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2010

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First Version: July 2007
Revised: June 2008

Abstract

This paper examines the mathematical abilities of 15-year olds in a range of countries which participated in the 2003 cycle of the OCED's Programme for International Student Assessment (PISA). Utilising information on the scores obtained by individual students in the mathematical part of the PISA assessment, we use a range of indicators from the literature on inequality and poverty to evaluate the "mathematical performance" of participating countries. Since data from PISA contained a wealth of information on the circumstances of the students, in terms of their home and school environment, we identify, and examine the relative influence of, factors which serve to enhance the mathematical performance of students in the PISA assessment.

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Acknowledgements: We are grateful to the Editor and to an anonymous referee for valuable comments.

1. Introduction

An interesting - but, arguably, not sufficiently appreciated – feature of the labour market is the positive association between a person's earnings and whether he or she had studied mathematics at school to an advanced level (Kenny et. al., 1979, Dolton and Vignoles, 2000, McIntosh and Vignoles, 2000). Dolton & Vignoles (2000, 2002) measured the returns an individual obtained from having an A-level in Mathematics (a Mathematics qualification taken in British schools at post-16 level) and found that the return became evident at a later, rather than earlier, stage of a person's career; they argued that the return was due to employers observing that employees with A-level mathematics had higher levels of productivity compared to those who did not have this qualification. They concluded that individuals with an A-level in mathematics earned 7%-10% more than similarly educated workers without this qualification, even after controlling for the initial ability of these individuals.

In a similar vein, Jenkins et al (2003) found that, for women, mathematical ability was particularly important in determining which of them would undertake lifelong learning that would lead to a qualification – an important consideration given the emphasis on lifelong leaning in recent UK government policy thinking such as the "new skills agenda". More recently, Kounine et. al. (2008) have bemoaned the decline of mathematics in the United Kingdom and argued that winning the battle of the "maths economy" will be crucial to the UK's future economic success. [1]

Murnane et al (1995) and Ingram & Neumann (2006) while supporting this claim, also presented evidence that, in the United States, mathematical qualifications had become increasingly important in determining wage rates: Ingram & Neumann (2006)

[1] For a contrary view on the usefulness of mathematics see Jenkins (2008).

found that the return to mathematical and verbal ability nearly doubled between 1980 and 1998.

What benefits the individual also benefits the economy: at a macroeconomic level, studies making use of international school test score data have found a link between mathematical ability and the economic growth of the state (Hanushek and Kimko, 2000). This linkage has been recognised by policy makers. For example, Alan Greenspan giving testimony before the Committee on Education and the Workforce of the U.S. House of Representatives in March 2004 noted that:

> "Research on wealth creation in both emerging and developed nations strongly suggests that it is the knowledge and the skill of our population interacting under our rule of law that determine our real incomes… A study conducted in 1995 revealed that, *although our fourth-grade students were above average in both math and science, by the time they reached their last year of high school they had fallen well below the international average*." (Greenspan, 2004, emphasis added).

Given the importance of the level of mathematical ability of a country's population, in determining both individual life chances and also macroeconomic performance, this paper asks two broad questions:

(i) How do levels of mathematical ability differ between countries and are levels of inter-country inequality in mathematical ability susceptible to analysis using the tools of inequality theory?

(ii) What are the factors that influence such ability and, in exercising such influence, what is the relative strength of the relevant factors?

We answer these questions using data from the OCED's Programme for International Student Assessment (PISA) which is one of a range of trans-national tests of student ability.[2] PISA is a collaborative effort, involving all OECD countries and a significant number of partner countries, to measure how well 15 year students "are prepared to meet the challenges of today's knowledge society". The PISA 2003 assessments consist of paper-and-pencil tests and the following domains are tested: (i) mathematical literacy; (ii) reading literacy; (iii) scientific literacy.

On the basis of these tests, each of 276,150 students in 41 countries was assigned a score for mathematics, reading, and science. In addition to this information about how well students fared in their assessments, the PISA data contains a wealth of information on the circumstances surrounding a student. These relate to *inter alia*: (i) his/her personal circumstances, living arrangements etc; (ii) parental attributes relating to education, class; (iii) home possessions and environment relating to books, computers, internet, place to study; (iv) school circumstances relating to amount of instruction, relationship with teachers, type of school etc.

The issue of inter-country differences in education achievement has been investigated by Maas and Criel (1982) who estimated Gini coefficients based on enrolment data for 16 East African countries and, more recently, by Thomas *et. al.* (2001, 2002) who studied inequality in educational attainments for 140 countries. The latter set of papers developed the concept of education Gini and argued that this could be used as an indicator of welfare complementing average educational, health, nutritional, and income attainments. In so doing, Thomas et. al.. (2001, 2002) were motivated by Sen's (1999) observation that concern with equity should not be confined to just income inequality but, indeed, should be extended to embrace all the dimensions which impinge on a person's ability to function effectively in society.

[2] See Hansen & Vignoles (2005) and Brown et al (2007) for a discussion of how PISA compares to other assessments.

Machin and Vignoles (2004) have studied educational inequality in the UK from the perspective of socio-economic groups. Comparing two cohorts of individuals, born in 1958 and 1970, they found that the links between educational achievement and parental income / social class strengthened over this period so that the labour market success or failure of individuals became more closely connected to their parents income. This work is complemented by that of Galindo-Rueda *et. al.* (2004) who found that in UK higher education, even before the introduction of tuition fees, children from poorer neighbourhoods had become less likely to participate in higher education since 1994-95, as compared to children from richer neighbourhoods.

In this paper we extend the contribution of Thomas *et. al.* (2001, 2002), to the study of educational inequality, in one respect. Following the work of Anand and Sen (1997) we argue that the average achievements of a country with respect to a particular welfare indicator –which in the case of this paper is "mathematical ability" as measured by PISA – should be tempered by considerations of inequality in the distribution of achievements between the individuals in the country's population. Anand and Sen (1997) referred to the resultant indicators of achievement as being "equity sensitive". In this paper we construct, for each country in the PISA sample, equity sensitive indicators of mathematical ability.

The heart of the paper lies, however, in answering the latter questions relating to the factors which enhance mathematical ability. Since, as noted above, data from PISA contained a wealth of information on the circumstances of the students, particularly in terms of their home and school environment, we identify, and examine the relative influence of, factors which serve to enhance the mathematical performance of students in the PISA assessment.

The determinants of the factors underlying educational achievement have been the subject of several studies. Jenkins *et. al.* (1993) examined the determinants of lifelong learning and concluded that those who left school with O-level qualifications or higher were much more likely to undertake lifelong learning. Okpala and Onocha (1988) examined the factors underlying student achievement in physics in Nigeria and examined the role of *inter alia* gender, home, interest in physics in shaping achievement in physics.

2. Equity Sensitive Indicators of Student Assessment Scores

In a paper prepared for the 1995 *Human Development Report*, Anand and Sen (1997) pointed out that a country's non-economic achievements were likely to be unequally distributed between subgroups of its population: for example, in terms of gender equality, which was the focus of their concern, the female literacy rate, or female life expectancy, was often lower than that for males. In the face of such inter-group inequality, they argued that a country's achievement with respect to a particular outcome should not be judged exclusively by its mean level of achievement (for example, by the average literacy rate for a country) but rather by the mean level *adjusted to take account of inter-group differences in achievements*.

Anand and Sen (1997) proposed a method, based on Atkinson's (1970) seminal work on the relation between social welfare and inequality, for making such adjustments and they termed the resulting indicators *equity sensitive indicators*. They further suggested that assessments of country achievements should be made on the basis of such equity sensitive indicators rather than, as was often the case, on the basis of its mean level of achievement. This would, then, allow a comparison between two countries, one of which had a lower mean achievement level, but a more equitable distribution of achievement, than the other.[3] In this section we apply these ideas to the student assessment scores (SAS) - hereafter, simply, "scores" – in the PISA data.

Suppose that $\bar{X}$ is the average score in a country where X_i is the score of student i $(i=1 \ldots N)$. We know that, because of inequality in the distribution of scores between students, the average score of a country will not be achieved by all its students.

[3] Anand and Sen (1997) compared the Honduras (with an average literacy rate of 75%, distributed between men and women as 78%, 73%) with China (with an average literacy rate of 80%, distributed between men and women as 92%, 68%) and asked which country should be regarded as having the "better" achievement with regard to literacy: China with a higher overall rate or the Honduras with greater gender equality?

Therefore, in assessing the SAS achievement of a country, by how much should we reduce its average SAS to take account of inequality in scores?

The answer to this question depends on how *averse we are to inequality*. In his seminal paper on income inequality, Atkinson (1970) argued that we (society) would be prepared to accept a reduction in average income, *provided the lower income was equally distributed*, from a higher average income which was unequally distributed.[4] The size of this reduction depended upon our degree of "inequality aversion" which Atkinson (1970) measured by the value of a (inequality aversion) parameter, $\varepsilon \geq 0$. When $\varepsilon = 0$, we are *not at all* averse to inequality implying that we would not be prepared to accept even the smallest reduction in average income in order to secure an equitable distribution. The degree of inequality aversion increases with the value of ε: the higher the value of ε, the more averse we would be to inequality and, in order to secure an equitable distribution of income, the greater the reduction in average income we would find acceptable.

These ideas can, equally well, be applied to student assessment scores. We can reduce the average score, $\bar{X}$, of a country by the amount of inter-student inequality in scores to arrive at X^e, a "group equity sensitive" score for the country, $X^e \leq \bar{X}$. We refer to X^e as the *equally distributed equivalent score.* The size of this reduction (as given by the difference, $\bar{X} - X^e$) depends upon our aversion to inequality: the lower our aversion to inequality, the smaller will be the difference and, in the extreme case in which there is no aversion to inequality ($\varepsilon = 0$), there will be no difference between the average and the equity sensitive scores. Three special cases, contingent upon the value assumed by ε, may be distinguished:

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[4] In the language of economics, the two situations would yield the same level of social welfare, i.e. be 'welfare equivalent'.

1. When $\varepsilon = 0$ (no inequality aversion), X^e is the *arithmetic mean* of the student scores: $X^e = \bar{X}$
2. When $\varepsilon = 1$, X^e is the *geometric mean* of the student scores: $X^e = \left[\prod_{i=1}^{N} X_i\right]^{1/N} < \bar{X}$.
3. When $\varepsilon = 2$, X^e is the *harmonic means* of the student scores: $X^e = \frac{N}{\sum_{i=1}^{N}(X_i)^{-1}} < \bar{X}$.

Table 1 shows the equity sensitive scores for each of the 41 countries in the PISA data, contingent upon the amount of inequality in the distribution of scores between *all* the 15 year olds in (the sample for) that country. This table shows three separate rankings for the 41 countries: first, the ranking that resulted entirely from the average score of countries when intra-country inequality was deemed not to matter; second, the ranking that resulted when inequality aversion was measured by $\varepsilon = 1$ or, in other words, the relevant average was measured by the geometric mean; lastly, the ranking that resulted when inequality aversion was measured by $\varepsilon = 2$ or, in other words, the relevant average was measured by the harmonic mean.

Table 1 shows that the rankings changed only slightly from no inequality aversion to inequality aversion: the first 8 places were unchanged; Australia slipped from 11th to 12th place while Canada rose from 12th to 11th place; Germany slipped from 19th to 20th to 21st under successively higher degrees of inequality aversion while Italy slipped from 24th to 25th place. In general, we can conclude that the use of equity-sensitive indicators did not add much to the rankings of countries by their average level of mathematical achievements.

3. Explaining Mathematical Proficiency

Pisa (2003) defined seven levels of proficiency in mathematics: *level 1*, for SAS ≤ 357.77; *level 2*, for 357.77 < SAS ≤ 420.07; *level 3*, for 420.07 < SAS ≤ 482.38; *level 4*, for 482.38 < SAS ≤ 554.68; *level 5*, for 554.68 < SAS ≤ 606.99; *level 6*, for 606.99 < SAS ≤ 669.3; *level 7*, for SAS > 669.3.

Table 2 shows that Tunisia and Brazil (52 percent), Indonesia (51 percent), Uruguay (30 percent), Mexico (26 percent), Turkey (26 percent), Thailand (18 percent), and Greece and Serbia (18 percent) had the largest proportion of students at the lowest level of proficiency in mathematics. In all the other countries, less than one in ten students – and, in many countries, less than one in twenty students – were at the lowest proficiency level.

At the other extreme, Table 3 shows that Hong Kong (32 percent), Netherlands and Belgium (27 percent), Liechtenstein and the Czech Republic (25 percent), and Japan and Korea (24 percent) had the largest proportion of students at the two highest levels of mathematical proficiency.

Table 4 details the results of a regression model which seeks to explaining the predicted Scores in Mathematics by using the socio-economic characteristics of the student and their family along with information about their education (both inside and outside the school environment) and a county-type control variable.

The socio-economic characteristic explanatory variables for the student and the family comprise

- The age of the student: we would expect an older student to perform better in tests, *ceteris paribus*; however, it is worth pointing out that given that the students in the survey are fairly tightly clustered in terms of age with a

- The type of family: single parent, a nuclear family (mother, father and children), mixed family or other. The traditional view would be that students who are members of a nuclear family would perform better in education.
- The level of parental education converted into years of schooling. Here we would expect that more schooling for the parent should improve the educational performance of the student..
- Whether the language used by the family at home is the same as the language used for the test. It is plausible that those students using a language at home other than the test language would under-perform in a test conducted in the test language but it is also possible that such students may have a predisposition to working harder to overcome this disadvantage which will outlay the expectation of underperformance.
- Students with parents who are white collar workers (i.e. non-manual) should perform better than students whose parents who are blue collar workers (i.e. manual). Within these two groups, students with parents who are high skilled workers should perform better than students with parents who are low skilled workers. This would be consistent with results on socio-economic educational inequality reported by Machin and Vignoles (2004) and Galindo-Rueda *et. al.* (2004).

The information about their education (both inside and outside the school environment) comprised the following items:

(i) The minutes of mathematical instruction at School: these were categorised by us into four quartiles ranging from lowest to highest, with the highest quartile taken as the residual or base category. We would expect that the test score would rise as the amount of instruction increased.

(ii) The interest in Mathematics: these were categorised by us into four quartiles ranging from lowest to highest, with the highest quartile taken as the residual or base category. We would expect that the test score to rise as the interest in Mathematics increased.

(iii) The availability of computing resources at home: these were categorised by us into four quartiles ranging from lowest to highest, with the highest quartile taken as the residual or base category. We would expect that the test score rises as the availability of computing resources increases; this is both due to computers being of use for improving educational attainment and the presence of computers being a partial proxy for higher household income.

(iv) The availability of other resources at home (such as a quiet place to study): these were categorised by us into four quartiles ranging from lowest to highest, with the highest quartile taken as the residual or base category. We would expect the test score to rise as the availability of other resources at home increased.

(v) The level of motivation: this was categorised by us into four quartiles ranging from lowest to highest, with the highest quartile taken as the residual or base category. We would expect that the test score to rise as the level of motivation increased.

(vi) The level of discipline in the classroom, categorised by us into four quartiles ranging from lowest to highest, with the highest quartile taken as the residual or base category. We would expect that the test score to rise as the level of discipline in the classroom increased.

(vii) The use of different learning strategies. Here we are comparing the use of *elaboration* learning strategies (which is the residual category - an example being to understand new concepts in mathematics by relating them to things already known) with *memorisation* learning strategies (example: learn the answers to problems off by heart) and *control* learning strategies (example: self testing as the students study, to see if they remember the work already done). Our *a priori* belief was that elaboration and control learning strategies were more conducive to a higher test score compared to memorisation strategies.

With this background, we use the regression estimates shown in Table 4 to highlight the main findings.

Country Effects. One would expect that, after controlling for other variables, the country in which a student lived would influence his/her results. This is because the country of residence would capture the general level of resources available to residents and which would buttress the more specific variables pertaining to the individual students. Ideally, the equations should have been estimated with a dummy for each country. However, given that there were 41 countries, we decided to group the countries as follows:

(i) OECD English speaking countries (Australia, Canada, Ireland, New Zealand, UK, and USA).

(ii) OECD Nordic countries (Denmark, Finland, Iceland, Norway, Sweden).

(iii) OECD West European countries (Austria, Belgium, France, Germany, Greece, Italy, Luxembourg, Netherlands, Portugal, Spain, Switzerland).

(iv) OECD East European countries (Czech Republic, Hungary, Poland, and Slovakia.

(v) Other OECD countries. (Japan, Korea, Mexico, and Turkey).

(vi) OECD Partner countries (Brazil, Hong Kong, Indonesia, Leichtenstein, Latvia, Macao, Russia, Thailand, Tunisia, Uruguay, and Serbia).

The results show that, with the OECD partner countries taken as the residual option, all the following OECD groups had positive coefficient estimates[5]: East Europe (38 points), West Europe (29 points), Nordic countries (23 points), English-speaking countries (22 points). However, the other OCED countries (which contained mathematically weak countries like Mexico and Turkey) reported a negative coefficient estimate of 16 points.

Family Type and Parental Occupation. The results showed very clearly that both family structure and parental occupation exercised a significant influence on student performance: *ceteris paribus* a student from a nuclear family was predicted to score approximately 6 more points, on average, than a student from a single parent family while students whose parents were high skilled white collar workers were predicted to score an average of 29 more points than students whose parents were low skilled blue collar workers.

This parental advantage persisted for students whose parents were lower down the occupational ladder: students whose parents were low skilled white collar workers were predicted to score 12 more points than students whose parents were low skilled blue collar workers while students whose parents were high skilled

[5] Meaning that, ceteris paribus, their average scores were higher than the OCED partner average by the coefficient estimate.

blue collar workers were predicted to score 2 more points than students whose parents were low skilled blue collar workers.

Both these results are consistent with received wisdom: Machin and Vignoles (2004) have drawn attention to the links between educational achievement and parental class; Gordon (1996) has argued that the current rates of success in the GCSE exams in Britain (taken at the age of 16) are associated with locations away from inner cities and the reason for this is the high concentration of lone parent families in inner cities.

Years of Schooling and Language at Home. The effect on scores of how well educated students' parents were was significant, but weak. A student, one of whose parents had received 17 years of parental schooling (approximately equivalent to a Master's degree) was predicted to score approximately only two more points than a student with 10 years of parental schooling (approximately equivalent to leaving at age 16). This suggests that, once parental occupational class was controlled for, there was not much additional role for the influence of parental educational.

Similarly, the language spoken at home did exert a large and significant effect on student scores: students, for whom the language used by the family at home was different from the language used for the test, scored an average of 16 fewer points compared to students for whom the language used by the family at home was the same as the language used for the test.

The School Environment. The environment at school affected student performance in a number of ways:

1. First, and most obvious, was the time which the school devoted to instruction in mathematics. There was a very clear correlation between student

performance and the time spent on mathematics. For example, compared to the average score of students in the highest quartile with respect to instruction time: *ceteris paribus* the average score of students in the lowest quartile of instruction time was 18 points lower; the average score of students in the second quartile of instruction time students was 10 points lower, and the average score of students in the third quartile of instruction time was 8 points lower.

2. Second, our results clearly showed that classroom discipline mattered. The effective learning of mathematics depended not just on the time devoted to is study but also on classroom management which sought to create a good learning environment in the classroom through high standards of discipline. *Ceteris paribus* compared to the average score of students who enjoyed the highest level of classroom discipline, students in the lowest quartile for the level of discipline in the classroom scored, on average, 31 fewer points. Nor was this poorer performance confined to the lowest level of discipline: students in the second quartile for the level of discipline in the classroom scored approximately 21 fewer points than students in the highest quartile, while students in the third quartile for the level of discipline in the classroom scored approximately 11 fewer points (compared to students in the highest quartile).

3. Third, the school could have been instrumental in making students interested in mathematics and motivating them to do well.[6] Of these two factors, once interest in mathematics had been controlled for, motivation did not have much of a role. Compared to the average score of students with the highest level of

[6] Though, in addition to the school, several factors –including, home, friends – could influence interest and motivation.

interest in mathematics, the average score of students with lowest level of interest was 27 points lower but, compared to the average score of students with the highest level of motivation to study mathematics, the average score of students with lowest level of motivation was only 4 points lower.

4. Fourth, the school had an influence on student scores through the choice of appropriate teaching and learning strategies. However, it must be admitted that after controlling for the other school factors, 1-3 above, the influence of learning strategies on student performance was weak.

The results relating to school environment have obvious implications for the efficiency with which schools are run. In this connection, Bee and Dolton (1985) offer evidence that there may be economies of scale to be reaped from having larger schools.[7]

The Home Environment: The results clearly pointed to the importance of computing resources at home as an aid to proficiency in mathematics. Students in the lowest quartile for the availability of computing resources at home scored, on average, approximately 47 points fewer than students in the highest quartile while students in the second quartile for the availability of computing resources at home scored approximately 23 points less than students in the highest quartile. Interestingly, there was hardly any difference in the average scores of students in the third and fourth quartiles of computing facilities thus suggesting that the marginal benefit of better facilities diminished very rapidly.

Non-computing resources - for example, in the form a quiet, separate place to study - were also important in influencing student scores. Students in the lowest quartile for the availability of other resources at home scored approximately 31

[7] See also Dolton (1991) for a study of the efficient provision of compting services in UK universities.

fewer points than students in the highest quartile; students in the second quartile for the availability of other resources at home scored approximately 15 fewer points than students in the highest quartile. With respect to non-computing resources, there was a 10 point difference in the average scores of students in the third and fourth quartiles of non-computing facilities suggesting that the marginal benefit of better non-computing facilities did not diminish as rapidly as did the marginal benefit of better computing facilities.

6. Conclusion

The proportion of mathematically weak students was lowest (3 percent or less) in Finland, Korea, the Netherlands, Macao, Canada, the Czech Republic, and Hong Kong and highest (over 50 percent) in Tunisia, Brazil, and Indonesia. At the other end of the scale, the proportion of mathematically strong students was highest (25 percent or more) in Hong Kong, the Netherlands, Belgium, Liechtenstein, and the Czech Republic.

In terms of the determinants of the point score, the strongest influences (defined as effecting, at least, a10 point increase in the point score) on mathematical performance – and in which where policy could play little or no role - were social class) and students being non-native language speakers .The areas in which schooling policy could have an effect were the amount of mathematical instruction at school and the level of classroom discipline. In terms of the home environment, the availability of a computer was important but equally important was the availability of non-computing facilities (quiet place to study, books etc.). Indeed, as the results showed the marginal benefit of computing facilities diminished much more rapidly than the marginal benefit from non-computing facilities.

A very important factor for high mathematical achievement is an interest in mathematics and here a number of factors – employment opportunities, school, home, friends – need to coalesce to create, sustain, and enhance interest in mathematics.

There is, therefore, evidence from this analysis of the PISA data that if policy makers wish to improve the level of mathematical ability of their 15 year olds, then a sensible policy regime would be to: (i) increase the amount of mathematical instruction at school and classroom discipline in general, (ii) increase the availability of computers at home (such as laptop borrowing schemes), (iii) increase the availability of other educational resources at home (such as by encouraging the borrowing of library books, CD-ROMs, DVD-ROMs and other educational material), (iv) take measures to develop an higher level of interest in Mathematics.

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Table 1
Equity Sensitive Assessment Scores in Mathematics of 15 year-olds, by Country

Country	Sample Size	Value of inequality aversion parameter			Rank under ε=0	Rank under ε=1	Rank under ε=2
		ε=0	ε=1	ε=2			
1. Hong Kong	4,478	555.9 (0.095)	547.1	537.2	1	1	1
2. Finland	5,796	542.8 (0.083)	536.8	530.4	2	2	2
3. Netherlands	3,992	542.1 (0.094)	534.3	525.9	3	3	3
4. Korea	5,444	540.7 (0.093)	533	524.8	4	4	4
5. Liechtenstein	332	536.5 (0.100)	527.4	517.5	5	5	5
6. Czech Rep	6,320	535.0 (0.102)	526.1	516.8	6	6	6
7. Japan	4,707	533.5 (0.102)	524	513.5	7	7	7
8. Belgium	8,796	533.2 (0.111)	521.9	509.3	8	8	8
9. New Zealand	4,511	525.6 (0.103)	516.6	507	9	9	10
10. Macao	1,250	522.8 (0.092)	515.6	508.1	10	10	9
11.Australia	12,551	522.3 (0.102)	513.3	503.6	11	12	12
12. Canada	27,953	521.4 (0.091)	514.3	506.9	12	11	11
13. Switzerland	8,420	518.2 (0.100)	509.7	500.6	13	13	13
14. Iceland	3,350	515.1 (0.095)	507.4	499.2	14	14	14
15. France	4,300	514.7 (0.096)	506.9	498.6	15	15	15
16. UK	9,535	514.4 (0.097)	506.5	498.1	16	16	16
17. Denmark	4,218	513.6 ((0.096)	505.9	497.6	17	17	17
18. Austria	4,597	511.9 (0.098)	503.9	495.5	18	18	18
19. Germany	4,660	508.4 (0.108)	498.4	487.6	19	20	21
20. Sweden	4,624	508.0 (0.101)	499.3	489.8	20	19	20
21. Ireland	3,880	504.7 (0.092)	497.8	490.6	21	21	19
22. Slovakia	7,346	504.2 (0.100)	495.9	487.1	22	22	22
23. Italy	11,639	496.0 (0.102)	487.3	477.7	23	25	25
24. Norway	4,064	495.6 (0.101)	487.4	478.8	24	24	24

25. Spain	10,791	494.8 (0.094)	487.6	479.8	25	23	23
26. Luxembourg	3,923	493.5 (0.101)	485.2	476.6	26	26	26
27. Poland	4,383	489.0 (0.100)	481.1	472.7	27	27	27
28. Hungary	4,765	488.6 (0.104)	480	470.9	28	28	29
29. Latvia	4,627	486.2 (0.097)	478.8	471.1	29	29	28
30. USA	5,456	481.5 (0.107)	472.6	463.3	30	30	30
31. Russian Fed	5,974	472.4 (0.105)	463.9	455.2	31	31	31
32. Portugal	4,608	465.2 (0.102)	457.4	449.2	32	32	32
33. Greece	4,627	440.9 (0.115)	431.2	420.9	33	33	33
34. Serbia	4,405	436.3 (0.106)	428.6	420.5	34	34	34
35. Turkey	4,855	426.7 (0.128)	415.7	404.8	35	35	36
36. Thailand	5,236	422.7 (0.107)	415.1	407.5	36	36	35
37. Uruguay	5,835	413.0 (0.137)	400.5	387.2	37	37	38
38. Mexico	29,983	405.4 (0.104)	398.3	390.8	38	38	37
39. Indonesia	10,761	361.5 (0.113)	354.1	346.5	39	39	39
4.0Brazil	4,452	360.4 (0.143)	348.6	336.3	40	40	41
41 Tunisia	4,721	359.3 (0.121)	351	342.5	41	41	40

Figures in parentheses are Gini coefficients

Table 2
Rates of Mathematical Weakness of 15-year olds, by Country*,
By Descending Order

Country	**Head Count Ratio (%)**
Tunisia	52.2
Brazil	51.5
Indonesia	50.5
Uruguay	30.2
Mexico	26.3
Turkey	25.8
Thailand	21.4
Greece	18.4
Serbia	17.8
Italy	12.2
Portugal	10.5
Russian Fed	9.9
USA	9.3
Germany	7.4
Hungary	7.1
Luxembourg	6.6
Latvia	6.4
Belgium	6.1
Poland	6
Norway	5.8
Spain	5.6
Sweden	5.4
Slovakia	5.1
Switzerland	4.5
Australia	4.4
France	4.4
Japan	4.4
Liechtenstein	4.2
New Zealand	4.2
Austria	4.1
Iceland	3.9
UK	3.9
Denmark	3.8
Ireland	3.8
Hong Kong	3
Czech Rep	2.8
Canada	2.7
Macao	2.6
Netherlands	2.5
Korea	2.2
Finland	1.2
All Countries	**12.2**

* Proportion of students at the lowest level of mathematical proficiency.

Table 3
Rates of Mathematical Strength of 15-year olds, by Country*
By Descending Order

Country	Head Count Ratio (%)
Hong Kong	32.4
Netherlands	27.4
Belgium	26.9
Liechtenstein	25.3
Czech Rep	25.1
Korea	23.8
Japan	23.6
Finland	22.3
New Zealand	20.7
Australia	19.3
Macao	17.2
Switzerland	17.1
Germany	16.1
Canada	16
France	15.1
UK	15.1
Sweden	14.8
Iceland	14.7
Denmark	14.6
Austria	14.5
Slovakia	12.9
Ireland	10.8
Italy	10.7
Norway	10.7
Hungary	9.7
Luxembourg	9.7
Poland	8.9
USA	8.6
Spain	8.1
Latvia	7.7
Russian Fed	6.8
Portugal	4.8
Turkey	4.7
Greece	3.1
Uruguay	2.6
Thailand	2
Serbia	2
Brazil	0.9
Mexico	0.36
Indonesia	0.11
Tunisia	0.11
All Countries	**11.8**

* Proportion of students at the highest two levels of mathematical proficiency.

Table 4

Regression Estimates for Explaining Predicted Scores in Mathematics

Family Type (residual- other type):	
Single parent family	13.24***
	(11.56)
Nuclear Family	18.65***
	(17.08)
Mixed Family	15.02***
	(11.48)
Highest Occupational Class of Parent (residual blue collar low skilled):	
White Collar, high skilled	28.60***
	(42.87)
White Collar, low skilled	12.15***
	(17.70)
Blue Collar, high skilled	1.70**
	(2.34)
Highest Educational Attainment of parent:	
Years of Schooling	1.75***
	(8.29)
(Years of Schooling)^2	0.007
	(0.77)
Language at home if different from test language	-15.61***
	(15.89)
Minutes of Mathematical Instruction at School (residual – fourth quartile):	
Minutes: lowest quartile	-17.85***
	(32.59)
Minutes: second quartile	-9.54***
	(14.14)
Minutes: third quartile	-7.62***
	(13.66)
Computing facilities at home (residual – fourth quartile):	
Facilities: lowest quartile	-46.91***
	(66.05)
Facilities: second quartile	-22.83***
	(31.05)
Facilities: third quartile	-1.51**
	(3.15)
Low home Educational Resources:	-2.92***
	(6.20)
Home Resources (residual – fourth quartile):	
Resources: lowest quartile	-30.86***
	(38.45)
Resources: second quartile	-14.46***
	(24.27)
Resources: third quartile	-9.68***
	(15.95)

Interest in mathematics (residual – fourth quartile):	
Interest: lowest quartile	-27.26***
	(49.91)
Interest: second quartile	-17.28***
	(35.60)
Interest: third quartile	-8.95***
	(14.52)
Motivation (residual – fourth quartile):	
Motivation: lowest quartile	-4.13**
	(6.16)
Motivation: second quartile	-2.19
	(4.17)
Motivation: third quartile	-2.57***
	(4.20)
Learning Strategies (residual – elaboration learning strategies):	
Memorisation/rehearsal learning strategies	1.01**
	(2.37)
Control learning strategies	3.57***
	(7.95)
Discipline in classroom (residual – fourth quartile):	
Discipline: lowest quartile	-31.20***
	(53.26)
Discipline: second quartile	-20.74***
	(40.89)
Discipline: third quartile	-10.78***
	(18.91)
Student's grade:	29.66***
	(93.32)
Country-specific variables (residual – OECD partner countries):	
OECD country: english speaking	22.23***
	(35.25)
OECD Nordic countries	23.12***
	(29.18)
OECD Western Europe	28.92***
	(48.55)
OCED Eastern Europe	38.49***
	(51.75)
OECD Others	-16.19***
	(22.49))
Intercept	*502.18****
	(271.93)
Observations	186612
R-squared (adjusted)	0.346

Absolute value of t statistics in parentheses
* significant at 10%; ** significant at 5%; *** significant at 1%