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The Peak of Oil Extraction and Consistency of the Government's Short- and Long-run Policies *

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Abstract The term "oil peak" usually is connected with the positive analysis problem, namely, with the problem of defining the year when the increase in the rate of oil extraction will be physically impossible. However, a normative approach to the problem of optimal extraction of a nonrenewable resource seems more important. We consider the economy which depends on the essential nonrenewable resource and the rate of the resource extraction increases over time. At some instant the government gradually switches to a sustainable (in sense of nondecreasing consumption over time) pattern of the resource extraction. Different approaches are offered for the construction of the paths of switching to decreasing resource use. Some seemingly attractive short-run policies of switching to decreasing extraction can run counter to long-run criteria. Reformulation of the short-run criterion can imply the optimal transition path consistent with the long-run government goals. It is shown analytically and numerically that there are values of parameters for the transition paths of extraction that consumption along these paths is asymptotically constant or infinitely growing. Numerical examples show for different reserve estimates that the "sustainable" peak of oil extraction must be earlier than the expected "physical" peak. A new approach to the Rawlsian maximin criterion which allows for growth of consumption is offered.

Keywords Nonrenewable resource · Intergenerational justice · Generalized Rawlsian criterion

JEL Classification Numbers Q32 · Q38

1 Introduction

The anxiety about limiting world resources has persisted since the famous work of Thomas Malthus, published in 1798. Many observers are trying to estimate the time of the peak in world's production of the nonrenewable resources (see e.g., works of D. Meadows et al. and theories based on the Hubbert's peak of oil production). Economic decline is assumed to follow. The report of Cambridge Energy Research Associates (CERA)[1] (November, 2006) contains a very optimistic evaluation of the world oil reserves (3.74 trillion barrels) in comparison with the estimates of "peak oil" theorist (1.29 trillion: Oil & Gas J., 2005, 103, 47: p. 25). CERA's scenario of oil extraction is also very encouraging, since it promises that the rates of extraction will grow for at least another 24 years before entering the "undulating plateau" followed by decline. In contrast, the "peak oil" forecasts predict that the world oil production has already peaked or will have a peak in the next 5-10 years.

But when we worry about the peak and the associated scarcity of the resource we think mostly about the influence of this impending shortage on the output of our economy and on our consumption and that of our descendents. And it is not obvious that we must adjust our demand for the nonrenewable resource strictly in accord with the "physical" peak of extraction. In other words a "sustainable"[2] peak may not coincide with the physical one.

For the modeling various scenarios of the world oil extraction we will use the transition paths, developed in (Bazhanov, 2006) which have been constructed for an economy with the growing rates of extraction with a switch to a hypothetical sustainable path. We assume that a rapid decrease in oil extraction can be extremely costly in terms of consumption foregone and

[1] See http://www.cera.com/aspx/cda/public1/home/home.aspx

[2] We will consider the simplest sustainability criterion meaning nondecreasing consumption over time.

this leads us to consider various more gradual transitions. This in turn leads us to reflect on welfare criteria for the case of significant changes in consumption levels across generations.

We consider the Hartwick saving rule (Hartwick, 1977) for the Solow (1974) model which implies that the economy must involve investing current exhaustible resource returns in reproducible capital in order to maintain constant per capita consumption over time. We review this for the case of a Cobb-Douglas technology with no capital depreciation, no technological progress, and zero extraction cost with production function

$$Q(t) = K^{\alpha}(t)R^{\beta}(t)L^{\gamma}(t) \tag{1}$$

where α, $\beta, \gamma \in (0, 1)$ are constants, $Q-$ output, $K-$ produced capital, R - current resource use, $L-$ labor (population). For simplicity we consider the case with zero population growth ($L(t) = L = const$) and $\gamma = 1 - \alpha$. Then by dividing equation (1) through by L we obtain $q = f(k, R) = k^{\alpha}R^{\beta}$ where lower-case variables are in per capita units, $R = -\dot{S}$, S - resource stock ($\dot{S} = dS/dt$). Prices of per capita capital and the resource are $f_k = \alpha q/k$, $f_R = \beta q/R$ where $f_x = \partial f/\partial x$. Per capita consumption is $c = q - \dot{k}$. The Hartwick savings rule implies $c = q - Rf_R$ or, substituting for f_R, $c = q(1 - \beta)$, which means that instead of $\dot{c} = 0$ we can check $\dot{q} = 0$.

From Hotelling rule $\dot{f}_R/f_R = f_k$ we have $\alpha\beta q/k + R(\beta - 1)/R = f_k = \alpha q/k$ which yields

$$\dot{R}/R = -\alpha q/k. \tag{2}$$

Then

$$\dot{q}/q = \alpha\dot{k}/k + \beta\dot{R}/R = \beta(\alpha q/k + \dot{R}/R) = 0, \tag{3}$$

which means that we really have $\dot{q} = \dot{c} = 0$ or $q = const$. Then $Rf_R = \beta q = const$ and we have $\dot{k} = \beta q = const$ for deriving $k(t)$ and (2) for deriving $R(t)$. We can find two constants of integration k_0 for $k(t) = k_0 + \beta q t$ and the constant of equation $\dot{R}/R = -1/\left(k_0/\alpha q + \beta t/\alpha\right)$ using initial conditions $R(0) = R_0$ and $S(0) = S_0$, where S_0 is the given resource stock which must be used for production over infinite time: $S_0 = \int_0^\infty R(t)dt$. Then we have

$$R(t) = R_0 \left[1 + R_0 \beta t / S_0(\alpha - \beta)\right]^{-\alpha/\beta}, \tag{4}$$

where $\alpha > \beta$ (Solow condition) and

$$\dot{R}(t) = -\alpha R_0^2 / S_0(\alpha - \beta) \left[1 + R_0 \beta t / S_0(\alpha - \beta)\right]^{-(\alpha+\beta)/\beta}. \tag{5}$$

Since we assume that our economy depends on the resource essentially, we obtain path $R(t)$, asymptotically approaching zero and the path of extraction changes $\dot{R}(t)$ (or negative acceleration of stock $S(t)$ diminishing) also approaching zero, but starting from the negative value $\dot{R}_0 = -\alpha R_0^2/[S_0(\alpha - \beta)]$. Note, that path (4), asymptotically approaching zero, is necessary, but not sufficient condition of following Hartwick rule for Cobb-Douglas economy under the Hotelling rule assumption. By definition of $f(k, R)$ it can be seen, that if economy is extracting resource in accord with (4) and resource rent is consuming (total investments are less than resource rent), then $q(t)$ and $c(t)$ are asymptotically approaching zero, but from a greater starting value $c(0)$. Assuming that our economy has some "additional" savings, besides resource rent, it is possible to relax the assumption of zero population growth (as in (Stiglitz, 1974) and (Asheim et al., 2005), or zero capital depreciation. But in any case, if we assume, that

1) economy at every instant of time depends on resource (even if we gradually introduce substituting technologies and this dependence asymptotically approaches zero), and

2) we really want to maintain nondecreasing per capita consumption,

then rate of extraction $R(t)$ must tend to zero.

Capital - resource substitution is a fundamental topic in energy economics and there is an empirical evidence (Nordhaus, 1972, Pindyck, 1979) which can support the assumption that the elasticity of substitution between natural resources and capital exceeds unity. This implies that resource can be inessential. Other investigations (Fuss, 1977, Magnus, 1979, and partly in Halvorsen and Ford, 1979) show that energy and capital are rather strong complements than substitutes (elasticity is less than unity) and some researches find that this value is rather close to unity (Griffin and Gregory, 1976, Pindyck, 1979). In any case empirical evidence is not a proof and as Dasgupta and Heal (1979, p. 207) noted "Past evidence may not be a good guide for judging substitution possibilities for large values of k/R". And so, we can assume that for the world economy oil is essential, especially taking into account that no adequate immediate substitutes are available for transportation fuels, a main area of oil use (Heinberg, 2003, Nemoto, 2005). However, as we can see, e.g., from oil extraction data in December issues of Oil and Gas Journal, rates of extraction are in fact both growing on the world level (see Fig. 1 before the year 2005) and for the leading oil producers, not declining. Per capita world oil extraction (Fig. 2) is also not declining though after the period of growth it is following an undulating plateau since the oil crisis of 1979-1980.[3]

Assume that the government after a period of oil-rent consumption and growing rate of extraction decided to conform to the intergenerational justice principle and switch at t_0 to some sustainable path of saving, e.g., to the Hartwick rule.[4] An example with $\alpha = 0.3$ and $\beta = 0.05$ gives us $R(t)$ and $\dot{R}(t)$ for world oil extraction in Fig. 1 and Fig. 3 after the year

[3] We took the world population in 2006 equal to unity and as a source of information for the world population dynamics we used http://www.census.gov/ipc/www/worldpop.html .

[4] "Strong policy action is needed to move the world onto a more sustainable energy path." [The World Energy Outlook 2006 Maps Out a Cleaner, Cleverer and More Competitive Energy Future, IEA Press Release, http://www.iea.org/Textbase/press/pressdetail.asp?PRESS_REL_ID=187]

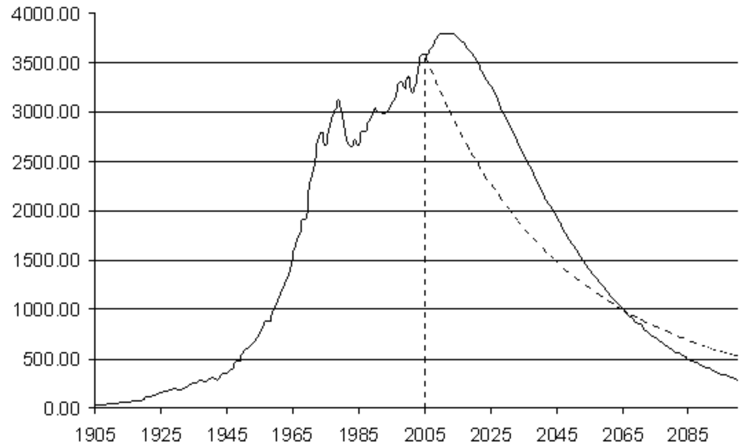

Figure 1: World oil extraction: historical data (before 2005); Hartwick curve (dotted); LA curve (solid)

2005 (dotted lines).

An abrupt switch to the Hartwick rule means that people in oil-producing countries must instantly forget about this principal source of income and in a moment substantially re-structure their living style. Moreover, countries must instantly reorganize their economies, because of the sharp decrease of consumption, which in turn leads to a decrease in production, a possible increase in unemployment, a further decrease of demand and so on. Thus, for an economy not following the Hartwick rule, the sudden invocation of intergenerational justice creates the dilemma of choosing between two awkward futures: diminishing consumption to zero in the future because of the inevitable shortage of essential exhaustible resources or diminishing consumption to a sustainable level right from the moment of switching to the Hartwick rule.

Solow's model implies that oil-rent is invested from the very beginning and that there is no time gap between the moment of oil extraction and correspondent increase of reproducible

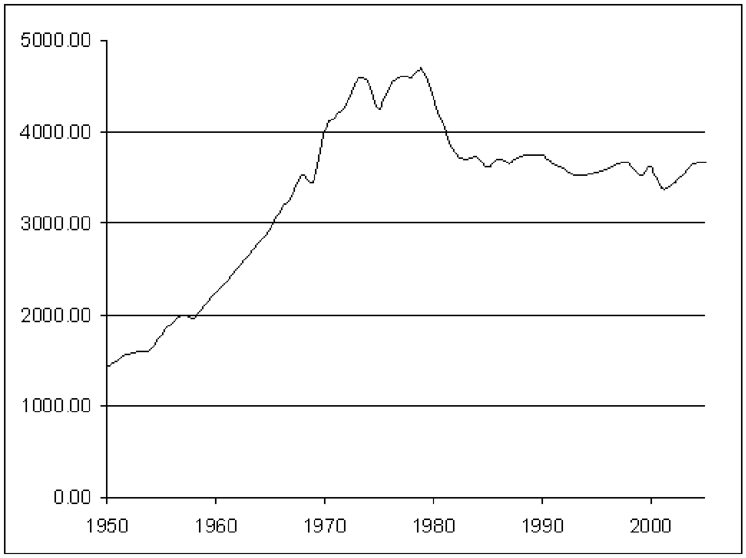

Figure 2: Per capita world oil extraction (historical data).

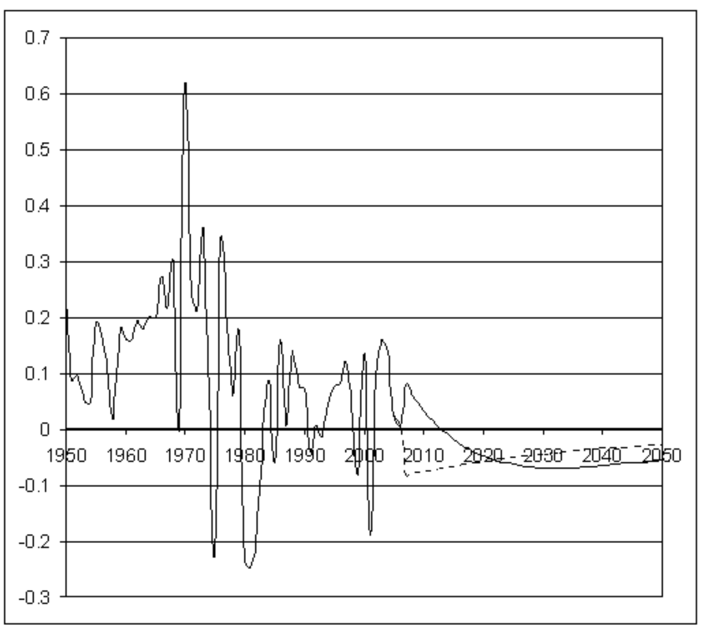

Figure 3: Per capita extraction accelerations: historical data (before 2005); Hartwick curve (dotted); LA curve (solid)

capital according to the Hartwick rule. We can consider it as an adequate model if we assume that reproducible capital is a fund of some high-return securities and oil profit can be instantly invested in some shares or bonds. But suppose that money bills are not able to substitute gasoline in engines of our cars when we have shortage of oil. And the shortage will be the inevitable result of growing demand because of economic growth and decreasing, according to (3), supply of oil. It means, that in order to sustain nondecreasing output with the same structure, we must invest at least part of oil profit into development of oil-substituting technologies. In other words, we must create an "anti-oil market" with the oil rent. And under this assumption the model of instant investment can not be really adequate because of the difficulties of a rapid re-structuring. Historical examples show that the development and the introduction of coal-based technologies took decades despite the obvious benefit of the new technologies for economy. The same can be said about the switch from a coal to an oil economy. Now we must consider the problem of switching to technologies, based on renewable resources not because they are economically more preferable but just because of anticipated shortage of profitable but exhaustible raw materials. And this process will occur over decades, not months.

The second dimension of the impossibility of an instant switch to the Hartwick rule is the awkward requirement of an abrupt and very substantial change of saving patterns for oil producing countries. As an illustration we can compare nonrenewable resource profit only from oil with the total amount of investments for a selection of countries. For example, oil gives Kuwait about 50% of GDP but gross fixed investments are only 6.6% of GDP. For Saudi Arabia these numbers are 45% and 16.3%, United Arab Emirates - 30% and 20.7%, Venezuela - 33% and 23.8%.[5] A very detailed analysis of the investment and consumption patterns in

[5] Source of information:
http://www.cia.gov/cia/publications/factbook/docs/profileguide.html (March 2006)

the world is in (Arrow et al., 2004). From leaders of oil producers only Norway can boast almost coinciding numbers (about 18.6%[6]), because of investing oil rent to Petroleum Fund, though there is no direct connection between this Fund and development of oil-substituting technologies.

However, the well-known empirical research of Kuznets (1946) tells us that consumer behavior is very persistent over time despite changes of governments and government policy. Subsequent analyses, for example, the work of Duesenberry (1949), tried to explain this phenomenon, and later papers examined why consumers do not react on "natural experiments" such as the Reagan cuts in taxes (Poterba, 1988). In any case, there is evidence that at least in the short run saving rate is very stable, and it is much more difficult to change it instantly, than to change a government policy toward maximin.

Hence, the problem of switching to sustainable path of essential resource extraction must take into account the next factors:

1) the path must have a period of a gradual slow-down in the rate of extraction;

2) there is a time lag between the moment of resource rent investment and correspondent increase in capital;

3) there is a non-zero period length for changing saving patterns from resource rent consumption to resource rent investment.

In this paper we suppose, for simplicity, that the third problem is already solved (as in Norway), and also we will temporarily neglect the influence of the second factor. So, we will concentrate on the question of the construction the trajectories for the transition period using various optimality criteria and examine consumption behavior along the paths.

[6]Source of information: http://www.ssb.no/en/indicators/ (March 2006)

2 Formulations of the problem

It is natural for the government, switching to a decreasing path of the resource extraction, to try to construct this path in such a way that the short-run negative social impacts along the path are minimal. According to (3) GDP percent change for our economy is

$$\dot{q}/q = \alpha\dot{k}/k + \beta\dot{R}/R. \tag{6}$$

Since the economy is going to enter the period with $\dot{R} < 0$, the government can try to minimize the negative influence of the decreasing extraction on output q.

We can apply a mechanical analogy, comparing the rate of extraction with the speed of some vehicle. The most unpleasant feelings in a vehicle trip occur when the vehicle has to reduce the speed very quickly. Then, using this analogy, we can consider

1) *the criterion of "smooth breaking"*, which gives the optimal path with the minimal in absolute value peak of negative acceleration $A(t) = \dot{R}(t)$. In other words we have a problem

$$\begin{aligned} F_1(R(t)) &= \min_t A(t) \to \max_{R(t)}, \\ s.t. \quad R(0) &= R_0, \\ \int_0^\infty R(t)dt &= S_0, \end{aligned} \tag{7}$$

where the last condition means that the resource is essential.

However, the negative acceleration $\dot{R}(t)$ influences the GDP percent change not directly but as a numerator of the fraction $\dot{R}(t)/R(t)$. Then it looks more relevant for economics consider

2) *the criterion of "minimum shock on GDP"*, namely

$$F_2(R(t)) = \min_t \left[\dot{R}(t)/R(t)\right] \to \max_{R(t)} \tag{8}$$

subject to the same initial conditions.

Finally, we can take into account the first term in (6) which can shift the point of maximum influence of negative acceleration, or, in other words, we can apply the maximin criterion to the whole expression $\dot{q}/q$. Hence, we will consider as the third criterion

3) *the criterion of the "maximin GDP percent change":*

$$F_3(R(t)) = \min_t \left[\alpha\dot{k}(t)/k(t) + \beta\dot{R}(t)/R(t)\right] \to \max_{R(t)} . \tag{9}$$

with the same constraints.

3 Solving the problem

The transition path can be constructed in the same class of rational functions as the Hartwick curve (4). The difference is in the numerator, which in the expression for acceleration $A = \dot{R}$ must depend on t with a negative coefficient to control "smooth breaking" in the neighborhood of $t = 0$. Namely, $A(t)$ must be in the form of

$$A(t, b, c, d) = \dot{R}(t) = (A_0 + bt)/(1 + ct)^d, \tag{10}$$

where $b < 0, c > 0, d > 1$ (for convergence $A(t) \to -0$ with $t \to \infty$). Corresponding to (10) $R(t)$ has a dependence on b, c, and d in

$$R(t) = \{-\left[A_0 + b/[c(d-2)]\right]/[c(d-1)] + bt/[c(2-d)]\}/(1+ct)^{d-1}.$$

Note, that a constant of integration for $\dot{R}(t) = A(t)$ must be zero for the convergence of $\int_0^\infty R(t)dt$, and also for the convergence, d actually must be greater than 3. Then we have $R_0 = -\left[A_0 + b/[c(d-2)]\right]/[c(d-1)]$, which can be used to express $b:$

$$b(c, d) = -c(d-2)\left[R_0 c(d-1) + A_0\right], \tag{11}$$

and then the transition curve has a dependence on c and d in

$$R(t) = R_0 \{1 + [c(d-1) + A_0/R_0]\, t\} / (1+ct)^{d-1}. \tag{12}$$

Coefficient c can be expressed from the condition that resource is finite $S_0 = \int_0^\infty R(t)dt$:

$$\begin{aligned} S_0/R_0 &= \int_0^\infty (1+ct)^{1-d}dt + [c(d-1) + A_0/R_0] \int_0^\infty t/(1+ct)^{d-1}dt \\ &= [1 + \{R_0 c(d-1) + A_0\} / \{R_0 c(d-3)\}] / [c(d-2)], \end{aligned}$$

which means that c is a solution of quadratic equation

$$c^2 S_0/R_0 - 2c/(d-3) - A_0/[R_0(d-3)(d-2)] = 0.$$

The only relevant root (because we are looking for $c > 0$) is

$$c(d) = \left[R_0/(d-3) + \left\{ R_0^2/(d-3)^2 + S_0 A_0/[(d-3)(d-2)] \right\}^{0.5} \right] / S_0. \tag{13}$$

Hence, we have a single independent parameter d which defines the shape of the curve (including its peak) and we can use this parameter as a control variable in some selected optimization problem

$$F\,[R(t,d)] \rightarrow \max_d$$

which can be connected with the short- or long-run policy in output or in consumption. In our numerical examples we used $A_0 = 0.08$ and as world oil reserves and extraction on January 1, 2006 (Oil & Gas J., 2005, 103, 47: p. 25.): $R_0 = 71,793.8$ [1,000 bbl/day] $\times 365 = 26,204,737$ [1,000 bbl/year] (or 3.58969 bln t/year); $S_0 = 1,292,549,534$ [1,000 bbl] (or 177.06 bln t). We use coefficient 1 ton of crude oil = 7.3 barrel. For our short-run criteria $F_1\,[R(t,d)]$, $F_2\,[R(t,d)]$ and $F_3\,[R(t,d)]$ we constructed the following "optimal" paths of extraction:

1) *the "smooth breaking" or the Least Acceleration (LA) curve* (the solution of (7), see Bazhanov, 2006) has $d = 36.8837$ which implies c =0.001459 and b =-0.0136. Plots of $R(t)$ and $A(t)$ are on the Fig. 1 and Fig. 3 after the year 2005 (solid lines). This path has a peak of extraction at $t_{\max} = 5.87$ with the $R_{\max} = 3.8016$ bln t/year. Numerical estimation of the qualitative behavior of GDP percent change (6) along this path yields results similar to those for in $\dot{R}(t)$ on Fig. 3 after the year 2005 (solid line). It has a minimum $[\dot{q}/q]_{\min} = -0.122\%$ at $t_{\min} = 271$, then it is negative in the long run, approaches zero, but never exceeds it. Note, that for the LA curve $d \to \infty$ with $A_0 \to +0$ and we failed to construct this curve for $A_0 \leq 0.06$.

2) *The "minimum shock on GDP" or the MS curve* (the solution of (8)) can be obtained by solving the first order condition $d(\dot{R}(t)/R(t))/dt = 0$ which gives us the only relevant root (since we are looking for $t^* > 0$) corresponding to a minimum in t :

$$t^*(d) = -\left\{A_0 + [\{A_0 - b(c(d), d)/c(d)\}\{A_0 + R_0 c(d)(d-2)\}]^{0.5}\right\}/b(c(d), d).$$

Then we substitute for $t^*(d)$ and solve the first order condition $d\left[\dot{R}(t^*(d))/R(t^*(d))\right]/d\,[d] = 0$. Numerical investigation gives us the single root which corresponds to the maximum in d : $d = 6.1178$ with c and b defined by (11) and (13). This curve qualitatively resembles the LA curve, but it has a peak of extraction earlier, at $t_{\max} = 3.52$ with the $R_{\max} = 3.7162$ bln t/year. The MS curve, unlike the LA curve, can be constructed for $A_0 < 0$.

3) *The "maximin GDP percent change" or the MM curve* can be constructed as a numerical solution of (9) only for the cases when we have the inevitable short-run minimum in $\dot{q}/q$ (period of crisis). For our numerical examples we had 2 different cases: a) GDP percent change is always positive, monotonically decreasing, and asymptotically approaching zero for $d \leq \alpha/\beta + 2$ (see Corollary 1 below); b) $\dot{q}/q$ decreases to some negative value (minimum) and then asymptotically approaches zero always remaining negative (for $d > \alpha/\beta + 2$, see Fig. 5). In the first case there

is no solution for (9) because minimum does not exist. In the second case, if we have the constraint on $d : d \geq \alpha/\beta + 2 + \varepsilon$, then for any positive ε the problem has a corner solution $d^* = \alpha/\beta + 2 + \varepsilon$ and we obtain the MM curve of extraction along which the consumption decreases to zero.

4 Consumption along transition curves

We are going to check if our short-run criteria are consistent with our long-term goal, namely, nondecreasing consumption. We will examine, for simplicity, the case when all the resource rent is always invested in capital (zero net investments) and there are no time lags between the moments of investment and the corresponding capital increase. The only reason for the government to change the pattern of extraction is that sustainable (in sense of constant consumption) path of the essential resource extraction must be decreasing and asymptotically approaching zero.

Note, that constant per capita consumption over time in this case is the result of

1) total investment of oil rent in capital (with no time lag) and

2) fulfillment of the Hotelling rule.

In this paper we are going to analyze the case when some reasons cause the deviation from an efficient path of extraction and we must find the optimal path across inefficient curves. We set down these assumptions below in the definitions 1 - 4, and the Propositions 1 and 2.

Definition 1 An *intertemporal program* $\langle f(t), c(t), k(t), R(t)\rangle_{t=0}^{\infty}$ is a set of paths $f(t)$, $c(t)$, $k(t)$, $R(t)$, $t \geq 0$ such that $f(t) = f[k(t), R(t)]$ and $c(t) = f(t) - \dot{k}(t)$.

Definition 2 For positive initial stock of capital and resource $(k_0, S_0) \gg 0$ the set of the programs $F = \{\langle f(t), c(t), k(t), R(t)\rangle_{t=0}^{\infty}\}$ is *a feasible sheaf* at $t = 0$ and each of the paths $f(t)$,

$c(t)$, $k(t)$, $R(t)$ is *a feasible path* if any program $\langle f(t), c(t), k(t), R(t)\rangle_{t=0}^{\infty}$ from F for all $t \geq 0$ satisfies the conditions:

1) $(f(t), c(t), k(t), R(t)) \gg 0$;

2) $R(t), k(t), c(t)$ are continuously differentiable and $\sup_t \left|\dot{R}(t)\right| \leq \dot{R}_{\max} < \infty$;

3) $f(t)$ is twice continuously differentiable;

4) $\int_t^{\infty} R(t)dt \leq S(t)$;

5) $k(0) = k_0, c(0) = c_0, R(0) = R_0, \dot{R}(0) = A_0 \leq \dot{R}_{\max}$.

Definition 1 is based on the definition of the interior feasible path in (Asheim et al., 2005). The differences reflect our assumptions: a) population is constant; b) the speed of change of the extraction rate $\dot{R}$ is limited and continuous for all t including $t = 0$. Henceforth, a "program" and a "path" will refer to a feasible program and a feasible path.

Definition 3 (Dasgupta, 1979, p. 214) A feasible program $\langle f(t), c(t), k(t), R(t)\rangle_{t=0}^{\infty}$ from F is *intertemporally inefficient* if there exists a program $\left\langle \overline{f}(t), \overline{c}(t), \overline{k}(t), \overline{R}(t)\right\rangle_{t=0}^{\infty}$ from F such that $\overline{c}(t) \geq c(t)$ for all $t \geq 0$ and $\overline{c}(t) > c(t)$ for some t.

Definition 4 (Dasgupta, 1979, p. 214) A set of feasible programs $E = \{\langle f(t), c(t), k(t), R(t)\rangle_{t=0}^{\infty}\}$ is a *set of efficient programs* if all the programs $\langle f(t), c(t), k(t), R(t)\rangle_{t=0}^{\infty}$ from E are not inefficient.

Proposition 1 *If $\dot{f}_R(0)/f_R(0) \neq f_k(0)$ then $F \cap E = \emptyset$ or all the feasible paths are inefficient.*

Proof. Since $f(t)$ is twice continuously differentiable at $t = 0$, then there exists $\varepsilon > 0$ such that for any $t \in [0, \varepsilon)$ and for any feasible program $\langle f(t), c(t), k(t), R(t)\rangle_{t=0}^{\infty} \in F$ the Hotelling rule is not satisfied: $\dot{f}_R(t)/f_R(t) \neq f_k(t)$. Necessity of the Hotelling rule for the efficiency of a program (see, e.g., Asheim et al., Dasgupta, 1979) follows the assertion of the Proposition.

Now we will show that in our assumptions (zero extraction cost) all the growing paths of extraction are inefficient.

Proposition 2 *For an economy with technology* $q = k^{\alpha} R^{\beta}$ *where* $\alpha, \beta \in (0, 1)$; $k(t), R(t) > 0$ *and* $\dot{k}(t) < q(t)$ *for all* t, *the path of extraction is inefficient if there is* $\bar{t} \geq 0$ *such that* $\dot{R}(\bar{t}) > 0$.

Proof. Since the Hotelling rule is a necessary condition for efficiency, it is enough to show that it does not hold for the growing rate of extraction. Indeed, we can write the Hotelling rule $\dot{f}_R(t)/f_R(t) = f_k(t)$ as $\dot{f}_R/f_R = R\beta \left[\alpha\dot{k}/kR + \beta q\dot{R}/R^2\right]/(\beta q) - \dot{R}/R = \alpha\dot{k}/k - (1-\beta)\dot{R}/R = \alpha q/k$ (since $f_k = \alpha q/k$). Then we have $\alpha\dot{k}/k + (\beta-1)\dot{R}/R = \alpha q/k$ or $(\beta-1)\dot{R}/R = \left(q - \dot{k}\right)\alpha/k$. The right hand side of the last equation is always positive and the left hand side can be positive only for $\dot{R} < 0$ for any $t \geq 0$ (since $(\beta - 1) < 0$ and $R > 0$).

So, the transition path (12) is not efficient (extraction grows in a neighborhood of $t = 0$) unlike the Hartwick curve (3) which is derived from the Hotelling rule and so satisfies it identically. Hence, to examine the consumption behavior in our case along some path we should check the fulfillment of the Hotelling rule along this curve. For the general case $\dot{q} = f_k\dot{k} + f_R\dot{R}$. Then $\dot{f}_R = \beta d\,(q/R)\,/dt = \beta\left[f_k\dot{k}/R + f_R\dot{R}/R\right] - \beta\dot{R}q/R^2$. Dividing on $f_R = \beta q/R$ we have $\dot{f}_R/f_R = R\beta\left[\alpha\dot{k}/kR + \beta q\dot{R}/R^2\right]/(\beta q) - \dot{R}/R = \alpha\dot{k}/k - (1-\beta)\dot{R}/R$. Since $f_k = \alpha q/k$ we have $\dot{f}_R/f_R = f_k\left[\dot{k}/q - (1-\beta)k\dot{R}/(\alpha q R)\right]$ and substitution for $\dot{k}$ the saving rule $\dot{k} = \beta q$ gives us

$$\dot{f}_R/f_R = f_k\left[\beta - (1-\beta)k\dot{R}/(\alpha q R)\right]. \tag{14}$$

Just to check, we can see, that for the Hartwick curve $[\cdot] \equiv 1$, because the Hotelling rule implies $\dot{R}/R = -\alpha q/k$.

Hence, if $[\cdot] < 1$, then $\dot{q} > 0$, because $\dot{f}_R/f_R < f_k$, which follows $-\dot{R}/R < \alpha q/k$ or $\alpha q/k + \dot{R}/R > 0$. And the latter, using expression in the left hand side of (2), means $\dot{q} > 0$.

In the same way, $[\cdot] > 1$ follows $\dot{q} < 0$ and, in general, $\operatorname{sgn}\dot{q} = \operatorname{sgn}\{1 - [\cdot]\}$. So, to examine long-run consumption $c = (1-\beta)q$ along the LA curve, we can check asymptotic behavior of $[\cdot]$ in (14).

Proposition 3 *If an economy with technology $q = k^{\alpha}R^{\beta}$ is such that $\alpha, \beta \in (0,1)$; $\beta < \alpha$ and*

1) resource rent is completely invested in capital;

2) there is no time lag between the moment of investment and the corresponding increase in capital;

3) rate of extraction $R(t)$ is such that

$$\dot{R}(t) = (A_0 + bt)/(1+ct)^d,\ b < 0,\ c > 0,\ d > 3,$$

then the asymptotic behavior in output q for different β is:

$$\lim_{t\to\infty} \operatorname{sgn}\dot{q}(t) = \begin{cases} -1, & \beta(d-2) \geq 1, \\ \operatorname{sgn} L(d,\alpha,\beta), & \beta(d-2) < 1, \end{cases} \tag{15}$$

where

$$L(d,\alpha,\beta) = \frac{[\alpha - \beta(d-2)]}{[\alpha - \alpha\beta(d-2)]}.$$

Proof of the Proposition is in (Bazhanov, 2006, Appendix).[7]

Corollary 1. Under the assumption of the Proposition 3 the consumption $c(t)$ is

1) asymptotically decreasing if $d > \alpha/\beta + 2$;

2) asymptotically constant if $d = \alpha/\beta + 2$;

3) asymptotically growing if $3 < d < \alpha/\beta + 2$.

Proof. Note that for $\beta(d-2) < 1$ or $d < 1/\beta + 2$ denominator of $L(d,\alpha,\beta)$ is positive. Then the sign of $L(d,\alpha,\beta)$ is defined by nominator. Since $c = (1-\beta)q$ and $\operatorname{sgn}\dot{c} = \operatorname{sgn}\dot{q}$ then

[7] The simplified expression for $L(d,\alpha,\beta)$ was obtained by direct substitution of expressions for b, c and ρ.

substituting the expressions for d into $L(d, \alpha, \beta)$ in (15) we obtain the assertion of the Corollary. In the case when $d \geq 1/\beta + 2$ or $\beta(d-2) \geq 1$ we define the sign of $\dot{c}$ by the first line in (15) which is included in the first case of the Corollary.

5 Numerical examples for the Oil & Gas Journal's reserve estimates

The Hartwick saving rule which we use in our economy implies that the consumption path is $c = q - \dot{k} = (1-\beta)q = (1-\beta)k^{\alpha}R^{\beta}$ where $R(t)$ is a known transition curve and $k(t)$ is an unknown path of capital. We can calculate $k(t)$ from the equation for the saving rule $\dot{k} = \beta k^{\alpha}R^{\beta}$ assuming that we have estimation of k_0. From (3) we have $\dot{q}/q = \beta(\alpha q/k + \dot{R}/R)$ which implies the expression for k_0, given $R_0, \dot{R}_0$, and our output percent change $(\dot{q}/q)_0$:

$$k_0 = \left\{ \left[\left(\frac{\dot{q}}{q}\right)_0 \frac{1}{\beta} - \frac{\dot{R}_0}{R_0} \right] / \left(\alpha R_0^{\beta}\right) \right\}^{\frac{1}{\alpha-1}} .$$

For the example with $\alpha = 0.3, \beta = 0.05$ we have $\alpha/\beta + 2 = 8$ and the Corollary implies that sustainable in the sense of nondecreasing consumption are the paths with $3 < d < 8$. Given $R_0, \dot{R}_{0=} A_0$ for the world oil extraction, we have the short-run optimal (in the sense of criterion F_1) value of $d^* = 36.8837$ (or $\beta(d-2) = 1.74 > 1$). This means, that consumption and output decrease in the long run along the LA curve. Using $(\dot{q}/q)_0 = 0.04$ which implies k_0 =0.2809628328 and $c_0 = 0.6919442652$, we obtained the consumption path shown on Fig. 4 (the numerical solutions for $k(t)$ here and below were obtained in Maple by the procedure $rkf45$) and output percent change (Fig. 5). For $\alpha = 0.2, \beta = 0.05$ (estimates from Nordhaus and Tobin, 1972) we also have consumption decreasing to zero in finite time.

We can fit the parameter d to obtain the "oil peak" around $t = 5$ (the forecast of "oil peak"

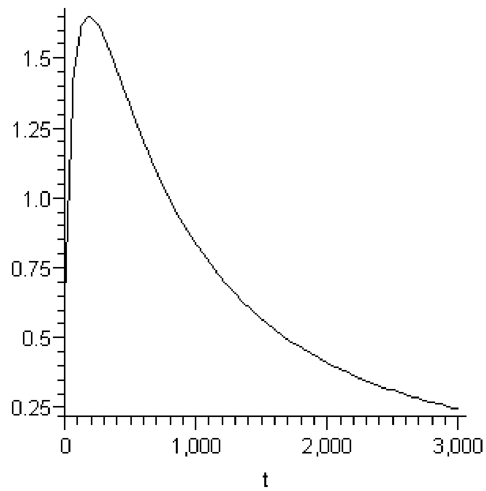

Figure 4: Consumption decrease along the LA curve

theorists). Then we have $d = 11$ and according to the Corollary it follows the consumption asymptotically decreasing to zero (as on Fig. 4) after the maximum $c_{\max} = 1.915$ at $t = 504$.

Corollary 1 shows that we can fit the single free parameter of the transition curve d and recalculate $c(d)$ and $b(d)$ using some welfare criterion, e.g., constant consumption over time in the long run (asymptotically constant consumption) instead of the short-run criterion F_1. An example with $\alpha = 0.3$ and $\beta = 0.05$ gives us $d = 8.0$. In this case the maximum negative output shock takes place a little bit earlier ($t_{\max} = 19.6$) in comparison with $t_{\max} = 25.136$ for the LA curve; the value of the shock is larger ($A_{\max} = -0.0716$) in comparison with $A_{\max LA} = -0.06959$, but the shock is weaker than for the curve (4), for which $A_{\max} = -0.08350$. The oil peak for this curve must be closer, namely, at $t = 4.27$ with $R_{\max} = 3.7433$.

To check that the level of consumption along this curve, which we will call the "Transition Constant Consumption" (TCC) curve, is far enough from zero, we can solve numerically for

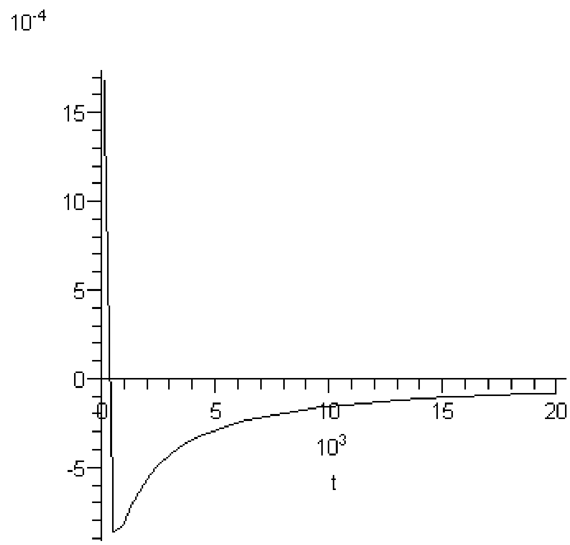

Figure 5: Output percent change ($\frac{\dot{q}}{q}$) along the LA curve

$k(t)$ and then plot $c(t)$ (Fig. 6). The value of constant consumption for the t, big enough, is around $c_{const} = 2.42801$. GDP percent change along this curve of extraction is always positive and asymptotically approaches zero.

Consumption along the F_2-optimal MS curve, according to the Corollary, must be asymptotically increasing since for this curve we have $d < \alpha/\beta + 2 = 8$. And indeed, numerical solution for the MS curve gives us the unlimited monotonical growth in consumption (Fig. 7) with the GDP percent change decreasing to zero but with slower rate than for the TCC curve. The only "cost" of this growth is that the oil peak for this case must be even more closer, at $t = 3.52$.

Note that we consider here patterns of infinite growth (limited or unlimited) just to show that for some paths of the resource extraction we can avoid the decrease of consumption to zero even along inefficient curves. Actually, for more realistic analysis of the cases with nondecreas-

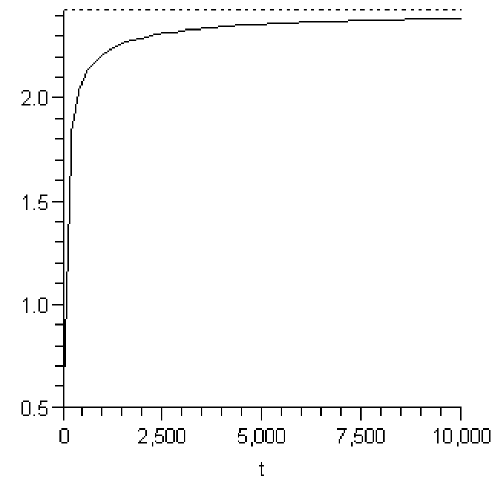

Figure 6: Consumption along the TCC curve

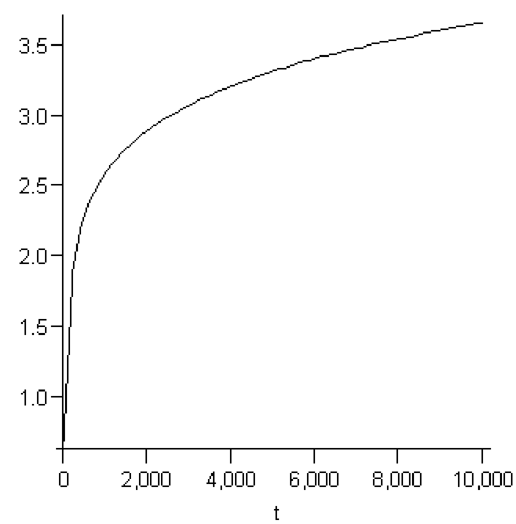

Figure 7: Unlimited growth.

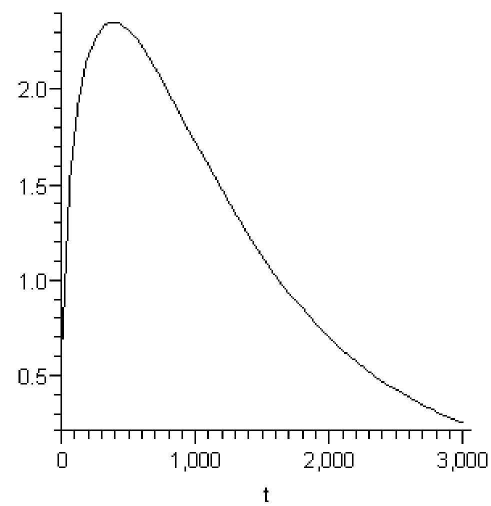

Figure 8: Consumption path for the CERA's scenario of oil extraction.

ing consumption we must introduce capital depreciation and some patterns of technological progress. Here we can interpret our model as a model with technological progress, compensating for the capital depreciation (see Section 8).

6 Numerical examples for the CERA's reserve estimates

In order to construct the transition path of extraction with the peak at $t = 24$ as in CERA's scenario, we must take a rather large value of d. For $\alpha = 0.3$ and $\beta = 0.05$ we must take $d = 10^{10}$ which (see Corollary 1) already means that this path (even without the undulating plateau) is unsustainable. Numerically it is expressed in the peak of consumption at $t = 375$ with $c_{\max} = 2.36$ (Fig. 8).

The next step of comparison involves constructing the path of extraction which is borderline

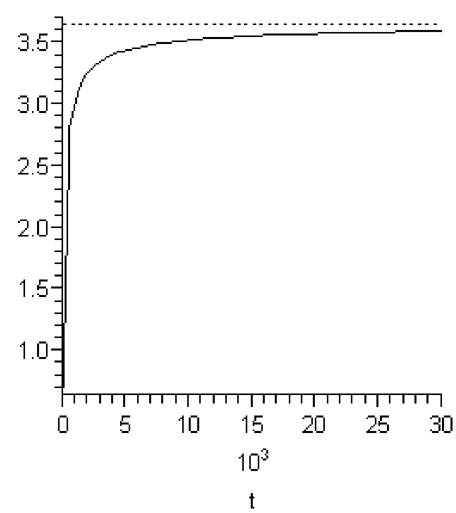

Figure 9: Consumption for the TCC curve of extraction for the CERA's reserves estimates.

between sustainable and unsustainable paths. As it was shown above, this path for $\alpha = 0.3$ and $\beta = 0.05$ has a value of $d = 8$. The only difference of this path (Fig. 9) from the one on Fig. 6 is that consumption approaches a higher asymptote with $\bar{c} = 3.64842$. The peak of oil extraction in this case must be at $t = 17.9$ ($R_{\max} = 4.187$ bln t/year) which is at least 6 years earlier than in CERA's scenario.

For $\alpha = 0.2$ and $\beta = 0.05$ which are recommended by Nordhaus and Tobin, a "borderline curve" needs $d = 6$ which implies the oil peak at $t = 15.15$ ($R_{\max} = 4.093$ bln t/year). For these parameters we have k_0 =0.1983598129, c_0 =0.7327691725 and the asymptote $\bar{c}$ =1.947.

We complete the comparison with the case when d is defined as a solution of the short-run problem with criterion F_1 (the least acceleration). For $\alpha = 0.3$ and $\beta = 0.05$ the larger reserves give us $d = 7.52$ which is already a sustainable value in comparison with the result obtained for the Oil & Gas Journal reserve estimates (see Fig. 4). In this case we have rather slow but unlimited growth of consumption like on Fig. 7. Oil peak in this case must be at $t = 17.4$. Note that for the same d but $\alpha = 0.2$ and $\beta = 0.05$ we have already an unsustainable pattern of extraction (decreasing to zero consumption in the long run) since the "borderline value" of d for such an economy is 6. Criterion F_2 gives us the MS curve with $d = 5.46$ which like for the Oil&Gas Journal S_0 implies the sustainable growth of consumption.

7 Technical restrictions and dynamic consistency

The examples in previous two sections show that the government's short-run policy can be consistent with the long-run goal. However, the criterion F_1 (the least acceleration of extraction) can be consistent or inconsistent depending on the actual amount of reserves S_0. Then we can assume that the criterion F_2 (minimum shock on GDP) can also give us unsustainable pattern of extraction for smaller S_0. However, attempts to find numerically such a value S_0 for which criterion F_2 gives us the MS curve with $d > \alpha/\beta + 2$ were unsuccessful. Though for very small[8] value of reserve $S_0 = 7$ (in our previous examples we have $S_0 = 177.06$ and 512.3242 bln. t) consumption enters the period of sustainable growth only after a period of decline (Fig. 10).

At first glance this example can support the idea that we can increase the rate of extraction for rather long time and then, having very small reserve, switch to sustainable path without substantial decrease in consumption in the short run. However, according to our formulation of the problem and the definition of the feasible path of extraction, we have the restriction on changes in extractions: $\sup_t \left|\dot{R}(t)\right| \leq \dot{R}_{\max} < \infty$. This condition means that the extraction can be reduced without losing consumption only with the rate not exceeding $\dot{R}_{\max}$ which is

[8] Given current rate of extraction it is not enough even for two years.

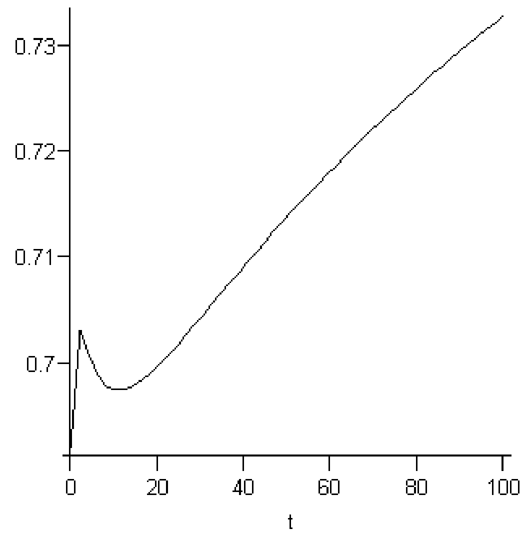

Figure 10: Consumption along the MS curve for $s_0 = 7$.

defined by the rate of introducing the substitute technology. For our numerical examples we can estimate $\dot{R}_{\max}$ from historical data (Fig. 3). Note that before 1980 $\dot{R}$ oscillated around 0.2. As a result of energy crises in 1973 and 1979-80 it was a period of introducing new technologies. Then after 1980 per capita accelerations oscillate already around zero. But energy crises followed by declines in output and consumption. Hence, since we consider the problem of switching to sustainable path without losing consumption we can take as a reasonable estimate $\dot{R}_{\max} = 0.1$. For the example with $S_0 = 7$ (Fig. 10) we obtained sustainable (in the long-run) path of extraction with $\sup_t \left|\dot{R}(t)\right| = 1.5$ which is infeasible. This means that if we are looking for the minimum reserve S_0 which still implies the sustainable MS curve of extraction, we will obtain the corner solution defined by our constraint in $\dot{R}(t)$.

We found the minimum amount of reserve $S_0 = 110$ which gives us the MS curve $R(t)$ with feasible $\dot{R}(t)$. This means that given O&GJ reserve estimate, oil can be extracted with the current per capita rate of extraction for $\frac{177.06-110}{R_0} = 18.7$ years. Then given the same R_0 and $A_0 = 0$ the economy can enter the decreasing, sustainable, feasible ($\sup_t \left|\dot{R}(t)\right| = 0.099$), and F_2-optimal MS curve.[9]

In our numerical experiments we did not manage to find any signs of dynamic inconsistency with respect to changes of initial conditions along the path. We tried to reconstruct the path using the same criterion (for example, asymptotically constant consumption, which implies $d = \alpha/\beta + 2$) for different initial points $\bar{t}$ along the path, recalculating initial conditions as $\overline{R}_0 = R(\bar{t}), \overline{A}_0 = \dot{R}(\bar{t})$, and $\overline{S}_0 = S_0 - \int_0^{\bar{t}} R(t)dt$. In all cases the new path coincided with the initial curve.[10]

8 Technological progress compensating for capital depreciation

We assumed that there is no capital depreciation and no technological progress in our simplified model. We can interpret these assumptions as an equivalent assumption about technological progress which compensates for the capital depreciation. Then for each scenario of the resource extraction we can construct this compensating technological progress and estimate its plausibility. In other words our assumption implies that

$$q(t) = T(t)k^{\alpha}R^{\beta} - \delta k$$

[9]Note again that the negative property of the MS curve is it's inefficiency because it is constructed for $\dot{r}_o \geq 0$. So it should be used only for some transition period followed by "smooth" switching to an efficient curve when the value of $\dot{r}$ along the MS curve allows to do it.

[10]It would be interesting to construct the path of extraction as a function of the rest of reserve $r = r(s(t))$ because different reports about reserve estimates are the source of large uncertainty in the path $s(t)$. Deviations of $s(t)$ will result in the dynamic inconsistency of the optimal paths constructed at different moments of time with the initial path.

and technological progress $T(t)$ is such that $T(t)k^{\alpha}R^{\beta} - \delta k = k^{\alpha}R^{\beta}$. Then we have

$$T(t) = 1 + \delta k^{1-\alpha}R^{-\beta}.$$

For the Hartwick's curve $R(t) = R_0\left[1 + R_0\beta t/S_0(\alpha-\beta)\right]^{-\alpha/\beta}$ we have $k(t) = \beta q(t) \equiv \beta q_0$ and so $k(t) = k_0 + \beta q_0 t$. Then

$$T_{Hart}(t) = 1 + \delta R_0^{-\beta}(k_0 + \beta q_0 t) \cdot \left[\frac{1 + \frac{R_0\beta}{S_0(\alpha-\beta)}t}{k_0 + \beta q_0 t}\right]^{\alpha}.$$

We can show that the compensating technological progress for the Hartwick's curve (Hartwick's technological progress) is asymptotically linear. Indeed,

$$\begin{aligned}
\lim_{t\to\infty}\dot{T}(t) &= \lim_{t\to\infty}\delta R_0^{-\beta}\left\{\beta q_0\left[\frac{1 + \frac{R_0\beta}{S_0(\alpha-\beta)}t}{k_0 + \beta q_0 t}\right]^{\alpha} + \delta R_0^{-\beta}(k_0 + \beta q_0 t) \cdot \frac{d}{dt}\left(\left[\frac{1 + \frac{R_0\beta}{S_0(\alpha-\beta)}t}{k_0 + \beta q_0 t}\right]^{\alpha}\right)\right\} \\
&= \delta R_0^{-\beta}(L_1 + L_2) = \frac{\delta\beta q_0^{1-\alpha}R_0^{\alpha-\beta}}{[S_0(\alpha-\beta)]^{\alpha}} = const
\end{aligned}$$

where

$$L_1 = \beta q_0 \lim_{t\to\infty}\left[\frac{1 + \frac{R_0\beta}{S_0(\alpha-\beta)}t}{k_0 + \beta q_0 t}\right]^{\alpha} = \beta q_0\left[\frac{R_0}{S_0 q_0(\alpha-\beta)}\right]^{\alpha} = const$$

and

$$L_2 = \alpha\left[\frac{R_0}{S_0 q_0(\alpha-\beta)}\right]^{\alpha-1}\lim_{t\to\infty}\left[\frac{R_0\beta}{S_0(\alpha-\beta)} - \beta q_0\frac{\frac{R_0\beta}{S_0(\alpha-\beta)}t}{k_0 + \beta q_0 t}\right] = 0.$$

The paths of the compensating technological progress for the rational transition curves can be constructed numerically (Fig. 11)[11]. The paths of $T(t)$ corresponding to sustainable extraction (with nondecreasing consumption) are located between the curve depicted with crosses ($T(t)$ for the rational curve with $d = \alpha/\beta + 2 = 8$) and the dotted line. Note that these paths are not linear. Like the Hartwick's technological progress (boxed line on the Fig.

[11] We constructed our examples for $\delta = 0.1$.

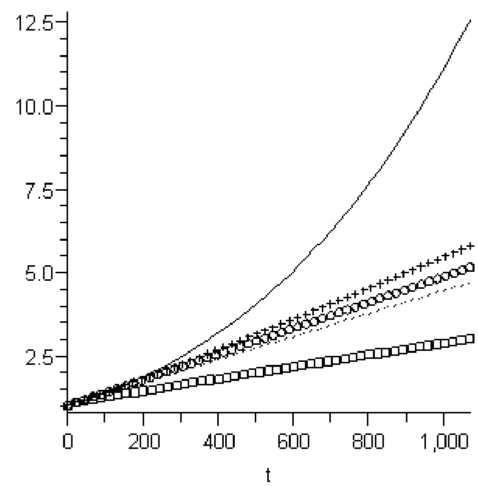

Figure 11: Technological progress compensating for capital depriciation

11), these paths can be linear only asymptotically because linear function constructed with the intersection $T_0 = T(0)$ and the slope $T_1 = \dot{T}(0)$ (T_1 does not depend on d) is depicted as the dotted line on the Fig. 11. The solid convex curve is the pattern of behavior of the compensating technological progress for unsustainable paths of extraction (here it is for $d = 100$). The curve depicted with circles corresponds to the technological progress for the rational path of extraction with $d = 4$. For $d = 3.0001$ the path of $T(t)$ almost coincides with the dotted line. Hence the sustainable paths of extraction require more plausible patterns of the compensating technological progress than the unsustainable ones.

Note that technological progress of the $AK-$model is not sufficient to compensate for the linear capital depreciation in the presence of essential nonrenewable resource. Indeed, if we consider technological progress as a human capital $H(t)$ in a model $Q = \widetilde{A}K^{\alpha}R^{\beta}(HL)^{\gamma} - \delta K$ with $\gamma = 1 - \alpha$ and $H \equiv K$ then per capita output is $q = \widetilde{A}kR^{\beta} - \delta k$. Since $\widetilde{A}$ is a constant, q

is declining to zero in finite time which is defined by the moment when the extraction $R(t)$ is small enough to satisfy the condition $\widetilde{A}R^{\beta} = \delta$.

9 The generalized Rawlsian maximin principle

According to Rawls's maximin principle, the patterns of sustainable growth of consumption are obviously the results of overinvestment. But actually Rawls (1971, p. 291) objected to applying his maximin principle to the questions of justice among generations because of unacceptable consequences. In (Bazhanov, 2006) we offer a generalized approach for the defining a "relevant position" in Rawls's theory which implies that we must take into account not only the values of some indicators of life quality in the present but rather such indicators combined with their time changes or differences in consumption from previous years. Then the utility in its simplest form is $u = u(c, \dot{c})$. Applying maximin principle, e.g., for u in additive form we have $u(c, \dot{c}) = wc(t) + (1 - w)\dot{c}(t) = \gamma = const$ for any $t > 0,\ w \in [0, 1]$ which with $c_0 = c(0)$ follows

$$c(t) = \left[\gamma - \exp\{-wt/(1 - w)\}(\gamma - c_0 w)\right]/w \tag{16}$$

or we have a case of limited growth (Fig. 12 compare with Fig. 9) for $\gamma > c_0 w$ and (16) is *desirable* in a sense "...that an extra bit of consumption at t is more valuable than the same *extra* bit at $t + 1$, since individuals will, in any case, have more consumption at $t + 1$" (Dasgupta and Heal, 1979, p. 284). Observe that (16) describes a limited decline for $\gamma < c_0 w$ and identically constant consumption (as in the Hartwick rule) for $\gamma = c_0 w$.

We do not claim that everybody favors this type of just path, particularly when it is apparent that rather small sacrifices in present can bring slow but *unlimited* growth in the long run (Fig. 7). For those, who prefer this form of intertemporal distribution, the more appropriate consumption utility function would be the function with essential factors, e.g., the

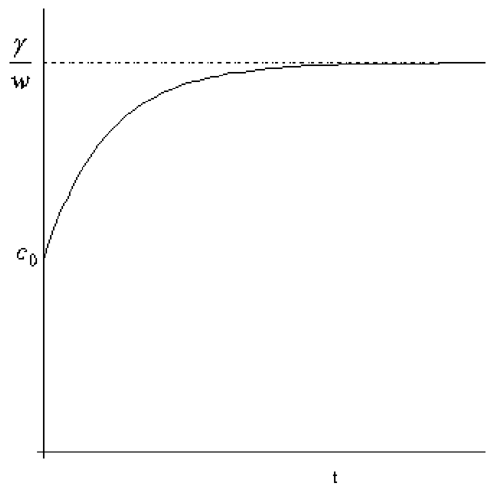

Figure 12: Generalized "fair Rawlsian growth" (16), $w = 0.5$

Cobb-Douglas case. Then the rule of intertemporal distribution is $c^w \dot{c}^{1-w} = \gamma = const$ which gives us $c(t) = c_0 (1 + \mu t)^{\varphi}$where $c_0 = c(0), \mu = (\gamma/c_o)^{1/\varphi} / \varphi, \varphi = 1 - w$ or a pattern of unlimited (quasi-arithmetic, Asheim et al., 2005, p. 5) growth which (for w close to 1) looks like the curve on Fig. 7.

In general, utility can be written as a CES function, or as a function with a variable elasticity where the elasticity parameter and w are to be chosen by the government. Then the specific just savings principle can be deduced for the specific utility function and the transition path of extraction can be adjusted to approach as close as possible (depending on constraints) the asymptotically optimal (in the long run) pattern of intertemporal distribution of consumption.

10 Concluding remarks

Using our transition path analysis, we have found for the Cobb-Douglas economy that the sustainable (in terms of nondecreasing consumption) or "normative" peak of oil extraction must be earlier than the "physical" peak when the growth of oil production is already technically impossible. Another result is that the short-run government's policy in choosing the path of switching to the sustainable resource extraction can be consistent with the long-run welfare criterion depending on the formulation of the short-run criterion and the amount of the resource reserves.

Analysis of the long-run consumption along the transition curves shows that even for the Oil & Gas Journal's world oil reserves estimates which are about three times less than CERA's estimates, there is a path of extraction with asymptotically constant (separated from zero) consumption over time. Moreover, a "worsening " of the short-run situation (shortening the period of transition and introducing a stronger negative shock on output) yields the possibility of slow, but unlimited growth of the consumption in the long run.[12]

The situation is brighter with the CERA's reserve estimates though the qualitative result is the same: the sustainable oil peak must be earlier than the "physical" one. The anxiety about possible violation the intergenerational justice criterion increases when we consider the examples with technological parameters α and β estimated by Nordhaus and Tobin. For the economy with these parameters the sustainable oil peak for the CERA's reserves estimates must be in the next 15 years.

For the cases of different patterns of consumption growth the transition curve (to be exact,

[12] We interpret our model as a model with technological progress, compensating for the capital depreciation. This assumption makes possible the opportunity of infinite growth in presence of the diminishing return production function ($\alpha < 1$) and the essential exhaustible resource.

the single free parameter - d) can be fitted to satisfy desirable qualitative behavior of consumption in accord with the various optimality criteria for the long run. And it again raises the long-standing question about the fairest ethical theory for the distribution of consumption across generations. If decreasing oil consumption is really necessary, which criterion must we follow?

Aside from equivocation on the main welfare criterion there are some other questions and limitations of the model we have presented.

(1) There is an interesting question of the path stability with respect to errors in estimations of parameters α and β. This question can be considered in the frame of construction of dynamically consistent path with respect to changes in S_0, α, and β.

(2) Transition curves can be constructed in a different class of functions, e.g., as a solution of calculus of variation problem.

We also assumed that:

(3) The cost of extraction is zero and population is constant though it would be interesting to consider the problem of transition when extraction costs are present.

(4) There is no time lag between the moment of oil extraction and the corresponding increment of capital; this is not true if the oil rent is invested in alternative technologies.

(5) All oil rent is invested into reproducible capital. In general, this is not observed and we should consider some period of increasing investments along some smooth (maybe hysteresis-like) curves and examine the influence of this curve on the long-run consumption behavior.

(6) We can consider the problem of smooth switching to the efficient path of extraction after using the transition curve for entering the decreasing path.

We think that all these questions need special careful consideration in separate papers.

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