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Abstract

For emerging market returns there is strong evidence that the departure from normality is primarily driven by kurtosis and not skewness. This paper investigates the empirical validity of a return generating process that includes quadratic and cubic market returns as factors of pricing for an emerging market. Following Barone-Adesi *et al*. (2004) a multivariate test of a three-moment pricing model is developed. The empirical evidence in the market returns support the stylized facts typical for an emerging market and reveal that any return generating process that includes only a quadratic term (coskewness) may be misspecified. However comparison of higher order market return factors with Fama French factors indicates that while risk exposure to these higher order co-moments factors especially cokurtosis is important the co-moments do not possess sufficient explanatory power to render Fama French factor redundant.

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1. Introduction and Literature Review

The failure of the conventional capital asset pricing model (CAPM) to adequately explain cross-sectional variation of risky asset return has spurred alternative explanations of asset pricing. The Arbitrage Pricing Theory (APT) of Ross (1976) is one such alternative. APT stipulates that under no arbitrage the expected returns of assets are expressed as a linear function of certain common factors though the theory does not specify the factors themselves. Many studies follow this lead. For example, based primarily on statistical considerations, Fama and French (1992) advocate inclusion of two factors mimicking the size and book-to-market value of the assets besides the systematic risk; the beta. Kraus and Litzenberger (1976) and Dittmar (2002) emphasize inclusion of higher co-moments namely coskewness and cokurtosis as explanatory variables of expected returns.

The unknown form of the return distribution is unlikely to be described by the first two moments only. See for example Chung, Johnson and Schill (2006) for more emphasis on this issue. Therefore incorporation of the higher-order co-moments such as coskewness and cokurtosis[1] besides the beta as additional risk factors is an important issue. The dynamics underlying higher order co-moments is consistent with the investor preference. Moreover Brockett and Kahane (1992) demonstrate by giving counter examples that ascertaining direction of preference for moments on the basis of the sign of the derivatives of the utility

[1] The covariance of an asset with the market returns measure to the contribution of the asset to the variability of a well diversified portfolio. In a similar way coskewness and cokurtosis measure the contribution of the asset to the overall skewness and kurtosis of the portfolio. An asset with positive coskewness with the market diminishes the sensitivity of a portfolio to large absolute market returns, (Barone-Adesi et al., 2004). Therefore *ceteres paribus* investors like assets with positive coskewness. Kurtosis here refers to the tail thickness of the return distribution. A higher cokurtosis therefore indicates higher likelihood that extreme returns jointly occur in the given asset and the market.

function is not justified. The utility based higher moments asset pricing is derived assuming both two-fund separation and a representative agent. The validity of these assumptions for emerging markets is questioned by Hwang and Satchell (1999).

Barone-Adesi (1985) and Barone-Adesi, Gagliardini and Urga (2004) (henceforth referred to as BA and BAGU respectively) provide an arbitrage based approach to test the restriction imposed by the APT on the system of a quadratic market model. The modelling and econometric testing framework adopted by BA and BAGU is important since it recasts the covariance-coskewness CAPM as the APT restriction on the system of quadratic market model for which the Gibbons (1982) multivariate methodology is readily applicable. Thus they reconcile the two important alternative explanations of expected returns. Their testing approach also avoids the errors-in- variables and multicollinearity problems of utility based asset pricing and makes better use of available information by employing the contemporaneous covariance among the asset returns in a multivariate setting. This approach of APT testing is also efficient. Some approaches involve pre-specified macroeconomic variables. BA and BAGU approach uses the information on the return on the stocks and the market portfolio only thereby being less dependent on external macroeconomic data. Despite its intuitive appeal there is no application of the BA and BAGU methodology on emerging markets.

Relevance of higher moments and their likely impact on expected returns are established to be different in emerging and developed markets. For example, Aggarwal el al. (1999) observes that generally the skewness in the return distribution of emerging market indices is negative while it is positive for developed markets. Moreover there has been evidence to suggest that for emerging market returns the departure from normality is primarily driven by kurtosis and not skewness. In a group of 17 emerging markets including Pakistan, Hwang and Satchell (1999) show that cokurtosis has at least as much explanatory power as coskewness. Further, the wide spread evidence of outliers in emerging market returns suggest that the extreme outcomes have a higher probability of occurrence in emerging markets. BA and BAGU point out the possibility of a missing systematic factor in their pricing model. They did not consider cokurtosis as a potential explanatory variable of asset returns[2]. According to the stylized facts of emerging markets returns it appears that cokurtosis might be a useful factor for such markets. The purpose of this paper is to extend the multivariate methodology of BAGU to incorporate cokurtosis and provide empirical evidence from an emerging market. BAGU adopt a Pseudo- maximum likelihood approach to estimate the model parameter and test the model through an asymptotic least square methodology that does not rely on the normality assumption. Non-normality of returns is an important consideration when modelling emerging market returns as the microstructure and relatively turbulent political and economic environment makes the normality assumption difficult to justify.

The APT does not prescribe the factors that should be included in the factor space[3]. Fama and French (1993) suggest size and book-to-market portfolios returns as potential APT factors. For developed capital markets several authors have compared the Fama-French factors and higher order co-moments in explaining asset returns. For example, in the US market using CRISP portfolios Chung, Johnson and Schill (2006) find that Fama-French factors ceased to be effective in explaining asset returns when the first 10 co-moments are incorporated. Therefore they conclude that Fama-French factors may proxy for higher order co-moments. Using Fama-French size portfolios, BAGU report that the size factor anomaly appears to be resolved by incorporating coskewness in the pricing model. On the other hand for a sample of UK data Hung, Shackleton and Xu (2004) provide limited evidence in favour of higher order market factors associated with coskewness and cokurtosis compared to the Fama-French factors. To our knowledge no study has investigated relative performance of different APT factors in emerging markets. In this paper we investigate whether the Fama-French factors or the higher co-moment market factors explain portfolio returns better for an

[2] The specification tests for the mean return equation in BAGU did not support a cubic market return factor.
[3] The BA and BAGU provide a heuristic approach of linking the quadratic market model with APT. Therefore the linear and quadratic market returns are deemed as APT factors.

emerging market. We compare the performance of alternative factor models statistically and on the basis of economic significance.

In the empirical analysis we consider the Karachi stock market which is the largest stock exchange in Pakistan[4]. This market has received considerable attention in recent years when in 2002 it was declared as the best performing stock market in the world in terms of the percentage increase in the local market index value. We investigate whether an asset pricing model with higher co-moments is able to explain risk-return relation in this emerging market.

The rest of the paper is organized as follows: Section 2 describes estimation and inference for a higher order co-moment model. Section 3 discusses the framework for comparing the higher order co-moment model with the Fama French alternative. Section 4 discusses the data. Empirical results are analysed in section 5 and section 6 provides some concluding remarks.

2. Framework for estimation and inference for higher co-moments model

We consider a specification of the return generating process with a quadratic and a cubic market return factor[5]. Let R_t denote an $N \times 1$ vector of N asset returns at time t and R_{mt} and R_{ft} represent the return of the market portfolio and the risk free rate respectively. The cubic market model can be expressed as:

$$r_t = \alpha + \beta\, r_{mt} + \gamma\, q_{mt} + \delta\, c_{mt} + \varepsilon_t \tag{1}$$

where $r_t = R_t - R_{ft}$ is the vector of excess returns, $r_{mt} = R_{mt} - R_{ft}$, $q_{mt} = R^2{}_{mt} - R_{ft}$ and $\mathrm{c}_{\mathrm{m_t}} = \mathrm{R}^3{}_{\mathrm{mt}} - \mathrm{R}_{\mathrm{ft}}$. The N intercepts are collected in vector α and each of β, γ and δ is $N \times 1$ vector of sensitivities. The ε_t is the vector of error term which is assumed to satisfy

[4] Karachi Stock Exchange is the largest of the three stock markets in Pakistan. On April 17, 2006 the market capitalization was US$ 57 billion which is 46 percent of Pakistan's GDP for the Fiscal Year 2005-2006 (Ref: Pakistan Economic Survey, 2005-06).

[5] The framework that we outline here is an extension of the BAGU approach on a return generating process with a quadratic term.

$$E(\varepsilon_t \mid I_t) = 0 \text{ and } E(\varepsilon_t \varepsilon_t' \mid I_t) = \Sigma \tag{2}$$

The information set I_t includes all current and past lagged values of R_m and R_f. Although γ and δ do not exactly correspond to the usual definition of coskewness and cokurtosis, BAGU argue that they are good proxies for these measures. Further, the multivariate methodology used in the estimation of the cubic market model avoids the problems of error-in-variable in measuring coskewness and cokurtosis and also avoids multicollinearity. Although the cubic market model is only a statistical description of the return generating process, following the arguments of the Kraus and Litzenberger (1976) and BA it can be stipulated that the cubic market model (1) is consistent with the four-moment CAPM. The APT approach of BA involves minimal assumptions about the investor's utility function which can be exploited for modelling the emerging markets risk-return relationship. According to BA the expected asset returns under APT is given by the following linear specification

$$E(r_t) = \beta \lambda_1 + \gamma \lambda_2 + \delta \lambda_3 \tag{3}$$

Where λ_1, λ_2 and λ_3 are expected excess return on portfolios whose return are perfectly correlated with r_m, q_m and c_m respectively. It is obvious that $\lambda_1 = E(r_m)$. Assets with a higher value of coskewness decrease the risk of a portfolio with respect to large absolute market returns. If skewness of the market returns is positive then $\lambda_2 < 0$. Following the arguments of Scott and Horvath (1980) we have $\lambda_3 > 0$.

Applying expectations to (1) and equating with (3) results in the following APT imposed restriction on the coefficients of the cubic market model,

$$\alpha = \gamma v_1 + \delta v_2 \tag{4}$$

where $v_1 = [\lambda_2 - E(q_m)]$ and $v_2 = [\lambda_3 - E(c_m)]$. Therefore the arbitrage equilibrium consistent with coskewness and cokurtosis results in the following expected returns:

$$E(r_t) = \beta r_{mt} + \gamma q_{mt} + \delta c_{mt} + v_1\gamma + v_2\delta \tag{5}$$

As in BAGU a Quasi-Maximal Likelihood (QML) approach for testing (4) can be invoked. In the present context the essential idea of the QML approach is that consistent and asymptotically normally distributed estimators of the parameters can be obtained by correctly specifying the first two moments of the error distribution given in (2). The normal log likelihood function for the restricted model can then be constructed to estimate the parameters and perform the inference. The consistency and asymptotic normality is guaranteed even if the likelihood is misspecified; see for example Mittelhammer *et al.* (2000). Thus this approach does not rely upon the assumption of normality of the errors. The wide spread evidence of non-normality of the returns and the compelling reasons to include the higher moments dictate the importance of this normality-robust feature in estimation and inference in an emerging market context.

Let $\hat{B} = [\hat{\alpha}\ \hat{\beta}\ \hat{\gamma}\ \hat{\delta}]$ be the $N \times 4$ matrix of the estimates of the parameters. Then QML implies that under assumption (2)

$$\sqrt{T}(\hat{B} - B) \xrightarrow{d} N(0, \Sigma \otimes E(F_t F_t')) \tag{6}$$

where $F_t = [1\ r_m\ q_m\ c_m]$. The joint statistical significance of the coefficients in the unrestricted system (1) can be carried out using a Wald test. For example, the test of the hypothesis $H_0 : \delta = 0$ results in the following test statistic:

$$W_1 = (T - N/2 - 3/2)\frac{1}{\hat{\Sigma}_f^{33}}\hat{\delta}'\hat{\Sigma}^{-1}\hat{\delta} \quad \xrightarrow{d} \quad \chi^2(N) \tag{7}$$

Here $\hat{\Sigma}_f^{33}$ represents the (3,3) element in the inverse of the covariance matrix of $f_t = [r_{m_t}\ q_{m_t}\ c_{mt}]'$. The Wald test statistic is adjusted with a finite sample correction suggested in Jobson and Korkie (1982). The constrained model (3) involves cross-equation restrictions. The restricted parameters are:

$$[\hat{\beta}'\ \hat{\gamma}'\ \hat{\delta}']' = [\sum_{t=1}^{T} r_t \hat{H}_t'][\sum_{t=1}^{T} \hat{H}_t \hat{H}_t'] \tag{8}$$

$$\hat{H}_t = [r_{mt} \quad q_{mt} + \hat{v}_1 \quad c_{mt} + \hat{v}_2]',\ \hat{Z} = [\hat{\gamma}\ \hat{\delta}] \tag{9}$$

$$[\hat{v}_1 \ \hat{v}_2]' = (\hat{Z}'\hat{\Sigma}^{-1}\hat{Z})^{-1}\hat{Z}'\hat{\Sigma}^{-1}(\bar{r}_t - \hat{\beta}\,\bar{r}_{mt} - \hat{\gamma}\,\bar{q}_{mt} - \hat{\delta}\,\bar{c}_{mt}) \tag{10}$$

The parameters can be iteratively estimated with starting values provided by their unrestricted counterparts. Alternatively the parameters in the restricted model can be estimated by non-linear Feasible Generalized Least Square as discussed in Gallant (1987) and Srivastava and Giles (1987). This approach is also robust to normality.

The restriction (4) is tested using an asymptotic least square statistics[6]

$$W_2 = (T - N/2 - 3/2)\frac{(\hat{\alpha} - \tilde{v}_1\hat{\gamma} - \tilde{v}_2\hat{\delta})'\hat{\Sigma}^{-1}(\hat{\alpha} - \tilde{v}_1\hat{\gamma} - \tilde{v}_2\hat{\delta})}{1 + \tilde{\lambda}'\hat{\Sigma}_f^{-1}\tilde{\lambda}} \xrightarrow{d} \chi^2(N-2) \tag{11}$$

where $\tilde{\lambda} = \hat{\mu} + [0 \ \ \tilde{v}_1 \ \ \tilde{v}_2]'$, $\hat{\mu} = [\bar{r}_{mt} \ \bar{q}_{mt} \ \bar{c}_{mt}]'$ and $[\tilde{v}_1 \ \tilde{v}_2]' = (\hat{Z}'\hat{\Sigma}^{-1}\hat{Z})^{-1}\hat{Z}'\hat{\Sigma}^{-1}\hat{\alpha}$.

In the QML based approach the moments must be correctly specified. This essentially translates into specifying the return generating process correctly. Therefore we consider two other alternative specifications of the return generating process; one that considers only the coskewness and another only the cokurtosis in addition to systematic beta risk. To select the most appropriate return generating specification a joint Wald test on the parameters of unrestricted system (1) is performed.

In the literature, the following three-factor Fama and French (1992) model is advocated as an alternative to the CAPM where the size (SMB) and the book-to-market (HML) factors are stipulated as another possible set of APT factors.

$$r_t = \beta_0 + \beta_1\, r_{mt} + \beta_2\, SMB_t + \beta_3\, HML_t + \varepsilon_t \tag{12}$$

3. Comparison of the factor models

To compare the relative performance of the higher order market factors and the Fama-French factors, we consider a J-type non-nested model testing approach originally adopted

[6] The unrestricted system has 4N parameters and the restricted system has 3N+2 parameters. The APT therefore imposes N-2 restrictions which are employed as the degrees of freedom.

by Chen (1983) for comparing CAPM and APT and applied by Faff (1992) in a multivariate context which result in a Likelihood Ratio Test.

One way of assessing economic significance alternative models is to compare their pricing errors from the equilibrium model. We compare the mispricing of the two competing models for the three set of portfolios constructed according the size, beta and the industry sorts. Finally the model's goodness of fit are compared through univariate and multivariate system based methods.

4. Description of the Data

The tests discussed in section III are applied to portfolios formed from a sample of stocks listed on the Karachi Stock Exchange (KSE). The sample period spans 13 ½ years from September 1992 to April 2006 and includes 162 monthly observations. The data consist of monthly closing prices of 101 stocks and the Karachi Stock Exchange 100 index (KSE-100) and are collected from the DataStream database. To investigate robustness in the empirical results we consider portfolios based on both beta and industry in addition to the size portfolio considered in BAGU. We construct seventeen equally weighted portfolios. Construction of the Fama French factors requires firm level data on shareholder equity, number of outstanding stocks and market capitalization. The State Bank of Pakistan's document "Balance sheet analysis of joint stock companies" comprise of annual data on balance sheet items for non-financial firms. For financial firms the data is obtained from the State Bank[7]. The market related data on the capitalization and the number of outstanding stocks are collected through the financial daily Business Recorder. As the market data are not available for the full sample period the data employed corresponds to roughly the middle of the sample period. The book value is obtained as the net assets of the firms excluding any preferred stocks. The mimicking portfolio of the size and book to market is constructed

[7] We are thankful to Mazhar Khan and Kamran Najam for his helpful cooperation in the balance sheet data access.

similar to the Fama and French (1993) methodology. For detail of construction of these factors see Iqbal and Brooks (2007).

Table 1 presents some descriptive statistics for excess returns on the size portfolios and the market portfolio[8]. The last two columns report the Jarque-Bera normality test statistic and the associated p-value. The skewness of the market return is negative. This is a common feature in emerging markets (Aggarwal et al., 1999). The returns are quite volatile as observed by their standard deviations. It is generally observed that the source of non-normality is the excess kurtosis.

5. Results of the empirical analysis

A. Higher-order co-moment model

Table 2 presents some goodness of fit measures of three alternative systems of unrestricted seemingly unrelated regression equations for the size, beta and industry portfolios. The results show that the model with cokurtosis (system 2) has a higher overall average adjusted r-square compared to the model with coskewness only (system 1). This is observed in all of the three types of portfolios. The system with both coskewness and cokurtosis (system 3) has a slightly higher explanatory power than systems 1 and 2 according to Glahn's (1969) squared composite correlation coefficient. Overall in terms of the goodness of fit system 3 is the preferred model. The bottom row of Table 3 reports Wald and F tests for the joint statistical significance of the set of coefficients in the unrestricted model of size portfolios. Jointly the beta coefficients are significantly different from zero.

Further there is strong evidence to suggest that cokurtosis is jointly significant. Coskewness as a measure of risk is not jointly significant.

[8] The results with other two portfolios are available from authors upon request.

Table 1: Descriptive statistics of size portfolio returns

Portfolio	Mean	SD	Skewness	Kurtosis	Jarque-Bera	P-value (JB)
1	-0.701	8.066	0.559	3.919	14.16	0.0008
2	-0.490	13.295	0.183	13.099	689.45	0.0000
3	-0.564	7.941	0.707	5.341	50.54	0.0000
4	-0.688	7.632	-1.037	8.969	269.54	0.0000
5	0.029	9.361	0.202	2.790	1.40	0.4950
6	-0.376	8.231	0.104	3.557	2.39	0.3010
7	0.233	8.380	0.938	7.094	136.97	0.0000
8	0.490	6.950	0.379	2.821	4.10	0.1287
9	0.199	9.561	0.363	3.700	6.87	0.0321
10	0.053	7.040	-0.169	3.429	2.02	0.3640
11	0.074	9.725	0.222	3.009	1.33	0.5136
12	-0.279	7.897	0.565	3.754	12.47	0.0019
13	0.407	7.090	0.388	3.267	4.55	0.1024
14	0.076	10.345	0.263	3.146	2.03	0.3632
15	0.383	9.433	0.398	3.319	4.98	0.0828
16	0.057	11.519	-0.178	3.548	2.88	0.2361
17	-0.239	11.716	0.130	3.709	3.85	0.1453
Market Portfolio	0.535	9.823	-0.437	4.787	26.74	0.0000

Thus the initial diagnostic tests points toward the importance of cokurtosis relative to the coskewness in modelling portfolio returns of the emerging market under consideration. Similar results are obtained with beta and industry portfolios.

Table 3 presents the seemingly unrelated regression parameters estimates for the size portfolios[9] from the unrestricted system of equation (1). The t-statistic of the coefficients for the individual equations is reported in the parenthesis. The intercept is significantly different from zero only in the largest size portfolio and at the 10% level. In all but one portfolio the

[9]. The results for these set of portfolios are broadly similar to the size portfolio.

estimated systematic risk (beta) is highly significantly different from zero. The coefficient for the coskewness is not significant in any portfolio. However, the cokurtosis coefficient is significantly different from zero in 12 of the 17 portfolios at the 5 per cent level.

Table 2: Goodness of fit measures of the alternative unrestricted system of SUR equations

	System 1	System 2	System 3
	Panel A: Size Portfolio		
Average $\overline{R}^2$	0.3203	0.3483	0.3486
System R^2	0.3535	0.3789	0.3831
	Panel B: Beta Portfolio		
Average $\overline{R}^2$	0.2923	0.3146	0.3155
System R^2	0.3625	0.3858	0.3896
	Panel C: Industry Portfolio		
Average $\overline{R}^2$	0.3138	0.3363	0.3362
System R^2	0.3725	0.3922	0.3957

Notes: System 1 (covariance – coskewness): $r_t = \alpha + \beta\, r_{mt} + \gamma\, q_{mt} + \varepsilon_t$

System 2 (covariance – cokurtosis): $r_t = \alpha + \beta\, r_{mt} + \delta\, c_{mt} + \varepsilon_t$

System 3 (covariance – coskewness – cokurtosis): $r_t = \alpha + \beta\, r_{mt} + \gamma\, q_{mt} + \delta\, c_{mt} + \varepsilon_t$

The system R-square is Glahn's (1969) squared composite correlation coefficient computed as $R^2 = \dfrac{tr(\hat{Y}' A \hat{Y})}{tr(YAY)}$ where $Y = [r_1\ r_2\ \ ...r_N]$ and $\hat{Y} = [\hat{r}_1\ \hat{r}_2\ \ ...\hat{r}_N]$ are each $T \times N$ matrices of excess returns and OLS fitted values of the excess returns respectively and $\mathrm{A} = [\mathrm{I}_{\mathrm{T}} - \frac{1}{\mathrm{T}}\mathrm{jj}']$ where j is a $T \times 1$ vector of ones.

This observation further highlights the importance of cokurtosis in explaining asset returns. Brooks and Faff (1998) and Holmes and Faff (2004) invoke the literature from market timing ability of managed funds to provide an interpretation of the sign of the coefficients in the higher order market model.

Table 3: Parameter estimates for the unrestricted SUR cubic market model for the size portfolios

Portfolio	**Intercept ($\hat{\alpha}$)**	**Market ($\hat{\beta}$)**	**Co-skewness ($\hat{\gamma}$)**	**Co-kurtosis ($\hat{\delta}$)**
1	-0.4950 (-0.70)	0.4839* (5.28)	-0.0049 (-1.29)	-0.0004* (-2.60)
2	-1.2391 (-1.04)	0.2994* (1.85)	0.0061 (0.91)	-0.0001 (-0.32)
3	-0.4575 (-0.68)	0.3609* (3.93)	-0.0031 (-0.83)	-0.0002 (-0.91)
4	-0.4557 (-0.67)	0.2774* (3.01)	-0.0040 (-1.05)	-0.0002 (-1.48)
5	-0.3178 (-0.46)	0.7354* (7.85)	-0.0005 (-0.14)	-0.0005* (-2.65)
6	-0.5365 (-0.86)	0.6474* (7.60)	-0.0020 (-0.57)	-0.0005* (-3.35)
7	0.5574 (0.81)	0.5566* (5.96)	-0.0065* (-1.69)	-0.0005* (-3.08)
8	0.2697 (0.49)	0.4591* (6.15)	-0.0003 (-0.09)	-0.0003* (-1.95)
9	0.0478 (0.07)	0.8764* (9.37)	-0.0034 (-0.88)	-0.0008* (-4.97)
10	0.0090 (0.02)	0.6234* (9.59)	-0.0031 (-1.14)	-0.0004* (-3.19)
11	0.2137 (0.29)	0.8040* (8.11)	-0.0060 (-1.47)	-0.0007* (-3.73)
12	-0.1532 (-0.27)	0.6726* (8.55)	-0.0051 (-1.58)	-0.0005* (-3.75)
13	0.2046 (0.40)	0.5734* (8.30)	-0.0011 (-0.39)	-0.0003* (-2.57)
14	-0.0161 (-0.02)	0.9927* (10.93)	-0.0047 (-1.24)	-0.0007* (-4.03)
15	-0.3587 (-0.73)	0.9351* (14.05)	0.0025 (0.90)	-0.0004* (-3.02)
16	-0.3076 (-0.59)	1.1704* (16.49)	-0.0028 (-0.95)	-0.0004* (-2.90)
17	-1.0245* (-1.82)	1.1166* (14.58)	0.0019 (0.60)	-0.0002* (-1.72)
F/Wald	0.778 [0.715]	483.10 [0.000]	17.543 [0.418]	44.59 [0.000]

** indicates significance at 10 percent level of significance*

Notes: the t-statistics of the parameter estimates are reported in the parenthesis and the p-values of the Wald test are given in the square bracket. The test for intercept is the F test proposed by Gibbons, Ross and Shanken (1989) which is robust to non-normality in small samples. The test used for the remaining coefficients are the Wald tests.

They consider that the fund's time-varying beta is related to the market return and the squared market return. The gamma coefficient measures the market exposure when the market returns are higher and a lower market exposure when the market returns are lower. The funds with this positive market timing ability are therefore attractive. Similarly the delta coefficient measures the volatility timing ability of the funds. A negative delta implies that investors do not experience any return compensation during high volatility periods and fund managers should seek to avoid market exposure at these times. Interestingly in most cases in Table 3 the gamma coefficients are negative and delta coefficients are negative and significant too- a result that was found for a majority of funds in Holmes and Faff (2004) and for a majority of countries in the international asset pricing study in Brooks and Faff (1998).

Table 4 reports the QML based test of the restriction imposed by the arbitrage equilibrium for the three portfolio schemes. In all three cases the arbitrage restrictions are not rejected suggesting the appropriateness of higher order co-moments in the emerging market under consideration.

Table 4: QML test statistic and the Chi Square P-values

Portfolio	QML test statistic	P-value
Size	10.412	0.793
Beta	12.295	0.656
Industry	12.991	0.527

B. Fama-French model

Table 5 presents the parameter estimate for the size portfolios in the three-factor Fama-French model[10]. The intercepts are not significantly different from zero indicating that the Fama-French model adequately explains the variation in returns. The big size portfolios tend to have larger coefficients for the systematic risk (beta) factor. All beta coefficients are significantly different from zero.

[10] The non-linearity of SMB factor is also observed in beta portfolios. Small beta firms have positive coefficients on SMB factor while they are negative for large beta firms. The industry portfolio results corroborate the importance of all the three factors i.e. the market, size and book-to-market in explaining the industry returns.

Table 5: Parameter estimates for the SUR system of Fama-French model for the size portfolios

Portfolio	**Intercept ($\hat{\beta}_0$)**	**Market ($\hat{\beta}_1$)**	**Size ($\hat{\beta}_2$)**	**Book-to-Market ($\hat{\beta}_3$)**
1	-0.6830 (-1.214)	0.4529* (5.994)	0.6371* (4.081)	-0.0955 (-0.870)
2	-0.0014 (-0.002)	0.6382* (5.478)	1.7726* (7.366)	-0.4438* (-2.625)
3	-0.2783 (-0.551)	0.3840* (5.654)	0.6651* (4.740)	-0.4785* (-4.853)
4	-0.6594 (-1.232)	0.4157* (5.778)	0.8644* (5.816)	0.0490 (0.469)
5	0.1057 (0.188)	0.6171* (8.178)	0.5743* (3.684)	-0.3826* (-3.499)
6	-0.1359 (-0.279)	0.4403* (6.732)	0.4723* (3.496)	-0.5597* (-5.896)
7	0.097 (0.162)	0.4329* (5.351)	0.3407* (2.038)	0.0254 (0.216)
8	0.5003 (1.078)	0.3810* (6.107)	0.2915* (2.262)	-0.2063* (-2.279)
9	0.1795 (0.2862)	0.4910* (5.822)	0.1660 (0.953)	-0.3270* (-2.671)
10	-0.0635 (-0.153)	0.3905* (6.983)	-0.1080 (-0.935)	-0.2210* (-2.722)
11	0.2661 (0.455)	0.3238* (4.119)	-0.1883 (-1.159)	-0.7386* (-6.474)
12	-0.4240 (-0.822)	0.3773* (5.445)	-0.1272 (-0.888)	-0.1708* (-1.698)
13	0.2083 (0.465)	0.4122* (6.842)	-0.0371 (-0.298)	-0.0581 (-0.664)
14	0.0854 (0.157)	0.4916* (6.722)	-0.3272* (-2.161)	-0.6555* (-6.175)
15	0.1215 (0.283)	0.6586* (11.427)	-0.2165* (-1.818)	-0.2780* (-3.323)
16	-0.4002 (-0.952)	0.7920* (14.013)	-0.6084* (-5.211)	-0.2835* (-3.456)
17	-0.5996 (-1.392)	0.7395* (12.768)	-0.6800* (-5.684)	-0.4439* (-5.281)
F / Wald	0.607 [0.882]	413.39 [0.000]	2387.06 [0.000]	193.612 [0.000]

** indicates significance at 10 percent level of significance*

Note: the t-statistics of the parameter estimates are reported in the parenthesis and the p-values of the Wald and F tests are given in the square bracket. The test for intercept is the F test proposed by Gibbons, Ross and Shanken (1989) which is robust to non-normality in small samples. The test used for the remaining coefficients are the Wald tests.

The coefficients associated with the SMB factor indicates that the returns are nonlinear with respect to this factor as the associated coefficients are positive for the small size portfolios while they are negative for the big size portfolios. This indicates a small firm premium effect. The HML factor is negatively related to portfolio returns in all cases and their association is significant in a majority of the size portfolios.

The bottom row of Table 5 presents multivariate tests. The joint significance of the intercept is tested through an F-test as developed by Gibbons, Ross and Shanken (1989). The test indicates an overall adequacy of the Fama French factor model and no indication of any abnormal returns. The joint significance of the coefficients of the size and book-to-market factors is tested by a Wald test similar to that in Tables 3. Jointly the coefficients of the three Fama French factors are significantly different from zero across portfolios.

Thus the multivariate tests appear to indicate a stronger joint relationship between portfolio return and the three factors. This is in contrast to the unrestricted coskewness–cokurtosis model (Table 3) where one of the factors namely the coskewness is found significant in the multivariate tests.

D. Non-nested test of alternative APT factors models

Here we discuss the results of the non-nested test described in section 3 for the two competing factor models for the size portfolios[11]. The estimated coefficients α_i and the corresponding t-statistics are reported in Table 6. The non-nested test corroborate the findings from the mispricing analysis in that for smaller size portfolios and a few larger portfolios the Fama-French model could not be rejected. The higher co-moment factor model is supported in 5 medium size portfolios. Overall the number of portfolios for which the Fama-French factor model is superior is twice the number for the higher co-moment model. Two portfolios do not favour either of the two models. The multivariate test results reported in the bottom row of Table 6 clearly supports the Fama-French alternative.

E. Goodness of fit measures of alternative models

Table 7 presents some goodness of fit measures of the two competing factor models. Once again the Fama-French alternative performs slightly better in explaining the variation in portfolio returns in all three types of portfolios. The average coefficient of determination and the composite correlation coefficient are higher by about 5 per cent for the Fama-French model compared to the higher co-moment alternative.

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[11] The other two sets of portfolios results in generally similar conclusion.

Table 6: Non-Nested tests of higher moment vs Fama-French models for size portfolios

Portfolio	Coefficient α_i	t-test for $H_0 : \alpha_i = 0$	t-test for $H_0 : \alpha_i = 1$	Favor HM?	Favor FF?
1 (smallest)	0.2662	1.374	-3.787*	No	Yes
2	0.0268	0.225	-8.158*	No	Yes
3	0.0101	0.074	-7.271*	No	Yes
4	0.0670	0.406	-5.650*	No	Yes
5	0.2395	1.517	-4.819*	No	Yes
6	0.2126	1.732*	-6.415*	No	Yes
7	0.6655	2.527*	-1.270	Yes	No
8	0.2784	1.129	-2.927*	No	Yes
9	0.7551	4.536*	-1.471	Yes	No
10	0.5993	2.408*	-1.609	Yes	No
11	0.2645	1.922*	-5.344*	No	Yes
12	0.8460	3.347*	-0.609	Yes	No
13	0.9745	2.683*	-0.070	Yes	No
14	0.3179	2.315*	-4.967*	No	No
15	0.5958	3.100*	-2.103*	No	No
16	0.2268	1.427	-4.863*	No	Yes
17 (largest)	0.1217	0.898	-6.482*	No	Yes

Multivariate Test: LRT = 122.624, P-value = 0.000

$$H_0 : \alpha_1 = \alpha_2 = ...\alpha_N = 1$$

*: Significantly t-statistic value at 10 % level

Table 7: Goodness of fit measures of the alternative system of the higher co-moment and Fama-French models

	Higher Moment	Fama-French
Panel A: Size Portfolio		
Average $\overline{R}^2$	0.3486	0.4072
System R^2	0.3831	0.4498
Panel B: Beta Portfolio		
Average $\overline{R}^2$	0.3155	0.3750
System R^2	0.3896	0.4407
Panel C: Industry Portfolio		
Average $\overline{R}^2$	0.3362	0.3750
System R^2	0.3957	0.4321

6. Conclusion

This paper extends the multivariate test of BAGU for arbitrage pricing with coskewness to incorporate the cokurtosis in the return generating process and provides empirical evidence for an emerging market. The empirical results support the three factor arbitrage pricing restrictions with the common factor representing the systematic risk, the coskewness and the cokurtosis. Comparing the relative importance of the higher comments it is shown that in a system of the cubic market model equation either unrestricted or carrying the arbitrage pricing restrictions the cokurtosis remains to be an important explanatory factor while the impact of coskewness is almost negligible.

In the literature Fama French factors are advocated as another possible set of APT factors. The comparative studies of the alternative APT factors are extremely rare for emerging markets. This paper compares the relative merit of the Fama French and the higher co-moment model employing Pakistan's stock market data as a case study. We provide the empirical evidence from a non-nested test and compare the pricing error resulting from the two competing models. Some goodness of fit measures for individual portfolio and the joint multivariate tests are also performed. The empirical analysis prefer the Fama French factors to the higher co-moment factors although the explanatory power of the later model is only slightly less than the former model. This conclusion is apparently contradictory to the US study of Chung, Johnson and Schill (2006) where the Fama French factors were no longer significant once the first ten co-moments were employed. In this study to keep consistency in the number of factors we have employed only first three co-moments. The results however generally points to the fact that rules of the game of the risk return analysis may be different in emerging markets.

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