The two sides of envy

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23 September 2010

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September 26, 2010

Abstract

The two sides of envy, destructive and competitive, give rise to qualitatively different equilibria, depending on economic, institutional, and cultural environment. If inequality is high, property rights are poorly protected, and social comparisons are strong, the society is likely to settle in the "fear equilibrium," in which better endowed agents restrain their efforts to prevent destructive envy of the relatively poor. In the opposite case, the standard "keeping up with the Joneses" competition arises, and individuals satisfy their relative standing concerns through suboptimally high efforts. The different nature of these equilibria leads to starkingly contrasting effects of envy on economic performance. From welfare perspective, adoption of better institutions may not be Pareto improving, since positional externality is curbed in the low-output fear equilibrium. The theory is consistent with broad empirical facts from social sciences and bridges the gap between separate lines of research on envy.

Keywords: Envy, Inequality, Positional Externalities, Property Rights

JEL Classification Numbers: D31, D62, D74, O10, O43, P26, Z13

*I am grateful to Oded Galor for his guidance and advice. Pedro Dal Bó, Geoffroy de Clippel, Peter Howitt, Mark Koyama, Nippe Lagerlöf, Ross Levine, Glenn Loury, Louis Putterman, Holger Strulik, Eytan Sheshinski, Ilya Strebulaev, and David Weil provided valuable comments.

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1 Introduction

Interpersonal comparisons and concern for relative standing are pervasive features of individuals interacting in a society. That is why social sciences including anthropology, sociology, psychology, political science, and economics take these phenomena seriously. This paper is an attempt to reconcile the results of theoretical and empirical research on the role of relative standing concerns in driving economic and institutional outcomes. It develops a theory which captures in a unified framework qualitatively different equilibria that can emerge in the presence of positional externalities, depending on economic, institutional and cultural environment. The theory is used to analyze in a novel way a number of important issues such as the role of redistributive mechanisms in mitigating destructive activities, the evolution of property rights protection, and inequality dynamics.

Throughout the paper the concern for relative standing is referred to as "envy." It is assumed that envy is part of preferences and individuals care about how their own consumption compares to that of their reference group.[1] The fundamental idea, supported by different strands of research on envy, is that relative concerns can be satisfied in two important ways: by increasing own outcome (competitive envy) and by decreasing the outcome of the reference group (destructive envy).[2] The active side of envy is determined by the environment reflected in three main factors: inequality of endowments, level of property rights protection, and strength of social comparisons. So, envy itself is in the utility function, and its manifestation is conditioned by the "budget constraint" shaped by economic, institutional and cultural factors.

The basic model is set up as a simple two-person (two-group) dynamic game consisting of two stages. In the first stage, each individual invests, and the outcome depends on effort and endowment. In the second stage, this investment outcome is used in production. Moreover, the initially disadvantaged agent may choose to spend part of his time to improve his relative position by disrupting the other agent's production process. Whether he chooses

[1] A number of evolutionary theoretic explanations have been proposed for why people have relative concerns in their preferences, recently by Samuelson (2004) and Rayo and Becker (2007); see Hopkins (2008, section 3) and Robson and Samuelson (2011, section 4.2) for an overview. Evidence in support for relative concerns is abundant and comes from empirical happiness research (Luttmer, 2005), experimental economics (Zizzo, 2003), neuroscience (Fliessbach et al., 2007), and surveys (Solnick and Hemenway, 2005); see Clark et al. (2008, section 3) and Frank and Heffetz (2011, section 3) for an overview.

[2] An alternative option is to give up and drop out from the competition for status (Banerjee, 1990; Barnett et al., 2010). Yet another possibility is to redefine the reference group (see Falk and Knell (2004) for a model with endogenous formation of reference standards).

to do so depends on the inequality of investment outcomes and tolerance for inequality, which is determined endogenously by the level of property rights protection (effectiveness of destruction technology) and the importance of relative concerns.[3] The better endowed individual anticipates the possibility of destructive envy and can prevent it by trading off output for leisure in the investment stage.

This game gives rise to different types of equilibria, with either competitive or destructive side of envy active. The first one is a familiar "keeping up with the Joneses" (KUJ) equilibrium, in which individuals compete with each other peacefully and consumption externality leads to suboptimally high output at a cost of reducing leisure. The second type is the "fear equilibrium," in which the better endowed individual anticipates destructive envy and prevents it by restricting effort in the investment stage. In this equilibrium the rich individual is at the corner solution and factors that aggravate the "fear constraint" lead to lower economic outcomes. Finally, if the initial inequality is very high or the tolerance for inequality is low, that is, property rights are poorly protected and relative concerns are strong, a destructive equilibrium arises, in which actual destruction takes place and part of the output is wasted to satisfy envy.

The rest of the paper is organized as follows. Section 2 reviews related literature and evidence motivating the need for a unified economic theory of envy. Section 3 lays out the basic model and examines its comparative statics. In Section 4 the theory is modified to allow voluntary transfers, and redistributive mechanisms are shown to prevent destruction in equilibrium. In Section 5 the basic model is used to analyze the incentives of envious agents to adopt better property rights protection that would eliminate the fear of destructive envy. It is shown that a positive institutional change may fail since individuals might prefer to "live in fear" rather than compete in a "rat race" for status. Section 6 extends the basic model to a dynamic framework by endogenizing the evolution of inequality. This extension allows to analyze transitions between different types of equilibria, as well as convergence to the long-run steady state. It also highlights the factors that can keep societies in the fear of envy trap or help them to get out of it. Section 7 concludes. All proofs are collected in Appendix.

[3]See Hirschman and Rothschild (1973) for a pioneering discussion of the factors affecting tolerance for inequality and of the "tunnel effect" as a force countervailing envy.

2 Related Literature and Evidence

Envy and its effects on people's incentives have been the subject of attention since ancient times. The distinction between the two sides of envy, destructive and competitive, goes back at least to Aristotle's "Rhetoric" (see Schoeck, 1969, p. 194).[4] Classical economists like Adam Smith and Richard Whately considered emulation to be a powerful engine of economic development.[5] Whately in his "Introductory Lectures on Political Economy" (1831) suggests that "as wealth increased, the continued stimulus of emulation would make each man strive to surpass, or at least not fall below his neighbors" (quoted in Kern, 2001, p. 355) and goes on to describe the mechanics of the KUJ competition. At the same time, Smith in "The Wealth of Nations" (1776) warns about the destructive side of envy and sees it as one of the reasons for the necessity of government's protection of private property: "The affluence of the few supposes the indigence of many, who are often both driven by want, and prompted by envy, to invade their possessions. It is only under the shelter of the civil magistrate that the owner of that valuable property... can sleep a single night in security" (quoted in Schoeck, 1969, p. 200).

This dichotomy has been discussed extensively by sociologists (Schoeck, 1969; Clanton, 2006), anthropologists (Foster, 1972), philosophers (D'Arms and Kerr, 2008), political scientists (Fernández de la Mora, 1987), psychologists (Smith and Kim, 2007; van de Ven et al., 2009), theologists (Malina, 2001, chapter 4) and economists (Elster, 1991; Zizzo, 2008). The terms that roughly correspond to the two sides of envy and are used by researchers in different contexts are: 1) destructive envy, black envy, envy proper, malicious envy, schadenfreude, resentment; 2) competitive envy, white envy, benign envy, emulation, keeping up with the Joneses. The connotation is negative for the first group and mostly positive for the second. In the economic theory of envy, proposed in this paper, the motive, concern for relative standing, is always the same, but, depending on the environment, its active side may be either destructive or competitive.

Despite the general consensus about the two sides, or behavioral consequences, of envy, most in-depth research has concentrated on its one particular aspect. The fear of envy, caused by its destructive potential, is examined in the works by anthropologists studying small-scale societies. Foster (1972) distinguishes between competitive and fear "axes" of envy focusing on the latter, along which a man "fears being envied for what he has, and

[4]For an overview of philosophical texts on envy see Schoeck (1969, chapter 11) and Fernández de la Mora (1987, part A). A recent philosophical examination of inequality and envy is by Ben-Ze'ev (1992).

[5]The treatment of relative standing concerns by classical economists is examined by Kern (2001).

wishes to protect himself from the consequences of the envy of others" (p. 166). He argues that the fear axis is predominant in peasant societies and links it to the model of cognitive orientation that he labeled the "Image of Limited Good.". In such societies "life is played as a zero-sum game, in which one player's advantage is at the expense of the other" (Foster, 1972, p. 168). Consequently, people are reluctant to exert effort or innovate since they expect sanctions such as forced redistribution, plain destruction, or casting the evil eye.

The evil eye belief is one of the interesting cultural phenomena associated with the fear of envy. According to the general view, it is the belief that people can project harm by looking at others or their property. Most theories about the nature of the evil eye belief link it to the fear of envy (Maloney, 1976). Furthermore, Aquaro (2004) *defines* it as a belief "that a person's eye produces harmful emanations when he or she feels envy towards another." The term "institutionalized envy" coined by Wolf (1955) summarizes the set of cultural control mechanisms related to the fear axis of envy which includes, apart from the evil eye belief, gossip, fear, and practice of witchcraft. According to Wolf, such mechanisms minimize "disruptive phenomena such as economic mobility, abuse of ascribed power, or individual conspicuous show of wealth" (p. 460).

Institutionalized envy can have important effects on economic incentives. Platteau (2000, chapter 5) analyzes the role of witchcraft as a deterrent to private accumulation of wealth in African rural societies. Schoeck (1969, chapter 5) cites other examples of how institutionalized envy restrains innovation and economic progress. A typical one is that of a peasant in an Indian village who refused to use a new fertilizer motivating it by the fear of the evil eye in case of especially good harvest. Schoeck generalizes this phenomenon and calls it the "envy-barrier" of developing countries, pointing out that this aspect of envy is often ignored: "The social sciences have put forward numerous theories on the assumption that the normal man seeks a maximum in production and in property. . . These theories, however, overlook the fact that in great many situations the object of human activity is diminution; that regularly recurring modes of human behaviour have as their object the lessening of assets, not just their replacement by other assets" (p. 59). This point is accounted for in the proposed theory of envy. In the formal model of Section 3 the fear of envy is neither ungrounded nor tied to supernatural sanctions: in equilibria reflecting the destructive side of envy actual destruction represents a credible threat.

Destructive or fear manifestations of envy are characteristic not only for simple pre-industrial societies. Mui (1995) focuses on two large industrial economies, Soviet Union and China, in the beginning of their transition to the free market. He brings up evidence on emerging cooperative restaurants and shops in the Soviet Union being regularly attacked by people resenting their success. He then cites (p. 313) a similar story from Chinese press about a peasant whose successful entrepreneurship provoked the envious neighbors to steal timber for his new house and kill his farm animals. "I dare not work too hard to get rich again" was his comment. The fear of destructive envy may be a serious issue in societies with socialist experience for two reasons. One is the ideology of leveling, which, contrary to the intentions of policymakers, may intensify invidious comparisons and lower tolerance for inequality. The second reason is the neglect of private property rights that makes destructive envy more likely.

In the proposed theory the fear of destructive envy is a deterrent to effort. However, restricting own outcomes is not the only way to reduce envy. One of the obvious methods of envy-avoidance is hiding assets from the sight of the envious or diminishing their value. In Ghana, a rich man reduces his relatives' envy by leaving unfinished a house he was building (Schoeck, 1969). In Bolivia, people of the Siriono tribe eat alone at night to avoid envious looks of the others (Holmberg, 1985). In a Mexican village, the fear of envy underlies the refusal to install glass windows in houses (Foster, 1979). In an Egyptian village, livestock is kept hidden in the recesses of the house to keep it from the evil eye (Ghosh, 1983). These are examples of how the fear of envy leads to productivity loss.

Taking all this evidence together, one could distinguish the salient features of an equilibrium showing the destructive side of envy. On the economic side: little or no innovation, low productivity and social mobility, envy-avoidance behavior. On the cultural side: ubiquitous fear of envy, "institutionalized envy." On the institutional side: poor protection of private property rights, redistributive mechanisms.

A very different equilibrium arises if the competitive side of envy is active. This is the well-known "keeping up with the Joneses" (KUJ) competition. In this case the relative standing concern is satisfied by exerting additional effort and investing in productive rather than destructive activities, which has very different implications of envy for economic performance. As demonstrated by Frank (1985), positional externality leads to overconsumption of positional good (consumption) versus nonpositional good (leisure) compared to cooperatively determined demands. Schor documents these phenomena for the modern U.S. society in "The Overspent American" (1998) and "The Overworked American" (1992), respectively.

Neumark and Postlewaite (1998) study the employment decisions of women using data from the U.S. National Longitudinal Survey of Youth and find evidence that labor supply is partly driven by relative income concerns. Bowles and Park (2005) use data on ten European OECD countries over the period 1963–1998 to show that greater earnings inequality is associated with longer work hours. They argue that the underlying cause is the Veblen effect of the consumption of the rich on the less wealthy, that is, emulation. Recognizing the effect of positional concerns on effort, Frank (2007) also warns about the potential adverse effects of competitive envy on savings and human capital investment and argues for introducing a progressive consumption tax to correct consumption externality.

The "hedonic treadmill," characteristic of the KUJ-type competition, has been the subject of recent papers in the field of happiness economics. Social comparison is one of the keys to understanding why happiness and material well-being may not always go together. Section 5 conducts the comparative welfare analysis of the KUJ and fear equilibria to address this issue.

To summarize, here are the salient features of an equilibrium with competitive envy. On the economic side: high productivity and mobility, consumer society. On the cultural side: emulation, KUJ, no fear of envy. On the institutional side: well-protected private property rights.

When is each of the described equilibria more likely to arise? As shown in the formal model of the next section, the type of equilibrium is determined jointly by a *set* of economic, institutional and cultural factors summarized by inequality of endowments, protection of property rights, and strength of positional concerns.

The two papers most directly related to this work are Mui (1995) and Mitsopoulos (2009). Mui (1995) constructs a model that captures an equilibrium in which innovations are not adopted because of expected envious retaliation. The essence of the fear equilibrium in the present theory is similar. However, Mui's setup ignores the competitive component of relative standing concerns and thus, does not account for the positional externality that is being curbed in the fear equilibrium. This leads to alternative welfare implications of improving institutional quality. In Mitsopoulos (2009) individuals who care about relative savings decide on the allocation of effort between productive and destructive activities. However, the static nature of the stage game and assumptions made on technologies do not allow to capture the *qualitatively different* equilibria associated with the two sides of envy, which is crucial for understanding the evidence presented above.

Finally, this paper is related to the strand of literature on conflict, appropriation, and the emergence of property rights.[6] The focus is, however, on envy as the sole cause of conflict (Sections 3 and 4) and the role that positional externalities may play in the decision to adopt better property rights protection (Section 5).

[6] See, e.g., Skaperdas (1992), Grossman and Kim (1995), Muthoo (2004).

3 The Basic Model

Environment. Consider two agents that may be thought of as representatives of equal-sized homogeneous groups of people. They differ only in the amount of broadly defined initial endowments, K_i, $i = 1, 2$. In particular,

$$K_1 = \lambda K, \qquad K_2 = (1 - \lambda)K, \tag{1}$$

where K is the total endowment in the economy and parameter $0 < \lambda < 1/2$ captures the degree of initial inequality: Agent 1 is "poor" and Agent 2 is "rich." They interact in the following two-stage game (see Figure 1).

In both stages each agent has a unit of time. In the first stage, time is spent on investment, say, education or innovation. Each agent exerts effort, $L_i \geqslant 0$, to produce an outcome, Y_i, according to the following investment function:

$$Y_i = F(K_i, L_i) = K_i L_i, \quad i = 1, 2. \tag{2}$$

This outcome may be thought of as a factor of production like human or physical capital, or potential output, to be realized in the second stage.

Unequal outcomes of the investment stage may lead to destructive envy on part of the poor agent in the second stage. Following Grossman and Kim (1996), assume that there is a "predator and prey" type of relationship between Agent 1 and Agent 2, such that the initially disadvantaged agent can engage in destruction after observing the outcomes of the first stage.[7] As will become clear, Agent 1 may choose to do that purely out of envy, if the level of inequality is high enough.

In the second stage, while the rich agent spends all the time on realization of potential output, the poor splits his time between destruction and production. The destruction technology is as follows. If Agent 1 allocates a fraction $d \in [0, 1]$ of his time to disrupt the productive activity of Agent 2, the latter retains only a fraction p of Y_2, where

$$p(d) = \frac{1}{1 + \tau d}. \tag{3}$$

As will become clear, the formulation with time allocation makes the model scale-free: optimal destruction intensity will be a function of posterior inequality (but not the scale)

[7] For simplicity, the rich are not allowed to invest in protection. A more general formulation would also allow for theft with partial destruction, without affecting the qualitative results. The implications of defense in a model of appropriation have been examined by Grossman and Kim (1995; 1996).

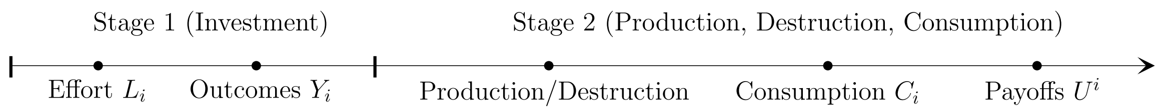

Figure 1: Timing of events in the envy game.

of first-stage outcomes, which captures the essence of destructive envy.[8] The function $p(d)$ has plausible properties: it is bounded, with $p(0) = 1$, decreasing, and convex. Parameter $\tau > 0$ measures the effectiveness of destruction and represents the overall level of private property rights protection. In particular, property rights are secure if τ is low.

At the end of stage 2 individuals consume whatever is left after production and destruction (if any) take place:

$$C_1 = (1-d)Y_1, \qquad C_2 = p(d)Y_2, \tag{4}$$

and payoffs are generated. The utility function is of the following form:

$$U^i = U(C_i, C_j, L_i) = v(C_i - \theta C_j) - e(L_i) = \frac{(C_i - \theta C_j)^{1-\sigma}}{1-\sigma} - L_i, \tag{5}$$

where $i, j = 1, 2$, $i \neq j$, $\sigma > 0$ and $0 < \theta < 1$.[9] It is increasing in own consumption ($U_1 > 0$) and decreasing in the other agent's consumption ($U_2 < 0$) and own effort ($U_3 < 0$). Parameter θ captures the strength of concern for relative standing. Agents are each other's reference points which is natural in the setup with two individuals. This utility function incorporates additive comparison which is one of the two most popular ways (the other being ratio comparison) to model envy.[10] Linearity in effort is assumed for analytical tractability. Overall, the form of the utility function is identical to that in Ljungqvist and Uhlig (2000), but in their model the reference point for each agent is the average consumption in the population.

A crucial property of this utility function is that the cross-partial derivative U_{12} is positive, leading to the "keeping up with the Joneses" kind of behavior, or emulation, due to complementarity between own and reference consumption. This is what Clark and Oswald

[8] If, apart from time, material resources or extra effort are needed to destroy, the scale effect may arise, which does not change the qualitative results of the basic model, but undermines analytical tractability.

[9] Assume for simplicity that $U^i = -\infty$ whenever $C_i \leqslant \theta C_j$. Since effort is unbounded, this will never be the relevant case in equilibrium.

[10] Additive comparison has been used, among others, by Knell (1999), Ljungqvist and Uhlig (2000) and Mitsopoulos (2009). For examples of models with ratio comparison see Boskin and Sheshinski (1978) and Carroll et al. (1997). Clark and Oswald (1998) examine the theoretical properties of both formulations.

(1998) call "comparison-concave" utility (since $v''(C_i - \theta C_j) < 0$). Intuitively it means that individuals are willing to match an increase in consumption of their reference group. The reason is that an increase in C_j reduces the consumption rank of individual i and, under concave comparisons, raises the marginal utility of own consumption. Comparison-concavity also implies that the effect of envy is stronger for the poorer agent.

Finally, the utility function reflects the assumption that consumption is a positional good, while leisure is not. This hypothesis has been consistently advocated by Frank (1985; 2007) and finds empirical support (Solnick and Hemenway, 2005).

Best responses. The dynamic structure of the model makes subgame perfect equilibrium a natural solution concept. The model is solved backwards, starting at stage 2, when effort is sunk. Given the outcomes of the investment stage, Y_1 and Y_2, Agent 1 chooses the intensity of destruction, d, to maximize his payoff:

$$v((1-d)Y_1 - \theta p(d) Y_2) \longrightarrow \max_d \qquad s.t. \quad 0 \leqslant d \leqslant 1. \tag{6}$$

This yields the optimal second-stage action[11]:

$$d^* = \begin{cases} 0, & \text{if} \quad Y_1/Y_2 \geqslant \tau\theta; \\ \frac{1}{\tau} \cdot \left(\sqrt{\tau\theta \frac{Y_2}{Y_1}} - 1 \right), & \text{if} \quad Y_1/Y_2 < \tau\theta. \end{cases} \tag{7}$$

Few things are to be noticed about this expression. First, without envy ($\theta = 0$) there is no destruction. If envy is present, the decision about engaging in destruction depends on the level of posterior inequality, Y_1/Y_2. If Y_1 is high enough compared to Y_2, Agent 1 finds it optimal to refrain from destruction. Otherwise, it is optimal to engage in destruction and its intensity is increasing in inequality, effectiveness of destructive technology and strength of envy. The threshold $\tau\theta$ is an endogenous measure of tolerance for inequality. Given the level of posterior inequality, destructive envy is more likely to occur if relative standing concerns are strong (large θ) and property rights are poorly protected (large τ). Assume from now on that $\tau\theta < 1$, which means that there is some tolerance for inequality and no destruction takes place if $Y_1 \geqslant Y_2$. The retention rate corresponding to d^* is

$$p^* = p(d^*) = \begin{cases} 1, & \text{if} \quad Y_1/Y_2 \geqslant \tau\theta; \\ \sqrt{\frac{Y_1}{Y_2} \cdot \frac{1}{\tau\theta}}, & \text{if} \quad Y_1/Y_2 < \tau\theta, \end{cases} \tag{8}$$

and, for the case with destruction, it is strictly decreasing in τ, θ, and Y_2/Y_1.

[11] Clearly, $d^* = 1$ is never optimal in equilibrium. Moreover, the first-stage effort will guarantee that $(1-d^*)Y_1 - \theta p(d^*) Y_2 > 0$. Hence, only the relevant case $0 \leqslant d^* < 1$ is considered.

The agents are forward-looking and anticipate second-stage outcomes when making their first-stage decisions. Although Agent 2 is passive at stage 2, he is perfectly aware of how his investment outcome will affect d^* and takes this into account at stage 1:

$$v(p^* Y_2 - \theta(1-d^*)Y_1) - Y_2/K_2 \longrightarrow \max_{Y_2} \tag{9}$$

For technical reasons, it is easier to analyze the best responses of both agents in terms of consumption levels, C_i, rather than first-stage outcomes Y_i. Note that these are different only if destruction actually takes place. In the latter case there is a one-to-one mapping between Y_i and C_i. To guarantee that the best-response functions are always well-defined the following assumption is maintained about τ throughout the paper: $1 < \tau < 1/\theta$. Moreover, assume that $\sigma > 1$, i.e., the elasticity of marginal utility with respect to relative consumption, which drives the emulative behavior, is high enough.[12]

Lemma 1 (BR of Agent 2). The best-response function of Agent 2, $\mathrm{BR}_2 \equiv C_2^*(C_1)$, has the following form:

$$C_2^*(C_1) = \begin{cases} K_2^{1/\sigma} + \theta C_1, & \text{if} \quad C_1 \geqslant \widetilde{C}_1; \\ C_1 \cdot \frac{1}{\tau\theta}, & \text{if} \quad \widehat{C}_1 \leqslant C_1 < \widetilde{C}_1; \\ C_2^d(C_1), & \text{if} \quad C_1 < \widehat{C}_1, \end{cases} \tag{10}$$

where

$$\widetilde{C}_1 \equiv \frac{\tau\theta}{1-\tau\theta^2} K_2^{1/\sigma}, \quad \widehat{C}_1 \equiv \frac{\tau\theta}{1-\tau\theta^2} \left(\frac{1+\theta^2}{2} K_2\right)^{1/\sigma},$$

and $C_2^d(C_1)$ is implicitly given by

$$C_2 - \theta C_1 = \phi \cdot \left(\frac{C_1 + \theta C_2}{C_2}\right)^{1/\sigma}, \quad \phi \equiv \left(\frac{1+\theta^2}{2\theta(1+\tau)} K_2\right)^{1/\sigma}. \tag{11}$$

The function $C_2^d(C_1)$ is strictly increasing and concave.

The best response function of Agent 2, depicted in the left panel of Figure 2, consists of three segments that correspond to the following cases. If the output of Agent 1 is high enough, Agent 2 is not constrained by destructive envy and displays the standard KUJ behavior. If the output of Agent 1 is in the intermediate range, the best response of Agent 2 would be not to allow destructive envy at stage 2 by constraining his own effort at stage 1. This intermediate region is the "fear segment," in which Agent 2 exerts the maximum

[12] This assumption is a convenient regularity condition that is not crucial for the main results, but resolves ambiguities in certain propositions.

possible effort that prevents destruction at stage 2. Finally, if the output of Agent 1 is low, it becomes too costly for Agent 2 to avoid destruction by constraining effort and the optimal action is to allow some of it.

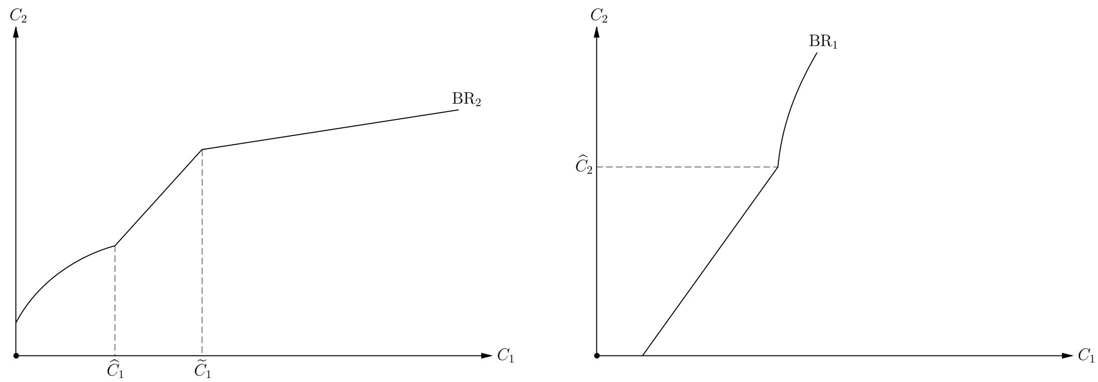

Figure 2: Best responses of Agent 2 (left) and Agent 1 (right).

Agent 1 is forward-looking and knows his own second-stage behavior when optimizing at the investment stage:

$$v((1-d^*)Y_1 - \theta p^* Y_2) - Y_1/K_1 \longrightarrow \max_{Y_1} \tag{12}$$

Lemma 2 (BR of Agent 1). The best-response function of Agent 1, $\mathrm{BR}_1 \equiv C_1^*(C_2)$, has the following form:

$$C_1^*(C_2) = \begin{cases} K_1^{1/\sigma} + \theta C_2, & \text{if} \quad C_2 \leqslant \widehat{C}_2; \\ C_1^d(C_2), & \text{if} \quad C_2 > \widehat{C}_2, \end{cases} \tag{13}$$

where

$$\widehat{C}_2 \equiv \frac{K_1^{1/\sigma}}{\theta(\tau-1)}$$

and $C_1^d(C_2)$ is implicitly given by

$$C_1 - \theta C_2 = \psi \cdot \left(\frac{C_1}{C_1 + \theta C_2}\right)^{1/\sigma}, \quad \psi \equiv \left(\frac{1+\tau}{\tau} K_1\right)^{1/\sigma}. \tag{14}$$

The function $C_1^d(C_2)$ is strictly increasing and convex.

The best response function of Agent 1, depicted in the right panel of Figure 2, consists of two segments. If the output of Agent 2 is low enough, Agent 1 prefers to catch up peacefully without causing destruction. This is the KUJ case. If, however, Y_2 is too high, destruction is the best response.

Equilibria. Depending on parameter values, the envy game may end up in three different types of equilibria which we consider in turn.

1. *Keeping up with the Joneses equilibrium* (KUJE). This equilibrium would always emerge if destruction were not possible (right panel of Figure 3). Its features are well-known in economics literature and have been formally analyzed by Frank (1985) and Hopkins and Kornienko (2004), among others. In the present model it arises when the "competitive segments" of the best response functions intersect (left panel of Figure 3) and is given by

$$C_i^{\mathrm{KUJ}} = Y_i^{\mathrm{KUJ}} = \frac{K_i^{1/\sigma} + \theta K_j^{1/\sigma}}{1-\theta^2}, \quad i, j = 1, 2, \quad i \neq j. \tag{15}$$

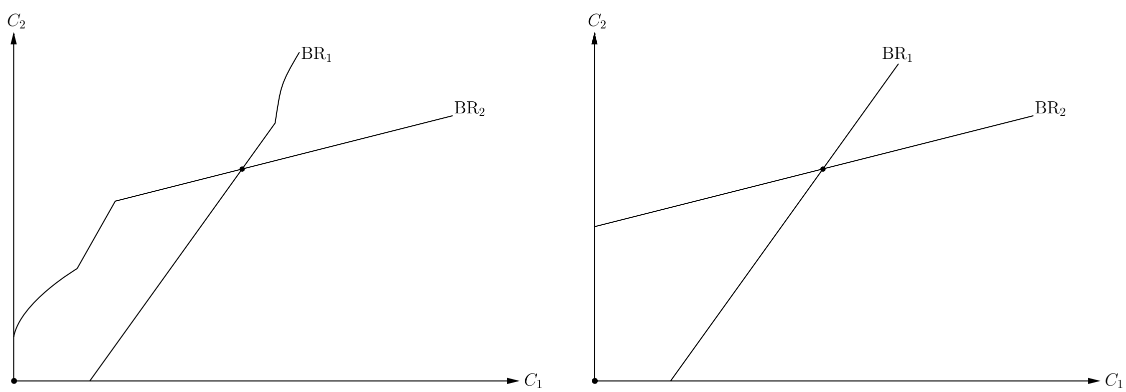

Figure 3: KUJE in the envy game with (left) and without (right) the second stage.

2. *Fear equilibrium* (FE). In this case the richer agent constrains his effort to avoid destructive envy at the second stage of the game. He consumes right at the point that makes it not optimal for the poorer agent to engage in destruction. This equilibrium resembles the features that have been documented by anthropologists and were discussed in Section 2. It arises when the "fear segment" of BR_2 intersects BR_1 at the kink point (left panel of Figure 4) and is given by

$$C_1^{\mathrm{F}} = Y_1^{\mathrm{F}} = \frac{\tau K_1^{1/\sigma}}{\tau - 1}, \quad C_2^{\mathrm{F}} = Y_2^{\mathrm{F}} = \frac{K_1^{1/\sigma}}{\theta(\tau - 1)}. \tag{16}$$

3. *Destructive equilibrium* (DE). In this equilibrium it is not optimal for Agent 2 to reduce effort to completely avoid destructive envy and he accommodates some of it. Agent 1 in response chooses his optimal destruction intensity. Equilibrium consumption levels are implicitly given by the system:

$$\begin{cases} C_2^{\mathrm{D}} = C_2^d(C_1^{\mathrm{D}}), \\ C_1^{\mathrm{D}} = C_1^d(C_2^{\mathrm{D}}). \end{cases} \tag{17}$$

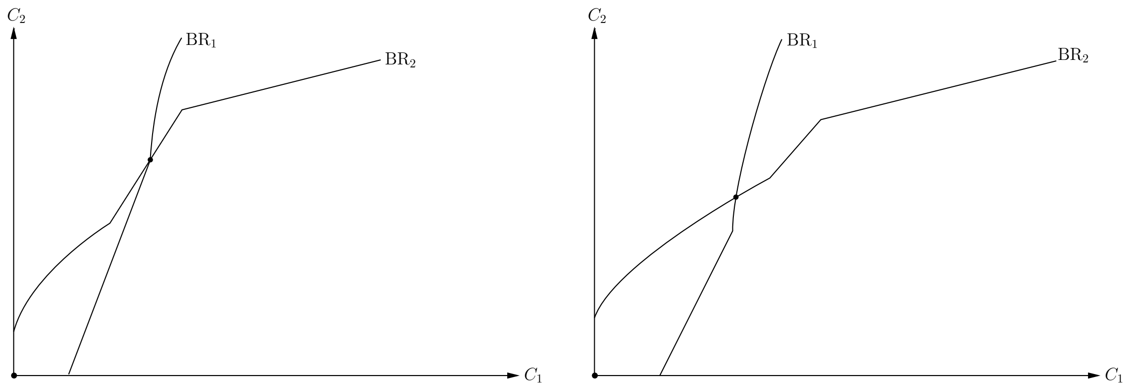

Figure 4: Fear (left) and destructive (right) equilibria.

The right panel of Figure 4 shows a generic destructive equilibrium, which occurs at the intersection of "destructive segments." It follows from the properties of $C_2^d(C_1)$ and $C_1^d(C_2)$ that they can only intersect once (see the proof of Proposition 1).

Now that it is clear what kind of equilibria can potentially arise as an outcome of the envy game, the question is under what conditions each of them is observed. The following proposition provides a full taxonomy of equilibria in terms of three exogenous parameters: λ, the initial level of inequality (economic environment), τ, the level of property rights protection (institutional environment), and θ, the strength of relative comparisons (cultural environment).

Proposition 1 (Equilibrium of the envy game). There exists a unique subgame perfect equilibrium of the envy game. Denote $k \equiv K_1/K_2$. Then, if $k \geqslant \tilde{k}$, it is the KUJ equilibrium (15). If $\hat{k} \leqslant k < \tilde{k}$, it is the fear equilibrium (16). Finally, if $k < \hat{k}$, it is the destructive equilibrium (17). The threshold values for k are

$$\tilde{k} \equiv \left[\frac{\theta(\tau-1)}{1-\tau\theta^2}\right]^\sigma, \quad \hat{k} \equiv \frac{(1+\theta^2)}{2}\tilde{k} < \tilde{k} < 1. \tag{18}$$

This result can be expressed in terms of regions for λ, τ, and θ. The relevant thresholds are uniquely determined by (18) accounting for the assumption that $1 < \tau < 1/\theta$.

Given the setup of the game, this result is very intuitive. Other things equal, the standard KUJ equilibrium is more likely to emerge if the level of inequality is not high, property rights are well-protected, and relative comparisons are not very strong. For the intermediate levels of these parameters the fear equilibrium emerges. Finally, poor property rights protection, high inequality, and strong relative comparisons make the destructive equilibrium more likely to arise.

It is important to emphasize, however, that the three exogenous parameters of interest (not counting σ) *jointly* determine the equilibrium type. For instance, just having low inequality is not enough to obtain the KUJ equilibrium. If, at the same time, institutions are very weak (destructive technology is very efficient) and/or relative standing concerns are very strong, the economy may still end up in the fear or destructive equilibrium. Note also that the model is scale-free in the sense that the type of equilibrium does not depend on the absolute level of total endowment, K.

Comparative statics. The next important issue to address is how the parameters of interest affect economic performance (measured by final outputs) across different equilibria. The nature of these alternative equilibria is very different, which generates opposite effects of envy on economic performance.

As follows from (15), in the KUJ equilibrium outputs do not depend on τ since destructive envy is not binding. The effect of θ is straightforward: increasing the strength of relative concerns in the KUJ equilibrium acts as additional incentive to work, which leads to higher levels of effort and output. Positional externality leads to overworking and overconsumption compared to what is socially optimal (Frank, 1985).

The effect of raising λ (increasing equality) on aggregate output depends crucially on σ. Equations (1) and (15) imply that the total output is

$$Y^{\mathrm{KUJ}} = Y_1^{\mathrm{KUJ}} + Y_2^{\mathrm{KUJ}} = \frac{\lambda^{1/\sigma} + (1-\lambda)^{1/\sigma}}{1-\theta} K^{\frac{1}{\sigma}}. \tag{19}$$

If $\sigma > 1$, it is a strictly concave and increasing function of λ in the interval $(0, 1/2)$. Concavity of outputs in endowments seems more natural than a kind of nondecreasing returns to scale that would emerge under $\sigma \leqslant 1$.

The total effect of redistribution on private outputs consists of two parts: wealth effect and comparison effect. Wealth effect is just the direct effect of making one agent poorer and the other richer. In case of increasing λ the wealth effect is positive for Agent 1 and negative for Agent 2. The total wealth effect on output is positive if $\sigma > 1$. Private output is a concave function of own wealth and so, redistribution from the rich agent to the poor has a positive net marginal effect. Comparison effects reflect the fact that the reference group becomes poorer for Agent 1 and richer for Agent 2. Consequently, comparison effect is negative for Agent 1 and positive for Agent 2. The total comparison effect has the same sign as the total wealth effect. Under $\sigma > 1$ private output is a concave function of the other agent's wealth and redistribution from the rich agent to the poor has a positive marginal effect, since the negative comparison effect on the output of the poor is outweighed by the positive effect on the output of the rich.

Next, consider the fear equilibrium (16). In this case the outcomes depend on τ, the measure of property rights protection. Raising τ (reducing the quality of institutions) decreases the tolerance of Agent 1 for inequality and aggravates the "fear constraint" of Agent 2. This means that with higher τ Agent 2 has to produce less to avoid destructive envy which leads to lower total output given by

$$Y^{\mathrm{F}} = Y_1^{\mathrm{F}} + Y_2^{\mathrm{F}} = \frac{\lambda^{1/\sigma}(1+\tau\theta)}{\theta(\tau-1)} K^{\frac{1}{\sigma}}. \tag{20}$$

The effect of raising θ is similar since it, too, decreases the tolerance for inequality. This is in stark contrast with the role of positional concern in the KUJ equilibrium. In the latter case it acts as an additional incentive to work while in the fear equilibrium it constrains productive effort by increasing the hazard of destructive envy. On the other hand, the effect of raising equality in the fear equilibrium is unambiguously positive. Increasing λ means a positive wealth effect for Agent 1 and, since the output of Agent 2 is tied to that of Agent 1, this translates into higher output of Agent 2. That is, in FE redistribution from the rich to the poor increases investment and final production by alleviating the fear constraint.

Destructive equilibrium is harder to analyze analytically. Multiple effects are at work which makes the aggregate comparative statics with respect to θ ambiguous. If inequality is high or tolerance for inequality is low, stronger envy leads to substantial destruction which may lower the consumption of Agent 2, as well as total final output (left panel of Figure 6). In contrast, if the destructive environment is not severe (or σ, the catching-up propensity, is high), the stimulating effect of envy dominates (right panel of Figure 6). Thus, comparative statics in DE combines the features of FE and KUJE. Moreover, it can be shown that, under $\sigma > 1$, higher τ and lower λ unambiguously decrease total consumption. The following proposition summarizes the comparative statics results of this section.

Proposition 2 (Comparative statics of the envy game). Across equilibria the effects of λ, θ, and τ on total final output are as follows.

1. KUJE:

   (a) $\partial Y/\partial\lambda > 0$; $\partial Y_1/\partial\lambda > 0$; $\partial Y_2/\partial\lambda > 0$ iff $\theta > [(1-\lambda)/\lambda]^{\frac{1-\sigma}{\sigma}}$.

   (b) $\partial Y/\partial\theta > 0$; $\partial Y_i/\partial\theta > 0$.

   (c) $\partial Y/\partial\tau = \partial Y_i/\partial\tau = 0$.

2. FE:

   (a) $\partial Y/\partial\lambda > 0$; $\partial Y_i/\partial\lambda > 0$;

   (b) $\partial Y/\partial\theta < 0$; $\partial Y_1/\partial\theta = 0$; $\partial Y_2/\partial\theta < 0$;

   (c) $\partial Y/\partial\tau < 0$; $\partial Y_i/\partial\tau < 0$.

3. DE:

   (a) $\partial C/\partial\lambda > 0$; $\partial C_1/\partial\lambda > 0$; $\partial C_2/\partial\lambda \lesseqgtr 0$;

   (b) $\partial C/\partial\theta \lesseqgtr 0$; $\partial C_1/\partial\theta > 0$; $\partial C_2/\partial\theta \lesseqgtr 0$;

   (c) $\partial C/\partial\tau < 0$; $\partial C_i/\partial\tau < 0$.

Proposition 2 makes clear that the mechanics of the qualitatively different equilibria of the envy game may imply contrasting comparative statics. Stronger envy is detrimental for economic performance in the fear equilibrium, increases total output under the KUJ-type competition, and has an ambiguous effect in DE. Higher inequality has a negative effect across all equilibria types. Well-protected private property rights are conducive to economic performance in both FE and DE and play no role beyond the KUJE threshold.

Overall, Proposition 2 establishes the relationship between aggregate economic performance and the parameters of the model, reflecting economic, institutional and cultural environment (see Figures 5 and 6).

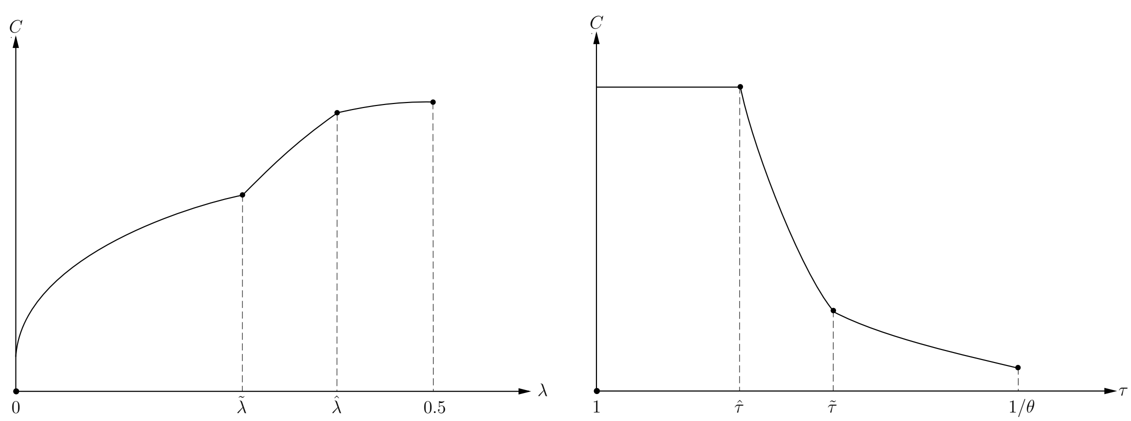

Figure 5: Inequality (left), property rights (right), and economic performance.

The analysis of this section implies that the potentially adverse effects of envy may create incentives for actions in order to overcome them. The following two sections address this issue.

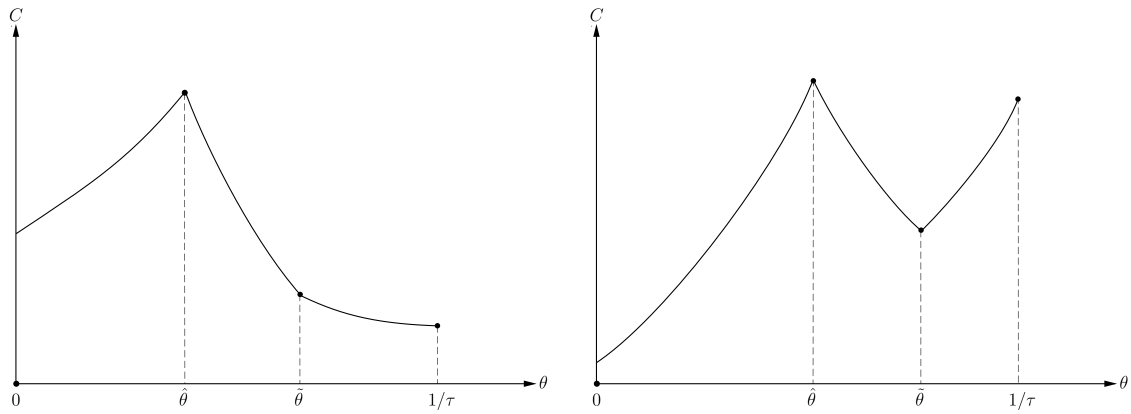

Figure 6: Envy and economic performance.

4 Voluntary Redistribution

One way to deal with destruction caused by envy is through redistribution of various forms. Cancian (1965) and Foster (1979) examine redistributive practices in Latin American peasant societies, such as ceremonial expenditures and sponsorship of religious fiestas by the rich (cargo system). In particular, Cancian (1965, p. 140) suggests that "service in the cargo system legitimizes the wealth differences that do exist and thus prevents disruptive envy." Platteau (2000, chapter 5) examines similar arrangements in Sub-Saharan Africa and Asian village communities, where "in seeking neighbourer's goodwill or in fearing their envy, incentives operate for the rich to redistribute income and wealth to the poor" (p. 235).

Mui (1995, section 4) discusses sharing in the emerging market economies of the Soviet Union and China manifested in contributions of the nascent entrepreneurial class to charity or local public goods, which he sees as an attempt to alleviate destructive envy. Mui then goes on to formally show how sharing can support the adoption of innovation in equilibrium. Some authors (Schoeck, 1969; Fernández de la Mora, 1987) even see the modern progressive taxation system as a remnant of "egalitarian envy."

Within the framework of Section 3, a natural question is whether the rich would be willing to share the fruits of their effort to avoid destructive envy and how this affects the possible equilibrium outcomes of this modified envy game. Consider the case of ex-post sharing, that is, after investment has taken place. Assume that Agent 2 can make a transfer to Agent 1 before stage 2 begins, as shown in Figure 7.

Consider the node of the game where the rich agent decides on transfer having seen the outcomes of the investment stage. Obviously, non-zero transfer may only be optimal

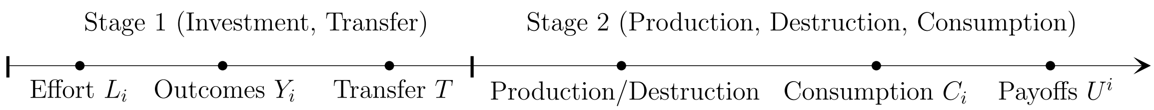

Figure 7: Timing of events in the envy game with transfers.

if $Y_1 < \tau\theta Y_2$, i.e., destructive envy is binding. In this case Agent 2 may want to make a positive transfer T to lower the intensity of destruction, d^*. Given that effort is sunk at this stage, Agent 2 maximizes [13]

$$v(p^*(T)(Y_2 - T) - \theta(1 - d^*(T))(Y_1 + T)) \longrightarrow \max_T \qquad s.t. \quad 0 \leqslant T \leqslant \bar{T}, \tag{21}$$

where $\bar{T} \equiv (\tau\theta Y_2 - Y_1)/(1 + \tau\theta)$ is the minimum transfer sufficient to completely avoid destruction. Lemma 3 characterizes the optimal transfer of Agent 2.

Lemma 3 (Optimal transfer). For $Y_1/Y_2 \leqslant \tau\theta$, the optimal transfer is $T = \bar{T}$ if and only if $\tau\theta \leqslant (1-\theta)/(1+\theta)$. Otherwise,

$$T = \begin{cases} T^*, & \text{if} \quad Y_1/Y_2 < \mu; \\ 0, & \text{if} \quad Y_1/Y_2 \geqslant \mu, \end{cases}$$

where T^* is given by the first-order condition of (21) for the interior solution and $0 < \mu < \tau\theta$ is a constant depending on τ and θ (see Appendix).

In what follows consider the simple case $\tau\theta \leqslant (1-\theta)/(1+\theta)$. Then, as established in Lemma 3, it is always optimal for Agent 2 to make the full transfer $\bar{T}$. Intuitively, the rich individual will be willing to do so if $\bar{T}$ is low. The next lemma establishes what the best response of Agent 2 looks like under full transfer.

Lemma 4 (BR of Agent 2 with full transfer). The best response function of Agent 2, $\mathrm{BR}_2^T \equiv Y_2^*(Y_1)$, has the following form:

$$Y_2^*(Y_1) = \begin{cases} K_2^{1/\sigma} + \theta Y_1, & \text{if} \quad Y_1 \geqslant \widetilde{Y}_1; \\ Y_1 \cdot \frac{1}{\tau\theta}, & \text{if} \quad \breve{Y}_1 < Y_1 < \widetilde{Y}_1; \\ (\gamma_2 K_2)^{1/\sigma} - Y_1, & \text{if} \quad Y_1 \leqslant \breve{Y}_1, \end{cases} \tag{22}$$

where

$$\widetilde{Y}_1 \equiv \frac{\tau\theta}{1 - \tau\theta^2} K_2^{1/\sigma}, \quad \breve{Y}_1 \equiv \frac{\tau\theta}{1 + \tau\theta}(\gamma_2 K_2)^{1/\sigma}, \quad \gamma_2 \equiv \left[\frac{1 - \tau\theta^2}{1 + \tau\theta}\right]^{1-\sigma}.$$

[13] Formally, $(1 - d^*)Y_1 - \theta p^* Y_2$ is positive if and only if $Y_2/Y_1 < (1+\tau)^2/4\tau\theta$. Otherwise, $U^1 = -\infty$ and Agent 1 is indifferent between any feasible destruction intensities. For concreteness, focus on d^* as the best response of Agent 1 in this case.

Now, instead of "destructive region" the best response of Agent 2 has a "transfer region" (left panel of Figure 8). For low levels of Y_1 it is optimal to prevent destruction through transfers rather than by producing less in the first stage.

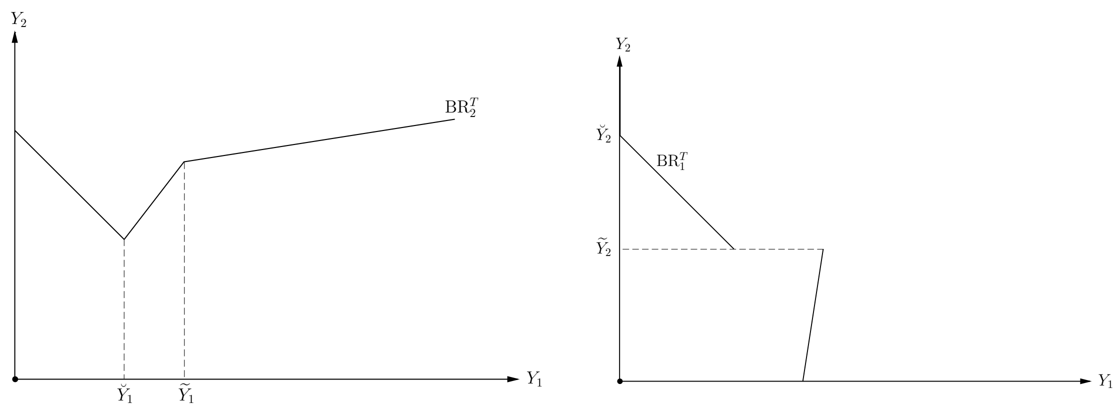

Figure 8: Best responses in the envy game with transfers.

Agent 1 anticipates to get the full transfer in case of high inequality, because he knows that the rich agent will be afraid of destructive envy. He takes this into account when choosing his first period effort. Intuitively, given the high potential output of the rich, Agent 1 has an incentive to invest as little effort as possible and cause a threat of destructive envy thereby provoking the rich to make a transfer. Lemma 5 gives a characterization of the best response function of Agent 1 for a special case that illustrates all possible kinds of equilibria that can emerge in a modified envy game.

Lemma 5 (BR of Agent 1 with full transfer). Under parametric conditions (A7) and (A8) provided in the Appendix the best response function of Agent 1, $\mathrm{BR}_1^T \equiv Y_1^*(Y_2)$, is given by

$$Y_1^*(Y_2) = \begin{cases} K_1^{1/\sigma} + \theta Y_2, & \text{if} \quad Y_2 \leqslant \widetilde{Y}_2; \\ (\gamma_1 K_1)^{1/\sigma} - Y_2, & \text{if} \quad \widetilde{Y}_2 < Y_2 < \check{Y}_2; \\ 0, & \text{if} \quad Y_2 \geqslant \check{Y}_2, \end{cases} \tag{23}$$

where

$$\widetilde{Y}_2 \equiv \frac{\sigma(1-\gamma_1^{1/\sigma})}{(1-\sigma)(1+\theta)} K_1^{1/\sigma}, \quad \check{Y}_2 \equiv (\gamma_1 K_1)^{1/\sigma}, \quad \gamma_1 \equiv \left[\frac{\theta(\tau-1)}{1+\tau\theta}\right]^{1-\sigma}.$$

The right panel of Figure 8 depicts this function. Note, in particular, that there is a discontinuity at point $\widetilde{Y}_2$, as it becomes optimal for Agent 1 to switch to the KUJ-type best response when Y_2 is low enough. Under alternative parametric restrictions BR_1^T looks very similar except that there may be no intermediate region, in which Agent 1 prefers to work more than the minimum required to make a credible threat.

Given the best responses of the agents, two qualitatively different types of equilibria may arise: fear equilibrium with transfers (FT) and KUJ equilibrium. Moreover, as shown in the bottom panels of Figure 9, multiplicity of equilibria cannot be ruled out. Proposition 3 characterizes the equilibria under conditions of Lemma 5.

Proposition 3 (Equilibria of the envy game with transfers). The envy game with transfers has: 1) a unique KUJ equilibrium (15) if $k > \gamma_2/\gamma_1$; 2) a unique fear equilibrium with full transfer of the form

$$\begin{aligned} \{Y_1^{\mathrm{FT}}, Y_2^{\mathrm{FT}}\} &= \{0, (\gamma_2 K_2)^{1/\sigma}\}; \\ \{C_1^{\mathrm{FT}}, C_2^{\mathrm{FT}}\} &= \{\varepsilon(\gamma_2 K_2)^{1/\sigma}, (1-\varepsilon)(\gamma_2 K_2)^{1/\sigma}\}, \end{aligned} \tag{24}$$

if $k < \omega$, where $\varepsilon \equiv [\tau\theta/(1+\tau\theta)]$ and $\omega \equiv [\sigma(1-\theta)(1-\gamma_1^{1/\sigma})/(1-\sigma)-\theta]^{-\sigma}$; 3) two equilibria (15) and (24) if $\omega < k < \gamma_2/\gamma_1$; 4) multiple equilibria with full transfer of the form

$$\begin{aligned} \{Y_1^{\mathrm{FT}}, Y_2^{\mathrm{FT}}\} &= \{Y_1^{\mathrm{FT}}, (\gamma_2 K_2)^{1/\sigma} - Y_1^{\mathrm{FT}}\}, \quad 0 \leqslant Y_1^{\mathrm{FT}} \leqslant \widetilde{Y}_2; \\ \{C_1^{\mathrm{FT}}, C_2^{\mathrm{FT}}\} &= \{Y_1^{\mathrm{FT}} + \bar{T}, Y_2^{\mathrm{FT}} - \bar{T}\}, \end{aligned} \tag{25}$$

if $k = \gamma_2/\gamma_1$, along with a KUJ equilibrium.

Destructive and fear equilibria of the basic model are replaced by "fear equilibrium with transfers," in which the rich agent redistributes part of his output to avoid destructive envy of the poor agent. Since inequality is what matters for the amount of destruction, by investing nothing in the first stage the poor agent creates the fear of envy forcing the rich to share. In turn, when the poor agent expects to receive a transfer, he does not need to work hard and chooses to produce the minimum amount needed to create a credible threat to destroy. This type of equilibrium arises when initial inequality is high. In contrast, if it is low, the only possible outcome of the game is the standard KUJ equilibrium. It is easy to see that in the baseline case $\sigma > 1$ both γ_2/γ_1 and ω are increasing in τ and θ, that is, reducing tolerance for inequality increases the likelihood of the redistributive equilibrium. This is intuitive and parallels the taxonomy of equilibria in Proposition 1.

Interestingly, the possibility of transfers gives rise to (stable) multiple equilibria for intermediate levels of inequality (bottom panels of Figure 9). This implies that societies with similar characteristics and moderate wealth differences may end up in one of the two alternative equilibria: one with redistribution and fear of destructive envy and the other with KUJ-type competition. The following proposition highlights the contrast between the two equilibria.

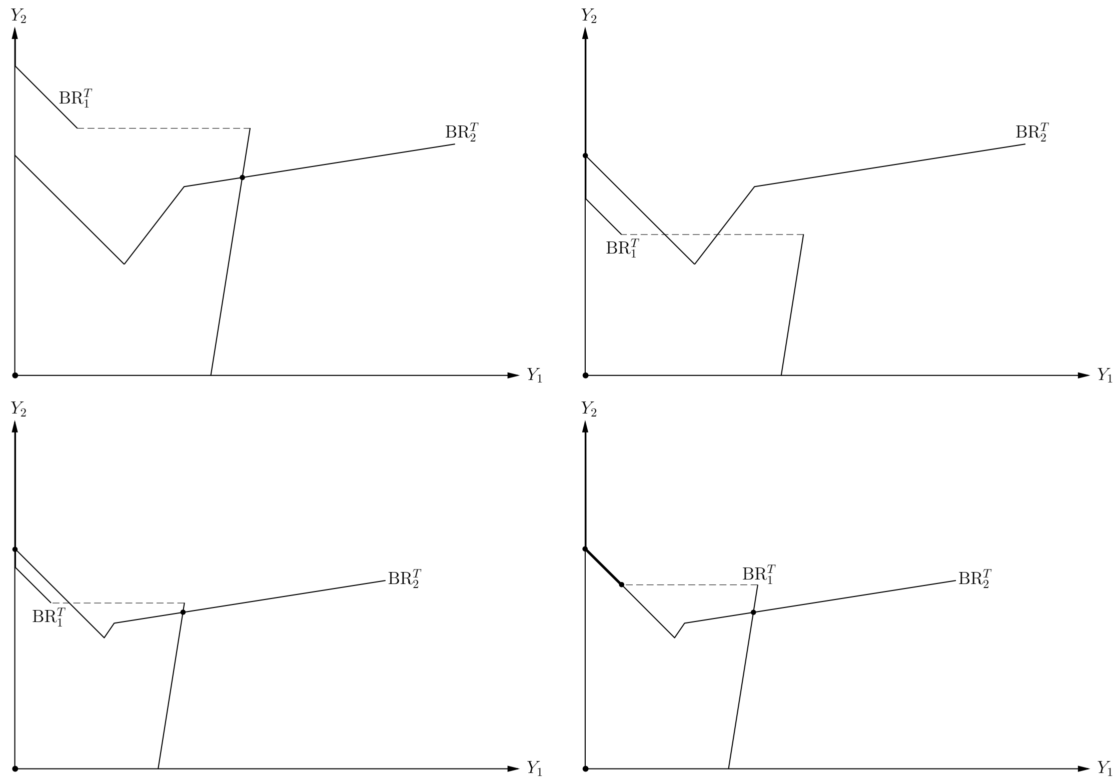

Figure 9: Equilibria in the envy game with transfers.

Proposition 4. Assume that $\omega < k < \gamma_1/\gamma_1$, that is, the envy game with transfers has an FT equilibrium (24) and a KUJ equilibrium (15). Then, the FT equilibrium is characterized by: 1) higher ex-post inequality, that is, $C_1^{\mathrm{FT}}/C_2^{\mathrm{FT}} < C_1^{\mathrm{KUJ}}/C_2^{\mathrm{KUJ}}$; 2) lower total output, that is, $Y_2^{\mathrm{FT}} < Y^{\mathrm{KUJ}}$.

Curiously, despite redistribution taking place in FT equilibrium, it turns out to be more unequal ex-post than the KUJE, in which catching up takes place. This is not surprising. In fact, anthropologists documented that egalitarian norms and redistributive mechanism often fail to achieve equality. For instance, in his account of the cargo system in a Maya society Cancian (1965) argues that "there is, in effect, sufficient leveling. . . to satisfy normative prescriptions, but not enough to produce an economically homogeneous community" (p. 140).

The second result of Proposition 4 is intuitive. In the KUJE, positional externality makes both agents work hard and leads to high total output. In FT equilibrium, the poor agent does not work at all to create a threat of destructive envy, while the rich provides consumption for both. The externality is thus neutralized at least for the poor agent. In fact, it can be shown that the poor agent always gets a higher utility in the FT equilibrium, precisely for this reason. For the rich agent welfare analysis is more complicated, and, in general, overworking under KUJ competition is not necessarily dominated by overworking under the fear of envy. However, in the case when the two equilibria are in fact Pareto rankable the FT equilibrium will be dominant.

The welfare analysis of the envy model is relevant not only in the situation with multiple equilibria. The following section goes back to the basic model to investigate in detail the welfare properties of different equilibria and the prospects of cooperation in choosing the institutional parameter, τ.

5 Welfare and property rights protection

This section examines how the degree of property rights protection affects the welfare of individuals in the basic model of Section 3. In particular, one might analyze in a crude form the possibility of institutional change represented by a change in τ. Assume that agents can agree to alter τ if this leads to a Pareto-superior equilibrium. Even such a simplistic approach allows to see the fundamental differences between alternative equilibria and uncover the nontrivial welfare implications of improving property rights protection in the presence of positional externalities.

The connection between the emergence of property rights and externalities was famously considered by Demsetz (1967). He argued that the main function of property rights is to internalize externalities, and institutional change in this domain is intended to cope with new externality problems. In the context of the present theory it is the consumption externality that makes property rights matter. As shown below, it is not the emergence of property rights protection but its *demise* that may internalize this externality.

We start by analyzing different types of equilibria separately. First, as follows from (15), the welfare of individuals in the KUJ equilibrium is independent of τ. In the fear equilibrium, however, τ matters, since it affects the effort and output of the agents. Moreover, it can be easily established that utilities of both agents in the fear equilibrium depend positively on τ. As follows from (16), these are given by

$$\begin{aligned} U_{\mathrm{F}}^1 &= \left[\frac{1}{1-\sigma} - \frac{\tau}{\tau-1}\right] K_1^{\frac{1-\sigma}{\sigma}}, \\ U_{\mathrm{F}}^2 &= \left[\frac{1}{1-\sigma} \cdot \left(\frac{1-\tau\theta^2}{\theta(\tau-1)}\right)^{1-\sigma} - \frac{1}{\theta(\tau-1)} \cdot \frac{K_1}{K_2}\right] K_1^{\frac{1-\sigma}{\sigma}}. \end{aligned} \tag{26}$$

Differentiation with respect to τ shows that $\partial U_{\mathrm{F}}^1/\partial\tau > 0$ always and $\partial U_{\mathrm{F}}^2/\partial\tau > 0$ if and only if $K_1/K_2 > (1-\theta^2)\cdot[\theta(\tau-1)/(1-\tau\theta^2)]^\sigma$ which, as follows from Proposition 1, is always true in the fear equilibrium.

The intuition behind this result is simple. In case of Agent 1, notice that the relative consumption term, $C_1 - \theta C_2 = K_1^{1/\sigma}$, is independent of τ, while effort, $L_1 = \frac{\tau}{\tau-1}K_1^{1/\sigma-1}$, decreases in τ since lower effort is required to produce a lower level of output caused by loosening the property rights protection. So, under higher τ he is able to maintain the same relative standing with lower effort. Agent 2 loses somewhat in his relative position, that is, $C_2 - \theta C_1$ is decreasing in τ, but he is still willing to trade it off for more leisure if K_1/K_2 is high enough. Large K_1 means that the reference group is richer which induces extra effort, while small K_2 means that maintaining relative standing requires more effort, again causing extra work.

Given this property of the FE, if we allow the agents to collectively choose the level of property rights protection, it will be a Pareto improvement for them to move from any fear equilibrium E towards the kink point E' marking the border between the fear and destructive regions (see Figure 10). Would they want to move inside the destructive region? It turns out that the welfare analysis in the destructive equilibrium is more complicated and in general yields ambiguous conclusions. Generically, two scenarios arise. In the first one, utility of Agent 1 is increasing in τ while utility of Agent 2 is decreasing. This is not surprising since higher τ means that it becomes easier for Agent 1 to destroy, while Agent 2 suffers more from destructive envy. In this scenario, the individuals have a conflict of interests over the level of property rights protection and, under Pareto criterion, they won't move deeper in the destructive region, staying at the kink point. This also means that once the agents are in a DE, they won't move towards the fear region, since this won't be profitable for Agent 1.[14] The other plausible scenario is that both agents prefer better property rights protection in the destructive region in which case there is movement towards the kink point E' from both sides.

Next, consider the following thought experiment. Assume the agents are in the fear equilibrium. Would they want to adopt a lower value of τ that would move them to the KUJ equilibrium? The following proposition provides conditions under which the agents will (not) want to adopt better property rights protection and find themselves in the KUJ "rat race."

[14] Note, however, that, as established in Proposition 2, total output decreases in τ in DE which opens a possibility of compensatory side payments on part of the rich.

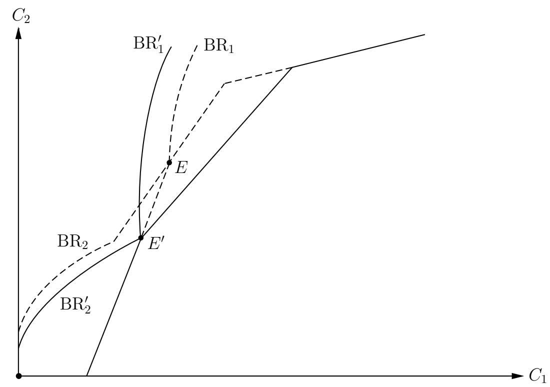

Figure 10: Pareto improving increase in τ in the fear equilibrium.

Proposition 5. Let $\hat{k} \leqslant k < \tilde{k}$, that is, agents are in FE. Then, $\exists!\ \bar{\theta} \in (0, 1)$ such that: 1) If $\theta > \bar{\theta}$, adopting a (lower) τ that would bring up the KUJ equilibrium is a Pareto worsening; 2) If $\theta \leqslant \bar{\theta}$, $\exists!\ \bar{k}(\tau) \in [\hat{k}, \tilde{k})$, such that adopting such τ is a Pareto worsening if and only if $k > \bar{k}$. Private outputs in the initial FE are always lower than those that would have been achieved by improving the property rights protection.

The intuition of Proposition 5 is the same as in the above discussion of choosing τ in the fear equilibrium. As shown in the Appendix, Agent 1 will always prefer FE. In particular, his relative standing is the same across equilibria, but the amount of leisure is always higher in FE. The rich agent will not want to move to KUJE if in that equilibrium he will have to work a lot to support his status. That happens if either relative comparisons are strong (θ is high) or there is low inequality (k is large). In these cases the rich would prefer to stay in FE, in which the fear of destructive envy restrains effort and curbs the consumption externality leading to overworking in the KUJ equilibrium. Thus, worse property rights protection corrects the distortion caused by positional concerns.

So, under conditions of Proposition 5, both agents are happier in an equilibrium with poor property rights protection and lower standard of living, rather than in a high-output KUJ equilibrium. This is reminiscent of Sen's (1983, p. 160) observation that "a grumbling rich man may well be less happy than a contented peasant, but he does have a higher standard of living," as well as what Graham (2010) calls the "paradox of happy peasants and miserable millionaires." Proposition 5 offers one possible explanation of the "paradox." Figure 11 illustrates the case, in which moving from FE to KUJE by improving property rights protection is a Pareto worsening (left panel) in spite of increased private outputs (right panel).

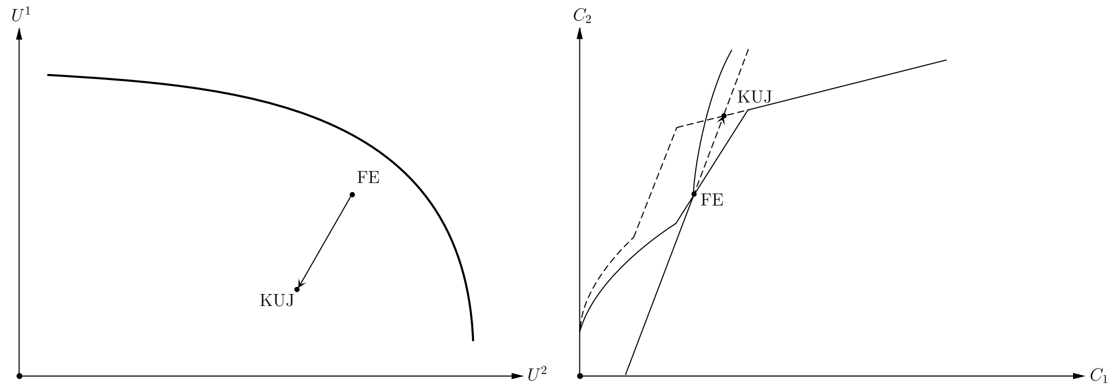

Figure 11: From FE to KUJE: Improving the property rights protection.

It is instructive to also read Proposition 5 in the "reverse" order, assuming that initially agents are in the KUJ equilibrium.

Proposition 5′. Let $k \geqslant \tilde{k}$, that is, agents are in KUJE. Then, $\exists!\ \bar{\theta} \in (0,1)$ such that: 1) If $\theta > \bar{\theta}$, adopting a (higher) τ that would bring up the fear equilibrium is a Pareto improvement; 2) If $\theta \leqslant \bar{\theta}$, $\exists!\ \bar{k}(\tau) \in (\hat{k}, \tilde{k})$, such that adopting such τ is a Pareto improvement if and only if $k > \bar{k}$. Private outputs in the initial KUJE are always higher than those that would be achieved by distorting the property rights protection.

Taken literally, Proposition 5′ may be seen as a formalization of a more than a century-old argument raised by Veblen (1891) in an attempt to explain the support for socialist movement, specifically, for the abolition of private property rights. Veblen argued that this demand is driven precisely by the discontent caused by the forces of emulation that is exacerbated in an industrial society (p. 65–66): "The ground of the unrest with which we are concerned is, very largely, jealousy, – envy, if you choose: and the ground of this particular form of jealousy that makes for socialism, is to be found in the institution of private property." Moreover, Veblen goes on to describe what in the model can be called a transition from KUJE ("keep up appearances") to FE ("socialism"): "Under modern conditions the struggle for existence has, in a very appreciable degree, been transformed into a struggle to keep up appearances. The ultimate ground of this struggle to keep up appearances by otherwise unnecessary expenditure, is the institution of private property. . . With the abolition of private property, the characteristic of human nature which now finds its exercise in this form of emulation, should logically find exercise in other, perhaps nobler and socially more serviceable, activities." He then emphasizes that the abolition of the KUJ competition would lessen the amount of labor and output required to support the economy. This is exactly what happens after transition to the fear equilibrium: output and labor supply fall, individuals enjoy more leisure and are happier with less output. The rich will prefer well-protected property rights if inequality is high enough since moving to FE would mean losing too much in terms of relative standing. Thus, in a KUJE with relatively high inequality there is a conflict of interests over this decision, or, in Veblen's terms, different sentiments towards the socialist movement.

The result that both agents may prefer inferior institutions is important in understanding why societies may be stuck for some time in the FE even if in the long-run the transition to KUJE is inevitable. This issue is explored in the dynamic framework of next section.

6 Envy and Dynamics of Inequality

So far, the analysis focused on the equilibria of the static model in which inequality of endowments was taken as fixed. In this section intergenerational links are introduced that allow to explore the dynamics of inequality. It is shown how an economy can evolve endogenously through different envy regimes following the dynamics of inequality. Incorporation of the insights from the static model into a dynamic framework allows to examine the changing role of envy in the transition process.

Environment. The economy is populated by a sequence of non-overlapping generations, indexed by $t \geqslant 0$. Time is discrete, and each generation lives for one period. The initial population consists of 2 homogeneous groups of people (representative agents), the poor and the rich, with initial endowments K_{10} and $K_{20} > K_{10}$. Each person has 1 child, so that in each time period two groups of people are descendants of the initially rich and initially poor. Parents care about their children and leave bequests, b_{it}, $i = 1, 2$, at the end of each period t.[15] In particular, they derive utility not just from relative consumption but the relative Cobb-Douglas aggregator of consumption, now denoted c_{it}, and bequest, b_{it}:

$$U_t^i = \frac{\pi(c_{it}^{1-\alpha} b_{it}^{\alpha} - \theta c_{jt}^{1-\alpha} b_{jt}^{\alpha})^{1-\sigma}}{1-\sigma} - L_{it}, \tag{27}$$

where $i, j = 1, 2$, $i \neq j$, $0 < \alpha < 1$ parameterizes the fraction of final output allocated to bequest, and $\pi \equiv [(1-\alpha)^{1-\alpha}\alpha^{\alpha}]^{\sigma-1}$ is a normalization constant. Consumption and bequest are made out of final output, C_{it}, so that $b_{it} + c_{it} = C_{it}$. This formulation leaves the workings of the basic model from Section 3 intact while introducing dynamic linkages.

[15] These bequests may represent any kind of investment in children that increases the productivity of their effort, for example, expenditure on human capital.

Dynamical system. The initial endowment of generation $t+1$, K_{it+1}, is assumed to depend on the endowment of their parents and parental investment in children:

$$K_{it+1} = K_{it}^{\beta} b_{it} = K_{it}^{\beta} \alpha C_{it}, \tag{28}$$

where $i = 1, 2$, and $0 < 1 - \beta < 1$ is the rate of geometric depreciation of parental endowment.[16] Note that Proposition 1 holds each period t, and the level of initial inequality in period $t+1$ is determined endogenously:

$$k_{t+1} \equiv \frac{K_{1t+1}}{K_{2t+1}} = k_t^{\beta} \cdot \frac{C_{1t}}{C_{2t}}, \tag{29}$$

with the initial condition $0 < k_0 < 1$. The joint dynamics of K_{1t} and K_{2t} depends on the type of equilibrium, in which the economy resides, and the equilibrium next period is, in turn, determined by the economic outcomes of the current period. Lemma 6 characterizes this dynamical system and follows directly from Proposition 1 and the law of motion (28).

Lemma 6 (Dynamics of endowments). The two-dimensional dynamical system for K_{1t} and K_{2t} is given by

$$\begin{bmatrix} K_{1t+1} \\ K_{2t+1} \end{bmatrix} = \begin{cases} \frac{\alpha}{1-\theta^2}[K_{1t}^{\beta}(K_{1t}^{1/\sigma} + \theta K_{2t}^{1/\sigma}), K_{2t}^{\beta}(K_{2t}^{1/\sigma} + \theta K_{1t}^{1/\sigma})], & \text{if} \quad k_t \geqslant \tilde{k}; \\ \frac{\alpha}{\theta(\tau-1)}[\tau\theta K_{1t}^{1/\sigma+\beta}, K_{2t}^{\beta} K_{1t}^{1/\sigma}], & \text{if} \quad \hat{k} \leqslant k_t < \tilde{k}; \\ \alpha[K_{1t}^{\beta} \cdot C_1^D(K_{1t}, K_{2t}), K_{2t}^{\beta} \cdot C_2^D(K_{1t}, K_{2t})], & \text{if} \quad k_t < \hat{k}, \end{cases} \tag{30}$$

where $C_1^D(K_{1t}, K_{2t})$ and $C_2^D(K_{1t}, K_{2t})$ are the final outputs in DE given by (17).

The thresholds $\tilde{k}$ and $\hat{k}$ divide the (K_{1t}, K_{2t}) phase plane into three regions according to the types of equilibria (see Figures 12 and 13): KUJ, F (fear) and D (destruction). In each of these regions the motion is governed by the corresponding part of the dynamical system (30). To rule out explosive dynamics it is assumed throughout this section that $\sigma(1-\beta) > 1$. This implies, in particular, that $\sigma > 1$ so that the results of the previous sections hold.

It is convenient to analyze a companion one-dimensional difference equation driving the dynamics of inequality. Some of its properties are established in the following lemma.

Lemma 7 (Dynamics of inequality). The dynamics of k_t is given by

$$k_{t+1} = \begin{cases} g_1(k_t) \equiv [k_t^{1/\sigma+\beta} + \theta k_t^{\beta}]/[1 + \theta k_t^{1/\sigma}], & \text{if} \quad k_t \geqslant \tilde{k}; \\ g_2(k_t) \equiv \tau\theta k_t^{\beta}, & \text{if} \quad \hat{k} \leqslant k_t < \tilde{k}; \\ g_3(k_t), & \text{if} \quad k_t < \hat{k}, \end{cases} \tag{31}$$

[16] Persistence of endowments is introduced to make the dynamics of the model more realistic. If $\beta = 0$, the main qualitative results remain unchanged but the economy will not stay in the fear equilibrium for more than 1 period.

where $g_3(k_t)$ is implicitly given by

$$k_t = \frac{\tau(1+\theta^2)}{2\theta(1+\tau)^2} \cdot \frac{(z_{t+1}+\theta)^2}{z_{t+1}} \cdot \left(\frac{z_{t+1}-\theta}{1-\theta z_{t+1}}\right)^\sigma, \quad z_{t+1} \equiv k_{t+1}/k_t^\beta. \tag{32}$$

Moreover, $k_{t+1} > k_t$ for all $0 < k_t < 1$.

In what follows the analysis focuses on the dynamics in the two empirically most relevant regions, the fear region and the KUJ region. The two lemmas below characterize the behavior of the system in these regions.

Lemma 8 (KUJ region dynamics). The system converges to a unique stable "equal" long-run steady state $\bar{K}_1^{\text{KUJ}} = \bar{K}_2^{\text{KUJ}} = \bar{K} = [\alpha/(1-\theta)]^{\frac{\sigma}{\sigma(1-\beta)-1}}$. The steady-state levels of output are equal to $\bar{Y}_1^{\text{KUJ}} = \bar{Y}_2^{\text{KUJ}} = \bar{Y} = \bar{K}^{1/\sigma}/(1-\theta)$. The evolution of endowments is determined by the loci

$$\Delta K_i \equiv K_{it+1} - K_{it} = 0: \quad K_{it}^{1/\sigma} + \theta K_{jt}^{1/\sigma} = K_{it}^{1-\beta} \cdot (1-\theta^2)/\alpha, \quad i,j = 1,2, \quad i \neq j,$$

such that $dK_{it}/dK_{jt} > 0$ and $d^2K_{it}/dK_{jt}^2 < 0$ for each locus.

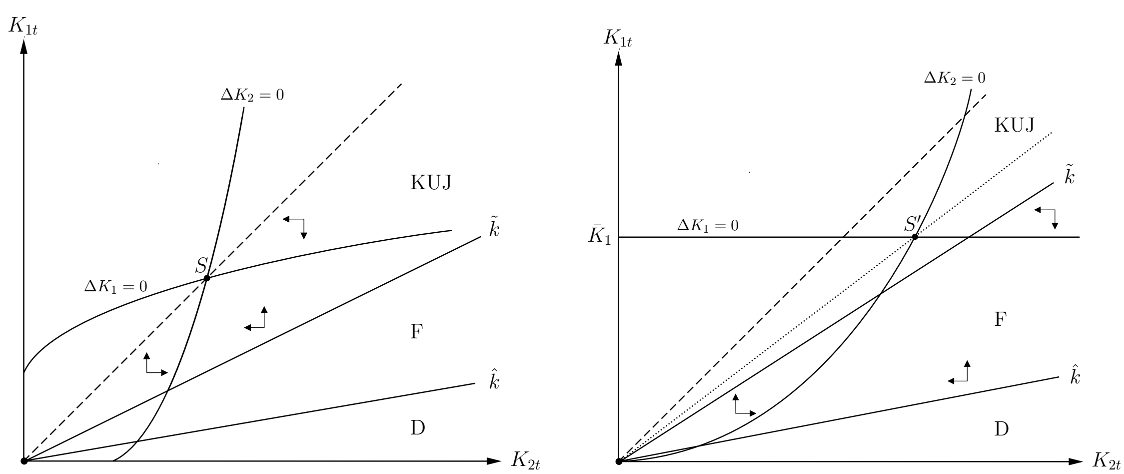

Figure 12: Dynamics and steady states in the KUJ (left) and fear (right) regions.

Figure 12 (left panel) depicts schematically the dynamics in the KUJ region. If $\sigma(1-\beta) > 1$, the initially poor eventually catch up with the initially rich, and in the steady state both are "running to keep in the same place" (Hopkins and Kornienko, 2004).

Lemma 9 (Fear region dynamics). In the fear region the system moves towards a unique stable "unequal" long-run steady state $\bar{K}_1 = [\alpha\tau/(\tau-1)]^{\frac{\sigma}{\sigma(1-\beta)-1}}$, $\bar{K}_2 = \bar{K}_1 \cdot (\tau\theta)^{\frac{1}{\beta-1}}$. The corresponding levels of output are $\bar{Y}_1 = [\tau/(\tau-1)]\bar{K}_1^{1/\sigma}$ and $\bar{Y}_2 = \bar{K}_1^{1/\sigma}/[\theta(\tau-1)]$.

This steady state is, however, unattainable, since it is located in the KUJ region, that is, the system moves to the KUJ region before reaching the fear steady state. The evolution of endowments is determined by the loci

$$\begin{aligned}\Delta K_1 = 0: &\quad K_{1t} = \bar{K}_1 = [\alpha\tau/(\tau-1)]^{\frac{\sigma}{\sigma(1-\beta)-1}};\\ \Delta K_2 = 0: &\quad K_{1t} = K_{2t}^{\sigma(1-\beta)} \cdot [\theta(\tau-1)/\alpha]^{\sigma}.\end{aligned}$$

Figure 12 (right panel) depicts schematically the dynamics in the fear region. It is instructive to look at the comparative statics of the long-run levels of output with respect to τ and θ. They resemble the results of the static model: in the KUJ steady state, outputs are increasing in θ and independent of τ; in the (unattainable) fear steady state, the output of group 1 is independent of θ and decreasing in τ while the output of group 2 is decreasing in θ and τ. Note also that, despite their qualitatively different nature, as $\tau\theta \to 1$, the two long-run equilibria get closer and coincide in the limit.

Given Lemmas 7–9, it is easy to establish how the possible development trajectories look. The long-run convergence result is stated in Proposition 6 and depicted in Figure 13.

Proposition 6 (Long-run convergence). Starting with any initial conditions $\{K_{10}, K_{20}\}$, such that $0 < k_0 < 1$, the endowments converge to a unique stable long-run "equal" steady state of the KUJ region, $\bar{K}$. Inequality decreases monotonically along the transition path.

Thus, if the economy starts off, say, in destructive region, it experiences a transition to the KUJ steady state, possibly passing through the fear region and staying there for a while. What allows to switch between alternative regimes is that inequality decreases over time causing phase transitions.

As established in Section 3, the effect of envy on economic performance is opposite in fear and KUJ regions, which implies the changing role of envy in the transition process. Matt (2003) examines the cultural change with regard to envy in the American consumer society in 1890–1930, the period when the phrase "keeping up with the Joneses" came into popular use. She quotes (p. 4) an essay from the Christian Advocate newspaper (1926), in which the "new version" of the tenth commandment is documented with dismay: "Thou shalt not be outdone by thy neighbor's house, thou shalt not be outdone by thy neighbor's wife, nor his manservant, nor his car, nor anything – irrespective of its price or thine own ability – anything that is thy neighbor's." The fear of envy goes away and the emotion is used by producers and sellers to advertise their products. Envy suppression paves the way to envy creation via marketing (Belk, 2008). As emulation becomes the engine of

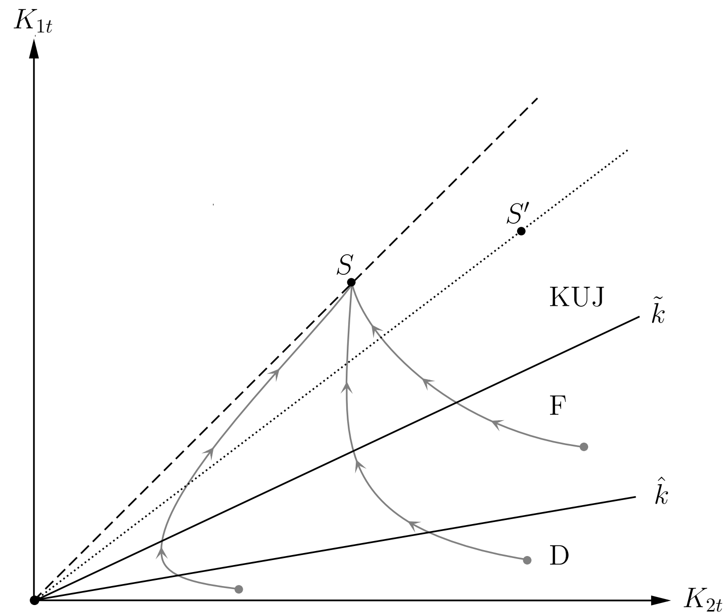

Figure 13: Evolution through envy regimes.

consumer society, the factors that make positional concerns more important contribute to higher outputs and overworking aggravating the consumption externality.

Religion and ideology are important cultural elements that may affect the intensity of social comparisons. Major world religions denounce envy. Buddhism considers envy to be an evil state of mind and teaches that a virtuous person will celebrate the good fortune of a neighbor (Clanton, 2006). Islam condemns Hasad, destructive envy, although permits Ghibtah, envy that is free from malice, or emulation.[17]

The condemnation of envy is probably the strongest in the Judeo-Christian tradition. Envy is one of the deadly sins and features prominently in the tenth commandment. In fact, Aquaro (2004) argues that envy is a core emotion driving most sinful behaviors that created the need for the ten commandments to combat these sins. Or, as Schoeck (1969) puts it, "a society from which all cause of envy had disappeared would not need the moral message of Christianity" (p. 160). He also argues that the Roman Catholic belief in the will of god, the Protestant work ethic, and the Calvinist concept of predestination were instrumental in combating envy and increasing tolerance for inequality.

In the present dynamic model such religious and moral teachings may be seen as causing downward pressure on θ. Assume that the economy is in the fear region and θ falls. Then,

[17]For analysis linking envy, inequality, and contemporary revival of Islam see Carvalho (2009).

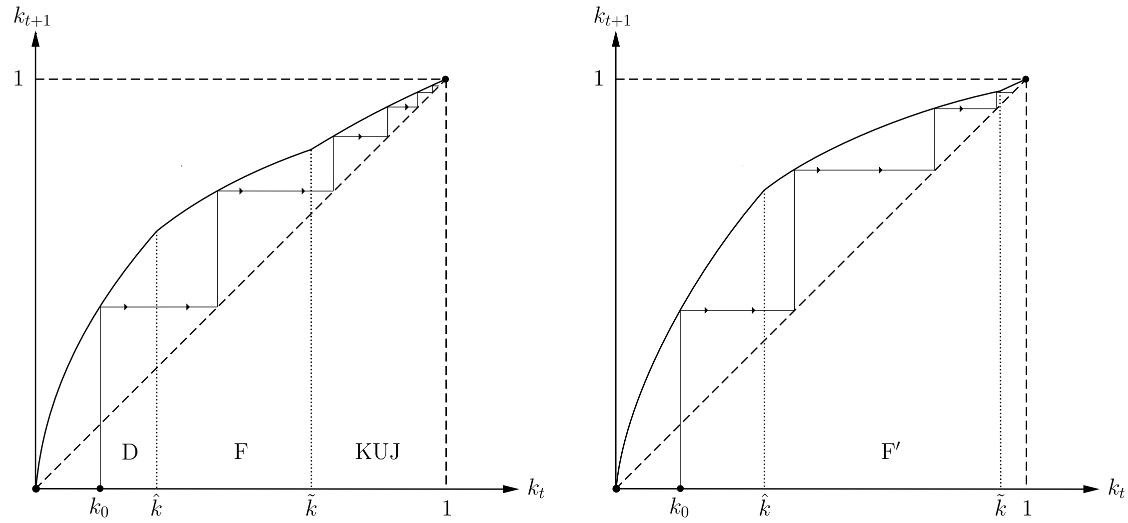

Figure 14: Dynamics of inequality under low (left) and high (right) τ.

as follows from Proposition 2, outputs will rise because the fear constraint of the rich is alleviated permitting higher effort without fear of destructive envy. Moreover, as follows directly from (18), a fall in θ lowers the inequality threshold $\tilde{k}$ contributing to a faster transition from FE to KUJE. As the economy enters the KUJ region, destructive envy turns into emulation, and the change in θ has an opposite impact on economic performance. In KUJ region, the same factors that drive the society out of the fear region by reducing the strength of relative concerns have a negative effect on output.

An example of ideology affecting θ is material egalitarianism. The concept of everyone being equal is effective in fostering social comparison and lowering tolerance for inequality (see discussion in Section 2).

It is tempting to incorporate the result of Section 5 in this simple dynamic framework. In particular, assume that the first generation that finds itself in the fear equilibrium is to decide on the level of property rights protection, and conditions of Proposition 5 hold. Then, it is optimal for both agents to increase τ. Then, it can be shown that: 1) The threshold $\tilde{k}$ increases; 2) The scope of the fear region, $(\tilde{k} - \hat{k})$, extends; 3) The long-run fear steady state moves closer to the long-run KUJ steady state. So, an increase in τ would endogenously prolong the presence of the economy in the fear region (see Figure 14), and at the same time will make the FE more egalitarian since the erosion of institutions decreases tolerance for inequality and exacerbates the fear constraint. This may explain the persistence of the fear equilibrium, along with such characteristics as poorly protected private property rights, fear of envy, and relatively low inequality.

7 Concluding remarks

This paper develops a unified framework for the economic analysis of envy by capturing its two main forces, destructive and competitive. The active side of envy is determined jointly by the level of and tolerance for inequality, which depends endogenously on the degree of property rights protection and the strength of relative concerns.

Competitive envy gives rise to the standard "keeping up with the Joneses" (KUJ) equilibrium characterized by suboptimally high effort and consumption due to positional externality. This equilibrium roughly corresponds to the evidence from modern industrialized economies, in which emulation is the main driver of consumer demand. On the other hand, low tolerance for inequality may lead to qualitatively different equilibria, in which the destructive side of envy is predominant. In the "fear equilibrium" the better endowed agents constrain their effort to prevent destructive envy of the relatively poor. This equilibrium resembles the features of pre-industrial societies documented by anthropologists, and some characteristics of socialist and transition economies, where political ideology coupled with poor protection of property rights were instrumental in decreasing tolerance for inequality and fostering the fear of envy.

The different nature of these equilibria yields contrasting comparative statics with respect to envy. In the KUJ equilibrium, envy enhances production by intensifying emulation, while in the fear equilibrium it reduces output by aggravating the fear constraint.

The basic model is applied to examine the interplay between institutions, welfare, and economic performance. In Section 4 it is shown how the possibility of voluntary transfer can mitigate destruction in equilibrium, which is in line with the evidence on the role of redistributive mechanisms. In Section 5 a comparative welfare analysis of the fear and KUJ equilibria is conducted. Despite the fact that KUJ equilibrium always yields higher outputs than the fear equilibrium, it may be Pareto inferior. In particular, consumption externality that causes inefficiency of the KUJ equilibrium is partially curbed in the fear equilibrium, which implies that adopting better property rights protection may not represent a Pareto improvement in the presence of positional externalities.

Finally, the static model is augmented with dynamic linkages to explore the evolution of the economy through different envy regimes. Starting at any initial condition, the economy converges to the long-run KUJ steady state and inequality monotonically decreases along the transition path. The dynamic model highlights the changing role of envy in the transition process and permits to analyze the role of institutions and culture with regard to envy. Factors that decrease tolerance for inequality prolong the presence in the fear region and delay transition to the KUJ steady state. Moreover, individuals may choose to stay longer in the fear region by voluntarily refusing to adopt better institutions. This may explain the persistence of poor property rights protection, the fear of envy and relatively low inequality in simple societies as well as countries with long socialist experience.

A promising direction for future research would be to incorporate the present theory of envy in a more sophisticated growth model which would allow to explore the feedback mechanisms between technology, institutions, inequality, and the two sides of envy.

Overall, the proposed framework offers a parsimonious way to account for envy in the economic analysis of institutions, development, and culture, and bridges the gap between separate lines of theoretical and empirical research on envy in social sciences.

Appendix

Proof of Lemma 1. Agent 2 is solving (9) subject to (7) and (8). Consider first the KUJ case, $\tau\theta Y_2 \leqslant Y_1$, in which

$$U^2 = \frac{(Y_2 - \theta Y_1)^{1-\sigma}}{1-\sigma} - \frac{Y_2}{K_2}.$$

It is strictly concave in Y_2, and the first-order conditions yield the following optimum:

$$Y_2 = \begin{cases} K_2^{1/\sigma} + \theta Y_1, & \text{if} \quad Y_1 \geqslant \widetilde{C}_1; \\ Y_1 \cdot \frac{1}{\tau\theta}, & \text{if} \quad Y_1 < \widetilde{C}_1, \end{cases} \tag{A1}$$

where $\widetilde{C}_1$ is defined in Lemma 1. That is, the left derivative of U^2 at point $Y_2 = Y_1/\tau\theta$ is positive (negative) iff $Y_1 < \widetilde{C}_1$ ($Y_1 > \widetilde{C}_1$). Next, consider the destructive case, $\tau\theta Y_2 > Y_1$, in which

$$U^2 = \frac{1}{1-\sigma} \cdot \left(\sqrt{\frac{Y_1 Y_2}{\tau\theta}} \cdot (1+\theta^2) - \frac{\theta(\tau+1)}{\tau} Y_1 \right)^{1-\sigma} - \frac{Y_2}{K_2}.$$

Again, U^2 is concave in Y_2, and the interior optimum is uniquely defined by the first-order condition:

$$\left(\sqrt{\frac{Y_1 Y_2}{\tau\theta}} \cdot (1+\theta^2) - \frac{\theta(\tau+1)}{\tau} Y_1 \right)^{-\sigma} \cdot \left(\frac{1+\theta^2}{2\sqrt{\tau\theta}} \cdot \sqrt{\frac{Y_1}{Y_2}} \right) - \frac{1}{K_2} = 0. \tag{A2}$$

It follows that the right derivative of U^2 at point $Y_2 = Y_1/\tau\theta$ is positive (negative) iff $Y_1 < \widehat{C}_1$ ($Y_1 > \widehat{C}_1$), where $\widehat{C}_1$ is defined in Lemma 1 and $\widehat{C}_1 < \widetilde{C}_1$, since $0 < \theta < 1$. Hence, allowing destruction is the best response iff $Y_1 < \widehat{C}_1$.

To rewrite (A2) in terms of consumption note that under destruction

$$\begin{cases} C_1 = (1-d^*)Y_1 = \dfrac{1+\tau}{\tau} Y_1 - \sqrt{\dfrac{\theta}{\tau} Y_1 Y_2}; \\ C_2 = p^* Y_2 = \sqrt{\dfrac{Y_1 Y_2}{\tau\theta}}. \end{cases} \implies \begin{cases} Y_1 = \dfrac{1+\tau}{\tau}(C_1 + \theta C_2); \\ Y_2 = \theta(1+\tau) \cdot \dfrac{C_2^2}{C_1 + \theta C_2}. \end{cases} \tag{A3}$$

Substituting this into (A2) yields (11). Next, applying the implicit function theorem to (11) gives

$$\frac{dC_2}{dC_1} = \frac{\theta + \frac{1}{\sigma} \cdot \frac{C_2 - \theta C_1}{C_1 + \theta C_2}}{1 + \frac{1}{\sigma} \cdot \frac{C_2 - \theta C_1}{C_1 + \theta C_2} \cdot \frac{C_1}{C_2}} > 0$$

since in equilibrium $C_2 > \theta C_1$. It is also straightforward to show that $dC_2/dC_1 < C_2/C_1$. Finally,

$$\frac{d^2 C_2}{dC_1^2} = \left(\sigma + \frac{C_2 - \theta C_1}{C_1 + \theta C_2} \cdot \frac{C_1}{C_2} \right)^{-1} \cdot \frac{C_2 - C_1 \cdot \frac{dC_2}{dC_1}}{C_2(C_1 + \theta C_2)} \cdot \left[\frac{(1+\theta^2)\left(C_1 \frac{dC_2}{dC_1} - C_2 \right)}{C_1 + \theta C_2} - \frac{\frac{dC_2}{dC_1}(C_2 - \theta C_1)}{C_2} \right] < 0,$$

since $C_2 > \theta C_1$ and $0 < dC_2/dC_1 < C_2/C_1$. Hence, $C_2^d(C_1)$ is strictly increasing and concave.

Proof of Lemma 2. Agent 1 is solving (12) subject to (7) and (8). In the KUJ case, $\tau\theta Y_2 \leqslant Y_1$,

$$U^1 = \frac{(Y_1 - \theta Y_2)^{1-\sigma}}{1-\sigma} - \frac{Y_1}{K_1}.$$

It is strictly concave in Y_1, and the first-order conditions yield the following optimum:

$$Y_1 = \begin{cases} K_1^{1/\sigma} + \theta Y_2, & \text{if} \quad Y_2 \leqslant \widehat{C}_2; \\ \tau\theta Y_2, & \text{if} \quad Y_2 > \widehat{C}_2, \end{cases} \tag{A4}$$

where $\widehat{C}_2$ is defined in Lemma 2. In the destructive case, $\tau\theta Y_2 > Y_1$,

$$U^1 = \frac{1}{1-\sigma}\cdot\left(\frac{1+\tau}{\tau}Y_1 - 2\sqrt{\frac{\theta}{\tau}Y_1Y_2}\right)^{1-\sigma} - \frac{Y_1}{K_1}.$$

Assumption $\sigma > 1$ is sufficient for U^1 to be concave in Y_1. In particular, it can be shown by simple differentiation that the sign of $\partial^2 U^1/\partial Y_1^2$ is the same as the sign of $f(x) \equiv -(1+\sigma)x^2 + \xi(2\sigma+1/2)x - \sigma\xi^2$, where $x \equiv \sqrt{\theta Y_2/\tau Y_1}$ and $\xi \equiv (\tau+1)/\tau$. Then, it is easy to show that $f_{\max} \propto 1 - 8\sigma$ and so, U^1 is concave if $\sigma > 1/8$.

Note that U^1 is differentiable at point $Y_1 = \tau\theta Y_2$ with $u_1'(\tau\theta Y_2) = ((\tau-1)\theta Y_2)^{-\sigma} - 1/K_1$. Next, consider two cases. If $Y_2 \leqslant \widehat{C}_2$, there is an interior optimum in the KUJ region and concavity of U^1 ensures that it is a global optimum. If $Y_2 > \widehat{C}_2$, there is a unique optimum in the destructive region given by the following first-order condition:

$$\left(\frac{1+\tau}{\tau}Y_1 - 2\sqrt{\frac{\theta}{\tau}Y_1Y_2}\right)^{-\sigma}\cdot\left(\frac{1+\tau}{\tau} - \sqrt{\frac{\theta}{\tau}\cdot\frac{Y_2}{Y_1}}\right) - \frac{1}{K_1} = 0. \tag{A5}$$

Finally, using (A3) rewrite equation (A5) in terms of consumption levels to get 14. Application of the implicit function theorem to (14) gives

$$\frac{dC_1}{dC_2} = \frac{\theta - \frac{\theta}{\sigma}\cdot\frac{C_1-\theta C_2}{C_1+\theta C_2}}{1 - \frac{\theta}{\sigma}\cdot\frac{C_1-\theta C_2}{C_1+\theta C_2}\cdot\frac{C_2}{C_1}} > 0,$$

since $\sigma > 1$ and in equilibrium $C_1 > \theta C_2$. It is also straightforward to show that $dC_1/dC_2 < C_1/C_2$. Finally,

$$\frac{d^2C_1}{dC_2^2} = \frac{\theta}{\sigma}\left(1 - \frac{\theta}{\sigma}\cdot\frac{C_1-\theta C_2}{C_1+\theta C_2}\cdot\frac{C_2}{C_1}\right)^{-1}\left[\frac{\theta\left(\frac{dC_1}{dC_2}\cdot\frac{C_2}{C_1}-1\right)^2}{C_1+\theta C_2} + \frac{(C_1-\theta C_2)\left(\frac{dC_1}{dC_2}+\theta\right)\left(1-\frac{dC_1}{dC_2}\cdot\frac{C_2}{C_1}\right)}{(C_1+\theta C_2)^2}\right] > 0,$$

which proves convexity.

Proof of Proposition 1. Private outputs in the KUJ equilibrium are given by (15). For this to be an equilibrium two conditions must hold: $Y_1^{\mathrm{KUJ}} \geqslant \widetilde{C}_1$ and $Y_2^{\mathrm{KUJ}} \leqslant \widehat{C}_2$. This yields $k \geqslant \tilde{k}$, where $\tilde{k}$ is defined in Proposition 1. In the fear equilibrium private outputs are given by (16). For Y_2^{F} to fall in the fear region it must be the case that $\widehat{C}_1 \leqslant Y_1^F < \widetilde{C}_1$, which yields $\hat{k} \leqslant k < \tilde{k}$ with $\hat{k}$ defined in Proposition 1. Finally for $k < \hat{k}$ the equilibrium is given by (17). To prove its existence and uniqueness note that, from Lemma 1, $C_2^d(C_1)$ is strictly increasing and concave and, from Lemma 2, $C_1^d(C_2)$ is strictly increasing and convex. Moreover, it is straightforward to show that $\widehat{C}_1 > C_1^d(\widehat{C}_1/\tau\theta)$. Then, the intermediate value theorem guarantees existence, and the properties of best response functions imply uniqueness. In particular, direct comparison shows that the slope of BR_1 is always steeper than that of BR_2, which ensures single crossing.

Proof of Proposition 2. Results for KUJE and FE follow directly from differentiation of (15) and (16). For DE, consider the system defining the equilibrium:

$$\begin{cases} f_1 \equiv C_1 - \theta C_2 - \psi[C_1/(C_1+\theta C_2)]^{1/\sigma} = 0; \\ f_2 \equiv C_2 - \theta C_1 - \phi[(C_1+\theta C_2)/C_2]^{1/\sigma} = 0. \end{cases}$$

Using the equilibrium conditions, it can be shown that

$$[D_C f]^{-1} = \frac{1}{\Delta}\begin{bmatrix} 1 + \frac{1}{\sigma}\cdot\frac{C_2-\theta C_1}{C_1+\theta C_2}\cdot\frac{C_1}{C_2} & \theta - \frac{\theta}{\sigma}\cdot\frac{C_1-\theta C_2}{C_1+\theta C_2} \\ \theta + \frac{1}{\sigma}\cdot\frac{C_2-\theta C_1}{C_1+\theta C_2} & 1 - \frac{\theta}{\sigma}\cdot\frac{C_1-\theta C_2}{C_1+\theta C_2}\cdot\frac{C_2}{C_1} \end{bmatrix},$$

where C is the vector of consumption levels, f is the vector of f_1 and f_2, and $\Delta \equiv \det(D_C f) = 1 - \theta^2 + [(C_1-\theta C_2)^2(C_2 - \theta C_1)]/[\sigma(C_1+\theta C_2)C_1C_2] > 0$. It follows from this expression that all elements of $[D_C f]^{-1}$ are positive if $\sigma > 1$.

For part (a), note that, by the implicit function theorem, $D_\lambda C = -[D_C f]^{-1}\cdot D_\lambda f$ and

$$D_\lambda f = -\begin{bmatrix} \frac{\psi_\lambda}{\psi}(C_1-\theta C_2) \\ \frac{\phi_\lambda}{\phi}(C_2-\theta C_1) \end{bmatrix} = \frac{1}{\sigma}\begin{bmatrix} -\frac{1}{\lambda}(C_1-\theta C_2) \\ \frac{1}{1-\lambda}(C_2-\theta C_1) \end{bmatrix}.$$

It follows that
$$\frac{\partial C_1}{\partial \lambda} \propto \left[1+\frac{1}{\sigma}\frac{C_2-\theta C_1}{C_1+\theta C_2}\frac{C_1}{C_2}\right](C_1-\theta C_2)-\frac{\theta\lambda}{1-\lambda}\left[1-\frac{1}{\sigma}\frac{C_1-\theta C_2}{C_1+\theta C_2}\right](C_2-\theta C_1).$$

Dividing (11) by (14) and denoting $x \equiv C_1/C_2 \in (\theta, 1)$, in equilibrium we have

$$\frac{\lambda}{1-\lambda} = \left(\frac{x-\theta}{1-\theta x}\right)^{\sigma} \cdot \frac{\tau(1+\theta^2)}{2\theta(1+\tau)^2} \cdot \frac{(x+\theta)^2}{x}.$$

Plugging this expression in the previous equation and making transformations, we get that the sign of $\partial C_1/\partial\lambda$ is determined by

$$\frac{\sigma(\theta+x)+x(1-\theta x)}{\sigma(\theta+x)-(x-\theta)} - \frac{\tau(1+\theta^2)}{2(1+\tau)^2} \cdot \left(\frac{x-\theta}{1-\theta x}\right)^{\sigma-1} \cdot \frac{(x+\theta)^2}{x}.$$

The first term is always greater than 1. The second term is increasing in x and at $x=1$ simplifies to $\tau(1+\theta^2)(1+\theta)^2/[2(1+\tau)^2]$. The latter expression is maximized at $\tau = 1$ in which case it is equal to $(1+\theta^2)(1+\theta)^2/4 \leqslant 1 \ \forall\ \theta \in (0,1)$. Hence, the second term is always less than 1 and $\partial C_1/\partial\lambda > 0$.

Similarly, the sign of the effect of λ on total consumption, $\partial C/\partial\lambda$, coincides with the sign of

$$\frac{\sigma(1+\theta)(x+\theta)+(1-\theta x)(1+x)}{\sigma(1+\theta)(x+\theta)-\theta(x-\theta)(1+1/x)} - \frac{\tau(1+\theta^2)}{2(1+\tau)^2} \cdot \left(\frac{x-\theta}{1-\theta x}\right)^{\sigma-1} \cdot \frac{(x+\theta)^2}{x}.$$

As above, the first term is always greater than 1, while the second is always less than 1. Hence, $\partial C/\partial\lambda > 0$.

For part (b), note that $D_\theta C = -[D_C f]^{-1} \cdot D_\theta f$ and

$$D_\theta f = -\begin{bmatrix} C_2 \cdot \left(\frac{1}{\sigma}\cdot\frac{C_1-\theta C_2}{C_1+\theta C_2}-1\right) \\ -C_1 + \frac{1}{\sigma}\cdot\frac{C_2-\theta C_1}{C_1+\theta C_2}\cdot\frac{(1-\theta^2)C_1-2\theta^3 C_2}{\theta(1+\theta^2)} \end{bmatrix}.$$

It follows that the sign of $\partial C_1/\partial\theta$ is determined by the sign of

$$C_2+\theta C_1 - \frac{1}{\sigma}\frac{C_2-\theta C_1}{C_1+\theta C_2}\left(\frac{(1-\theta^2)C_1-2\theta^3C_2}{1+\theta^2}-C_1\right).$$

Since $(1-\theta^2)C_1 - 2\theta^3 C_2 - (1+\theta^2)C_1 = -2\theta^2 C_1 - 2\theta^3 C_2 < 0$, we get $\partial C_1/\partial\theta > 0$.

For part (c), note that $D_\tau C = -[D_C f]^{-1} \cdot D_\tau f$ and

$$D_\tau f = -\begin{bmatrix} \frac{\psi_\tau}{\psi}(C_1-\theta C_2) \\ \frac{\phi_\tau}{\phi}(C_2-\theta C_1) \end{bmatrix}$$

with both elements positive, since $\psi_\tau < 0$ and $\phi_\tau < 0$. It follows that the elements of $D_\tau C$ are negative meaning that C_1 and C_2 are decreasing in τ.

Proof of Lemma 3. Using (7), rewrite (21) as

$$v\left((1+\theta^2)\sqrt{\frac{(Y_1+T)(Y_2-T)}{\tau\theta}} - \frac{\theta(\tau+1)}{\tau}(Y_1+T)\right) \longrightarrow \max_T \qquad s.t. \quad 0 \leqslant T \leqslant \bar{T}.$$

The objective function is concave in T. The first-order condition for interior solution, T^*, is

$$\frac{1+\theta^2}{2\sqrt{\tau\theta}} \cdot \frac{Y_2-Y_1-2T^*}{\sqrt{(Y_1+T^*)(Y_2-T^*)}} = \frac{\theta(\tau+1)}{\tau}, \quad 0 < T^* < \bar{T}.$$

It is straightforward to check that $v'(\bar{T}) \geqslant 0$ iff $\tau\theta \leqslant (1-\theta)/(1+\theta)$ which is the condition for full transfer. On the other hand, $v'(0) < 0$ iff $a(1-x^2) < bx$, where $x \equiv \sqrt{Y_1/Y_2}$, $a \equiv (1+\theta^2)/2\sqrt{\tau\theta}$ and $b \equiv \theta(\tau+1)/\tau$. Solving this inequality yields the condition $Y_1/Y_2 \geqslant \mu$, where $\sqrt{\mu} \equiv [\sqrt{\tau^2(1+\theta^2)^2+\theta^2(\tau+1)^2\tau\theta} - \theta(\tau+1)\sqrt{\tau\theta}]/[\tau(1+\theta^2)]$, and $0 < \mu \leqslant \tau\theta$ if $\tau\theta \leqslant (1-\theta)/(1+\theta)$.

Proof of Lemma 4. In the KUJ case, $\tau\theta Y_2 \leqslant Y_1$, as established in the proof of Lemma 1, the optimal action of Agent 2 is (A1). In the case with full transfer the utility of Agent 2 is

$$U^2 = \frac{\gamma_2(Y_1+Y_2)^{1-\sigma}}{1-\sigma} - \frac{Y_2}{K_2},$$

where γ_2 is defined in Lemma 4. It is concave in Y_2 and the unique optimum is given by $Y_2 = (\gamma_2 K_2)^{1/\sigma} - Y_1$, which is interior iff $Y_1 < \check{Y}_1$, where $\check{Y}_1$ is defined in Lemma 4, and $\check{Y}_1 < \widetilde{Y}_1$, as direct comparison shows. Putting this together yields (22).

Proof of Lemma 5. In the KUJ case, $\tau\theta Y_2 \leqslant Y_1$, as established in the proof of Lemma 2, the optimal action of Agent 1 is (A4). Now consider the case in which Agent 2 makes a full transfer under a credible threat of destruction, $\tau\theta Y_2 > Y_1$. The utility of Agent 1 is

$$U^1 = \frac{\gamma_1(Y_1+Y_2)^{1-\sigma}}{1-\sigma} - \frac{Y_1}{K_1},$$

where γ_1 is defined in Lemma 5. This yields the following solution:

$$Y_1 = \begin{cases} b_1 \equiv \tau\theta Y_2, & \text{if} \quad Y_2 < (\gamma_1 K_1)^{1/\sigma}/(1+\tau\theta); \\ b_2 \equiv (\gamma_1 K_1)^{1/\sigma} - Y_2, & \text{if} \quad (\gamma_1 K_1)^{1/\sigma}/(1+\tau\theta) \leqslant Y_2 < \check{Y}_2; \\ b_3 \equiv 0, & \text{if} \quad Y_2 \geqslant \check{Y}_2, \end{cases} \tag{A6}$$

where $\check{Y}_2$ is defined in Lemma 5. First, note that if the optimum is not interior in the KUJ region, that is, if $Y_2 > \widehat{C}_2$, the global optimum is determined by (A6) since in this case the left derivative of U^1 at point $Y_1 = \tau\theta Y_2$ is less than the (negative) right derivative. Consider for simplicity only the case $\check{Y}_2 < \widehat{C}_2$ (the other case can be analyzed in exactly the same way) which holds if

$$\sigma < \bar{\sigma} \equiv \frac{\ln[(1+\tau\theta)/(\theta(\tau-1))]}{\ln(1+\tau\theta)}. \tag{A7}$$

Then, as follows from (A6), $Y_2 \geqslant \widehat{C}_2$ implies that the best response will be b_3. If, however, $Y_2 < \widehat{C}_2$, the optimum in the KUJ region is interior and needs to be compared to the best response in the transfer region. The utility generated by the former is

$$U^2_{\text{KUJ}} = \frac{\sigma}{1-\sigma} K_1^{\frac{1-\sigma}{\sigma}} - \frac{\theta Y_2}{K_1}.$$

If $\check{Y}_2 \leqslant Y_2 < \widehat{C}_2$, b_3 is the best response in the transfer region and the corresponding utility is

$$U^2_{\text{FT}} = \gamma_2 \cdot \frac{Y_2^{1-\sigma}}{1-\sigma}.$$

Then, it is easy to show that $U^2_{\text{FT}} > U^2_{\text{KUJ}}$ in the region $\check{Y}_2 \leqslant Y_2 < \widehat{C}_2$ if the following restriction holds:

$$\left[\frac{1}{1-\sigma} + \theta\right] \cdot \left[\frac{\theta(\tau-1)}{1+\tau\theta}\right]^{\frac{1-\sigma}{\sigma}} > \frac{\sigma}{1-\sigma}. \tag{A8}$$

Assume that it holds. Then, consider the region $(\gamma_1 K_1)^{1/\sigma}/(1+\tau\theta) \leqslant Y_2 < \check{Y}_2$, in which b_2 is the best response in the case with transfer and the corresponding utility is

$$U^2_{\text{FT}} = \frac{\sigma}{1-\sigma}\gamma_1^{1/\sigma} K_1^{\frac{1-\sigma}{\sigma}} + \frac{Y_2}{K_1}.$$

Since (A8) holds and KUJ optimum is interior (that is, b_1 cannot be a best response) it must be the case that in this region there exists a value of Y_2 such that $U^2_{\text{FT}} = U^2_{\text{KUJ}}$. Direct computation shows that this value is $\widetilde{Y}_2$ as defined in Lemma 5. So, if $Y_2 \leqslant \widetilde{Y}_2$, KUJ best response is the global optimum; otherwise, it is b_2.

Proof of Proposition 3. For (15) to be an equilibrium, as follows from Lemma 4 and Lemma 5, the following conditions must hold: $Y_1^{\text{KUJ}} \geqslant \widetilde{Y}_1$ and $Y_2^{\text{KUJ}} \leqslant \widetilde{Y}_2$. This yields the condition $k \geqslant \max\{(\gamma_2/\gamma_1)^{\sigma/(\sigma-1)}, \omega\}$. If the best responses intersect in their transfer regions, we get outputs as in (24). For it to be an equilibrium it must be the case that $Y_1^{\text{FT}} \leqslant \check{Y}_1$ and $Y_2^{\text{FT}} \geqslant \check{Y}_2$ which yields the condition $k \leqslant \gamma_2/\gamma_1$. To get the partitioning in Proposition 3 observe that under assumptions of Section 4 the following is true: $(\gamma_2/\gamma_1)^{\sigma/(\sigma-1)} < \omega < \gamma_2/\gamma_1$. From direct comparison and re-arrangement of terms, the former part of this inequality holds iff $[(1+\tau\theta)/(\theta(\tau-1))]^\alpha < 1+[\alpha(1+\theta)]/[\theta(\tau-1)]$, where $\alpha \equiv (\sigma-1)/\sigma \in (0,1)$. This is always true since the left-hand side is a convex function of α for $0 < \alpha < 1$ and at $\alpha = 0$ and $\alpha = 1$ both sides of the inequality coincide. Similarly, $\omega < \gamma_2/\gamma_1$ iff

$$\alpha \left[\frac{1-\tau\theta^2}{\theta(\tau-1)}\right]^\alpha < (1-\theta)\left(\left[\frac{1+\tau\theta}{\theta(\tau-1)}\right]^\alpha - 1\right) - \alpha\theta.$$

It is straightforward to show that both sides of this inequality are strictly increasing, convex functions that coincide at $\alpha = 0$ and $\alpha = 1$. Moreover, the derivative of the right-hand side exceeds that of the left-hand side at $\alpha = 0$ iff $\ln([1+\tau\theta]/[\theta(\tau-1)]) > \theta/(1-\theta)$. This is true since $1 < \tau < (1-\theta)/[\theta(1+\theta)]$. Note that the latter inequality also implies that $\sigma(1-\theta)(1-\gamma_1^{1/\sigma})/(1-\sigma) > \theta$, that is, ω is well-defined.

Finally, it can be shown that the fear equilibrium without transfers (as in the basic model) does not exist. For (16) to be an equilibrium, condition $Y_2^{\mathrm{F}} = \hat{C}_2 \leqslant \widetilde{Y}_2$ must be satisfied, but restriction (A7) rules this out since $\widetilde{Y}_2 < \check{Y}_2 < \hat{C}_2$. Putting everything together yields the statement of Proposition 3.

Proof of Proposition 4. It follows from (15) and (24) that

$$\frac{C_1^{\mathrm{KUJ}}}{C_2^{\mathrm{KUJ}}} = \frac{k^{1/\sigma}+\theta}{1+\theta k^{1/\sigma}}, \qquad \frac{C_1^{\mathrm{FT}}}{C_2^{\mathrm{FT}}} = \tau\theta.$$

Then, inequality is higher in FT equilibrium iff $k > [(1-\tau\theta^2)/\theta(\tau-1)]^{-\sigma}$, and it is sufficient to show that $\omega > [(1-\tau\theta^2)/\theta(\tau-1)]^{-\sigma}$ to prove part 1. The latter holds iff $[(1+\theta)/\theta(\tau-1)] > [\sigma(1-\gamma_1^{1/\sigma})/(1-\sigma)]$. Denote the right-hand side of this inequality as $f(\alpha)$, where $\alpha \equiv (\sigma-1)/\sigma \in (0,1)$. Then, it is straightforward to show that $f'(\alpha) > 0$ and $\lim_{\alpha\to 1} f(\alpha) = [(1+\theta)/\theta(\tau-1)]$, which completes the proof of part 1.

It follows from (15) and (24) that $Y^{\mathrm{KUJ}} > Y_2^{\mathrm{FT}}$ iff $(k^{1/\sigma}+1)/(1-\theta) > \gamma_2^{1/\sigma}$. Since $k > \omega$,

$$\frac{k^{1/\sigma}+1}{1-\theta} > \frac{\omega^{1/\sigma}+1}{1-\theta} = \frac{1-\sigma\gamma_1^{1/\sigma}}{\sigma(1-\gamma_1^{1/\sigma}) - \theta(1-\sigma\gamma_1^{1/\sigma})}.$$

This expression exceeds $\gamma_2^{1/\sigma}$ iff $1-\theta < (\sigma-1)/(\sigma\gamma_1^{1/\sigma}-1)$. Denote the right-hand side of the inequality as $g(\sigma)$. Then, $g'(\sigma) < 0$ and $\lim_{\sigma\to\infty} g(\sigma) = 1-\theta$, which completes the proof of part 2.

Proof of Proposition 5. The utilities of individuals in FE are given by (26). In KUJE, as follows from (15), they are

$$U_{\mathrm{KUJ}}^i = \left[\frac{1}{1-\sigma} - \frac{1}{1-\theta^2}\right] \cdot K_i^{\frac{1-\sigma}{\sigma}} - \frac{\theta}{1-\theta^2} \cdot \frac{K_j^{\frac{1}{\sigma}}}{K_i}, \quad i,j = 1,2,\ i \neq j.$$

For Agent 1, $U_{\mathrm{KUJ}}^1 > U_{\mathrm{F}}^1$ iff $k > [\theta(\tau-1)/(1-\tau\theta^2)]^\sigma$ which is, as shown in Proposition 1, the threshold for KUJE vs. FE. This implies that Agent 1 always prefers staying in FE. For Agent 2, $U_{\mathrm{KUJ}}^2 > U_{\mathrm{F}}^2$ iff

$$\left[\frac{1}{1-\sigma} - \frac{1}{1-\theta^2}\right] \cdot \left(\frac{K_2}{K_1}\right)^{\frac{1}{\sigma}} + \frac{1-\tau\theta^2}{\theta(\tau-1)(1-\theta^2)} > \frac{1}{1-\sigma} \cdot \left[\frac{1-\tau\theta^2}{\theta(\tau-1)}\right]^{1-\sigma} \cdot \frac{K_2}{K_1}. \tag{A9}$$

Let $L(x)$ and $R(x)$ denote the left and right-hand sides of (A9), respectively, where $x \equiv K_2/K_1$. Then, $L(1/\tilde{k}) = R(1/\tilde{k})$ and

$$f(\theta) \equiv L(1/\hat{k}) - R(1/\hat{k}) \propto \frac{1}{1-\sigma} \cdot \left[\left(\frac{2}{1+\theta^2}\right)^{1/\sigma} - \frac{2}{1+\theta^2}\right] - \frac{1}{1-\theta^2} \cdot \left[\left(\frac{2}{1+\theta^2}\right)^{1/\sigma} - 1\right].$$

Consider first the case $\sigma > 1$, in which $L'(x) < 0$ and $L''(x) > 0$. It can be shown that $\exists!\ \bar{\theta} \in (0,1)$ such that $L(1/\hat{k}) - R(1/\hat{k}) > 0$ iff $\theta < \bar{\theta}$. Let $z \equiv 2/(1+\theta^2) \in (1,2)$, and $\gamma \equiv 1/\sigma \in (0,1)$. Next, define $g(z) \equiv 2(z-1)f(z) = [2\gamma/(\gamma-1)] \cdot (z^{1+\gamma} - z^\gamma - z^2 + z) - (z^{1+\gamma} - z)$. It is clear that $f(\theta)$ has a unique root $\bar{\theta} \in (0,1)$ iff $g(z)$ has a unique root $\bar{z} \in (1,2)$ apart from $z = 1$. To prove the latter notice that: 1) $g(1) = 0$, $g(2) \propto 1+\gamma-2^\gamma > 0$; 2) $g'(1) = -\gamma < 0$; 3) $g''(z) = 4/(1-\gamma) - [(1+\gamma)^2/(1-\gamma)]z^{\gamma-1} - 2\gamma z^{\gamma-2} > 4/(1-\gamma) - (1+\gamma)^2/(1-\gamma) - 2\gamma = 3-\gamma > 0$. It can be shown similarly that Proposition 5 also holds for $0 < \sigma < 1$.

Proof of Lemma 7. The functional forms of $g_i(k_t)$, $i = 1, 2$, follow directly from (15), (16), and (29). The form of $g_3(k_t)$ follows from (11), (14) and (29). Next, it is straightforward to establish by differentiation that $g_1(k)$ and $g_2(k)$ are strictly increasing and concave, given that $\beta < 1$ and $\sigma(1-\beta) > 1$. Also, $g_1(1) = 1$, $g_1(\tilde{k}) = g_2(\tilde{k}) > \tilde{k}$, and $g_2(\bar{k}) = g_3(\bar{k}) > \bar{k}$. The two last properties follow from assumptions that $\theta < 1$, $\tau\theta < 1$, $\sigma > 1$ and $\beta < 1$. Together this implies that $k_{t+1} > k_t$ for $k_t \geqslant \hat{k}$. Finally, if there exists a steady state $\bar{k}$ in the destructive segment, it is given by $g_3(\bar{k}) = \bar{k}$. Making a substitution $\kappa = \bar{k}^{1-\beta}$ and rearranging terms yields the equation defining the steady state:

$$\frac{\kappa^{2-\beta}}{(\kappa+\theta)^{2(1-\beta)}} = \pi \left(\frac{\kappa-\theta}{1-\theta\kappa}\right)^{\sigma(1-\beta)}, \tag{A10}$$

where $\pi \equiv [\tau(1+\theta^2)/2\theta(1+\tau)^2]^{1-\beta}$ and in equilibrium $\kappa > \theta$. It is easy to show that $R(\kappa)$, the right-hand side of (A10), is strictly increasing and convex. Also, $L(\kappa)$, the left-hand side of (A10), is strictly increasing. To show that it is concave for $\kappa > \theta$, note that the sign of $L''(\kappa)$ is determined by the sign of $\tilde{L} \equiv (\kappa+\theta)(2-\beta)(\beta\kappa+\theta(1-\beta)) - \kappa(3-2\beta)(\beta\kappa+\theta(2-\beta))$. At $\kappa = \theta$ this expression is negative, since $\beta < 1$. Moreover, it is strictly decreasing for the same reason. Hence, $L''(\kappa) < 0$ for $\kappa > \theta$. This implies that there exists at most one solution to (A10). If there is no solution, the proof is finished. Assume now that $\bar{k}$ exists. In this case $\bar{k} > \hat{k}$. To see this, note first that $\hat{k} < (\tau\theta)^{1/(1-\beta)}$. Next, $\kappa > \tau\theta$ since $L(\tau\theta) > R(\tau\theta)$. Hence, $\bar{k} \equiv \kappa^{1/(1-\beta)} > (\tau\theta)^{1/(1-\beta)} > \hat{k}$, i.e., $k_{t+1} > k_t$ for $0 < k_t < \hat{k}$.

Proof of Lemma 8. It follows from the properties of $g_1(k_t)$ that k_t monotonically converges to 1 in the KUJ region. The expression for $\Delta K_i = 0$ comes from (30) and may be rewritten as $K_{jt} = K_{it}[(\rho K_{it}^{1-\beta-1/\sigma} - 1)/\theta]^{\sigma}$, where $\rho \equiv (1-\theta^2)/\alpha$. Then, by differentiation we get that $dK_{jt}/dK_{it} > 0$ and $d^2K_{jt}/dK_{it}^2 > 0$, since $\sigma(1-\beta) > 1$. This implies the stated properties of the $\Delta K_i = 0$ loci. The expression for $\bar{K}$ follows from solving $\Delta K_1 = \Delta K_2 = 0$ and (15) then gives $\bar{Y}$.

Proof of Lemma 9. The equations for $\Delta K_i = 0$ come from (30). Since in the fear region $K_{1t+1} = \alpha\tau K_{1t}^{1/\sigma+\beta}/(\tau-1)$ and $\sigma(1-\beta) > 1$, K_{1t} converges to $\bar{K}_1$. Plugging this in $\Delta K_2 = 0$ yields $\bar{K}_2$. The output levels follow from (16). To see that the long-run fear equilibrium is in the KUJ region, note that the implied steady-state level of inequality is $(\tau\theta)^{1/(1-\beta)}$ which exceeds $\tilde{k} = [\theta(\tau-1)/(1-\tau\theta^2)]^{\sigma}$, since $\sigma(1-\beta) > 1$ and $[\theta(\tau-1)/(1-\tau\theta^2)] < \tau\theta < 1$.

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