# An econometric model to quantify benchmark downturn LGD on residential mortgages

Morone, Marco and Cornaglia, Anna

Intesa Sanpaolo

28 May 2010

# AN ECONOMETRIC MODEL TO QUANTIFY BENCHMARK DOWNTURN LGD ON RESIDENTIAL MORTGAGES

**Marco Morone**[1]
Risk Management
Intesa Sanpaolo, via Monte di Pietà, 26, Torino, 10121 Italy
Email: marco.morone@intesasanpaolo.com

**Anna Cornaglia**
Risk Management
Intesa Sanpaolo, via Monte di Pietà, 26, Torino, 10121 Italy
Email: anna.cornaglia@intesasanpaolo.com

First Draft: May 28th, 2010

[1] The opinions expressed in this paper are those of the authors and do not necessarily reflect views of IntesaSanpaolo bank.

## ABSTRACT

*The paper describes a theoretical approach to determine the downturn LGD for residential mortgages, which is compliant with the regulatory requirement and thus suited to be used for validation, at least as it can give benchmark results. The link between default rates and recovery rates is in fact acknowledged by the regulatory framework as the driver of the downturn LGD, but data constraints do not usually allow for direct estimation of such a dependency. Both default rates and LGD parameters can anyway be related to macroeconomic variables: in the case of mortgages, real estate prices are the common driver. Household default rates are modelled inside a Vector Autoregressive Model incorporating a few other macroeconomic variables, which is estimated on Italian data. Assuming that LGD historical data series are not available, real estate prices influence on recovery rates is described through a theoretical Bayesian approach: possession probability conditional to Loan to Value can thus be quantified, which determines the magnitude of the effect of a price increase on LGD.*

*Macroeconomic variables are then simulated on a five years path in order to determine the loss distribution (default rates times LGD per unit of EAD), both in the case of stochastic price dependent LGD and of deterministic LGD (but still variable default rates). The ratio between the two measures of loss, calculated at the 99.9$^{th}$ percentile for consistency with the regulatory formulas, corresponds to the downturn effect on LGD. In fact, the numerator of the ratio takes into account correlations between DR and LGD.*

*Some results are presented for different combinations of average LGD and unconditional possession probability, which are specific for each bank.*



KEYWORDS: **downturn LGD, default and recovery rates correlation, mortgage, Loan to Value, real estate price, possession probability, Bayesian approach, stress testing, Vector Autoregression**

## 1. Introduction

Basel 2 framework document requires that estimated *LGD* parameters must "reflect economic downturn conditions where necessary to capture the relevant risks". In fact, the potential for realized recovery rates to be lower than average during times of high default rates may be a material source of unexpected credit losses for some exposures or portfolios.

*PD* parameters are prudentially transformed inside the regulatory formulas, according to the hypothesis of a normal distribution of the underlying asset, of which the 99.9$^{th}$ percentile is taken for capital calculation. The same does not happen for the *LGD* parameter, which is thus required to be by itself conservative enough to be consistent with *PD*.

Downturn LGD validation represents a challenging task. In fact, how to determine downturn *LGD* was not explicitly defined in Basel 2 document, thus leaving space for different methodologies to be developed and compared. Some further guidance (*BCBS*, 2005) went more into details, but still preferred a principle based approach, which identified the basic components or steps to be followed in the process of estimating the downturn *LGD*, namely: identifying the appropriate downturn conditions for each asset class and jurisdiction; identifying adverse dependencies, if any, between default rates and recovery rates; incorporating such dependencies in order to produce parameters which are consistent with identified downturn conditions. It was thus stated as a milestone that the relationship between *LGD* and default rates (and implicitly *PDs*, which should be correct estimators of default rates) should drive the downturn *LGD* calculation, and this represents a leading principle to be taken into account through validation. Still, limited data availability represents a challenge for downturn *LGD* estimation and validation, as it makes very difficult to transpose the regulatory indications into a best practice.

Data constraints do in fact condition estimation of *LGD* in general, as it needs historical series covering the entire default duration for each exposure, and downturn *LGD* in particular. This in fact also requires to identify dependencies between parameters (*LGD* and *DR*) which are by nature different from each other, at least as far as time horizon is concerned.

For mortgage portfolios specifically, historical losses are strongly downward biased because of the extremely favorable performance of variables like house prices and interest rates in the available observation window. This scenario has already experienced a downturn, but a time gap will elapse before recession effects can be incorporated into the *LGD* parameters estimates. Furthermore, the source of recoveries shows up to be relevant in order to understand the link between the economic cycle and losses on mortgage exposure, but the length of the observation window is not enough for collecting a sufficient number of observations, under downturn conditions, on facilities terminated by foreclosure on the mortgaged asset. It is thus difficult for a bank to directly obtain reliable internal estimates of all the required parameters using only internal data.

Some alternative methodology should thus be used in order to determine the downturn *LGD* while still be compliant with the regulatory requirements. This represents an evolving area, which is expected to generate new approaches and to lead to new best practices in the sector. Up to now, the procedures used in practice can be retrieved to two main approaches: the ones which are based on future flow estimates under downturn conditions and the ones which stress *LGD* explanatory variables. The model which is outlined in the rest of the document can be inserted in this second stream of work.

The link to macroeconomic variables is in fact established for both default rates and *LGD*: even if this is done according to two different models, the use of a common driver which is the real estate price allows us to stress the two parameters simultaneously, implicitly deriving a relation between them.

The dynamics of macroeconomic variables (GDP, interest rates, real estate prices) and household default rates are modeled through a Vector Autoregressive model, built on quarterly data (paragraph 3). Once estimated, the model makes it possible to simulate the residuals (structural shocks) in order to get a joint distribution of the included variables. Householders default rate and real estate price are in this case crucial, as they are the key variables to establish the relationship between *PD* and *LGD*.

As far as *LGD* is concerned, a theoretical approach is needed to overcome data constraints that prevent us from direct estimation (paragraph 4): the idea here is to catch the effect of real estate prices on *LGD* taking into account that the only recovery component they influence is the one derived from foreclosure termination. But linked to real estate value is also the weight attributed to this source of recovery, or the possession probability, which is in fact conditional on Loan to Value: a decrease in real estate value will reduce equity the borrower holds in the property, so that he will be encouraged to give the asset to the lender rather than repaying the debt.

Again, direct estimation of possession probability dependency on Loan to Value is not usually possible due to data shortage. A Bayesian approach is thus followed here, whose parameters can be calculated on external or internal data.

Simulated values of real estate prices are used to feed *LGD* calculation according to the outlined approach: this allows us to associate to each simulated value for default rate a simulated value for LGD (paragraph 5) and to multiply them in order to derive a loss distribution (per unit of *EAD*) in the case of stochastic (and correlated with PD) *LGD*. If a deterministic *LGD* value, e.g. equal to its expected value, is used for calculating the losses corresponding to each simulated default rate, the resulting distribution will not take into account *LGD* sensitivity to macroeconomic conditions. The ratio between the 99.9$^{th}$ percentiles of the two distributions can be interpreted as the downturn effect on *LGD*, consistent with Basel 2 framework.

The model as such seems to respect the regulator's intention when addressing the problem of the margin of conservatism that should characterize *LGD* estimated parameters, at least as it takes into account the link between *LGD* and the credit cycle, while giving results that are perfectly suited to be inserted into the capital requirement calculation.

In the present application, while the *VAR* model is estimated on Italian systemic data, the theoretical model for *LGD* is not given a parameterization relying on external or internal datasets. Rather, we preferred here to present results for a range of different, but realistic, values of expected *LGD*, possession probability and average *LTV*. This allows us to perceive the estimated downturn effect sensitivity to parameters that are specific for each bank. Also, the parameters, relying on a macroeconomic model that must be country-specific, can represent a benchmark to be used while validating LGD results, even if obtained through different methods and starting from different assumptions.

## 2. Literature review

The framework to which all studies on downturn *LGD* refer is the *BCBS* (2005) document, that aimed at providing guidance on how to meet the regulatory requirement that *LGD* parameters used in Pillar I capital calculations must reflect economic downturn conditions. This defined some principles or guidelines to be respected, inside which all approaches, even if quite different from each other, can be collocated.

Apart from theoretical works, some review of the practices which are actually followed is presented in the *CEBS* (2008) document on Basel II implementation issues.

Being the dependency between default rates and recovery rates crucial for downturn *LGD* quantification, a first stream of literature directly analyzes the correlation, and its implication on prociclicality and economic/regulatory capital calculation. This can be done on aggregate default rates and on the *LGD* implicit in corporate bond price after default. In this respect, Altman et al. (2005), find a positive correlation resorting to an econometric multivariate model which includes also macroeconomic indicators and some proxies of high-yeld bond market behavior. Among others, Dullman and Trap (2004) such as Hillebrand (2006), Barco (2007) all propose a model for quantifying the dependence of recoveries and default rates on a common systematic risk: their estimations rely on *single factor model* and on time series collected from rating agency database.

The present work uses two different approaches for modelling the effect of the economic conditions to respectively default rates and *LGD*.

Thus, part of the literature that inspired it just concerns the relationship between default rates and macroeconomic variables through simultaneous equation systems or Vector Auto Regressive approach. In particular, Virolainen (2004) analyzed Finnish default data with a *SUR* model: sectorial default rates were simultaneously explained by GDP deviation from its trend, short interest rate and indebteness level.

As far as Italian case is concerned, Marcucci and Quagliariello (2005) provided evidence of a link between Italian household arrears and some macroeconomic variables such as output gap, household indebtedness, inflation and short interest rate using a quarterly *VAR* model. Our results, although based on a slightly different time frame, variable selection and lag structure, appear to be quite consistent with their findings in terms of default rate dynamics and response to GDP and to interest rate.

Hoggarth et al. (2005) explained the aggregate corporate and household bank's write-off on UK data: for the latter they included real income, inflation rate, short interest rate and real estate inflation. Wong J. (2006) applied a VAR system to credit exposures of Hong Kong's retail banks employing GDP growth, interest rate and property prices, the latter used to generate *LGD* values in an expected loss simulation exercise. More recently, Avouyi et al. (2009) found a quite robust relationship among French corporate default rates, GDP growth rate, short interest rates and corporate average spreads as market default predictors.

The link of *LGD* on mortgages to the macroeconomic environment, as expressed by the real estate price, is here established by means of a theoretical model. The approach adopted to relate *LGD* and collateral value through foreclosure (possession) probability is supported by the Bank of Spain validation document (2007) and by an FSA (2008) survey meant to quantify downturn *LGD* on UK residential mortgage.

As far as downturn *LGD* calculation is concerned, the task usually consists in solving for the downturn *LGD* input so that it gives a value of regulatory/economic capital identical to the model with systematically correlated default rate and *LGD*. Altman et Al. (2001), Miu and Ozdemir (2006), Chabaane et Al. (2004) but also some of the aforementioned authors, despite the use of different correlation structures and parameterization, explore this issue and outline the relevant impact of downturn evaluation.

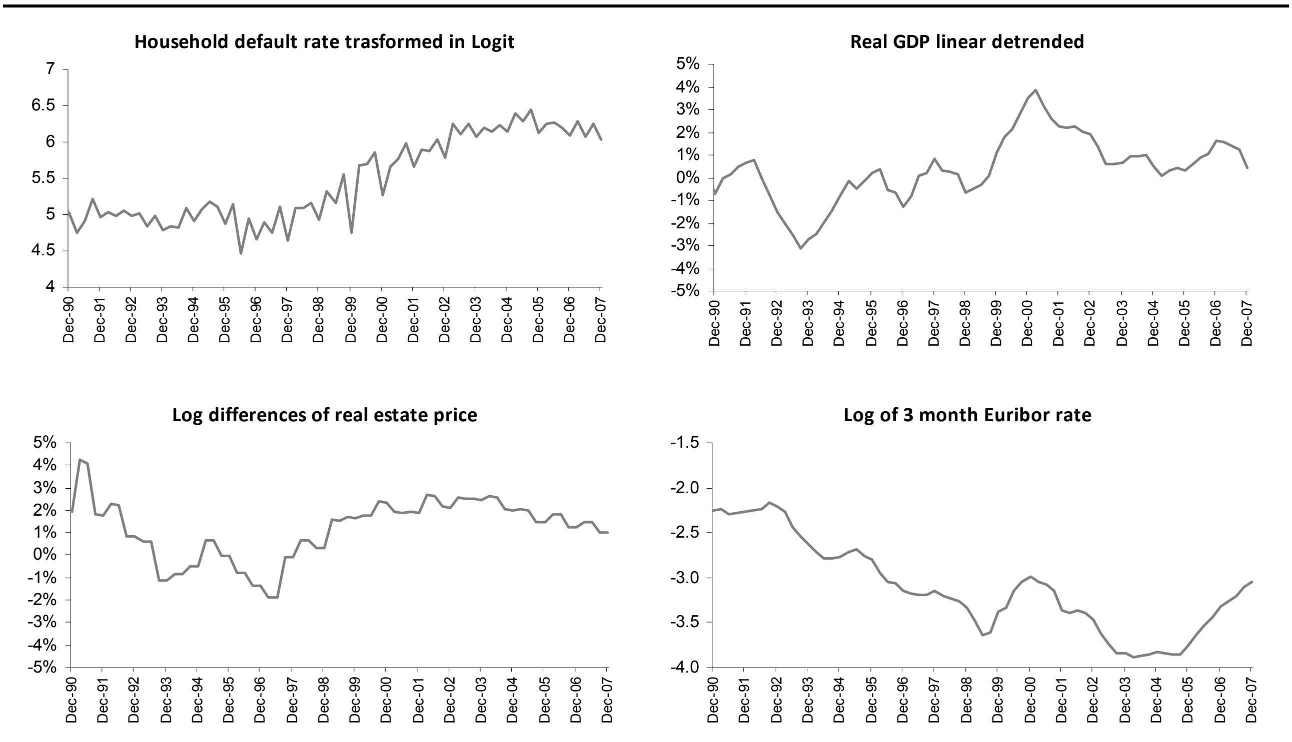
## 3. The relation between default rates, real estate price and macroeconomic variables

To analyze the transmission mechanism of macro-financial shocks to household default rates and real estate price, a Vector Auto-Regressive (*VAR*) model was formulated. *VAR* models are generalized form of simple autoregressive process for *n* variables: they allow to investigate mutual relationship among variables even in case of time series non-stationarity:

$$Y_t = C + \beta \cdot S + \mathrm{A}_1 \cdot Y_{t-1} + \ldots + \mathrm{A}_p \cdot Y_{t-p} + \varepsilon_t \qquad \textbf{[3.1]}$$

In [3.1] representation, $C$ is a constant vector, $S$ a matrix of seasonal dummy variables, $\beta$ and $\mathrm{A}_i$ are matrices of parameters, $\varepsilon_t$ a vector of correlated residuals/shocks. $Y_t$ is the vector of endogenous variables including the household aggregate default rates (transformed in Logit), the Italian real GDP (linearly detrended), the log changes in real estate price index and the short term interest rate (logarithm of three month Euribor rate)[2].

**Figure 1: Macroeconomic variable used in *VAR* system**




Modeling the dynamics of the aforementioned macroeconomic variables using a *VAR* has the advantage that impulse response analysis can be figured out: once the system parameters are estimated, it becomes possible to simulate a number of shocks to the macroeconomic variables

[2] Inside household default rate equation, three point dummy variables on 1996Q2, 1999Q4 and 2000Q4 were introduced for the estimation: the aim was removing the detected outlier effect during these three mentioned quarters.

and consider the feedback from these shocks to the householder default rates and real estate price. In this work, the latter are in fact the key variables to establish the relationship between *PD* and *LGD*.

The time series used for the *VAR* estimation are quarterly and extend from 1990 second quarter to 2007 fourth quarter[3], resulting in 71 available observations depicted in Figure 1. As mentioned before, each variable was treated as follows:

- a Logit transformation for **Italian household default rate** (LHDR) was set in order to limit its range between 0 and 1. The default rate is collected by Bank of Italy and can be defined as the ratio between the number of Italian loans classified as bad debts in the reference quarter and the number of performing loans outstanding at the end of the previous one;
- **Italian real GDP** was detrended ($GDP_{det}$) using a linear trend from 1990_Q2 to 2009_Q3: the new computed variable can also be called output gap, which is frequently used in the empirical literature aimed to explain the default rate behavior, as the ability of agents to service their debt;
- **real estate price index** (based on the average price of Italian dwelling) was expressed in terms of log differences (ΔLogRE). This variable is crucial in explaining household default rate: in fact, real estate represents the major source of collateral and so, if its value declines, the incentive to continue servicing the debt will weaken;
- finally, **three months Euribor rate** was expressed in logarithm (LogR3M) and directly affects the burden of the debt.

Uniroot tests such as *Augmented Dickey Fuller* and *KPSS* were conducted and all the selected variables appeared to be non-stationary (see tables A1 and A2): since *Johansen* Cointegration test performed on the system (see table A3) did not reject the existence of one cointegrated relationship, it is most likely that any stochastic common trend will be picked up by the *VAR*.

As far as the estimation technique is concerned, *OLS* method was used and 3 time lags ($p$=3) were chosen on the basis of *AIC* (Akaike Info Criterion) and *HQC* (Hannan-Quinn Criterion) lag selection tests (see table A4).

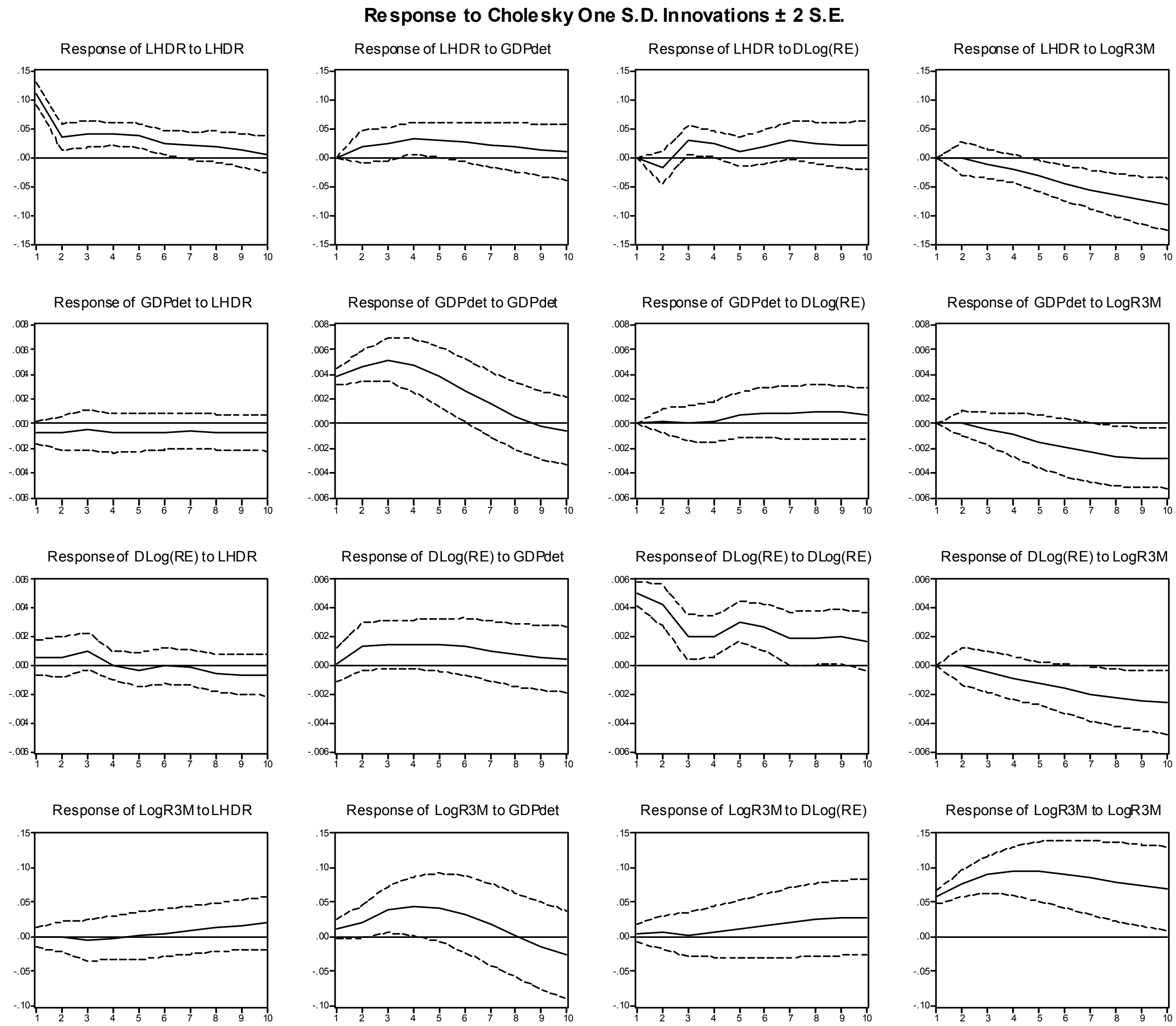
The impulse response functions provide a detailed picture of the dynamics of the variables. Figure 2 shows the impulse responses of each variable to one standard deviation shocks of the others: since variance-covariance matrix of *VAR* residual/shocks[4] is unlikely to be diagonal, the residuals need to be orthogonalized, e.g. using a *Cholesky* decomposition as done in this case.

This kind of analysis visually suggests some dependencies among the variables:

[3] 2008 and the first three quarters of 2009 were excluded from the estimation sample to perform on them out-of-time tests.

[4] Besides, $e_t$ residuals appear to follow a Gaussian distribution as confirmed by *Jarque-Bera* tests in Table A5, except for real estate which tends to reject the normality assumption at 5% confidence level. See Figure A1 for their graphical representation.

**Figure 2: Impulse response functions with +/- two standar error bands**




- default rate (Logit DR) depends negatively (positively) on real GDP shock and decreases (increases) significantly within the first four quarters;
- a positive shock in real estate returns implies a significant reduction in household default rate;
- the response of default rate to short term interest rate is positive and rather significant after the fourth quarter;
- as far as GDP is concerned, there is evidence of a strong negative relationship with short term rate;
- the real estate returns response is positive with respect to GDP and negative to interest rate: looking at +/-2 standard errors bounds, the feedback effect of default rate can't be strongly supported;

- finally, what emerges from the last group of pictures is a positive reaction of interest rate to real GDP.

Table 1 and 2 indicate the percentage of the forecasting errors explained by each variable at a given time horizon for householder default rate and real estate price changes, also known as variance decomposition:

**Table 1: Logit(Default Rate) variance decomposition**

| Forecast horizon | LHDR | $GDP_{det}$ | ΔLog(RE) | LogR3M |
|---|---|---|---|---|
| 1 | 100% | 0% | 0% | 0% |
| 2 | 94% | 3% | 3% | 0% |
| 3 | 85% | 7% | 7% | 1% |
| 4 | 75% | 12% | 9% | 3% |
| 5 | 69% | 16% | 9% | 6% |
| 6 | 61% | 19% | 9% | 12% |
| 7 | 52% | 19% | 10% | 19% |
| 8 | 45% | 19% | 10% | 26% |

The default rate variable is mainly explained by the default rate itself: at the fourth quarter horizon, 12% of LHDR forecast error is explained by GDP, 9% by real estate and 3% by interest rate. After 2 years, a reduction in LHDR power can be observed, in favor of the other macroeconomic variables, with a slight predominance of short term Euribor

**Table 2: ΔLog(RE) variance decomposition**

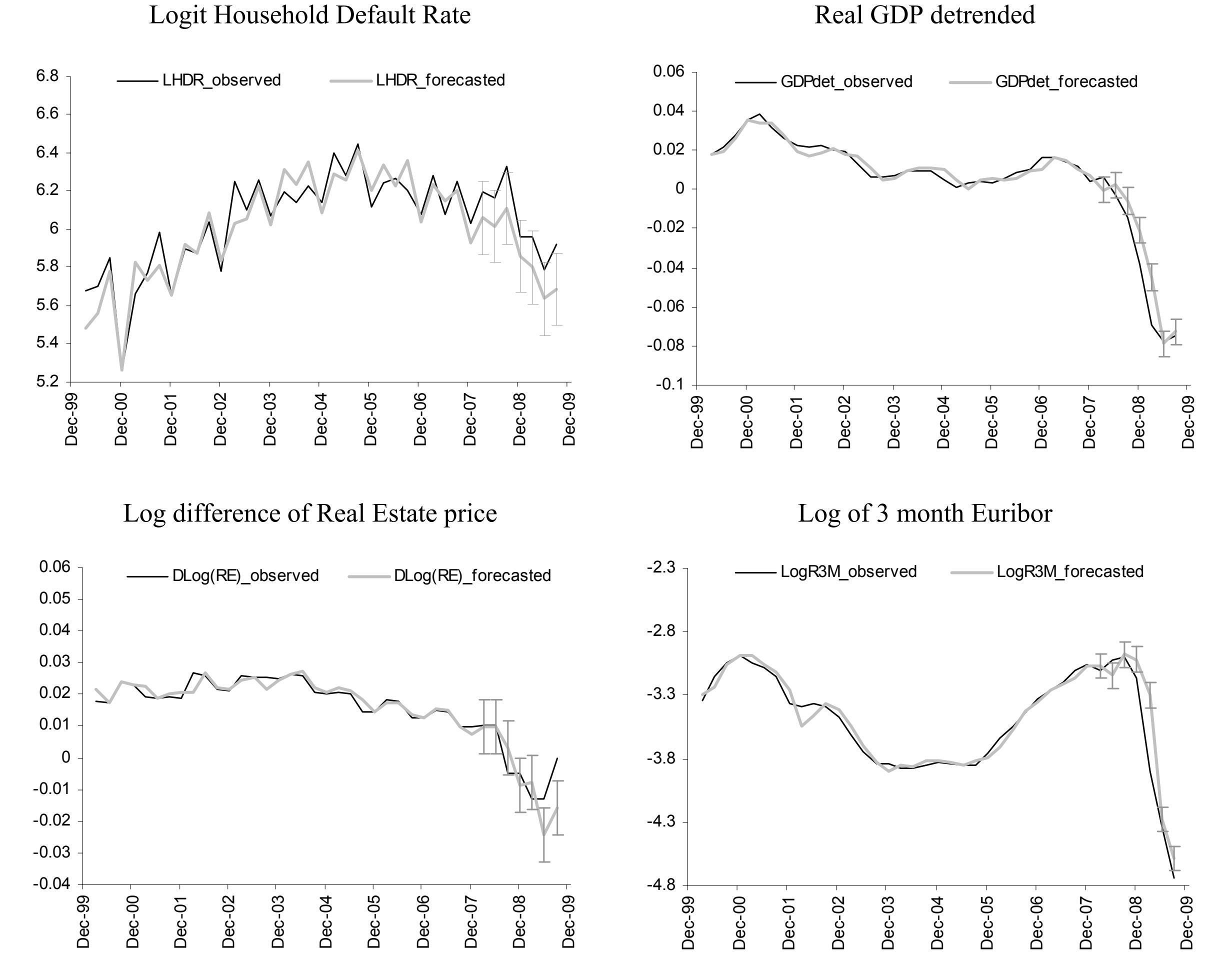
| Forecast horizon | LHDR | $GDP_{det}$ | ΔLog(RE) | LogR3M |
|---|---|---|---|---|
| 1 | 0% | 0% | 100% | 0% |
| 2 | 0% | 3% | 96% | 0% |
| 3 | 2% | 7% | 91% | 0% |
| 4 | 1% | 11% | 86% | 1% |
| 5 | 1% | 12% | 83% | 3% |
| 6 | 1% | 13% | 80% | 5% |
| 7 | 1% | 14% | 76% | 9% |
| 8 | 2% | 14% | 73% | 12% |

In Table 2, the high persistency of Δlog(RE) in explaining itself can be noticed: however, at the eighth quarter time horizon real GDP appears to be the second powerful variable with 14% explained variance.

An out of sample exercise on the *VAR* model was also conducted in order to check for its robustness through the last downturn period, experienced during the years 2008 and 2009.

Figure 3 depicts the dynamic one-quarter ahead forecast (the “true” out of sample observations are used for lagged value in the system) for each variable, starting from the last quarter of 2007.

**Figure 3: Dynamic one quarter ahead forecast with 95% error bands**[5]




As shown in the graphs, the forecasted time frame can be deemed as a factual structural break, especially for real detrended GDP, which fall down to around -8% causing an overestimation (underestimation) of household default rates (Logit): this results in observed default rates (Logit) lying near the lower (upper) 95% confidence bounds, which however grant a conservative forecast of the model in case of severe recession phases.

Finally, iterating forward the *VAR* system several times with the information set at 2007_Q4, we found a converging long run solution for each variable shown in table 3 and 4 (in which LHDR was transformed into default rate HDR and LogR3M into the three-month Euribor rate R3M):

**Table 3: Quarterly long run solution**

| Quarter | HDR | $GDP_{det}$ | $\Delta$LogRE | R3M |
|---|---|---|---|---|
| 1 | 0.30% | 0.08% | 0.64% | 3.42% |
| 2 | 0.32% | 0.01% | 0.53% | 3.43% |
| 3 | 0.29% | 0.06% | 0.21% | 3.43% |
| 4 | 0.37% | 0.04% | 0.38% | 3.45% |

[5] The black line represents the observed variable, the grey the forecast; segments indicate the 95% confidence bounds.

**Table 4: Long run solution: annualized values**

| HDR | $GDP_{det}$ | ΔLogRE | R3M |
|---|---|---|---|
| 1.28% | 0.05% | 1.76% | 3.43% |

Since the outcomes frequency is quarterly based, with dummies seasonal effect (variable *S* in equation [2.1]), household default rates and real estate returns were transformed to hold a one-year time horizon[6] (see Table 4).

## 4. A Bayesian approach to assess *LGD* and real estate prices connection

Recovery flows, which determine the observed *LGD* once a default cycle is closed, are not all deriving from the same source. For the purpose of our analysis, the relevant distinction is between recoveries deriving from foreclosure on the mortgage asset, i.e. the sale of the real estate property, and other sources of recovery (e.g. the counterparty paying back part of his debt in order to protect his investment, or cured positions).

From the bank point of view, only the first recovery source is affected by a change in real estate prices. But its weight on *LGD* depends on possession probability (or foreclosure probability), which in turn depends on Loan to Value.

To quantify the sensitivity of residential mortgage Loss Given Default to the systematic risk movements, an equation that establishes the relationship between *LGD* random variable[7] and real estate price should be formulated[8]:

$$LGD \mid \Delta\%re_t = 1_{P(Po|LTV,\Delta\%re_t)} \cdot (LGD \mid Po, \Delta\%re_t) + [1 - 1_{P(Po|LTV,\Delta\%re_t)}] \cdot LGD \mid \overline{Po} \qquad \textbf{[4.1]}$$

$$(LGD \mid Po, \Delta\%re_t) = \max\{[1-(1-LGD \mid Po)\cdot(1+\Delta\%re_t)], 0\} \qquad \textbf{[4.2]}$$

where:

- $1_{P(Po|LTV,\Delta\%re_t)}$ is the possession random variable, which takes 1 with probability $P(Po \mid LTV, \Delta\%re_t)$ and 0 otherwise (*Po* stands for possession event, *LTV* represents the Loan To Value random variable on defaulted loan portfolio, $\Delta\%re_t$ is the real estate percentage change at time *t*);

[6] A survival probability rule was adopted for DR: $\prod_{q=1}^{4}(1-HDR_q) = 1 - HDR$. Quarterly real estate returns were capitalized within the year $\prod_{q=1}^{4}(1+\Delta LogRE_q) = 1 + \Delta LogRE$. $GDP_{det}$ and R3M were averaged within the year.

[7] Capital letter indicates the random variables, small letter their realizations.

[8] Approach also suggested by the Bank of Spain (Banco De Espana (2007)) for downturn LGD estimation.

- $P(Po \mid LTV, \Delta\%re_t)$ is the possession probability conditional to Loan To Value and to systematic factor $\Delta\%re_t$. If *LTV* decreases, borrowers will have more equity in the property (collateral) and the probability of forced sales or foreclosure will reduce since they can raise money or guarantee to preserve their investment. If *LTV* exceeds 100% for instance because of a sharp downturn in real estate market, the negative equity will encourage borrowers to loose the property and let the lender to recover through foreclosure. Thus, during a downturn phase, the proportion of defaulted facilities expected to be terminated by foreclosure will be much higher than the long run rate[9]: $\Delta\%re_t$ works in fact as systemic risk modifying the mean of *LTV* distribution.
- $(LGD \mid Po, \Delta\%re_t)$ is the *LGD* conditional to foreclosure termination and to $\Delta\%re_t$ realization as in [4.2];
- $LGD \mid Po$ is the *LGD* conditional to foreclosure termination observed in the long run period. Its distribution should be estimated by the bank, exploiting the defaulted positions database (including the costs of foreclosure procedures);
- $LGD \mid \overline{Po}$ is the *LGD* conditional to the final "cure" of defaults or other situations complementary to foreclosure such as further borrower flows after default. By assumption it does not depend on real estate market behavior. Also in this case, its distribution can be figured out by banks non-performing loans dataset;
- $\Delta\%re_t$ is the percentage change in real estate price coming from $\Delta\%RE_t$ distribution[10], evaluated by means of *VAR* model described in the previous paragraph which affects both $P(Po \mid LTV, \Delta\%re_t)$ and $(LGD \mid Po, \Delta\%re_t)$ as stated in equations [4.1] and [4.2];

Now, since our focus is to model the systemic behavior of stochastic *LGD*, if we assume a fully granular defaulted loan portfolio following the strong law of large numbers and consider the conditional independence between possession event $1_{P(Po \mid LTV, \Delta\%re_t)}$ and $(LGD \mid Po, \Delta\%re_t)$ variates, equation [4.3] holds:

$$\begin{aligned} elgd_t = E(LGD \mid \Delta\%re_t) &= P(Po \mid \Delta\%re_t) \cdot E(LGD \mid Po, \Delta\%re_t) \\ &\quad + [1 - P(Po \mid \Delta\%re_t)] \cdot E(LGD \mid \overline{Po}) \end{aligned} \qquad \textbf{[4.3]}$$

[9] The use of *LTV* as a key variable for the modeling of possession probability is consistent with the option theory approach (see for instance Van Order, R. (2008)) used for valuing debt, whereby the equity holders (household in this case) are considered to hold a put option on the value of the collateral where the strike price is debt. The rationale is that if the asset value falls below the debt value the equity holders are better off by giving the asset to the lenders rather than repaying the debt.

[10] Here, the percentage changes correspond to log differences used in *VAR* system.

It describes the whole portfolio *LGD* ($elgd_t$) conditional to the macroeconomic environment in *t*: $E(LGD \mid Po, \Delta\%re_t)$ and $E(LGD \mid \overline{Po})$ are the average portfolio *LGD* conditional respectively to possession and non-possession events, $P(Po \mid \Delta\%re_t)$ is the average possession probability across all the counterparties. Here, the main insight is that, given a certain economic cycle realization, total *LGD* results in the sum of the average *LGDs* conditional on different types of termination, weighted by their respective probabilities of occurrence. Real estate channel will affect both the $E(LGD \mid Po, \Delta\%re_t)$, by increasing or decreasing the collateral value (see [4.2], replacing $LGD \mid Po$ with $E(LGD \mid Po)$ in the right side), and the average possession probability $P(Po \mid \Delta\%re_t)$ through Loan To Value distribution change in mean.

As far as $P(Po \mid LTV, \Delta\%re_t)$ is concerned, it could be estimated on internal data, if available, by logistic approach, using the 0/1 event of foreclosure as dependent variable and *LTV* coupled with other variables as independent. If internal loss data are not complete enough to yield to such an estimation (that is, if a sufficiently long historical window containing observations under downturn conditions and a sufficient number of facilities terminated by foreclosure are lacking in database), a Bayesian approach could be useful, possibly resorting to external data source:

$$P(Po \mid LTV) = \frac{f_{Po}(LTV, \mu_{Po}, \sigma_{Po}) \cdot P(Po)}{f_{Po}(LTV, \mu_{Po}, \sigma_{Po}) \cdot P(Po) + f_{\overline{Po}}(LTV, \mu_{\overline{Po}}, \sigma_{\overline{Po}}) \cdot [1 - P(Po)]} \qquad \textbf{[4.4]}$$

$P(Po \mid LTV)$ is the long run possession probability conditional to *LTV* distribution. The denominator, that is the long run unconditional *LTV* distribution $f(LTV, \mu, \sigma)$, can be obtained from internal data, fitting the empirical distribution or using a parametric density (*Normal*, *Lognormal*, *Beta*) with mean ($\mu$) and variance ($\sigma$) corresponding to observed *LTV* statistics. The first component of numerator, which represents the long run *LTV* distribution conditional to foreclosure event, could be computed by examining historic possessions from external source or by some reasonable and prudential assumption about internal *LTV* distribution. Finally, $P(Po)$, defined as the long run possession probability (prior in Bayesian formulation), can be based on internal figures, on recession-based forecasts or again on external data statistics.

In order to determine $P(Po \mid \Delta\%re_t)$ in equation [4.3], and thus to incorporate a time-dependency behavior in $P(Po)$, we admit time-dependence only via the change in mean of unconditional *LTV* density[11]:

[11] See Tashe (2006), according to which it is possible to deal with cyclical effect modifying (shifting) either the unconditional distribution as we did or the prior. In the latter case we should have enough historical information and data to develop a model aimed at *P*(*Po*) forecasting.

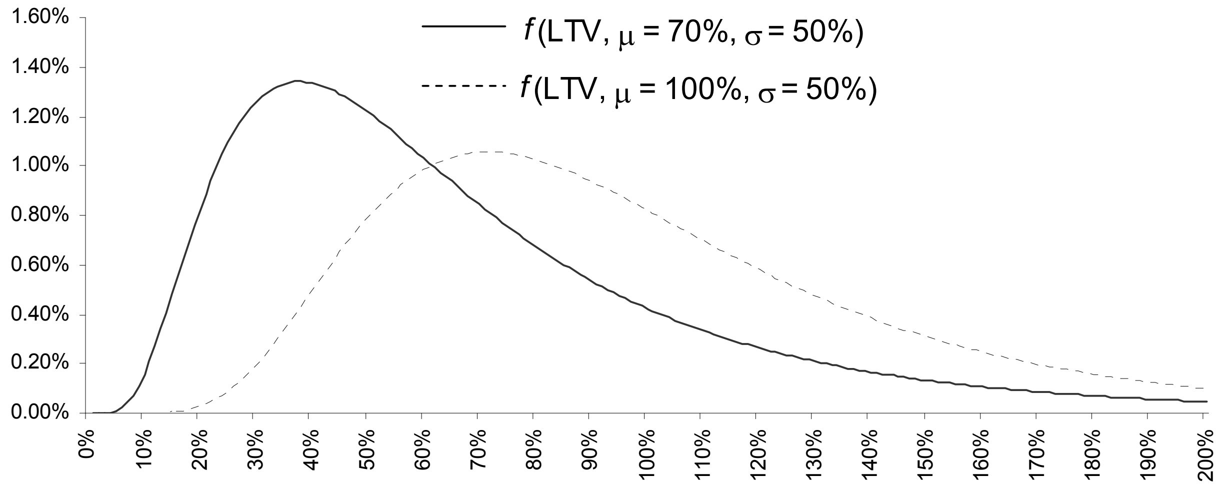
$$P(Po \mid \Delta\%re_t) = \int_0^{\infty} P(Po \mid LTV) \cdot f(LTV, \mu/(1+\Delta\%re_t), \sigma) \cdot dLTV \qquad \textbf{[4.5]}$$

while if $\Delta\%re_t = 0$ we catch exactly the average long run foreclosure probability

$$P(Po) = \int_0^{\infty} P(Po \mid LTV) \cdot f(LTV, \mu, \sigma) \cdot dLTV$$

In this fashion, economic up or downturns are only reflected by a shift of the *LTV* distribution towards the best or worse *LTVs*.

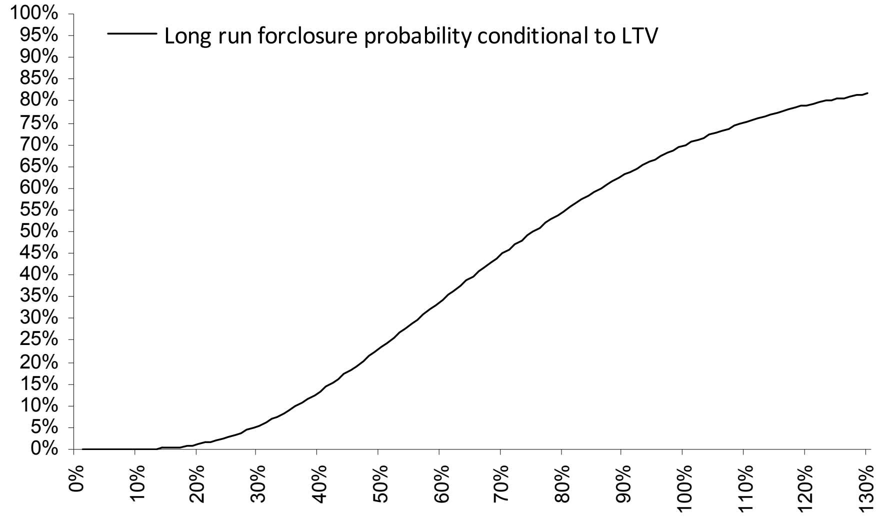
**Figure 4: Long run $f$ and $f_{Po}$ distributions used**




In the present work, to describe the unconditional *LTV* density, we adopted a lognormal distribution with mean and standard deviation respectively equal to 70% and 50% (although further tests with 50%, 60% and 80% $\mu$ values, leaving the volatility fixed, were implemented), trying to render a typical long run *LTV* distribution at default, on an Italian mortgage portfolio. Concerning the foreclosure conditional *LTV*, the same distribution was prudentially shifted in order to reach a 100% $\mu_{Po}$ value (Figure 4), assuming that borrowers are willing to loose the property when *LTV* amounts to 100% on average. The left side of this distribution (*LTV* < 100%), referring to positive equity positions, encompasses for unexpected events that make it difficult to avoid forced sales such as job loss, a significant change in health status, and changes in family structure (see Qi, M., Yang, X. (2007) for more details about *LTV* relevance in *LGD* estimation).

Regarding $P(Po)$, some options were experimented: following the shape of the two aforementioned distributions and thus equation [4.4] for long run possession probability, it was assumed alternatively being equal to 30%, 35% and 40% (see Figure 5). 35% is the median value for possession probability derived from a Financial Services Authority study (FSA 2008), conducted on a sample of 10’000 UK mortgage residential exposures.

**Figure 5: *P(Po | LTV)* with *P(Po)* set to 35%, $\mu$ = 70%, $\mu_o$ = 100%, $\sigma = \sigma_{Po}$ = 50%**




For the sake of simplicity, although banks should distinguish between $E(LGD \mid Po)$ and $E(LGD \mid \overline{Po})$ inside [4.3], for this exercise a unique value of long run *LGD* was adopted (*ELGD*), assuming that on average (i.e. in the long period where also $\Delta\%re_t = 0$), the bank recovery process is able to guarantee $E(LGD \mid Po)$ and $E(LGD \mid \overline{Po})$ neutrality. Furthermore, from a computational point of view, equation [4.5] was determined by numerical integration.

## 5. Quantifying downturn LGD through simulation

After having defined a macroeconomic model for household default rates and collateral value (approximated by real estate price index changes) and properly quantified the sensitivity of *LGD* to collateral movements, the algorithm described here explains the main steps adopted to compute the downturn *LGD (DLGD)*. The aim is to incorporate, by means of *VAR* system simulations, any adverse dependency between default and recovery rates into the *DLGD*.

Let's define the loss of an infinitely granular portfolio with unit exposure and stochastic *LGD* as:

$$L \mid Y_t = HDR_t \cdot ELGD_t \qquad \textbf{[5.1]}$$

$HDR_t$ is the aggregate household default rate random variable at year *t*, whose dynamics depends on the quarterly *VAR* system estimated before ($y_t$ *VAR* realizations), $ELGD_t$ is the portfolio Loss Given Default at *t*, also depending on *VAR* and particularly on real estate price changes through equation [4.3].

The [5.1] can be compared to the loss with deterministic *LGD* set to its downturn value $\overline{DLGD(\alpha)}_t$, computed for each *t*:

$$L^* \mid Y_t = HDR_t \cdot \overline{DLGD(\alpha)}_t \qquad \textbf{[5.2]}$$

Since our aim is to capture in downturn *LGD* all the risk coming from the *HDR-LGD* correlation and thus to compensate for the fact that regulatory capital calculation does not incorporate systematic *LGD*, the $\alpha$ (e.g. set to 99.9% following Pillar I regulatory formula) quantiles of $L|Y_t$ and $L^*|Y_t$ should be equated and then solving for $\overline{DLGD(\alpha)}_t$:

$$q_\alpha[L \mid Y_t] = q_\alpha[L^* \mid Y_t]$$

$$q_\alpha[L \mid Y_t] = q_\alpha[DR_t] \cdot \overline{DLGD(\alpha)}_t$$

$$\overline{DLGD(\alpha)}_t = \frac{q_\alpha[L \mid Y_t]}{q_\alpha[DR_t]} \qquad \textbf{[5.3]}$$

Dividing [5.3] by *ELGD*, a measure of downturn LGD mark-up $\nu(\alpha)_t$ is figured out:

$$\nu(\alpha)_t = \frac{q_\alpha[L \mid Y_t]}{q_\alpha[DR_t] \cdot ELGD} \qquad \textbf{[5.4]}$$

The algorithm to compute the percentiles $q_\alpha$ and thus $\nu(\alpha)_t$ can be summarized as follows:

1) Simulation of the *VAR* system over a five years period, that is a 20 quarters economic scenario. $Y_{t,q}$ random variables ($t$ stands for years and $q$ for quarters) are generated starting from their long run values (Table 3) and drawing normally distributed $e_{t,q}$ shocks: since their correlation matrix was discovered to be not statistically significant, as shown by the large value of the computed standard errors (Table A6), they were drawn as independent;
2) transformation of default rate and real estate return horizon from quarterly into annual. Household default rates were transformed using a simple survival probability rule:

$$\prod_{t,q=1}^{4}(1 - HDR_{t,q}) = 1 - HDR_t \qquad \textbf{[5.5]}$$

where $q$ is the quarter number from 1 to 4, $t$ indicates the year from 1 to 5.

For real estate, the annual increase of average simulated prices as indicated by [5.6] was adopted:

$$RE_{t=1,q=1} = (1 + \Delta\% RE_{t=1,q=1})$$

$$RE_{t=1,q=2} = RE_{t=1,q=1} \cdot (1 + \Delta\% RE_{t=1,q=2})$$

$$\ldots$$

$$RE_{t=5,q=4} = RE_{t=5,q=3} \cdot (1 + \Delta\% RE_{t=5,q=4})$$

$$\Delta\% RE_t = \overline{RE_{t,q}} / \overline{RE_{t-1,q}} - 1 \qquad \textbf{[5.6]}$$

3) computation of annual $HDR_t$, $LGD_t$ and thus $L|y_t$ for $t$ going from 1 to 5 (years);

4) the iteration of steps 1-3 (in this case 30'000 iterations) allows to compute percentiles $q_{\alpha}(HDR_t)$, $q_{\alpha}(L|Y_t)$, and as a consequence $\overline{DLGD(\alpha)}_t$ and $\nu(\alpha)_t$.

As it can be noticed, the simulation described above is run on a five years time path, which is a length consistent with a full economic cycle, starting from long period values: however, the calculated downturn *LGD* time horizon is still annual since the algorithm provides for a one-year theoretical loss due on real estate prices changes. The lengthening of the simulation horizon serves the objective to let the simulated economic system take all plausible real-world values smoothing the persistence of the starting, long period, values.

Tables 5, 6 and 7 show the results of the simulation related to one-year household default rate ($HDR_t$), annual change in average real estate price ($\Delta RE\%_t$) and $LGD_t$: the outcomes are displayed in terms of percentiles at risk (95%, 99% and 99.9%), for each simulation year.

Regarding real estate price (Table 6), the percentiles are computed prudentially removing the long run trend specified in Table 3 and 4 and thus adding a margin of conservatism in describing real estate price dynamics.

As far as *LGD* is concerned (Table 7), its calculation was done assuming an *ELGD* equal to 15%, with $\mu$ = 70% and a prior for possession probability *P(Po)* equal to 35%: to be noticed is the maximum increase in *LGD* value from 15% to 20% during the fifth year, which corresponds to a percentage change of about 33%.

**Table 5: Percentiles of one year household Default Rate ($HDR_t$)**

| | | Simulated years starting from long run period | | | | |
|---|---|---|---|---|---|---|
| $\alpha$ | LR | **1** | **2** | **3** | **4** | **5** |
| 95% | 1.28% | 1.46% | 1.62% | 1.79% | 1.96% | 2.12% |
| 99% | 1.28% | 1.55% | 1.78% | 2.05% | 2.33% | 2.58% |
| 99.9% | 1.28% | 1.65% | 2.02% | 2.47% | 2.85% | 3.30% |

**Table 6: Percentiles of one-year changes in Real Estate price ($\Delta RE\%_t$)**

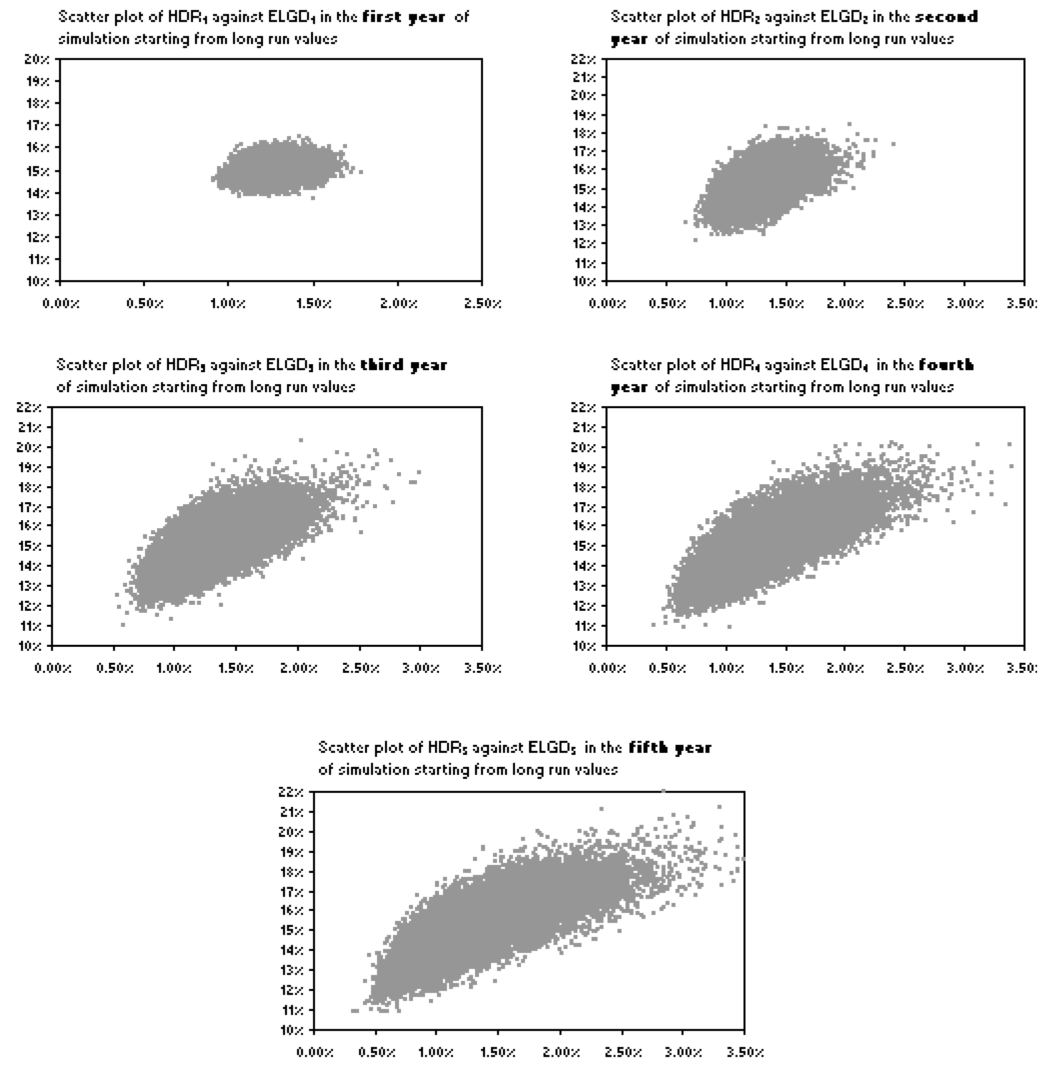
| | Simulated years (t) starting from long run period | | | | |
|---|---|---|---|---|---|
| $\alpha$ | **1** | **2** | **3** | **4** | **5** |
| 95% | -1.85% | -4.24% | -5.72% | -6.65% | -7.17% |
| 99% | -2.61% | -5.94% | -8.08% | -9.30% | -10.03% |
| 99.9% | -3.46% | -7.81% | -10.79% | -12.25% | -13.35% |

**Table 7: Percentiles of $ELGD_t$ distribution with $P(Po)$ = 35% (unconditional forclosure probability), $ELGD$ = 15%, $\mu$ = 70%, $\mu_o$ = 100%, $\sigma = \sigma_{Po}$ = 50%**

| | | Simulated years (t) starting from long run period | | | | |
|---|---|---|---|---|---|---|
| $\alpha$ | LR | **1** | **2** | **3** | **4** | **5** |
| 95% | 15.0% | 15.6% | 16.3% | 16.9% | 17.2% | 17.4% |
| 99% | 15.0% | 15.8% | 16.9% | 17.7% | 18.2% | 18.5% |
| 99.9% | 15.0% | 16.1% | 17.6% | 18.8% | 19.4% | 19.8% |

In order to appraise the joint behavior of default rate and *LGD* by year, Figure 6 displays the positive correlation coming out from the simulation: this is essentially due to the collateral price common risk driver which affects both default and recovery rate.

**Figure 6: Correlation between $HDR_t$ (x-axis) and $ELGD_t$ (y-axis) over the five years of simulation. The assumptions are $P(Po)$ = 35%, $ELGD$ = 15%, $\mu$ = 70%, $\mu_o$ = 100%, $\sigma = \sigma_{Po}$ = 50%**



Thus, the graphical inspection presented here allows us to support that adverse dependencies between the two variables are correctly identified: their magnitude will depend on the one side on the *VAR* system coefficients (equation [3.1]), on the other on equation [4.3] which states the *LGD* – collateral value dependency. Furthermore, this framework makes it possible to identify, indirectly through simulation, downturn conditions (e.g. periods of negative GDP growth, lower collateral value and higher default rate) and transfer them into downturn *LGD* as pointed out by Tables 8 and 9.

At 99.9%, as indicated by the last column of Table 8, the average downturn *LGD* is 17.4%, with an average downturn mark-up of 16% ranging from 2.1% during the first year to 25.3% at the fifth year: these figures are better shown in the last row of Table 9.

**Table 8: Downturn *LGD* computed with *P(Po)* = 35%, *ELGD* = 15%, $\mu$ = 70%, $\mu_o$ = 100%, $\sigma = \sigma_{Po}$ = 50%**

| $\alpha$ | *ELGD* | $\overline{DLGD(\alpha)}_1$ | $\overline{DLGD(\alpha)}_2$ | $\overline{DLGD(\alpha)}_3$ | $\overline{DLGD(\alpha)}_4$ | $\overline{DLGD(\alpha)}_5$ | *Average* $\overline{DLGD(\alpha)}$ |
|---|---|---|---|---|---|---|---|
| 95% | 15.0% | 15.2% | 15.8% | 16.4% | 16.8% | 17.0% | 16.2% |
| 99% | 15.0% | 15.3% | 16.2% | 17.0% | 17.7% | 17.8% | 16.8% |
| **99.9%** | **15.0%** | 15.3% | 16.5% | 18.0% | 18.3% | **18.8%** | **17.4%** |

Both $\overline{DLGD(\alpha)}$ and $\nu(\alpha)$ are increasing with confidence level since, as $\alpha$ increases, the $\alpha$ quantile of the loss distribution $q_\alpha[L \mid Y_t]$ occurs at a more severe downturn $\overline{DLGD(\alpha)}_t$, which also results in a larger *LGD* mark-up. As outlined in Barco (2007), the implication is that different downturn *LGDs* would be required for regulatory capital calculations done at 99.9% confidence and economic capital calculations with other confidence level. Another relevant feature to be highlighted is the enhancement of $\overline{DLGD(\alpha)}_t$ and $\nu(\alpha)_t$ with time (*t*), due to the higher probability of observing adverse chained scenarios, different from long run starting values. As the length of the simulation horizon is also consistent with the five years of data required by the regulator for *LGD* estimation, the choice of the last year result is probably the most prudential estimate, for a given confidence level, of the downturn effect.

**Table 9: *LGD* mark-up $\nu(\alpha)_t$**

| $\alpha$ | $\nu(\alpha)_1$ | $\nu(\alpha)_2$ | $\nu(\alpha)_3$ | $\nu(\alpha)_4$ | $\nu(\alpha)_5$ | *Average* $\nu(\alpha)$ |
|---|---|---|---|---|---|---|
| 95% | 1.3% | 5.6% | 9.4% | 11.8% | 13.4% | 8.3% |
| 99% | 1.9% | 8.1% | 13.3% | 17.9% | 18.4% | 11.9% |
| **99.9%** | 2.1% | 9.8% | 19.8% | 22.3% | **25.3%** | **15.9%** |

Table 10 outlines some further results using different *ELGD* values from 5% to 25%, leaving fixed the other parameters and setting $\alpha$ equal to 99.9%: as found by other empirical studies, the higher the *ELGD*, the lower the mark-up $\nu(\alpha)_t$. For increasing *ELGD* values, the contribution of default rate in explaining the loss $L|Y_t$ becomes higher and thus reduces $\nu(\alpha)_t$, also viewed as the systemic impact of *LGD* on the loss. As listed in the last column of table 10, the average mark-up computed across the five years decreases in fact from 59% to 7.3% whilst during the fifth year from 90% to 11%.

**Table 10: *LGD* mark-up computed for different value of *ELGD* at 99.9% percentile**

| *ELGD* | $\nu(\alpha)_1$ | $\nu(\alpha)_2$ | $\nu(\alpha)_3$ | $\nu(\alpha)_4$ | $\nu(\alpha)_5$ | *Average* $\nu(\alpha)$ |
|---|---|---|---|---|---|---|
| 5% | 11.6% | 40.5% | 73.0% | 80.2% | 89.8% | 59.0% |
| 10% | 3.8% | 17.7% | 32.9% | 37.1% | 41.3% | 26.6% |
| 15% | 2.1% | 9.8% | 19.8% | 22.3% | 25.3% | 15.9% |
| 20% | 1.1% | 6.1% | 13.9% | 14.7% | 16.7% | 10.5% |
| 25% | 0.8% | 4.4% | 9.7% | 10.4% | 11.4% | 7.3% |

Finally, in Table 11 some sensitivity tests were conducted, moving the average loan to value $\mu$ with a step of 10% from 50% to 80% and leaving unchanged all the other conditions (e.g. *P(Po)* = 35%, *ELGD* = 15%, $\mu_o = 100\%$, $\sigma = \sigma_{Po} = 50\%$, $\alpha = 99.9\%$): the main insight founded here is the negative, although negligible, relationship between $\mu$ and downturn effect. The latter appears in fact to be slightly more significant as the loans portfolio shows a better quality, that is a lower $\mu$ (the average magnitude is 16.3% when $\mu$ amounts to 50%). This tendency is explained by the steepness of $P(Po \mid LTV)$ in [4.4], which depends positively on how much separate the $Po$ and $\overline{Po}$ *LTV* groups are: the greater the difference in mean between the two, the higher the $P(Po \mid \Delta\%re_t)$ volatility, causing a larger $ELGD_t$ systemic contribution. This feature is also consistent with the view according to which the level of conservatism tends to be higher for more secured portfolio, where possible shock to the collateral is likely to have significant effects on the *LGD*.

**Table 11: *LGD* mark-up computed for different values of $\mu$ and *P*(*Po*)**

| *P(Po)* | $\mu$ | $\nu(\alpha)_1$ | $\nu(\alpha)_2$ | $\nu(\alpha)_3$ | $\nu(\alpha)_4$ | $\nu(\alpha)_5$ | *Average* $\nu(\alpha)$ |
|---|---|---|---|---|---|---|---|
| 30% | | 1.7% | 8.0% | 17.5% | 19.0% | 22.3% | 13.7% |
| 35% | 50% | 2.1% | 10.0% | 20.3% | 23.0% | **25.9%** | **16.3%** |
| 40% | | 2.7% | 12.0% | 23.0% | 26.7% | 29.3% | 18.7% |

| *P(Po)* | $\mu$ | $\nu(\alpha)_1$ | $\nu(\alpha)_2$ | $\nu(\alpha)_3$ | $\nu(\alpha)_4$ | $\nu(\alpha)_5$ | *Average* $\nu(\alpha)$ |
|---|---|---|---|---|---|---|---|
| 30% | | 1.7% | 8.0% | 17.4% | 18.9% | 22.0% | 13.6% |
| 35% | 60% | 2.1% | 10.0% | 20.2% | 22.8% | **25.8%** | **16.2%** |
| 40% | | 2.7% | 11.9% | 22.9% | 26.4% | 29.1% | 18.6% |
| 30% | | 1.7% | 7.9% | 17.3% | 18.5% | 21.3% | 13.3% |
| 35% | 70% | 2.1% | 9.8% | 19.8% | 22.3% | **25.3%** | **15.9%** |
| 40% | | 2.7% | 11.8% | 22.8% | 25.8% | 28.7% | 18.3% |
| 30% | | 1.7% | 7.7% | 16.8% | 17.9% | 20.0% | 12.8% |
| 35% | 80% | 2.1% | 9.5% | 19.2% | 21.2% | **24.4%** | **15.3%** |
| 40% | | 2.7% | 11.5% | 22.2% | 24.9% | 27.9% | 17.8% |

## 6. Conclusion

The leading principle to be followed while validating downturn *LGD* is that its quantification should rely on incorporating into *LGD* estimates the adverse dependencies which are eventually identified between default rates and recovery rates. This should anyway be translated into the choice of the estimation method, which is heavily conditioned by data constraints.

Literature on this subject usually concentrate on the analysis of correlations between observed system-wide corporate default rates and *LGDs* which are implicit in bonds prices after default. Banks internal series on workout *LGD* are in fact generally not long and complete enough to cover a full economic cycle, or at least a downturn period which can ensure reliable estimates of the relationship with internal default rates. Furthermore, the length of default, or the time span which elapses between the default event and the default termination, can be quite long for some countries, among which Italy, thus weakening the evidence of correlation.

The present document describes a methodology for downturn effect estimation which still starts from the relation between *LGD* and *DR*: this is here derived implicitly, through the identification of common risk drivers, which can thus be simulated in order to get the paths of both variables under different economic conditions.

While default rates are modeled inside a Vector Auto-Regressive system that links together several macroeconomic variables, whose parameters can be estimated on systemic data, the lack of workout *LGD* reliable time series prevents us from doing the same on recovery side. A theoretical Bayesian approach was then adopted in order to catch the effect of the macroeconomic situation on *LGD*: as the focus of the analysis is the mortgage segment, the relevant driver is in this case the real estate price.

The proposed framework, which puts together different techniques soundly based in the literature, has the advantage of overcoming data constraints while still relying on a workout *LGD* concept as input. It is thus perfectly tailored on the mortgage portfolio, to which the alternative approach based on distressed bonds prices is not applicable, and whose relation with a specific macro impulse is intuitive. Its application could anyway be extended to other collateralized portfolios, if a specific theoretical model of their link to the economic cycle can be designed.

The approach we describe in the paper aims to be fully consistent with the regulatory requirements: it is in fact based on identification of adverse dependencies between default rates and recovery rates, and it captures in downturn *LGD* all the risk coming from this correlation. Furthermore, as the downturn measure is to be used to feed the regulatory capital formulas, which do not account for systematic risk in *LGD* as on the contrary they implicitly do for *PDs*, we focused on the same 99.9% percentile at risk which is considered for the default probability parameter.

As the approach is sufficiently general, it can serve not only to compute bank specific *LGD* parameters, but also to give benchmark values for estimates obtained through different approaches. This is why we chose to present the results for a range of different, but still realistic, assumptions on parameters values (we specifically refer here to average Loan to Value, average *LGD*, unconditional possession probability). This allows us to perceive the estimated downturn effect sensitivity to the parameters that are specific for each bank: in this regard, we found that the downturn *LGD* effect strongly decreases for increasing values of *ELGD*, while it is far less sensible to changes in the other parameters. For instance, if we consider the last year of the simulation to be the best estimate of the downturn effect, in order to allow the simulated economic system to fully display its effects, in the case of a portfolio with *ELGD* equal to 15%, the downturn mark up amounts to around 25%.

# Appendix A

**Table A 1: Augmented-Dickey-Fuller unit root tests (1990Q2 – 2007Q4) on variables included in *VAR***

| Variable | Lag | Statistic | Critical values 10% | 5% | 1% |
|---|---|---|---|---|---|
| LHDR | 3 | -0.106 | -2.57 | -2.86 | -3.43 |
| $GDP_{det}$ | 1 | -2.211 | -2.57 | -2.86 | -3.43 |
| ΔLog(RE) | 2 | -1.836 | -2.57 | -2.86 | -3.43 |
| LogR3M | 1 | -1.800 | -2.57 | -2.86 | -3.43 |

Note: The null hypothesis is the presence of unit root. Akaike Info Criterion was used for lag selection.

**Table A 2: Kwiatkowski-Phillips-Schmidt-Shin test for stationarity (1990Q2 – 2007Q4) on variables included in *VAR***

| Variable | Lag | Statistic | Critical values 10% | 5% | 1% |
|---|---|---|---|---|---|
| LHDR | 3 | 3.155*** | 0.347 | 0.463 | 0.739 |
| $GDP_{det}$ | 1 | 1.446*** | 0.347 | 0.463 | 0.739 |
| ΔLog(RE) | 2 | 0.453* | 0.347 | 0.463 | 0.739 |
| LogR3M | 1 | 2.837*** | 0.347 | 0.463 | 0.739 |

Note: The null hypothesis is the absence of unit root. * rejection at 10% level, ** at 5% level, *** at 1% level

**Table A 3: Johansen cointegration rank test on VAR with 3 time lags as in [3.1]**

| n° of CE | Eingenvalue | Statistic | Critical Value | Prob |
|---|---|---|---|---|
| None * | 0.356 | 63.175 | 54.079 | 0.63% |
| At most 1 | 0.219 | 33.238 | 35.192 | 8.00% |
| At most 2 | 0.135 | 16.390 | 20.261 | 15.69% |
| At most 3 | 0.091 | 6.4882 | 9.1645 | 15.62% |

Note: Trace test indicates 1 cointegrating equation at the 0.05 level. * denotes rejection of the hypothesis at the 0.05 level. Tests are based on 2 lags Error Correction representation with LHDR, $GDP_{det}$, ΔLogRE), LogR3M variables and restricted constant in the cointegrating equations.

**Table A 4: Selection tests up to the fourth lag.**

| Lag | Akaike Iinformation citerion | Final Prediction Error | Shwarz Criterion | Hannan Quinn |
|---|---|---|---|---|
| 0 | -9.885 | 6.00E-10 | -8.9638 | -9.5206 |
| 1 | -18.967 | 6.89E-14 | -17.519 | -18.394 |
| 2 | -19.660 | 3.51E-14 | -17.685* | -18.878 |
| 3 | -19.914* | 2.81e-14* | -17.414 | -18.925* |
| 4 | -19.684 | 3.73E-14 | -16.656 | -18.486 |

**Figure A 1: Residuals of *VAR* system estimation. Time series plot**

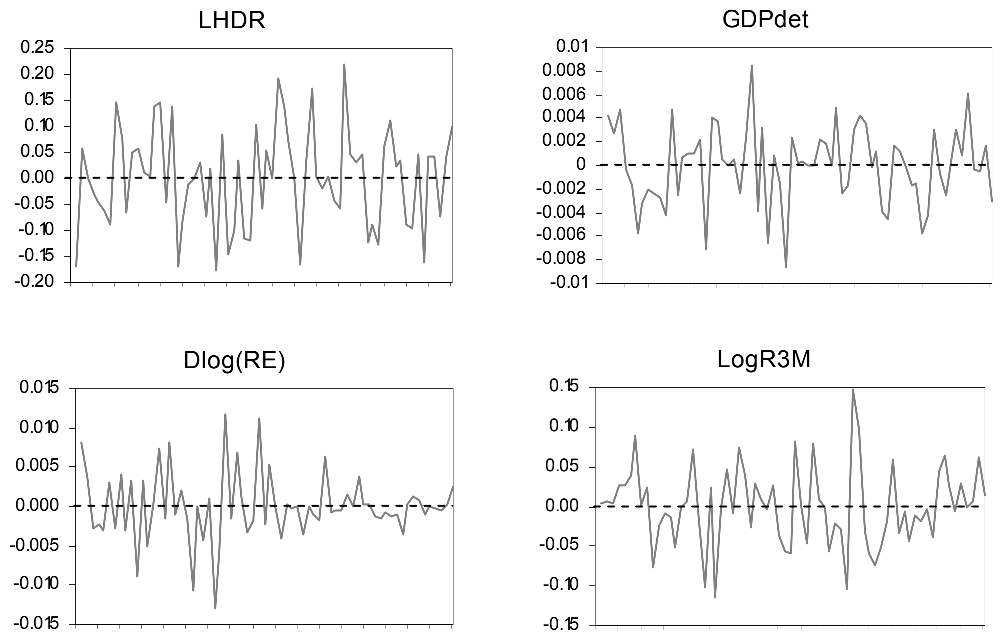



**Table A 5: Normality test for residual $e_t$**

| Residual ($e_t$) | Statistic | P-value | Skewness | Kurtosis |
|---|---|---|---|---|
| LHDR | 1.1903 | 0.551 | 0.087 | 2.376 |
| $GDP_{det}$ | 0.1719 | 0.917 | -0.06 | 2.789 |
| $\Delta$Log(RE) | 6.4259 | 0.040 | 0.193 | 4.455 |
| LogR3M | 3.3934 | 0.183 | 0.450 | 3.622 |

Note: P-value refers to the normality null hypothesis tested with *Jarque-Bera* test.

**Table A 6: *Cholesky* decomposition of Variance-Covariance $e_t$ matrix**

| | LHDR | $GDP_{det}$ | $\Delta$Log(RE) | LogR3M |
|---|---|---|---|---|
| LHDR | 0.0948 | | | |
| | (0.0081) | | | |
| $GDP_{det}$ | -0.0007 | 0.0035 | | |
| | (0.0004) | (0.0003) | | |
| $\Delta$Log(RE) | 0.0005 | 0.0000 | 0.0043 | |
| | (0.0005) | (0.0005) | (0.0004) | |
| LogR3M | -0.0023 | 0.0120 | 0.0025 | 0.0551 |
| | (0.0069) | (0.0068) | (0.0067) | (0.0047) |

Note: Standard errors in brackets.

## 6. References

Altman, E., Resti A. and Sironi A. (2001). *Analyzing and explaining default recovery rates*. Report submitted to the International Swaps and Derivatives Associations, London, December.

Altman, E., Brooks B., Resti A. and Sironi A. (2005). *The link between default and recovery rates: theory, empirical evidence and implications*. Journal of Business 78 (6), 203-227.

Avouyi-Dovi, S., Bardos, M., Jardet, C., Kendaoui, L., Moquet, J. (2009). *Macro stress testing with a macroeconomic credit risk model : Application to the French manufacturing sector*. Banque de France. Document de travail 238.

Banco De Espana (2007). *Loss given default estimates under downturn conditions (DLGD) in mortgage loan portfolios in Spain*. Directorate General Banking Supervision. Validation document1

Barco, M. (2007). *Going Downturn*. Cutting Edge, Credit Risk Model, 39-75.

Basel Committee on Banking Supervision (2005). *Guidance on Paragraph 468 of the Framework Document*. Bank for International Settlements.

Chabaane, A., Laurent, J.P., Salomon J. (2004). *Double impact: Credit Risk Assessment and Collateral Value.* Revue Finance, 25, 157-178.

Committee of European Banking Supervisors (2008). *Range of practices on some Basel II implementation issues*.

Financial Services Authority (2008). *Residential Mortgage Downturn LGDs*. FSA paper.

Dullmann, K., Trapp M. (2004). *Systematic risk in recovery rates – an empirical analysis of US corporate credit exposures*. Deutsche Bundesbank discussion paper 2.

Hillebrand, M. (2006). *Modelling and estimating dependent loss given default*. Risk September, 120-125.

Hoggarth, G., Sorensen, S., Zicchino, L. (2005). *Stress tests of UK banks using a VAR approach. Bank of England*. Working Paper 282.

Marcucci, J., Quagliariello, M. (2005). *Is Bank Portfolio Riskiness Procyclical? Evidence from Italy using a Vector Autoregression*. The University of York. Discussion Papers in Economics, 2005/09.

Miu P., Ozdemir B. (2006). *Basel requirements of downturn LGD: modeling and estimating probability of default and loss given default correlations*. Journal of Credit Risk 2 (2), 43-68.

Qi, M., Yang, X. (2007). *Loss Given Default of High Loan-to-Value Residential Mortgages*. Economics and Policy Analysis Working Paper 2007-4.

Tasche D. (2006). *Validation of internal rating systems and PD estimates*. Working paper.

Van Order, R. (2008). *Modelling and evaluating the credit risk of mortgage loans: a primer*. The Journal of Risk Model Validation 2(2), 63-82.

Virolainen, K. (2004). Macro *stress-testing with a macroeconomic credit risk model for Finland*. Bank of Finland. Bank of Finland discussion Papers 18.

Wong, J., Choi, k., Fong, T. (2006). *A framework for macro stress testing the credit risk of banks in Hong Kong*. Hong Kong Monetary Authority Quarterly Bullettin.