B(\xi)\cdot W(\xi)\cdot W(\xi) - (B(\xi)\cdot(B(\xi)\cdot W(\xi) + (\xi - \phi)\cdot(F(\xi)\cdot W(\xi) - B(\xi)\cdot E(\xi))))}{B(\xi)\cdot W(\xi) + (\xi - \phi)\cdot(F(\xi)\cdot W(\xi) - B(\xi)\cdot E(\xi))}$$

Divide (6^{15}) by $B(\xi)\cdot W(\xi)$

(6^{16}) $$g = \frac{W(\xi) - B(\xi)\cdot\dfrac{B(\xi)\cdot W(\xi) + (\xi - \phi)\cdot(F(\xi)\cdot W(\xi) - B(\xi)\cdot E(\xi))}{B(\xi)\cdot W(\xi)}}{\dfrac{B(\xi)\cdot W(\xi) + (\xi - \phi)\cdot(F(\xi)\cdot W(\xi) - B(\xi)\cdot E(\xi))}{B(\xi)\cdot W(\xi)}}$$

$$E(\xi) = \left(\frac{d}{d\xi}W(\xi)\right) \qquad F(\xi) = \left(\frac{d}{d\xi}B(\xi)\right)$$

(6^{17}) $$v(\xi) := 1 + (\xi - \phi)\cdot\left(\frac{F(\xi)}{B(\xi)} - \frac{E(\xi)}{W(\xi)}\right)$$

(6^{18}) $$g(\xi) := \frac{W(\xi)}{v(\xi)} - B(\xi)$$

$$z(\xi) := B(\xi) + g(\xi) \qquad \tau(\xi) := \frac{z(\xi)}{W(\xi)}$$ Final definition, p.22

$$u(\xi) := \xi - \tau(\xi)\cdot(\xi - \phi)$$ Defined according to (3^{1}) as level of basic goods (social minimum) at p. 22, Observation 1,