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Model Based Monte Carlo Pricing of Energy and Temperature Quanto Options*

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Abstract

Weather derivatives have become very popular tools in weather risk management in recent years. One of the elements supporting their diffusion has been the increase in volatility observed on many energy markets. Among the several available contracts, Quanto options are now becoming very popular for a simple reason: they take into account the strong correlation between energy consumption and certain weather conditions, so enabling price and weather risk to be controlled at the same time. These products are more efficient and, in many cases, significantly cheaper than simpler plain vanilla options. Unfortunately, the specific features of energy and weather time series do not enable the use of analytical formulae based on the Black-Scholes pricing approach, nor other more advanced continuous time methods that extend the Black-Scholes approach, unless under strong and unrealistic assumptions. In this study, we propose a Monte Carlo pricing framework based on a bivariate time series model. Our approach takes into account the average and variance interdependence between temperature and energy price series. Furthermore, our approach includes other relevant empirical features, such as periodic patterns in average, variance, and correlations. The model structure enables a more appropriate pricing of Quanto options compared to traditional methods.

Keywords: weather derivatives, Quanto options pricing, derivative pricing, model simulation and forecast.

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1. Introduction

In many economic sectors, weather conditions may significantly affect the demand for goods or services, or influence regular working paths. Accordingly, weather risk has a strong impact on sales or production levels (meaning that they are correlated to the weather) and significant impact on financial results. Furthermore, given that the weather risk affects the volume of sales or production, it is often called a volumetric risk.

The development of financial engineering led to the diffusion of weather derivatives as a tool for transferring weather risks off the balance sheet. However, the first generation contracts only dealt with weather risk and just covered exposure to temperature changes. However, most energy companies are characterised by a strong weather exposure and cannot effectively hedge weather risk with just simple weather derivatives. In fact, there is a low, but significant, correlation between outdoor temperature and energy price. Hence, the real weather exposure in many energy companies is non-linear and the payout from classic weather derivatives does not enable a complete coverage of revenue shifts caused by weather conditions and their impact on energy prices. Weather therefore has a direct impact on energy companies through revenues (as weather affects energy demand); as well as an indirect impact (by affecting energy prices). As a result, to hedge weather exposure more appropriately, non-linear contracts that take into account both energy price and weather conditions should be used. Quanto options are an example of the type of contract that enables an improvement in the weather risk management process.

Quanto options (abbreviation of 'quantity adjusting options') originally appeared in currency-related markets, where the price of a financial instrument quoted in a given currency is converted to another currency at a fixed rate (see Zhang, 2001, for additional details). Within the energy market, Quanto options take into account the volumetric impact of weather conditions on energy price. For instance, when the winter is colder than expected in a north European country, the energy market suffers an increase in demand and an increase in the energy price. In this case, energy producers should hedge the volume risk but also take into account the benefits of price increases. In addition, Ho et al. (1995) show that hedging with Quanto contracts is much cheaper and more efficient than through simple combinations of two separate plain vanilla options written on energy prices and temperatures, respectively.

Unfortunately, several elements affect the correct pricing of Quanto options. Initially, we might consider several payoff designs in the knowledge that for many of these designs the closed-form pricing formula derived within a Black-Scholes framework is unavailable. A relevant computational effort is required for the pricing of such contracts. Secondly, the peculiar features of energy price and temperature time series (jumps, long-memory, periodic patterns in mean and variance, non-Gaussian distributions) raise some doubts regarding the appropriateness of a simple geometric Brownian motion as a reference model. To overcome these limitations, contract prices are determined by brokers using a variety of approaches, and so a huge dispersion of prices is observed in OTC markets.

The aim of this paper is to present a pricing methodology for Quanto options that is based on Monte Carlo simulations from an econometric model for the underlying time series. The

approach we propose (in two variants) enables pricing a range of instruments (including Quanto options) based on the underlying modelled variables, and can be used to hedge the non-linear price and volume exposures that are typical of the energy sector.
In this study we motivate the need for Quanto options in the energy sector, and we provide a pricing approach for these contracts. Our method has several advantages: it includes the stylised facts characterising energy log-prices and temperature data (namely, periodic patterns, long-memory, heteroskedasticity, and correlation dynamics); the method improves on univariate approaches since it includes interdependence of energy from temperature; and it allows correlations to depend on a periodic function. The novel aspects of our contribution depend thus on the model, on the empirical evidence of periodic structure in energy and temperature correlation, and on the proposal of a Financial approach for pricing Quanto options.
The paper proceeds as follows. Section 2 describes energy markets, their correlation with weather, and the peculiar features of energy prices and temperature time series. Furthermore, Section 2 contains a motivating example on the use and specification of contingent derivatives in practice. Section 3 touches on the problem of pricing weather contracts from a general viewpoint. In addition, the section highlights the advantages and disadvantages of a number of approaches. Section 4 presents the model specification, and discusses the estimation and simulation issues. Section 5 contains the empirical results: model estimation and pricing of a Quanto option based on Oslo energy price and temperature data. Section 6 contains a summary and conclusions.

2. Quanto options for energy and weather markets

To provide a rationale for the introduction of Quanto options based on weather data and energy prices, we start with a brief introduction on energy and weather derivative markets. Later, we focus on economic motivations for the use of Quanto options, and we provide a short introduction on possible designs for Quanto payoffs.

2.1 Energy markets

The deregulation of electricity markets started in the early 1990s in the US and some European countries. One of the most important electricity markets that led the way in liberalisation was the Nordic Power Exchange (known as Nord Pool) which included Sweden, Norway, Finland and Denmark. Nord Pool was established in January 1993 in Norway and was progressively expanded to include the other Nordic nations. Nord Pool organises the Elspot a day-ahead electricity market ('spot physical market') as well as an electricity derivatives market ('financial market'). In the spot market, 24 hourly power contracts are traded for physical delivery in one specific hour during the next day. A price per Megawatt Hour (MWh), named the system price, is determined separately for each hour for the whole market area through a uniform-price auction, without considering capacity limits in the transmission lines. A wide range of derivative contracts are traded at Nord Pool, including a variety of forwards, futures, and options. All forward and futures contracts refer to a base load of one Megawatt (MW) during every hour for a given ‘delivery period’ (ranging from one day to one year in length), and all contracts are settled in cash daily against the system price during the ‘delivery period’. Nord Pool, now also termed Nord Pool ASA, is considered one of the most liquid wholesale markets in the world.

There are several bidding areas for which the transmission system operator defines the capacity allocated for Elspot. When the flow of power between bidding areas exceeds the allocated capacity, the areas may have different prices. If power flows are within the defined limits, the energy price becomes common across the areas.

In Norway, there are six different spot price areas: Bergen, Kristiansand, Kristiansund, Oslo, Tromso, and Trondheim. In this study, we make use of data from one specific bidding area, that of Oslo, for which we consider both the area daily energy price, and the Oslo daily average temperature. As the econometric model used in this paper will allow for the interaction between energy prices and temperature, the fact that both variables are geographically located is very attractive and unique in the doctrine.

One additional remarkable feature of the Nordic Electricity Market for financial asset valuation purposes, is the existence of wide-ranging futures/forward contract maturities (daily, weekly, monthly, quarterly, and yearly). This extensive variety of contracts means the forward curve can be obtained for the whole market by using the system price (the main reference of the market) as the underlying asset. Further to this, each bidding area has its own forward contracts (‘Contracts for Differences’) enabling the estimation of specific forward curves for each area. For more details on Nord Pool ASA we refer interested readers to www.nordpool.com.

2.2 Weather derivative markets

The first contracts linked to weather data appeared in late 90s in the US as an effect of energy market liberalisation. At that time, several energy companies realised that outdoor temperature is one of the key factors responsible for profit and loss in the energy sector.

In general terms, weather derivatives represent a wide class of financial contracts (traded on exchanges or over-the-counter) whose settlement directly depends on weather variables (mainly quantitative such as temperature, wind speed, or precipitation) at a given meteorological station. The Chicago Mercantile Exchange (CME) hosts the only exchange market actually trading weather contracts. The CME quotes futures and options on these futures for 41 locations around the world. The most traded contracts depend on temperature indices. The two most popular indices are the Daily HDD (Heating Degree Day) and the Daily CDD (Cooling Degree Day) which capture, respectively, the winter and summer exposure to temperatures. These indices are defined as:

$$Daily\ HDD = max\ (65°F – average\ daily\ temperature,^{1}\ 0) \quad (1)$$

$$Daily\ CDD = max\ (average\ daily\ temperature – 65°F,\ 0) \quad (2)$$

Cumulated CDD (between May-October) and cumulated HDD (between October-April) are the reference quantities for contracts covering 18 US and 6 Canadian cities. The CME uses similar indices for 9 European locations, with two small differences: the threshold temperature level is set to 18 Celsius degrees, and the Cumulative Average Temperature (CAT[2]) replaces the CDD because of the cooler summer temperatures in Europe compared to those in the US. To evaluate contracts, each index point (tick) has a theoretical value of 20 USD for US and Canadian location-based contracts and 20 GBP for European-based contracts. The CME also quotes a Frost Day Index based contract for Amsterdam; temperature-based contracts for Japanese locations (these are based on different temperature indices); and snowfall-based contracts (depending on snowfall indices within a day at a specific location).

Despite recent increase in the volume of weather contracts traded at the CME, the contracts still appear to be illiquid, especially for European locations. There are several elements driving such evidence, with one element playing a key role: sellers customise contracts to end-user needs, and therefore the standardisation (even with respect to the currency of the payout) is perceived as a negative element that creates additional risks for end-users. As a result, most contracts are exchanged over-the-counter. The writers of these contracts (mostly banks, insurance companies, and other financial institutions) sell weather protection to their clients, and often define contracts payouts using several weather indices at the same time (to hedge weather risk at several locations, or at a location not covered by a meteorological station). Furthermore, it is quite common to have contracts based on products or other non-linear functions of many indices.

2.3 An economic motivation for Quanto options

To show how Quanto options can be used for hedging we present a simple case of a power operator, a company that is responsible for providing energy (via the grid) to a set of firms and households. Firstly, we need to make some assumptions about the activity of the retail operator. We assume that the retail operator, using historical data, estimates that in the next heating season (from November to March) the HDD index for a given area will reach the value of 2500 points with a standard deviation of 300 points. This represents its expectation about the future HDD. Using such a forecast, the retail operator defines the amount of energy that it should buy through long-term contracts with energy producers.

The power retail operator also knows that changes in average daily temperature during the heating season have significant impacts on power demand. The retailer estimates that the impact is equal to 100 MWh per HDD point. Therefore, if the heating season is colder

[1] The average temperature is equal to the arithmetic mean of the observed maximum and minimum temperatures during one full day at a given meteorological station.

[2] The daily CAT is simply the daily average temperature.

(warmer) than expected, the retail operator will have to buy (sell) energy on the spot market, being thus exposed to energy price changes. Let's assume that at the beginning of the heating season, the expected average spot market price for the entire heating season is €45/MWh with an estimated standard deviation of €8/MWh. Furthermore, retail prices are sticky and must be maintained over the heating season. Therefore, the retail operator faces a risk associated with temperature variations, as well as with energy price changes. Table 1 presents the changes in the total cost suffered by the retail operator caused by deviations in temperature and associated with a range of possible values of the spot market energy price (we used a range equal to ± two standard deviations). Note the total cost increases if the retail operator needs to buy additional power in the spot market, or decreases if it sells excess power in the spot market.

In Table 2 we report the total revenues from customers under the assumption that the retail price is fixed at €49.5/MWh (assume this price already includes a margin 10% for the retail operator). Note that revenues increase with increases in the HDD index, since the retail operator sells larger amounts of energy to retail customers. Table 3 reports the changes in the margin obtained by the retail operator (revenues of Table 2 minus costs in Table 1). We note that, under the assumption of constant retail prices, the impact of temperature is offset under some price states. However, in order to secure profit for the retail operator, these changes should be hedged with a proper contract, possibly eliminating the risk implicit in Table 3. Quanto options could be used for that objective.

INSERT HERE TABLES 1 TO 3

2.3 Quanto options description

Quanto options belong to the wide class of correlation exotic options and are very popular on OTC and exchange markets, see Zhang (2001) for a survey. In general, a Quanto option (also called *product option or flexo option*) is a derivative contract, where the payoff depends on the product of two indices. As an example, consider a European based investor (with a Euro denominated wealth) with a position on a USD-denominated ETF tracking the S&P500 index. To offset the risk of a negative return (which is the combination of the ETF and the foreign exchange rate returns), the investor may follow alternative approaches. The first possibility is static hedging with two separate vanilla options (on the S&P500 and the Euro-Dollar rates). However, this coverage is not efficient due to the non-linear exposure of the uro-denominated wealth of the investor to the FX and S&P500 risks. Furthermore, this strategy requires the payment of two possibly expensive premiums. A second choice is the use of dynamic hedging. This strategy suffers because of the same limitations as the static strategy, namely, a loss in efficiency and a high cost. Quanto options, as the third solution, are the most appropriate choice, see Ho et al. (1995). A derivative based on the products of the S&P500 index and of the Euro-Dollar exchange rate is cheaper and more efficient.

If we focus on energy markets, we noted in the previous section that the revenues of power retail operators (and thus their risk exposure) are a non-linear function (a product) of two elements: energy price and temperature. Such a feature motivates the use of Quanto options for static hedging.

Let's continue with the example given in previous section, and recall that Table 3 reports the overall risk exposure of the retail operator to energy price and temperature changes. We can ideally divide Table 3 into four sections (the four corners), each representing a scenario with different deviations from the Temperature HDD expected values and the energy spot market price. Each section has only positive or negative values. To receive from a derivative a positive payoff associated with energy and temperature values higher than expected (lower-right corner) we can use the following double call option:

$$max(0, E-K_1)\times\tau\times max(0, HDD-K_2) \tag{4}$$

where K_1 and K_2 denote the strike values for energy and weather, respectively, $E = \frac{1}{m}\sum_{i=1}^{m} E_i$, E_i is the average daily energy price, m is the number of days in the heating season, $HDD = \sum_{i=1}^{m} \max(0, 18 - T_i)$, T_i is the average daily temperature level, and τ is the tick value, the change in energy demand per unit change in the HDD index. In our example, we fix the tick value at 100 MWh per point of HDD. Hence the payout of let's say 5 HDD points corresponds to a money transfer equivalent to the price of 500 MWh.

The opposite scenario (upper-left corner) corresponds to the following double put expression:

$$max(0, K_1-E)\times\tau\times max(0, K_2-HDD) \tag{5}$$

Clearly, appropriate formulae also exist for mixed scenarios (when one index is below the expected value and the other index above the expected value). For the joint offset of negative corners of Table 3, we could use the following compositions of (4) and (5) that identify Quanto options:

$$max((E-K_1)\times\tau\times (HDD-K_2),0) \tag{6}$$

$$max((K_1-E)\times\tau\times (K_2-HDD),0) \tag{7}$$

Note that (6) and (7) represent the same Quanto option, ensuring protection with respect to the negative states included in Table 3. The Quanto option depends on the product between two quantities (energy and temperature), each in deviation from a proper strike price, and with the introduction of a tick-value that translates the underlying option into a monetary quantity. We report in Table 4 the design of a Quanto option for the power retail operator example introduced in Section 2.2, and the option payoff in Table 5. By combining the outcomes of Table 3 and Table 5 we note that if the power retail operator buys a Quanto, its revenues will not be affected by negative states (without considering the Quanto premium).

INSERT HERE TABLES 4 AND 5

3. Energy and weather derivatives valuation

Theoretically, the most accurate price of a financial instrument is the market price. Such a rule also refers to all derivatives based on weather variables. However, if the market for these instruments is limited, illiquid, and highly inefficient, then a suitable model should support the pricing process, see VanderMarck (2003).
A contract writer could decide to determine the theoretical contract price by standard methods such as the Black and Scholes (1973) model and its numerous extensions. However, Dischel (2002), highlights that the Black and Scholes model (B&S thereafter) cannot be used for the pricing of temperature-based contracts for a number of reasons: the underlying process governing the evolution of temperature is far from being a geometric Brownian motion since it includes long and short memory behaviours, as well as seasonal patterns; the market is extremely illiquid and shallow (it is mainly driven by reinsurance companies); temperature indices may not be adequately modelled by the Gaussian distribution; the underlying variable is not a traded asset and therefore pricing by replication is impossible. As a result, the Actuarial pricing approach dominates this market. Below we briefly introduce this method and later provide a link to a financial pricing approach we could follow to improve the Actuarial approach. Note that both methods are based on model simulations, and thus could be considered as Monte Carlo option pricing methods.

3.1 The Actuarial approach

Writers of weather-based contracts generally define the price using a range of approaches. One of the most common is the Actuarial approach, which is a popular methodology in the insurance sector. For an introduction to the methodology and some examples, see Zeng (2000), Davis (2001), Brix et al. (2005), among others.
The Actuarial approach depends on the forecasts of the distribution of contract outcomes obtained from historical data and, if available, short- and medium-term forecasts. The Actuarial price equals the average expected payout coming from the predicted density (this is often called the 'fair value'), plus a margin that, beside remuneration, also covers the costs of the contract writer (such as fixed costs, and risk-loading factors for model and market uncertainty, see Henderson, 2002).
To produce the distribution of contract outcomes we can follow various methods. The literature classifies these methods into three groups: Historical Burn Analysis (HBA); Index Modelling (IM); Daily Modelling (DM). The first method, HBA, defines the distribution of contract payouts using historical weather indices evaluated using weather data. Given its simplicity, it is often used as a preliminary pricing method. Index Modelling extends HBA by adding a distributional hypothesis to the weather indices. It thus enables the capture of the tails and asymmetry of weather indices. The distribution is fitted on historical data and used within a Monte Carlo approach to determine the contract fair value. Unfortunately, both HBA and IM suffer from several drawbacks, as pointed out by Nelken (2000). In particular, they may be inaccurate if used with a limited number of historical observations over weather

indices (these are generally evaluated monthly, quarterly, or yearly). Furthermore, these methods may not be appropriate for the pricing of products based on a non-traded asset (weather) and a traded quantity (such as energy, but also natural gas, or EUA).
The Daily Modelling method overcomes the previous problems and is thus becoming the most popular, see Brix et al. (2005). This method begins by fitting a model on daily weather data and then using that model to produce forecasts of weather indices. Given its structure, DM enables many features of weather data to be taken into account, and can also be easily extended to contracts based on many underlying assets. The use of daily data also simplifies the pricing of contracts with very short maturities (for instance weekly), given that the approach can incorporate short-term meteorological forecasts.
The Daily Modelling method bases its results on a time series model to replicate the empirical features of the underlying index data. In this study we focus on models for daily temperature such as the most popular indices in the weather derivative market that are commonly used in Quanto structures. For this purpose, we use the ARFIMA-FIGARCH model with deterministic components that include trends and seasonality in mean and variance. Models in this shape are widely used on the market and the general idea was presented by Beine and Laurent (2003).
This model is then used to predict the future evolution of these variables and of the associated indices (HDD or CDD). By using Monte Carlo methods, a large number of paths or scenarios are simulated and used to recover the contract payout density. The expected value of the density then defines the contract fair value. In standard practice, the price charged to the contract buyer includes a margin calibrated to the Value-at-Risk of the contract payouts, and is also discounted by using a risk-free rate (for instance the Euribor rate).
In this paper, we apply an Actuarial pricing procedure that matches the methods available in the literature and consists of the following steps:

1) Propose a model that jointly captures the dynamic of electricity prices and temperature;
2) Simulate a number of paths for electricity and temperature using the model developed in step (1);
3) Estimate the average pay-off of a specific option using the simulated path and given the contract parameters;
4) Increase the above value by a risk-loading factor computed as 5% of the pay-off Value-at-Risk at the 95% confidence level;
5) Discount the above future value using the appropriate Euribor rate.

3.2 A Financial approach

In this subsection we present a financial solution to the Quanto valuation using some well known facts in derivative valuation. The underlying assumption is that electricity derivatives are driven by two state variables: electricity price and weather (proxy of load). The wide range of maturities and periods available in the Nord Pool for electricity prices enables estimation of the forward curve for every day. We can interpret the forward curve as the risk-neutral trend of electricity prices.[3] If we assume that the market price of risk of the weather variable is zero, then all the risk adjustment in the derivative valuation will come from the electricity price risk.[4] Under this assumption, we propose to determine a risk-neutral valuation of energy and temperature Quanto options following these steps:

1) Estimate the forward curve with a range of futures/forward energy prices;
2) Propose a model jointly capturing the dynamic of electricity prices and temperatures;
3) Simulate a number of paths for electricity and temperature using the model developed in step (2), but replacing the electricity price estimated trend with the forward curve;
4) Estimate the average pay-off of a specific option using the simulated path and the contract parameters;
5) Discount the above future value using the appropriate Euribor rate.

We stress that step 2 above and the step 1 of the Actuarial approach provide exactly the same model. The relevant difference is in the simulation, where the Actuarial approach uses the estimated, or 'real' trend, while the Financial approach employs the forward curve by turning the Actuarial pricing into a Financial risk-neutral pricing which also accounts for changes in outdoor temperature.

We note that in a recent contribution Pirrong and Jermakyan (2008) proposed a model for electricity derivatives valuation using two state variables: demand (load) and fuel price. Given that electricity demand is closely related with weather conditions Pirrong and Jermakyan (2008) suggest the introduction of weather as an additional state variable as a possible extension of their framework. Despite the fact that their proposal would enable the evaluation of assets whose payoffs depend on power prices, loads, and weather, we stress that this requires a very sophisticated mathematical framework that employs numerical techniques without providing closed form solutions. The valuation framework we propose in this paper has the advantage of providing a model describing a number of stylised facts regarding the joint evolution of energy prices and temperature. Some of these facts have not been previously studied (see Section 4.1 below) and can be considered as an extension of the Actuarial approach – with a direct link to Financial pricing methods.

4. Methodology of Quanto option pricing

This section presents the models and methods we use in the evaluation of Quanto options both under the Actuarial and Financial approaches. We first present the model for the joint evaluation of energy price and temperature dynamics. We then describe the estimation of the

[3] See Hull (1997), pages 297-298.

[4] Benth and Benth (2007) assume the market price of risk to be zero in their application for the Stockholm temperature derivative valuation. When two risk factors are considered in a derivative valuation, some authors assume that the market price of risk of one of these factors to be zero, see for example Gibson and Schwartz (1990).

energy price forward curve. Finally, we briefly discuss the simulation approach we follow to generate future paths of energy price and temperature.

4.1 An econometric model for energy and temperature data

We propose here a model that describes the joint evolution of energy log-prices and average temperature – including a number of stylised facts and features characterising the variables. In particular, we take into account: seasonality patterns in means and variances for both series; day-of-the-week effects in the energy log-price mean and variances; log-memory in both series means and variances; weekly seasonal auto-regressive patterns in energy log-prices; auto-regressive patterns with spillovers from temperature levels to energy log-prices; heteroskedasticity with variance spillovers from temperature to energy; dynamic correlations with seasonal evolution. In the following, we present the several components of the model in detail.

Denote by x_t the energy log-price and by y_t the average temperature level. The following dynamic system governs the mean evolution of both series:

$$\Phi(L)\Xi(L)\Delta(L)\left(\begin{bmatrix} x_t \\ y_t \end{bmatrix} - \Upsilon Z_t\right) = \Theta(L)\begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \tag{8}$$

where,

$$\Phi(L) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \sum_{j=1}^{p} \begin{bmatrix} \phi_{1,1,j} & \phi_{1,2,j} \\ 0 & \phi_{2,2,j} \end{bmatrix} L^j, \quad \Xi(L) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \sum_{j=1}^{P} \begin{bmatrix} \xi_{1,1,j} & \xi_{1,2,j} \\ 0 & \xi_{2,2,j} \end{bmatrix} L^{Sj},$$

$$\Delta(L) = \begin{bmatrix} (1-L)^{d_1} & 0 \\ 0 & (1-L)^{d_2} \end{bmatrix}, \qquad \Upsilon Z_t = \begin{bmatrix} \beta_1' \\ \beta_2' \end{bmatrix} D_t + \begin{bmatrix} \delta_1' \\ 0 \end{bmatrix} W_t,$$

$$\Theta(L) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \sum_{j=1}^{q} \begin{bmatrix} \theta_{1,1,j} & \theta_{1,2,j} \\ 0 & \theta_{2,2,j} \end{bmatrix} L^j, \qquad \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \Big| I^{t-1} \sim D(0, \Sigma_t),$$

and: *L* denote the backshift operator, $\Phi(L)$ is a Vector Auto Regressive (VAR) polynomial of order *p* with a restricted structure enabling an effect of lagged temperature on energy log-prices; $\Xi(L)$ is a Seasonal Vector Auto Regressive (S-VAR) polynomial of order *P*, needed to capture the stochastic weekly patterns (and thus S=7); $\Delta(L)$ is a long-memory matrix inducing long-range dependence over temperature and energy log-prices; ΥZ_t is a deterministic mean component that can be partitioned into a vector of sinusoidal trend components D_t (including sine and cosine waves, a constant, and a polynomial trend), and a matrix W_t of day-of-the-week dummies and holiday dummies (which affects only the energy log-prices); $\Theta(L)$ is a Vector Moving Average (VMA) polynomial of order q with a restricted structure similar to that of VAR; the innovation process follows a conditional distribution with a time-varying covariance matrix Σ_t that will be defined below; and I^{t-1} is time *t-1* information set. Furthermore, we assume that all parameter matrices satisfy the constraints ensuring stationarity and invertibility.

Before moving to the second-order moment structure, we will report several comments on the mean structure. Firstly, the feedback from temperature to energy is not direct, but temperature enters into the energy equation in deviation from its unconditional mean $\beta_2' D_t$ and with a long-memory style impact. In fact, the energy log-price equation has the following configuration:

$$\begin{aligned}\Phi_{1,1}(L)\Xi_{1,1}(L)(1-L)^{d_1}\left(x_t-\beta_1' D_t-\delta_1' W_t\right)+\Phi_{1,2}(L)(1-L)^{d_2}\left(y_t-\beta_2' D_t\right)= \\ =\Theta_{1,1}(L)\varepsilon_{1,t}+\Theta_{1,2}(L)\varepsilon_{2,t}\end{aligned} \tag{9}$$

Thus, the structural evolution of the temperature matters for the evolution of energy log-prices. We motive this choice by the fact that the unconditional mean could be easily captured and anticipated by the market (it is purely deterministic). In addition, the stochastic long-range dependence characterising temperature is well-known and plays a role in determining the movements of energy, thus we also introduce long-memory feedbacks.

The mean residual vector follows an unspecified conditional density characterised by heteroskedasticity, with a covariance matrix decomposed into volatility and correlation elements:

$$\Sigma_t = V_t R_t V_t \tag{10}$$

where V_t is a diagonal matrix of conditional volatilities and R_t is a dynamic correlation matrix.

We model conditional variances by long-memory log-GARCH(1,d,1) processes with variance spillovers and feedbacks, and with deterministic components. The outer elements in (10) are represented as:

$$V_t=\begin{bmatrix} h_{1,t}^{0.5} & 0 \\ 0 & h_{2,t}^{0.5}\end{bmatrix}, \quad \mathrm{dg}\left(V_t V_t\right)=H_t=\begin{bmatrix} h_{1,t} \\ h_{2,t}\end{bmatrix}=\begin{bmatrix} \ln\left(\sigma_{1,t}^2\right) \\ \ln\left(\sigma_{2,t}^2\right)\end{bmatrix} \tag{11}$$

where the conditional variances are logarithms of underlying quantities that obey the following dynamic equations:

$$\begin{bmatrix}\sigma_{1,t}^2\\ \sigma_{2,t}^2\end{bmatrix}=\begin{bmatrix}\omega_1\\ \omega_2\end{bmatrix}+\begin{bmatrix}\gamma_1'\\ \gamma_2'\end{bmatrix}D_t+\begin{bmatrix}\varphi_1'\\ 0\end{bmatrix}W_t+\boldsymbol{\xi}\begin{bmatrix}\sigma_{1,t-1}^2\\ \sigma_{2,t-1}^2\end{bmatrix}+\boldsymbol{\alpha}\begin{bmatrix}\ln\left(\varepsilon_{1,t-1}^2\right)-E\left[\ln\left(z_{1,t-1}^2\right)\right]\\ \ln\left(\varepsilon_{2,t-1}^2\right)-E\left[\ln\left(z_{2,t-1}^2\right)\right]\end{bmatrix} \tag{12}$$

In (12), coefficient matrices have the following representation:

$$\boldsymbol{\alpha}=\begin{bmatrix}\alpha_{1,1} & \alpha_{1,2}\\ 0 & \alpha_{2,2}\end{bmatrix},\quad \boldsymbol{\xi}=\begin{bmatrix}\xi_{1,1} & \xi_{1,2}\\ 0 & \xi_{2,2}\end{bmatrix},$$

and D_t and W_t are the same deterministic matrices used in (8) (the day-of-the-week dummies affect only the evolution of log-energy variances). Furthermore, we restrict model orders to 1 for simplicity (higher orders can be easily introduced, but in our experience they are not needed); $z_{i,t}$, $i=1,2$ are the standardised residuals defined as $z_{i,t}=\varepsilon_{i,t}h_{i,t}^{-0.5}$ and with $E\left[\ln\left(z_{1,t}^2\right)\right]=-1.27$ under Gaussianity. Finally, we note that the ARCH and GARCH matrices enable a dependence of energy variances on temperature variances and innovations. As a result, if the off-diagonal coefficients in the ARCH and GARCH matrices are jointly equal to zero, the two conditional variances evolve as two independent log-GARCH processes. In the proposed model, the introduction of a log-transformation enables the removal of the constraints for positivity of conditional variances, thus simplifying the model estimation. The conditional variance dynamic could follow alternative specifications, starting from the seminal contributions of Engle (1982) and Bollerslev (1986), to the long-memory model of Baillie et al. (1996), to the more advanced specifications such as the periodic long-memory GARCH of Bordignon et al. (2007, 2009). For a survey of possible GARCH specifications see Bollerslev et al. (1992, 1994), and Bollerslev (2009).

Finally, we describe the dynamic evolution of the conditional correlations R_t, which have been introduced to account for the dynamic we observed in preliminary exploratory analysis of energy and temperature data. This is further confirmed by the estimate of rolling correlations over the variance standardised residuals $z_{i,t}$, $i=1,2$ of our empirical data, see Figure 1 and the following section for additional details. Dynamic conditional correlations models are now quite common in the literature, and their introduction is due to the works of Engle (2002), and Tse and Tsui (2002). Despite the huge number of studies proposing correlation models (see the surveys by Bauwens et al., 2006, and Silvennoinen and Terasvirta, 2009), few specifications enable the introduction of exogenous variables in the correlation equation without requiring excessive parameter constraints. We adopt here the model of Christodoulakis and Satchell (2002) which proposes a dynamic equation for the Fisher transformation of the correlation. The model has a relevant limitation, it cannot be generalised to system dimensions higher than two, yet it perfectly fits our bivariate framework. The conditional correlation matrix R_t and the Fisher transformation of the correlation are equal to:

$$R_t = \begin{bmatrix} 1 & r_t \\ r_t & 1 \end{bmatrix} \qquad \rho_t = \frac{1}{2}\ln\left(\frac{1+r_t}{1-r_t}\right) \tag{13}$$

We then model ρ_t as follows:

$$\rho_t = \psi_0 + \psi_1\rho_{t-1} + \psi_2\left(z_{1,t-1}z_{2,t-1}\right) + \boldsymbol{\psi}_3' D_t + \boldsymbol{\psi}_4' W_t \tag{14}$$

where D_t and W_t are the usual dummy matrices of deterministic components, and the innovation is given by the cross-product of GARCH standardised residuals. Given the estimates of the parameters in (14), we recover the conditional correlation matrix by inverting the Fisher transformation in Equation (13):

$$r_t = \frac{\exp(2\rho_t)-1}{\exp(2\rho_t)+1} \tag{15}$$

Equation (14) could be also slightly modified to allow for correlation targeting, by acting on the intercept

$$\varphi_0 = \left(1-\varphi_1-\varphi_2-\gamma_1' E[D_t]-\gamma_2' E[W_t]\right)\rho \tag{16}$$

where ρ is the Fisher transformation of the sample correlation between GARCH standardised residuals. Finally, we denote by $\boldsymbol{\eta}_t$ the uncorrelated residuals, equal to

$$\boldsymbol{\eta}_t = \begin{bmatrix} 1 & r_t \\ r_t & 1 \end{bmatrix}^{-0.5} \begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix} \tag{17}$$

The model outlined in the previous paragraphs potentially contains many parameters. The introduction of long-memory in both the mean and variance further increases the computational complexity. Therefore, we chose to estimate the model in four steps, at the cost of loss in estimation efficiency. At first, we estimated the deterministic mean specification using the least squares method, and on the residuals we estimated the dynamic mean components by Quasi-Maximum Likelihood while assuming a constant variance matrix for the innovations. We then estimated the covariance part following Engle (2002) in another two steps. Therefore, we filter out the conditional variance dynamic, and finally we estimate the correlation parameters.

4.2 Estimating the energy forward curve

Benth et al. (2007) propose a method to construct a smooth curve from observed forward prices. Since electricity prices are seasonally dependent, they propose to decompose the curve into a seasonal component and a correction term. The correction term is defined as a polynomial spline function with a maximum smoothness property, so that the constructed curve perfectly replicates the observed market prices. The seasonal component is a parametric function which is estimated by least squares. Following Benth et al. (2007) the relationship between average forward price and fixed delivery forwards is defined as follows:

$$F(0,T^s,T^e) = \int_{T^s}^{T^e} \frac{1}{T^e - T^s} f(t)dt \qquad (17)$$

where: $F(0,T^s,T^e)$ represents the price at time 0, today, for receiving a unit of electricity (a Megawatt) at a continuous flow during the period (T^s,T^e); T^s and T^e denote, respectively, the start and the end of the settlement period; and $f(t)$ represents the price of a forward at time 0 with delivery at the fixed time $t \geq 0$. Under the risk-neutral measurement $f(t)$ will be equal to the expected value of the underlying asset to be delivered in t, that is the electricity instantaneous forward price.

Assume that m futures/forward contracts are observed at time 0. Let t_0 be the start of the settlement period for the contract with the shortest time to delivery, and denote by t_n the end of the settlement period for the contract going furthest in the future. The forward price is split into two parts

$$f(t) = s(t) + \varepsilon(t), \; t \in [t_0, t_n] \qquad (18)$$

where $s(t)$ and $\varepsilon(t)$ are two continuous functions representing the seasonality of the forward curve and the adjustment function that measures the forward curve's deviation from the seasonality, respectively. As the adjustment function is less sensitive to time as the forward maturities go ahead, this suggests that a time-varying ε should be flat at the long end and therefore, $\varepsilon'(t_n) = 0$ is assumed. Benth et al. (2007) define the 'smoothness' criteria as the function $\varepsilon(t)$ minimising the mean square value of its second derivative on $[t_0, t_n]$, $\int_{t_0}^{t_n}[\varepsilon''(t)]^2 dt$, over the set of continuously twice differentiable functions. Benth et al. (2007) show that the smoothest adjustment function with the above properties is a polynomial spline of order four. Specifically, this function can be written as

$$\varepsilon(t) = \begin{cases} a_1 t^4 + b_1 t^3 + c_1 t^2 + d_1 t + e_1, \; t \in [t_0, t_1] \\ a_2 t^4 + b_2 t^3 + c_2 t^2 + d_2 t + e_2, \; t \in [t_1, t_2] \\ \vdots \\ a_n t^4 + b_n t^3 + c_n t^2 + d_n t + e_n, \; t \in [t_{n-1}, t_n] \end{cases} \qquad (19)$$

where $\{t_0, t_1, \dots, t_n\}$ is the list of dates where overlapping contracts are split into sub-periods and the parameters to be found define the following vector

$x^T = [a_1\ b_1\ c_1\ d_1\ e_1 \cdots a_n\ b_n\ c_n\ d_n\ e_n]$. We determine parameters by solving the following equality constrained convex quadratic programming problem

$$\min_x \int_{t_0}^{t_n} \left[s''(t)\right]^2 dt \tag{20}$$

subject to the connectivity and smoothness constraints of derivatives at the knots, $j = 1, \dots, n-1$,

$$(a_{j+1} - a_j)t_j^4 + (b_{j+1} - b_j)t_j^3 + (c_{j+1} - c_j)t_j^2 + (d_{j+1} - d_j)t_j + e_{j+1} - e_j = 0$$
$$4(a_{j+1} - a_j)t_j^3 + 3(b_{j+1} - b_j)t_j^2 + 2(c_{j+1} - c_j)t_j + d_{j+1} - d_j = 0$$
$$12(a_{j+1} - a_j)t_j^2 + 6(b_{j+1} - b_j)t_j + 2(c_{j+1} - c_j) = 0$$

and

$$s'(t_n; x) = 0$$

$$F_i(0, T_i^s, T_i^e) = \int_{T_i^s}^{T_i^e} \frac{1}{T_i^e - T_i^s}(s(t) + \varepsilon(t))dt$$

for $i = 1, \dots, m$. This is a minimisation problem with $3n + m - 2$ constraints. The solution can be obtained by using the Lagrange Multiplier Method and solving the following unconstrained minimization problem

$$\min_{x,\lambda} x^T H x + \lambda^T (Ax - b) \tag{21}$$

The solution is obtained by solving the following linear equation

$$\begin{bmatrix} 2H & A^T \\ A & 0 \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix} = \begin{bmatrix} 0 \\ b \end{bmatrix} \tag{22}$$

where

$$H = \begin{pmatrix} h_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & h_n \end{pmatrix}, \quad h_j = \begin{pmatrix} \frac{144}{5}\Delta_j^5 & 18\Delta_j^4 & 8\Delta_j^3 & 0 & 0 \\ 18\Delta_j^4 & 12\Delta_j^3 & 6\Delta_j^2 & 0 & 0 \\ 8\Delta_j^3 & 6\Delta_j^2 & 4\Delta_j^1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ and } \Delta_j^i = t_{j+1}^i - t_j^i$$

and the matrix A and b are obtained by formulating all the constraints in a equation system $Ax = b$ where A is a $(3n + m - 2) \times 5n$ matrix and b is a $(3n + m - 2) \times 1$ vector.

Finally, we must define a seasonal function $s(t)$. We suggest estimating a sinusoidal function similar to that used by Benth et al. (2007):

$$s(t) = \beta_0 + \beta_1 \cos(t + \psi)\frac{2\pi}{Y} \quad (23)$$

where β_0, β_1, and ψ are parameters to be estimated, while Y should be calibrated to the year length (either 365 or 366 days).
The difference between the sinusoidal trend and the forward prices are then fitted with a polynomial spline function using the maximum smoothness criteria. The solution given above to the constrained optimisation problem can be difficult to solve because many restrictions in the optimisation problem are close to being linear combinations, thus producing explosive results. An example could be given by the presence of monthly contracts within a specific quarter, as well as quarterly contracts. If the quarterly contract prices is very close to the linear combination of the corresponding monthly contract prices, a solution may not be found. Benth et al. (2008) propose solving the problem by using the QR factorisation. Alternatively, some contracts could be dropped from the analysis, thereby creating an over-identification problem.

4.3 Simulating Quanto pay-offs

Following the steps mentioned in Sections 3.1 and 3.2, the generation of the Quanto option pay-off is based on the simulation of a number of possible paths of energy prices and average temperatures. The central element is thus given by the model outlined in Section 4.1.
To generate the Quanto pay-off we start with the estimated parameters of the model in equations (8)-(14) using a sample from time *1* to *T*. Assuming that the maturity date is in time *T+h*, we follow these steps to generate one possible option payoff:
i) generate the uncorrelated residuals $\boldsymbol{\eta}_t$ for $t=T+1, T+2, \ldots T+h$; in order to avoid misspecification errors in the joint distribution of model residuals, we suggest generating these series by resampling from the in-sample model residuals;
ii) given the uncorrelated residuals we proceed backward and simulate the variance standardised but correlated residuals $\mathbf{z}_t = \left[z_{1,t}\ z_{2,t}\right]'$, the mean residuals $\boldsymbol{\varepsilon}_t = \left[\varepsilon_{1,t}\ \varepsilon_{2,t}\right]'$, the average temperature and energy possible paths x_t, and y_t; all these quantities will be simulated for $t=T+1, T+2, \ldots T+h$, and will represent a possible future evolution of the observed paths up to time *T* (thereby being conditional on the real data available up to time *T*);
iii) in the case of the Financial approach, the real trend will be now replaced by the forward curve estimated in accordance with Section 4.2;
iv) given the simulated paths of energy price and average daily temperature, determine the value of the HDD index in the range *T+1* to *T+h*, and the average energy price in the same range;

v) determine the possible pay-off of the option for the simulated path, using the Quanto formula in (6).
We then repeat steps i)-v) several times and so recover a density for the Quanto pay-off. The option price will then be determined following the steps in Sections 3.1 and 3.2.

5. Empirical results

5.1. Data description

The empirical part of this research makes use of daily time series of energy prices provided by the Nord Pool electricity spot market (Nordic Elspot) and of daily average air temperatures provided by the National Climatic Data Center (United States). Time series of daily mean temperature for Oslo (Norway) refer to the period 1 January 1978 to 31 December 2008. Time series of energy prices refer to the period 1 January, 1999[5] to 31 December, 2008. The daily energy price is equal to the arithmetic average of the hourly prices within a specific day.[6] The reference currency for the entire market is the euro, and prices refer to one Megawatt per hour (MWh). Furthermore, in order to apply the Financial approach for Quanto valuation we consider a range of futures/forward prices that we report in Table 6.

5.2. Preliminary analysis

Table 7 reports the descriptive statistics of electricity returns in the Oslo area and Table 8 reports the autocorrelation statistics for electricity prices. One of the most discussed features of energy prices is the so-called 'holiday effect', the impact on electricity returns of the weekends. We have obtained a list of Norwegian holidays and weekends from www.timeanddate.com. Using the information provided by this website, we define four subsets of returns within our sample: the returns obtained between two working days (WW), a working day and a holiday (WH), a holiday and a working day (HW), and two consecutive days of holiday (HH), respectively. Descriptive statistics for these subsets are reported in columns 3 to 6 of Table 7. The last four columns of the same table refer to meteorological seasons, where the winter includes the months of December, January and February, and the other seasons follow. By looking at Table 7 we can make a number of considerations. At first, we note that the Oslo data is characterised by zero average return, and a strong Monday effect (increase in prices after holidays) which is observed in column HW. The reverse effect, associated with column WH, shows evidence of negative returns. The remaining two partitions of the whole sample, WW and HH, are less relevant for the purpose of the actual paper; nevertheless, we note that both provide an average negative return. Furthermore,

[5] Older observations are unavailable.
[6] Note that every year, due to the transition from standard time to daylight savings time and vice versa, there is a day in spring with 23 hours, as well as an autumn day with 25 hours.

energy returns are right skewed and leptokurtic, and the variance of electricity price is higher in summer and lower in autumn. In addition, the winter and spring variance is similar to the variance calculated for the entire sample. Finally, when focusing on dynamic properties of the series, the unit root hypothesis cannot be rejected. Table 8 shows that electricity returns and squared returns have several significant autocorrelation patterns: daily, weekly, quarterly, and yearly. These elements will be important in the time series modelling.
Table 9 reports a descriptive analysis of Oslo temperature data. We define meteorological seasons on a monthly basis, winter contains the months of December, January and February, Spring includes April, May and June and so on. In Table 9, the mean increases from winter to summer, while the volatility decreases. Furthermore, temperature distribution is not normal, the unit root hypothesis cannot be accepted, and data shows a strong autocorrelation pattern both in mean and variance (Table 10).
Finally, given the purpose of jointly modelling energy and temperature, we also examine the relation between the two series from a descriptive viewpoint. We observe a significant negative correlation between time series of log-changes in energy prices and raw temperatures during winter and autumn (Table 11). If we consider changes in energy prices and changes in daily temperature then correlations are statistically significant for all seasons and over the entire sample. In particular, we note that the correlations are statistically significant and negative with only the exception of summer (positive and significant). Our model confirms this preliminary finding and shows evidence of periodic patterns in the correlations between energy and temperature (see following section).

INSERT HERE TABLES 7-11

5.3. Results of model estimation and simulation

We estimate the model presented in Section 4.1 for Oslo energy price and temperature, and report the results in Tables 12 to 15. We also estimate the model with the multi-step approach described in Section 4.1. This method clearly does not achieve full efficiency but enables a relatively fast model estimation (less than two hours). We estimate the model using data up to December 2007 for the purpose of pricing a contract that matures at the end of 2008.
Estimated parameters show that the deterministic component of energy includes a trend and a yearly cosine wave, see Table 12. The temperature periodic component seems much more relevant, and it is governed by a combination of sine and cosine waves. Energy prices also show evidence of Friday, Sunday and Non-Working day effects. In these cases, the average energy price is lower than in the other days. Notably, the Monday effect is not present.
Table 13 reports the coefficients driving the stochastic mean dynamics. Both energy and temperature series, in deviation from their deterministic components, show long-memory effects. Energy long-range dependence is stronger than that of temperature. Furthermore, energy prices have a short-term dynamic that depends on lags of energy (lags 1, 3, 5, and 6) and temperature (lags 2, 4, 5, 6, and 7). This last finding supports the dependence of energy prices on temperature deviations with respect to its deterministic component. Such a result is further confirmed by the significance of the limited, and weekly S-VAR impact of

temperature on energy prices (energy S-VAR is also statistically significant). Temperature dynamic has a mild short-term component with significant lags 1, 5, 7, together with the SAR term. Finally, both series have a statistically significant MA term, and energy does not depend on lagged temperature innovations.

The variance dynamic of temperature (see Table 14) does not depend on periodic components, and has a low persistence compared to the financial time series. This was an expected result. Energy variances depend on lagged temperature variances (further supporting the joint modelling of the two variables), on a yearly cosine wave, and on Monday and Friday dummies.

Finally, we focus on the correlation dynamic shown in Table 15. In Section 4.1 we propose a model including periodic elements in the correlation dynamic. To support our choice, Figure 1 reports the 60 days of rolling correlations between the energy and temperature standardised residuals $\mathbf{z}_t = \left[z_{1,t} \; z_{2,t} \right]'$: the graph shows evidence of a strong periodic pattern. Therefore, it is not surprising to see in Table 15 verification that the correlation dynamic is highly persistent and shows relevant sine and cosine waves.

INSERT HERE TABLES 12-15 AND FIGURE 1

To implement the Financial valuation approach we also estimated for the electricity prices in Oslo between 1999-2007 a sinusoidal function similar to that used by Benth et al. (2007) and reported in Section 4.2, namely, Equation (23). We obtain the 2008 values of the sinusoidal function from the following equation:

$$\underset{}{s(t)} = \underset{(0.247)}{27.268} + \underset{(0.349)}{4.800} \cdot \underset{(4.221)}{\cos(t + 21.605)} \frac{2\pi}{366} \tag{24}$$

where in parenthesis we report the coefficient standard errors.

We then fitted the difference between the sinusoidal trend in (24) and the forward prices for maturities included in Table 6 with a polynomial spline function by using the maximum smoothness criteria (see Section 4.2). To overcome an overidentification problem, we eliminated some closely related contracts.[7] Figure 2 displays the forward curve on 28 December, 2007; the last trading day in 2007 for the Nord Pool derivative market.

INSERT HERE FIGURE 2

Using the estimated model and forward curve, we proceed to the simulation of 10000 possible paths for the energy price and the average temperature for 2008. We decided to limit the

[7] Specifically, we drop the contracts with tickers: ENOW02-08, ENOW03-08, ENOW04-08, ENOQ1-08, ENOQ2-08, ENOYR-09, SYOSLJAN-08, SYOSLFEB-08, SYOSLMAR-08, SYOSLQ4-08. These contracts are not included as restrictions in the forward curve estimation because they generate overidentification problems. For instance, this is the case of the monthly forward contracts corresponding to January, February and March and the first quarter forward contract. The sum of the three forward prices in January, February and March is almost equal to three times the first quarter forward price (they differ by only a small amount).

number of simulations to this value because the benefits of increasing the number would have been very marginal. We report in Table 16 and in Figures 3 and 4 some descriptive statistics of the Monte Carlo simulations.

INSERT HERE FIGURES 3-4 AND TABLE 16

The distribution of HDD indices for the analysed periods appears to be symmetric and close to the Gaussian. The simulated energy prices have a strong right-sided asymmetry, coherent with the real energy time series. This feature is very important and it will have a relevant impact on the premiums of Quanto options. Summarising, the simulated data closely replicates the features of real temperature and energy time series.

5.4. Valuation of Quanto options

We used the simulations above for pricing a Quanto option under both an Actuarial and a Financial approach. We considered a set of different Quanto options where the underlying factors are the HDD index (based on the average daily temperature) and the average delivery period energy price for the Oslo area. We set the strikes for these options at the historically average value for temperature and to the closing forward prices for electricity. Therefore, these options can be perceived as ‘at the money’ options. Tables 17 and 18 include the contract specification and risk-free rates used in the pricing process, while Table 19 contains the pricing results.

INSERT HERE TABLES 17 - 18

For the January delivery period the Actuarial and Financial methods provide similar prices, and the difference is less than 2.5%. For other delivery periods the Actuarial approach overestimates the option premiums, especially for option type II, where the difference in premiums for April is about 60%. On average, premiums provided by the Actuarial approach are 14% higher than those provided by the Financial approach for the type I option, and 27% higher for the type II option. The reason of such huge differences lays in the calibration of the energy price evolution by the forward curve. In this way, the Financial approach introduces market expectation about the future evolution of energy into the Quanto prices; that is to say, risk adjusted expected electricity prices.

It is important to note, that in both approaches, the real market prices will be slightly higher due to the inclusion of additional factors, e.g. current portfolio structure, remuneration, costs of contract writing and exchange transactions, risk-loading factors for model and market uncertainty, etc.

6. Conclusions

In this paper we propose a bivariate model capturing some well-known stylised facts of energy log-prices and temperature, together with their interdependence. Furthermore, we show evidence that the correlation between these quantities has a periodic behaviour, an element not yet discussed in the literature. This econometric model was employed for the purpose of Quanto options pricing using two approaches; an Actuarial approach and a Financial approach (the latter approach differing from the former approach because it includes market expectations about the evolution of energy prices). We provide an empirical application showing the benefits of the model proposed and of the Financial approach for pricing Quanto contracts. We demonstrate that premiums are, in most cases, lower for the Financial approach, a positive outcome which is coupled with transparency and coherency as the approach benefits from all the available information on the market.

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Table 1. Additional cost (€) under different spot market price (€/MWh) and temperature (HDD) scenarios

Temperature (HDD)	€29/MWh	€37/MWh	€45/MWh	€53/MWh	€61/MWh
1900	-1,740,000	-2,220,000	-2,700,000	-3,180,000	-3,660,000
2050	-1,305,000	-1,665,000	-2,025,000	-2,385,000	-2,745,000
2200	-870,000	-1,110,000	-1,350,000	-1,590,000	-1,830,000
2350	-435,000	-555,000	-675,000	-795,000	-915,000
2500	0	0	0	0	0
2650	435,000	555,000	675,000	795,000	915,000
2800	870,000	1,110,000	1,350,000	1,590,000	1,830,000
2950	1,305,000	1,665,000	2,025,000	2,385,000	2,745,000
3100	1,740,000	2,220,000	2,700,000	3,180,000	3,660,000

Red negative numbers denote an increase in cost, while black positive numbers identify a decrease in total cost.

Table 2. Additional revenues (€) under different spot market price (€/MWh) and temperature (HDD) scenarios

Temperature (HDD)	€29/MWh	€37/MWh	€45/MWh	€53/MWh	€61/MWh
1900	-2,970,000	-2,970,000	-2,970,000	-2,970,000	-2,970,000
2050	-2,227,500	-2,227,500	-2,227,500	-2,227,500	-2,227,500
2200	-1,485,000	-1,485,000	-1,485,000	-1,485,000	-1,485,000
2350	-742,500	-742,500	-742,500	-742,500	-742,500
2500	0	0	0	0	0
2650	742,500	742,500	742,500	742,500	742,500
2800	1,485,000	1,485,000	1,485,000	1,485,000	1,485,000
2950	2,227,500	2,227,500	2,227,500	2,227,500	2,227,500
3100	2,970,000	2,970,000	2,970,000	2,970,000	2,970,000

Red negative numbers denote a decrease in revenues, while black positive numbers identify an increase in revenues.

Table 3. Deviations of the power retailer margin (€) under different spot market price (€/MWh) and temperature (HDD) scenarios

Temperature (HDD)	€29/MWh	€37/MWh	€45/MWh	€53/MWh	€61/MWh
1900	-1,230,000	-750,000	-270,000	210,000	690,000
2050	-922,500	-562,500	-202,500	157,500	517,500
2200	-615,000	-375,000	-135,000	105,000	345,000
2350	-307,500	-187,500	-67,500	52,500	172,500
2500	0	0	0	0	0
2650	307,500	187,500	67,500	-52,500	-172,500
2800	615,000	375,000	135,000	-105,000	-345,000
2950	922,500	562,500	202,500	-157,500	-517,500
3100	1,230,000	750,000	270,000	-210,000	-690,000

Red negative numbers denote losses while black positive numbers identify profits.

Table 4. Specification of a Quanto option based on HDD and energy price

Protection period	1 November 2009 – 31 March 2010
Temperature index HDD	Index cumulated during the protection period
Strike HDD (K_2)	2500
Energy Index (E)	Average price of Nord Pool spot price (arithmetic) during protection period
Strike energy (K_1)	49.5 €/MWh
Tick value	€ 100 MWh / HDD
Payout formula	MAX(0,(K_2 - HDD)*tick*(K_1 - E))
Maximum payoff (cap)	€ 1,000, 000

Table 5. Payoff (€) from the Quanto option in Table 4 under different spot market price (€/MWh) and temperature (HDD) scenarios

Temperature (HDD)	**€29/MWh**	**€37/MWh**	**€45/MWh**	**€53/MWh**	**€61/MWh**
1900	1,230,000	750,000	270,000	0	0
2050	922,500	562,500	202,500	0	0
2200	615,000	375,000	135,000	0	0
2350	307,500	187,500	67,500	0	0
2500	0	0	0	0	0
2650	0	0	0	52,500	172,500
2800	0	0	0	105,000	345,000
2950	0	0	0	157,500	517,500
3100	0	0	0	210,000	690,000

Table 6. Market data from Nord Pool (December 28, 2007)

This table displays the closing prices of futures contracts (weekly maturities) and forward contracts (monthly, quarterly, and yearly maturities) for electricity seasoned at Nord Pool on the last trading day of 2007. Contracts at the bottom of the table whose ticker begins with 'SYOSL' are the *Contracts for Differences* referred to in the Oslo bidding area – and enable the estimation of specific forward curves for this area. The remaining contracts are those whose underlying is the system price for the whole Nordic area. Columns with headings 'Startdate' and 'Enddate' define the delivery period for each contract.

Ticker	Closing price	Startdate	Enddate
ENOW01-08	47.79	2007-12-31	2008-01-06
ENOW02-08	50.50	2008-01-07	2008-01-13
ENOW03-08	51.50	2008-01-14	2008-01-20
ENOW04-08	52.48	2008-01-21	2008-01-27
ENOMJAN-08	51.11	2008-01-01	2008-01-31
ENOMFEB-08	52.95	2008-02-01	2008-02-29
ENOMMAR-08	49.90	2008-03-01	2008-03-31
ENOMAPR-08	49.60	2008-04-01	2008-04-30
ENOMMAY-08	47.60	2008-05-01	2008-05-31
ENOMJUN-08	47.98	2008-06-01	2008-06-30
ENOQ1-08	51.55	2008-01-01	2008-03-31
ENOQ2-08	48.55	2008-04-01	2008-06-30
ENOQ3-08	49.00	2008-07-01	2008-09-30
ENOQ4-08	53.93	2008-10-01	2008-12-31
ENOQ1-09	56.15	2009-01-01	2009-03-31
ENOQ2-09	48.90	2009-04-01	2009-06-30
ENOQ3-09	48.70	2009-07-01	2009-09-30
ENOQ4-09	52.90	2009-10-01	2009-12-31
ENOYR-09	51.70	2009-01-01	2009-12-31
ENOYR-10	50.88	2010-01-01	2010-12-31
ENOYR-11	50.10	2011-01-01	2011-12-31
ENOYR-12	50.17	2012-01-01	2012-12-31
SYOSLJAN-08	-1.00	2008-01-01	2008-01-31
SYOSLFEB-08	-1.00	2008-02-01	2008-02-29
SYOSLMAR-08	-1.00	2008-03-01	2008-03-31
SYOSLQ1-08	-1.00	2008-01-01	2008-03-31
SYOSLQ2-08	-1.00	2008-04-01	2008-06-30
SYOSLQ3-08	-0.50	2008-07-01	2008-09-30
SYOSLQ4-08	-0.50	2008-10-01	2008-12-31

Table 7: Summary statistics of electricity prices

This table reports the descriptive statistics of electricity returns in Oslo (basic data provided by the Nord Pool market). Each column heading contains between brackets the number of observations. We have examined the holiday effect on electricity returns. The WW, WH, HW and HH headings refer to the returns obtained between two working days, a working day and a holiday, a holiday and a working day, and two consecutive days of holiday, respectively. The last four columns refer to meteorological seasons, where the winter is defined as the months of December, January, February and so on. *Kruskal-Wallis* statistics test equality between the whole sample and medians in each column time series. *Levene* statistics test equality between the whole sample and variances in each column time series. *Skewness* means the skewness coefficient and has the asymptotic distribution N(0; 6/T) under normality, where T is the sample size. The null hypothesis tests whether the skewness coefficient is equal to zero. *Kurtosis* means the excess kurtosis coefficient and has an asymptotic distribution of N(0 ; 24/T) under normality. The hypothesis tests whether the excess kurtosis is equal to zero. The *ADF* and *PP* refers to the Augmented Dickey and Fuller (1981) and Phillips and Perron (1988) unit root tests on the original log-price time series. One-sided *p*-values computed following Mackinnon (1996) for the *ADF* and *PP* test are displayed as 〈.〉 (corresponding to the process with intercept and trend). The number of lags in the ADF test and the truncation lag in the PP test are obtained by information criteria (Schwarz and Newey and West, respectively). Marginal significance levels are displayed as [.] in the remaining tests.

	WHOLE (3652)	WW (1970)	WH (548)	HW (549)	HH (585)	Winter (902)	Spring (920)	Summer (920)	Autumn (910)
Mean x 100 [=0]	0.03 [0.86]	-0.58 [0.00]	-4.60 [0.00]	9.49 [0.00]	-2.42 [0.00]	-0.08 [0.78]	-0.33 [0.27]	0.33 [0.38]	0.19 [0.25]
Median x100 [kruskal-wallis]	-0.28	-0.39 [0.13]	-3.27 [0.00]	6.57 [0.00]	-1.14 [0.00]	-0.47 [0.21]	-0.69 [0.01]	0.08 [0.03]	-0.03 [0.11]
SD [levene]	0.09	0.07 [0.00]	0.08 [0.97]	0.10 [0.00]	0.07 [0.00]	0.09 [0.47]	0.09 [0.06]	0.11 [0.00]	0.05 [0.00]
Skewness [=0]	-0.01 [0.74]	-2.02 [0.00]	-2.56 [0.00]	3.67 [0.00]	-6.36 [0.00]	2.34 [0.00]	0.70 [0.00]	-1.90 [0.00]	1.25 [0.00]
Kurtosis [=0]	35.72 [0.00]	61.38[0.00]	20.49 [0.00]	27.61 [0.00]	89.66 [0.00]	51.93 [0.00]	7.81 [0.00]	30.14 [0.00]	7.52 [0.00]
Minimum	-1.19	-1.19	-0.81	-0.12	-1.12	-0.81	-0.53	-1.19	-0.18
Maximum	1.25	0.80	0.36	1.25	0.32	1.25	0.61	0.80	0.36
ADF	-4.28 〈0.00〉								
PP	-4.97 〈0.00〉								

Table 8: Autocorrelation statistics for electricity prices

This table reports the autocorrelation statistics of electricity returns for Oslo in the Nord Pool market. The column *ρ(.)* reports the autocorrelation coefficient, while the *Q(.)* and $Q^2(.)$ labels identify columns containing the Ljung-Box tests for serial correlation on the levels and on their squares, respectively. At lag *k*, both test statistics are distributed as a Chi-square with *k* degrees of freedom. We report the p-values in brackets.

Lags	*ρ*(.)	*Q(.)*	Q^2(.)
1	-0.04 [0.02]	5.57 [0.02]	182.59 [0.00]
7	0.25 [0.00]	404.68 [0.00]	855.68 [0.00]
14	0.26 [0.00]	717.46 [0.00]	1297.75 [0.00]
21	0.29 [0.00]	1120.58 [0.00]	1438.98 [0.00]
63	0.21 [0.00]	2618.56 [0.00]	1596.19 [0.00]
364	0.21 [0.00]	12149.22 [0.00]	1816.69 [0.00]

Table 9. Summary statistics of temperature in Oslo - raw data [°C]

This table reports the descriptive statistics of temperature data in Oslo (raw data provided by the National Climate Data Center). Each column heading contains between brackets the number of observations. We have also examined meteorological seasons, where the winter is defined as the December, January and February months and so on. *Kruskal-Wallis* statistic tests equality between the whole sample and medians in each column time series. *Levene* statistic tests equality between the whole sample and variances in each column time series. *Jarque-Bera* statistic tests whether the time series is normally distributed. The reported probability [*p-value*] is the probability that a Jarque-Bera statistic exceeds (in absolute value) the observed value under null hypothesis of a normal distribution. The *ADF* and *PP* refer to the Augmented Dickey and Fuller (1981) and Phillips and Perron (1988) unit root tests on the original log-price time series. One-sided *p*-values computed for the *ADF* and *PP* test are displayed as ⟨.⟩ (corresponding to the process with intercept and trend). The number of lags in the ADF test and the truncation lag in the PP test are obtained by information criteria (Schwarz and Newey and West, respectively).

	WHOLE (11323)	Winter (2798)	Spring (2852)	Summer (2852)	Autumn (2821)
Mean	4.48	-5.52	3.92	14.70	4.62
Median [Kruskal-Wallis]	4.64	-4.64 [0.0]	3.89 [0.0]	14.50 [0.0]	5.11 [0.17]
SD [Levene]	8.95	6.17 [0.0]	5.95 [0.0]	3.06 [0.0]	5.78 [0.0]
Skewness	-0.37	-0.61	-0.27	0.19	-0.36
Kurtosis	2.61	2.93	3.28	2.68	2.71
Minimum	-27.94	-27.94	-18.86	5.50	-15.50
Maximum	23.64	7.36	19.31	23.64	18.36
Jarque-Bera [p-value]	328.3 [0.00]	175.2 [0.00]	43.2 [0.00]	29.7 [0.00]	71.2 [0.00]
ADF [p-value]	-6.93 [0.00]	-14.7 [0.00]	-13.11 [0.00]	-17.70 [0.00]	-14.45 [0.00]
PP [p-value]	-13.51 [0.00]	-16.54 [0.00]	-13.08 [0.00]	-17.32 [0.00]	-14.68 [0.00]

Table 10. Autocorrelation statistics of temperature data

This table reports the autocorrelation statistics of time series of daily temperature in Oslo after removing trend and seasonality in mean. The column *ρ(.)* reports the autocorrelation coefficient, while the *Q(.)* and $Q^2(.)$ labels identify columns containing the Ljung-Box tests for serial correlation on the levels and on squares, respectively. At lag *k*, both test statistics are distributed as a Chi-square with *k* degrees of freedom. We report the p-values in brackets.

Lags	*ρ(.)*	*Q(.)*	$Q^2(.)$
1	0.80 [0.00]	7298.93 [0.00]	5732.10 [0.00]
2	0.61 [0.00]	11517.06 [0.00]	8342.09 [0.00]
3	0.49 [0.00]	14223.61 [0.00]	9981.17 [0.00]
4	0.4 [0.00]	16070.93 [0.00]	11169.77 [0.00]
5	0.34 [0.00]	17373.74 [0.00]	12033.06 [0.00]
10	0.19 [0.00]	20672.91 [0.00]	14949.36 [0.00]
20	0.14 [0.00]	23438.71 [0.00]	17490.67 [0.00]
50	0.06 [0.00]	26204.58 [0.00]	21723.53 [0.00]
100	0.00 [0.00]	26405.62 [0.00]	22595.09 [0.00]
183	0.00 [0.00]	26470.80 [0.00]	28367.76 [0.00]
364	0.05 [0.00]	27662.87 [0.00]	42254.09 [0.00]

Table 11. Correlation between electricity and temperature

	WHOLE	WINTER	SPRING	SUMMER	AUTUMN
Energy log-price changes and temperature levels	-0.0235	-0.1317*	-0.0500	0.0160	-0.1024*
Energy log-price changes and temperature changes	-0.2102*	-0.2644*	-0.2880*	0.1146*	-0.1163*

* indicates significant at the 5% of significant level.

Table 12. Coefficients in the mean deterministic component

	Energy			Temperature		
	Coeff.	St.dev.	T-stat	Coeff.	St.dev.	T-stat
$\beta_{i,1}$: Constant	2.314	0.148	15.683	5.477	0.202	27.074
$\beta_{i,2}$: Linear trend	0.332	0.062	5.388			
$\beta_{i,3}$: Yearly cosine wave	0.211	0.073	2.888	-10.242	0.297	-34.486
$\beta_{i,4}$: Yearly sine wave				-3.513	0.269	-13.043
$\delta_{1,1}$: Friday	-0.021	0.006	-3.600			
$\delta_{1,3}$: Sunday	-0.031	0.011	-2.935			
$\delta_{1,3}$: Non-working days	-0.065	0.014	-4.681			

The table reports estimated coefficients together with their standard errors and T-statistics. Note that i=1 for Energy and i=2 for temperature. The last three coefficients enter only in the energy equation.

Table 13. Coefficients in the mean dynamic component

	Energy			Temperature		
	Coeff	St.dev.	T-stat	Coeff	St.dev.	T-stat
d_i :Memory	0.397	0.059	6.731	0.189	0.046	4.101
$ø_{1,1,1}$: Energy (t-1)	0.134	0.050	2.693			
$ø_{i,2,1}$: Temperature (t-1)	-0.004	0.003	-1.279	0.194	0.095	2.050
$ø_{1,1,2}$: Energy (t-2)	-0.001	0.046	-0.024			
$ø_{i,2,2}$: Temperature (t-2)	0.109	0.002	52.716	0.001	0.070	0.012
$ø_{1,1,3}$: Energy (t-3)	0.060	0.017	3.487			
$ø_{i,2,3}$: Temperature (t-3)	0.000	0.001	0.031	-0.026	0.014	-1.809
$ø_{1,1,4}$: Energy (t-4)	0.001	0.018	0.043			
$ø_{1,2,4}$: Temperature (t-4)	0.037	0.001	57.233	0.001	0.017	0.044
$ø_{1,1,5}$: Energy (t-5)	0.356	0.017	21.546			
$ø_{i,2,5}$: Temperature (t-5)	0.006	0.001	10.242	0.230	0.015	15.529
$ø_{1,1,6}$: Energy (t-6)	0.203	0.017	11.853			
$ø_{i,2,6}$: Temperature (t-6)	-0.004	0.001	-7.114	0.026	0.017	1.558
$ø_{1,1,6}$: Energy (t-7)	-0.007	0.032	-0.222			
$ø_{i,2,7}$: Temperature (t-7)	0.017	0.002	8.031	-0.168	0.043	-3.908
$\xi_{1,1,1}$: Energy SAR(1)	-0.243	0.034	-7.056			
$\xi_{i,2,1}$: Temperature SAR(1)	-0.007	0.002	-3.074	0.189	0.043	4.450
$\theta_{1,1,1}$: MA(1) Energy	0.372	0.061	6.059			
$\theta_{i,2,1}$: MA(1) Temperature	0.001	0.003	0.340	0.454	0.096	4.733

The table reports estimated coefficients together with their standard errors and T-statistics. Note that i=1 for Energy and i=2 for temperature. Non-significant coefficients are reported in italics.

Table 14. Coefficients of the variance dynamic

	Energy			Temperature		
	Coeff	St.dev.	T-stat	Coeff	St.dev.	T-stat
ω_i: intercept	-0.684	0.317	-2.156	0.645	0.366	1.764
$\alpha_{i,i}$: ARCH	0.176	0.022	7.889	0.042	0.013	3.271
$\alpha_{1,2}$: temperature innovations (t-1)	-0.023	0.014	-1.667			
$\xi_{i,i}$: GARCH	0.730	0.057	12.837	0.618	0.225	2.748
$\xi_{1,2}$: temperature variances (t-1)	0.066	0.029	2.303			
$\gamma_{i,1}$: Yearly Cosine wave	-0.206	0.096	-2.141	0.186	0.117	1.597
$\gamma_{i,2}$: Yearly Sine wave	0.007	0.023	0.285	0.029	0.031	0.944
$\varphi_{1,1}$: Monday	0.754	0.184	4.092			
$\varphi_{1,2}$: Friday	0.498	0.242	2.057			
$\varphi_{1,3}$: Saturday	-0.182	0.461	-0.396			
$\varphi_{1,4}$: Non-working days	0.401	0.264	1.519			

The table reports estimated coefficients together with their standard errors and T-statistics. Note that i=1 for Energy and i=2 for temperature. Non-significant coefficients are reported in italics.

Table 15. Coefficients of correlation dynamic

	Coeff	St.dev.	T-stat
ψ_0: intercept	-0.186	0.013	-14.080
ψ_1: innovations	-0.013	0.006	-2.213
ψ_2: persistence	0.943	0.027	35.001
$\psi_{3,1}$: yearly cosine wave	-0.014	0.005	-2.563
$\psi_{3,2}$: half-yearly cosine wave	0.006	0.002	2.470
$\psi_{3,3}$: quarterly cosine wave	0.005	0.003	2.002
$\psi_{4,1}$: yearly sine wave	*0.001*	*0.002*	*0.127*
$\psi_{4,2}$: half-yearly sine wave	*0.001*	*0.002*	*0.604*
$\psi_{4,3}$: quarterly sine wave	-0.004	0.002	-2.239

The table reports estimated coefficients together with their standard errors and T-statistics. Non-significant coefficients are reported in italics.

Table 16. Monte Carlo simulations of weather and energy variables

This table reports the mean, minimum, maximum, positive and negative semi-deviation of Monte Carlo simulated weather and energy variables appearing in Equation (15) that will be used to compute Quanto options payouts. In Panels A and B the model appearing in Section 4 and estimated in Section 5.2 is used to obtain 10,000 simulations for the whole of year 2008. In Panel C, the Monte Carlo simulation of the energy variable is carried out after the estimated trend of the process for energy prices is substituted with the risk-neutral trend appearing in Figure 2.

Panel A. HDD simulations with the real estimated distribution.

Statistics	January	February	March	April	Autumn
Minimum	332.36	323.43	256.62	128.63	937.57
Mean	688.87	638.02	571.98	397.31	1,528.51
Maximum	1,029.20	970.53	948.53	660.40	2,212.05
Semi-deviation (-)	-53.12	-54.84	-50.10	-40.01	-96.61
Semi-deviation (+)	53.78	53.71	48.98	40.00	98.60

Panel B. Average energy prices simulations with the real estimated distribution.

Statistics	January	February	March	April	Autumn
Minimum	12.95	8.97	13.39	10.11	24.96
Mean	43.50	42.73	38.66	34.98	46.47
Maximum	244.75	198.56	119.97	108.18	106.61
Semi-deviation (-)	-10.09	-13.13	-8.59	-7.25	-5.44
Semi-deviation (+)	4.40	5.54	4.44	4.02	3.79

Panel C. Average energy prices simulations with the risk-neutral (calibrated) distribution.

Statistics	January	February	March	April	Autumn
Minimum	12.75	12.68	18.35	19.42	26.23
Mean	43.30	46.43	43.62	44.28	47.73
Maximum	244.55	202.27	124.94	117.48	107.87
Semi-deviation (-)	-10.09	-13.13	-8.59	-7.25	-5.44
Semi-deviation (+)	4.40	5.54	4.44	4.02	3.79

Table 17. Specification of Quanto options based on HDD and energy price for Oslo

Temperature index (HDD)	Index with base 18°C cumulated during protection period basing on readings from meteorology station in Oslo (WMO 1384)				
Energy Index (E)	Nord Pool spot price (monthly arithmetic average) during protection period				
Tick value	€10 × MWh / HDD				
Maximum payout	No limit				
Payout formula for Quanto option I	MAX[0,(K_2-HDD)*tick*(K_1 - E)]				
Payout formula for Quanto option II	MAX[0,(HDD-K_2)*tick*(K_1 - E)]				
Protection period	January 2008	February 2008	March 2008	April 2008	Autumn 2008
Strike energy (K_1)*	50.11	51.95	48.9	49.6	53.43
Strike HDD (K_2)**	683	622	589	400	1579

* Forward prices (€/MWh) for Oslo electricity prices at Nord Pool in 28 December, 2007; to be used as strikes for energy prices. These prices are closing prices. The Autumn protection period refers to the 4th quarter: October, 1; to December, 31.

** 10 years average HDD decreased by 2 points for each month and decreased by 5 points for each quarter to be used as strikes for weather index with dealer margin included.

Table 18. Euribor(%) December 31st, 2007

Period	act/360	log. Rate
1 week	4.141	4.139
2 weeks	4.175	4.172
3 weeks	4.228	4.223
1 month	4.288	4.280
2 months	4.494	4.477
3 months	4.684	4.657
4 months	4.698	4.662
5 months	4.702	4.657
6 months	4.707	4.652
7 months	4.713	4.649
8 months	4.716	4.643
9 months	4.725	4.643
10 months	4.732	4.641
11 months	4.739	4.639
12 months	4.745	4.636

Table 19. Quanto options valuation

The last row in each panel reports the final price for each given type of option. In both approaches the final price is the mean value discounted with the risk-free rates. Panels A and B represent pure Actuarial approach, while panels C and D represent the Financial approach where in the Monte Carlo simulation the estimated trend of the energy prices is calibrated with the risk-neutral trend (forward curve of energy prices) appearing in Figure 1 and then discounted with the risk-free rates. Panel E shows the the real pay-offs of Quanto options using the true temperature and electricity prices in 2008.

Panel A. Actuarial approach to Quanto option I

Statistics	January	February	March	April	Autumn
Mean	€ 4,966	€ 6,674	€ 7,069	€ 5,385	€ 10,365
Standard deviation	€ 10,227	€ 13,292	€ 10,589	€ 8,671	€ 15,700
Value at Risk 95%	€ 22,430	€ 30,625	€ 29,197	€ 23,441	€ 43,707
Value at Risk 99%	€ 40,050	€ 56,395	€ 46,442	€ 38,416	€ 69,091
Maximum	€ 283,731	€ 243,060	€ 99,101	€ 89,969	€ 133,427
Final price	**€ 4,948**	**€ 6,625**	**€ 6,987**	**€ 5,302**	**€ 9,896**

Panel B. Actuarial approach to Quanto option II

Statistics	January	February	March	April	Autumn
Mean	€ 2,160	€ 3,201	€ 1,730	€ 2,818	€ 2,148
Standard deviation	€ 4,221	€ 5,781	€ 3,834	€ 4,886	€ 5,101
Value at Risk 95%	€ 10,652	€ 15,607	€ 9,898	€ 13,408	€ 12,465
Value at Risk 99%	€ 18,904	€ 26,024	€ 17,688	€ 20,995	€ 24,613
Maximum	€ 95,235	€ 56,792	€ 39,657	€ 43,951	€ 61,797
Final price	**€ 2,152**	**€ 3,178**	**€ 1,710**	**€ 2,774**	**€ 2,050**

Panel C. Financial approach to Quanto option I

Statistics	January	February	March	April	Autumn
Mean	€ 5,000	€ 6,322	€ 5,591	€ 3,450	€ 9,483
Standard deviation	€ 10,260	€ 13,158	€ 8,874	€ 6,013	€ 14,606
Value at Risk 95%	€ 22,590	€ 28,781	€ 23,532	€ 15,081	€ 40,265
Value at Risk 99%	€ 40,455	€ 55,778	€ 39,572	€ 26,760	€ 64,671
Maximum	€ 283,443	€ 251,101	€ 112,137	€ 98,287	€ 127,495
Final Price	**€ 4,982**	**€ 6,275**	**€ 5,526**	**€ 3,397**	**€ 9,054**

Panel D. Financial approach to Quanto option II

Statistics	January	February	March	April	Autumn
Mean	€ 2,206	€ 2,256	€ 1,097	€ 1,133	€ 1,905
Standard deviation	€ 4,280	€ 4,564	€ 2,784	€ 2,459	€ 4,633
Value at Risk 95%	€ 10,846	€ 11,725	€ 6,575	€ 6,201	€ 11,303
Value at Risk 99%	€ 19,078	€ 21,457	€ 13,239	€ 11,490	€ 22,413
Maximum	€ 94,892	€ 49,769	€ 44,689	€ 36,211	€ 56,804
Final price	**€ 2,198**	**€ 2,239**	**€ 1,084**	**€ 1,116**	**€ 1,818**

Panel E. Real Quanto pay-offs in 2008

Quanto option I	€4,697	€15,051	0	€3,247	0
Quanto option II	0	0	€317	0	€1,729

Figure 1. 60 days rolling correlations between energy and temperature variance standardised residuals.

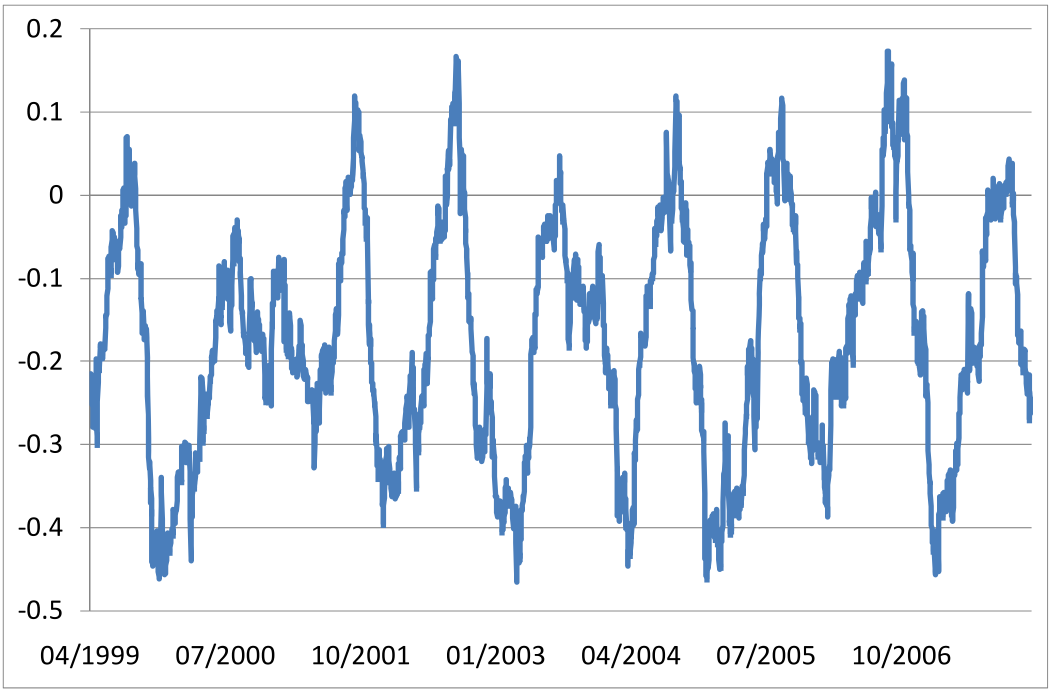


Figure 2. Oslo forward curve for the year 2008 computed on 28 December, 2007.

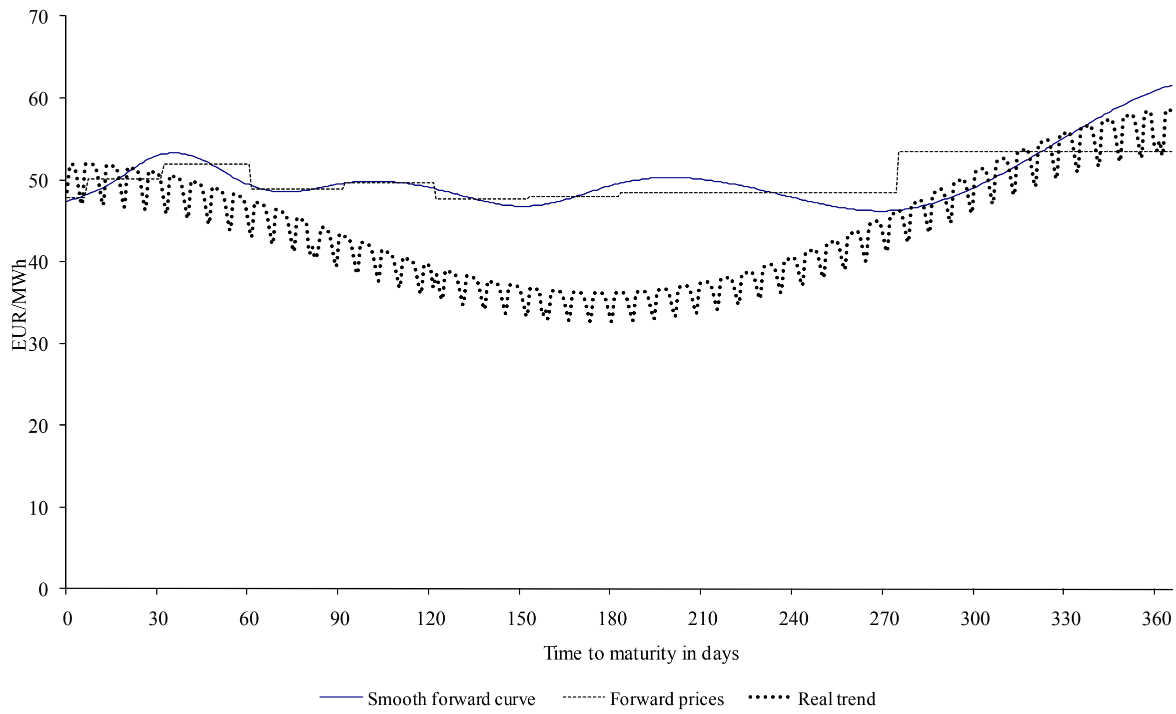


Figure 3. Distribution of simulated HDD values for Oslo for different contract option periods.

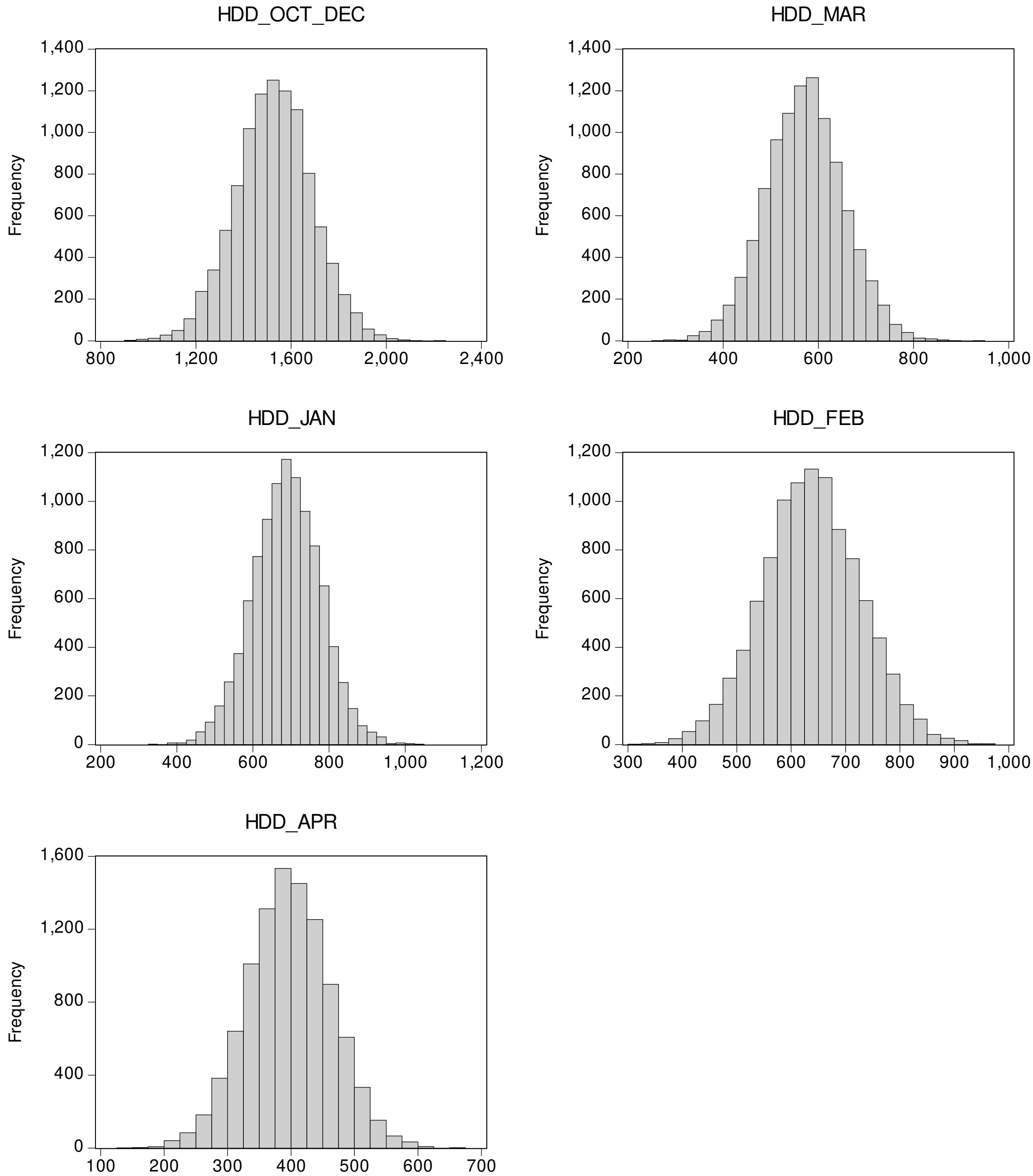
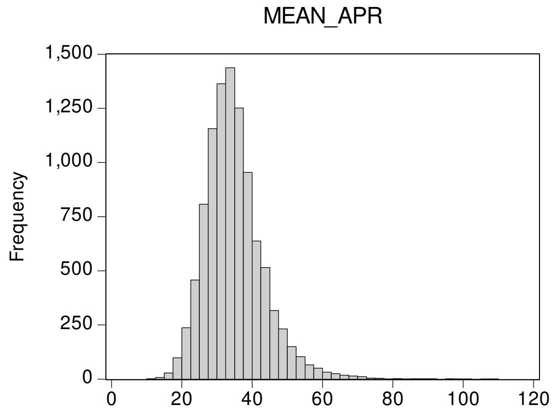


Figure 4. Distribution of simulated energy meanprices for Oslo for different contract option periods.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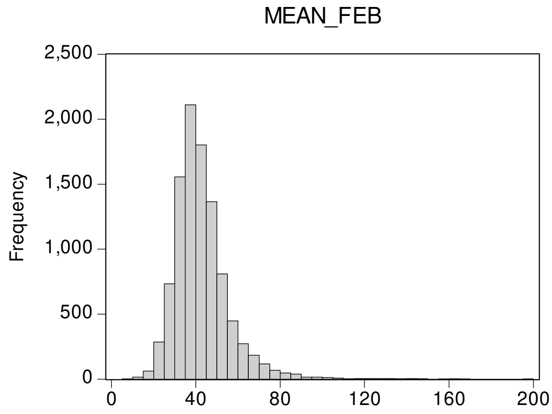
80
100
120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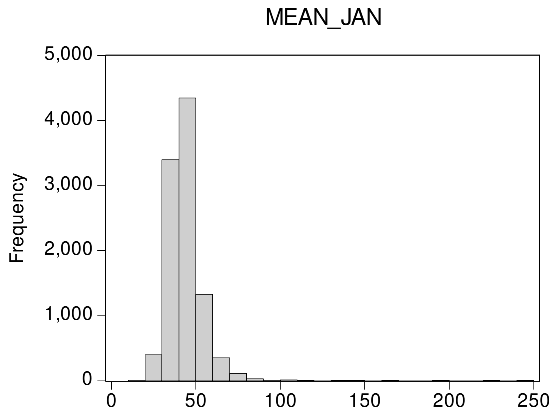
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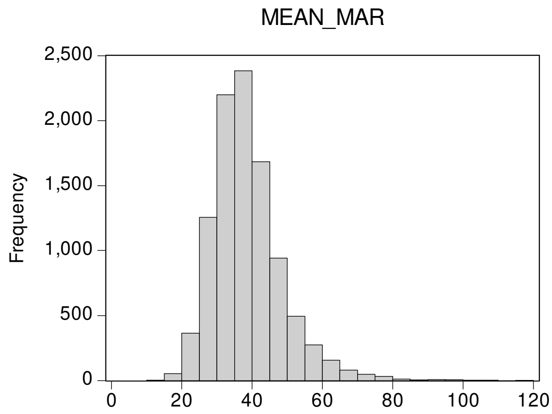
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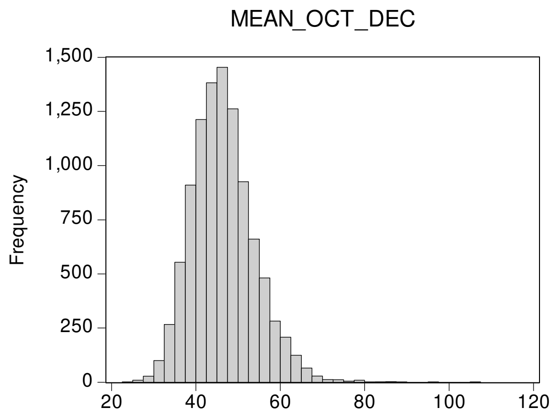
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