Rational indecisive choice

Gerasimou, Georgios

University of Cambridge

27 September 2010

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Georgios Gerasímou*

Faculty of Economics AND King's College, University of Cambridge

September 27, 2010

Abstract

This paper proposes and characterises two preference-based choice rules that allow the decision maker to choose nothing if the criteria associated with them are satisfied by no feasible alternative. Strict preferences are primitive in the first rule and weak preferences in the second. Each of them includes the corresponding utility-maximisation theory of rational choice as a special case. The first one explains changes in the magnitude of context effects observed in experiments that allow for indecision. The second offers one explanation of experimental findings suggesting that choice is more likely to be made from small rather than from large sets. The general conclusion in both cases is that an individual conforms to meaningful and testable principles of choice consistency whenever assumed to be occasionally indecisive.

A person is defined to be rational if he does the best he can, using reason and all available information, to further his own interests and values.

Truman Bewley (1986/2002)

A mode of behavior is rational for a given person if this person feels comfortable with it, and is not embarrassed by it, even when it is analyzed for him.

Itzhak Gilboa (2010)

1 Introduction

Experiments on consumer behaviour have provided empirical support to the introspectively obvious claim than people facing a choice problem often avoid choosing when given the opportunity to do so. Reasons for such behaviour include a positive bias for the status quo, uncertainty about one's own preferences regarding the available courses of action, conflict induced by these available options that is difficult to resolve, a deliberate effort to defer choice that may be linked to beliefs that the future will bring better opportunities, inaction inertia,

*I thank Robert Evans, Paola Manzini, Marco Mariotti, Romans Pancs and workshop audiences at Cambridge and Amsterdam for useful comments.

and also anticipatory feelings such as regret about a potentially bad choice.[1] By way of the standard convention of a choice correspondence that is well-defined everywhere in its domain and the core axiom of preference completeness, most of decision theory predicts that a person will always be able to make a choice from any set of feasible options. Reconciling theory with the real-world phenomenon of severe indecisiveness of the type that leads to absolutely no choice being made is nevertheless desirable.

Working within the general framework of abstract static choice theory, this paper proposes and characterizes two choice rules, together with some of their special cases, that provide certain preference-based criteria which, if not met by any feasible alternative in a given set, render the decision maker (DM) indecisive and lead her to avoid choosing from that set. The first rule takes strict preferences as primitive and predicts that a feasible option will be chosen if and only if it is preferred to at least one other feasible option and dispreferred by none, where preferences are generally asymmetric and, as a special case, also transitive. At the heart of the characterisation is an intuitive weakening of the Weak Axiom of Revealed Preference (WARP), the *Reduced* WARP, which postulates that if an option x is chosen over an option y when only the two of them are feasible, then y cannot be chosen from another set where x is also feasible. This choice procedure provides a formal explanation for the strengthening of the "attraction effect" and weakening of the "compromise effect"[2] that has been observed in experiments where subjects were freely allowed to abstain from choosing (Dhar and Simonson (2003)).

The second rule takes weak preferences as primitive and allows for indifference to be distinguished from incomparability. A DM here chooses an alternative from a set if and only if it is preferred or indifferent to all other alternatives in that set, where preference-or-indifference is a reflexive and transitive relation that is generally incomplete. Standard WARP is central in the characterisation of this rule, which provides a framework for thinking about cases where an individual, although happy to choose an alternative x when only x is feasible or y when only y is feasible, may nevertheless avoid choosing between x and y. More generally, this procedure may be suitable in modeling situations where choices are made from sets with relatively few alternatives but not from sets with many, a phenomenon that was first documented by Iyengar and Lepper (2000) and attributed to the increased complexity of the choice problem in the latter case.

[1] A detailed survey and discussion of experimental findings on decision avoidance can be found in Anderson (2003).

[2] Both these effects (discussed below) are robust empirical observations that, according to a common interpretation, suggest systematic WARP violations in certain contexts.

Obviously, the two choice rules studied here are not utility-maximising ones. Instead, it would probably be accurate to label them *reason-based*, as they are similar in spirit to the procedures that Shafir, Simonson, and Tversky (1993) discussed when coining this term. In any case, an important theoretical conclusion to be drawn from the present paper is that to assume that an individual is occasionally indecisive as if she followed a reason-based choice rule such as those studied here is equivalent to assuming that she obeys meaningful and normatively appealing principles of choice consistency. In particular, it is by no means necessary that a DM be decisive in order for her to be acting rationally on the one hand, and it is by all means reasonable to hypothesize that once her indecision is respected in empirical applications, then any choices she actually makes will in fact be "more rational" than when it is not. The latter assertion is also consistent with the well-known argument of Luce and Raiffa (1957) that forcing a DM to choose "*between inherently incomparable alternatives*" could subsequently lead her to intransitive choices. To the extent that intransitivity implies irrationality, upon reversing this argument it becomes clear that *not* forcing a DM to choose when she is not actually willing to do so eliminates one source of possible intransitivities and, by extension, irrationalities. In this respect, the view taken here differs from that in Mandler (2005) where it is claimed that, despite preference incompleteness, *"agents can be forced to choose from any set of options."*

As far as methodology is concerned, indecisive choice in this paper is modeled by correspondences (and functions) that are *empty-valued* (i.e. undefined) at those sets of feasible alternatives where none of the latter meets the prespecified selection criterion. One may argue that, in the broad sense, "no choice" is itself a choice and therefore one need not resort to partially defined choice mappings to model indecision. Instead, the argument goes, an explicit alternative standing for "no choice" may be included in the original domain, making choice everywhere well-defined by definition. Although both approaches aim to model exactly the same phenomenon, in this paper where choice is interpreted in the narrow and pragmatic sense of choice from *physical* alternatives, the former approach appears more natural and analytically more convenient. We note further that implicit support for this point of view also comes from Arrow (1959, p. 122) where, although this line of research was not actually pursued, we find that "*(choice) is of course not necessarily defined for all possible sets (...)*".

With regard to the relevant literature, it seems that the only paper considering the possibility of indecisive choice in the same way as we do here is that by Clark (1995). Although both works share the same view that indecisiveness and rationality in choice are compatible, Clark (1995) does not axiomatise the preference-based choice rules that we do here and does not relate the analysis with experimental findings. On the other hand, the recent literature on choice with incomplete preferences (e.g. Mandler (2005, 2009), Masatlioglu and Ok (2005), Eliaz and Ok (2006), Danan (2010)), which is often axiomatic in nature, always builds on the assumption that the DM's set of choosable alternatives is nonempty for every menu. Thus, it is assumed there that any hesitation caused by preference incompleteness is somehow resolved once the DM is presented with a choice task. This, in particular, is also true for the model of status-quo bias suggested by Masatlioglu and Ok (2005), where the DM's behaviour is affected by the presence of a default option in a menu, but her optimal sets are always nonempty, i.e. either the default option or something preferred to it gets to be chosen from that menu. The theory proposed in the present paper may also be interpreted as one modeling a DM who occasionally preserves her status quo, but one where the latter does not explicitly enter the choice problem in the form of a default option as in Masatlioglu and Ok (2005).

2 Indecisive Choice and Strict Preferences

We start with some definitions. Let X denote the consumption set, which is understood to be the universal set of alternatives and $\mathcal{M}^*$ a collection of subsets of X that necessarily includes all those with exactly two elements. A set $M \in \mathcal{M}^*$ is a *menu* of alternatives that are feasible for the consumer in a particular situation. A *choice correspondence* $C : \mathcal{M}^* \twoheadrightarrow X$ is a set-valued mapping such that for all $M \in \mathcal{M}^*$, $C(M) \subseteq M$. If $C(M) \neq \emptyset$ for all $M \in \mathcal{M}^*$, then C is a *decisive* choice correspondence, otherwise it is *indecisive*. If C is single-valued whenever non-empty, then it is a *choice function* and we will use the notation $c : \mathcal{M}^* \to X$ in that case. If, in addition, c is well-defined everywhere in $\mathcal{M}^*$, then it is a *decisive* choice function. As far as notational abuses are concerned, we write $C(x, y) = x$ instead of $C(\{x, y\}) = \{x\}$ whenever it is understood that the consumer chooses x from the menu $\{x, y\}$. We say that a DM, whose choices are captured by a choice correspondence C, is *behaviourally indecisive* at a menu $M \in \mathcal{M}^*$ if $C(M) = \emptyset$.

For $x, y, z \in X$, a preference relation $\succ$ on X is said to be *asymmetric* if $x \succ y$ implies $y \not\succ x$, *total* if $x \neq y$ implies $x \succ y$ or $y \succ x$, and *transitive* if $x \succ y$ and $y \succ z$ together imply $x \succ z$. If $x \not\succ y$ and $y \not\succ x$ for some $x, y \in X$, then we write $x \mathrel{\widehat{\sim}} y$. The relation $\mathrel{\widehat{\sim}}$ is *reflexive* ($x \mathrel{\widehat{\sim}} x$) and *symmetric* ($x \mathrel{\widehat{\sim}} y$ implies $y \mathrel{\widehat{\sim}} x$) but not transitive in general. Assuming that only trivial indifference exists (i.e. each alternative is indifferent to itself only), then $\mathrel{\widehat{\sim}}$ is to be interpreted as the relation that captures the DM's *psychological indecision*, i.e. if $x \mathrel{\widehat{\sim}} y$, then she is psychologically indecisive between alternatives x and y. On the other hand, if indifference is nontrivial, then it is some reflexive and symmetric subrelation of $\widehat{\sim}$, with psychological indecision being the irreflexive and symmetric relation that complements indifference in $\widehat{\sim}$.

Define the *revealed preference relation* $\succ^*$ associated with a choice correspondence C by $x \succ^* y$ if $x \in C(M)$ and $y \in M \setminus C(M)$ for some $M \in \mathcal{M}^*$. Define further the *reduced revealed preference relation* $\succ^\#$ by $x \succ^\# y$ if $C(x, y) = x$. Clearly, $\succ^\#$ is asymmetric by definition and contained in $\succ^*$, so that $x \succ^\# y$ implies $x \succ^* y$, while the converse is not true in general. Our definition of $\succ^*$ coincides with the one in Arrow (1959), but other papers assume that $\succ^\#$ captures revealed preferences instead.

2.1 Multiple Choosable Alternatives

We first study a choice rule that is based on strict preferences and allows for none or more than one alternatives to be chosen. Hence, the rule defines an indecisive choice correspondence $C : \mathcal{M}^* \twoheadrightarrow X$. We start by considering the following axioms that C will later be assumed to satisfy.

Reduced Weak Axiom of Revealed Preference (ReWARP)

If $M \in \mathcal{M}^*$, $x, y \in M$ and $C(x, y) = x$, then $y \notin C(M)$.

It is straightforward that ReWARP generalises WARP by placing fewer restrictions in the hypothesis part of the axiom. More specifically, when formulated for choice correspondences WARP states that whenever x is chosen over y in an arbitrary menu M, then y cannot be chosen in another menu M' where x is also feasible. By contrast, ReWARP requires that the same conclusion be true only when x is chosen over y in the particular menu $\{x, y\}$. This is intuitive: When only x and y are feasible, choice is more focused, it cannot be framed by the presence of third options, it requires fewer cognitive resources and it is therefore likely to be a more accurate reflection of the consumer's underlying attitude towards x and y than it would be had there been more feasible alternatives to choose from. Furthermore, the fact that x is chosen over y in this focused problem where choosing nothing is also an option adds further support to the presumption that this behaviour reveals a genuine preference for x over y hence making it more reasonable to expect that no weak reversal of this preference will be observed in other choices involving x and y.

Decision-Driven Choice (DDC)

If $M \in \mathcal{M}^*$, $x, y_1, \ldots, y_k \in M$ *and* $C(x, y_1) = \ldots = C(x, y_k) = \emptyset$, *then* $x \notin C(M)$.

Suppose that a DM is behaviourally indecisive between an alternative x and every alternative in a collection $y_1, y_2, \ldots, y_k$, when considering them in pairs. Two possible scenarios for this state of affairs is that decision conflict arises in every pairwise comparison (e.g. in a multi-attribute setting) and that she is unfamiliar with x and therefore reluctant to make a choice even if she is familiar with each y_i. In the latter case, a cautious DM may prefer to wait until more information becomes available on x before making a choice in order to avoid missing a potentially good opportunity. Whichever the case, there seems to be no compelling reason why x should be chosen from the larger menu that includes all these options, and this is precisely what DDC requires.

Weakly Dominant Choice (WDC)

If $M \in \mathcal{M}^*$, $x, y, z_1, \ldots, z_m \in M$, $C(x, y) = x$ *and* $C(x, z_i) \neq z_i$ *for all* $i \leq m$, *then* $x \in C(M)$.

If x is chosen over y from $\{x, y\}$ and z_i is not chosen over x from $\{x, z_i\}$, $i \leq m$, either because no choice is made or because x is chosen over some z_i instead, then WDC states that x should in fact be chosen from $M := \{x, y, z_1, \ldots, z_m\}$ too, if M is itself feasible. The justification is that x has been revealed strictly better to at least one other element of M in the purest of possible choice situations involving the two of them (keeping in mind that not choosing anything was also possible), while it has also been revealed not strictly worse than any other alternative in M in the same context. WDC postulates that the DM currently being modeled believes that these two facts provide sufficient reason for x to be chosen from the larger set that includes all the above alternatives.

Pairwise Choice Transitivity (PCT)

If $x, y, z \in X$, $C(x, y) = x$ *and* $C(y, z) = y$, *then* $C(x, z) = x$.

PCT is a consistency requirement on pairwise choices. As such, it is more likely to be satisfied when not choosing is permissible than when it is not, because when x is chosen over y from $\{x, y\}$ and y over z from $\{y, z\}$ in this setting, it is understood that x is unambiguously better than y and y than z; if there was any sort of ambiguity as to which one was better in each case, the DM would have chosen nothing instead. Thus, expecting that x will also be chosen over z from $\{x, z\}$ is, as usual, normatively sound, but in this case it may also be a requirement that enjoys greater descriptive validity compared to a situation of forced choice.

Proposition 1

An indecisive choice correspondence $C : \mathcal{M}^* \twoheadrightarrow X$ satisfies ReWARP, DDC and WDC if and only if there exists an asymmetric relation $\succ$ on X such that for all $M \in \mathcal{M}^*$

$$x \in C(M) \Longleftrightarrow \begin{cases} x \succ y & \text{for some } y \in M \\ & \text{and} \\ z \not\succ x & \text{for all } z \in M. \end{cases} \tag{1}$$

Furthermore, C also satisfies PCT if and only if $\succ$ is, in addition, transitive.

Proof:

Let $C : \mathcal{M}^* \twoheadrightarrow X$ satisfy (1). Suppose $M \in \mathcal{M}^*$ and $x, y \in M$ are such that $C(x, y) = x$. It holds that $x \succ y$. Let $y \in C(M)$. By definition, $z \not\succ y$ for all $z \in M$. Therefore, $x \not\succ y$ too, a contradiction. Next, let $M := \{x, y_1, y_2, \ldots, y_k\} \in \mathcal{M}^*$ and $C(x, y_1) = C(x, y_2) = \ldots = C(x, y_k) = \emptyset$. Clearly, $x \not\succ y_i$ for all $i \leq k$. If $x \in C(M)$, then $x \succ y_i$ for some $i \leq k$, a contradiction. Finally, suppose $M \in \mathcal{M}^*$ and $x \in M$ are such that $C(x, y) = x$ for some $y \in M \setminus \{x\}$ and $C(x, z) \neq z$ for all $z \in M \setminus \{x\}$. It holds that $x \succ y$ and $z \not\succ x$ for such $y, z \in M$, from which follows directly that $x \in C(M)$. Therefore, ReWARP, DDC and WDC are satisfied. It is clear that PCT will also be satisfied if $\succ$ is transitive.

For the converse implication, suppose that C satisfies the three axioms and consider the (asymmetric) relation $\succ^\#$ on X as defined above. Let $x \in C(M)$ for some $M \in \mathcal{M}^*$. If $y \succ^\# x$ for some $y \in M$, then ReWARP is violated. Hence, no such $y \in M$ exists. Suppose next that $x \succ^\# y$ holds for no $y \in M$. Since it is now known that $y \not\succ^\# x$ for all $y \in M \setminus \{x\}$, this implies that $C(x, y) = \emptyset$ for all $y \in M \setminus \{x\}$. But DDC ensures that $x \notin C(M)$, a contradiction. Thus, for all $M \in \mathcal{M}^*$, $x \in C(M)$ implies $x \succ^\# y$ for some $y \in M$ and $y \not\succ^\# x$ for all $y \in M$. In the other direction, if for some $M \in \mathcal{M}^*$ there exists $x \in M$ such that $x \succ^\# y$ for some $y \in M$ and $z \not\succ^\# x$ for all $z \in M$, then it follows trivially from WDC that $x \in C(M)$. Finally, it is again clear that PCT implies that $\succ^\#$ is also transitive. ■

In words, under this choice rule the DM selects an alternative from a menu if she considers that alternative better than at least some other alternative in that menu and worse than none, the dominance based on an asymmetric or asymmetric and transitive preference relation. If no feasible option satisfies this choice criterion, then nothing is chosen and the DM is behaviourally indecisive. If there are more than one options satisfying the criterion, then any one of them will be chosen. Importantly, it does not suffice here that an option be undominated according to the preference relation for it to be choosable (c.f., for instance, Richter (1971)). At the same time, choosable alternatives need not be preferred to all non-choosable ones either. Hence, the model is in accordance with Statement 2 in Rubinstein and Salant (2008) suggesting that *"economists should be also looking at models in which the observed choice leads to conclusions other than that the chosen element is always mentally preferred to the other elements in the set."*

Let us now take note of the following facts about Proposition 1, which are true in its general version that does not require PCT to be satisfied by the choice correspondence C.

Observation 1:
For all $M \in \mathcal{M}^*$, $C(M) \neq \emptyset$ implies $C(M) \subset M$. $\Diamond$

Indeed, if $C(M) = M$ for some $M \in \mathcal{M}^*$, then $x \in M$ is such that $x \succ y$ for some $y \in M$, which implies $y \notin C(M) = M$, a contradiction. This observation points to the meaningfulness of the concept of choice under this rule, because an alternative is selected only at the cost of another one not being selected. In particular, there is no way that all elements of a menu can ever be chosen, which is in sharp contrast to models following the standard, decisive approach to choice that allow for this possibility when all alternatives in a menu are undominated with respect to a preference relation. Although the informational content revealed in both these cases in terms of how the feasible options compare to each other is minimal, the two approaches carry quite distinct intuitions and have very different implications.

Observation 2:
For all $x, y, z \in X$, if $C(x, y) = \emptyset$ and $C(y, z) = y$, then $y \in C(x, y, z)$.
If, in addition, $x \notin C(x, z)$, then $y = C(x, y, z)$. $\Diamond$

For the first part, $y \notin C(x, y, z)$ implies $x \succ y$ or $z \succ y$. But $C(x, y) = \emptyset$ implies

$x \mathrel{\widehat{\sim}} y$, and $C(y,z) = y$ implies $y \succ z$, a contradiction. For the second part, suppose that $x \in C(x,y,z)$. Then, since $x \mathrel{\widehat{\sim}} y$, it follows that $x \succ z$. But $C(x,z) \neq x$ implies either $z \succ x$ or $x \mathrel{\widehat{\sim}} z$, a contradiction. Furthermore, $y \succ z$ implies $z \notin C(x,y,z)$ too. Thus, if the consumer is indecisive between x and y in $\{x,y\}$ and chooses y over z in $\{y,z\}$, then she must also choose y from $\{x,y,z\}$. If it is also true that x is not chosen from $\{x,z\}$ either because z is chosen or because she is indecisive between them, then y is uniquely chosen from $\{x,y,z\}$.

Observation 3:
For all $x,y,z \in X$, if $C(x,y) = C(y,z) = C(x,z) = \emptyset$, then $C(x,y,z) = \emptyset$. $\Diamond$

Suppose $x \in C(x,y,z)$. Then $x \succ y$ and $z \not\succ x$ or $x \succ z$ and $y \not\succ x$. $C(x,y) = x$ holds in the first case and $C(x,z) = x$ holds in the latter, a contradiction. Similarly, $y, z \in C(x,y,z)$ is precluded. Intuitively, if the consumer can't make a choice from $\{x,y\}$, $\{y,z\}$ and $\{x,z\}$ then she won't be able to choose from $\{x,y,z\}$ either. This implication may be thought of as a consistency requirement for the consumer's indecision.

Example 1

Suppose $X = \{w,x,y,z\}$, $\mathcal{M}^* = \{M \subseteq X : |M| > 1\}$ and the consumer has the following preferences:

$$\begin{array}{ccc} x \succ w, & y \succ w, & y \succ z, \\ x \mathrel{\widehat{\sim}} y, & x \mathrel{\widehat{\sim}} z, & w \mathrel{\widehat{\sim}} z. \end{array}$$

If this consumer proceeds according to the rule of Proposition (1), then she will make choices that are captured by an indecisive choice correspondence C as follows:

$$\begin{aligned} &C(w,x) = C(w,x,z) = x, \\ &C(w,y) = C(y,z) = C(x,y,z) = C(w,y,z) = y, \\ &C(x,y) = C(x,z) = C(w,z) = \emptyset, \\ &C(X) = C(w,x,y) = \{x,y\}. \end{aligned}$$

Although these choices conform with ReWARP, the fact that y is chosen over x from $\{x,y,z\}$ and x is chosen in the presence of y from $\{w,x,y\}$ shows that they violate WARP despite there being no $\succ$-cycles. In fact, it is precisely the psychological indecision between x and y that leads to WARP violations in this example. ♦

Clearly, with the additional requirement that $\succ$ in Proposition 1 also be *total* we obtain as a special case the standard result that a choice correspondence $C : \mathcal{M}^* \twoheadrightarrow X$ satisfies (1) where $\succ$ is asymmetric, total and transitive, if and only if C is decisive, single-valued (hence a choice *function* c) and satisfies WARP. This is so because once totality is assumed in addition to asymmetry and transitivity, an alternative x in a menu M is dispreferred by no other alternative $y \in M$ if and only if it is in fact preferred to *all* other alternatives in M.

Finally, a natural question to ask is whether the axioms of Proposition 1 are equivalent to utility-maximising behaviour once the choice correspondence is assumed decisive. The following example shows that the answer is "no", i.e. even in a decisive-choice setting the implied behaviour is not a utility-maximising one: Let $X = \{w, x, y, z\}$, $C(w, x) = x$, $C(w, y) = y$, $C(w, z) = z$, $C(x, y) = y$, $C(x, z) = \{x, z\}$, $C(y, z) = \{y, z\}$, $C(w, x, z) = \{x, z\}$, $C(w, x, y) = y$, $C(x, y, z) = y$, $C(w, y, z) = \{y, z\}$ and $C(X) = \{y, z\}$. These choices satisfy ReWARP, WDC, PCT and (trivially) DDC. However, the fact that y is chosen over z from $\{x, y, z\}$ and z is chosen in the presence of y from $\{w, y, z\}$ shows that WARP is violated.

2.2 Attraction and Compromise Revisited

Consider the following three situations of choice from two-attribute alternatives (these may be lotteries on a zero versus a strictly positive monetary prize, the two attributes being the amount of the non-zero prize and its probability of being won). First, the consumer chooses between alternatives x and y, each dominating the other in one attribute. Next, she chooses from a set where to x and y is added a third alternative z that is dominated in both attributes by y, dominates x in one attribute and is dominated by x in another. Finally, she chooses from a set where an alternative z' is added to x and y such that z' dominates both x and y in a fixed attribute and is dominated by both of them in the other, making y the intermediate option.

Subjects in experiments on individual choice are typically forced to choose from each one of the above three sets of alternatives. Forced choice in the first problem usually leads to choice probabilities close to $1/2$ for both alternatives x and y. The *attraction effect*, originally due to Huber, Payne, and Puto (1982), is the empirical observation that the choice probability of y in forced-choice experiments is significantly higher in the second problem where it is the asymmetrically dominating alternative, than in the first where it is not. Within the same experimental setup, the *compromise effect*, originally due to Simonson (1989), is the observation that the choice probability of y in the third problem where it is the intermediate (or "compromise") option is again significantly higher than in the first problem where it is not.

One approach in interpreting these choices is to accept that the representative individual chooses both x and y in the first case and only y in the second and third. When these choices are captured by a decisive choice correspondence C_d that sets $C_d(x,y) = \{x,y\}$, $C_d(x,y,z) = y$ and $C_d(x,y,z') = y$ respectively, it becomes clear that they violate WARP, since y is chosen over x in the second and third problems and x is chosen in the presence of y in the first (Figure 1).

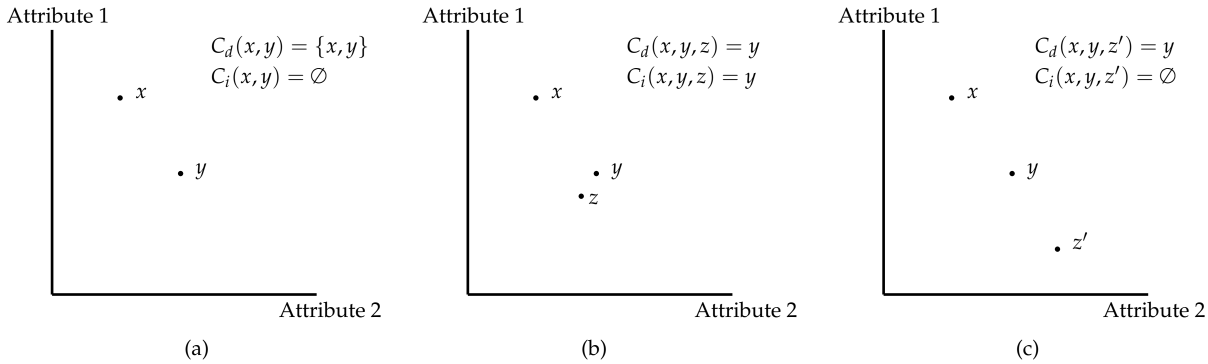

FIGURE 1

Attraction (b) and compromise (c) with decisive and indecisive choice

Is it actually true, however, that an individual facing the above choice problems would act "irrationally" in a more realistic situation where she would be freely allowed to abstain in the event that, for whatever reason, she would be indecisive? To answer this question suppose the consumer has a non-total preference relation $\succ$ over two-attribute alternatives that is captured by the asymmetric part of the usual partial ordering in $\mathbb{R}^2$. It will then be true for her that $x \mathrel{\overset{\frown}{\sim}} y$, $x \mathrel{\overset{\frown}{\sim}} z$, $y \succ z$, $x \mathrel{\overset{\frown}{\sim}} z'$ and $y \mathrel{\overset{\frown}{\sim}} z'$. Under the rule studied above these preferences translate into pairwise choices captured by an indecisive choice correspondence C_i with $C_i(x,y) = \varnothing = C_i(x,z)$, $C_i(y,z) = y$, and $C_i(x,z') = \varnothing = C_i(y,z')$. In view of Observations 2 and 3, these choices also imply $C_i(x,y,z) = y$ and $C_i(x,y,z') = \varnothing$. In other words, the asymmetrically dominating alternative y is again uniquely chosen in the second problem whereas neither the compromise alternative y nor x or z' is chosen in the third problem.

The reason-based model of Proposition 1 therefore predicts that the attraction effect will be *stronger* when the consumer's indecision is respected. First, the choice probability of y when only x and y are feasible is likely to be low due to the conflicting attribute values which may lead the individual to choose nothing. At the same time, the choice probability of y in the second problem is likely to be high because y meets the criteria of choice from $\{x, y, z\}$ according to the rule followed (i.e. y dominates z and is undominated by x) while x and z don't (z is dominated by y and x dominates neither y nor z). The model also predicts a *weakening* of the compromise effect when indecision is allowed. In the menu $\{x, y, z'\}$ the consumer is unable to make comparisons in all three pairs of alternatives, again due to the conflicting attribute values. Since, in particular, no two alternatives are ordered by strict preference, there is no reason for a choice to be made from this set if the individual proceeds in the way described above. Hence, the choice probability of y is predicted to be low in both sets $\{x, y\}$ and $\{x, y, z'\}$.

These two predictions generated by the above choice rule find empirical support in the experimental study of Dhar and Simonson (2003), who examined the effect that allowing subjects to be indecisive has on the magnitude of both the attraction and compromise effects. More specifically, these authors found evidence that when abstaining is a permissible option, then the attraction effect is strengthened and the compromise effect weakened and they did so by comparing the difference in choice probabilities of the asymmetrically dominating and the compromise options between the groups where choice was forced and those where it wasn't. The explanation they provided for the moderation of the compromise effect in this case is that since the intermediate option is likely to be chosen in situations of preference uncertainty or decision conflict, the no-choice option serves this purpose even better and is therefore a powerful substitute.

2.3 Unique Choosable Alternatives

When a model of individual choice predicts that more than one alternatives in a given menu satisfy the optimality criterion for being chosen, it is understood that each one of them will be chosen eventually if the individual is to face the particular problem sufficiently many times. It is also understood that at any given time when the choice problem presents itself, the one alternative chosen among the optimal ones in the end will be selected in a way not captured formally by the model. Yet, it is often desirable that a theory be able to make sharper predictions in terms of which option gets to be chosen from a menu. For that to happen, however, the choice correspondence of the primitive model must somehow reduce to a choice function. This is the task carried out in this subsection, where it is shown how the indecisive choice correspondence $C : \mathcal{M}^* \twoheadrightarrow X$ that was characterised above reduces to an indecisive choice function $c : \mathcal{M}^* \rightarrow X$. For the task at hand, the following axioms will be essential.

Weak Axiom of Revealed Preference (WARP)

If $M, M' \in \mathcal{M}^*$, $x = c(M)$, $y \in M \setminus \{x\}$ and $y = c(M')$, then $x \notin M'$.

As noted above (and as well-known), a decisive choice function $c_d : \mathcal{M}^* \rightarrow X$ satisfies WARP if and only if it is rationalised by an asymmetric, total and transitive preference relation $\succ$, so that for all $M \in \mathcal{M}^*$, $c_d(M) = x$ if and only if $x \succ y$ for all $y \in M \setminus \{x\}$. WARP's intuitive appeal is arguably greater in the context of indecisive choice than it is in that of decisive choice, because, as already argued, it is reasonable to expect that a DM will make choices more consistently if she's not forced to choose.

Binary Inductive Decisiveness (BID)

If $M \in \mathcal{M}^*$, $x = c(M)$ and $y \in M \setminus \{x\}$, then $c(x, y) \neq \emptyset$.

Suppose that x is chosen over y in some menu. Since the consumer was allowed not to choose if she wanted to, one may infer from this choice that she was able to evaluate sufficiently well both x and y. Hence, given that the consumer has been "revealed informed" over both x and y, it is natural to expect that she will also be able to make a choice from the menu that consists of exactly x and y. BID generalizes the $\alpha 2$ property of Sen (1977) when the latter is formulated for choice functions, by not explicitly requiring x to be chosen from $\{x, y\}$ if it has previously been chosen from a set where y was also feasible. If c is decisive, then WARP implies $\alpha 2$. If c is indecisive, then WARP and BID together imply $\alpha 2$.

Weak Expansion (WEXP)

If $M, M' \in \mathcal{M}^*$ and $c(M) = x = c(M')$, then $c(M \cup M') \neq \emptyset$.

WEXP says that if the same alternative is chosen in two different menus, then, for reasons analogous to those above, some choice should be made from their union too. WEXP is a weak generalisation of the "Expansion" axiom due to Manzini and Mariotti (2007), which, in particular, requires that the same alternative also be chosen from the union of the two original menus, i.e. for M,

$M' \in \mathcal{M}^*$, $C(M) = x = C(M')$ implies $C(M \cup M') = x$. If c is decisive, then WARP implies Expansion. If c is indecisive, then WARP and WEXP together imply Expansion.

Ternary Inductive Decisiveness (TID)

If M, $M' \in \mathcal{M}^*$, $x = c(M)$, $y \in M \setminus \{x\}$, $y = c(M')$ and $z \in M' \setminus \{y\}$, then $c(x,y,z) \neq \emptyset$.

As with BID, if a consumer chooses x over y in one case, and chooses y over z in another case, then given that not choosing was also permissible in both situations, one may interpret these choices as suggestive of the consumer's familiarity with all of x, y and z. Once familiarity of the consumer with all three alternatives x, y and z has been acknowledged, assuming as in TID that some choice will be made from the menu consisting of exactly these alternatives seems plausible.

Proposition 2

An indecisive choice function $c : \mathcal{M}^* \to X$ satisfies WARP, BID and WEXP if and only if there exists an asymmetric relation $\succ$ on X such that for all $M \in \mathcal{M}^*$

$$c(B) = x \qquad \Longleftrightarrow \qquad x \succ y \text{ for all } y \in M \setminus \{x\}. \tag{2}$$

Furthermore, c also satisfies TID if and only if $\succ$ is, in addition, transitive.

Proof:

Let $c : \mathcal{M}^* \to X$ satisfy (2) with respect to an asymmetric $\succ$. Suppose there are $M, M' \in \mathcal{M}^*$ such that $x = c(M)$, $y \in M \setminus \{x\}$, $y = c(B')$ and $x \in B'$. Then $x \succ y$ and $y \succ x$ both hold, contradicting the asymmetry of $\succ$. Thus, WARP is satisfied. Let there $M \in \mathcal{M}^*$ be such that $c(M) = x$ and $y \in M \setminus \{x\}$. Since this implies $x \succ y$, it follows that $\emptyset \neq c(x,y) = x$ and hence that BID is also satisfied. Finally, let $M, M' \in \mathcal{M}^*$ have the property that $c(M) = x = c(M')$. It is true that $x \succ w$ for all $w \in M \setminus \{x\}$ and $x \succ z$ for all $z \in M' \setminus \{x\}$. Hence, $x \succ y$ for all $y \in M \cup M'$ such that $x \neq y$, and therefore $x = c(M \cup M') \neq \emptyset$ too. Thus, WEXP is satisfied. Suppose now that $\succ$ is also transitive and that there are M, $M' \in \mathcal{M}^*$ such that $x = c(M)$, $x \neq y \in M$, $y = c(M')$ and $y \neq z \in M'$. We have $x \succ y$, $y \succ z$ and, from transitivity, $x \succ z$. Hence, $x = c(x,y,z)$ and therefore TID is also satisfied.

Suppose now that c obeys WARP, BID and WEXP. Consider the revealed strict preference relation $\succ^*$. WARP ensures that $\succ^*$ is asymmetric. Let $c(M) = x$ for some $M \in \mathcal{M}^*$. By definition, $x \succ^* y$ for all $y \in M \setminus \{x\}$. Conversely, suppose there exists $M \in \mathcal{M}^*$ such that $x \succ^* y$ for all $y \in M \setminus \{x\}$. BID ensures that $c(x,y) \neq \emptyset$ for all such y and WARP ensures that $c(x,y) = x$ is actually true. From WEXP then follows that $c(M) \neq \emptyset$. If $c(M) \neq x$, then WARP is violated. Hence, c satisfies (2). Finally, if TID is also assumed, then $x \succ^* y$, $y \succ^* z$ and WARP together imply $x = c(x,y,z)$ and therefore $x \succ^* z$, so that $\succ^*$ is also transitive. ∎

Thus, the DM portrayed in Proposition 2 proceeds rather cautiously, compares all possible pairs of alternatives in a menu and chooses the one that is preferred to all others, if such an alternative exists. If not, then she chooses nothing. In particular, the general version of the rule does not require the consumer to order (totally or otherwise) each menu according to the preference relation. Thus, a choice can be made from a menu even if some alternatives cannot be compared and even if preference cycles are also involved. In the strict version that requires transitivity, of course, preference cycles are ruled out but the former property continues to apply.

Proposition 2 is a special case of Proposition 1, because if the option that is chosen from a set is preferred to all others in that set, then it is clearly preferred to at least one and dispreferred by none. Thus, WARP, BID and WEXP together imply ReWARP, DDC and WDC, and the first triple together with TID implies the second together with PCT. The tightness of the axiomatic system formed by WARP, BID, TID and WEXP is established in the Appendix. We finally note that in view of the second part of Proposition 2 these four axioms together imply the *Strong Axiom of Revealed Preference* (SARP): if $x_1 = c(M_1)$, $x_2 \in M_1 \setminus \{x_1\}$, $x_2 = c(M_2)$, $x_3 \in M_2 \setminus \{x_2\}, \ldots, x_k = c(M_k)$, then $x_1 \notin M_k$.

The next example illustrates Proposition 2 in its general version.

Example 2

Suppose $X = \{a,b,c,d,e\}$ and $\mathcal{M}^* = \{M \subseteq X : |M| > 1\}$. Assume further that the DM has the following preferences on X:

$$\begin{aligned}
&b \succ a, \;\; b \succ c, \;\; b \succ d, \\
&a \succ c, \;\; c \succ d, \;\; d \succ a, \\
&a \,\widehat{\approx}\, e, \;\; b \,\widehat{\approx}\, e, \;\; c \,\widehat{\approx}\, e, \;\; d \,\widehat{\approx}\, e.
\end{aligned}$$

Following WARP, BID and WEXP (but since $\succ$ is intransitive, clearly not TID), she will be observed making the following choices:

$$C(a,b,c,d) = C(a,b,d) = C(b,c,d) = C(a,b) = C(b,c) = C(b,d) = b,$$
$$C(a,c) = a, \qquad C(c,d) = c, \qquad C(d,a) = d,$$
$$C(M) = \emptyset \qquad \text{for all other } M \in \mathcal{M}^*.$$

Thus, she chooses b whenever b is present in a menu and e is not. More specifically, b is strictly preferred to all other alternatives in X except e, which, in particular, is incomparable by strict preference to all other elements of X. Thus, since the choice rule of Proposition 2 is dominance-based and e is dominated by no alternative in X, every effort of choice in its presence is doomed. Finally, one notes that no choice is made from $\{a,c,d\}$ either because the preference cycle in this set ensures that no dominant alternative exists. Thus, preference cycles constitute one additional source of behavioural indecision.[3] ♦

3 Indecisive Choice and Weak Preferences

The choice rule studied in this section is based on weak preferences and therefore allows for indifference and psychological indecision to co-exist and be distinguished. Given the consumption set X, the domain of choice here is a family of subsets of X, denoted $\mathcal{M}$, that includes all sets up to three elements. The DM's weak preference relation on X will be denoted $\succsim$, with $\succ$ being the asymmetric and $\sim$ the symmetric part of $\succsim$, capturing strict preference and indifference, respectively. The relation $\succsim$ is *complete* if $x \succsim y$ or $y \succsim x$ holds for all $x,y \in X$, and *incomplete* otherwise. Whenever $x \not\succsim y$ and $y \not\succsim x$ is true for some $x,y \in X$ we write $x \curlywedge y$ and interpret $\curlywedge$ as the relation that captures *psychological indecision* in this context. Finally, given a choice correspondence $C : \mathcal{M} \twoheadrightarrow X$, the consumer's *revealed weak preference relation* $\succsim^*$ is defined by $x \succsim^* y$ if there exists $M \in \mathcal{M}$ such that $x \in C(M)$ and $y \in M$. We consider now the following axioms for C.

Weak Axiom of Revealed Preference (WARP)

If $M, M' \in \mathcal{M}$, $x \in C(M)$ and $y \in M \setminus C(M)$, then $y \in C(M')$ implies $x \notin M'$.

Arrow (1959) proved that WARP is satisfied by a *decisive* choice correspondence on a domain that includes all subsets of X up to three elements if and only if it

[3]This point is also made in Clark (1995).

is *rationalised* by a complete and transitive weak preference relation, so that the chosen alternatives in every menu coincide with the maximum elements of the relation in that menu (see Chapter 1 in Mas-Colell, Whinston, and Green (1995)). Hence, decisiveness of the choice correspondence and WARP are jointly equivalent to utility maximising behaviour in this context. WARP is undoubtedly the reference point among the various criteria of choice consistency that have been proposed. It has already been argued above that it is even more reasonable and normatively appealing to expect that choice consistency principles in general (and therefore also WARP) will be satisfied when abstaining is also an option.

Ternary Inductive Decisiveness (TID)

If $M, M' \in \mathcal{M}$, $x \in C(M)$, $y \in M$, $y \in C(M')$ and $z \in M'$, then $C(x, y, z) \neq \emptyset$.

TID is the axiom introduced in Section 2.3, rewritten here in a way that makes it applicable to choice correspondences. Notice, however, that a choice has to be made from $\{x, y, z\}$ even if x is not chosen *over* y and y *over* z, but instead *in the presence* of y and z, respectively. The justification of the axiom in the current setup is analogous to the one in Section 2.3.

Choice from Singletons (CS)

If $x \in X$, then $C(x) = x$.

CS requires that whenever a DM faces the problem of choosing between a single alternative, whatever that is, and not choosing anything at all, she does the former. Clearly, it rules out the possibility that the DM might want to abstain from choosing if the only feasible alternative is one that happens to be unfamiliar to her, and as such it is a restrictive axiom in this setting.

Consistent Decisiveness (CD)

If $M, M' \in \mathcal{M}$ and $M \supset M'$, then $C(M) \neq \emptyset$ implies $C(M') \neq \emptyset$.

It may be argued that it is normatively sound to expect from an individual who is capable of making a choice from one menu to also be able to choose from every submenu of that menu. After all, the argument goes, making a choice in the former situation requires more effort and presupposes that a certain level of familiarity with all available options exists. Since this familiarity is necessarily carried over the smaller menu, it seems plausible at first sight that a choice should be made there too. This is the restriction that CD imposes. Clearly, the discussion following Proposition 1 demonstrates that there are cases for which there seems to be no reason that this axiom should be satisfied.

Proposition 3

An indecisive choice correspondence $C : \mathcal{M} \twoheadrightarrow X$ satisfies WARP, TID and CS if and only if there is a reflexive and transitive relation $\succsim$ on X such that, for all $M \in \mathcal{M}$

$$x \in C(M) \iff x \succsim y \text{ for all } y \in M. \tag{3}$$

Furthermore, C also satisfies CD if and only if

$$C(M) \neq \emptyset \iff x \succsim y \text{ or } y \succsim x \text{ for all } x, y \in M. \tag{4}$$

Proof:

For the first part, let C be consistent with (3). Since $\succsim$ is reflexive, CS is trivially satisfied. Consider $M, M' \in \mathcal{M}$ with $x \in C(M)$, $y \in M \setminus C(M)$, $y \in C(M')$ and $x \in M'$. It holds that $x \succ y$ and $y \succsim x$, a contradiction. Hence, WARP is also satisfied. Finally, let $M, M' \in \mathcal{M}$ be such that $x \in C(M)$ $y \in M$, $y \in C(M')$ and $z \in M'$. It is true by assumption that $x \succsim y$ and $y \succsim z$. Hence, $x \succsim z$ by transitivity of $\succsim$, and therefore $x \in C(x, y, z) \neq \emptyset$, which proves that TID is satisfied too.

Conversely, let C satisfy WARP, TID and CS, and consider the revealed weak preference relation $\succsim^*$ on X defined above. CS ensures that $\succsim^*$ is reflexive. Suppose $x \succsim^* y$ and $y \succsim^* z$. TID implies $C(x, y, z) \neq \emptyset$. If $z \in C(x, y, z)$, then since $y \succsim^* z$, WARP implies that $y \in C(x, y, z)$ too. But then WARP and $x \succsim^* y$ together imply $x \succsim^* z$. Hence, $\succsim^*$ is transitive. By definition of $\succsim^*$, if $x \in C(M)$ for some $M \in \mathcal{M}$, then $x \succsim^* y$ for all $y \in M$, and therefore (3) is satisfied.

For the second part, let C obey (4) too. Define

$$\mathcal{F} := \{M \in \mathcal{M} : \ x \succsim y \text{ or } y \succsim x \text{ for all } x, y \in B\},$$

$$\mathcal{F}_k := \{M \in \mathcal{F} : |M| = k\},$$

$$\mathcal{F}^* := \operatorname{argmax}\{|M| : M \in \mathcal{F}\}.$$

Suppose that $M \in \mathcal{F}^*$ implies $|M| = n$. It holds that $\mathcal{F}_k$ is well-defined for $k = 1, \ldots, n$ and, from (4), that $C(M) \neq \emptyset$ if and only if $M \in F_k$ for $k \leq n$. Thus, if $M \in \mathcal{F}_k$ and $M' \subset M$, then $M' \in \mathcal{F}_{k'}$ with $k' < k$, which proves that CD is satisfied.

Now assume that C is consistent with WARP, TID, CS and CD. Again, it is known from above that $\succsim^*$ satisfies (3). Assume that there exists $M \in \mathcal{M}$ such that $C(M) \neq \emptyset$ and $x \not\succsim^* y$, $y \not\succsim^* x$ for some $x, y \in M$. It holds by definition that $x, y \notin C(M)$ and $C(x,y) = \emptyset$. But since $\{x,y\} \subset M$ and $C(M) \neq \emptyset$, this contradicts CD. Conversely, if there is $M \in \mathcal{M}$ such that $w \succsim^* z$ or $z \succsim^* w$ for all $w, z \in M$, then clearly, since $\succsim^*$ is a preorder, there exists $x \in M$ such that $x \succsim^* y$ for all $y \in M$ so that $C(M) \neq \emptyset$ from (3). This establishes (4). ■

Thus, a DM following (3) makes choices that satisfy WARP, TID and CS and have the property that some alternative is selected from a menu if and only if this alternative is preferred or indifferent to all others according to a weak preference relation that is captured by an incomplete preorder. Again, if there are many such alternatives then one of them will be chosen by some unmodeled procedure, whereas no choice is made if none meets this criterion. Hence, this rule provides the weak-preference analogue of the strong version (which requires transitivity of $\succ$) of the choice rule that is captured in Proposition 2.

Furthermore, if she proceeds according to (4) as well, then her choices also satisfy CD and the sharp prediction of the rule is that a choice will actually be made only from those sets that she's able to completely preorder. In other words, even if there is one alternative that is weakly preferred to everything else in a menu, the existence of at least two incomparable options generate behavioural indecision. The following property is also possessed by C in this case:

Observation 4

If (4) holds, then $C(M) = \emptyset$ and $M' \supset M$ together imply $C(M') = \emptyset$.

Thus, under WARP, TID and CS an indecisive choice correspondence C satisfies Consistent Decisiveness if and only if it also satisfies *Consistent Indecisiveness* (CI) in the above sense. Again, the tightness of WARP, TID, CS and CD is proved in the Appendix.

Below is an example that illustrates the choice rule in its general form when only (3) is necessarily satisfied.

Example 3

Suppose $X = \{v, w, x, y, z\}$, $\mathcal{M} = \{M : M \subseteq X\}$ and the DM's preferences are such that

$$v \succ w, \;\; v \succ x, \;\; v \succ y,$$
$$v \succ z, \;\; w \succ x, \;\; w \succ y,$$
$$x \sim y, \;\; w \curlywedge z, \;\; x \curlywedge z, \;\; y \curlywedge z.$$

Proceeding according to (3), the DM will be observed making the following choices:

$$\begin{aligned}
&C(M) = v \;\text{ for all } M \in \mathcal{M} \text{ such that } v \in M,\\
&C(w) = C(w,x) = C(w,y) = C(w,x,y) = w,\\
&C(x) = x, \qquad C(y) = y, \qquad C(z) = z,\\
&C(x,y) = \{x,y\},\\
&C(M') = \varnothing \;\text{ for all other } M' \in \mathcal{M}.
\end{aligned}$$

Interpret the elements of X as money lotteries with the following properties: (i) v first-order stochastically dominates (FOSD) all the other lotteries in X, (ii) w FOSD both x and y, (iii) there is no FOSD relation between x and y but the prize/probability tradeoffs are insignificant, (iv) there is no FOSD relation between z and each of w, x and y but in all three cases the prize-probability tradeoffs are significant, making her psychologically indecisive (Tversky and Shafir (1992) report relevant experimental results). It is therefore "rational" that the DM chooses v from X even though she can't compare z with w, x and y in that set. Furthermore, despite choosing w from $\{w, x, y\}$ and from each subset of that set, adding z to $\{w, x, y\}$ again leads to no choice due to the plethora of decision conflicts generated with its insertion. ♦

A trivial implication of property (3) of Proposition 3 is that in a situation where the DM has to choose between x and y and her mental attitudes toward these alternatives dictate $x \curlywedge y$, she will choose nothing (i.e. $C(x, y) = \varnothing$) despite the fact that each one of x and y would have been chosen had they been the only feasible alternatives (i.e. $C(x) = x$ and $C(y) = y$). It may be the case that this will seem paradoxical at first sight, but this simple prediction of the model might be suitable in modeling the following situation reported by Tversky and Shafir (1992): *"A case in point was described to us by Thomas Schelling, who some time ago had decided to buy an encyclopedia for his children. To his chagrin, he discovered that two encyclopedias were available in the bookstore. Although either one would have been satisfactory, he found it difficult to choose between the two, and as a result bought neither."* Schelling's story was coupled by experimental results conducted by Tversky and Shafir that involved CD players, first considered individually and then in a setting that generated decision conflict, which pointed to the same conclusion.

Iyengar and Lepper (2000), in carefully designed field experiments, found evidence that decision makers are more likely to choose an alternative from a menu when this menu is "small" and contains relatively few alternatives than when it is "large" and contains many more. The intuitive explanation they offered for this finding is that the choice task is more complex in the large menu than in the small one, and the magnitude of complexity is apparently an important determinant of (in)decisiveness in choice. Although this connection was not made in that paper, it may be plausible to hypothesise that one source of the observed *behavioural* indecision in these situations is the occurrence of *psychological* indecision between alternatives in the large menu, which becomes more likely as more options are added to a set where no such indecision originally existed.

More specifically, assuming again a representative subject with a choice correspondence C, the findings of Iyengar and Lepper suggest that $C(M) \neq \emptyset$ for some "small" M and $C(M') = \emptyset$ for some "large" $M' \supset M$. When the individual's choices satisfy WARP, TID, CS and CD, then both (3) and (4) apply, in which case it is understood that the individual is able to completely preorder menu M with $\succsim$ but she is unable to do so for menu M'. So there necessarily exist $x, y \in M'$ such that $x \curlywedge y$, and either x or y (or both) is feasible in M' but not in M. Thus, the interpretation of complexity proposed by this model is based on psychological indecision as captured by the incompleteness of $\succsim$. Finally, support to this indecision-based interpretation of complexity comes from Observation 4 and CI, which ensures that the desirable property of complexity respecting monotonic increases in the size of the menu is also attained.

The final example is one that highlights the properties of the choice rule in its strong version where both (3) and 4) are satisfied.

Example 4

Let $X = \{a, b, c, d, e, f\}$, $\mathcal{M} = \{M : M \subseteq X\}$ and consider the following preferences for the DM:

$$\begin{gathered} a \succ b,\ \ b \sim c,\ \ a \succ c,\ \ c \succ d,\ \ b \succ d, \\ a \succ d,\ \ e \succ f,\ \ a \curlywedge e,\ \ a \curlywedge f,\ \ b \curlywedge e, \\ b \curlywedge f,\ \ c \curlywedge e,\ \ c \curlywedge f,\ \ d \curlywedge e,\ \ d \curlywedge f. \end{gathered}$$

If the DM conforms with WARP, TID, CS and CD, these preferences lead to the following choices:

$$\begin{aligned}
&C(a) = C(a,b) = C(a,c) = C(a,d) = C(a,b,c) = C(a,b,d) = \\
&= C(a,c,d) = C(a,b,c,d) = a, \\
&C(b) = C(b,d) = b, \\
&C(c) = C(c,d) = c, \\
&C(d) = d, \\
&C(e) = C(e,f) = e, \\
&C(f) = f, \\
&C(b,c) = C(b,c,d) = \{b,c\}, \\
&C(M) = \varnothing \qquad \text{for all other } M \in \mathcal{M}.
\end{aligned}$$

So this DM is able to choose from $\{a,b,c,d\}$, $\{e,f\}$ and each one of their submenus because she can completely preorder all of these with $\succsim$, which is nevertheless an *incomplete* preorder in X. Adding an element of the first menu to the second and vice versa leads to indecision-based complexity and ultimately to no choice. ♦

4 Conclusion

This paper proposed and axiomatised two preference- and reason-based choice rules, together with some of their special cases, that allow the decision maker to choose nothing in the event that none among the feasible alternatives satisfies the selection criterion associated with these rules. This theory can accommodate experimental findings that cannot be explained by models assuming decisiveness of choice. It also formalises in very specific ways the idea that decisiveness is not a necessary condition for rationality. In particular, both rules rely either on WARP itself or on its intuitive generalisation, ReWARP, along with some other behavioural axioms that are generally weak and intuitive. As far as future work is concerned, a dynamic theory of individual choice as well as social choice when agents are rationally indecisive are likely to be two interesting problems.

Appendix

Claim 1
For an indecisive choice correspondence $C : \mathcal{M}^* \twoheadrightarrow X$ each of ReWARP, DDC, WDC and PCT is independent of the other three.

Proof:

Suppose $X = \{w, x, y, z\}$ and $\mathcal{M}^* = \{M \subseteq X : |M| > 1\}$.

C_1 below satisfies ReWARP, DDC and WDC but violates PCT.

$$\begin{aligned}
&C_1(x,y) = x, \qquad C_1(y,z) = y, \qquad C_1(x,z) = \emptyset,\\
&C_1(x,y,z) = C_1(w,x,y) = C(X) = x,\\
&C_1(w,y,z) = y,\\
&C(M) = \emptyset \text{ for all other } M \in \mathcal{M}^*.
\end{aligned}$$

C_2 below satisfies ReWARP, DDC and PCT but violates WDC.

$$\begin{aligned}
&C_2(w,x) = C_2(w,y) = w, \qquad C_2(w,x,y) = \emptyset,\\
&C_2(w,x,z) = w,\\
&C_2(M) = \emptyset \text{ for all other } M \in \mathcal{M}^*,
\end{aligned}$$

C_3 below satisfies ReWARP, WDC and PCT but violates DDC.

$$\begin{aligned}
&C_3(w,x) = C_3(w,y) = \emptyset, \qquad C_3(w,x,y) = w,\\
&C_3(x,y) = x, \qquad C_3(y,z) = y, \qquad C_3(x,z) = x,\\
&C_3(x,y,z) = x,\\
&C_3(M) = \emptyset \text{ for all other } M \in \mathcal{M}.
\end{aligned}$$

C_4 below satisfies DDC, WDC and PCT but violates ReWARP.

$$\begin{aligned}
&C_4(w,x) = w, \qquad C_4(x,y) = x, \qquad C_4(w,y) = w,\\
&C_4(w,z) = C_4(x,z) = C_4(y,z) = \emptyset,\\
&C_4(w,x,y) = \{w,x\},\\
&C_4(w,x,z) = C_4(w,y,z) = w,\\
&C_4(x,y,z) = \{x,y\},\\
&C_4(X) = \{w,x,y\}.
\end{aligned}$$

■

Claim 2

For an indecisive choice function $c : \mathcal{M}^* \to X$ each of WARP, BID, TID and WEXP is independent of the other three.

Proof:

Suppose X and $\mathcal{M}^*$ are as in the proof of the previous claim.

c_1 below satisfies BID, TID and WEXP but violates WARP.

$$\begin{aligned}
&c_1(x,y) = c_1(w,y) = y, \qquad c_1(w,x,y) = x,\\
&c_1(w,x) = c_1(w,z) = c_1(w,x,z) = w,\\
&c_1(x,z) = x, \qquad c_1(y,z) = z, \qquad c_1(x,y,z) = x,\\
&c_1(w,y,z) = w, \qquad c_1(X) = \emptyset.
\end{aligned}$$

c_2 below satisfies BID, TID and WARP but violates WEXP.

$$\begin{aligned}
&c_2(x,y) = c_2(x,z) = x, \qquad c_2(x,y,z) = \emptyset,\\
&c_2(x,w) = c_2(w,z) = c_2(x,w,z) = w,\\
&c_2(w,y) = y, \qquad c_2(y,z) = c_2(w,y,z) = z,\\
&c_2(w,x,y) = c(X) = \emptyset.
\end{aligned}$$

c_3 below satisfies BID, WARP and WEXP but violates TID.

$$\begin{aligned}
&c_3(w,y) = y, \qquad c_3(y,z) = z, \qquad c_3(w,y,z) = \emptyset,\\
&c_3(x,y) = c_3(x,z) = c_3(x,y,z) = x,\\
&c_3(w,x) = c_3(w,z) = c_3(w,x,z) = w,\\
&c_3(w,x,y) = c_3(X) = \emptyset.
\end{aligned}$$

c_4 below satisfies TID, WARP and WEXP but violates BID.

$$\begin{aligned}
&c_4(w,x) = c_4(w,z) = \emptyset, \qquad c_4(w,x,z) = w,\\
&c_4(w,y) = y, \qquad c_4(y,z) = c_4(w,y,z) = z,\\
&c_4(w,x) = c_4(x,z) = c_4(x,y,z) = x,\\
&c_4(w,x,y) = c_4(X) = \emptyset.
\end{aligned}$$

■

Claim 3

For an indecisive choice correspondence $C : \mathcal{M} \twoheadrightarrow X$ each of WARP, ID, CS and CD is independent of the other three.

Proof:

Let $X = \{w,x,y,z\}$ and $\mathcal{M} = \{M : M \subseteq X\}$.

C_1 below satisfies WARP, CS and ID but violates CD.

$$\begin{aligned}
&C_1(M) = w, \text{ if } w \in M,\\
&C_1(x) = x, \qquad C_1(y) = y, \qquad C_1(z) = z,\\
&C_1(x,y) = C_1(y,z) = C_1(x,z) = C_1(x,y,z) = \emptyset.
\end{aligned}$$

C_2 below satisfies WARP, CD and CS but violates ID.

$$\begin{aligned}
&C_2(w) = w, \quad C_2(x) = x, \quad C_2(y) = y, \quad C_2(z) = z,\\
&C_2(w,x) = C_2(w,y) = C_2(w,z) = C_2(w,x,z) = C_2(w,y,z) = w,\\
&C_2(x,y) = x, \quad C_2(y,z) = y, \quad C_2(x,z) = z,\\
&C_2(w,x,y) = C_2(x,y,z) = C(X) = \emptyset.
\end{aligned}$$

C_3 below satisfies WARP, CD and ID but violates CS.

$$\begin{aligned}
&C_3(x) = x, \quad C_3(y) = y, \quad C_3(z) = z,\\
&C_3(x,y) = C_3(x,z) = C_3(x,y,z) = x,\\
&C_3(y,z) = y,\\
&C(B) = \emptyset \text{ for all other } B \in \mathcal{M}.
\end{aligned}$$

C_4 below satisfies CD, CS and ID but violates WARP.

$$\begin{aligned}
&C_4(w) = w, \quad C_4(x) = x, \quad C_4(y) = y, \quad C_4(z) = z,\\
&C_4(w,x) = w, \quad C_4(x,y) = x, \quad C_4(w,y) = y,\\
&C_4(w,x,y) = x,\\
&C_4(M) = \emptyset \text{ for all other } M \in \mathcal{M}.
\end{aligned}$$

■

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