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Abstract

This paper examines whether the CPI and real GDP for the U.S. exhibit nonlinear reversion to trend as recently concluded by Beechey and Österholm [Beechey, M. and Österholm, P., 2008. Revisiting the uncertain unit root in GDP and CPI: testing for non-linear trend reversion. Economics Letters 100, 221-223]. The wild bootstrap is used to correct for non-normality and heteroscedasticity in a nonlinear unit root test. Test results are found to be sensitive to the sample period examined.

Keywords: Nonlinear unit root test; Wild Bootstrap, non-normality
JEL Classification: C22, E31, E32

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1. Introduction

The Nelson and Plosser (1982) analysis of the stationarity properties of fourteen U.S. time series spawned a large literature of similar studies and motivated numerous advancements in time series analysis. Despite methodological progress in testing for unit roots, the issue of stationarity of important macroeconomic series remains unresolved. In a recent contribution, Beechey and Österholm (2008) reject a unit root in U.S. real GDP and the CPI using the nonlinear unit root test developed by Kapetanios et al. (2003). Their result suggests that allowing for nonlinear reversion to trend results in a clear rejection of a unit root in both series. Their findings have two important implications. First, if real GDP is trend stationary then shocks have only temporary effects on real GDP, thus casting doubt on the importance of permanent, real shocks as a source of business cycles. Second, a trend stationary CPI is consistent with central bank targeting of the aggregate price level rather than the inflation rate.[1]

This study shows that the Beechey and Österholm conclusion of trend stationary real GDP is reversed if the sample period is extended to include recent data. The finding of a trend stationary CPI is considerably weakened for the original sample period using critical values corrected for heteroscedasticity and non-normality through use of the wild bootstrap. Finally, a unit root cannot be rejected for the post-WWII CPI.

2. Methodology

Kapetanios et al. (KSS) test the null hypothesis of a unit root versus an alternative of a mean reverting process with nonlinear, exponential smooth transition

[1] A finding of a non-stationary price level and a stationary inflation rate would be consistent with inflation targeting by the central bank.

autoregressive (ESTAR) dynamics. As demonstrated by Taylor (2001), standard augmented Dickey-Fuller (ADF) tests have low power if a time series actually is stationary with nonlinear adjustments toward mean. An appealing property of the KSS test is that it offers potential gains in power that are highest in the region of the null hypothesis. Although fully parameterized ESTAR models are quite complicated and may suffer from identification problems, a first-order Taylor approximation yields the simple test equation:

$$\Delta y_t = \delta y_{t-1}^3 + \sum_{j-1}^{p} \gamma_j \Delta y_{t-j} + \varepsilon_t \qquad (1)$$

When examining a series with potential trend reversion, the test series (y_t) is constructed as the set of residuals from a preliminary OLS regression of the unadjusted time series $\tilde{y}_t$ on a constant and linear time trend. The null hypothesis $H_0 : \delta = 0$ then is tested versus an alternative $H_1 : \delta < 0$ using the t-statistic on the estimated $\hat{\delta}$ coefficient. The null hypothesis of a unit root is rejected in favor of nonlinear reversion to trend if the test statistic lies beyond a lower critical value.

Asymptotic lower critical values for the test are provided by KSS. However, these critical values are derived under the assumption that the test equation innovation series (ε_t) is normally distributed and homoscedastic. Van Dijk et al. (1999) and Engel et al. (2005) demonstrate that a nonlinear model may be incorrectly selected if large outliers are present in the data. In other words, a linear unit root may be incorrectly rejected in equation (1) in favor of nonlinear trend reversion if the data are not normally distributed. Arghyrou and Gregoriou (2008) show that KSS test results also may be affected if the innovations are autoregressive conditional heteroscedastic (ARCH). They then demonstrate that the wild bootstrap is appropriate for generating critical values of the KSS test when the test residuals are non-normal and/or heteroscedastic of an unknown form.

The wild bootstrap procedure involves estimating equation (1) by OLS and retaining the estimated residuals ε_t as well as the t-statistic for testing the null hypothesis. In implementing the wild bootstrap 100,000 sets of new residuals ε_t^* are generated according to:

$$\varepsilon_t^* = \varepsilon_t u_t \tag{2}$$

The u_t variable is drawn from the two-point distribution suggested by Mammen (1993):

$$u_t = \frac{1-\sqrt{5}}{2} \text{ with probability } p = \frac{1+\sqrt{5}}{\sqrt{20}}, \text{ and} \tag{3}$$

$$u_t = \frac{1+\sqrt{5}}{2} \text{ with probability } (1-p).$$

The KSS test equation with the null hypothesis imposed:

$$\Delta y_t = \sum_{j-1}^{p} \gamma_j \Delta y_{t-j} + \varepsilon_t \tag{4}$$

is used to create 100,000 artificial data sets. Each artificial data set is constructed by combining estimates of the γ_j coefficients with one of the generated ε_t^* series.

The null hypothesis is true by construction for each artificial data set. In addition, the u_t terms are mutually independent drawings from a distribution that is independent of the original data and has the properties $E(u_t) = 0$, $E(u_t^2) = 1$, and $E(u_t^3) = 1$. These properties imply that any non-normality or ARCH present in the original ε_t residuals from equation (1) remains in the generated ε_t^* for each artificial data set. In the wild bootstrap procedure, the artificial data sets are subjected to the KSS test to generate a vector of ordered test statistics. This vector then is used to construct the empirical distribution of the test statistics under the null hypothesis. The lower (wild bootstrapped) critical values of the test are based upon this empirical distribution.

3. Empirical Results

The U.S. GDP data are quarterly, seasonally adjusted values, while the U.S. CPI series is composed of monthly, not seasonally adjusted observations. Both series are logged prior to testing.[2] The KSS unit root test is applied to various samples, including the periods studied by Beechey and Österholm. The Akaike Information Criterion (AIC) determines the number of lagged differenced terms in the KSS test equation (1).[3] The KSS test residuals are subjected to a Jarque-Bera normality test and Engle's (1982) test for ARCH.[4] Normality is rejected for all test equations, and homoscedasticity is rejected for some.

In the initial step, the KSS test is applied to the sample periods considered by Beechey and Österholm, 1947.1 through 2005.2 for U.S. real GDP and 1914.01 through 2005.07 for the CPI. Results are displayed in the top panel of Table 1. The test statistics reject the unit root null using a 1% asymptotic critical value for real GDP and a 5% asymptotic critical value for the CPI.[5] However, conclusions based on asymptotic critical values are questionable due to rejection of normality for both sets of test residuals and rejection of homoscedasticity for the CPI test residuals. Wild bootstrapped critical values verify rejection of a unit root at a 1% level for real GDP over this sample. However, the unit root null no longer can be rejected for the CPI over this sample using the wild bootstrapped 5% critical value.

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[2] Both data series were obtained from the FRED database of the Federal Reserve Bank of St. Louis.
[3] Beechey and Österholm select the lag length to be equal to that minimizing AIC for a standard ADF test. Use of their approach does not alter any conclusions in this study.
[4] Only first order ARCH was considered in these tests.
[5] Asymptotic critical values are -3.40 (5%) and -3.93 (1%).

It appears that the finding by Beechey and Österholm of nonlinear reversion to trend of U.S. real GDP is robust to correction for non-normality and ARCH. However, additional data have become available since their study. The KSS test results with data for 1947.1-2009.3 for GDP are presented in the second panel, left column of Table 1. Non-normality and ARCH impel comparison of the test statistic to wild bootstrapped critical values. With the additional data, which includes the most recent recession, the evidence no longer supports nonlinear reversion of real GDP to trend.[6]

Finally, we consider whether the CPI results are affected by the inclusion of pre-WWII data. Perhaps a unit root can be rejected for the post-war era, a period of a more activist Federal Reserve. The KSS test is applied to the monthly CPI for 1947.01-2009.11. Results are shown in the final column of the second panel of Table 1. Wild bootstrapped critical values are used due to rejections of normality and homoscedasticity. A unit root cannot be rejected for the CPI over the post-WWII sample.

4. Conclusions

In contrast to the Beechey and Österholm finding, the use of wild bootstrapped critical values to correct for non-normality and heteroscedasticity results in failure to reject a unit root in the U.S. CPI for the January 1914-July 2005 sample at the 5% level. This result demonstrates the potential sensitivity of KSS test conclusions based on asymptotic critical values to violations of the assumptions of normality and homoscedasticity. Nor can the unit root null be rejected in the U.S. CPI for the post-WWII era. Overall, test results for the CPI are not consistent with price level targeting

[6] A unit root can be rejected using the 1% wild bootstrapped critical value for a sample period of 1947.1 through 2007.3 that eliminates only those observations of real GDP associated with the recession. For brevity, further details of this result are not presented in this paper, but are available upon request from the authors.

by the Fed. Rather, failure to reject a unit root for either sample is more consistent with central bank targeting of the inflation rate.

Evidence from the 1947.1-2005.2 period examined by Beechey and Österholm suggests that U.S. real GDP is a trend stationary series. This result is unchanged when wild bootstrapped critical values are used to correct for non-normality. However, once data from the recession beginning in the fourth quarter of 2007 are included, the unit root null cannot be rejected. Failure to reject a unit root is consistent with the original conclusion by Nelson and Plosser that U.S. real GDP is non-stationary. Diebold and Senhadji (1996) emphasize the sensitivity of linear unit root test results to the sample period considered. Our findings for real GDP suggest that nonlinear unit root test results also are sensitive to alterations in the data sample.

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Table 1
KSS Test Results

	Real GDP	CPI
Sample	1947.1-2005.2	1914.01-2005.07
KSS Test Statistic	-4.084**	-3.608
5% Critical Value	-3.296	-3.710
1% Critical Value	-3.989	-4.059
Jarque-Bera Statistic	22.01**	7572.03**
ARCH Statistic	1.68	8.90**
Sample	1947.1-2009.3	1947.01-2009.11
KSS Test Statistic	-2.858	-2.479
5% Critical Value	-3.310	-3.033
1% Critical Value	-3.912	-3.411
Jarque-Bera Statistic	17.89**	402.56**
ARCH Statistic	4.56*	42.19**

* Denotes significance at a 5% level.
**Denotes significance at a 1% level.