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OPTIMALITY CRITERIA OF HYBRID INFLATION-PRICE LEVEL TARGETING

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This paper provides a comprehensive analysis of the relative performance of inflation targeting, price level targeting, and hybrid targeting of them in a simple three-period steady state to steady state economy facing transmission lag, and derives optimal policies implementing commitment solution under all set of hybrid expectations, social preference, and cost-push shock persistence. The main intention of the examination is to reveal the nature of the interrelations between economic and policy parameters.

JEL: E50, E52, E58

Keywords: inflation targeting, price level targeting, hybrid targeting, optimal policy

The difference between the policy of inflation targeting and price level targeting can be captured in their attitude to shocks affecting price level. In most general terms, inflation targeting attempts to maintain a targeted inflation path, and does not care for unanticipated misses in the past which implies the rising uncertainty around the future price level. Price level targeting attempts to maintain a targeted price path which implies that uncertainty around the price level does not increase with the progress of time.¹ In others words, inflation targeting is a regime without memory, while price level targeting is a history-dependent policy.

A fixed price level has reasonable benefits. Planning and contracting becomes easier as nominal values become real values. The information message of prices is without any distortion, since their realignments would purely reflect scarcity, which enhances the resource allocation mechanism. The continuous transfer of welfare from the cash holders to the government using inflation device (not just surprise inflation!) is also wiped out. However, the idea of price level targeting has been criticized from both practical and theoretical standpoint.² The “conventional wisdom”, as Svensson (1999) named it, states that the consequence of price level stabilization is the higher volatility of inflation and output gap.³ “The intuition is straightforward: In order to stabilize the price level under price-level targeting, higher-than-average inflation must be succeeded by lower-than-average inflation. This should result in higher inflation variability than inflation targeting, since in the latter case, base level drift is accepted and higher-than-average inflation need only be succeeded by

¹ With a constant inflation target, inflation is stationary, while the (log of) price level has a unit root. If the inflation target is zero, the (log of) price level follows random walk, if it is a positive value, then the (log of) price level follows a stochastic trend. With a constant price level target, the (log of) price level is stationary around the targeted value, and inflation is stationary, too, around zero. If the (log of) targeted price path has a constant positive slope, then the (log of) price level is trend stationary as it follows a deterministic trend, and inflation becomes stationary around the slope of the (log of) targeted price path.

² Fischer (1994) argues that since indexed financial assets and nominal contracting are given, targeted price level has not too much sense. McCallum (1999) claims that the abovementioned benefits would not be significant in the United States.

³ Svensson (1999) referred to the writings of Lebow, Roberts and Stockton (1992), Fischer (1994), and Haldane and Salmon (1995).

average inflation. Via nominal rigidities, the higher inflation variability should then result in higher output variability.”⁴ Svensson (1999) pointed out that the root of the conventional comprehensive results arises from the usage of postulated reaction functions instead of endogenous decision rules. He showed, using a New Classical Phillips curve, that under discretion price level targeting provides lower inflation variability, than inflation targeting does, without affecting the output gap variability at the same time, if there is sufficiently high persistence in the output gap. He called this “free lunch”. Vestin (2006) made this comparison in a New Keynesian economy, assuming perfectly credible central banks. He demonstrated that free lunch result holds, even if there is no endogenous output gap persistence, and if there is no persistence in cost-push shocks, price level targeting can implement commitment solution. With exogenous inflation persistence, price level targeting can be also better than inflation targeting, though the key issue is the assignment of proper preference weight in the loss function of the central bank.⁵

As Woodford (2000) emphasized, the optimal policy under commitment is history-dependent in the case of forward looking expectations. However, since it is generally time-inconsistent, it does not provide a too realistic solution.⁶ The point is to implement such a discretionary policy, that can incorporate the past in the decision making process. A predetermined price level target operates as a solid nominal anchor, and incorporates history dependent policy. Under New Classical Phillips curve, current inflation-output gap trade off is not affected by inflation expectations, as they are predetermined. However, as Barnett and Engineer (2001) explained, with the existence of sufficiently high persistence in the output gap, rational actors indirectly form their expectations in a forward looking manner, as they know the future persistency effects of the current output gap affecting the trade off. In a New Keynesian economy the relation is more straightforward, as here inflation expectations affect inflation-output gap trade-off right in the present, and so the gains of a credible price level target arise immediately.

Since efficiency of price level targeting and inflation targeting is sensitive to the key assumptions on, for instance, expectations, several examinations were concluded with creating more generalized economic environments and implementing new hybrid policies. Batini and Yates (2003) analyzed such a hybrid regime that combines the characteristics of inflation targeting and price level targeting, incorporating the weighted average of inflation and price level target into the central bank’s loss function. Under hybrid inflation expectations, they concluded that hybrid targeting is good when policy rules are set in a forward looking manner. Cecchetti and Kim (2005) also showed that in an economy represented by a New Classical Phillips curve with high output gap persistence, an appropriately chosen hybrid target results in optimal policy. Nessen and Vestin (2005) demonstrated, using a New Keynesian Phillips curve, that a policy targeting the average inflation of several forthcoming periods provides better performance than inflation targeting, but worse than price level targeting.⁷ They also showed, using a hybrid Phillips curve, that if backward pricing more and more characterizes the economy, then the benefits of price level targeting deteriorates, and average inflation targeting will offer the best solution out of the three discretionary regimes, until the economy becomes fully backward looking.

⁴ Svensson (1999), p. 278

⁵ Svensson (1999) derived his results using the same preference weight in the loss functions of price level targeting and inflation targeting.

⁶ Woodford’s (1999) concept of “timeless perspective” ignores the initial conditions of the regime, eventuate in commitment policy that is time-consistent.

⁷ Note, price level targeting is a policy that targets average inflation of infinite periods, while inflation targeting aims the “average” of one period ahead only.

The model to be introduced in Section 1 presents a frame that incorporates multiple economic phenomena inducing quite different policy implications. Section 2 and 3 provides comprehensive analysis under different economic circumstances, beginning with the least realistic case up to the most general case. Section 4 gives a brief empirical outlook related to the results, while Section 5 concludes.

1. The model

1.1. The economy

Suppose that inflation in the economy is determined by factors presented in the following hybrid Phillips curve,⁸

$$\pi_t = \phi\pi_{t+1|t} + (1-\phi)\pi_{t-1} + \delta x_t + u_t, \quad (1)$$

$$x_t = f(\text{instruments}_{t-1}),$$

where (in a logarithm) π_t is the inflation rate, $\pi_{t+1|t}$ is the forward looking rational inflation expectation, π_{t-1} is the backward looking adaptive inflation expectation, x_t is the output gap, u_t is an AR(1) disturbance term, $u_t = \rho u_{t-1} + \varepsilon_t$, where ε_t is an i.i.d. with zero mean and variance of σ^2 , and ϕ, δ, ρ are constants ($0 \leq \phi \leq 1$, $\delta > 0$); ϕ gives the composition of the expectations of the actors in economy, while δ shows the slope of the Phillips curve, and ρ indicates the persistence of the exogenous supply shock.⁹

What does this Phillips curve consider and what does it not cover? Inflation is influenced by three factors on a general basis: expectations, shocks, and cyclic factors. The model captures various expectation structures, exogenous and persistent supply shocks, and it also considers monetary transmission lag. As shocks affecting potential output and aggregate demand are not modelled, the value of x_t is unambiguously determined by the monetary instruments set before the period, namely on the basis of the information in period t-1. Although x_t is under the perfect control of the central bank, actually, it can respond to a current shock only in the next periods only. It follows that contrary to the prevalent assumption of the topic literature, the central bank has not perfect control over inflation in periods when a cost-push shock occurs.¹⁰ Therefore, as there is no uncertainty around the output gap, it is supposed that the output gap itself is the instrument.

Both endogenous and exogenous inflation persistence stand in accordance with the general perception that the inflation process has inertia, however, they presume widely different policy implications. Furthermore, the lowered reaction capability of the central bank reflects that monetary actions exert their full impact in a longer time. On the other hand, this model implies that the clearing of labour market works at its best: without monetary intervention, the output immediately returns to its natural level, meaning that the output gap is not persistent.

⁸ Notation q_{ij} is used instead of $E_j q_i$. Both denote the expected value of the variable q at time i conditional upon information available in time j .

⁹ First-order autoregressive persistent shock term is presented in Clarida et al. (1999).

¹⁰ Svensson (1999), Cecchetti and Kim (2003), Nessen and Vestin (2005), Vestin (2006) all assumed that the central bank has perfect control over inflation with the concurrent change of the output gap, while Batini and Yates (2003) used backward looking IS function.

1.2. The regimes

We consider four regimes, the theoretical benchmark and three discretionary solutions, namely inflation targeting with commitment, inflation targeting, price level targeting, and a hybrid regime of the latter two. Standard quadratic loss functions used in the literature generally incorporate the inflation, output gap, and seldom the nominal interest rate variability, and can be derived from a general equilibrium model with monopolistically competitive firms.¹¹ Svensson (1999) replaced the inflation target to price level target in the standard loss function, which eventuated in price level targeting.

Under inflation targeting with commitment (ITC), the central bank makes a precommitment to its future actions, and optimizes in the initial period only. This theoretical benchmark solution minimizes the intertemporal social loss function itself, which is

$$E \sum_{t=1}^{\infty} \beta^{t-1} L_t = E \sum_{t=1}^{\infty} \beta^{t-1} \frac{1}{2} [(\pi_t - \pi_t^*)^2 + \lambda(x_t - x_t^*)^2]. \quad (2)$$

In the case of inflation targeting (IT) the central bank tries to pursue the targeted value of inflation and output gap, namely to minimize the expected loss

$$E \sum_{t=1}^{\infty} \beta^{t-1} \hat{L}_t = E \sum_{t=1}^{\infty} \beta^{t-1} \frac{1}{2} [(\pi_t - \pi_t^*)^2 + \hat{\lambda}(x_t - x_t^*)^2] \quad (3)$$

in every period. In a price level targeting regime (PT), the central bank tries to neutralize the divergence from the targeted price level and output gap, namely aims to minimize the loss function

$$E \sum_{t=1}^{\infty} \beta^{t-1} \tilde{L}_t = E \sum_{t=1}^{\infty} \beta^{t-1} \frac{1}{2} [(p_t - p_t^*)^2 + \tilde{\lambda}(x_t - x_t^*)^2] \quad (4)$$

in every period. If inflation, output gap, and price level are incorporated in such a way that the weighted mixture of the inflation and price level targets are used in the loss function, we obtain hybrid targeting (HT). In this case, the loss function to be minimized in every period is

$$E \sum_{t=1}^{\infty} \beta^{t-1} \check{L}_t = E \sum_{t=1}^{\infty} \beta^{t-1} \frac{1}{2} [((1-\theta)\pi_t + \theta p_t) - ((1-\theta)\pi_t^* + \theta p_t^*)]^2 + \check{\lambda}(x_t - x_t^*)^2], \quad (5)$$

where $0 < \theta < 1$. It is perceivable that if $\theta = 0$, we get to IT, and if $\theta = 1$, then we get to PT.¹² The λ reflects the relative importance of output gap variability compared to the importance of inflation variability. Different notations express that the central bank's preference weight can and usually do differ from the society's preference weight.

¹¹ See Rotemberg and Woodford (1997).

¹² As Batini and Yates (2003) noted referring to Larry Ball and Frank Smets, this weighting method incorporates the covariance term between inflation and price level into the loss function.

1.3. *The three-period analytical frame*

The frame to be presented is examining a steady state to steady state economy which is hit by a single shock (ε_t) at the beginning of the first period. Suppose that the significance of the events in each period is equivalent ($\beta = 1$). In period 0, the economy is in steady state, where the variables (π_t and x_t) are equal to zero, and for simplicity, price level (p_t) starts from zero, too.¹³ Another simplifying assumption is the zero inflation target (π_t^*), in the case of ITC and IT, and the zero price level target (p_t^*) in the case of PT. Supposing that economy works on its long time potential, neither discretionary regimes endeavour to aim an output level differing from the potential one in any case creating surprise inflation ($x_t^* = 0$), namely there is no inflation bias described by Kydland and Prescott (1977) and Barro and Gordon (1983).¹⁴ Thus, the scope of the examination is on stabilization bias, originating also from dynamic inconsistency. In practice, none of the monetary regimes can temporize the pursuing of previously communicated targets without deteriorating credibility, but the scale is largely depend on what the economic actors surmise on the reliability of the central bank.¹⁵ Eventually, excess recalibration of the path of the targeted variable erodes the trust in the declared policy for sure. In this sense, the model covers the time horizon where declared targets should be achieved maintaining credibility, with other words where the immanent characteristics of the regimes are clearly revealed, nothing but in ‘low resolution’. Accordingly, the three discretionary regimes focus solely on the declared goals, that is inflation, price level, and hybrid targets ($\hat{\lambda} = \tilde{\lambda} = \check{\lambda} = 0$); it provides the most consistent way of assuming high credibility gains in the absence of commitment technology. Thus, suppose that discretionary regimes have the credibility in point of reaching their final goals considering rational actors, that is the forward looking economic actors fully understand the nature of the regime, and trust in the pursuing of the declared target.¹⁶ The expectations can be formalized to

$$\pi_{t+1|t} = p_{t+1|t} - p_t, \quad (6)$$

where $p_{t+1|t}$ means the expected price level of the next period. These expectations manifest in different manner, depending on the characteristics of the discretionary regime. In IT, the actors expect that a zero inflation target is pursued, namely that the price level of next period will be the same as in the concurrent period ($p_{t+1|t} = p_t$). In PT, it is believed that monetary actions are in order to assure the targeted price level, namely zero ($p_{t+1|t} = p^* = 0$). What do the actors of the economy expect in HT? It depends what emphasis the price level target bears, namely

¹³ Setting the initial price level and the target to zero theoretically implies the existence of negative prices. However, it serves only the better comparability of inflation and price level responses.

¹⁴ Since there are no market imperfections causing higher market clearing unemployment rate compared to the “natural” one, no incentive remains at all to aim an output level differing from the potential one.

¹⁵ In the 1990s, early inflation targeters were criticized because inflation target, which meant disinflation at that time, was in the foreground causing higher unemployment rate; however that was the way of gaining credibility. After successful disinflation, secondary goals (e.g. output, interest rate, exchange rate) started to move into the foreground. With built-up credibility, the counter-actions to mitigate a potential shock as soon as possible were and are used less frequently, and gradual approach is emphasized.

¹⁶ It is quite obvious that an inflation target or a constant price level target is easier to be communicated than a positive-slope price level target or even more than a hybrid target. More transparency may result more credibility, however we do not draw any distinction between the examined discretionary regimes in this regard.

from the grade of ‘history dependency’. In every period, they expect that θ proportion of the inflation occurred in the first period will be undone, or in other words, their expected price level target will be

$$p_{t+1|t} = p_1 - \theta\pi_1 = (1-\theta)\pi_1. \quad (7)$$

With this model specification, since backward pricing excludes credibility matters, expectations are driven by policy framework and formed exogenously at same time. With the assumptions given, at latest in the fourth period steady state should be achieved. Model calibration has been summarized in Table 1, while the solutions of the model are presented by Table 2 (details in Appendix A).

Table 1
Set of variables

Period (t) \ Variable	0	1	2	3	4
π_t	0	π_1	π_2	0	0
x_t	0	0	x_2	x_3	0
p_t	0	π_1	$\pi_1 + \pi_2$	$\pi_1 + \pi_2$	$\pi_1 + \pi_2$
π_t^*, x_t^*, p_t^*	0	0	0	0	0
ε_t	0	ε_1	0	0	0
u_t	0	ε_1	$\rho\varepsilon_1$	$\rho^2\varepsilon_1$	0

In period 3, because of the existence of lagged price term, inflation should be zero in order to ensure steady state in period 4. However, it is only a constraint for the commitment solution, and not for any discretionary case, as they strictly achieve their declared monotargets at latest at the end of period 2 (see Appendix A).

As the disturbance term is an AR(1) process, it calms down within the progress of time. Here, a ‘quick’ calm down feature is used, as its effects from period 4 are disregarded. It has no relevance in our analysis, since from period 4, it would affect all regimes in the same manner.

Table 2
Results

	IT	PT	HT
π_1	ε_1	$\frac{\varepsilon_1}{1+\phi}$	$\frac{\varepsilon_1}{1+\theta\phi}$
π_2	0	$-\frac{\varepsilon_1}{1+\phi}$	$-\varepsilon_1 \left(\frac{\theta}{1+\theta\phi} \right)$
x_2	$-\frac{\varepsilon_1}{\delta}(1-\phi+\rho)$	$-\frac{\varepsilon_1}{\delta} \left(\frac{2-\phi}{1+\phi} + \rho \right)$	$-\frac{\varepsilon_1}{\delta} \left(\frac{\theta+1-\phi}{1+\theta\phi} + \rho \right)$
x_3	$-\frac{\varepsilon_1}{\delta}\rho^2$	$\frac{\varepsilon_1}{\delta} \left(\frac{1-\phi}{1+\phi} - \rho^2 \right)$	$\frac{\varepsilon_1}{\delta} \left(\frac{\theta(1-\phi)}{1+\theta\phi} - \rho^2 \right)$
ITC			
π_1	$\varepsilon_1 \left[\left(\left(\frac{1-\phi}{1-\phi(1-\phi)} - \delta \frac{1+\phi^2 + \frac{\lambda}{\delta^2}(1-\phi)^2}{\delta(1+\phi^2 + \frac{\lambda}{\delta^2}(1-\phi)^2) + \frac{\lambda}{\delta}(1-\phi(1-\phi))^2} (1-\phi) + \phi \right) + \rho \left(\frac{1}{1-\phi(1-\phi)} - \delta \frac{1+\phi^2 + \frac{\lambda}{\delta^2}(1-\phi)^2}{\delta(1+\phi^2 + \frac{\lambda}{\delta^2}(1-\phi)^2) + \frac{\lambda}{\delta}(1-\phi(1-\phi))^2} + \rho \frac{\lambda}{\delta^2}(1-\phi) \right) \right) + 1 \right]$		
π_2	$\varepsilon_1 \left[\frac{1-\phi}{1-\phi(1-\phi)} - \delta \frac{1+\phi^2 + \frac{\lambda}{\delta^2}(1-\phi)^2}{\delta(1+\phi^2 + \frac{\lambda}{\delta^2}(1-\phi)^2) + \frac{\lambda}{\delta}(1-\phi(1-\phi))^2} (1-\phi) + \phi + \rho \left(\frac{1}{1-\phi(1-\phi)} - \delta \frac{1+\phi^2 + \frac{\lambda}{\delta^2}(1-\phi)^2}{\delta(1+\phi^2 + \frac{\lambda}{\delta^2}(1-\phi)^2) + \frac{\lambda}{\delta}(1-\phi(1-\phi))^2} + \rho \frac{\lambda}{\delta^2}(1-\phi) \right) \right]$		
x_2	$-\varepsilon_1 \left[\frac{\frac{1+\phi^2 + \frac{\lambda}{\delta^2}(1-\phi)^2}{1-\phi(1-\phi)} (1-\phi) + \phi + \rho \left(\frac{1+\phi^2 + \frac{\lambda}{\delta^2}(1-\phi)^2}{1-\phi(1-\phi)} + \rho \frac{\lambda}{\delta^2}(1-\phi) \right)}{\delta \frac{1+\phi^2 + \frac{\lambda}{\delta^2}(1-\phi)^2}{1-\phi(1-\phi)} + \frac{\lambda}{\delta}(1-\phi(1-\phi))} \right]$		
x_3	$\frac{\varepsilon_1}{\delta} (\phi-1) \left[\left(\frac{1-\phi}{1-\phi(1-\phi)} - \delta \frac{1+\phi^2 + \frac{\lambda}{\delta^2}(1-\phi)^2}{\delta(1+\phi^2 + \frac{\lambda}{\delta^2}(1-\phi)^2) + \frac{\lambda}{\delta}(1-\phi(1-\phi))^2} (1-\phi) + \phi \right) + \rho \left(\frac{1}{1-\phi(1-\phi)} - \delta \frac{1+\phi^2 + \frac{\lambda}{\delta^2}(1-\phi)^2}{\delta(1+\phi^2 + \frac{\lambda}{\delta^2}(1-\phi)^2) + \frac{\lambda}{\delta}(1-\phi(1-\phi))^2} + \rho \frac{\lambda}{\delta^2}(1-\phi) \right) \right] - \rho^2$		

2. Comparing the regimes

When judging various regimes, we let society decide, therefore, when the social loss function indicates a lower value, it is considered to be the better policy. A discretionary policy is denoted to be the optimal, if it can replicate the commitment solution. First, we examine results under specific conditions, and then step by step losing constraints, eventually we obtain general results. Keeping in mind that λ and δ have inverse relationship to social loss, suppose for simplicity that a change in the output gap puts an equal impact on the inflation ($\delta=1$). Thus, examination implicitly follows the implications of the altering slope of the Phillips curve (see Appendix B).

2.1. No exogenous persistence ($\rho = 0$)

2.1.1. Only inflation matters ($\lambda = 0$)

If society focuses solely on inflation variability, the dynamic response of inflation, output, price level, and the level of loss, when expectation structure is forward looking, is showed in Figure 1 ($\theta = 0.5$ is used in HT).¹⁷

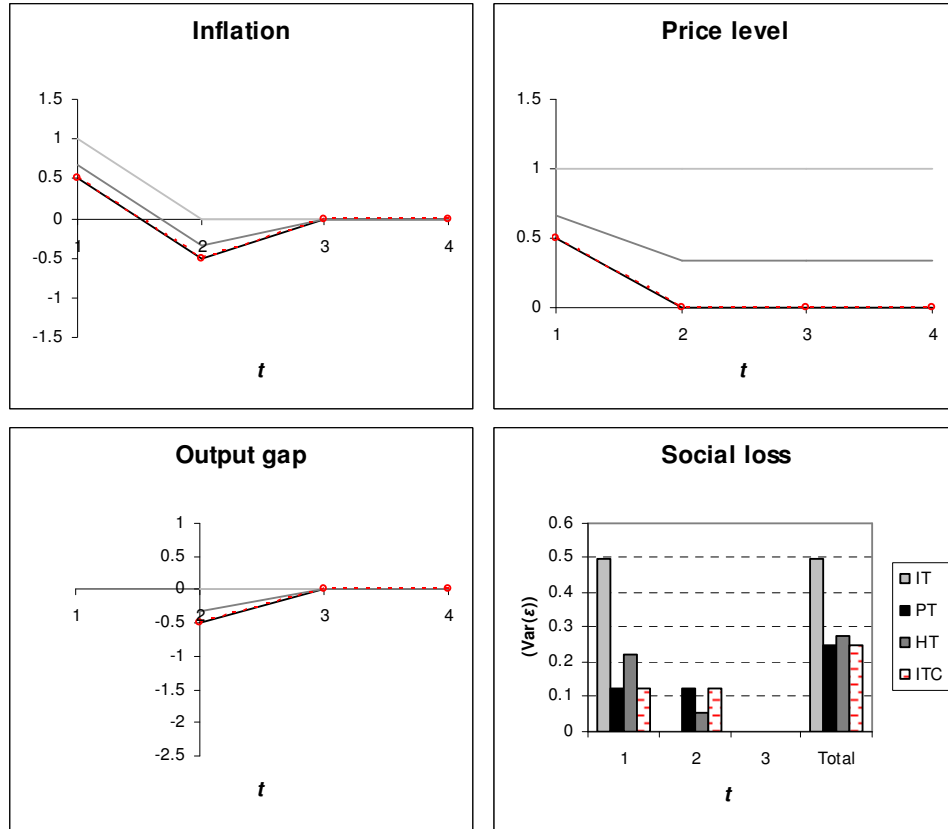


Fig. 1. *Dynamic response of variables and social loss ($\phi = 1, \lambda = 0, \rho = 0$)*

If expectations are forward looking, ITC, the theoretical benchmark implements price stability. Along such expectations, PT provides the best performance out of discretionary solutions; it achieves a total expected loss equal to the benchmark, which is half the size than in the case of IT. Due to the forward looking expectations, price level target proved itself to be useful, and just like in the case of ITC, only the half of the shock appeared in the inflation of the first period ($0.5\epsilon_1$). In the case of HT, the inflation of the first period was higher ($0.667\epsilon_1$), the correction in the second period was the half of it ($-0.334\epsilon_1$, since $\theta = 0.5$), and resulted in a moderate price level drift. Conspicuous, that the expected loss was largely diminished by the partial presence of price level target.

¹⁷ In the response diagrams, the scales of the ordinates are fixed in order to ease comparability of the different cases.

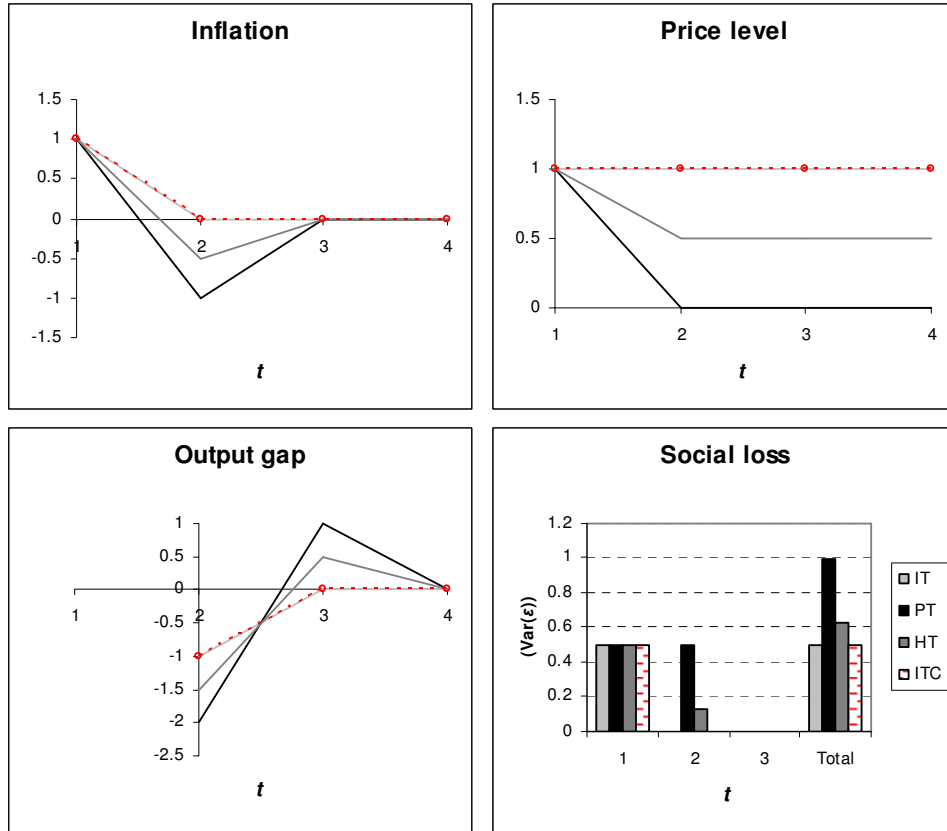


Fig. 2. Dynamic response of variables and social loss ($\phi = 0, \lambda = 0, \rho = 0$)

If the expectation structure is purely backward looking, we have totally different result (Figure 2). The order turns around with PT performing at its low by creating an expected loss double as much as IT does. In this case, ITC means a price level drift equal to the size of the shock, which is the same as what happened in the IT regime. What is the root of this? The PT regime has to maintain the price level target. Due to the purely backward looking expectations, price level target plays no orienting role at all; therefore, the shock gets integrated in the inflation of the first period at maximum extent. In the second period, a price rise of the same magnitude would occur due to lagged pricing. The task of the monetary authority is eventually to neutralize this inflationary pressure and to undo the price rise of the previous period, which means higher inflation volatility compared to IT.¹⁸

Inflation expectations perceivable in reality are not characterized by these extreme structures.¹⁹ In order to conduct comprehensive analysis, let us take a look at Figure 3, which shows the losses of regimes as a function of expectations, more precisely as the degree of forward lookingness; $\hat{S}(\phi)$, $\tilde{S}(\phi)$, $\bar{S}(\phi)$, and $S(\phi)$ denote social loss indicated under IT, PT, HT, and ITC, respectively. (Variability of inflation and output gap are discussed extensively in Chapter 3. Note that social loss reflects the linear combination of the variance of inflation and output gap.)²⁰

¹⁸ At the same time, it means higher output gap volatility, too, but now it has no relevance for society.

¹⁹ Chapter 4 provides a broader empirical outlook.

²⁰ *Proof:* The two variables are

$$\pi_t = c_t \varepsilon_t \text{ and}$$

$$x_t = d_t \varepsilon_t,$$

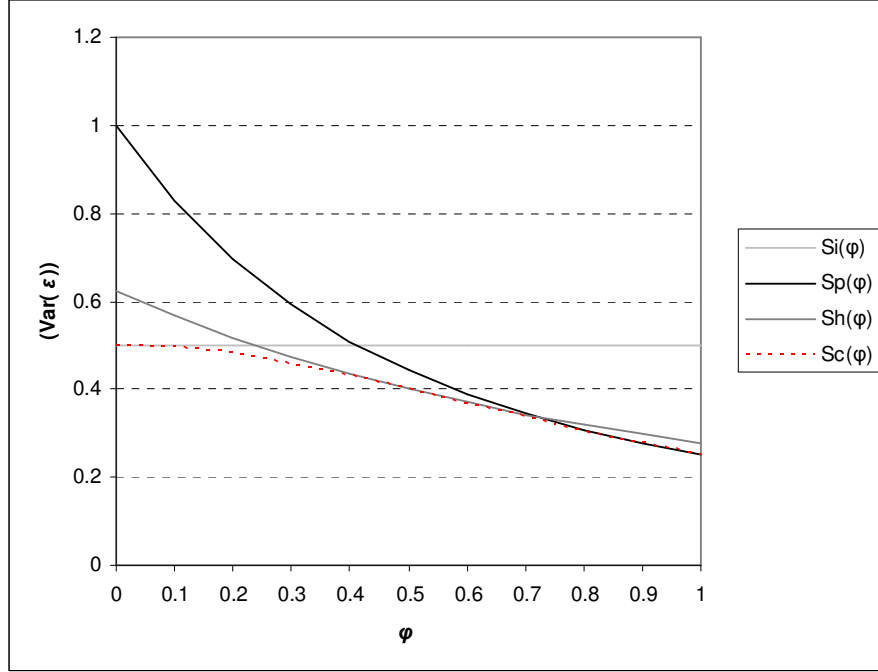


Fig. 3. *Social loss* ($\lambda = 0, \rho = 0$)

This figure shows that, depending on expectations, what discretionary policy is more adequate and where they are optimal. With the circumstances given, expected loss of IT is not influenced by the expectations, and in the case $\phi = 0$, its performance coincides with the one perceived by ITC. The more forward looking economy actors are, the better PT performs, and in the case of $\phi = 1$, it will be equivalent with the benchmark. The performance of HT depends on the value of θ : if $\theta \rightarrow 1$, then $\tilde{S}(\phi)$ embeds in the $\tilde{S}(\phi)$ curve; if $\theta \rightarrow 0$, then it embeds in the $\hat{S}(\phi)$ curve.

Table 3 sums up the order of regimes under various expectations and policy mixes, i.e. when it is better to apply a certain policy, and gives the optimality criteria.

The expected period loss is

$$E(L_t) = \frac{1}{2} [E(\pi_t^2) + \lambda E(x_t^2)] = \frac{1}{2} [E(c_t^2 \varepsilon_t^2) + \lambda E(d_t^2 \varepsilon_t^2)]$$

Since $Var(\varepsilon_t) = E(\varepsilon_t^2) - E^2(\varepsilon_t) = E(\varepsilon_t^2)$, the loss will be

$$E(L_t) = \frac{1}{2} [c_t^2 \sigma^2 + \lambda d_t^2 \sigma^2]$$

where both terms are the appropriate variances, namely

$$E(L_t) = \frac{1}{2} [Var(\pi_t) + \lambda Var(x_t)].$$

Table 3
Order of strength and optimal policies

Expectation	Policies	Loss
$0 < \phi < \sqrt{2} - 1$	$\frac{2\phi}{1-\phi^2} < \theta < 1$	$\hat{S}(\phi) < \bar{S}(\phi) < \tilde{S}(\phi)$
	$\theta = \frac{2\phi}{1-\phi^2}$	$\hat{S}(\phi) = \bar{S}(\phi) < \tilde{S}(\phi)$
	$0 < \theta < \frac{2\phi}{1-\phi^2}$	$\bar{S}(\phi) < \hat{S}(\phi) < \tilde{S}(\phi)$
$\phi = \sqrt{2} - 1$	$\exists \theta$	$\bar{S}(\phi) < \hat{S}(\phi) = \tilde{S}(\phi)$
$\sqrt{2} - 1 < \phi < 1$	$\frac{\phi^2 - 2\phi - 1}{1 - \phi^2 + 2\phi} < \theta < 1$	$\bar{S}(\phi) < \tilde{S}(\phi) < \hat{S}(\phi)$
	$\theta = \frac{\phi^2 - 2\phi - 1}{1 - \phi^2 + 2\phi}$	$\bar{S}(\phi) = \tilde{S}(\phi) < \hat{S}(\phi)$
	$0 < \theta < \frac{\phi^2 - 2\phi - 1}{1 - \phi^2 + 2\phi}$	$\tilde{S}(\phi) < \bar{S}(\phi) < \hat{S}(\phi)$
$\phi = 0$	$\theta = 0$	$\hat{S}(\phi) = S(\phi)$
$0 < \phi < 1$	$\theta = \phi$	$\bar{S}(\phi) = S(\phi)$
$\phi = 1$	$\theta = 1$	$\tilde{S}(\phi) = S(\phi)$

In the case of IT or PT with the circumstances given, the reproduction of the commitment solution will occur only at the two extreme expectation structures. In the case of hybrid expectation, HT stands for solution. With the proper balance between price level and inflation target, it is possible to create a result that is equivalent to ITC under every value of ϕ . With the current criteria given, emphasis shall be taken on price level target exactly at the same extent as the degree of forward lookingness (e.g. see Figure 3). What stated above can be seen graphically in Figure 4.

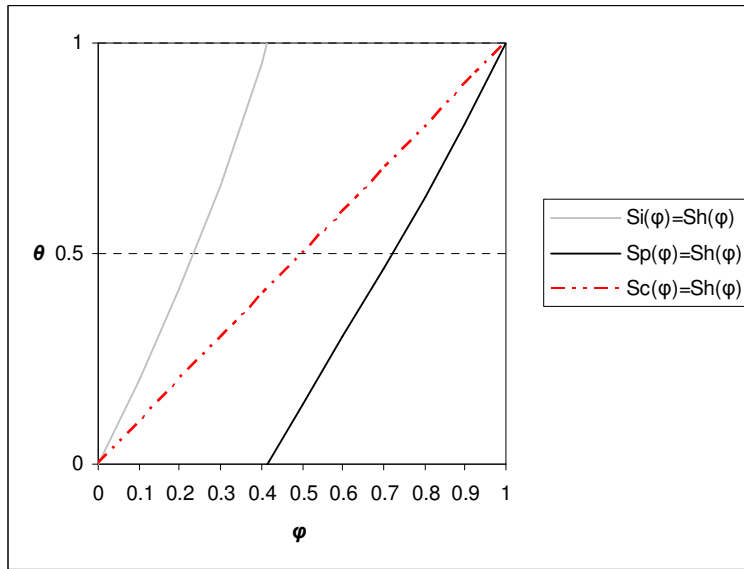


Fig. 4. Order of strength and optimal policies

The grey curve shows what mix leads IT and HT to equivalent results. The black curve means the same relation between PT and HT. The area above the grey curve means the unambiguous dominance of IT. The area below the grey curve and above the black curve shows the superiority of hybrid policies over IT and PT regimes, while the area below the black curve means the unambiguous superiority of PT. The relative effectiveness of PT and IT depends on expectations, with the previous one being a better option if the ratio of forward looking expectations is a bit over 40 per cent ($\phi > \sqrt{2} - 1$). ITC can be achieved in any expectation structure, and optimal solutions fall on the diagonal. One important question left to be cleared is the relation of HT policies under different expectation structures, which is shown in Figure 5.

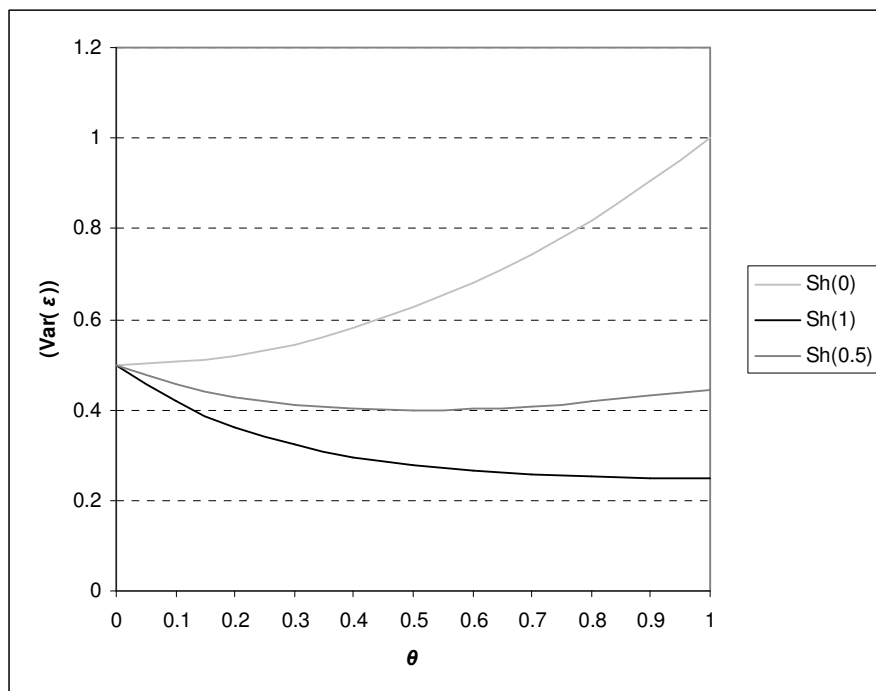


Fig. 5. Social loss under different policy mixes ($\lambda = 0, \rho = 0$)

In the New Keynesian case, adding some price level target to an IT loss function results in a notable decline in social loss. In the extremely backward looking case, incorporating some inflation target into a PT loss function decreases the loss as well; however, this latter decline is more significant, as we put more and more weight on the newly incorporated target. These relations are not linear, since the gains are decreasing. Moreover, concerning expectations, linearity does not stand either: if expectations of the economic actors shift from fully backward looking behaviour, it ameliorate the general performance of HT more, than a same shift would do it close to the fully forward looking case.²¹

²¹ As presented previously, Figure 5 also reveals that in the forward looking case it is PT, in the backward looking case it is IT, and in the mean it is HT with equal weight on price level and inflation target, that is the best discretionary policy, and that different set of expectations affect performance of PT the most, and do not affect IT at all.

2.1.2. Multigoal society ($\lambda > 0$)

First, we are about to examine a society that considers the inflation and the output gap divergence from their preferred values equally harmful ($\lambda = 1$).

We knew right at the beginning that the dynamic response of the variables of the three discretionary regimes will not change compared to the preceding but their social loss levels. However, in the case of ITC the optimal values of variables and the loss are also affected, since it considers ‘real’ social preference.

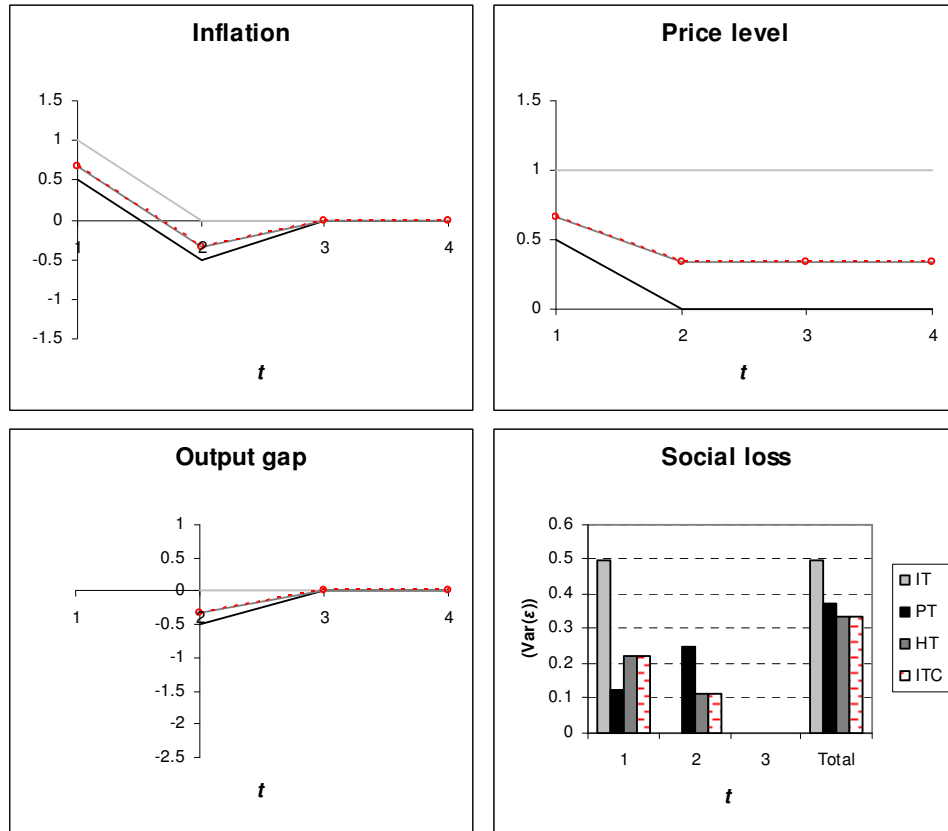


Fig. 6. Dynamic response of variables and social loss ($\phi = 1, \lambda = 1, \rho = 0$)

In the case of purely forward looking expectations, inflation in the first period became higher than in the case of $\lambda = 0$ ($0.667\epsilon_1$ instead of $0.5\epsilon_1$), and inflation shock was adjusted only partially ($-0.334\epsilon_1$ instead of $-0.5\epsilon_1$), since the opening of the output gap was dampened. The result is a price level drift that is less than the shock itself ($0.334\epsilon_1$).

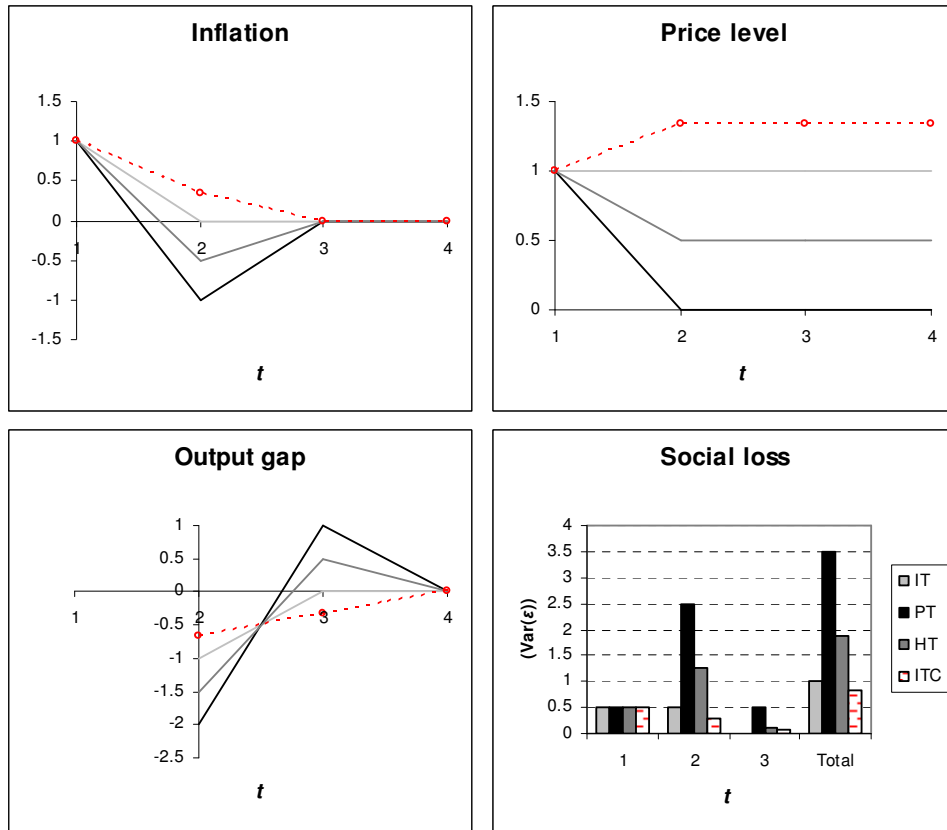


Fig. 7. Dynamic response of variables and social loss ($\phi = 0$, $\lambda = 1$, $\rho = 0$)

In the case of fully backward pricing, the total shock builds into price level in the first period, and in the second period, ITC neutralized inflation persistence thereof only partially ($-0,334\epsilon_t$), i.e. there was no correction, which resulted in a further price level drift.

Figure 8 shows the social loss in the environment of the hybrid Phillips curve.

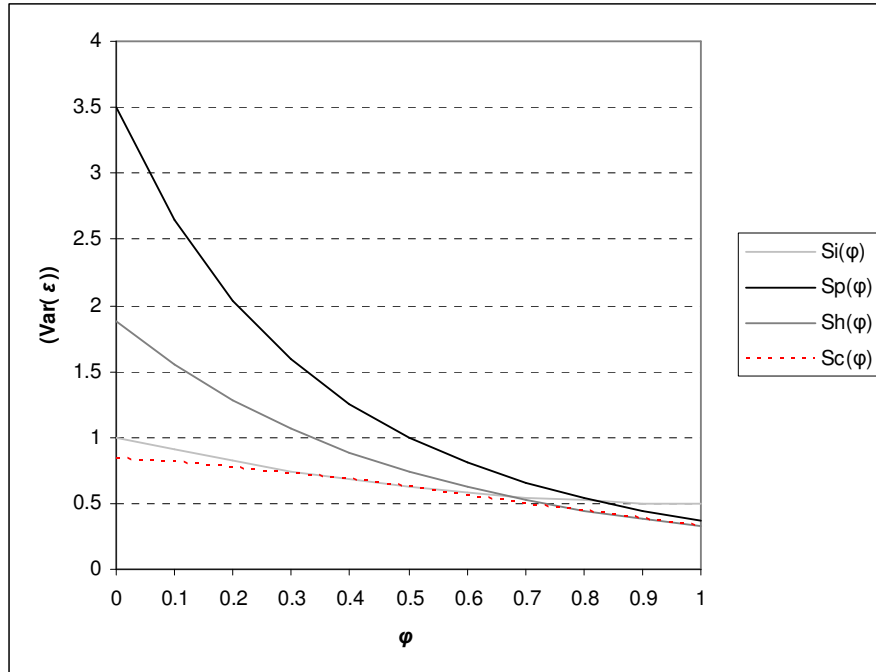


Fig. 8. *Social loss* ($\lambda = 1, \rho = 0$)

The order of strength among regimes has been reset. The disadvantages of the PT regime are plainer to see, especially when expectations are more and more backward looking. The reason behind is that the readjustment of the price level needed heavy intervention in the case of significant endogenous persistence: larger output gap had to be made, which is now penalized by the social loss function. Adding the importance of output gap volatility, IT has gained a relatively better position over PT. What even more important is that PT cannot replicate the commitment solution under any expectation structure.

However, the reproduction of the benchmark result by IT exists on a theoretical level. With any given λ , there is only one expectation structure where IT can perform this, i.e. only by certain dot pairs (λ', ϕ') . The probability of the existence of a proper pair is zero, while there are very limited instruments of the economic policy to influence these variables.

HT could mean a solution to this dilemma. This regime has the advantage to unbind the constraints of the expectation structure by using weighted inflation and price level target mix, thus only the social preference weight remains the independent variable. Under certain circumstances by picking the right θ , it enables to achieve the ITC solution at a positive output gap preference weight, note, without having an output gap weight in the central bank's loss function differing from zero. The freedom of the hybrid policy is limited by the position of IT, which means that with a given λ , it is capable to do so where

$$\hat{S}^{-1}(\phi') = S^{-1}(\phi') < \phi \leq 1. \quad (8)$$

If the significance of the variance of the output gap becomes higher, the lower bound of inequality (8) will be satisfied by higher values of ϕ , which means that the latitude of HT keeps on diminishing, analytically

$$\hat{S}^{-1}(\phi') = S^{-1}(\phi') \rightarrow 1, \text{ when } \lambda \rightarrow \infty,$$

and so it is necessary that $\theta \rightarrow 0$. The rising of λ enables HT to achieve the ITC solution on a shrinking (more and more forward looking) spectrum. To sum it up: in the range of ϕ where inequality (8) is not satisfied, IT is the suboptimal policy, and in the range of ϕ satisfying the inequality, HT is the optimal policy, since $\forall \lambda > 0: \hat{S}(1) \neq S(1)$.²² The following table summarizes optimal policies.

Table 4
Optimal policies ($\lambda > 0, \rho = 0$)

Expectations	Best policy	Remarks
$0 < \phi < \hat{S}^{-1}(\phi') = S^{-1}(\phi')$	IT, suboptimal	$\hat{S}(0) \neq S(0)$, if $\lambda > 0$
$\phi = \hat{S}^{-1}(\phi') = S^{-1}(\phi')$	IT, optimal	
$\hat{S}^{-1}(\phi') = S^{-1}(\phi') < \phi < 1$	HT, optimal	$\hat{S}(1) \rightarrow S(1)$, if $\lambda \rightarrow \infty$

Considering what has been said before, let us take a look again at Figure 8. It shows that in the case of purely forward looking expectations ($\phi = 1$) and that of inflation and output gap volatility having the same importance to society ($\lambda = 1$), the optimal combination to achieve the theoretical minimal loss is $\theta = 0.5$. Generally, in the in the fully forward looking case, the optimal values of θ belonging to the various values of λ are

$$\theta = \frac{2\lambda^2 + 6\lambda + 4 + \sqrt{(2\lambda^2 + 6\lambda + 4)^2 - 4(\lambda^3 + 4\lambda^2 + 5\lambda + 2)(2 + \lambda)}}{2(\lambda^3 + 4\lambda^2 + 5\lambda + 2)}, \quad (9)$$

which is shown in Figure 9.²³

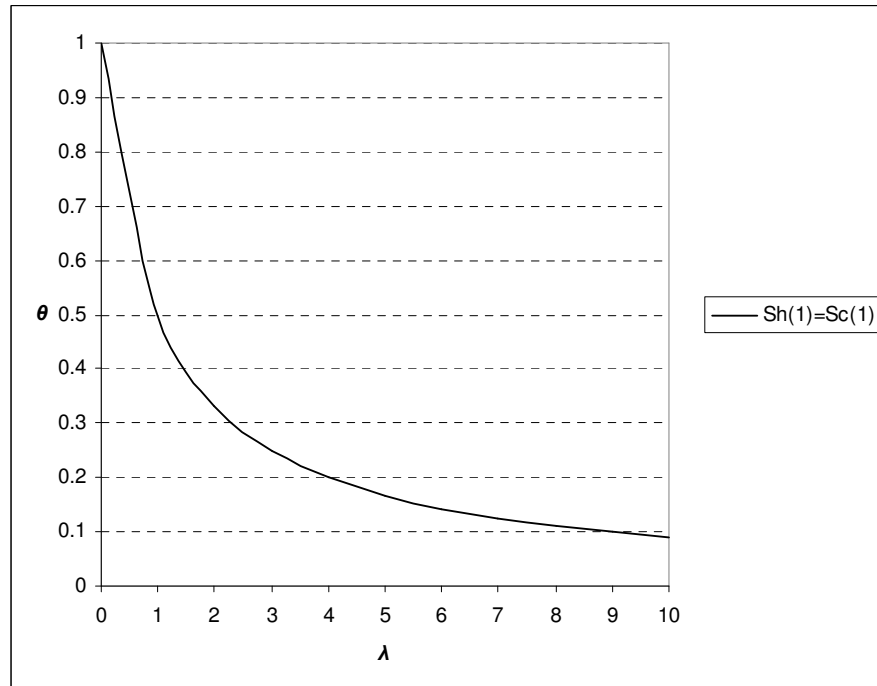


Fig. 9. Optimal policies at various social preference weights ($\phi = 1, \rho = 0$)

²² With elementary calculus $\hat{S}(1) = S(1)$, if $\lambda = -2$.

²³ Naturally, this relation can be derived at any other values of ϕ satisfying inequality 8.

This figure highlights that the rise of the preference weight, particularly by its lower values, (and/or the decline in the slope of the Phillips curve) drastically worsen the usefulness of incorporating significant price level target aside inflation target into the loss function of the central bank, even in the forward looking case. The reason is that society does not like larger output gap variability needed for eliminating price level drift, and this is even more obvious with the increase of lagged pricing at the expense of forward looking behaviour, as the interventions required are even heavier. Two of the previous results can also be seen from a different perspective. In the New Keynesian case without exogenous persistence, PT can replicate ITC only if society does not concern the output gap variability, and under same expectations, IT can not replicate the commitment solution if preference weight tends to infinity (and/or the slope of the Phillips curve tends to zero).

2.2. The role of the exogenous persistence ($\rho > 0$)

In the previous analysis, the exogenous persistence effect of the shock was not considered. There is the question whether its presence changes our previous results, and if it does, then what way. One should not forget that the already perceived lagged inflation is endogenously determined, while supply shock persistence is exogenously given, since it is not affected by policy. Let us see what our model indicates with moderate persistence ($\rho = 0.5$).

2.2.1. The output does not matter ($\lambda = 0$)

First, the more simple case is considered when society cares about the inflation variability only.

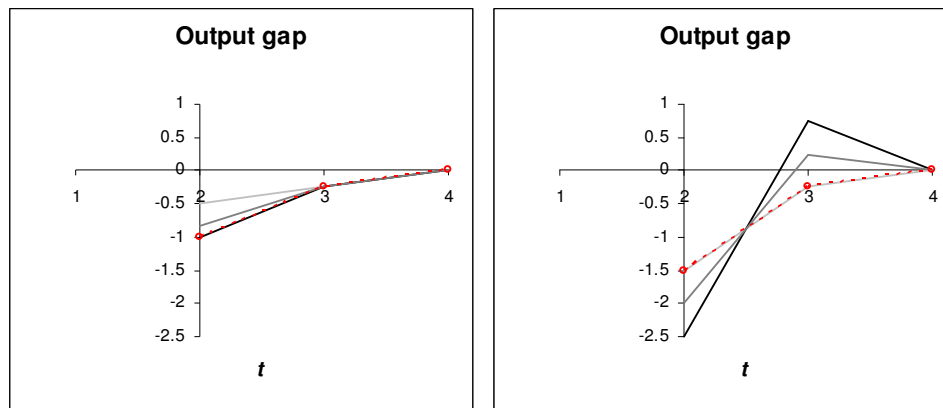


Fig. 10. Dynamic response of the output gap ($\phi = 1$, $\phi = 0$)

Figure 10 shows the dynamic response of the output gap only, since there was no quality shift at all when compared to the case without persistence ceteris paribus. If we take a look again at the results of the model (Table 2), exogenous persistence effects can apparently be identified in every solution. The effect of the exogenous persistence increasing inflation would have been $0.5\varepsilon_1$ in the second period, and $0.25\varepsilon_1$ would have been in the third period. Every regime had to intervene at a higher scale when compared to the case without persistence: its absolute value is as much as higher the persistence would have contributed to the increase of inflation, thus the result is the shifting of the output gap into negative direction. Since it does not affect losses, they are equivalent to the case without persistence.

2.2.2. The output gap matters, too ($\lambda > 0$)

Again, suppose that the inflation and output gap variability have the same importance to society, $\lambda = 1$. It is clear that this change affects only the dynamics of the variables of ITC and, naturally, the loss of all regimes.

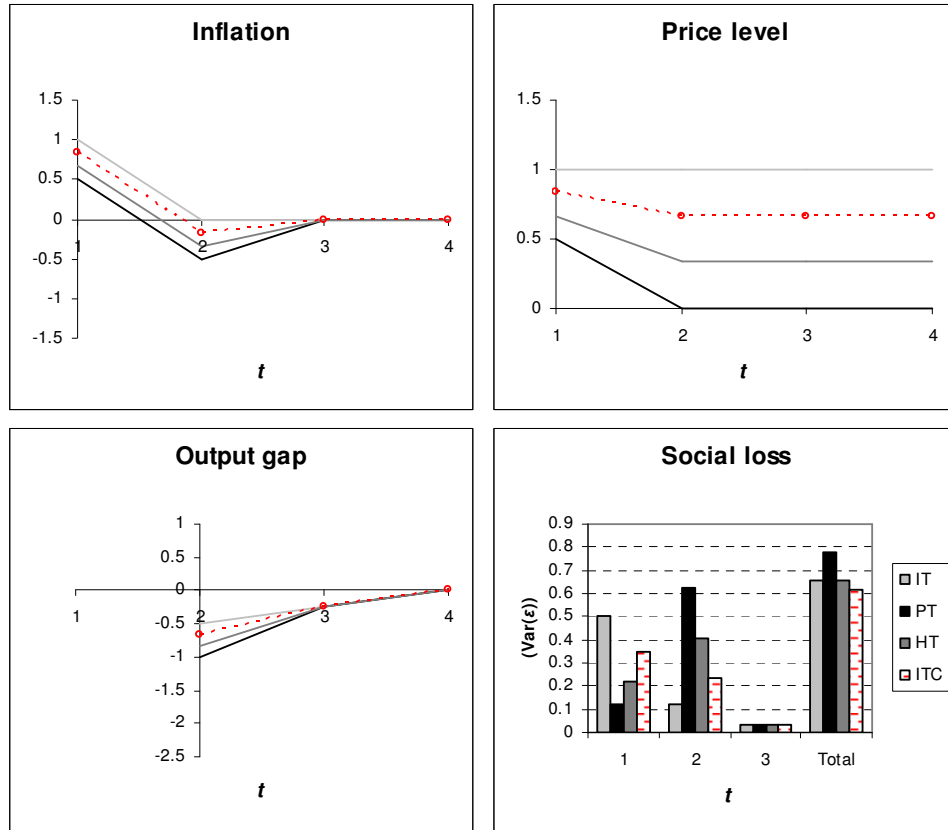


Fig.11. *Dynamic response of variables and social loss* ($\phi = 1, \lambda = 1, \rho = 0.5$)

In the case of fully forward looking expectations, although inflation is set higher by ITC when compared to the case without persistence *ceteris paribus*, its increase is smaller than the pressure from persistence (the inflation in the first period is $0.8667\epsilon_1$ instead of $0.667\epsilon_1$, and in the second it is $-0.1667\epsilon_1$ instead of $-0.334\epsilon_1$). The reason behind this is that commitment solution has countered the shock by widening the output gap ($-0.667\epsilon_1$ instead of $-0.334\epsilon_1$). Eventually, price level moved higher, although by the two third of the persistence effect of the first period only, when compared to the price level drift without persistence.

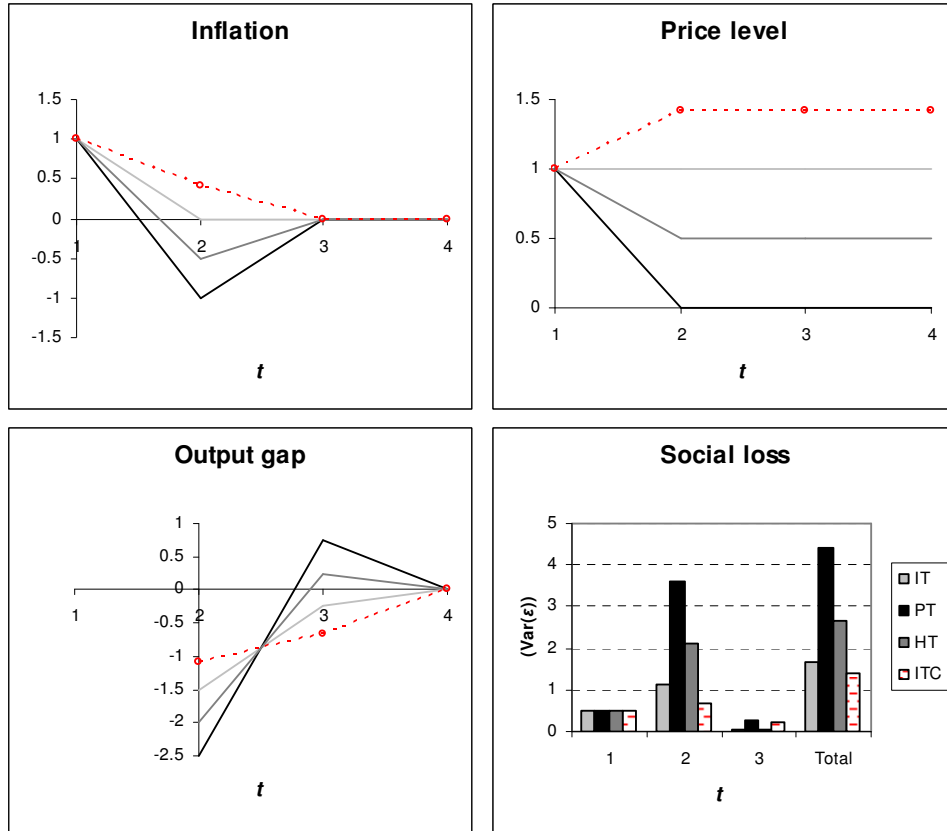


Fig. 12. *Dynamic response of variables and social loss* ($\phi = 0, \lambda = 1, \rho = 0.5$)

In the case of purely backward looking inflation expectations, the shock appears in the price level of the first period completely in the commitment solution. In the second period, ITC neutralizes around the three fourth of the inflation pressure originating from exogenous persistence and backward pricing (which is $1.5\epsilon_1$). This means that the drift in price level is larger by one sixth of the persistence effect when compared to the case without persistence. In order to analyze the hybrid expectation structure, let us take a look at Figure 13.

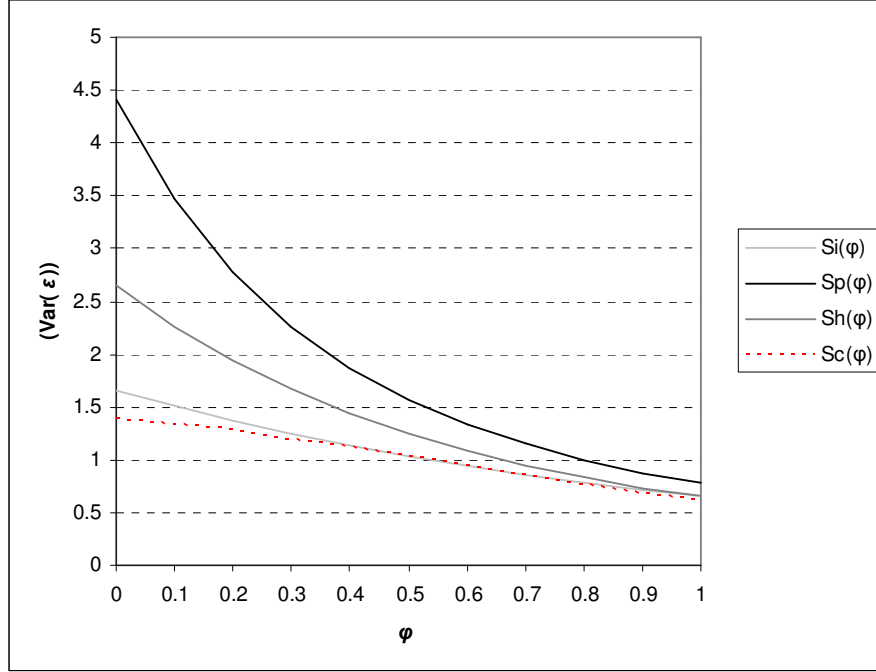


Fig. 13. *Social loss* ($\lambda = 1, \rho = 0.5$)

Larger interventions due to the exogenous persistence have the similar loss effect as if the importance of the output gap variance (λ) had increased (and/or the slope of the Phillips curve had declined). Contrary to the case without persistence, ITC cannot be reproduced by a discretionary regime at all if the increase of λ goes beyond a certain point, while this point appears during the persistence increase at a lower λ , i.e. its higher value nullifies the latitude of monetary policy concerning hybrid strategies sooner. The opportunity for HT is limited by the position of the IT which is affected by the change in λ and ρ at different scale. Theoretical optimum can be achieved by HT, where

$$\hat{S}^{-1}(\phi') = S^{-1}(\phi') < \phi \leq 1 \quad (10)$$

can be satisfied. If $\lambda \leq 1$, solutions always exist under any degree of persistence; however, if $\lambda > 1$, inequality (10) can not be satisfied unconditionally. This criterion implicitly determines a proper subset W of the vector space $V = \{x = (\lambda, \rho, \delta) : \lambda, \rho \in \mathbb{R}_0^+, \delta \in \mathbb{R}^+\}$, where IT and HT have the capability to implement the commitment solution. Combinations generating the boundary of this subset in the critical interval of $\lambda > 1$ are satisfying equality (11),

$$\rho = g(\lambda) = \frac{4\lambda + 2\lambda^2 + \sqrt{(4\lambda + 2\lambda^2)^2 - 4(\lambda^3 + 2\lambda^2)(2 + \lambda)}}{2(\lambda^3 + 2\lambda^2)}. \quad (11)$$

In such situations when the economy is characterized by these combinations, only IT can reproduce the benchmark solution, namely in the fully forward looking case. Beyond this boundary, none of the discretionary regimes can achieve ITC, and in that case, it is IT that provides the best policy, even though suboptimal one only, in the whole range of

expectations. Table 5 summarizes optimal policy criteria, while Figure 14 shows the abovementioned subset.

Table 5
Optimal policies ($\lambda > 0, \rho > 0$)

Expectations	Best policy	Criteria	Remarks
$0 \leq \phi < \hat{S}^{-1}(\phi') = S^{-1}(\phi')$	IT, suboptimal	$\forall \rho$, if $\lambda \leq 1$; $\rho < g(\lambda)$, if $\lambda > 1$	If $\rho = g(\lambda)$ and $\phi = 1$, IT is optimal ($\hat{S}(1) = S(1)$); If $g(\lambda) < \rho$, IT is suboptimal, $0 \leq \forall \phi \leq 1$
$\phi = \hat{S}^{-1}(\phi') = S^{-1}(\phi')$	IT, optimal		
$\hat{S}^{-1}(\phi') = S^{-1}(\phi') < \phi \leq 1$	HT, optimal		

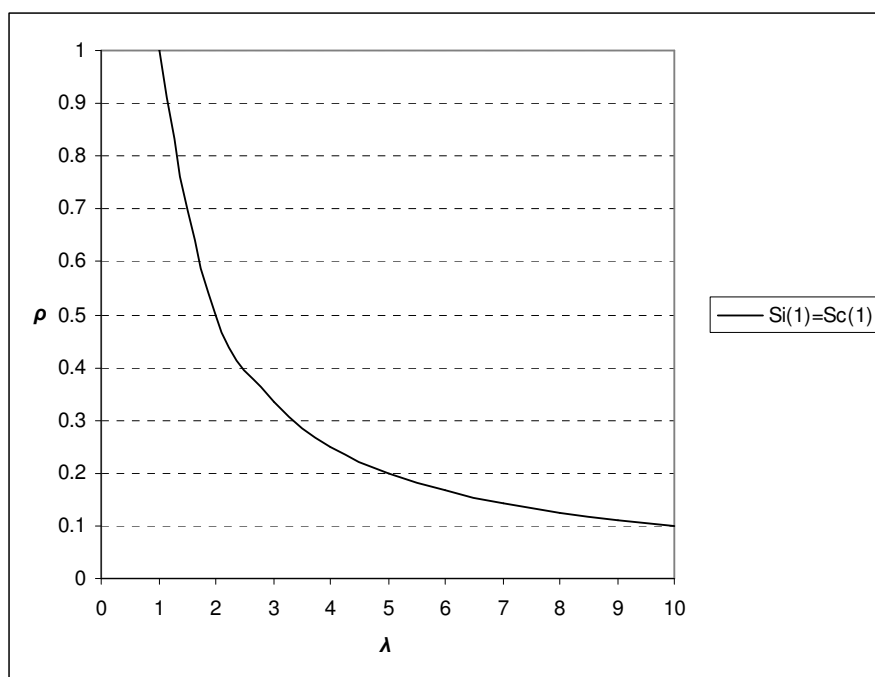


Fig. 14. Boundary of subset W ($\delta = 1$)

Figure 14 shows, if $\lambda \leq 1$, then persistence per se is not a constrain at all for the existence of optimal hybrid policies.²⁴ For instance, in the case of $\rho = 0.5$, HT has a relevance, if $\lambda < 2$, which means that it is capable, through a narrowing expectation range with the increase of λ at the same time, to achieve the theoretical optimum. If $\lambda = 2$, then $\hat{S}(1) = S(1)$, i.e. only IT can reproduce ITC, and only in the New Keynesian case; however, if $\lambda > 2$, then none of the discretional regimes is capable of that. Hence, IT gives the solutions closest to ITC throughout the whole expectation spectrum.

²⁴ At $\lambda = 1$, $\hat{S}(1) = S(1)$ would require $\rho = 1$, what can not be.

3. Inflation and output gap variability

The presented model always indicates higher output gap variability in the case of PT related to IT, however, it shows lower inflation variability under certain circumstances.²⁵ The reason is the implementation of the transmission lag. In the fully forward looking case, IT creates no output gap in the period following the supply shock, as it has no reason, while PT must shepherd the price level back to its targeted value. It is more straightforward in those cases, where the expectations are more and more backward looking, since the expectation driving effect of using a price level target deteriorates more and more, hence the initial jump in inflation is higher.²⁶

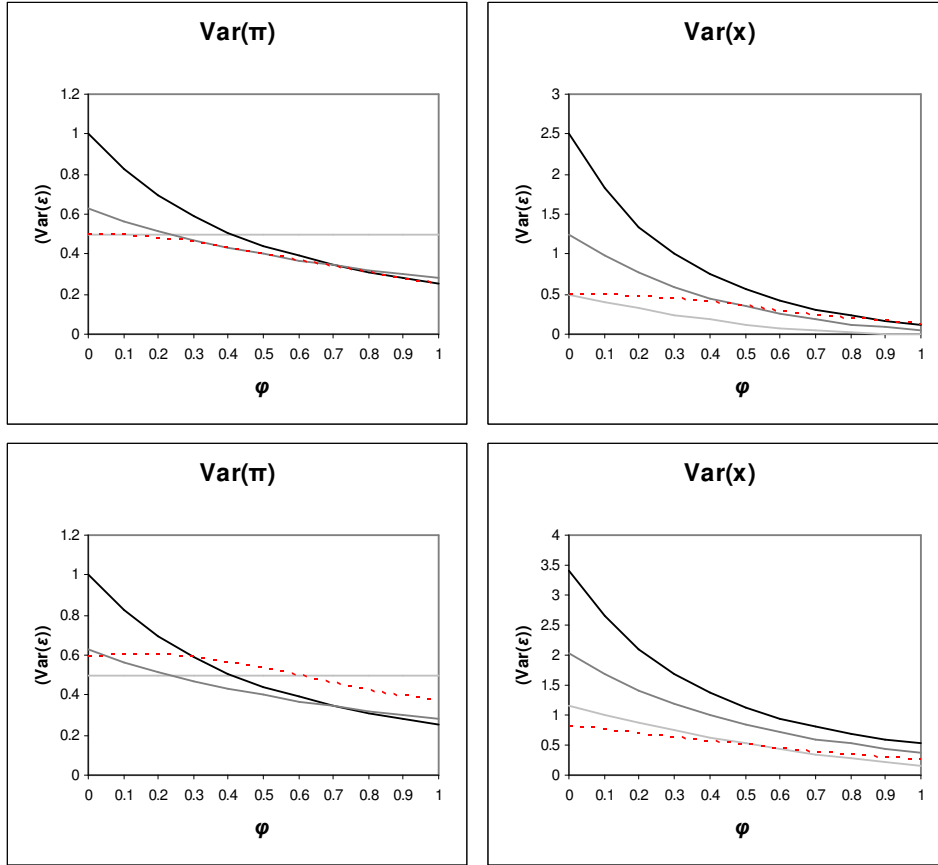


Fig. 15. Inflation and output gap variability, $\lambda = 0, \rho = 0$ and $\lambda = 1, \rho = 0.5$ ($\delta = 1$)

The output gap variability moves inversely to the slope of the Phillips curve which has the same effect under IT and PT, and moves along with exogenous inflation persistence, but, on the contrary, this latter one does not have the same impact on IT and PT. According to PT, it causes additional intervention requirement in period 2, however, it *helps* to counter the backward pricing effect for one period after the deflationary phase. On the other hand, when expectations are rather forward looking, it may mean *cost* in every period. The situation in the

²⁵ The simulation of Fillion and Tetlow (1994) also reported that PT creates lower inflation variability but higher output gap variability than IT, but as Svensson (1999) already noted, they did not give explanation beyond that these results indicate strong serial correlation of the price level.

²⁶ Considering strict targeting, if there was no transmission lag, the output gap variability would be lower in PT compared to IT in the fully forward looking case, and would be the same in the fully backward looking case.

case of IT is simpler, since exogenous inflation persistence always means additional intervention requirement on the whole range of expectations. Thus, with the rise of the exogenous persistence, the difference between output gap variances is the dependent of these full impacts on output gap variability (see Figure 16).

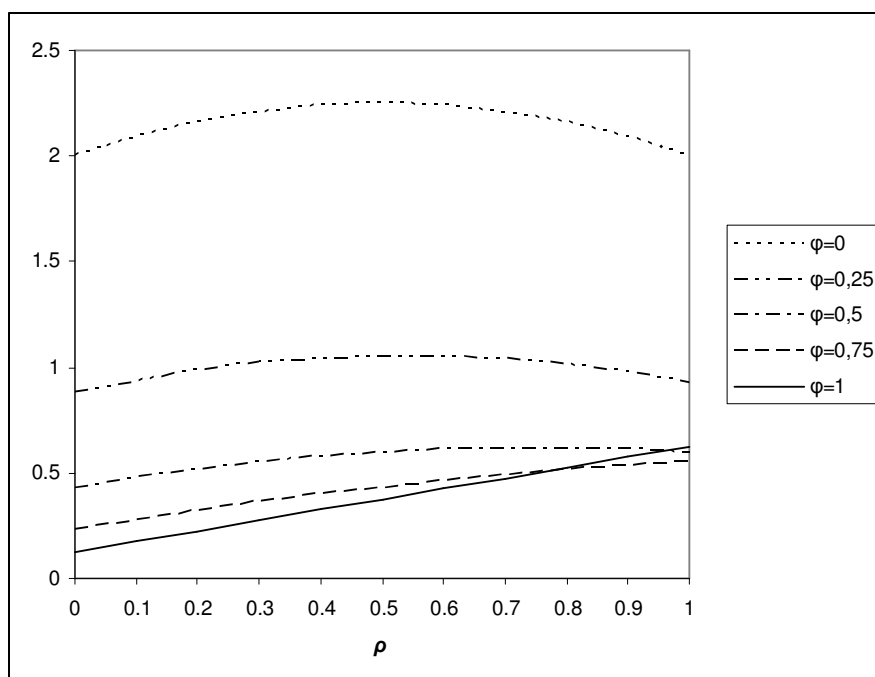


Fig. 16. Differences in output gap variability of IT and PT, ($\delta = 1$)

4. Empirical outlook

The estimation of Fuhrer and Moore (1995) using a sticky price model showed near equivalent forward and backward looking behaviour, while Galí and Gertler (1999) demonstrated that the forward looking behaviour is more dominant (0,68-0,87).²⁷ Also, Galí and Gertler (1999) emphasized the sluggish behaviour of real marginal cost, which might be a good explanation of the slow inflation response to output gap, hence, the high and costly output gap needed for making inflation move. This flattening tendency of the Phillips curve is also demonstrated by Sbordone (2007). She found that global competition affecting US economy decreases the sensitivity of inflation to marginal cost. Continuous supply shocks due to oil and food prices seem to be a long course, too. Backward pricing, declining slope of the Phillips curve, and persistent cost push shocks are not too favourable background for targeting a constant price level, though could be for hybrid policy according to the presented model.²⁸ However these conditions are not petrified. In the 1960s-70s, uncertainty around monetary transmission was high, since the lag was long and variable. That was the reason why

²⁷ Galí and Gertler (1999) argued that therefore the New Keynesian Phillips curve gives a good first approximation of the inflation dynamics. Rudd and Wheelen (2005) and Lindè (2005) claimed that it is a result of specification bias, while Kurmann (2005) pointed out the uncertainty around the estimation procedure. In a recent paper, Galí et al. (2005) stand out for their results.

²⁸ The latitude and applicability of hybrid policy is also restricted by all of these tendencies, and at the same time, as Cechetti and Kim (2003) noted, this regime has the disadvantage that a hybrid target is very hard to be communicated, which would be a key issue of conducting credible monetary policy.

Friedman (1968) emphasized the impossibility of price stabilization, but added that it could be otherwise, if the “understanding of monetary phenomena advances”. Au contraire, it was not the case at the beginning of the 20th century. According to the observation of Fisher (1912) on the US economy, the monetary transmission reliably exerted its full effect on prices in 3 month, again contrary to the experienced 1.5-2 years of our time. Another prevailed argument of our days is that creating deflation leads to financial instability, and so price level targeting is not favourable, which was emphasized by Fisher (1994) and Mishkin (2001). However, all of these phenomena may reflect the policy-affected economy of its era. The Swedish episode of the 1930’s showed that maintaining a constant price level target is feasible without falling into the pit of the zero bound problem. As Berg and Jonung (1999) emphasized the lessons learned, price level targeting helped to raise inflation expectations despite the persistent worldwide deflationary pressure during the Great Depression. This historical evidence is a fine support of Lucas critique, as the change in the policy modified the expectations of the economic actors, and so it is revealed that some of the ‘axiomatic causalities’ were only the manifest of a reigning paradigm.²⁹

5. Conclusions

In the presented three-period steady state to steady state framework it is showed that strict inflation, price level and hybrid targeting can all achieve theoretical optimum, inflation targeting with commitment, under certain circumstances. Considering transmission lag, the model indicated that price level targeting always creates higher output gap variability than inflation targeting, whereas the relation in inflation variability, and so the social loss implication, is an open issue, sensitively depending on the conditions.

Inflation targeting proved far more robust than price level targeting, while hybrid targeting had the best adaptability. It is showed that without exogenous inflation persistence, inflation targeting and hybrid targeting can always reproduce commitment solution on a descending, more and more forward looking range of expectations with the rise of the social preference weight on the output gap (and/or with the decline of the slope of the Phillips Curve), while in the most general and realistic case, the existence of exogenous persistence makes the possibility of implementation to the function of the social preference weight (and/or the slope of the Phillips curve).

The examination demonstrated the non-linear interrelation of economic and policy parameters. Depending on the policy framework, the impact of parameters on inflation variability, output gap variability and social loss manifest in a different way, moreover, not always in monotonic fashion.

Appendix

A. Model solutions

A.1. *Inflation targeting with commitment*

The expected loss to be minimized subject to the constraints given by Phillips curve is

²⁹ Mishkin (2006) reconsidered his earlier sceptical view contemplating the case of Japan, and concluded that PT can be favourable in an economy experiencing deflationary pressure. Models of Eggertson and Woodford (2003) and Wolman (2005) showed that implementing rules in order to maintain stationary price level helps to evade hitting the zero bound.

$$E_0 \sum_{i=1}^{\infty} \frac{1}{2} [\pi_i^2 + \lambda x_{i+1}^2]$$

which is $E_0 \frac{1}{2} [\pi_1^2 + \pi_2^2 + \lambda x_2^2 + \lambda x_3^2]$. Since the central bank has full credibility, it endogenizes inflation expectations during its optimizing process, thus $\pi_{t+1|t} = \pi_{t+1}$. The Lagrangian is

$$\begin{aligned} \Omega(\pi_1, \pi_2, x_2, x_3) = & \frac{1}{2} [\pi_1^2 + \pi_2^2 + \lambda x_2^2 + \lambda x_3^2] - \eta_1 (\phi \pi_2 + \varepsilon_1 - \pi_1) - \\ & - \eta_2 ((1-\phi)\pi_1 + \delta x_2 + \rho \varepsilon_1 - \pi_2) - \eta_3 ((1-\phi)\pi_2 + \delta x_3 + \rho^2 \varepsilon_1) \end{aligned}$$

whose first order conditions are

$$\begin{aligned} \frac{\partial \Omega}{\partial \pi_1} &= \pi_1 + \eta_1 - \eta_2 (1-\phi) = 0, \\ \frac{\partial \Omega}{\partial \pi_2} &= \pi_2 - \eta_1 \phi + \eta_2 - \eta_3 (1-\phi) = 0, \\ \frac{\partial \Omega}{\partial x_2} &= \lambda x_2 - \eta_2 \delta = 0, \\ \frac{\partial \Omega}{\partial x_3} &= \lambda x_3 - \eta_3 \delta = 0, \\ \frac{\partial \Omega}{\partial \eta_1} &= \phi \pi_2 + \varepsilon_1 - \pi_1 = 0, \\ \frac{\partial \Omega}{\partial \eta_2} &= (1-\phi)\pi_1 + \delta x_2 + \rho \varepsilon_1 - \pi_2 = 0, \\ \frac{\partial \Omega}{\partial \eta_3} &= (1-\phi)\pi_2 + \delta x_3 + \rho^2 \varepsilon_1 = 0. \end{aligned}$$

Simple rearrangements and substitutions lead to the optimal solutions.

A.2. Inflation targeting

The expected loss to be minimized is

$$E_t \sum_{i=1}^{\infty} \frac{1}{2} [\pi_{t+i}^2]$$

which is $E_1 \frac{1}{2} [\pi_2^2 + \pi_3^2]$ in period 1, and $E_2 \frac{1}{2} [\pi_3^2]$ in period 2. Inflation values minimizing the loss are

$$\pi_2 = 0 \text{ and } \pi_3 = 0.$$

According to equations (6) and (7), $\pi_{t+1|t} = 0$. With simple substitutions into the conditions given by the Phillips curve, solutions are obtained.

A.3. Price level targeting

The expected loss to be minimized is

$$E_t \frac{1}{2} \sum_{i=1}^{\infty} p_{t+i}^2 = E_t \frac{1}{2} \sum_{i=1}^{\infty} \left(\sum_{j=1}^{t+i} \pi_j \right)^2$$

which is $E_1 \frac{1}{2} [p_2^2 + p_3^2] = E_1 \frac{1}{2} [(\pi_1 + \pi_2)^2 + (\pi_1 + \pi_2 + \pi_3)^2]$ in period 1, and $E_2 \frac{1}{2} [(\pi_1 + \pi_2 + \pi_3)^2]$ in period 2. Inflation values minimizing the loss are

$$\pi_1 + \pi_2 = 0 \text{ and } \pi_3 = 0.$$

According to equations (6) and (7), $\pi_{t+1|t} = p^* - p_t = -p_t$. With simple substitutions into the conditions given by the Phillips curve, solutions are obtained.

A.4. Hybrid targeting

Using the transformation of

$$\theta p_t + (1-\theta)\pi_t = \theta p_{t-1} + \theta\pi_t + \pi_t - \theta\pi_t = \pi_t - \theta p_{t-1},$$

the loss function to be minimized is

$$E_t \frac{1}{2} \sum_{i=1}^{\infty} (\pi_{t+i} + \theta p_t)^2 = E_t \frac{1}{2} \sum_{i=1}^{\infty} \left(\pi_{t+i} + \theta \sum_{j=1}^i \pi_j \right)^2$$

which is $E_1 \frac{1}{2} [(\pi_2 + \theta p_1)^2 + (\pi_3 + \theta p_2)^2] = E_1 \frac{1}{2} [(\pi_2 + \theta\pi_1)^2 + (\pi_3 + \theta(\pi_1 + \pi_2))^2]$ in period 1, and $E_2 \frac{1}{2} [(\pi_3 + \theta(\pi_1 + \pi_2))^2]$ in period 2. Inflation values minimizing the loss are

$$\pi_2 + \theta\pi_1 = 0 \text{ and } \pi_3 = 0.$$

According to equations (6) and (7), $\pi_{t+1|t} = (1-\theta)\pi_1 - p_t$. With simple substitutions into the conditions given by the Phillips curve, solutions are obtained.

B. Social preference weight and the slope of the Phillips curve

Considering the commitment solution, one can recognize the relation scheme³⁰

$$\frac{1}{\delta + \frac{\lambda}{\delta}} = k$$

in the case of the output gap, and

$$\frac{\delta}{\delta + \frac{\lambda}{\delta}} = \delta k$$

in the case of inflation.

What does that mean? If the output gap does not matter at all ($\lambda = 0$), and the central bank should focus on the inflation target alone, then only the slope of the Phillips curve determines the size of the output gap necessary to achieve the goals. If it is not just the inflation, but the output gap matters ($\lambda > 0$), then it can be seen that the rules reduce the output level divergence from the potential one.

Considering the inflation rule, there are equivalent outcomes in any cases where $\frac{\lambda}{\delta} = 0$, since $\delta k = 1$ at the same time. If $\delta = \infty$, then the Phillips curve is vertical, which means that the central bank can make, with minimal intervention, the inflation move infinitely, and therefore, the preference weight on the output gap is not relevant. If $\lambda = 0$, the inflation rule is independent from the slope of the Phillips curve, as the scale of the intervention does not matter. These situations result in the same social loss; the only difference between the two cases can be captured in the size (and the variability) of the output gap. In discretionary regimes, since they are strict targeters, the social loss implications of these relations are more straightforward.

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³⁰ For perspicuity, the scheme of the new Keynesian case is demonstrated. Similar would be the case of the discretionary regimes as well, if they were not strict targeters.

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