Active Hedging Greeks of an Options Portfolio integrating churning and minimization of cost of hedging using Quadratic Linear Programing

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Abstract

This paper proposes a methodology for active hedging Greeks of an option portfolio integrating churning and minimization of cost of hedging. In the first section, hedging strategy is implemented by taking positions in other available options, while simultaneously minimizing the net premium paid for the hedging and then churning the portfolio to take into account the changed value of Greeks in the new portfolio. In the second section, the paper extends the model to incorporate the transaction cost while hedging the portfolio and churning it in Indian Scenario. Both constant and nonlinear shape of transaction cost has been considered as per the Security Transaction Tax and Brokerage charges in India. A quadratic programming has been presented which has been approximated by a linear programming solution. The prototype software has been developed in MS Excel using Visual Basic.

1. Introduction

Options are one of the most essential instruments being used by the hedgers to reduce the volatility of their portfolio. However options themselves are volatile. The change in the value of option portfolio can be described using the Greek variables Delta, Gamma, Vega, Theta and Rho. Due consideration to these values is essential to formulate any hedging strategy using options portfolio. This is reflected in the research done by Papahristodoulou [1] who presented a linear programming model to hedge options while incorporating the Greeks so that one doesn't need to track the movement of option portfolio with the underlying stocks. Horasanlı [2] is his work extended the above model to incorporate multi-asset portfolio of options and stocks. In the past Hull [3] and Rendleman [4] have also discussed ways to hedge an option portfolio. Sinha & Johar [5] presents a model to hedge Greeks of an existing portfolio by making use of the other available options in the market while minimizing the premium paid for the construction of hedge.

There are two things which haven't been discussed in the above mentioned references. Firstly a static view of portfolio has been considered and secondly transaction costs have been ignored. The value of Greeks changes continuously over a period of time and hence there is a need to rebalance the portfolio to incorporate the changed value of Greeks while taking the previous hedged portfolio as input. Transaction costs can play a significant part in determining whether the benefits of hedging are more than the costs of hedging or not. In the subsequent sections, we extend the methodology of Sinha & Johar [5] to rebalance the option portfolio while taking into account the changed value of Greeks and we have further augmented our model by incorporating transaction costs in Indian Scenario. We present a quadratic programming solution which has been approximated by a linear programming solution for which a prototype has been developed using MS-Excel and Visual Basic.

2. Greeks Used & Calculations

We have hedged the option portfolio for Delta, Vega and Gamma. The intent is to present a general model which can be later extended to incorporate the other Greek values as well. Consider a portfolio of *n* options. The table below presents some of the notations that we have used in this paper in regard to the options,

N_i	Number of options of i^{th} option
D_i	Delta of a option of i^{th} option
V_i	Vega of a option of i^{th} option
G_i	Gamma of a option of i^{th} option

The overall Delta, Vega and Gamma of the portfolio have been calculated using the following formula,

$$D_{portfolio} = \sum_{i=1}^{n} D_i * N_i$$

$$V_{portfolio} = \sum_{i=1}^{n} V_i * N_i$$

$$G_{portfolio} = \sum_{i=1}^{n} G_i * N_i$$

3. Greek Neutral Portfolio

Once we have the Greek values for our original portfolio we can create take position in the options available for hedging in the market such that the net Greek values become zero,

$$D_{net} = 0$$

$$V_{net} = 0$$

$$G_{net} = 0$$

Suppose we need to take position in m options to hedge Greeks in our original portfolio. Here we will add a new notation P_j,

P_j	*Premium paid to buy an option of j^{th} option (Paid in taking a long position and vice versa*

The overall Delta, Vega and Gamma can be calculated as,

$$D_{net} = D_{portfolio} + \sum_{i=1}^{n} D_i * N_i$$

$$V_{net} = V_{portfolio} + \sum_{i=1}^{n} V_i * N_i$$

$$G_{net} = G_{portfolio} + \sum_{i=1}^{n} G_i * N_i$$

Now while hedging using the available m options we have to make sure that the cost of setting up the hedge is minimal which implies that the net premium paid $\sum \boldsymbol{N_i P_i}$ is minimized. At this juncture we will introduce a new variable,

$$X_j = \begin{cases} 0, if\ j^{th}\ lot\ of\ option\ is\ not\ selected \\ 1, if\ j^{th}\ lot\ of\ option\ is\ selected \end{cases}$$

We now present the quadratic model as a solution to create a Greek neutral portfolio in an active hedging environment. We have used a methodology similar to the once used by Li, Z.F., Wang, S.Y., & Deng, X.T.[6].

Minimize,

$$\sum_{j=1}^{m} (N_j - N_{j-1}) * P_j * X_j$$

Subject to,

$$-D_{portfolio}\ (1 + Var_{Delta}) \leq \sum_{j=1}^{m} (N_j - N_{j-1}) * X_j * D_j \leq\ -D_{portfolio}\ (1 - Var_{Delta})$$

$$-D_{portfolio}\ (1 + Var_{Gamma}) \leq \sum_{j=1}^{m} (N_j - N_{j-1}) * X_j * G_j \leq\ -G_{portfolio}\ (1 - Var_{Gamma})$$

$$-D_{portfolio}\ \left(1 + Var_{Vega}\right) \leq \sum_{j=1}^{m} (N_j - N_{j-1}) * X_j * V_j \leq\ -V_{portfolio}\ (1 - Var_{Vega})$$

$$N\ \in Integer$$

$Var_{Delta,}$ Var_{Gamma} and Var_{Vega} are the variances allowed in the values of Delta, Gamma and Vega of the original portfolio to keep the scenario more realistic as perfect hedge is almost impossible in real world.

We can also put a constraint on the maximum number of options that we can use to make our portfolio Greek neutral. Suppose that we can use a maximum of Z options then we have an additional constraint,

$$\sum X_i \leq\ Z$$

The above model is quadratic constrained quadratic program which is typically NP-Hard to solve. This coupled with the fact that we have constrained N to be an integer which also typically causes problems to be NP-Hard.

We now present the approximations as has been used by Sinha & Johar [5] which will simplify the model,

- Drop the constraint of choosing at most Z options.

- Instead of constraining N to be integers we can use the following constraint to impose variance limits on N,

$$\left|N_j - Round(N_j)\right| \leq \ Var_N * N_j$$

4. Final Model (Without Transaction Cost)

Minimize,

$$\sum_{j=1}^{m} (N_j - N_{j-1}) * P_j * X_j$$

Subject to,

$$-D_{portfolio}\ (1 + Var_{Delta}) \leq \sum_{j=1}^{m} (N_j - N_{j-1}) * X_j * D_j \leq\ -D_{portfolio}\ (1 - Var_{Delta})$$

$$-D_{portfolio}\ (1 + Var_{Gamma}) \leq \sum_{j=1}^{m} (N_j - N_{j-1}) * X_j * G_j \leq\ -G_{portfolio}\ (1 - Var_{Gamma})$$

$$-D_{portfolio}\ \left(1 + Var_{Vega}\right) \leq \sum_{j=1}^{m} (N_j - N_{j-1}) * X_j * V_j \leq\ -V_{portfolio}\ (1 - Var_{Vega})$$

$$\left|N_j - Round(N_j)\right| \leq \ Var_N * N_j$$

Since have dropped the constraint of choosing at most Z options and we have used MS-Excel for solving the linear programming problem we need a limit on the number of options to be used for the purpose of creating a Greek neutral portfolio. Hence we have used the following constraint,

$$\sum_{i=1}^{n} N_i \leq \sum_{j=1}^{m} N_j$$

By using the above constraint we have restricted the number of options used for hedging to be less than the number of options in our original portfolio.

5. Final Model (With Transaction Cost in Indian Scenario)

In Indian scenario transaction consists of the following two components,

1. Security Transaction Tax (STT) – As per the Finance Act 2004, and modified by Finance Act 2008 (18 of 2008) STT on the transactions executed on the Exchange is as under, NSE[6].

Sale of an option in securities	0.017%	Paid by Seller
Sale of an option in securities, where option is exercised	0.125%	Paid by Purchaser

   Hence STT per option can be considered to be constant and doesn't change with the number of contracts and would only depend on the premium paid or received.

2. Brokerage – Brokerage is charged by the various brokers and is often negotiable. Brokerage charges generally decrease as the volume of the trade increases.

To incorporate transaction cost in the model discussed in the previous section an additional constraint is provided to limit the transaction cost below a certain value which can be specified by the user. If ***S*** is the maximum amount that the hedger would like to spend on the transaction cost then the additional constraint for this model is,

$$(C_B + C_{STT}) * \left(N_j - N_{j-1}\right) \leq S$$

6. Illustration

Prototype software was developed using MS-Excel and Visual Basic to solve the above model.

For the purpose of illustration we have used the same original portfolio as used by Papahristodoulou [1]. The following two sets of Ericsson options were available as of 13th Feb, 2001. The stock price was trading at SEK 96 at the Stockholm Stock Exchange. The first set of options corresponds to April options (days to expire were 66) and the second set corresponds to June options (days to expire were 122). The risk free rate of interest was 6%. Implicit volatility was estimated as 57% for April options and 55% for June options. The three and six month volatilities were 68% and 65% respectively.

Our aim is to establish a portfolio for a trader who wishes to hedge a portfolio formed using April options, as per his trading strategy, with June options.

Option Type	Number of Options	Strike Price	Premium Paid & Received	Delta	Gamma	Vega
Call	0	95	10.25	0.5815	0.01407	15.944
Call	32	100	7.75	0.5118	0.01436	16.279
Call	0	105	6	0.4442	0.01423	16.126
Call	0	110	4.5	0.3816	0.01373	15.563
Call	28	115	3.1	0.3246	0.01295	14.685
Call	0	120	2.5	0.2736	0.01199	13.586

Put	0	95	8	-0.4185	0.01407	15.944
Put	0	100	10.75	-0.4887	0.01436	16.279
Put	-25	105	14.25	-0.5558	0.01423	16.126
Put	-25	110	17	-0.6184	0.01373	15.563
Put	0	115	21	-0.6754	0.01295	14.685
Put	0	120	25	-0.7264	0.01199	13.586

1 Original Portfolio (April 2001 Options)

Option Type	Options strike price	Premium Paid & Received	Delta	Gamma	Vega	STT	Constant Brokerage	Total
Call	110	7.5	0.4448	0.011	21.93	0.001275	2	2
Call	115	6.5	0.3986	0.0107	21.42	0.001105	2	2
Call	120	4.75	0.3556	0.0103	20.67	0.0008075	2	2
Put	110	20	-0.555	0.011	21.93	0.0034	2	1.9
Put	115	22.25	-0.601	0.0107	21.42	0.0037825	2	1.9
Put	120	26.75	-0.644	0.0103	20.67	0.0045475	2	1.9
Put	120	24	0.5	0.0103	25	0.00408	3	1.7

2 June 2001 options available for hedging

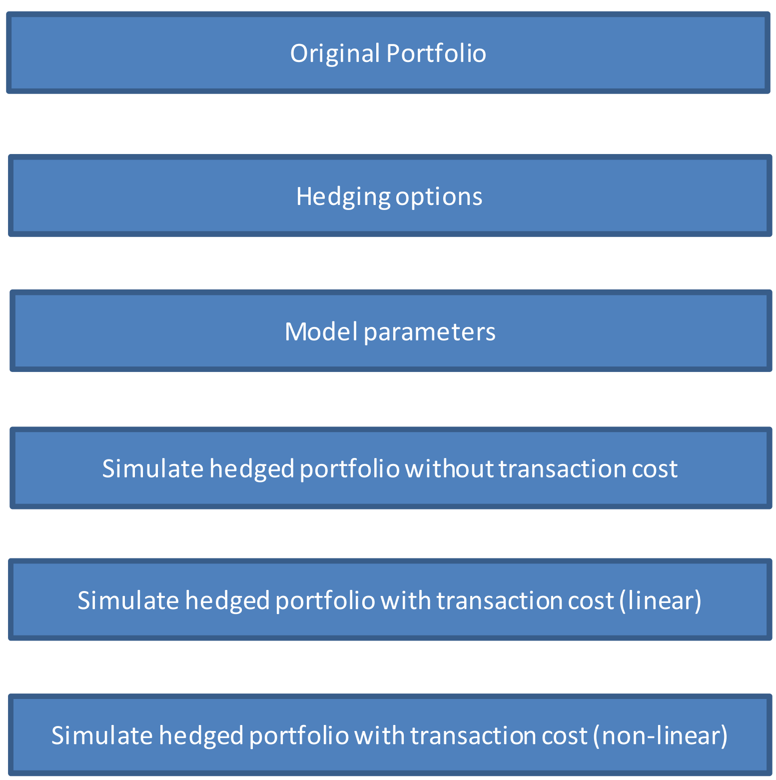

Figure 1 Main Interface

1. *Without transaction cost*

We have constrained the number of options used for hedging to be less than the number of options in the original portfolio and hence the limit on the number of options which can be used is 110. Now simulating the hedge without transaction cost for the first time gives us the following results,

Model Interface

Simulate hedged portfolio without transaction cost

Simulate hedged portfolio with transaction cost (linear)

Simulate hedged portfolio with transaction cost (non-linear)

	Premium Paid/ Received	Delta	Gamma	Vega	Transaction cost	Total options long/short
Before Hedging	0	54.8214	0.12312	139.883	0	0
After previous Hedging cycle	0	54.8214	0.12312	139.883	0	0
After last Hedging cycle	762	-0.548214	0.0525194	-1.39883	0	110
New option portfolio	762	-55.369614	-0.070601	-141.2818	0	110

Previous options portfolio used for hedging

Option Type	Strike Price	Premium Paid/ Received per lot	Number of options long/short
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

New options portfolio to be used for hedging

Option Type	Strike Price	Premium Paid/ Received per lot	Number of options long/short
Call	110	7.5	-54.67275501
Call	115	6.5	-2.661842637
Call	120	4.75	2.076781014
Put	110	20	14.26269874
Put	115	22.25	12.76270731
Put	120	26.75	23.48622426

New options to be transacted

Option Type	Strike Price	Premium Paid/ Received per lot	Number of options long/short
Call	110	7.5	-55
Call	115	6.5	-3
Call	120	4.75	2
Put	110	20	14
Put	115	22.25	13
Put	120	26.75	23

The magnified view of the results table is,

	Premium Paid/ Received	Delta	Gamma	Vega	Transaction cost	Total options long/short
Before Hedging	0	54.8214	0.12312	139.883	0	0
After previous Hedging cycle	0	54.8214	0.12312	139.883	0	0
After last Hedging cycle	762	-0.548214	0.0525194	-1.39883	0	110
New option portfolio	762	-55.369614	-0.070601	-141.2818	0	110

Since we haven't considered the transaction cost in the first simulation hence the transaction cost is zero. The first row in the table show the various parameters for the option portfolio, the second row show the results after the second last hedging cycle, the third row show the results after the last hedging cycle, and the last row show the parameters for the new option portfolio used to hedge the previous portfolio.

Since this is the first simulation the values of the portfolio in the second row is the same as the value in the first row.

The table below shows the options wise positions that we need to take in order to make our portfolio Greek neutral,

New options portfolio to be used for hedging

Option Type	Strike Price	Premium Paid/ Received per option	Number of options long/short
Call	110	7.5	-54.67275501
Call	115	6.5	-2.661842637

Call	120	4.75	2.076781014
Put	110	20	14.26269874
Put	115	22.25	12.76270731
Put	120	26.75	23.48622426

The results obtained are same as obtained by Sinha & Johar [5].

Churning the Portfolio without transaction cost

Now the option parameters have changed after a week and we want to rebalance the portfolio to make it Greek neutral. The following table represents the changed parameters for the option available for the options that are available for hedging.

Option Type	Strike Price	Premium Paid & Received	Delta	Gamma	Vega	STT	Constant Brokerage	Total
Call	110	7.5	0.4448	0.011	21.93	0.001275	2	0
Call	115	6.5	0.555	0.0107	21.42	0.001105	2	0
Call	120	4.75	0.3556	0.0103	20.67	0.0008075	2	0
Put	110	20	-0.555	0.02	21.93	0.0034	2	0
Put	115	18	-0.601	0.0107	24	0.00306	2	0
Put	120	21	-0.644	0.0103	20.67	0.00357	2	0

The values in red reflect the changed values of the parameters available for hedging. In reality all values will change including the values of the options used in the original portfolio. Our model is robust enough to incorporate all such changes. The table below summarizes the results obtained after churning the portfolio with the changed values,

	Premium Paid/ Received	Delta	Gamma	Vega	Transaction cost	Total options long/short
Before Hedging	0	54.8214	0.12312	139.883	0	0
After previous Hedging cycle	762	-0.96452619	0.1815968	31.503429	0	110
After last Hedging cycle	688	-0.548214	0.002502	-1.39883	0	110
New option portfolio	-74	-55.369614	-0.120618	-141.2818	0	110

Now as we can the premium value given in the second row was for the first round of hedging, while the premium given in the third row represents the total premium paid/received till now, while the premium value in the fourth row gives us the premium paid/received during the last hedging cycle. Here we can see that we have received some premium but the total number of options in which we have taken positions have remained constant. The table below shows the option wise positions that we need to take in addition to the positions that we took in the previous hedging cycle, New positions in options to be taken in addition to the original positions

Option Type	Strike Price	Premium Paid/ Received per option	Number of options long/short
Call	110	7.5	0
Call	115	6.5	0
Call	120	4.75	0
Put	110	20	-16
Put	115	18	16
Put	120	21	-2

2. *With transaction cost in Indian Scenario*

There are two components to a transaction cost in Indian Scenario as already discussed,

a. Brokerage – Brokerage depends on the number of options transacted. In the current case we have assumed stepwise decreasing function of brokerage with respect to transaction cost. The table below shows a sample of brokerage costs that we have assumed with respect to the number of options transacted,

Number of Options Transacted	Brokerage Charged Per Option (INR)
0-10	2
11-20	1.9
21-30	1.8
31-40	1.7

Our model is robust enough to incorporate constant brokerage per option irrespective of the volume of trade.

b. Security Transaction Cost – Security transaction cost per option remains constant irrespective of the number of options transacted. We have assumed the following STT for our simulation.

Option Type	Option Strike Price	Premium Paid & Received	STT (Per Option) = (0.17*Premimum)/100
Call	110	7.5	0.001275
Call	115	6.5	0.001105
Call	120	4.75	0.0008075
Put	110	20	0.0034
Put	115	22.25	0.0037825
Put	120	26.75	0.0045475

Now we will again consider the same original portfolio and same options available for hedging. However the difference would be to limit the transaction cost to INR 1000. Simulating the portfolio we get the following results,

	Premium Paid/ Received	Delta	Gamma	Vega	Transaction cost	Total options long/short
Before Hedging	0	54.8214	0.12312	139.883	0	0
After previous Hedging cycle	0	54.8214	0.12312	139.883	0	0
After last Hedging cycle	762	-0.548214	0.0525194	-1.39883	185.2	110
New option portfolio	762	-55.369614	-0.070601	-141.2818	185.2	110

Since the transaction cost limit was set to INR 1000 while to obtain a Greek neutral portfolio only INR 185.2 were required hence the results are similar as in the case of hedging without transaction costs.

Now we will limit the transaction cost to INR 150 and rerun the model from the scratch, i.e. considering the original portfolio again. We get the following results with the limit,

	Premium Paid/ Received	Delta	Gamma	Vega	Transaction cost	Total options long/short
Before Hedging	0	54.8214	0.12312	139.883	0	0
After previous Hedging cycle	0	54.8214	0.12312	139.883	0	0
After last Hedging cycle	579.75	10.5464938	0.0525675	-1.39883	150	86
New option portfolio	579.75	-44.2749062	-0.070552	-141.2818	150	86

Now since our transaction cost was limited to INR 150 our Greeks in the hedged portfolio haven't been hedged completely and hence showing a more realistic scenario. The options positions that we need to take in order to hedge our portfolio are,

New positions in options to be taken in addition to the original positions

Option Type	Strike Price	Premium Paid/ Received per lot	Number of options long/short
Call	110	7.5	-45
Call	115	6.5	-1
Call	120	4.75	0
Put	110	20	13
Put	115	22.25	13
Put	120	26.75	14

Now we will churn the portfolio again by changing some of the parameters as in the previous case without transaction costs. The transaction cost has been again given a limit of INR 150. The table below shows the changed parameter values of the options available for hedging,

Option Type	Option strike price	Premium Paid & Received	Delta	Gamma	Vega
Call	110	7.5	0.4448	0.011	21.93
Call	115	6.5	0.555	0.0107	21.42
Call	120	4.75	0.3556	0.0103	20.67
Put	110	20	-0.555	0.02	21.93
Put	115	18	-0.601	0.0107	24
Put	120	21	-0.644	0.0103	20.67

Simulating the hedging we get the following results after churning the portfolio,

	Premium Paid/ Received	Delta	Gamma	Vega	Transaction cost	Total options long/short
Before Hedging	0	54.8214	0.12312	139.883	0	0
After previous Hedging cycle	579.75	10.4478249	0.1689846	33.131687	150	86
After last Hedging cycle	573.5	0.414	-4E-05	-1.253	267.1	106
New option portfolio	-6.25	-54.4074	-0.12316	-141.136	117.1	106

We can see that the transaction cost in the second round has been INR 117.1 and which is again less than the limit of INR 150 and the portfolio is almost Greek neutral. The new positions that we need to take in addition to the original positions are,

New positions in options to be taken in addition to the original positions

Option Type	Strike Price	Premium Paid/ Received per lot	Number of options long/short
Call	110	7.5	-9
Call	115	6.5	-3
Call	120	4.75	1
Put	110	20	-13
Put	115	18	28
Put	120	21	-8

Hence we can conclude that the option portfolio can be made Greek neutral using the methodology described above. In addition to a static view our methodology incorporates an active hedging environment and transaction costs in Indian Scenario.

7. References

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