Experience and Worker Flows

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Abstract

This paper extends the literature on learning in labor markets by parameterizing the amount of learning that transfers across jobs. Previous models have assumed that learning is either job specific as in Jovanovic (1979) or perfectly transferable across jobs as in Gibbons et al. (2005). By allowing some but not all learning to be transferred, this model generates novel predictions of a decline in job finding rates with age and a decline in the variance of wages with experience that are consistent with observed worker outcomes.

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1 Introduction

Learning models account for many important features of labor market behavior. Jovanovic’s (1979) early work explains broad features of worker turnover behavior: a hump shaped hazard of separation from a job by tenure and declining separation rates with age. Recent models use learning to understand wage dispersion, wage growth, and occupational mobility (for example Moscarini (2005), Farber and Gibbons (1996), Gibbons et al. (2005), and Papageorgiou (2007)).

For analytical tractability, the literature on learning focuses on models that make stark assumptions about the form of learning. On one hand, matching models like Jovanovic (1979) assume that all learning is specific to a particular job. A worker’s performance on a particular job provides information only about that job. On the other hand, sorting models take learning to be about a worker’s ability. In these models, a worker’s performance on one job generates knowledge about her performance on all other jobs equally. Workers then use their current belief about their ability to sort themselves into the most profitable job. These assumptions are stark as workers learn about their ability on a particular job and some but not all of this information is useful in determining their productivity in other potential endeavors.

This paper constructs a search model to bridge the gap between these extreme assumptions in the literature. The model extends the matching framework to allow agents to learn not only about their current match, but also allow past learning to be useful in discerning the quality of their prospective matches when unemployed. This initial screening is similar to Jovanovic (1984), however the amount of information contained in the signal depends on worker’s past experience. Workers learn rapidly about their ability on a particular match and some but not all of this learning carries over into future matches. The model parameterizes how much learning from one job carries over to understanding how productive the worker will be in other job opportunities.

\footnote{For a summary of the literature on learning and a summary of the stylized facts on the distribution of labor earnings see Neal and Rosen (2000).}
The model explains how young workers transition from rapid turnover to stable employment over the life cycle. During the first ten years of labor market experience, workers transition from high job turnover into stable employment and have rapid wage growth. About two-thirds of lifetime job turnover and wage growth occurs during these early years (see Topel and Ward (1992), Flinn (1986)). Initial high turnover manifests itself in both high job finding and separation rates for young workers. The model is able to explain these patterns of behavior through learning and experience.

Additionally, the model captures the well known decline in worker turnover with age (see Clark and Summers (1982)). Past models of turnover have focused on explaining the decline in job separation rates. However, less focus has been paid to the observed decline in job finding rates. The model in this paper replicates attractive features of previous learning models: the decline in unemployment and job separation rates with age and the rise in wages with labor market experience among other predictions. Allowing experience to generate differential amounts of learning about current and future jobs generates novel predictions about the patterns of job finding rates by age and wage dispersion by experience.

The calibrated model generates declining job finding rates with age as experience allows workers to distinguish between good and bad job offers. For inexperienced workers jobs are experience goods; they only learn about the quality of the match by trying it out. However, as workers gain experience jobs become inspection goods. Market experience influences decisions by unemployed workers about which jobs to accept. As their experience grows, they reject more bad jobs causing the job finding rate to decline. The past literature on learning does not generate any prediction on job finding rates. In matching models, learning is completely job specific so employment forms a renewal process as workers are in an identical situation each time they become unemployed. In sorting models, although information transfers between jobs, perfect transfer of information means that workers direct their search to the job that best fits their abilities. Embedding learning across jobs into a matching framework generates a mechanism for past experience to alter a workers search behavior and change their job finding rates.
The calibrated model is then used to generate novel predictions about the volatility of wages in new jobs. The model predicts wage volatility declines with experience. Intuitively, more experience from previous jobs generates more information about new matches. This implies that wages should vary less for workers starting a new job with more past experience. This new implication from the learning model is confirmed by examining wage data from the National Longitudinal Survey of Youth 1979 (NLSY79) data.

While there is a large literature that studies wage variation\(^2\), few have looked at the wage variation within job spells\(^3\). Learning models have been successful in accounting for these facts. Farber and Gibbons (1996) consider a baseline model where all learning is public to provide a theory of observed wage dynamics\(^4\). This paper looks at how past experience influences wage volatility within job spells\(^5\). While the cross sectional literature finds that wage variation grows as cohorts age (see Rubinstein and Weiss (2006) and Gibbons et al. (2005)), the result that more past experience reduces individual wage volatility implies that some of this cross sectional variation is predictable from the individual perspective\(^6\).

Related to the literature on learning is an empirical literature that examines the transferability of human capital across jobs. McCall (1990) explores whether human capital is job or occupation specific in a similar search model. While my model does not have occupation specific human capital it does capture McCall’s (1990) finding that longer tenure in the first job implies lower hazard rates in future employment as experience allows workers to reject poor second matches. Altonji and Shakotko (1987), Topel (1991), and Altonji and

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\(^3\)An exception is the literature on careers in organizations. Baker et al. (1994) summarizes the stylized facts on wages within the firm. While these facts conflict with traditional models of the labor market, enriching standard models of learning have been helpful to explain the data. Gibbons and Waldman (2006) provide a nice summary of the literature and a learning model that can account for many of the features in the data.

\(^4\)There is also a small literature that deals with learning, occupational choice, and wages. For examples see: Miller (1984) and Antonovics and Golan (2008).

\(^5\)Jun and Munasinghe (2005) use volatility to try to predict job switching behavior, but do not try to explain observed volatility.

\(^6\)A large literature attempts to distinguish between permanent and transitory income shocks. To correctly measure these shocks it is important to control for both observable and unobservable heterogeneity in the wage process as in Meghir and Pistaferri (2004). Understanding factors that reduce volatility for an individual worker while cross sectional inequality rises can help to better understand the wage process.
Williams (2005) examine the extent to which wages rise with tenure in a given job rather than through total job market experience. Learning in my model has both a firm specific and a general effect. While some experience transfers to allow individuals to better identify the quality of future matches, workers learn about the quality of their current job at a faster rate. Although much of wage growth can be accounted for by career experience, there is still a premium for job tenure. Finally, Mincer and Jovanovic (1982) and Bartel and Borjas (1982) explore the relationship between turnover and wage growth. They find that much of wage growth is due to general experience while smaller portions can be attributed to firm experience and mobility choices. These findings are all consistent with model predictions.

The model draws closely from Moscarini (2005) who assumes that jobs are drawn from a distribution of only two types. Moscarini (2005) and Moscarini (2003) use this trick to embed Jovanovic’s (1979) model into a general equilibrium framework and explore implications for the wage distribution. Papageorgiou (2007) extends these models to explore occupational choices. An empirical literature related to Papageorgiou (2007) on career and job specific matches seeks to explain the decline in turnover during the life cycle. Neal (1999) presents a model where workers search for both a career and job specific match. The empirical implications of career and job matches for job turnover and wages are explored in Pavan (2007) and Pavan (2006) respectively. My model generates observed declines in job finding and separation rates without adding the complexity of a second type of career match.

The paper proceeds as follows. Section 2 presents the model. Section 3 describes how the parameters of the model are chosen. Section 4 presents the results from the calibrated model about job finding and separation rates, unemployment and wage growth. Section 5 shows that the model predictions about wage volatility are consistent with data from NLSY79. Section 6 concludes.
2 Model

This section describes the economic environment of an individual making optimal decisions when faced with uncertain production opportunities (jobs). She searches for production opportunities and when confronted with one she learns about its quality.

2.1 Production

The infinitely lived worker has preferences given by:

\[ U = \sum_{t=1}^{\infty} \beta^{t-1} c_t \]

There is no storage technology. The worker makes two decisions: when matched with an opportunity she decides between quitting to search for a new opportunity and continuing to produce and when unmatched she choose to accept or reject opportunities as she finds them.

Production occurs when a worker is matched with a productive opportunity. In each period, a match of type \( \mu \) produces output:

\[ x_t = \mu + z_t \]

where \( z_t \sim N(0, \sigma^2) \) is independently and identically distributed noise on the output process. Therefore, \( x_t \sim N(\mu, \sigma^2) \).

As in Moscarini (2005), the economy is composed of two types of opportunities: \( \mu \in \{\mu_h, \mu_l\} \). Let \( \mu_h > \mu_l \) so that \( \mu_h \) denotes the productivity of a good opportunity and \( \mu_l \) denotes the productivity of a bad one. All production opportunities are drawn independently from the same distribution where a fraction \( p_0 \) of them are of type \( \mu_h \).

2.2 Learning

The worker is uncertain about the quality of her current production. She learns about the quality of the match in two ways. First, while employed she observes her output in the
current production opportunity and updates her beliefs about the quality of the match using Bayes’ rule. Second, when an unmatched worker finds a new opportunity she receives a signal about its quality that depends on her past experience.

While matched, workers observe the output they produce in each period and update their beliefs. Given the normality of output noise, for any current belief, \( p \), the expected distribution of output is given by:

\[
\psi(x|p) = p \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x - \mu_h}{\sigma} \right)^2} + (1 - p) \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x - \mu_l}{\sigma} \right)^2}
\]

With probability \( p \) output is drawn from a normal distribution with mean \( \mu_h \) and variance \( \sigma \), while with probability \( 1 - p \) it is drawn from a normal with mean \( \mu_l \) and the same variance.

Using this known distribution of output, the worker observes her production and uses it to update her belief about the probability that she has a good match using Bayes’ rule. Given any current belief, \( p \), and observed output for a given period, \( x \), the updated belief, \( p' \), is formed by conducting a probability ratio test:

\[
f(p, x) \equiv p' = \frac{p e^{-\frac{1}{2} \left( \frac{x - \mu_h}{\sigma} \right)^2}}{p e^{-\frac{1}{2} \left( \frac{x - \mu_h}{\sigma} \right)^2} + (1 - p) e^{-\frac{1}{2} \left( \frac{x - \mu_l}{\sigma} \right)^2}}
\]

Here the numerator is proportional to the joint probability of observing output \( x \) and the match being good where the denominator is the total probability of observing output \( x \).

With this updating function, define the inverse function \( f^{-1}(p'|p) \) to be the \( x \) required to have posterior \( p' \) given prior \( p \). This function is given by:

\[
f^{-1}(p'|p) = \frac{\sigma^2}{\mu_h - \mu_l} \ln \left( \frac{p'(1 - p)}{(1 - p')p} \right) + \frac{\mu_h + \mu_l}{2}
\]

Define the distribution \( G(p'|p) \) as the distribution of updates beliefs after observing one period of output given a current belief \( p \). Then the p.d.f. of the \( G \) distribution, \( g \), is given
by:

\[ g(p'|p) = \psi(f^{-1}(p'|p)|p) \left| \frac{df^{-1}(p'|p)}{dp'} \right| = \psi(f^{-1}(p'|p)|p) \left( \frac{\sigma^2}{p'(1-p')(\mu_h - \mu_l)} \right) \]

The process of on the job learning can be generalized beyond the specified output process to be some distribution \( G \) that depends on the value of the current belief, \( p \), so the distribution of updated beliefs, \( p' \), is given by \( G(p'|p) \). For a general learning process, two restrictions are made on \( G \). First, \( G \) is non-degenerate so that the signal conveys some information about \( p \). Second, \( G \) is restricted so that \( p \) is a martingale. This is a natural restriction since \( G \) is used to update an individual’s current beliefs.

When meeting a new match the worker gets an initial signal about the quality of the match that depends on her past experience. She receives a signal that is equivalent to observing \( \alpha \tau + 1 \) observations from the output process. Where \( \tau \) is months of past work experience and \( \alpha \in [0,1] \) determines the fraction of experience that carries over from past jobs into information about new offers. This assumption normalizes the information that a worker with no experience gets to be equivalent to observing one period of output from the production process. The normality assumption makes non-integer observations well defined. Moreover, normality implies that to update beliefs after viewing \( t \) observations the worker only needs to know her prior belief \( p \), the average value of the observation \( \bar{x} \), and the number of observations observed \( t \), not the entire list of observations \( x_1, x_2, \ldots, x_t \). For a worker who observes \( t \) periods of output, the distribution of the average output per period, \( \bar{x} \), is given by:

\[ \tilde{\psi}(\bar{x}; p, t) = p \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\bar{x} - \mu_h}{\sigma \sqrt{t}} \right)^2} + (1 - p) \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\bar{x} - \mu_l}{\sigma \sqrt{t}} \right)^2} \]

Using the same updating strategy, the posterior after observing the output from \( t \) periods
is computed as:
\[
\tilde{f}(p, \bar{x}, t) = \frac{pe^{-\frac{1}{2}\left(\frac{\bar{x} - \mu_h}{\sigma}\right)^2}}{pe^{-\frac{1}{2}\left(\frac{\bar{x} - \mu_h}{\sigma}\right)^2} + (1 - p)e^{-\frac{1}{2}\left(\frac{\bar{x} - \mu_l}{\sigma}\right)^2}}
\]

Again, inverting \(\tilde{f}\) gives the value of \(\bar{x}\) needed to generate posterior \(p\): \(\tilde{f}^{-1}(p', p, t) = \bar{x}\). Define \(H(p'|\tau)\) as the distribution of initial beliefs from a new production opportunity. Hence the p.d.f. of the \(H\) distribution, \(h\), is given by:
\[
h(p'|\tau) = \psi(\tilde{f}^{-1}(p', p0, \alpha \tau + 1); p0, \alpha \tau) \left(\frac{\sigma^2}{p'(1 - p')(\mu_h - \mu_l)}\right)
\]
where \(\alpha\) and \(p_0\) are parameters. \(p_0\) is the prior probability that any new opportunity is good.

The distribution \(H(p'|\tau)\) can be generalized beyond the specific normality assumptions described above. In general, for \(H\) to provide more information about jobs it must be weakly increasing in \(\tau\) in terms of second order stochastic dominance. This means that for \(\tau_1 > \tau_2\):
\[
\int_0^x H(p'|\tau_1) - H(p'|\tau_2)dp' \geq 0 \quad \forall \ x \in [0, 1]
\]
For higher values of \(\tau\) workers get more initial information about the quality of a job. This increasing information for experienced unemployed workers is the novel feature of the model. A sufficient condition for second order stochastic dominance is that if \(\tau_1 > \tau_2\) then \(H(p'|\tau_1)\) is a mean preserving spread of \(H(p'|\tau_2)\).

### 2.3 Wages

Following Jovanovic (1979), the worker’s period payoffs from the production in the model are given by the expected value of output in each period. Given this output process, the wage received from a worker is given by:
\[
w(p) = p\mu_h + (1 - p)\mu_l
\]
This wage process is an equilibrium in an environment where there are a continuum of production opportunities (firms) that have no cost of entering the market. The production opportunities must make zero expected profits. Under these assumptions any wage process that pays the average wage along with any matching rate, $\lambda$, between workers and production opportunities can be sustained as an equilibrium outcome.

2.4 Value Functions

This section defines the value functions for the worker’s general problem. When employed the worker consumes her wage, $c_t = w(p)$, that depends on the probability that her job is good. The worker can separate from the job for two reasons. First, she could receive an unfavorable signal about the job quality and decide to quit. Second, with exogenous probability $\delta > 0$ an employed worker becomes separated from the job in each period. $\delta$ captures reasons for job separations not captured by the endogenous quits that arise from learning. Possible examples include plant closures or geographic relocation by the worker.

Let $V(p, \tau)$ be the value function for an employed worker with belief $p$ and experience $\tau$. The value is written as:

$$V(p, \tau) = w(p) + \beta \delta U(\tau + 1) + \beta (1 - \delta) \int_0^1 \max \{U(\tau + 1), V(p', \tau + 1)\} G(dp'|p)$$

A worker with belief $p$ and experience $\tau$ gets her expected output $w(p)$. In the next period, she is separated from her job with probability $\delta$, becoming unemployed with experience $\tau + 1$. With probability $1 - \delta$ she is not separated from her job and receives her updated belief from the distribution $G$. Depending on the realization of her updated belief she can choose to remain employed with belief $p'$ and experience $\tau + 1$ or quit to become unemployed with experience $\tau + 1$.

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7This equilibrium concept assumes that production opportunities (firms) are unable to separate workers of different levels of experience when matching. The focus of this paper is to understand the implications of learning on worker’s job decisions. Another interesting question would be to understand how firm’s ability to select workers of different experience levels impacts equilibrium employment outcomes. Such a contribution is beyond the scope of this paper.
Unemployed workers consume the unemployment value $c_t = b$. $b$ is high enough that if a worker knows for certain that a job is bad it is optimal to quit and low enough so that if the worker knows that the job is good that she will work. These assumptions ensure that the worker’s search problem is non-trivial.

When unemployed, the worker with experience $\tau$ gets an offer from the distribution of jobs $H(p'|\tau)$ with exogenous probability $\lambda$. She must choose between remaining unemployed and becoming employed with belief $p'$. If she does not receive a job offer she remains unemployed with the same experience.

Let $U(\tau)$ be the value function for an unemployed worker with experience $\tau$. The value function is given by:

$$U(\tau) = b + \beta(1 - \lambda)U(\tau) + \beta \lambda \max_{p'} \{U(\tau), V(p', \tau)\} H(dp'|\tau)$$

The final assumption is that experience can only be accumulated for a maximum of $T$ periods. This assumption allows the model to be computed. It can be justified on two separate grounds. First, $T$ can be chosen to be large enough so that workers already have nearly perfect information about new production opportunities after $T$ periods of past experience. Second, the finite nature of individual working lives means that workers only accumulate a finite amount of experience before retirement. The assumption implies that the marginal value of additional periods of experience is zero once a worker reaches $T$.

## 2.5 Model Characterization

The general learning framework described above embeds the learning models of Jovanovic (1979) and Gibbons et al. (2005) into a matching framework so the implications of worker job finding and separation rates can be explored. When $\alpha = 0$ there is no learning across jobs and the model is similar to that of Jovanovic (1984) where workers search and get an initial signal about the quality of a match. In the case of $\alpha = 1$, all learning from a particular job carries over to future jobs in that a worker gets a signal about the quality of an initial
opportunity of equal strength to all that they have learned in all past jobs. This embeds the model of Gibbons et al. (2005) into a matching framework. Different choices of $0 \leq \alpha \leq 1$ parameterizes how much information from one opportunity transfers to future ones.

The solution consists of a reservation level of productivity that depends on experience, $\bar{p}(\tau)$ such that workers will accept jobs or continue working as long as $p \geq \bar{p}(\tau)$ and reject offers or quit otherwise. The general model is rich enough to allow for different relationships between experience, the reservation productivity level, the job finding rate, and wages that are explored quantitatively in the next sections of the paper. The rest of this section characterizes these relationships to build intuition about the workings of the model.

First, the reservation productivity level is solved for by setting the value function of a matched worker with the reservation productivity equal to the value of an unmatched worker for each level of experience. That is $\bar{p}(\tau)$ solves:

$$V(\bar{p}(\tau), \tau) = U(\tau)$$

The sign of $\bar{p}'(\tau)$ is indeterminate.

To see why $\bar{p}(\tau)$ might be decreasing in $\tau$ consider the following example. If a worker gets no extra information about the quality of jobs until she gains $t$ units of experience then she gets a perfect signal after, there will be a space of experience just before $t$ that the worker will be willing to accept worse and worse opportunities just to get the payoff from getting $t$ units of experience. In this case, the option value of experience outweighs the current value to the worker and can generate decreasing reservation values.

The reservation productivity level will be increasing if the marginal value of information while employed at the reservation belief is less than the marginal value of information when unemployed. The reservation value increases when $U'(\tau + 1) \leq U'(\tau)$ because extra experience can only impact a worker when unemployed seeking a new job. This condition can be interpreted as requiring that the marginal value of experience for unmatched workers is declining. The direct benefit from the additional unit of experience has to be greater than the option value of the unit of experience for getting more experience later in life.
This intuition is formalized in the following proposition:

**Proposition 1** If $U' (\tau + 1) \leq U' (\tau)$, then $\bar{p}' (\tau) > 0$ for all $\tau \in \{ 0, 1, \ldots , T \}$.

**Proof.** See Appendix. ■

Although, Proposition 1 does not reduce the sign of $\bar{p}' (\tau)$ to restrictions on model parameters, it provides clear intuition for when the reservation belief will be increasing in experience. The condition that guarantees $\bar{p}' (\tau) > 0$ is:

$$V_\tau (\bar{p}(\tau), \tau) \leq U' (\tau)$$

Using the results on the worker’s reservation decision above, it is useful to consider the behavior of the job finding rate. In the model, the job finding rate is determined by the exogenous rate of matches combined with the workers willingness to accept production opportunities. The job finding rate as a function of experience, $f(\tau)$, is given by:

$$f(\tau) = \lambda (1 - H(\bar{p}(\tau), |\tau|))$$

Taking the derivative with respect to $\tau$ gives:

$$f'(\tau) = -\lambda h(\bar{p}(\tau)|\tau)p'(\tau) - \lambda H_\tau (\bar{p}(\tau), |\tau)$$

where $h(\bar{p}(\tau)|\tau)$ is the pdf of $H$. For the job finding rate to be decreasing in $\tau$, it is sufficient that $\bar{p}'(\tau) \geq 0$ and $\bar{p}(\tau) \leq p_0$. The first condition guarantees that the first term is negative and the second condition guarantees that the second term is negative because $H(p|\tau)$ is a mean preserving spread around $p_0$. These conditions generate declining job finding rates early in workers lives where $\bar{p}(\tau) \leq p_0$ as workers accept most jobs to gain experience.

The final implications of the model involve the process of the worker’s current belief $p$ while employed. Given the binary structure of productive opportunities in the model, the precision of a worker’s beliefs is $\frac{1}{\bar{p}(1-\bar{p})}$. As in Moscarini (2005), this precision does
not necessarily increase over time. However, as the worker gets additional signals about her current match quality the precision increases on average. This implies that beliefs will change by a greater degree the further they are from 0.5 as \( p(1 - p) \) is maximized at that value. While the model in discrete time does not provide a closed form solution for the standard deviation of \( G(p'|p) \), the above intuition shows that the standard deviation is decreasing in \( p \) if \( p > 0.5^8 \).

Given the wage process:

\[
w(p) = p\mu_h + (1 - p)\mu_l
\]

the behavior of \( p \) can be used to make predictions about the standard deviation of wages. The novel feature of the model is that a worker with more experience who starts in a new match will have more information about the quality of that match than a worker with less experience. In the case where \( \bar{p}(\tau) > 0.5 \) and is increasing, the model would predict that more experience translates to a higher value of \( p \) at the start of a new job and a lower variation in the path of future wages. These implications are quantitatively evaluated with simulations of the model.

3 Calibration

To parameterize the model, assume that there are a large number of workers facing identical decision problems. Each worker faces a different history of idiosyncratic shocks. Averaging outcomes across workers, aggregate data are constructed from the model. In computations, simulated data over a 40 year career is compared to actual worker outcomes. The period length is one month so that parameters are chosen to match monthly data on job finding and separation rates in the United States.

To compute the model there are ten parameters that must be chosen: the maximum amount of experience \( T \), the discount factor \( \beta \), the job offer rate \( \lambda \), the expected output

\[8\]In a continuous time analog of the model, the process for \( p \) depends on \( p(1 - p) \) and the signal to noise ratio of the output process, \( \frac{\mu_h - \mu_l}{\sigma} \). The term \( p(1 - p)\frac{\mu_h - \mu_l}{\sigma} \) closely approximates the standard deviation of \( G(p'|p) \) in the discrete time model, but differs slightly due to the boundary effects.
from a good match $\mu_h$, the expected output from a bad match $\mu_l$, the probability that a match is good $p_0$, the variance of output noise $\sigma$, the proportion of experience used for new matches $\alpha$, the exogenous separation rate $\delta$, and the value of leisure $b$.

$\mu_h$ is normalized to one and $\mu_l$ is normalized to zero. Given these normalizations the evolution of $p$ will be determined by the variance of output noise, $\sigma$. The evolution of $p$ is fully determined by the signal to noise ratio: $\frac{\mu_h-\mu_l}{\sigma}$. Because the model period is one month, $\beta$ is set to 0.9966 which corresponds to an annual interest rate of 4%. $T = 480$ to corresponding to a maximum level of experience of 40 years. This is a reasonable upper bound as it corresponds to the normal length of work for individuals in the U.S. Increasing the maximum level of experience has no effect on the results.

The remaining parameters are chosen to match features of the decline in job finding and separation rates in the U.S. The left panel of Figure 1 shows the decline in the job separation rate with age in the U.S. for workers aged 18-57\(^9\). The separation has a sharp initial decline from age 18-25 followed by a gradual decline later in life.

The right panel of Figure 1 shows the decline in the job finding rate. Similarly, the job finding rates fall fastest for the first 8-10 years, but the initial decline is less dramatic than the separation rate and finding rates continue to decline at a greater rate for the remainder of the workers’ careers. Notice that while job separation rates fall by about a factor of 10, the job finding rates only decline by about a factor of 2 over the life cycle. Taken together, the steeper decline in the separation rate implies that the unemployment rate declines with age.

$\lambda$ is chosen to match the worker’s rate of job offers. In the data, 17-year-old workers have a job finding rate of 0.57. In the model workers with little experience will accept nearly any productive opportunity that they find. So since workers start at age 18, this provides an upper bound on the job finding rate. To match this feature of the data, $\lambda$ is set to 0.6.

\(^9\)This data was constructed by Robert Shimer using CPS monthly microdata from 1976 to 2005. The procedure used follows Shimer (2007) to create a time series of job separation and finding rates for individuals of each age. The time series is used to create average unemployment, job finding, and job separation rates for each age group. For additional details, please see Shimer (2007) and his webpage http://robert.shimer.googlepages.com/flows.
Figure 1: *Monthly job separation rate in left panel and monthly job finding rate in right panel by age for the U.S. economy.*

$p_0$ determines the portion of good jobs in the economy. Since a worker with perfect information about the quality of jobs will only accept good ones, $p_0$ determines the amount of decline in the job finding rate over the worker’s life. $p_0$ is chosen to match the decline in the job finding rate found in the data. It is set to 0.7 which allows the model to match the job finding rate of 0.30 for 57 year old workers in the data.

Next, $\sigma$ is the amount of output noise. Higher values of $\sigma$ imply that workers learn slowly about the quality of their matches. In the limit, $\sigma = 0$ implies that workers perfectly observe the quality of the match with one observation while as $\sigma \to \infty$ workers have no learning. $\sigma = 4$ is chosen to match the shape of the decline in job finding rates. Higher values of $\sigma$ imply that workers learn more slowly. Slower learning implies that it takes longer to distinguish bad matches, and generates a longer decline in job finding rates. Higher values of $\sigma$ imply that the decline in job finding rates is quick and they then level off.

$\alpha$ determines the amount of experience that carries over in learning about new job opportunities. It is natural to restrict $\alpha$ to be in $[0, 1]$. $\alpha = 0$ is analogous to the standard *Jovanovic (1979)* model where individuals learn nothing about future jobs and the employ-
ment is a pure renewal process. $\alpha = 1$ is the limit where all learning carries over to future jobs. Higher values of $\alpha$ imply that workers learn faster about future jobs and therefore have a steeper decline in both job finding and separation rates. Model results for various values of $\alpha$ are shown. With the model period set to be a month, $\alpha = \frac{1}{30}$. This corresponds to getting one month worth of information about a new job for every two and a half years of labor market experience. Higher values of $\alpha$ predict a steeper initial decline followed by less learning later. This parameter is sensitive to the choice of $\sigma$. The chosen value of $\sigma$ implies that individuals learn quickly by observing output. Surprisingly, very low values of $\alpha$ generate large changes in the patterns of job finding rates.

$\delta$ is the rate of exogenous job separations. An upper bound on the value of $\delta$ is lowest observed monthly job finding probability in the data is 0.014 for 59-year-olds. A lower value of $\delta = 0.0075$ is chosen.

The final parameter is $b$. This parameter determines the relative desirability of being employed in a bad job compared to searching for a new job. Higher values of $b$ make unemployment more attractive. $b = 0.3$ is chosen to match the level of unemployment over a worker’s lifetime.

Table 1 summarizes the chosen parameters and their values.

<table>
<thead>
<tr>
<th>Name</th>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max Age</td>
<td>$T$</td>
<td>480</td>
<td>Max 40 Years Experience</td>
</tr>
<tr>
<td>Discount Factor</td>
<td>$\beta$</td>
<td>0.9966</td>
<td>4% Interest Rate</td>
</tr>
<tr>
<td>Job Offer Rate</td>
<td>$\lambda$</td>
<td>0.6</td>
<td>Peak of Finding Rate</td>
</tr>
<tr>
<td>Good Output</td>
<td>$\mu_h$</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>Bad Output</td>
<td>$\mu_l$</td>
<td>0</td>
<td>Normalization</td>
</tr>
<tr>
<td>Probability of Good Job</td>
<td>$p_0$</td>
<td>0.7</td>
<td>Decline in Finding Rate</td>
</tr>
<tr>
<td>Output Noise</td>
<td>$\sigma$</td>
<td>4</td>
<td>Curvature in Finding Rate</td>
</tr>
<tr>
<td>Experience Rate</td>
<td>$\alpha$</td>
<td>$\frac{1}{30}$</td>
<td>Various Values Shown</td>
</tr>
<tr>
<td>Exogenous Separation Rate</td>
<td>$\delta$</td>
<td>0.0075</td>
<td>Minimum of Separation Rate</td>
</tr>
<tr>
<td>Value of Leisure</td>
<td>$b$</td>
<td>0.3</td>
<td>Level of Unemployment</td>
</tr>
</tbody>
</table>
4 Simulated Results

This section documents the implications from the calibrated model. The novel feature of allowing learning to transfer between jobs through work experience is that past experience now has implications for workers while unemployed through their job search behavior. To document this, the value functions are computed to generate reservation probabilities for workers at each experience level. Using these decision rules, employment outcomes are simulated for individual workers. Monthly employment, job finding rates, job separation rates, wages, tenure, and total experience are recorded in the simulations. The outcomes for 10,000 simulated workers are computed from the date that workers enter the labor force.

To compare outcomes with labor force data, outcomes by age are constructed by entering workers into the labor market at the age they get their first full time employment. Topel and Ward (1992) compute the percentage of workers who enter the labor force at a given age by assuming that workers enter when they attain their first employment that lasts at least 2 quarters. This measure leaves out workers who take summer jobs and then return to school. Table 2 replicates their table showing the percentage of workers who enter the labor force at each age. When constructing the data from the model, all workers in the \( \leq 18 \) category enter at age 18 and all workers in the \( \geq 25 \) category enter at age 25.

The remainder of this section compares the simulated data from the calibrated model with the data.

### 4.1 Job Finding Rate

The first result is that the calibrated model is able to match the decline in job finding rates with age. Since experience allows individuals to learn about the quality of new matches,
experienced workers are more selective about which jobs they choose to accept. This feature allows the job finding rate to decline over a worker’s lifetime.

Figure 2 plots the decline in job finding rates from the simulated model against the data. Simulated job finding rates start out slightly higher and decline to match the rates observed in the data. The two series are almost identical after age 30. The calibrated model is able to capture the initial steep decline in job finding rates and continued gradual decline later in life. No previous models of learning generated any change in job search behavior so their predicted job finding rate is constant.

To compare the fits of the model with the data a goodness of fit is computed:

\[
Fit = 1 - \frac{\sum_{a=18}^{57} (\varepsilon_a - \bar{\varepsilon})^2}{\sum_{a=18}^{57} (y_a - \bar{y})^2}
\]
This is similar to an $R^2$ measure, where $\varepsilon_a$ is the difference between the model and the data for age $a$, $\bar{\varepsilon}$ is the average difference, $y_a$ is the level of the data for age $a$, and $\bar{y}$ is the average level of the data. The numerator give the sum of squared errors between the data and the model and the denominator gives the sum of squared deviations in the data. The calibrated model has a fit of 0.95.

To see how changes in $\alpha$ affect the results of the model, Figure 3 plots the job finding rate from the calibrated model for different values of $\alpha$. Low values of alpha imply a steeper initial decline in job finding rates as the worker is more quickly able to distinguish between good and bad jobs. In the cases close to the calibrated value of $\alpha = \frac{1}{30}$ the simulated job finding rates decline throughout the worker’s simulated working life. However, for high values of $\alpha$ the simulated job finding rate is not monotonic. In particular, when $\alpha = 1$ there

Figure 3: Job finding rate by age for various values of $\alpha$. 
is a large portion of the worker’s life for which the job finding rate is increasing. Recall that for a worker with perfect information, the job finding rate is determined by the arrival rate of productive opportunities $\lambda$ multiplied by the fraction of those opportunities that are good $p_0$. This gives a job finding rate of 0.42 for the current calibration. The initial rapid decline occurs as some information about the quality of the job initially makes the worker much pickier about which jobs to accept. Over time better information pushes a greater portion of jobs above the threshold to increase the job finding rate. The case of $\alpha = \frac{1}{48}$ shows that even small amounts of learning across jobs can have dramatic effects on the predicted worker search behavior over the life cycle. Finally, the case of $\alpha = 0$ is shown to be flat. This corresponds to the Jovanovic (1979) model where no learning transfers across jobs and workers have a flat job finding rate for their entire life.

### 4.2 Job Separation Rate

Figure 4 shows the decline in separations for the calibrated model compared with the data. The model exhibits an initial decline in the separation rate that is steeper than the data, but is unable to generate the highest levels of separations for young workers. Some of the high rates are due to workers moving in and out of the labor force for schooling that is not captured by the model. The decline in job separations happens in the model for two reasons. First, as in Jovanovic (1979) the job separation rate declines as workers sort themselves to good jobs which last longer on average than bad jobs. Second, experience allows older workers to match with better jobs than younger workers reducing the chance of separation for new jobs acquired later in life.

The fit of the calibrated model is 0.68. Despite not capturing the high level of initial separations in the data, the model with learning is still able to capture most of the decline in separations. This is consistent with the predictions in other models of learning.
4.3 Unemployment

It is well known that young workers face higher unemployment rates than prime aged workers. The model is able to capture this decline in unemployment with age.

Figure 5 shows the average annual unemployment rate by age. The dots depict the decline in unemployment found in the data where the solid line depicts the results from the calibrated model. The data show a steady decline in unemployment with age. Unemployment declines from about 17% for 18-year-old workers to between 3.5 and 4% for prime aged workers. The calibrated model captures a similar decline over the life cycle, with 18-year-old workers experiencing unemployment of 23% and declining to 4.1%. The initial decline in unemployment is steeper in the calibrated model than in the data reflecting all 18 year old worker entering the market unemployed. The fit calibrated model on the unemployment
The predicted decline in unemployment from the model can be understood by combining the results about declining job separation and job finding rates. The decline in job separation rates drives most of the decline in unemployment while the decline in job finding rates tends to slightly increase the unemployment rate. However, the decline in separations dominates as it goes from about 7.5% to 1.3% over the worker’s life while the job finding rates only decline by about a factor of 2 from about 56% to 30%.

4.4 Wage Growth

Flinn (1986) argues that wage growth and turnover are related for young workers. This model presents a theory that accounts for both phenomena. Topel and Ward (1992) doc-
ument a number of features of wage profiles during worker’s first 10 years of experience. They document that the first 10 years of the career account for two-thirds of lifetime wage growth. Job changes explain about one-third of wage growth. Moreover, wages on the job approximate a random walk. The model qualitatively replicates the behavior of wages over the life cycle.

Figure 6 shows the average annual wages by age from the model. The pattern of wage growth from the model is endogenous. The model generates rapid wage growth during the first 10 years of experience and then levels off. While the model matches the general pattern of wage growth, it doesn’t generate quite as much wage growth as found in the data where wages about double over the lifetime. This should be expected as the model generates only wage growth from workers moving to better matches with firms and does not include wage
growth from learning by doing or other forms of human capital gained while working. A model would need to include these other forms of wage growth to fully account for wages over the life cycle.

5 Wage Volatility

This section quantitatively evaluates the model’s predictions on wage variation using data on wages and job switching from NLSY79. The novel feature of this model is that workers who start jobs with more experience have better information about the quality of their new job. For standard parameterizations, this information means that experienced worker’s on average start with a higher \( p \) and hence their wages should display less variation in subsequent periods. These predictions are first confirmed using observations simulated from the model then the same results are documented using data from the NLSY79.

The NLSY79 is a nationally representative longitudinal survey conducted by the Bureau of Labor Statistics that samples 12,686 individuals who were between the ages of 14 and 22 years old when first surveyed in 1979. The individuals continued to be surveyed every year until 1994 when the survey switched to every two years. The data are restricted to before 1994 so that appropriate measure of annual wage volatility within jobs can be constructed. NLSY79 provides a rich set of panel data for tracking worker’s career outcomes. To avoid miscalculation of past experience, the sample is limited to workers who are 17 years old or younger at the time of the first interview.

To construct job variables the NLY79 provides a variable for the total number of past jobs that the respondent has held. In the NLSY a job is defined as a relationship between an individual employer and the worker. That is changes in position within a firm are not considered new jobs. If the total number of jobs in year \( t \) is greater than in year \( t - 1 \), then there is a new job observation. For each job observation the wage in each year is given by the CPS wage\(^\text{10}\). Finally, experience can be constructed by taking a cumulative sum of the

\(^{10}\)Since wages are only recorded each year it is possible that the wages could have already changed from the initial wage at the time of first observation. Despite this measurement issue, the same issue arises when
weeks worked in the past year variable. The number of past weeks worked is multiplied by 52 so that results can be presented in terms of years of experience. To compare observed outcomes from the NLSY79 with the model, 25 years of annual observations are simulated for the worker’s employment status, past experience, accumulated job number, and wage from the model. To make the samples comparable, both the simulated and NLSY79 data are restricted to jobs where workers start with less than 15 years of prior experience.

Each worker’s employment history is broken into jobs that are characterized by a wage for each year of tenure on the job and the initial experience level when starting the job. Wage volatility is measured as the absolute deviation from the worker’s expected wage growth path. The simplest measure of wage volatility is to take the absolute value of the difference in log wages at each tenure level from the initial wage on the job. However, this measure does not control for the expected levels of wage growth that occur at different levels of tenure and experience. To control for this, the wage volatility measure used is:

\[ v_t = | \log(w_t) - \log(w_0) - \bar{\bar{w}}_{et} | \]

Where \( v_t \) is the volatility of wages at tenure \( t \) on a given job\(^{11} \). \( w_t \) is the wage observed at tenure \( t \), \( w_0 \) is the initial wage observed on the job, and \( \bar{\bar{w}}_{et} \) is the median log wage deviation \((\log(w_t) - \log(w_0))\) observed for the two year initial experience group \( e \) and tenure level \( t \)\(^{12} \). By construction the volatility is zero for the initial wage observation (worker’s tenure of zero). Note that the concept of wage volatility here is at the individual level rather than a annual data is taken from the simulated model. In the simulated model a worker’s wages change every month based on their updated beliefs. Treating the simulated data the same as the NLSY79 observations should yield similar biases.

\(^{11} v_t \) is the \( t \) year measure of volatility, so \( v_3 \) measures the volatility of wages over a three year increment from starting the job. Another measure of volatility, \( v(t) \) can be defined as the one year volatility for each year \( t \) from the previous years observed wage:

\[ v(t) = | \log(w_t) - \log(w_{t-1}) - \bar{w}_{et} | \]

Where \( \bar{w}_{et} \) is the median log wage deviation \((\log(w_t) - \log(w_{t-1}))\) for the two year initial experience group \( e \) and tenure level \( t \). The rest of the analysis focuses on the first year volatility on a job, so these two measures are identical.

\(^{12} \)The median is a more appropriate measure here as it is robust to truncation. This is especially important as there is a zero lower bound of observed wage volatility. The results are similar if the mean is used.
cross section across individuals. Higher volatility implies that a given individual experiences larger changes in her wages on a given job. By subtracting $\bar{w}_{et}$ the measure of volatility used in this paper controls for median wage gains in each year of tenure at a particular job and with experience. While most workers get wage increases from year to year, subtracting of the expected wage growth at the tenure and experience level means that many workers are both above and below the expectation.

While the effect of experience on wages has been explored by a large theoretical literature (See Neal and Rosen (2000), Gibbons et al. (2005)), the previous literature has not explored the impact of experience on individual within job wage volatility. Understanding the features of the individual income process is important to explain a wide array of individual behavior (see Meghir and Pistaferri (2004)). This paper shows that past job experience has a predictable effect on individual wage volatility. Experience is shown to decrease individual level uncertainty about wages while cross sectional heterogeneity may increase within group wage variation. The model is able to account for the individual level declines in volatility.

The novel prediction of the model is that volatility should decline with more past job

Figure 7: Median wage volatility ($v_1$) by years of past experience. Simulated model in left panel NLSY79 data in right panel.
experience. To explore this prediction, it is sufficient to just look at the first year wage volatility on each new job, $v_1$. The left panel of Figure 7 plots the median wage volatility for the first year in each job binned by years of past experience. The model generates a decline in wage volatility for workers with more past work experience. Note that the model predicts that for inexperienced workers the one year change in wages will be about 7%. Median volatility declines to under 6% for workers with 10 years of experience. The right panel of Figure 7 plots the median wage volatility for the first year in each job with experience binned into yearly groups for the NLSY79 data. Just as in the simulation the data shows a decline in wage volatility for workers with more past work experience. For inexperienced workers, the one year change in wages is about 12%. Volatility declines at a steeper rate to under 10% for workers with 10 years of experience. The patterns of volatility with experience are consistent with those found in the model. Note that there is a scale shift between the two panels in the figure. Since the magnitudes of wage volatility are higher in the data, the model does not capture all of the observed wage volatility. However, the similar declines with experience imply that the model explains a large portion of the interaction between experience and volatility found in the data.

To more formally assess the model’s predicted effect of experience on wage volatility, the year one wage volatility measure $v_1$ is regressed on the past experience associated with each new job event. Quantile regressions are valuable for two reasons. First, they are robust to the lower bound issues in observed volatility. Second, they provide a more details predictions about how experience influences volatility at different points in the distribution so that the model predictions can be more closely compared to the evidence found in the data. This gives additional feedback about how experience influences workers at different parts of the volatility distribution. Quantile regressions are run for the 5th, 10th, 25th, 50th, 75th, 90th, and 95th percentiles.

The left panel of Figure 8 shows the results for the OLS regression plotted against the quantile regression results. The graph shows that both the OLS and median quantile regres-

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13See Koenker and Hallock (2001) for other examples of the quantile regression procedure.
Figure 8: Plot of quantile regression of first year wage volatility on past experience with error band. Thin lines gives OLS estimate with dashed error bands. Simulated model in left panel NLSY79 data in right panel.

regression confirm a negative and significant effect on wage volatility. The quantile regressions at other points in the distribution show that at higher percentiles of the distribution experience causes larger decreases in wage volatility. The right panel of Figure 8 plots the regression results of wage volatility on experience from the NLSY79 data. Again, the patterns in the regressions confirm the model predictions. Both the OLS and median quantile regression show a significant negative effect of experience on wage volatility. The magnitude of the decline is between 23 and 32 basis points per year of experience depending on using the median quantile regression or the standard OLS estimate.

The regression results for both the model and NLSY79 data are presented in Table 3. The estimated effect for the median and mean in the model is that an extra year of experience decreases the volatility of wages by about 13 basis points. The coefficient on experience is negative and significant in all cases except for the 0.05 quantile. This is expected as wage volatility is close to zero for the low quantiles and hence cannot decrease much further. The larger declines in the upper quantiles imply that there is a greater effect of experience for jobs
with higher wage volatility. The results from the NLSY79 data confirm this general pattern. The OLS estimate indicates that a year of experience decreases volatility by about 33 basis points while the median decreases by about 24. The data are negative and significant for all points in the quantile regression except for the .05 and .1 quantiles with slightly higher magnitudes than generated from the model.

Finally, Table 4 presents additional regression specifications for the NLSY79 data. Specification I shows the baseline results from above. Specification II includes a female dummy variable. The estimate remains similar and the result shows that women have about 1.6% less volatility than men. Finally, specification III includes education and race dummies. None of the dummies are significant and the effect of experience remains unchanged. The table confirms a robust negative relationship between past work experience and observed wage volatility in the first year on a new job.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.256***</td>
</tr>
<tr>
<td></td>
<td>(0.0120)</td>
</tr>
<tr>
<td>Experience</td>
<td>-0.00326**</td>
</tr>
<tr>
<td></td>
<td>(0.00136)</td>
</tr>
<tr>
<td>Female</td>
<td>-0.0160*</td>
</tr>
<tr>
<td></td>
<td>(0.00802)</td>
</tr>
<tr>
<td>College Degree</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Graduate Degree</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Race Dummies</td>
<td>No</td>
</tr>
</tbody>
</table>

*** p<0.01, ** p<0.05, * p<0.1.
Robust standard errors clustered by individual in parentheses.

Table 4: OLS regression results for NLSY79 data with controls.

6 Conclusion

This paper presents a model of learning that can explain changes workers’ job finding rates over their life cycle. Workers’ learning about the quality of their match is important for both observed outcomes while employed like wages and employment durations and outcomes while unemployed. This insight motivates the model where experience gives workers both knowledge about the quality of their current job and the ability to distinguish between good and bad jobs when unemployed.

A model with learning about both the quality of the current match and future matches has rich implications for labor market outcomes. It is consistent with the age profiles of unemployment, job finding rates, job separation rates, hazard rates of separation with tenure, wage dispersion, and wage growth. Having a model that has consistent predictions about a broad range of labor outcomes makes it ideal to analyze the effects of policy on these outcomes. The model is used to generate new predictions about individual worker’s wage volatility on jobs based on their past level of experience. The prediction of lower volatility with more past experience is found to hold in NLSY79 data.

While learning both within and across jobs accounts for many of the observed patterns.
found in the evidence on individual labor earnings, it generally does not capture the entire wage growth observed over the life cycle. Learning that transfers between jobs can be thought of as one specific type of human capital that agents acquire while working. To account for the entire wage patterns observed in the data it is important to distinguish between learning and other forms of specific human capital.
A Proof of Proposition 1

Claim 1 If $U'((\tau + 1) \leq U'(\tau)$, then $p'(\tau) > 0$ for all $\tau \in \{0, 1, \ldots, T\}$.

Proof. Differentiating equation (3) with respect to $\tau$ gives:

$$p'(\tau)V_p(\bar{p}(\tau), \tau) + V_r(\bar{p}(\tau), \tau) = U'(\tau)$$

Then $p'(\tau) > 0$ if $V_r(\bar{p}(\tau), \tau) \leq U'(\tau)$. It suffices to show that $V_r(p, \tau) \leq U'(\tau + 1)$ for all $\tau \in \{0, 1, \ldots, T\}$ and $p \in [0, 1]$. We will proceed by backward induction starting from $\tau = T$.

For $\tau = T$:

$$V_r(p, T) = U'(T + 1) = U'(T) = 0$$

For $T - 1$:

$$V_r(p, T - 1) = [\beta \delta + \beta(1 - \delta)G(\bar{p}(T)|p)] U'(T) + \beta(1 - \delta) \int_{\bar{p}(T)}^{1} V_r(p', T - 1) G(dp'|p)$$

$$= 0 = U'(T)$$

For $T - 2$:

$$V_r(p, T - 2) = [\beta \delta + \beta(1 - \delta)G(\bar{p}(T - 1)|p)] U'(T - 1)$$

$$+ \beta(1 - \delta) \int_{\bar{p}(T - 1)}^{1} V_r(p', T - 1) G(dp'|p)$$

$$= [\beta \delta + \beta(1 - \delta)G(\bar{p}(T - 1)|p)] U'(T - 1)$$

$$= \beta [\delta + (1 - \delta)G(\bar{p}(T - 1)|p)] U'(T - 1) < U'(T - 1)$$

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Finally, assuming $V_\tau(p, T - n) \leq U'(T - n + 1)$, we can solve for $T - n - 1$:

$$V_\tau(p, T - n - 1) = [\beta \delta + \beta (1 - \delta) G(\bar{p}(T - n)|p)] U'(T - n)$$

$$+ \beta (1 - \delta) \int_{\bar{p}(T - n)}^{1} V_\tau(p', T - n) G(dp'|p)$$

$$\leq [\beta \delta + \beta (1 - \delta) G(\bar{p}(T)|p)] U'(T - n)$$

$$+ \beta (1 - \delta)(1 - G(\bar{p}(T - n)|p)) U'(T - n + 1)$$

$$\leq [\beta \delta + \beta (1 - \delta) G(\bar{p}(T)|p)] U'(T - n)$$

$$+ \beta (1 - \delta)(1 - G(\bar{p}(T - n)|p)) U'(T - n) = U'(T - n)$$

The first inequality comes from the induction and the second comes from the hypothesis that $U'(\tau + 1) \leq U'(\tau)$ for all $\tau \in \{0, 1, \ldots, T\}$. ■
References


