The production of science

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Abstract

I develop a model of science production where researchers are endogenously allocated to different sectors (for profit or non profit), and to different labs. The main assumption is that firms invest in research to increase their *absorptive capacity*: the ability to use and understand scientific findings produced elsewhere. Firms create absorptive capacity by building labs and hiring researchers in a competitive market. Because of externalities, firms underinvest in labs. More interestingly, researchers and labs are substitutes in the revenue function, even though they are complements in the research production function. This implies a novel form of inefficiency: for any given investment, the allocation of researchers to firms is non optimal. I introduce into the model universities and I assume that their mission is to produce science. I show that universities attract the best researchers, and that the allocation of researchers to labs within the university system maximizes the research output.
KEYWORDS: Organization of Scientific Research, Externality, Policy, Absorptive Capacity, Matching with Investment.
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1 Introduction

The way scientists are allocated across countries, firms, and sectors affects, among many other things, the production of new knowledge, the pace of technological innovation, the rate of economic growth, and the effectiveness of public spending in research. It is therefore topical to understand the determinants of this allocation.

I develop a model of science production where heterogeneous scientists endogenously join either the for-profit research sector or the university sector. Within each sector, researchers are allocated to labs of different sizes in order to produce science. This allocation depends on whether and where the two inputs are substitutes or complements, that, in turns, is determined by the curvature of the payoff functions of firms and universities. The main insight of the paper is that, because firms and universities carry out research for distinct reasons, the competitive market allocation of researchers to labs is different than the allocation emerging in the university sector. In particular, I will show that within the university sector researchers are allocated to labs in order to maximize the total science produced, while this may not be the case in the for-profit sector. I also show that the private sector allocation of researchers to labs is not efficient.

To better illustrate this point, I assume that firms invest in research mainly to increase their *absorptive capacity*: the ability to use outside science. Research provides "a ticket of admission to an information network":[1] it allows firms to be always up to date with the science produced by other firms and universities. Also, science is difficult: only scientists that are actively engaged in research can read and understand several papers in a timely fashion. In other words, using publicly available science can be costly to firms; this cost is lower when firms produce more in-house research.

Since researchers are heterogeneous in their ability levels, the absorptive capacity assumption generates a strong form of allocative inefficiency. In the model, firms produce science by building labs and hiring researchers. As in the standard model of science, firms underinvest in labs because they cannot capture the full benefit of research. However, in this context, there is a second source of inefficiency. Firms carry out research to use outside science at a lower cost. In order to achieve this goal, firms can either invest in labs or hire a very productive researcher: researchers and labs are substitutes in the revenue function. The substitutability implies that the competitive market allocates researchers to labs according to a Negative Assortative Matching (NAM) rule: the worst researcher works with the biggest lab.

Intuitively, firms produce science so that their in-house researchers can be part of the scientific community. Once they reach the required scientific output, doing additional research

[1] Rosenberg (1990), p.170

generates little extra benefit: firms do not want to produce Nobel-prize-winning research; it is pointless for firms to invest a lot in labs and, at the same time, hire very productive researchers. More formally, producing science is a form of cost reduction. It follows that the marginal benefit of research decreases rapidly. This is why labs and researchers are substitutes in the revenue function.

At the same time I assume that researchers and labs are complements in the research production function; the total science produced is maximized under a Positive Assortative Matching (PAM) rule assigning the best researchers to the biggest labs. Therefore, in the allocation of researchers to labs, there is a trade-off between producing science and using science. Since firms aim at using science, for any given investment in labs the private sector minimizes the amount of science produced. The decentralized allocation of researchers to labs is inefficient.

I introduce universities into the model and I assume that their mission is to produce science. I also assume that academic scientists can work as consultants for the private sector. The job of a consultant is to help a firm using the available stock of science. Under these assumption, I show that the best researchers are hired by universities, and within universities researchers are allocated according to PAM: better researchers get to work with bigger labs. Therefore, the main observable difference between the private sector and the university sector is how resources are allocated to researchers.

Finally, I extend the model by assuming that researchers care about reputation, which is built by producing science. I show that, if reputation concerns are strong enough, the equilibrium in the private sector may switch from NAM to PAM. Intuitively, researchers are willing to receive lower wages in order to work in firms with big labs. In addition, for a given lab, productive researchers are willing to forfeit a bigger portion of their wages than unproductive researchers. In the new competitive equilibrium, productive researchers work in big labs, but may be paid less than unproductive researchers because they receive a higher reputation reward.

1.1 Relevant Literature.

The fact that the allocation of talented agents across sectors and occupations can have important aggregate consequences is not new. However, the argument is usually that, by joining different sectors, productive agents will be doing different things (see, for example, Baumol (1996) and Murphy, Shleifer, and Vishny (1991)), or they will be subject to a different set of incentives (see Aghion, Dewatripont, and Stein (2008)). In my model, scientists are always doing research, according to the same production function, no matter in what sector they work. Despite this, it does matters whether a given researcher joins the for-profit research sector or the university sector, because resources are organized differently in different sectors.

The literature on absorptive capacity is broad, especially its empirical branch. The first formulation of the concept is due to Tilton (1971), who analyzes the semiconductor industry during the '50s and '60s. At that time, this sector was composed of several firms, with about 45 producers in the US alone. Tilton observes that, for these firms, investing in research was a form of insurance: they were always guaranteed to be up to date with the latest scientific breakthrough. The term *absorptive capacity* was introduced by Cohen and Levinthal (1989), who provide both the first theoretical model of this concept and its first empirical test. Other important empirical works are Cockburn and Henderson (1998), Gambardella (1992) and Griffith, Redding, and Reenen (2004). On the theory side, several researchers explored the strategic implications of absorptive capacity (see, for example, Hammerschmidt (2006), Kamien and Zang (2000) and Leahy and Neary (2007)). In particular, Leahy and Neary (2007) derive some policy implications by showing that research joint ventures may decrease the amount of research carried out by firms. The reason is that firms invest in research partly to be able to use outside science. When the access to science is made easier by the creation of a joint venture, there is no need to carry out much research anymore.

My paper relies on the assumption that, because of absorptive capacity, there is a strong form of decreasing returns in science. This particular point is supported by Gittelman and Kogut (2003). The goal of their paper is to establish whether valuable science leads to valuable patents. The authors measure the quality of the scientific output by counting the number of citations received by papers produced within a given firm. Similarly, they measure patent quality by adding all the citations received by patents produced by the same firm. They find that "scientific knowledge and patents are related, but good publications and good patents are not."[2] In other words, producing some science deliver some benefit, but producing a lot of science does not (actually, in some of their specifications, the relationship between valuable patents and valuable science is negative).

Finally, The existing empirical investigations on the allocation of resources to researchers deal exclusively with specific public institutions. For example, Arora, David, and Gambardella (1998) analyze the funding allocation decisions of the Italian CNR (equivalent to the NSF) and they show that the reputation (past publication record) is the main explanatory variable. I am not aware of any study that looks at the same problem in the private sector.

In the next section, I describe the model. In the second section, I characterize the equilibrium for a given distribution of labs. In the third section, I derive the distribution of labs, formally define the equilibrium, and prove its existence. In the fourth section I

[2] Gittelman and Kogut (2003), p. 380.

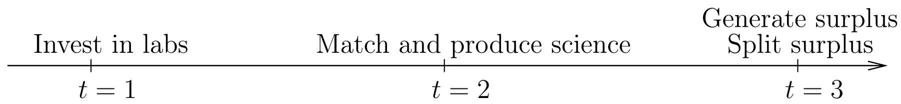

Fig. 1: Timeline

discuss the normative aspect of the model. I introduce universities in the fifth section, and reputation in the sixth section. In the last section I conclude by discussing possible empirical tests, policy implications, and extensions.

2 The Model

The economy is populated by a continuum of firms and a continuum of researchers. Firms differ in their productivity p, continuously distributed over $P = [0, \bar{p}]$, and researchers differ in their ability a, continuously distributed over $A = [0, \bar{a}]$. All agents have the same outside option assumed to be zero. The economy runs for three periods.

2.1 Investing in Labs.

In period $t = 0$ firms build labs. If a firm p sets up labs of size L it bears a cost $c(p, L)$ continuous, positive, with continuous first and second derivative, increasing in L, decreasing in p, with $\frac{\partial^2 c(p,L)}{\partial L^2} \geq 0$ and $c(p, 0) = 0 \, \forall p$.

2.2 Producing Science.

In period $t = 1$, each researcher is hired by one firm and works in the firm's lab. The amount of research produced within each match is:

$$R(a, L) = af(L)$$

where $f(L) \geq 0$, $f'(L) > 0$, and $f''(L) < 0$. Note that the two inputs are complements in the research production function. This implies that, for given distribution of labs, the allocation of researchers to labs that maximizes the production of science is Positive Assortative Matching (PAM): the most productive researcher should work in the biggest lab.

In the model, labs and researchers are the only two inputs in the production of science. Therefore the lab size should be interpret as the collection of all the elements that increase the chance of a discovery for given researcher's ability. Clearly, this include physical machines (a bigger telescope, a more powerful microscope, a state of the art DNA sequencing machine), but also the number of technicians and post-docs working in the lab. The fact that some of these inputs do not require an investment ex-ante but can be purchased after hiring the researcher will turn out to be irrelevant. In the next section, I will show that firms invest taken as given the researcher allocated to them. This implies that the timing could be reversed with no effect on the equilibrium investment.

Finally, in real life researchers work in team. This can be incorporated into the model by defining a as the research team's average quality. A previous matching state will determine how researchers form research teams, and how from a distribution of individual ability we can derive the distribution of a. To avoid introducing this additional matching stage, I will not pursue this interpretation further.

2.3 Using Science.

At the beginning of the last period ($t = 2$) there is a stock of new science available in the economy. Each firm's goal is to develop a new product out of the new science. To achieve this goal, each firm asks its researcher to search the literature, read and understand several papers, find the ones most relevant to the firm, and explain them to the other members of the staff. These activities are costly to the researcher and this cost is lower the more research he carried out in the previous period. Assume that the stock of science available in the economy has an expected commercial value V. The expected surplus generated when researcher a works for a firm owning a lab of size L is:

$$S(a, L) = V - g(af(L))$$

where $g()$ (continuous and differentiable, $g() > 0$, $g'() < 0$ and $g''() > 0$) represents the disutility the researcher has to endure in order for the firm to use the available science. Finally, V is taken as given by the firm but will be determined endogenously.

Note that, in the description of the model, the only reason for a firm to invest in research is to increase its absorptive capacity. Obviously, most firms invest in research both to use outside science at a lower cost and to increase their knowledge about a specific topic. In other words, the production of science has some direct value to the firm. This consideration can be readily incorporated into the model by defining $g(R(a, L)) = \tilde{g}(R(a, L)) - \hat{g}(R(a, L))$, with the interpretation that $\tilde{g}()$ is the cost of using science and $\hat{g}()$ is the direct benefit from doing research.

I make the following assumption on the curvature of the cost function $g()$.

Assumption 1.

$$-\frac{g''(af(L))}{g'(af(L))} > \frac{\frac{\partial^2 R}{\partial a \partial L}}{\frac{\partial R}{\partial a}\frac{\partial R}{\partial L}} = \frac{1}{af(L)}$$

To understand assumption 1, imagine that $g()$ is an isoelastic function. In this case, $-\frac{g''(x)}{g'(x)}x = k > 1$ is equivalent to requiring that $g()$ is bounded above. This is quite natural if there is no production motive and $g()$ only represents the cost of understanding. In this case, firms invest in labs to reduce their cost. It follows that the benefit a firm receives from carrying out research is never above V. However, it may be restrictive if the production motive is particularly strong.

Assumption 1 is meant to capture the following intuition. Firms produce science so that their in-house researchers can be part of the scientific community. Let's say that this is achieved by attending conferences. It follows that a firm will want to produce enough science so that its researcher can attend conferences, but producing even more science provides little extra value to the firm. Therefore, the marginal benefit a firm's enjoy from doing research is decreasing rapidly, and the surplus function is very curved.

2.4 Endogenous Science.

The value of science is taken as given by firms but it is determined endogenously aggregating all the research carried out in the economy. Call ν the expected commercial value of a unit of research and $h(L)$ the $p.d.f$ of L. The expected value of the stock of science is given by:

$$V = \nu \int m(L)f(L)h(L)dL \tag{1}$$

where the function $m(L) : \mathbb{R}^+ \to \{A, \emptyset\}$ assigns labs to researchers, with the convention that $m(L) = \emptyset$ represents an unmatched firm. The function $m(L)$ is determined in equilibrium.

3 The Equilibrium for Given Investment in Labs and for Given Aggregate Science.

In this section, I derive the equilibrium arising in period $t = 1$, when firms have already invested in labs. I analyze the problem taking the total amount of science produced in the economy V as given.

At this stage, firms differ only in the size of the lab they own. Once the distribution of labs is determined the productivity parameter p does not affect the equilibrium anymore. Let's introduce the following notation:

- $x(L) : \mathbb{R}^+ \to \mathbb{R}^+$, the payoff of a firm with lab L.
- $w(a) : A \to \mathbb{R}^+$, the payoff of a researcher with ability a.

Note that $w(a)$ represents the net payment received by a researcher. The total monetary transfer (the wage) received by a researcher a working in a lab L includes a compensation for the cost incurred and it is equal to $w(a) + g(R(a, L))$.

Definition 2. For given V, the job market for researchers is in equilibrium if:

- Feasibility: $x(L) + w(m(L)) \leq S(m(L), L)\ \forall L$.
- Stability: $x(L) + w(m(L)) \geq S(m(L'), L)\ \forall L, L'$.

The existence of a unique equilibrium for given V is a standard result in matching theory (see, for example, Kamecke (1992)).

Proposition 3. *Negative assortative matching (NAM) in the job market for researchers: the most productive researchers work in the smallest labs and the least productive researchers work in the biggest labs.*

Proof. It is easy to show that assumption 1 implies:

$$\frac{\partial^2 S(a, L)}{\partial a \partial L} < 0$$

By standard matching theory the equilibrium matching pattern is NAM. $\square$

From the firms' point of view, researchers and labs are substitutes. Since the private sector allocates researchers to labs so to maximize their marginal product, it follows that, in equilibrium, the most productive researchers will work in the smallest labs. However, labs and researchers' ability are complements in the research production function. The matching rule maximizing the total stock of science is PAM: the best researcher should work in the biggest lab. Therefore, the private sector, for a given distribution of labs, is minimizing the value of science V. There is a trade-off between maximizing science and maximizing the use of science. Since the private sector only considers the latter, the decentralized equilibrium is inefficient.

Proposition 4. *For given distribution of labs, if ν is high enough, the matching pattern emerging in the private sector is inefficient.*[3]

[3] The equilibrium concept used in this model is called F-core, and the type of externality is called widespread externality. For a theoretical analysis of the inefficiencies of an F-core economy with widespread externalities see Hammond, Kaneko, and Wooders (1989) and Hammond (1995).

3.1 Discussion.

Some reader may find assumption 1 too restrictive. It is important to notice, however, that the logic behind proposition 4 carries over even if the $g()$ function satisfies assumption 1 somewhere but not everywhere. Proposition 4 holds if over some range with positive mass of researchers and labs the social welfare function is supermodular while the private-surplus function is submodular. In this case, over that specific range the equilibrium matching will be NAM, but welfare can be improved by implementing PAM. Therefore, even in situations where assumption 1 does not hold, it is possible for the private sector matching pattern to be inefficient. However, in this case, the exact allocation of labs to researchers arising in the market can only be determined numerically.

On the other hand, there may be situations where absorptive capacity is not the driving force behind firms' investment decision. This is likely to be the case if a sector is dominated by few big firms. In these cases, assumption 1 may fail everywhere: the two inputs are global complements in the private-surplus function, and there is no inefficiency in the matching stage. Lemma 6 in the next section will show that firms underinvest in labs because they do not fully appropriate the benefit of new science. Therefore, if assumption 1 fails everywhere, the model collapses back to a standard model of science production where the only source of inefficiency is the firms' underinvesment.

Finally, the way the private surplus function depends on research may differ from the simple cost-reduction form I used. Note, however, that the arguments I just gave for the existence of substitutability in the private sector revolve around the shape of the private surplus function, and not so much on the particular functional form chosen. No matter the particular way science affects private surplus, absorptive capacity implies that $S(a, L) = \Pi(af(L))$, that $\Pi()$ is curved, and if it is curved enough the two inputs are substitutes.

4 The Ex-Ante Equilibrium

Before defining an equilibrium in the economy with an endogenous investment in labs, let's introduce some notation:

- $i(p) : P \rightrightarrows \mathbb{R}^+$, the equilibrium investment in labs made by a firm p.
- $\tilde{m}(p) \equiv m(i(p)) : P \rightrightarrows A$, the matching rule on the equilibrium path (for investment performed by some firms) mapping firms to researchers.[4]
- $\tilde{x}(p) \equiv x(i(p)) : P \to \mathbb{R}^+$, the payoff of firms on the equilibrium path.

[4] Both $i(p)$ and $\tilde{m}(p)$ are correspondences if all firms are identical and are functions otherwise. $m(L)$ is always a function.

- $l(a) \equiv i(\tilde{m}^{-1}(a))$ the lab a researcher of ability a receives in equilibrium.

The definition of equilibrium I use is similar to the one in Cole, Mailath, and Postlewaite (2001). The main difference is that, here, only one of the two sides invests.

Definition 5. The quadruple $\{i(.), m(.), x(.), w(.)\}$ constitutes an equilibrium if:

1. The investment is optimal:

$$i(p) = \arg\max_{L \geq 0}\{x(L) - c(p, L)\}$$

2. Ex post, the matching $\{i(.), \tilde{m}(.), \tilde{x}(.), w(.)\}$ is feasible and stable:
   - Feasibility: $\tilde{x}(p) + w(\tilde{m}(p)) \leq S(\tilde{m}(p), i(p))\ \forall p \in P$.[5]
   - Stability: $\tilde{x}(p) + w(\tilde{m}(p')) \geq S(\tilde{m}(p'), i(p))\ \forall p, p' \in P$.

3. For $L \notin \{L : L = i(p) \text{ for some } p \in P\}$ (investments off the equilibrium path):

$$x(L) = \max_a \{S(a, L) - w(a)\}$$

To understand the definition, assume that there is an equilibrium, and consider deviations made by a single firm. Since we are in a large economy, any action this firm may take has no impact on the equilibrium $x()$ and $w()$. Therefore, if p' imitates some other firm investing $i(p'')$, its payoff is $\tilde{x}(p'') \equiv x(i(p''))$: the equilibrium payoff of firm p''. Similarly, suppose that this firm is considering investing $L \notin \{L : L = i(p) \text{ for some } p \in P\}$: an investment not made by any of the firms. Since the equilibrium $w()$ is unchanged, this firm can match with any researcher a provided that it leaves him $w(a)$. Therefore the payoff of such deviation is $x(L) = \max_a \{S(a, L) - w(a)\}$.

Lemma 6. *In equilibrium, for $L \geq 0$:*

$$\frac{\partial x(L)}{\partial L} = \frac{\partial S(a, L)}{\partial L}\Big|_{a=m(L)}$$

Proof. See appendix. □

Lemma 6 implies that firms' investment solves:

$$\frac{\partial c(p, L)}{\partial L} = \frac{\partial S(a, L)}{\partial L}\Big|_{a=m(L)}$$

[5] The general definition of feasibility is more complicated (see Cole et al. (2001)). However, in the cases I consider here it is possible to use this simpler version.

In other words, firms maximize surplus taking V and the researchers they will be matched with as given. Since the social planner would take into account the impact of the individual investment on the total stock of science, lemma 6 implies that the investment is inefficient. Finally, note that the matching pattern expected to emerge in the following period affects the investment decisions. It follows, for example, that any policy attempting to change the allocation of researchers to labs will affect the investment and may turn out to be counterproductive.[6]

4.1 Identical Firms.

To prove the existence of the competitive equilibrium, I make two simplifying assumptions.

Assumption 7. *Firms are identical:* $c(p,L) = c(L)\,\forall p$.

Assumption 8. *It is impossible to understand a new piece of science if no research is carried out:* $\lim\limits_{x\to 0} g(x) = \infty$

Proposition 9. *Under assumptions 7 and 8, an equilibrium with zero research always exists. If the commercial value of research* ν *is high enough, there are also equilibria where a positive amount of science is produced. In these equilibria, researchers belonging to the set* $[\underline{a},\overline{a}]$ *match with firms investing* $l(a)$*, where:*

$$l(a) = max\left\{\left\{L \in \mathbb{R}^+ : \frac{\partial S}{\partial L} = \frac{\partial c}{\partial L}\right\}, 0\right\} \tag{2}$$

$$\underline{a} : \nu \int_{\underline{a}}^{\overline{a}} af(l(a))z(a)da = c(l(\underline{a})) + g(\underline{a}f(l(\underline{a}))) \tag{3}$$

and $z(a)$ *is the p.d.f. of* a.

Proof. See appendix. □

Figure 2 illustrates the case of two positive investment equilibria, given by the intersection of $V(\underline{a})$ and $\underline{a}(V)$, where:

$$V(\underline{a}) = \nu \int_{\underline{a}}^{\overline{a}} af(l(a))z(a)da$$

represents the aggregate science produced as a function of the measure of researchers employed, and

$$\underline{a}(V) = \{\underline{a} : V - g(\underline{a}f(l(\underline{a}))) = c(l(\underline{a}))\}$$

[6] Gall, Legros, and Newman (2009) analyze this problem in a different context.

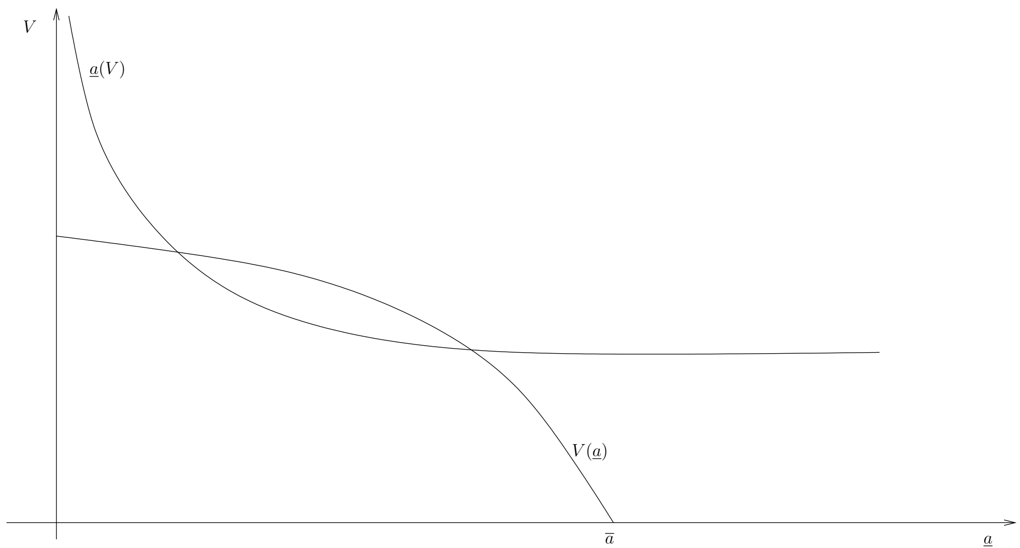

Fig. 2: Equilibrium $\underline{a}$ and V.

represents the worst researcher employed in the economy for given aggregate science V. Of the two equilibria represented in figure 2, one can be considered stable (the high V, low $\underline{a}$ one) and the other unstable.

By focusing on the stable equilibrium, it is possible to make a few comparative static exercises. If the value of a discovery ν increases, $V(\underline{a})$ moves upward: more researchers are matched and more research is produced. I can also introduce an exogenous stock of science V^f, science produced, for example, by a foreign country. The graph should be modified by writing on the vertical axes V^h instead of V, and by shifting $\underline{a}(V^h)$ downward: home country is producing more research as well. Obviously, all the comparative statics are reversed if we consider the unstable equilibrium.

5 The First Best

The social welfare generated within each match is:

$$SW(a, L) = \nu af(L) - g(af(L))$$

This function is neither globally supermodular nor globally submodular. It follows that the optimal allocation of researchers to labs can only be derived numerically, and it may involve implementing PAM over some range, and NAM over some other range. Intuitively, the social planner may, over some range, give priority to the production of science, and over some other to the use of science.

However, we know that the social planner problem has a unique solution. This implies that the first best allocation can be easily implemented if transfers based on the amount of science produced by each firm are feasible.[7]

Proposition 10. *The first best is implementable announcing the following rule: every firm producing some science receives a transfer equal to the value of the science produced by that firm minus V.*

Since there is a mass 1 of firms, V is the value of the average amount of science produced. Therefore, firms producing more than the average receive a subsidy, while the others are taxed. However, even if scientific output is observable, it is usually non contractible and, therefore, non taxable. For this reason, the first-best implementation has little practical interest.

Given the technical difficulties in dealing with the first best, in the next sections I switch to a positive analysis. I will introduce into the model new elements: universities, the government, and reputation concerns for researchers. I will then describe how they interact with the private sector and the decentralized equilibrium, and I will show that these policies and institutions play an important role in determining how resources are allocated to researchers.

6 The University Research Sector

Thus far I excluded from the analysis the sector that, in most countries, produces the vast majority of new science: universities. In this section I introduce universities and explore their impact on the private research sector and on the overall production of science.

6.1 Consultants.

As before, let's start analyzing the problem taking the distribution of labs as given. Universities are made up of labs. If a researcher a works in a university, he receives a lab of size $l^u(a)$. Researchers working in a university in period $t = 1$ can then work as consultants in period $t = 2$.

This is motivated by the literature on *star scientists.* Zucker, Darby, and Brewer (1998) show that the birth of the biotechnology industry during the 1970s in a particular region can be explained by the presence of star scientists: researchers with an outstanding research track in genetics. These scientists worked in academia, and, at the same time, were active as consultants, were part of the board of companies, and sometimes even created their own

[7] See Hammond (1995).

start-ups. Doing so, they brought into these private labs the public science they contributed to create.[8] For simplicity, I will refer to all these activities as simply consulting.

If an academic researcher works as consultant, the disutility the researcher has to incur in period $t = 2$ is equal to $g(R(a, l^u(a)))$, so that the total surplus created by a match between a firm and a researcher/consultant is:

$$S = V - g(R(a, l^u(a)))$$

Therefore, researchers (and firms) prefer the researcher to work in the university sector if $l^u(a) \geq l(a)$: in the university sector the researcher works in a lab bigger than the one she would work in if she had stayed in the private sector. Because of NAM in the private sector, for any $l^u(a)$, the most productive researchers are willing to join the university sector.

6.2 University Labs and Subsidies.

In order to derive the size of the university sector endogenously, I introduce into the model a government, and I assume that its objective is to maximize the total stock of science under an exogenous resource constraint.[9] It is important to stress here that my goal is to perform a positive analysis and not a normative one. In most countries, the government plays a crucial role in determining the amount of research carried out within the economy. My goal is to introduce it into the model in the most reasonable way and to analyze the impact of its policies on the overall production of science.

I assume that the government can employ its resources either to subsidize the production of labs, or to build a university research sector. Subsidies are cheaper than financing universities since they build on top of what firms are already investing. However, subsidies have no impact on the matching phase. Instead, building universities, although more expensive, allows the government to choose the optimal allocation of researcher to labs. Note that in the standard public good model of science there is little difference between direct provision of science or subsidies to private research. Here these two policies achieve different goals at different costs: depending on the conditions, the government will use one, the other or both.

The introduction of subsidies and universities changes the private sector equilibrium only marginally. Before the investment phase begins, the government announces $l^u(a)$, the lab a given researcher will receive if he joins the university sector. If a firm expects to be matched

[8] Note how both the star scientists literature and the absorptive capacity literature focus on sectors where, for firms, it is crucial to be up to date with the latest scientific discovery. For example, in biotechnology, once a piece of science reaches a textbook and becomes accessible without any absorptive capacity building or star scientist help, this very same piece of science is typically useless to firms.

[9] In the model, the government is uniquely characterized by its objective function. Readers may safely substitute the word "government" with, for example, "foundations."

with a researcher that, by moving to the university sector, would work in a lab bigger than the one the firm owns, this researcher should work in a university lab and then act as a consultant. In the anticipation of this event such a firm does not invest at all.

As a consequence, the market for consultants will be in equilibrium. Assume that there are researchers in the economy that are not matched. In the matching stage a firm that invested expects a positive payoff, while a firm that did not invest expects no payoff. In this case, some of the firms that did not invest will not be matched, so that the payoff of firms that did not invest but are matched with a consultant must be equal to zero. If instead all the researchers are employed, we can assume that firms that invested zero receive zero. In both cases, in equilibrium, all the surplus generated by a consultant will go to the consultant; a firm that did invest is better off without a consultant.

Finally, suppose that each firm receives from the government a transfer $\tau(L)$, continuous and differentiable. The private surplus function is now $S(a, L) + \tau(L)$. By lemma 6 in equilibrium $\frac{\partial S(a,L)}{\partial L} + \frac{\partial \tau}{\partial L} = \frac{\partial c}{\partial L}$. In the same way, the constrained efficient investment equilibrium exists and the worst researcher matched is given by $S(\underline{a}, l(\underline{a})) + \tau(l(\underline{a})) = c(l(\underline{a}))$. As far as $\tau(l(\underline{a})) = 0$, finding the equilibrium V and $\underline{a}$ is analogous to the problem solved in the previous section.

The government problem can be formalized in the following way:

$$\max_{L^u(a), \tau(l(a))} \{\nu \int_{\underline{a}}^{\overline{a}} a f(\hat{l}(a)) z(a) da\} \tag{4}$$

$$\text{s.t.} \begin{cases} \hat{l}(a) = \max\{l(a), l^u(a)\} & \text{(I)} \\ G = \int_{\underline{a}}^{\overline{a}} \left(\tau(l(a)) + l^u(a)\right) z(a) da & \text{(II)} \\ l(a) = \left\{L : \frac{\partial S(a,L)}{\partial L}\big|_{a=m(L)} + \frac{\partial \tau}{\partial L} = \frac{\partial k}{\partial L}\right\} & \text{(III)} \\ \frac{\partial l(a)}{\partial a} \leq 0 & \text{(IV)} \\ \underline{a} : \nu \int_{\underline{a}}^{\overline{a}} a f(l(a)) z(a) da = c(l(\underline{a})) + g(\underline{a} f(l(\underline{a}))) & \text{(V)} \\ \tau(L) \geq 0 & \text{(VI)} \end{cases}$$

where $\hat{l}(a)$ are the labs in use in the economy, some of which are private $l(a)$ and some of which belong to universities $l^u(a)$. The first constraint says that whenever researchers can choose between universities and private labs, they will work in the biggest lab. The second line is the government budget constraint. The following three say that the investment in labs induced by the government by means of subsidies is an equilibrium. The last line restricts $\tau(L)$ to be a subsidy rather than a tax.

It is possible to characterize the solution to the government problem.

Proposition 11. *In the university research sector, better researchers work in bigger labs.*

Proof. In building university labs, the only constraint that matters is constraint (II). Therefore, the government will set:

$$f'(l^u(a)) = \left(\frac{a'}{a}\right) f'(l^u(a'))$$

for all a and a' working in the university sector. □

Proposition 12. *All firms receiving subsidies invest the same amount.*

Proof. The allocation of labs in the university research sector is not achievable using subsidies because of constraint (IV). Therefore if the government uses subsidies, constraint (IV) is binding:

$$l(a) = \bar{l}$$

for all $l(a)$ receiving a positive subsidy. □

Proposition 13. *University labs are bigger than subsidized private labs.*

Proof. If this were not the case, the government could save money by turning some university labs into subsidized private labs. It also implies that the government will allocate the best researchers to the university sector. □

Figure 3 provides a careful illustration of the problem. In the top graph, the shaded area represents the cost borne by the government. In the bottom graph, the shaded area represents the increase in V due to government intervention.

The government problem is too complicated to be solved analytically. Therefore, I resort to numerical methods in order to determine when the government should subsidize, build universities or do both (the details of the simulation are in the appendix). The results are reported in figures 4 and 5.

In figure 4 different quadrants report the optimal distribution of labs for different values of $\bar{a}$ and G (G increases going from left to right, and $\bar{a}$ increases going from the top down). Figure 5 summarizes the results of the same exercise for a wider range of $\bar{a}$ and G. In both figures it is evident that, if the quality of the best researcher increases, the government is more likely to build university labs. When a researcher is very productive, the lab that he would work with in the private sector is very small: building universities allows the government to allocate more resources on the most productive researchers. Finally, figure 5 shows that when the government has more resources, it is more likely to use a mix of university labs and subsidies, rather than only one of the two policies.

The government's policies increase the equilibrium V. Compared to the economy without a government, now more entrepreneurs invest and more researchers are matched. This is

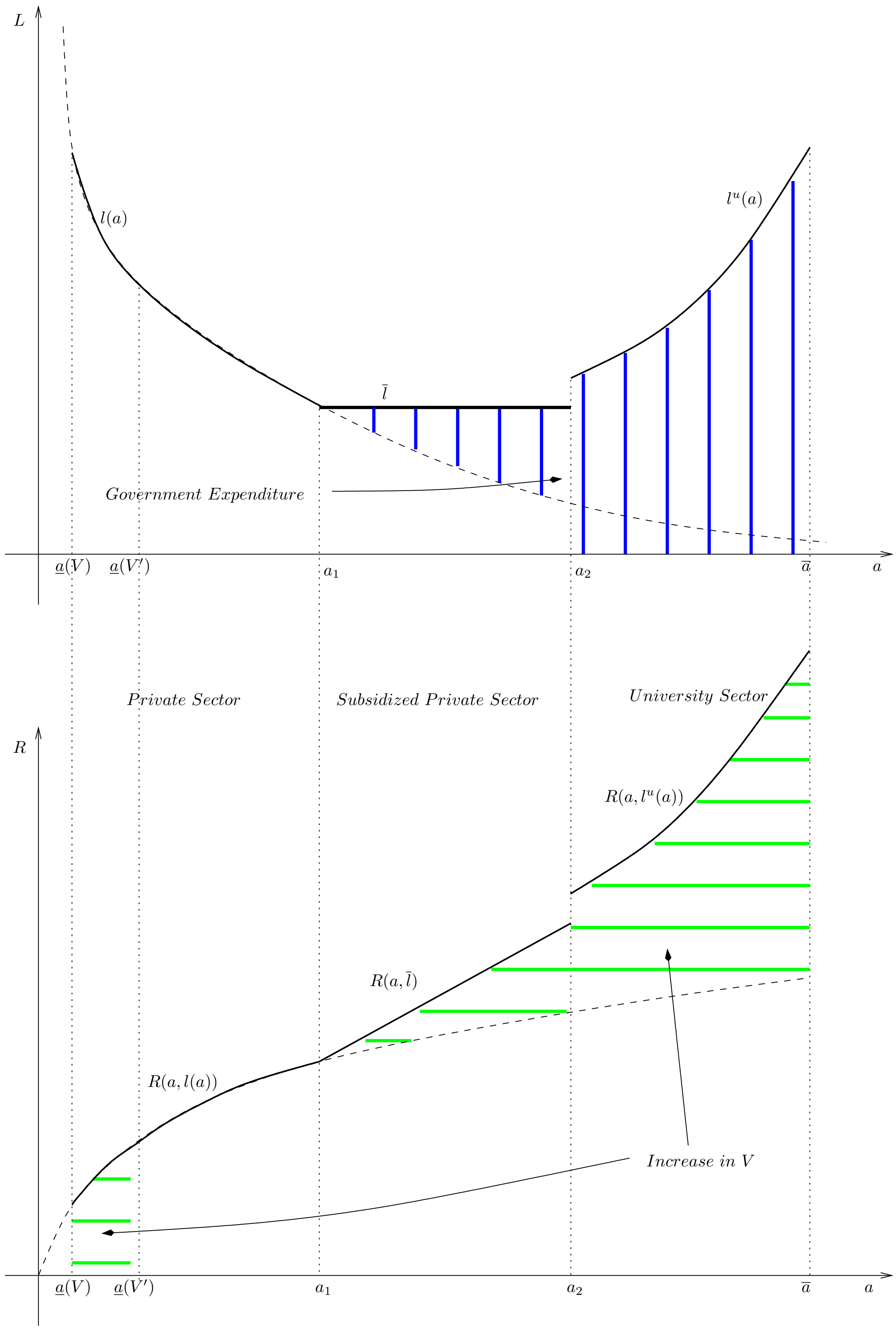

Fig. 3: Cost and Benefit of Government Intervention.

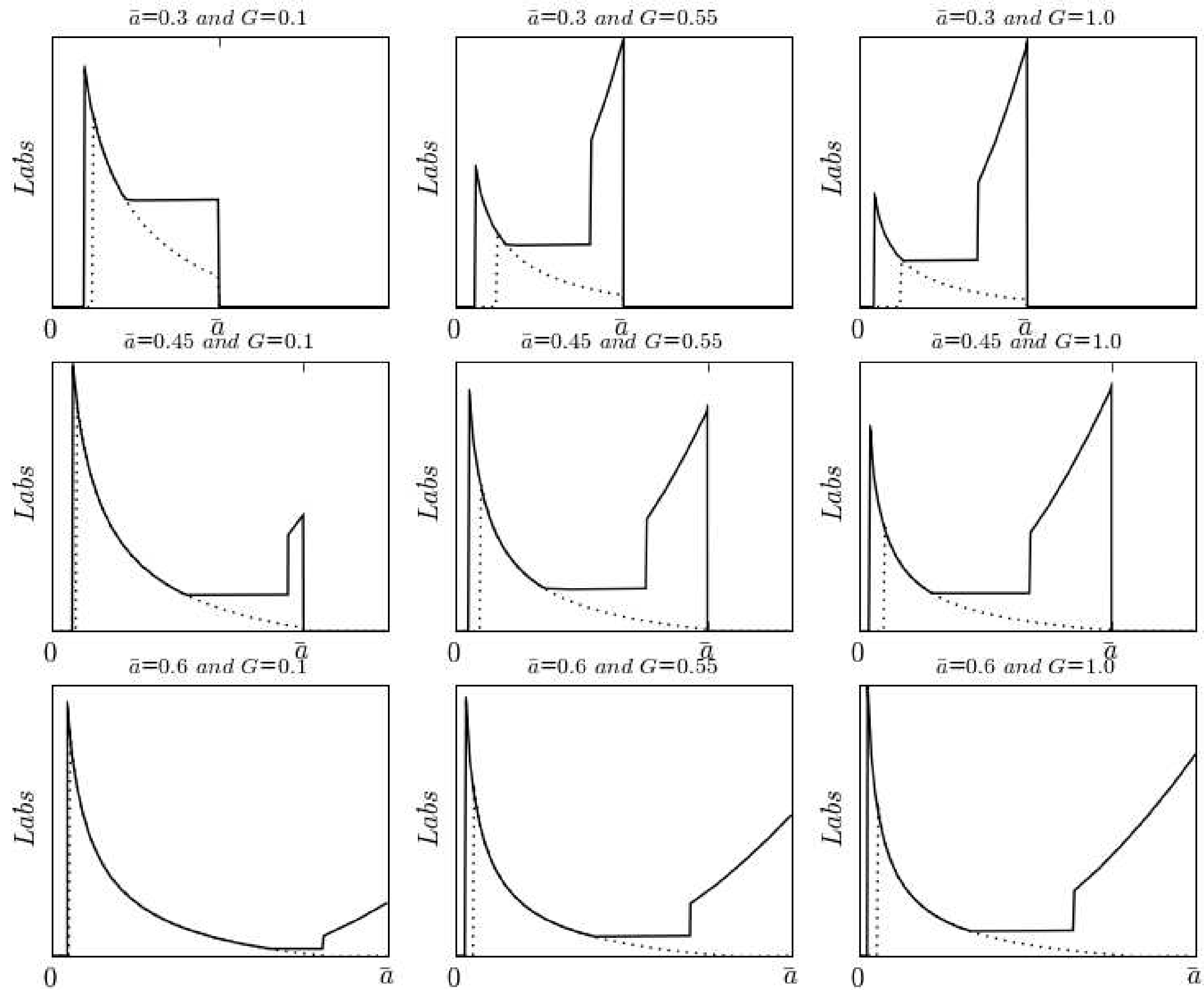

Fig. 4: Optimal Distribution of Labs (dotted line, no government; solid line, with government).

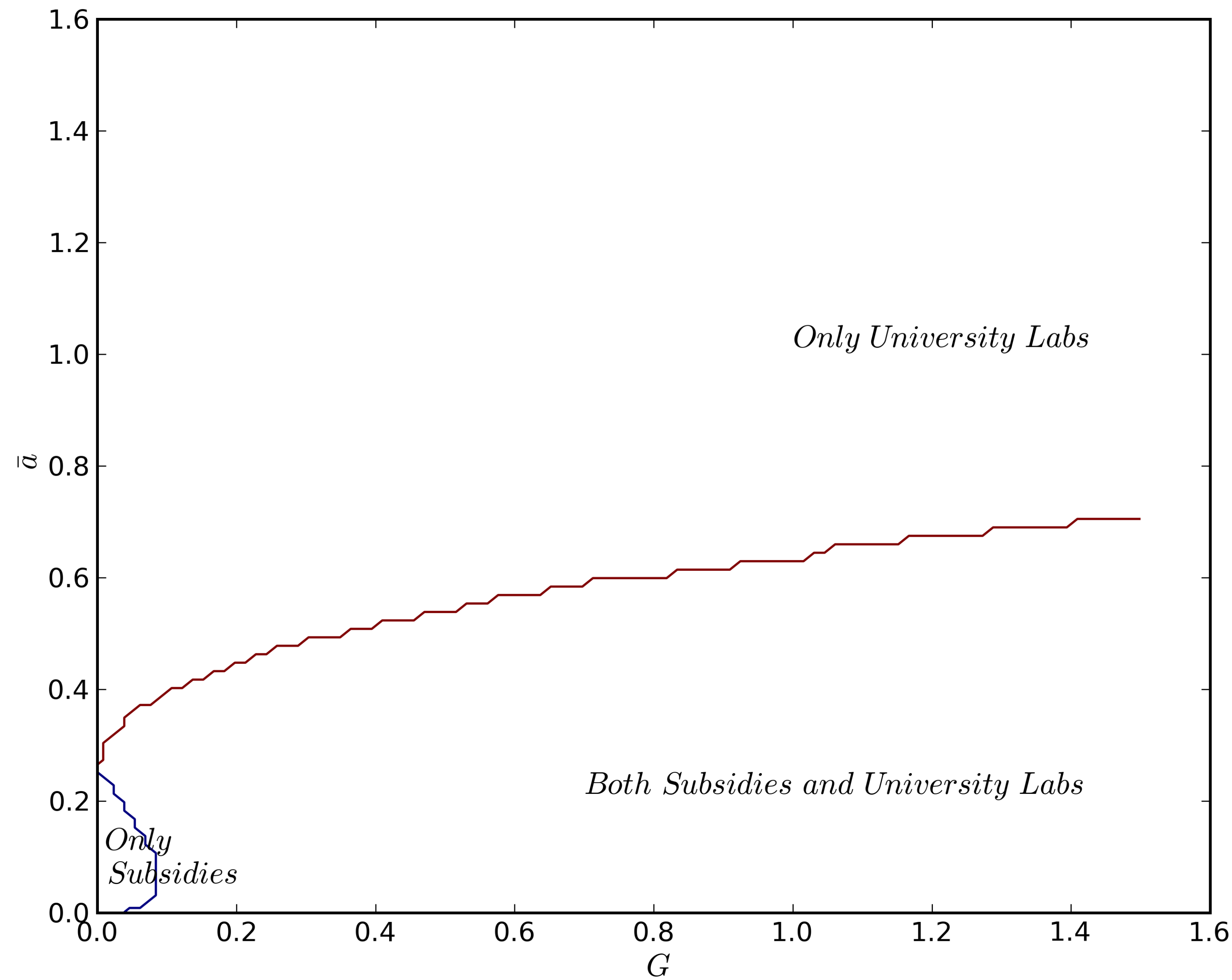

Fig. 5: Optimal Policy.

represented in the bottom graph of figure 3 by a decrease in $\underline{a}$ from $\underline{a}(V')$ (where V' is the stock of knowledge before government intervention) to $\underline{a}(V)$. Whether university research complements or substitutes private research depends on the number of new firms investing in research compared to the number of firms that stop investing because of the creation of university labs. In figure 4, the researchers joining the university sector would work with small labs in the private sector, so there is little decrease in private investment if the government increases its expenditure. Simulations (not reported) carried out for several parameters values always found private and university research to be complements. These findings are consistent with the empirical literature. David, Hall, and Toole (2000) review the existing econometric evidence trying to establish if university and private research are substitutes or complements. They report that most of the papers looking at aggregate measures find a complementarity effect, while, at the single firm level, there seems to be evidence of a substitution effect.

7 Reputation

Since the work of Merton (1957), it is well known that researchers care about reputation. Merton calls it the *race for priority*: scientists want to be recognized as the first to discover something. The role of reputation in science has already been explored in the economic literature by Dasgupta and David (1985). The general conclusion is that, on the one hand, reputation motivates researchers. This is very important because an incentive scheme based exclusively on the quality of scientific output would be very hard to implement. Second, it fosters openness. This guarantees the circulation of ideas and generates a faster pace of scientific progress. Here I will show that reputation may have an additional effect. If researchers care about science, they may be willing to accept a lower payment to work in a firm with a big lab. In equilibrium, good researchers may outbid bad researchers for the right to work in a given firm, therefore changing the matching pattern in the private sector.

Let's assume that the researchers' utility is:

$$U(a) = w(a) + \rho(R(a, l(a)))$$

where $w(a)$ is the net payment received working for the firm, and $\rho()$ is the *reputation concern*: the utility derived from doing science. Researchers may care about science because their future earning depend on it (through the reputation they build today), or simply because they like science. The following lemma shows that if reputation concerns are strong enough the equilibrium allocation of researchers to firms will change.

Lemma 14. *Assume that an equilibrium with positive investment exists. If:*

$$\rho'(x) \geq 0 \, \forall x \tag{5}$$

$$\rho''(x) = g''(x) \, \forall x \tag{6}$$

the equilibrium is PAM.

Proof. See appendix. □

Intuitively, researchers are willing to give up part of their payment in order to work in a firm with a bigger lab. Because of the complementarity between labs and researchers, a productive researcher is always willing to give up more than an unproductive researcher for the right to work in a firm with a given lab. Therefore, the final allocation of researchers to labs depends on the first derivative of $\rho()$: how fast the utility grows with the amount of research produced. Note also that a similar conclusion will be true even if condition 6 is not satisfied. In this case $\rho'(x)$ should be greater than a very complicated expression involving both $g''(x)$ and $\rho''(x)$ (see the appendix for more details).

To conclude, I show that, for any $\rho()$ that satisfies lemma 14 under assumptions 7 and 8, there exists an equilibrium.

Proposition 15. *Consider a $\rho()$ that satisfies lemma 14. Under assumptions 7 and 8, an equilibrium with zero research always exists. If the commercial value of research ν is high enough, there are also equilibria where a positive amount of science is produced.*

Proof. See appendix. □

It is easy to derive the net payment schedule that should emerge in the market when reputation concerns have the form described in lemma 14.

Lemma 16. *Consider a $\rho()$ that satisfies lemma 14. In the economy:*

$$w'(a) = -g'(R(a, l(a)))R_1(a, l(a)) - R_2 l'(a)\rho'(R(a, l(a)))$$

Since $g' < 0$, lemma 16 says that if $\rho'()$ is high enough, $w'(a)$ will be negative. In other words, if reputation concerns are strong, good researchers will receive a lower net payment than unproductive researchers because they expect to receive a higher future reward. Since, when the allocation is PAM, the disutility $g()$ is decreasing in ability, this implies that the equilibrium gross payment (the wage) is decreasing in a.

Therefore, the model is consistent with Stern (2004). In his paper "Do Scientists Pay to be Scientists?" the author collects data on job offers received by a sample of biology

Ph.D. job market candidates. He finds that firms engaged in science offer wages 25% lower than firms that are not engaged in science. The author interprets his results against the absorptive capacity hypothesis: firms giving a positive value to the production of science should pay researchers that are involved in science more. The alternative explanation is based on reputation concerns: firms do science as a way to reward scientists by letting them build their reputation. Lemma 16 shows that the two explanations can coexist.

Finally, it is possible to sketch what happen in a model with reputation concerns, universities, and subsidies. Clearly, if reputation concerns satisfy lemma 14, there is no need for universities and the government can spend all its resources in subsidies. However, if the lemma does not hold, the private sector allocation will be NAM over some range and PAM over some other. Universities may still be necessary to make sure the best researchers receive the biggest labs.

8 Conclusions

There are several reasons for firms to invest in research. The one proposed most often in the economic literature is *production*: firms invest in research because they want to increase the stock of science. Since science is a public good, the private sector will underinvest. There is a need for the government to intervene and take the production of science partially under its control.

A second explanation has been recently proposed. Using outside science is costly to a firm. This cost is lower if firms produce science. Therefore, firms invest in research to enhance their *absorptive capacity*, which is the ability to use the publicly available stock of science.

I build a model where firms build absorptive capacity in order to use outside science. I show that the private sector allocation is inefficient. In the model, there are researchers of different ability levels and firms owning labs of different sizes. The private sector allocates researchers and firms according to NAM: the best researcher works in the smallest lab. However, this matching pattern minimizes the total research produced in the economy.

I modify the baseline model in two ways. First, I introduce universities. I show that the best researchers work in university labs, and that, within the university sector, better researchers work with bigger labs in order to maximize the total amount of research produced.

Finally, I explore the effect of reputation. If researchers care about doing research, the market allocation of researchers to firms may change. In particular, I show that if the reputation concerns are strong enough, the matching pattern emerging in the private sector is PAM: good researchers work in big labs.

The conclusions of the model can be tested in several ways. For example, it should be possible to check whether labs and researchers are substitutes in the private sector. Substitability implies that the increase in revenues following an increase in expenditure in research facilities should be greater in firms with researchers that are less productive. Alternatively, one could check the market allocation of researchers to firms. In this case, however, the test should take into consideration the strength of the reputation concerns. Without reputation, the model predicts NAM. If reputation concerns exist and have the features I derived, we should observe PAM. For example, assuming that old researchers are less sensitive to reputation than young ones, the model predicts that productive young researchers should work in big labs and unproductive young researcher should work in small labs, while productive old researchers should work in small labs and unproductive old researchers should work in big labs.

Introducing absorptive capacity opens interesting policy questions. For example, in this context access to science is a policy instrument. Suppose that firms can learn about new discoveries only by sending their researchers to conferences. A rule that allows researchers from the private sector to participate in conferences only if their scientific contribution is above a certain threshold, may increase the amount of research carried out by the private sector.[10] Also, the way researchers are rewarded is an important determinant of the amount of science produced. It should be possible to transform all the different prizes and awards a researcher may receive during his career into a coherent policy instrument.

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Appendix

Proof of Proposition 4.

The social welfare generated within each match is equal to:

$$SW(a, L) = \nu a f(L) - g(af(L))$$

one obvious difference between the first best allocation and the private sector allocation is in who is matched. In the private sector, researchers and labs are matched if $V \geq g(af(L))$. Note that V is determined endogenously, and that there are multiple equilibria. However, the private sector condition for being matched is, in general, different than the social optimal one.

Going back to the matching pattern, note that NAM is inefficient only under some conditions on ν. To see this, imagine that the economy is so unproductive (low ν) that both from the social point of view and from the private point of view, nobody should be matched. In this case any matching pattern will lead to the same welfare (zero) so that NAM is trivially efficient.

It is easy to show that $SW_{12} > 0$ if:

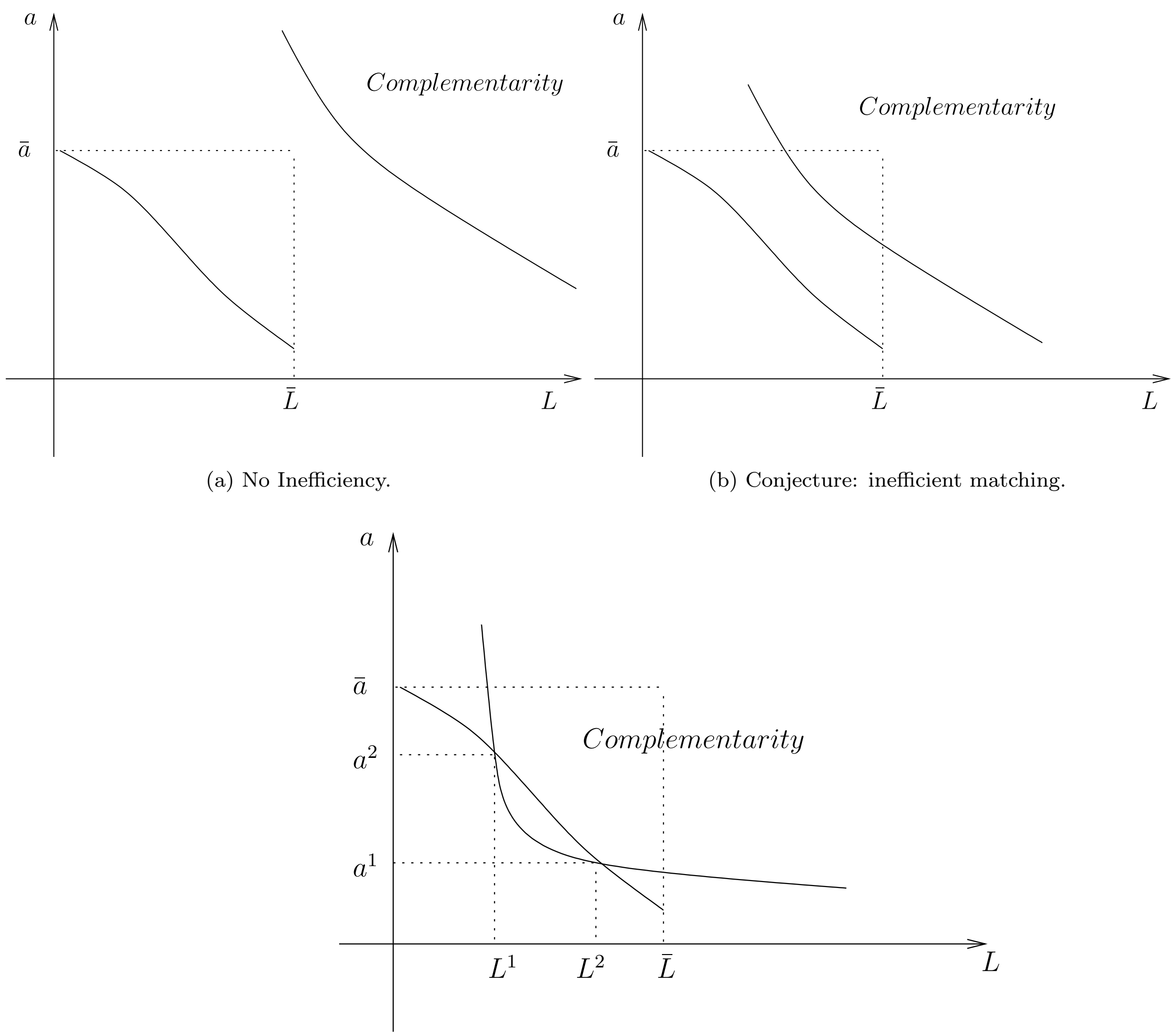
$$\nu > g'(x) + xg''(x)$$

Given this, we can be in one out of three possible situations. The first one is illustrated in figure 6a. In this case there is no complementarity in the relevant range of the social welfare function and NAM is efficient. Imagine now to increase ν. The area of complementarity expands, and eventually we reach the situation illustrated in figure 6b. In this case, it is possible for the social planner to reallocate some researchers and some labs in order to have an area of PAM. However, this leaves some unmatched agents, that should be re-matched somehow. Whether this deviation increases social welfare or not is left to be determined in future works. If ν is even higher, eventually the economy will reach the situation depicted in figure 6c. In this case it is possible to rematch researchers between a^1 and a^2 with labs from L^1 and L^2 according to PAM and increase the social welfare.

Proof of Lemma 6.

Define the following set

$$\Lambda = \{i(p) | p \in P\}$$



(a) No Inefficiency.

(b) Conjecture: inefficient matching.

(c) Inefficient matching.

Fig. 6: Complementarity range and matching function.

as the set containing all the L build by some firm. Consider two different L and L' in Λ. Stability and feasibility imply:

$$S(m(L),L) - x(L) \geq S(m(L),L') - x(L')$$

and:

$$S(m(L'),L') - x(L') \geq S(m(L'),L) - x(L)$$

taken together:

$$S(m(L),L) - S(m(L)L') \geq x(L) - x(L') \geq S(m(L'),L) - S(m(L'),L')$$

Write $L' = L + \epsilon$ and divide by ϵ. Let $\epsilon \to 0$. It implies that $x(.)$ is differentiable and:

$$\frac{\partial x(L)}{\partial L} = \frac{\partial S(a,L)}{\partial L}|_{a=m(L)}$$

Consider now L not in Λ. For example, imagine that a firm is thinking of investing $L > \sup\{\Lambda\}$. Because of NAM next period, such a firm would know that it will end up matched with $\underline{a}$, the lowest a matched in the economy, having to pay $w(\underline{a})$, since the deviation off the equilibrium path of a single firm does not affect the equilibrium $x(.)$ and $w(.)$. The expected payoff is:

$$x(L) = S(\underline{a},L) - w(\underline{a})\, \forall L > \sup\{\Lambda\}$$

that implies again:

$$\frac{\partial x(L)}{\partial L} = \frac{\partial S(a,L)}{\partial L}|_{a=\underline{a}}$$

The cases $L \leq \inf\{\Lambda\}$, and $\inf\{\Lambda\} \leq L \leq \sup\{\Lambda\}$ but $L \notin \Lambda$, lead to the same conclusion using the same logic.

Proof of Proposition 9.

For the first part, note that if firms expect $V = 0$, they have no reason to invest in research. Therefore, the total science produced will be zero.

Consider an equilibrium with positive investment. In general, if all the researchers and all the entrepreneurs in the economy were matched, the worst member of each group could enjoy a strictly positive payoff. In our case, since the worst researcher in the economy is $a = 0$ and $\lim\limits_{a \to 0} S(a,L) = -\infty$, there is always someone that is not matched. In addition, since all firms are identical ex-ante, they should all earn zero profits in equilibrium. Consider the match between the firm that invested the most and the worst researcher. The firm and the researcher both receive zero. The equilibrium $\underline{a}$ and V are the solutions to:

$$\underline{a} = \{a : S(a, L(a)) = c(L(a))\} \tag{7}$$

and:

$$V = \nu \int_{\underline{a}}^{\overline{a}} af(L(a))z(a)da \tag{8}$$

The equilibrium with positive investment exists if there is a $\{\underline{a}, V\}$ solution to equations 7 and 8.

Note that equation 8 has a finite value at $\underline{a} = 0$, is equal to zero at $\underline{a} = \overline{a}$, and is strictly decreasing. Finally, equation 7 can be rewritten as:

$$V = c(L(\underline{a})) + g(\underline{a}f(L(\underline{a}))) \tag{9}$$

Because of assumption 8, if $\underline{a} \to 0$ the solution to 9 diverges to infinity, has finite values for $\underline{a} \in (o, \overline{a}]$, and is continuous. Therefore, if ν is high enough, equations 7 and 8 will cross.

Proof of Proposition 10.

The social welfare generated in each match is equal to:

$$SW(a, L) = \nu af(L) - g(af(L))$$

the private surplus is:

$$S(a, L) = V - g(af(L))$$

clearly, a transfer like the one described transforms the private surplus into the social welfare function. Finally, because of lemma 6, when firms invest they equate marginal cost to marginal benefit. In this case, it implies that firms' investment is efficient.

Details of the Simulation.

I choose the following functional forms:

- $c(L) = (1 + r)L$
- $R(a, L) = af(L) = a(1 + L)^{\frac{1}{2}}$
- $g(R(a, L)) = \frac{1}{a(1+L)^{\frac{1}{2}}}$

and I assume that $\tau(l(\underline{a})) = 0$: the firm matched with the worst researcher receives no subsidy. This can be seen as a restriction on the amount of resources the government has. Note that all firms are identical.

The government problem can be written as:

$$\max_{l^u(\overline{a}),\bar{l},a_1,a_2} \{\int_{a_1}^{a_2} a(1+\bar{l})^{\frac{1}{2}}\,da + \int_{a_2}^{\overline{a}} a^2(1+l^u(\overline{a}))^{\frac{1}{2}}\,da - \int_{a_1}^{a_2} a(1+l(a))^{\frac{1}{2}}\,da\} \tag{10}$$

$$\text{s.t.} \begin{cases} \bar{l} = \left(\frac{1}{2(1+r)a_1}\right)^{\frac{2}{3}} - 1 & (1) \\ a_1 \leq a_2 \leq \overline{a} & (2) \\ \left(\frac{a_2}{\overline{a}}\right)(1+l^u(\overline{a})) - 1 \geq \bar{l} & (3) \\ \int_{a_1}^{a_2} \left(\bar{l} - \max\left\{\left(\frac{1}{2(1+r)a}\right)^{\frac{2}{3}} - 1, 0\right\}\right) da + \int_{a_2}^{\overline{a}} \left[\left(\frac{a}{\overline{a}}\right)^2 (1+l^g(\overline{a})) - 1\right] da = G & (4) \end{cases}$$

Figure 3 on page 17 represents it graphically. The objective function is the extra research produced thanks to the policy in place (the shaded area in the lower axes) at a cost summarized by constraint (4) and represented by the shaded area in the upper axis. Note that the increase in research at the bottom of the distribution of labs (between $\underline{a}(V)$ and $\underline{a}(V')$) can be safely ignored since it is an increasing function of the extra research V produced by the rest of the economy.

The simulation simply compares values of the objective function at different a_2 and $\bar{l}$. The aim is not to determine the exact optimal policy, but to check whether there is an interior solution (both subsidies and university labs) or one of the two corner solutions (only subsidies, only university labs).

I construct a grid $\{0, ..., \overline{a}\}$ containing all possible values of a_2. For every value of a_2, I construct a grid of possible value of $\bar{l} \in \left\{\left(\frac{1}{2(1+r)a_2}\right)^{\frac{2}{3}} - 1, ..., \tilde{l}\right\}$ where $\tilde{l}$ is an appropriate large number. For every a_2 and $\bar{l}$ I compute $l^u(\overline{a})$ using constraint (4) of 10. I consider the pair a_2 and $\bar{l}$ admissible if $l^u(a_2) = \left(\frac{a_2}{\overline{a}}\right)(1+l^u(\overline{a})) - 1 \geq \bar{l}$. Finally, I compute the value of the objective function. The final solution is the admissible pair $\{a_2, \bar{l}\}$ returning the highest value.

Finally, in the standard simulation, the value for r is 0.01 and for ν is 100. When checking for the complementarity or substitability of private and university research, the parameters I tried are: $\underline{a} \in [0, 5]$, $G \in [0, 5]$, $r \in \{0.01, 0.1, 1\}$ and $\nu \in \{75, 100, 150\}$; technical reasons restricted the choice of ν; the other parameters were picked arbitrarily.

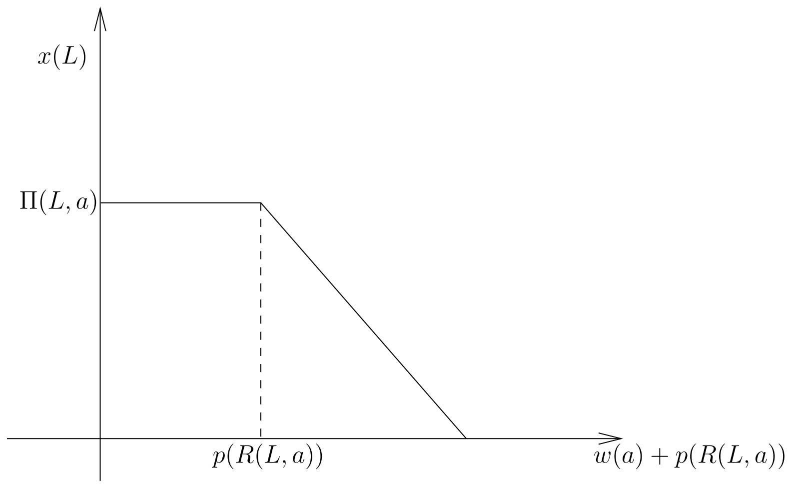


Fig. 7: Utility Possibility Frontier.

Proof of Lemma 14.

Figure 7 represents the utility possibility frontier of a match. For a given distribution of labs, whenever the equilibrium payoffs lie on the 45 degrees part, under lemma 14 the equilibrium matching is PAM. The reason is that the total surplus function $S(a,L)+\rho(R(a,L))$ (perfectly transferable between researchers and firms) is supermodular: firms with bigger labs are better off by matching with more productive researchers, and vice versa.

However, since the wage cannot be negative, the utility possibility frontier has a kink. At the kink, researchers receive $\rho(R(a,L))$ and firms receive $S(a,L)$. Again, for given distribution of labs, the payoff of each side is increasing in the other side's type. This implies that the equilibrium is PAM for these agents as well.

Finally, note that both sides prefer to be matched with a high type than with a low type, even when it means switching from the kink region to the the 45 degree region. This implies that the equilibrium is PAM overall.

To conclude the proof, I need to show that the investment made by firms is consistent with PAM. Once again, if the equilibrium split is on the 45 degree line, the model is a standard transferable utility one and the optimal investment solves:

$$l(a)=\left\{L|\frac{\partial\left[S(a,L)+\rho(R(a,L))\right]}{\partial L}|_{a=m(L)}=\frac{\partial c(p.L)}{\partial L}\right\}$$

using the implicit function theorem it is possible to show that $l'(a)>0$. On the other hand, when there is a corner solution, the optimal investment solves:

$$l(a) = \left\{ L | \frac{\partial S(a,L)}{\partial L} + \frac{\partial S(a,L)}{\partial a} m'(L) = \frac{\partial c(p,L)}{\partial L} \right\} \tag{11}$$

Note that $m'(L)$ is just $[l'(a)]^{-1}$. Therefore the optimal investment is given by:

$$l'(a) = \frac{\frac{\partial S(a,L)}{\partial a}}{\frac{\partial c(p,L)}{\partial L} - \frac{\partial S(a,L)}{\partial L}}$$

that is positive, since the denominator is greater than zero by 11.

Proof of Proposition 15.

Since lemma 14 imposes restrictions only on the slope of $\rho()$ I can normalize $\rho(R(\underline{a}, l(\underline{a}))) = 0$. This implies that, as before, the worst researcher matched is given by:

$$S(\underline{a}, l(\underline{a})) = c(l(\underline{a}))$$

and the value of the total stock of science in the economy is given by:

$$V = \nu \int_{\underline{a}}^{\overline{a}} af(l(a))z(a)da$$

This problem is identical to the one solved in the proof of proposition 9.

Proof of Lemma 16.

By stability, whenever $w(a) > 0$:

$$S(a, l(a)) + \rho(R(a, l(a))) - x(l(a)) \geq S(a, l(a')) + \rho(R(a, l(a'))) - x(l(a'))$$

Write the same condition for a', and take limits for $a' \to a$:

$$x'(L) = \frac{\partial S}{\partial L} + \rho' \frac{\partial R}{\partial L}$$

By feasibility:

$$S(a, l(a)) = x(l(a)) + w(a)$$

Differentiate both sides with respect to a and use the stability condition. The conclusion follows by simple algebra.