Fiscal policy rules in practice

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Abstract

This paper analyzes German and Spanish fiscal policy using simple policy rules. We choose Germany and Spain, as both are Member States in the European Monetary Union (EMU) and underwent considerable increases in public debt in the early 1990s. We focus on the question, how fiscal policy behaves under rising public debt ratios. It is found that both Germany and Spain generally exhibit a positive relationship between government revenues and debt. Using Markov-switching techniques, we show that both countries underwent a change in policy behavior in the light of rising debt/output ratios at the end of the 1990s. Interestingly, this change in policy behavior differs in its characteristics across the two countries and seems to be non-permanent in the case of Germany.

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1 Introduction

1.1 Motivation

The econometric evaluation of monetary policy with the help of simple policy rules has been subject of extensive research in recent years. This research has shown that monetary policy under discretion is suboptimal compared to a rule-based policy behavior. As a consequence of this strand of research monetary policy has substantially changed over the last three decades. Interest rate decisions of central banks have generally become more explicit and systematic.

In opposition to monetary policy rules, fiscal policy rules have received much less scrutiny in economics. Nonetheless, the design and performance of different fiscal policy rules remains an important element of macroeconomic policy analysis for a variety of reasons. One particular reason is that recent literature has discovered a link between fiscal policy and the price level. The relevance of this link between fiscal policy and prices depends crucially on the design of the policy rule.

This paper analyzes German and Spanish fiscal policy. Thereby, the principal objective of this paper is to investigate fiscal policy empirically in these two countries using simple policy rules. We choose Germany and Spain, as both are Member States in the European Monetary Union (EMU) and underwent considerable increases in public debt outstanding, particularly in the early 1990s. While other studies such as Taylor (2000) focus on the role of automatic stabilizers in fiscal policy behavior, we want to highlight the link between public debt and fiscal instruments. In particular, we want to answer the question, how fiscal policy behaves under rising public debt ratios.

It is found that both Germany and Spain generally exhibit a positive relationship between government revenues and debt. Using Markov-switching techniques, we show that both countries underwent a change in policy behavior in the light of rising debt/output ratios at the end of the 1990s. Interestingly, this change in policy behavior differs in its characteristics across the two countries and seems to be non-permanent in the case of Germany.

1.2 Literature Review

In 1993 John B. Taylor proposed a simple monetary policy rule linking the instrument of the central bank, i.e. interest rates, positively to inflation and output deviations. Since then this so-called Taylor rule has attracted a lot of attention. One reason for the popularity of Taylor rules is obviously their simple form and their potential to differentiate between discretionary and rule-based policy behavior easily. In this sense Taylor rules may serve as a benchmark for monetary policy evaluation. Unfortunately, they do not allow for any statements in terms of optimality, as they are ad-hoc and not derived from any welfare-theoretic considerations. What is also often criticized is the fact that a central bank with dozens of well-trained economists is unlikely to follow a simple decision rule such as proposed by Taylor (1993). Actually, central banks have developed complex decision processes based on numerous variables. They make considerable effort in collecting information directly from a large number of businesses and organizations[1]. A simple mechanical concept like the Taylor rule is hardly compatible with such a decision process. Already Taylor (1993) mentioned that policymakers do not follow policy rules mechanically. Central banks need more than a simple policy rule to conduct policy. Particularly it requires judgment to deal with special scenarios, which are not captured in a mechanical formula like the Taylor rule. But in opposite to pure discretion, the settings for the instruments are not determined from scratch each period. In this sense, policy rules are not more but also not less than a tool in identifying the basics behind policy actions, as it is neither desirable nor likely that a central bank starts from scratch each period.

The reason, why we start this literature review with monetary policy issues, is because policy rules have so far found less application in fiscal policy analysis. Nonetheless, they offer a way to think about fiscal policy systematically. Numerous papers deal with the question of cyclical properties of fiscal policy and its ability to stabilize the economy using simple policy rule specifications. Examples would be Gali and Perotti (2003), who assess the cyclical properties of fiscal policy in EMU before and after the introduction of the Maastricht Treaty, Taylor (2000), who investigates the reaction of automatic stabilizers in the United States, and Fatas and Mihov (2001), who analyze the relationship between government size and business cycle volatility in OECD countries.

Another strand of literature uses fiscal rules to test for the link between prices and public debt, as induced by the fiscal theory of the price level (FTPL) and for the sustainability of fiscal policy in general. Bohn (1998) finds out that U.S. fiscal surpluses have responded positively to debt. He argues that this provides evidence that U.S. fiscal policy has been sustainable. For the EMU, Afonso (2002) demonstrates, applying a panel data approach, that the FTPL is not supported for the EU-15 countries during the period 1970-2001, as Member States tend to react with larger future surpluses to increases in government liabilities. A recent paper by Davig and Leeper (2005) analyzes regime switches in fiscal policy for the U.S. They show that there have been periods of time, when government revenues have been positively and negatively affected by changes in the debt-output ratio.

This paper follows the approach of Davig and Leeper (2005). We investigate the relationship between fiscal instruments and public debt in a Markov-switching model, as a crucial difference between the analysis of monetary and fiscal rules arises from the heterogeneity of fiscal policy. In contrast to monetary policy fiscal policy is substantially affected by political flavors. With different political responsibilities we may expect at least some change in fiscal policy behavior. For this reason, we propose that any econometric analysis of fiscal rules should allow for changes in the underlying coefficients, as these are generally unlikely to be stable over time.

The remainder of this paper is organized as follows. In section 2 we introduce a simple framework for fiscal policy analysis using policy rules. We then give a brief survey of the methodology that is applied in this paper. After a description of the data used in the analysis, we provide the reader with the results in section 2.3. Finally, we make a systematic comparison of the country-specific results and check for their plausibility in section 2.4 and 2.5. Finally, section 3 summarizes the results and concludes.

[1]For further details the interested reader is referred to Svennson (2001).

2 A Simple Framework for Analyzing Fiscal Policy

The fundamental idea of policy rules is to evaluate and recommend certain types of policy behavior. In a nutshell this means that we want to identify rules, which link the instrument of policy authorities to some exogenous variables and finally turn out to be advantageous over other rules. The question then is, what is the instrument of fiscal policy? Until now, there is no comprehensive framework to analyze fiscal policy rules empirically. Basically fiscal policy has two instruments, the tax rate and the benefit rate. The tax rate determines government revenues, the benefit rate determines government spendings. We decided to follow the approach of Davig and Leeper (2005) and use government revenues as the dependent variable for the policy rule, as we think that it best serves for our purpose in investigating the reaction of fiscal policy to rising debt/GDP ratios.

Our fiscal policy rule takes the following form

$$\tau_t = \text{constant} + \gamma_Y(S_t^F)Y_t + \gamma_G(S_t^F)G_t + \gamma_B(S_t^F)B_{t-1} + \sigma(S_t^F)\varepsilon_t, \tag{2.1}$$

where τ_t denotes the ratio between government revenues and GDP in period t, Y_t represents the output gap, G_t is the expenditure/GDP ratio and B_{t-1} stands for debt/GDP ratio in period $t-1$. We decided to use B in period $t-1$ for two reasons. On the one hand we would run into an endogeneity problem, when including B in period t, and other other hand it is extremely unlikely that fiscal policy can immediately react to a change in B due to lags in the decision process of fiscal authorities. Therefore, we think that it is plausible to include a lagged value for the debt/GDP ratio. S_t^F denotes the state of fiscal policy at time t. It emphasizes that the coefficients and the variance of the error term, ε_t, are state dependent. We allow for regime switches to occur in fiscal policy behavior for the reasons given in the last section. We assume fiscal regimes to evolve according to a Markov chain with transition matrix P^F. We allow for two different states of the parameters, which should be sufficient for the purpose of the analysis[2].

In terms of the parameters in (2.1) it requires for fiscal policy to be sustainable that $\gamma_B > 0$ and sufficiently large such that a larger stock of public debt outstanding significantly increases government revenues so that the path of government debt itself is stabilized.

To make inference about fiscal policy behavior, we estimate (2.1) using a Bayesian Markov switching model. We decided to use Bayesian techniques, as the approach delivers easily interpretable credible intervals[3], which are not subject to asymptotic theory. Particularly, it allows for an evaluation of policy behavior in terms of most likely actions, as the coefficients are regarded as being random itself.

[2]We also did the analysis with a higher number of regimes, which did not deliver any reasonable results.

[3]Further details on credible intervals and the difference to the classical confidence interval may be found in Koop (2003).

2.1 Bayesian Analysis of Markov-Switching Models

The Bayesian analysis of Markov-switching models goes back to McCulloch and Tsay (1994). They show that Bayesian estimation of Markov-switching models is kept relatively simple when using the Gibbs sampler, as it solves the problem of drawing samples from a multivariate density function by drawing successive samples from the corresponding univariate density functions. The exposition given in the following is based on Harris (1999) and Krolzig (1997).

We consider the following simple univariate model, where the parameters can take on k different states S,

$$y_t = \mu(S_t) + X_t B(S_t) + \varepsilon_t(S_t), \tag{2.2}$$

where X_t is the vector of explanatory variables and $\varepsilon_t(S_t)$ a normally distributed *i.i.d.* error term with mean zero and regime-dependent covariance matrix $\Omega(S_t)$. $\mu(S_t)$ denotes a constant and $B(S_t)$ is the vector of coefficients in state S_t. Furthermore, we define the transition probabilities for a switch from regime i to regime j as $p_{ij} = p(S_t = j|S_{t-1} = i)$. We summarize these probabilities in the transition matrix P with size $(k \times k)$.

Let λ denote the set of all unknown parameters, i.e.

$$\lambda = [\mu(1), \ldots, \mu(k), B(1), \ldots, B(k), \Omega(1), \ldots, \Omega(k), P].$$

In partitioned notation this boils down to $\lambda = [\Theta, P]$. Inference on λ depends on the posterior distribution

$$p(\lambda|Y) \propto \pi(\lambda) p(Y|\lambda), \tag{2.3}$$

where $Y' = (y_1, \ldots, y_T)$ is the vector of observations and $\pi(\lambda)$ the prior for the parameter vector. As we are in a Markov-regime switching environment, we have additional unknown parameters given by the unobservable states. Therefore, the posterior density (2.3) is obtained by the integration of the joint probability distribution with respect to the state vector S, i.e.

$$p(\lambda|Y) = \int p(\lambda, S|Y) dS. \tag{2.4}$$

The problem arising from (2.4) is that the posterior distribution of λ depends on an unknown multivariate distribution $p(\lambda, S|Y)$. The Gibbs sampler offers a solution to this problem, as it allows to draw successive samples from univariate distributions for λ and S, namely $p(S|Y, \lambda)$ and $p(\lambda|Y, S)$, instead of the multivariate distribution $p(\lambda, S|Y)$. The Gibbs sampler constructs a Markov chain on (λ, S) such that the limiting distribution of the chain is the joint distribution of $p(\lambda, S|Y)$. There are two types of Gibbs sampler, single-move and multi-move, which differ in the way the states S are generated. We apply multi-move sampling as it - according to Liu et al. (1994) - will lead to a faster convergence than single-move sampling.

The idea of multi-move Gibbs sampling is to draw all states in S at once conditional on the observations. The starting point is to make use of the structure of the underlying Markov chain, i.e.

$$p(S|Y,\lambda) = p(S_T|Y,\lambda)\prod_{t=1}^{T-1} p(S_t|S_{t+1},y_t,\lambda). \tag{2.5}$$

The probabilities $p(S_T|Y,\lambda)$ can be calculated using the filter introduced by Hamilton (1989), after having chosen initial values for $p(S_0|Y)$. As we are not able to say anything about S_t for $t < 1$, we assume that the economy was in a steady state in $t = 0$. This enables us to choose steady-state probabilities for $p(S_0|Y)$, which are easy to compute[4]. We then may generate $p(S_T|Y,\lambda)$, which allows us to compute $p(S_t|S_{t+1},y_t,\lambda)$ by

$$p(S_t|S_{t+1},y_t,\lambda) = \frac{p(S_t,S_{t+1}|y_t,\lambda)}{p(S_{t+1}|y_t,\lambda)} = \frac{p(S_{t+1}|S_t)p(S_t|y_t,\lambda)}{p(S_{t+1}|y_t,\lambda)}. \tag{2.6}$$

The regimes can now be jointly generated according to (2.5). It is then possible to draw the unknown parameters from the conditional densities

$$p(\Theta_j|S,\Theta_{-j},Y) \propto L(Y|S,\lambda)\cdot p(\Theta_j) \tag{2.7}$$

$$p(P|S,\Theta,Y) \propto p(S_q|P)\prod_{t=q+1}^{T} p(S_t|S_{t-1},P)\cdot p(P). \tag{2.8}$$

Some further details on the mathematical backgrounds of Bayesian analysis of Markov-switching models may be found in the appendix.

2.2 Data

All data used corresponds to statistics of the International Monetary Fund except for German GDP, which is taken from the Federal Statistical Office Germany. All data is denoted in nominal terms and has a quarterly frequency. For τ_t and G_t we use total government revenues and expenditures. Government debt B_t is represented by total government debt, which includes in the case of Germany both debt of federal and federal state authorities. We use the Hodrick-Prescott filter to detrend GDP data. Output deviations Y are then given by the percentage deviation of GDP from its trend component[5]. The data starts for Germany with the 1^{st} quarter 1970 and ends with the 4^{th} quarter 2003. Unfortunately, the corresponding data for Spain is only partially available before 1986 so that the analysis of Spanish fiscal policy has to rely on the period 1986-2003. As all data is not seasonally adjusted and the seasonal pattern is also not offset by the division by GDP. Therefore, we introduced a set of seasonal dummy variables to capture the seasonal pattern.

[4] The procedure is explicitly described in section 4.1.3 of the appendix.

[5] We applied the Hodrick-Prescott filter in the case of Germany separately to the period before and after its reunification to avoid a bias in the trend component.

2.3 Results

In the following we provide the estimated coefficients of the fiscal policy rule as well as the temporal distribution of the regimes. For the estimation we use a Matlab code, which takes 30,000 draws from the corresponding posterior distribution. We allowed for two regimes to occur. The prior probability density function (pdf) of the transition probabilities p_{ij} is assumed to follow a β-distribution, as it is restricted to the interval $[0, 1]$. For the prior pdf of the coefficients we decided to take a normal distribution with mean $\gamma_{0,1} = 0$ in state 1, which would depict an active policy regime as revenues become exogenous, and $\gamma_{0,2} = 1$ in state 2, which would correspond to a passive policy regime as in particular the debt/output ratio has an impact on the revenue/GDP ratio. For the two prior pdfs we choose the same variance $\Sigma_{\gamma,0} = 1$ so that they are strongly overlapping each other[6]. This prior specification implies that we initially believe that there are no fundamental changes in policy behavior.

In case that the coefficients are not significantly different across the two regimes, i.e. the credible intervals given by the 2.5% and 97.5% quantiles are strongly overlapping each other, we decided to regard the corresponding coefficient as being not state-dependent. This has the advantage that we need to estimate less coefficients, while it is easier to identify changes in the underlying regimes. We do the same with the initially state-dependent variance of the error terms. Finally, the error terms were checked for their properties and may be considered as white noise.

2.3.1 Germany

Figure 1 shows the probability for each of the two potential fiscal regimes in Germany during the period 1970-2003. We can basically see that regime 1 played no role in describing Germany's fiscal policy until the late 1990s. The estimated coefficients given in section 4.2 of the appendix show that the two regimes differ only in the influence of the debt/GDP ratio and the size of the constant. All other coefficients are not regime-dependent. The same is true for the variance of the error terms, which turned out to be almost identical across the two regimes so that we decided to keep the variance fixed. Starting with the regime-invariant coefficients, we may say that Germany's fiscal policy is countercyclical, as the revenue/GDP ratio increases with positive output deviations. This reflects the role of automatic stabilizers[7]. Furthermore, we see that higher expenditure/GDP ratios are matched by growing revenue/GDP ratios, as the credible interval for γ_G is strictly positive. More interesting insights in the fundamentals of Germany's fiscal policy are given by the regime-dependent parameters. In regime 2 we observe a much larger constant combined with a less stronger reaction of the revenue/GDP ratio to increases in the debt/output ratio. The opposite is true for regime 1. Here we find a constant which is almost symmetrically distributed around zero, while $\gamma_B(S_t^F = 2)$ is substantially larger than in regime 1. To illustrate the difference between the two regimes we reduced them to a 2-dimensional problem

[6]We also did the estimation with other prior specifications, which left the results basically unchanged.

[7]One should note again at this point that this analysis is built on total government revenues and expenditures, which included social security contributions.

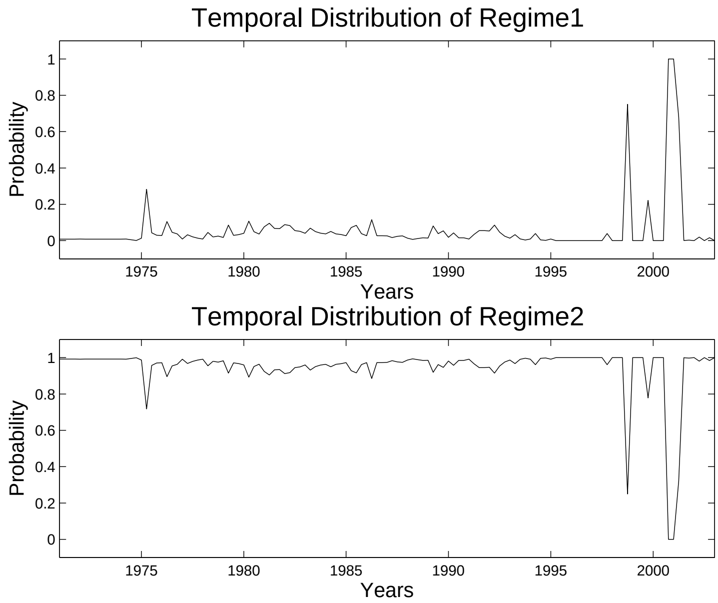


Figure 1: *Germany, Fiscal Policy Rule, Temporal Distribution of Regime Probabilities, 1970-2003.*

between revenues/GDP and debt/GDP. These may easily be plotted using the estimated median values of the coefficients for $\gamma_B(S_t^F = 1)$, $\gamma_B(S_t^F = 2)$ and the two constants. This delivers a straight line in debt/output, revenue/output space for each of the two regimes. A graphical representation is given in figure 3 and will be discussed in detail in section 2.4. In regime 1 we observe larger values for the revenue/GDP ratio than in regime 2, when the debt/GDP ratio exceeds a value of about 140 %. When looking at the data for Germany's debt/output ratio, we find values of almost 250 % toward the end of the sample[8]. From this it follows that regime 1 leads to larger revenue/output ratios in comparison to regime 2 at the end of the sample, when fiscal authorities have accumulated a substantial level of debt.

This shows that there seems to be a tendency toward a more sustainable fiscal policy in Germany with respect to debt in the late 1990s. Nonetheless, this shift in fiscal policy behavior is not permanent, as we observe fluctuations between the two regimes till the end of the sample period in figure 1. The analysis suggests so far that German fiscal policy underwent changes in its fundamentals, as the switch to regime 1 means a decrease in autonomous government expenditures, as depicted by the constant, combined with a more reactive behavior to increases in government debt.

[8]One should note that we divide total debt by quarterly GDP. Therefore, the debt/output ratio on an annual basis would be four times smaller.

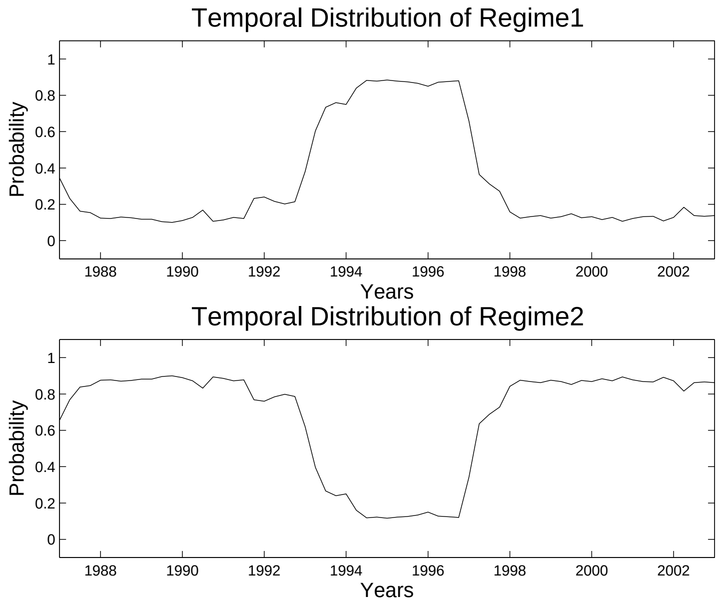


Figure 2: *Spain, Fiscal Policy Rule, Temporal Distribution of Regime Probabilities, 1986-2003.*

2.3.2 Spain

Figure 2 shows the temporal distribution of the two fiscal regimes for Spain. As indicated by the regime probabilities, we see a one-time shift in the fiscal regime during the mid-1990s. The difference between regime 1 and 2 is founded by the size of the constant. All other coefficients as well as the variance of the error term turned out not to differ across the two regimes. Regime 1 is characterized by a constant, which is almost symmetrically distributed around zero, while the constant in regime 2 takes on a value of about 1.9 with a strictly positive credible interval. Hence, regime 2 leads to larger revenue/GDP ratios given everything else. As in the case of Germany we find a countercyclical behavior of fiscal policy, as indicated by the estimated γ_Y, combined with a positive reaction of the revenue/GDP ratio to increases in the expenditure/GDP ratio. The debt/output ratio has also a positive impact on the revenue/output ratio. The estimated coefficient takes a value of about 0.02 and lies almost exactly between Germany's regime 1 and 2.

Also the Spanish results suggest that there has been a shift toward a more sustainable fiscal policy toward the end-1990s. In contrast to Germany this shift seems to be rather permanent, as depicted in figure 2. The estimated variance of the error terms is considerably larger than in the case of Germany. This means that in average our policy rule specification (2.1) fits Spanish data worse than German data. One could interpret this as greater uncertainty in Spain's fiscal policy. When bringing the regime switch in 1992 together with the data, we can see that it occurred simultaneously to the rise in the debt/output ratio. That means that the accumulation of government debt was caused by a drop in revenues as the substantially smaller constant in regime 1 suggests. With the regime switch in 1997 toward regime 2 the debt/output ratio starts

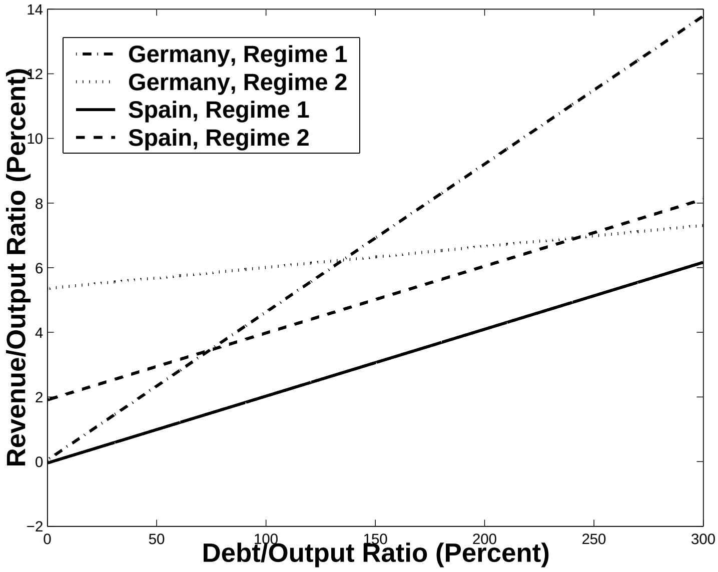


Figure 3: *Empirical Relationship between Debt/Output and Revenue/Output Ratios (excluding Government Expenditures and Output Deviations).*

falling again. This means that Spain did not undergo a fundamental change in policy behavior like Germany, instead it increased its autonomous government revenue to return to a sustainable debt/output path.

2.4 Comparison: Germany vs. Spain

Given the previous results we are now able to make a systematic comparison of fiscal behavior in Germany and Spain. From the estimated coefficients it becomes clear that Spanish fiscal policy exhibits a stronger countercyclical pattern as indicated by γ_Y, whose median value is twice as large in Spain than in Germany. Furthermore, we see that revenue/output ratios react much stronger to increasing expenditure/output ratios in Spain than in Germany. Of greater interest for our analysis is the relationship between public debt and revenues. This relationship can easily be analyzed under the exclusion of government expenditures in 2-dimensional space. Figure 3 plots revenue/output vs. debt/output ratios for each of the two regimes in the two countries. We assigned the constant to the straight lines, as we think that it fundamentally determines the way a government reacts to rising debt/output ratios for the reasons given above. We can see that Germany's fiscal regime 1 leads to substantially higher revenue/output ratios in the presence of high debt/output ratios than regime 2. Further, we can see that Spain's regime 1 leads to the lowest revenue/output ratios in comparison with all other regimes in the two countries. As Spain's debt/output ratio has never fallen below 130 % during the sample period, the lower

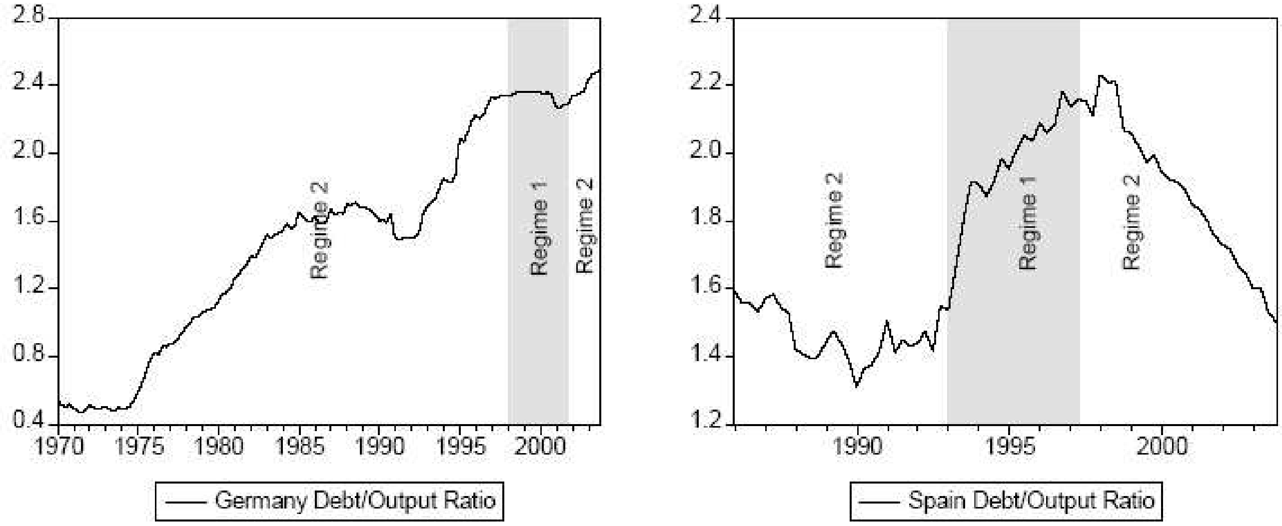


Figure 4: *Germany and Spain, Debt/GDP Ratios related to Regime Classification.*

interval for debt/output ratios in figure 3 is not relevant for the practical policy analysis. This means then that Spain's fiscal policy runs higher revenue/output ratios than Germany only for extremely large debt/output ratios of more than 250 % given that Germany is in regime 2 at the same time. Finally, Germany's regime 1 leads to the largest revenue/output ratios in the relevant interval of debt/output ratios.

When focusing on the debt/output ratio and excluding the other variables that determine the revenue/GDP ratio, we observe a shift toward a more sustainable fiscal policy in both countries during the end-1990s. This change in fiscal policy behavior has been more distinct in Germany than in Spain, as Germany's fiscal policy underwent a change in both autonomous government revenues and its reaction to the debt/GDP ratio. But in contrast to Spain Germany's fiscal policy change has not been permanent. When we solely focus on the debt/GDP ratio, we neglect the fact that Spain exhibits a much stronger reaction of revenues to changes in expenditures than Germany. Furthermore, when looking at the process of debt/output ratios in the two countries, as given in figure 4 , we can basically see that Germany, whose fiscal policy we generally interpreted as being more sustainable in terms of debt than Spain's, underwent considerable increases in the debt/output ratio at least till the late 1990s. At the same time Spain shows a substantial decrease in its debt/GDP ratio. We interpret this fact not as a lack of sustainability with respect to debt but as a lack of sustainability with respect to expenditures.

2.5 Plausibility of the Results

Figure 4 relates the run of debt/GDP ratios in the two countries to the underlying fiscal policy regimes[9]. The graphical representation shows that the regime changes also translate into the process of debt/GDP ratios, which is absolutely reasonable, as a weak or insignificant reaction of the revenue/GDP ratio to increases in the expenditure or the lagged debt/GDP ratio would lead to an increase of government debt in the current period and vice versa. We see a stabilization of public debt in relation to GDP in Germany, during the more sustainable periods of fiscal policy, while the debt/GDP ratio even starts shrinking in the case of Spain.

Besides this rather intuitive and informal plausibility check of our estimates, one can find formal evidence for the robustness and plausibility of our results. Thams (2007) analyzes the relevance of the FTPL for Germany and Spain using data for the period 1970-1998. The analysis provides evidence for non-Ricardian fiscal behavior in Spain, while the opposite is true for Germany. These results coincide with those given here. In Spain we find a very little reaction of revenues to changes in public debt during that period. Instead the path of public debt is stabilized by a one-time shift in autonomous government revenues. Although Germany shows an almost continuous increase of public debt over the sample period, revenues have been frequently adjusted to match the rising level of debt. This would militate in favor of a Ricardian fiscal policy.

[9]The choice for one regime or the other is determined by the corresponding regime probabilities. We say that fiscal policy is in regime 1 at time t, if the probability for regime 1 at time t is larger than 0.5. In the case of Germany we summarized the period of multiple regime changes at the end of the 1990s to a single regime change.

3 Conclusions

The analysis has uncovered changes in fiscal policy behavior using simple policy rules. In Spain we find a much stronger response of revenues to changes in government expenditures, while the relationship between public debt and revenues is weaker than for Germany's fiscal policy. Generally, Spain's fiscal policy deviates in average more strongly from the policy rule specification than Germany. This may be interpreted as evidence for higher uncertainty in Spain's fiscal policy.

In both countries we find evidence for a change toward a more sustainable fiscal policy at the end of the 1990s. This change does only turn out to be permanent in the case of Spain. Nonetheless, the difference between the two regimes seems to be more drastic in Germany than in Spain, as we find a drop in the autonomous government revenues combined with an even stronger response to changes in public debt. Spain exhibits a one-time shift in autonomous government revenues leaving the general relationship between revenues and debt unchanged.

Hence, we may say that both Germany and Spain underwent a switch in fiscal behavior in the light of rising debt/output ratios. Interestingly, this change in fiscal behavior exhibits different characteristics. While Germany tries to stabilize the debt/output ratio by a more fundamental change in fiscal behavior, Spain embarks on a different strategy by rising the overall revenue/output ratio. This result is remarkable, as both countries are subject to the same restrictions imposed by the Stability and Growth Pact. The analysis shows that the way fiscal authorities deal with these restrictions may well differ across countries.

4 Appendix

4.1 Bayesian Analysis of Markov-Regime Switching Models

We consider the more general case of a VAR[10] of order p given by

$$x_t = \mu(S_t) + \sum_{h=1}^{p} B^h_{S(t)} x_{t-h} + \varepsilon_t(S_t), \tag{4.1}$$

where $\varepsilon_t(S_t)$ is an normally distributed *i.i.d.* error term with mean zero and regime-dependent covariance matrix $\Omega(S_t)$. $\mu(S_t)$ denotes a constant and $B^h_{S(t)}$ is the matrix of coefficients for the h^{th} lag included. As indicated by S_t we assume that both the parameters included in B as well as the covariance Ω can adopt k different states. In any period parameters and covariance matrix may switch to a new state with a probability $p_{ij} \geq 0$. We define the transition probabilities for a switch from regime i to regime j as $p_{ij} = p(S_t = j|S_{t-1} = i)$. We then summarize these probabilities in the transition matrix P with size $(k \times k)$.

The aim is then to estimate the set of unknown parameters given by

$$\lambda \equiv \{\mu_1, \ldots, \mu_k, B_1, \ldots B_k, \Omega_1, \ldots, \Omega_k, P\}.$$

In partitioned notation this expression reduces to $\lambda \equiv \{\Theta, P\}$. Furthermore, it is convenient to rewrite (4.1) in stacked form as a VAR(1) model, i.e.

$$X_t = \bar{\mu}(S_t) + B(S_t) X_{t-1} + \bar{\varepsilon}_t(S_t), \tag{4.2}$$

where

$$X_t = \begin{bmatrix} x_t \\ x_{t-1} \\ \vdots \\ x_{t-p+1} \end{bmatrix}, \bar{\mu}(S_t) = \begin{bmatrix} \mu(S_t) \\ 0 \\ \vdots \\ 0 \end{bmatrix}, B(S_t) = \begin{bmatrix} B^1_{S(t)} & B^2_{S(t)} & \ldots & B^p_{S(t)} \\ I_m & 0 & \ldots & 0 \\ 0 & I_m & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix}, \bar{\varepsilon}(S_t) = \begin{bmatrix} \varepsilon_t(S_t) \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

Furthermore, let $X = (x_1, \ldots, x_T)$ be the vector of all observations.

4.1.1 The Likelihood Function

The contribution of the t^{th} vector of observations x_t to the likelihood conditional on the regime S_t is given by

[10]The procedure may be directly applied to the univariate case.

$$l(x_t|S_t, X_{-1}, \lambda) = (2\pi)^{-m/2}|\Omega^{-1}(S_t)|^{1/2} \cdot exp\left\{-\tfrac{1}{2}\varepsilon_t'(S_t)\Omega^{-1}(S_t)\varepsilon_t(S_t)\right\} \quad \text{with } \varepsilon_t(S_t) = x_t - \mu(S_t) - \textstyle\sum_{h=1}^{p} B_{S(t)}^h x_{t-h}. \tag{4.3}$$

As before, m denotes the number of variables in x_t and $X_{-1} = \{x_1, \ldots, x_{t-1}\}$, i.e. all observations up to period $t-1$. Next, we exploit the recursiveness of (4.2) for the first p observations by substituting for X_{t-1}. This yields

$$\begin{aligned} X_p &= \bar{\mu} + BX_{p-1} + \bar{\varepsilon}_p \\ &= \bar{\mu} + B(\bar{\mu} + BX_{p-2} + \bar{\varepsilon}_{p-1}) + \bar{\varepsilon}_p \\ &= \bar{\mu} + B\bar{\mu} + B\bar{\varepsilon}_{t-1} + \bar{\varepsilon}_t + B^2X_{p-2} \\ &= \ldots \\ &= \sum_{\tau=0}^{\infty} B^\tau \bar{\mu} + \sum_{\tau=0}^{\infty} B^\tau \bar{\varepsilon}_{t-\tau} \end{aligned} \tag{4.4}$$

under the assumption that there is no regime shift prior to p. This enables us to write the unconditional mean of X_p as

$$E[X_p] = \sum_{\tau=0}^{\infty} B^\tau \bar{\mu}.$$

For the existence of $E[X_p]$ it requires that all eigenvalues of B have absolute values less than one. For the variance of X_p it follows

$$\begin{aligned} Var[X_p] &= E(X_p - E(X_p))(X_p - E(X_p))' \\ &= E\left(\sum_{\tau=0}^{\infty} B^\tau \bar{\varepsilon}_{t-\tau}\right)\left(\sum_{\tau=0}^{\infty} B^\tau \bar{\varepsilon}_{t-\tau}\right)' \\ &= E\left(\sum_{\tau=0}^{\infty} B^\tau \bar{\varepsilon}_{t-\tau}\bar{\varepsilon}_{t-\tau}'(B^\tau)'\right) \\ &= E\left(\sum_{\tau=0}^{\infty} B^\tau \bar{\Omega}(B^\tau)'\right) \\ &= V(\Omega, B). \end{aligned} \tag{4.5}$$

We are now able to approximate $l(X_p|S_p, \lambda)$, which is the contribution of the first p data vectors to the likelihood, by

$$l(X_p|S_p, \lambda) = (2\pi)^{-(mp)/2}|V(\Omega, B)^{-1}(S_p)|^{1/2} \cdot exp\left\{-\frac{1}{2}X_p'(S_t)V(\Omega, B)^{-1}(S_p)X_p\right\}. \tag{4.6}$$

For the full likelihood conditional on the regimes we obtain, using (4.3) and (4.6)

$$L(X|S_T,\lambda) = l(X_p|S_p,\lambda)\prod_{t=p+1}^{T} l(x_t|S_t,X_{-1},\lambda). \tag{4.7}$$

Integrating over all possible states gives the unconditional likelihood

$$L(X|\lambda) = l(X_p|\lambda)\prod_{t=p+1}^{T} l(x_t|X_{-1},\lambda). \tag{4.8}$$

4.1.2 Generating the Regimes S using Gibbs-Sampling

We generate the regimes S with the help of multi-move Gibbs sampling. The idea is to obtain the T elements in S within one draw conditional on λ and the observed data X. The starting point is to make use of the structure of the underlying Markov chain. The density of the regimes $p(S|X,\lambda)$ can easily be rearranged in a multiplicative relationship as

$$\begin{aligned}
p(S|X,\lambda) &= p(S_1,\ldots,S_T|X,\lambda) \\
&= p(S_T|X,\lambda)p(S_{T-1},\ldots,S_1|S_T,X_{-1},\lambda) \\
&= p(S_T|X,\lambda)p(S_{T-1}|S_T,X_{-1},\lambda)p(S_{T-2},\ldots,S_1|S_{T-1},X_{-2},\lambda) \\
&= p(S_T|X,\lambda)\prod_{t=1}^{T-1} p(S_t|S_{t+1},x_t,\lambda). \qquad (4.9)
\end{aligned}$$

Knowing $p(S_T|X,\lambda)$ and $p(S_t|S_{t+1},x_t,\lambda)$ we could first draw S_T. Conditional on S_T it would then possible to obtain S_{T-1}, and again conditional on S_{T-1} we could draw S_{T-2} etc.

With some algebra one can show that

$$p(S_t=i|S_{t+1}=j,x_t,\lambda) = \frac{p_{ij}p(S_t=i|x_t,\lambda)}{\sum_{z=1}^{k} p_{zj}p(S_t=z|x_t,\lambda)}. \tag{4.10}$$

That means that given the matrix of transition probabilities P, it only requires $P(S_t|x_t,\lambda)$ to compute $p(S_t|S_{t+1},x_t,\lambda)$. $p(S_t|x_t,\lambda)$ can in turn be determined using the filter proposed by Hamilton (1989). This procedure demands initial values for S_0. We will briefly outline in the following how these may reasonably be chosen.

4.1.3 Deriving the Initial Probabilities

Using the filter of Hamilton (1989) to compute $p(S_T|X,\lambda)$ requires initial values for $p(S_0|X)$. By assuming that the economy was in a steady state in $t=0$, we may use steady-state probabilities for $p(S_0|X)$. The general condition for a steady-state probability is given by

$$P\cdot p(S_0|X) = p(S_0|X), \tag{4.11}$$

where P denotes the matrix of transition probabilities. This equation can be rearranged to

$$(I - P)p(S_0|X) = 0, \tag{4.12}$$

with I being a $(k \times k)$ identity matrix. We know that by construction the k probabilities in the vector of $p(S_0|X)$ add up to one. Thus, with $\iota = (1, \ldots, 1)'$ we may express this fact in vector notation as

$$\iota p(S_0|X) = 1. \tag{4.13}$$

In matrix notation (4.12) and (4.13) can be rewritten as

$$\underbrace{\begin{bmatrix} I - P \\ \iota \end{bmatrix}}_{\equiv M} p(S_0|X) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{4.14}$$

We premultiply this expression by $(M'M)^{-1}M'$ and obtain for the initial probabilities

$$p(S_0|X) = (M'M)^{-1}M' \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{4.15}$$

4.1.4 Generating the Parameters

After having generated S, we are now able to formulate the conditional density of the parameters, which is generally given by

$$p(\lambda_j|S, \lambda_{-j}, X) \propto L(X|S, \lambda) \cdot p(S|\lambda) \cdot p(\lambda_j), \tag{4.16}$$

where λ_{-j} denotes the set of all parameters except for λ_j.

4.2 Numerical Results of the Regime-Switching Approach

Germany

- Fiscal Policy Rule: 1970-2003, $(-)$ indicates that the coefficient is not state-dependent.

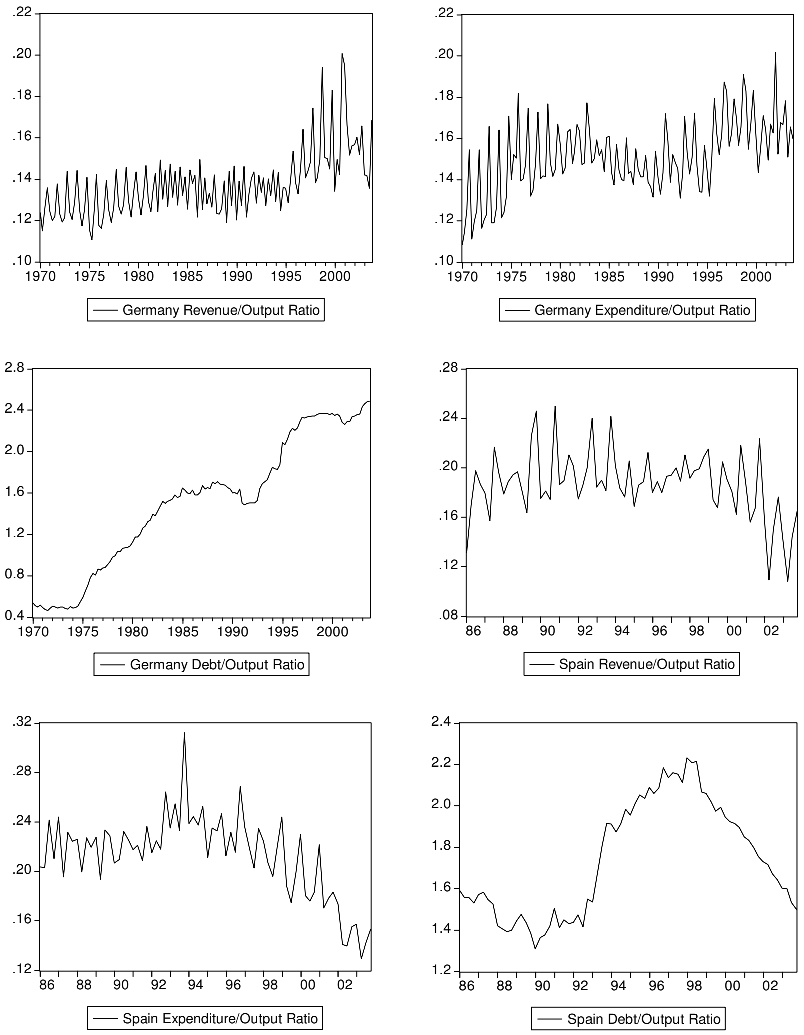
	2.5%	Median	97.5%
Constant($S_t^F = 1$)	-1.2090	0.0467	1.3407
Constant($S_t^F = 2$)	4.3338	5.3577	6.3301
$\gamma_Y(-)$	0.0146	0.1014	0.1871
$\gamma_G(-)$	0.4292	0.4976	0.5696
$\gamma_B(S_t^F = 1)$	0.0372	0.0458	0.0542
$\gamma_B(S_t^F = 2)$	0.0043	0.0065	0.0085
$\sigma^2(-)$	0.5564	0.6664	0.7953
p_{11}	0.2491	0.4955	0.7707
p_{12}	0.2293	0.5045	0.7509
p_{21}	0.0113	0.0345	0.0804
p_{22}	0.9196	0.9655	0.9887

Spain

- Fiscal Policy Rule: 1986-2003, $(-)$ indicates that the coefficient is not state-dependent.

	2.5%	Median	97.5%
Constant($S_t^F = 1$)	-1.0973	-0.0439	1.1760
Constant($S_t^F = 2$)	0.8891	1.9087	2.8654
$\gamma_Y(-)$	0.0018	0.2111	0.3969
$\gamma_G(-)$	0.6217	0.6959	0.7642
$\gamma_B(-)$	0.0121	0.0207	0.0291
$\sigma^2(-)$	1.8446	2.3238	3.0440
p_{11}	0.7221	0.8716	0.9588
p_{12}	0.0412	0.1284	0.2779
p_{21}	0.0139	0.0526	0.1475
p_{22}	0.8525	0.9474	0.9861

4.3 The Data


.22
.20
.18
.16
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1970 1975 1980 1985 1990 1995 2000
Germany Revenue/Output Ratio
.22
.20
.18
.16
.14
.12
.10
1970 1975 1980 1985 1990 1995 2000
Germany Expenditure/Output Ratio
2.8
2.4
2.0
1.6
1.2
0.8
0.4
1970 1975 1980 1985 1990 1995 2000
Germany Debt/Output Ratio
.28
.24
.20
.16
.12
.08
86 88 90 92 94 96 98 00 02
Spain Revenue/Output Ratio
.32
.28
.24
.20
.16
.12
86 88 90 92 94 96 98 00 02
Spain Expenditure/Output Ratio
2.4
2.2
2.0
1.8
1.6
1.4
1.2
86 88 90 92 94 96 98 00 02
Spain Debt/Output Ratio

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