Alliance Partner Choice in Markets with Vertical and Horizontal Externalities

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Abstract

This study investigates the choice between complementary and parallel alliances in a market with vertical and horizontal externalities. One composite goods firm competes with two components producers, each providing a complementary component of a differentiated composite good. Although the joint profits from a parallel alliance between the composite goods firm and a components producer are always larger than those from a complementary alliance between components producers, through Nash bargaining, a components producer prefers the complementary (parallel) alliance when the degree of product differentiation is sufficiently large (small). Combined with the result that a complementary alliance is socially preferable, our findings provide meaningful implications for antitrust policy.

Keywords: Complementary alliance, Parallel alliance, Nash bargaining, Antitrust policy
JEL classification number: L11, L13, L41.

1 Introduction

Today, strategic alliances are increasingly and widely used by firms in network-oriented industries such as airline, shipping, multimodal transport, telecommunications, and logistics industries, as well as firms in non-network markets of compatible components such as personal computers and

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software, ATMs and bankcards, and markets for vacations comprising of airline transportation and resort hotel stay.

One common feature of these markets is that multiple firms produce competing and/or complementary products. This feature raises an interesting managerial question of who is the best partner for an alliance. Furthermore, as these firms act independently, the well known concept of vertical and/or horizontal externalities exist in the markets. Then, the partner choice by firms, which consequently leads to different types of alliances, causes different impacts on welfare. Therefore, the partner choices for strategic alliance also raise welfare concerns.

This study investigates the partner choices for strategic alliances in a composite market, from both a positive and a normative perspective. We consider a simple but common market including a representative consumer and three firms. The consumer purchases a composite good offered solely by one composite goods firm, and another differentiated composite good comprising of two components, each produced by one components producer.¹ We examine the consequences of an alliance between the two complementary components producers and the alliance between one components producer and the composite goods firm. The former is characterized as a complementary alliance, and the latter as a parallel alliance. In particular, we argue whether or not one components producer should ally with another components producer (its complementary partner), or ally with the composite goods firm (its rival that produces a substitutive composite good), to derive a comparatively larger profit. We explicitly include a profit sharing negotiation by considering Nash bargaining solutions.

First, we find that a complementary alliance may be unprofitable, whereas a parallel alliance is always profitable than remaining independent for the allied firms. As Cournot (1838) pointed out, if there is no rival composite goods firm in the market, a complementary alliance between two complementary components producers corrects for a vertical externality by jointly pricing, and thus, it increases their profits. This is well known as the double marginalization problem in vertically related markets. However, if there is a rival firm, the correction for the vertical externality also has an adverse effect by enhancing the price competition between the two composite goods. When the two goods are less differentiated (i.e., the competition is substantial), this adverse effect makes the complementary alliance unprofitable. On the other hand, the parallel alliance is always profitable because it relaxes competition against the rival product by correcting horizontal externalities.

Further, we find that the joint profits obtained by the parallel alliance are always larger than those obtained by the complementary alliance. However, total profitability alone is not a sufficient determinant of the type of alliance that will emerge. Through the standard approach of Nash bargaining, we find that when the degree of product differentiation for the two composite goods is sufficiently large (small), a components producer prefers the alliance with its complementary partner (the com-

¹This market structure has a strong similarity with Park (1997), Lin (2005), as well as the structure under parallel vertical integration in Economides and Salop (1992). As Economides and Salop (1992) indicated, this type of market structure is perhaps the most common one (p. 107).
posite goods firm). This results from the fact that the bargaining position of each firm is different, depending on which firm it allies with and on market conditions. In particular, if a components producer allies with its complementary partner, they bargain for the joint profits on an equal footing and independently of the degree of product differentiation. Alternatively, if the components producer allies with the composite goods firm (its rival), the components producer’s bargaining position (or profit on the threat point) is weaker than its rival’s, and this disadvantage becomes larger as the degree of product differentiation becomes substantial. In other words, the components producer may prefer a type of alliance that yields a lower total profit but a higher share for itself when the degree of product differentiation is significant.

Our findings on the preference for an alliance partner provide clear and useful managerial implications. In short, for the composite goods firm, allying with a component producer is always profitable. For a component producer, it is sensible to ally with its complementary partner if the degree of product differentiation is sufficiently large; otherwise, allying with its rival, the composite goods firm, is preferable.

Furthermore, because the complementary alliance leads to lower prices by both eliminating double marginalization and intensifying competition, it is welfare-enhancing. However, the parallel alliance is welfare-decreasing because it relaxes competition. Combining these results, our findings on firms’ alliance partner choices also offer important implications for competition policies, such as the antitrust policy. First, when the degree of product differentiation is sufficiently large, a regulatory policy concerning the alliance formation might not be necessary because the welfare-enhancing complementary alliance will be spontaneously formed. Second, when the degree of product differentiation is intermediate, interestingly, the prohibition of forming a parallel alliance can encourage components producers to form a complementary alliance. Finally, when the degree of product differentiation is sufficiently small, the prohibition of forming a parallel alliance can prevent welfare reduction. However, this alone cannot encourage the formation of a complementary alliance due to its non-profitability. In this case, an additional policy (e.g., subsidy policy) for implementing the complementary alliance can also be socially desirable.

A number of previous studies have considered strategic alliances in composite good markets where there are both vertical and horizontal externalities. The studies that focused on the complementary alliance have been provided by Beggs (1994), Lin (2004), Zhang and Zhang (2006), among others. The studies dealing with parallel alliances in airline networks include Brueckner (2001), Shy (2001, Ch.9), and Lin (2005) and (2008). However, each of the previous studies assumed that the alliance partners are exogenously decided and the share rate of the joint profits between the allied partners is taken as given. Unlike these previous studies, the present study explicitly investigates the issue of partner choice, and considers the corresponding joint profits sharing problem by introducing the Nash-bargaining process.²

²Regarding the profitability of a horizontal merger, Salant et al. (1983) construct a Cournot model with homogenous goods, while Deneckere and Davidson (1985) construct a Bertrand model with product differentiation. Their focus is on
This paper is organized as follows: In Section 2, the market structure is presented, and the equilibrium of the benchmark case (without an alliance) is derived. Then, the outcomes of complementary and parallel alliances with the Nash-bargaining solutions are derived respectively. In Section 3, the effects of the two types of alliances are examined. The alliance partner preference for each firm is investigated, and then a welfare comparison is provided. Concluding remarks follow in Section 4.

2 The Model

Consider a market with two differentiated composite goods. Firm 1 provides a composite good, composed of components A and B, at a single price \( p_1 \). Firm 2 (3) provides component A (B) at price \( p_2 (p_3) \). Because we are concerned about strategic alliances in a composite goods market, the markets for component A or B individually are not included in our model. Following Economides and Salop (1992), the marginal costs for each firm are constant and assumed to be zero.

Consumers of composite goods can either purchase the composite good (good 1) provided by firm 1, or combine the two components in a fixed proportion (one unit of each) to form a composite good (good 23). The utility function of a representative consumer is defined by the following quasi-linear type:

\[
U(q_1, q_{23}) = a(q_1 + q_{23}) - \frac{1}{2} \left( (q_1)^2 + 2b q_1 q_{23} + (q_{23})^2 \right),
\]

where \( q_1 \) and \( q_{23} \), respectively, represent the demand for the two differentiated composite goods. Parameter \( a > 0 \), and the condition for the concavity of the function is \( 0 < b < 1 \).

The consumer surplus is measured by \( U(q_1, q_{23}) - [p_1 q_1 + (p_2 + p_3) q_{23}] \). Note that good 23 is available at total price \( (p_2 + p_3) \). Given the prices, the representative consumer chooses the quantity that maximizes his/her consumer surplus. Now the demand functions for the two composite goods the profitable merger size (i.e., the number of merger firms), and the corresponding bargaining problem among the merger firms is not included.

3Our model also applies to the case where firm 1’s composite good is one single product, as well as to the case where its composite good is combined from several components.

4As Zhang and Zhang (2006) indicate that alliance firms can remain separate business entities and retain their decision-making autonomy under strategic alliances, our assumption can be justified in a situation where firms can use discriminating price strategies for each single component market. Under discriminating price strategies, the alliance partners can cooperatively set a total price for the composite goods, while retaining separate decision-making in their own single component market. For these strategies in airline network markets, see Brueckner (2001), Lin (2004) and (2008) among others.

5The results are identical for positive constant marginal costs with “price” reinterpreted as the difference between price and marginal cost.

6This type of utility function has been used in a number of differentiated duopoly studies (e.g., Spence (1976), Dixit (1979), Vives (1984) and (1985), Sutton (1997), Bárıáez-Ruiz (2007), among others).
can be derived and written as follows:\footnote{Although the derived demand functions are linear, it yields a number of suggestive conclusions that are likely to hold true in general (Varian 1992, p. 294). Also note that under these demand functions, it is implicitly assumed that the demand for composite good $23$ is the same size as the demand for composite good $1$. This assumption is based on the full compatibility between the two components. The same assumption has been used by Economides and Salop (1992).}

\[ q_1 = \frac{a(1-b) - p_1 + b(p_2 + p_3)}{1-b^2}, \quad q_{23} = \frac{a(1-b) - (p_2 + p_3) + bp_1}{1-b^2}. \] (1)

Parameter $b$ can be used as an index of product differentiation. When parameter $b$ is close to zero (unity), the cross-price effect is extremely small (sufficiently large), which implies that the two differentiated composite goods are almost independent (perfect substitutes).

### 2.1 Equilibrium of the Independent Case (No Alliance)

We derive the Bertrand-Nash equilibrium for the benchmark case (Case-I) where no alliance is formed and each firm independently sets the price of its own product. The profit function for firm 1 can be defined as $\pi_1 = p_1 \cdot q_1(p_1, p_2, p_3; b)$, and the profit function for firm $i$ ($i = 2, 3$) is defined as $\pi_i = p_i \cdot q_{23}(p_1, p_2, p_3; b)$. Each firm chooses a price that maximizes its own profits, taking the prices of the other firms as given. Then, the profit-maximization of each firm is respectively characterized by the following first-order conditions:

\[
\frac{\partial \pi_1}{\partial p_1} = \frac{a(1-b) - 2p_1 + b(p_2 + p_3)}{1-b^2} = 0, \quad \text{for } i = 1, 2, 3.
\] (2)

\[
\frac{\partial \pi_i}{\partial p_i} = \frac{a(1-b) - 2p_i - p_j + bp_1}{1-b^2} = 0, \quad \text{for } i = 2, 3, \ i \neq j.
\] (3)

These first-order conditions, which are indeed the reaction functions for the firms, describe the strategic complementarity and substitutability relationship among the three firms.\footnote{For the definition of strategic substitutes and complements, Bulow et al. (1985) is useful.} The composite good of firm 1 and the composite good composed of the components of firms 2 and 3 are substitutes. Thus, $p_1$ and $p_2$ (as well as $p_3$) are strategic complements. On the other hand, because the components of firms 2 and 3 are complements, $p_2$ and $p_3$ are strategic substitutes. Note that the two components producers are symmetric in our model.

Solving the equation system (2)-(3), we obtain the outcome of Case-I shown in Table 1. The superscript “I” stands for the corresponding equilibrium values.

### 2.2 Complementary Alliance and Nash Bargaining over the Gains

We consider the case of a complementary alliance (Case-C) where firm 2 forms an alliance with its complementary partner (firm 3) to compete with their rival (firm 1), by cooperatively setting a total price for their composite good 23. It is assumed that a single decision-maker chooses a total price for their composite good 23 (denoted by $p_{23}$) to maximize the allied firms’ joint profits (denoted
Table 1: Bertrand-Nash Outcomes in Three Cases

<table>
<thead>
<tr>
<th>Prices</th>
<th>Demands</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case-I: Independent ownership</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p_1^I = \frac{a(1-b)(3+2b)}{2(3-b^2)} )</td>
<td>( q_1^I = \frac{a(3+2b)}{2(1+b)(3-b^2)} )</td>
<td>( \pi_1^I = \frac{a^2(1-b)(3+2b)^2}{4(1+b)(3-b^2)^2} )</td>
</tr>
<tr>
<td>( p_2^I = \frac{a(1-b)(2+b)}{2(3-b^2)} )</td>
<td>( q_2^I = \frac{a(2+b)}{2(1+b)(3-b^2)} )</td>
<td>( \pi_2^I = \frac{a^2(1-b)(2+b)^2}{4(1+b)(3-b^2)^2} )</td>
</tr>
<tr>
<td>Case-C: Complementary alliance (firm 2 allies with firm 3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p_1^C = \frac{a(1-b)}{2(2-b)} )</td>
<td>( q_1^C = \frac{a}{(1+b)(2-b)} )</td>
<td>( \pi_1^C = \frac{a^2(1-b)}{(1+b)(2-b)^2} )</td>
</tr>
<tr>
<td>( \pi_2^C = \frac{a(1-b)}{2(2-b)} )</td>
<td>( q_2^C = \frac{a}{(1+b)(2-b)} )</td>
<td>( \pi_2^C = \frac{a^2(1-b)}{(1+b)(2-b)^2} )</td>
</tr>
<tr>
<td>Case-P: Parallel alliance (firm 2 allies with firm 1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p_1^P = \frac{a}{2} )</td>
<td>( q_1^P = \frac{a(3+b)}{6(1+b)} )</td>
<td>( \pi_1^P = \frac{a(12+5b)}{36(1+b)} )</td>
</tr>
<tr>
<td>( p_2^P = \frac{a(1-b)}{3} )</td>
<td>( q_2^P = \frac{a}{3(1+b)} )</td>
<td>( \pi_2^P = \frac{a^2(1-b)}{9(1+b)} )</td>
</tr>
</tbody>
</table>

by \( \pi_{23} \), and then these joint profits will be shared through Nash bargaining. In Case-C, the demand functions change to

\[
q_1 = \frac{a(1-b) - p_1 + b(p_{23})}{1 - b^2}, \quad q_{23} = \frac{a(1-b) - (p_{23}) + bp_1}{1 - b^2}.
\]

(4)

Correspondingly, the profit function of firm 1 becomes \( \pi_1 = p_1 \cdot q_1(p_1, p_{23}; b) \), and the joint profits of the allied firms can be written as \( \pi_{23} = p_{23} \cdot q_{23}(p_1, p_{23}; b) \). In Case-C, there exists only the strategic complementarity relationship between firm 1 and the allied firms. Similarly, solving the profit-maximization problems for the two agents in their price setting, we obtain the outcome of the complementary alliance case shown in Table 1 (Case-C). The superscript “C” stands for the corresponding equilibrium values.

Now, we turn to solving the Nash bargaining problem for the allied firms. We assume a one-time-only alliance negotiation, which means that each firm has only one chance to negotiate with other firms (i.e., one firm). This could be justified by considering a situation where each firm does not have enough opportunity for bargaining because, for example, the deadline for reaching an agreement is approaching or there are some business practices which restrict free negotiation with different firms.\(^9\)

Therefore, the disagreement (threat) point of the bargaining problem is defined as

\(^9\)The assumption of one-time-only negotiation excludes the possibility of the following recurrence problem: when bargaining an alliance with firm 3, the threat point of firm 2 may be its profit from an alliance with firm 1, rather than its profit from staying independent. However, the profit from the parallel alliance with firm 1, as derived later, also depends upon firm 2’s bargaining power, which in turn, depends upon its bargaining position against firm 3. In other words, the threat point in forming a complementary alliance depends on the threat point when bargaining the parallel alliance, which in turn, depends on the threat point when bargaining the complementary alliance, and so forth. Our assumption enables us to avoid the recurrence problem concerning a threat point of bargaining.
the profits of firms 2 and 3 obtained in Case I, that is \((\pi_2^I, \pi_3^I)\). In Case-C, firm 2’s share \(\pi_C^2\) and firm 3’s share \(\pi_C^3\) of the joint profits (the Nash bargaining solution) can be derived and shown as follows:\(^{10}\)

\[
\pi_C^j = \pi_I^j + \frac{1}{2} \left[ \pi_{23}^C - \left( \pi_I^2 + \pi_I^3 \right) \right] = \frac{a^2(1-b)}{2(1+b)(2-b)^2}, \quad \text{for } j = 2, 3. \tag{5}
\]

Due to the symmetry between firms 2 and 3, each firm obtains an equal share of the joint profits (i.e., \(\pi_{23}^C/2\)) through the bargaining.\(^{11}\)

### 2.3 Parallel Alliance and Nash Bargaining over the Gains

We turn to the case of a parallel alliance (Case-P). In Case-P, firm 2 allies with its rival (firm 1), and a single decision-maker chooses the price of composite good 1 as well as the price of component A, to maximize their joint profits denoted by \(\pi_{12}\).\(^{12}\) In this case, the demand functions remain the same as in (1). The profit function for the unallied firm (firm 3) does not change, and its first-order condition for profit-maximization is still represented by (3). The joint profits of the allied firms can be written as \(\pi_{12} = p_1 \cdot q_1(p_1, p_2, p_3; b) + p_2 \cdot q_{23}(p_1, p_2, p_3; b)\), and the corresponding first-order conditions are:

\[
\frac{\partial \pi_{12}}{\partial p_1} = \frac{a(1-b) - 2p_1 + 2bp_2 + bp_3}{1-b^2} = 0, \quad \frac{\partial \pi_{12}}{\partial p_2} = \frac{a(1-b) + 2bp_1 - 2p_2 - p_3}{1-b^2} = 0. \tag{6}
\]

The aforementioned strategic complementarity and substitutability relationships between the allied firms and the unallied firm in Case-P are also reflected in (6). Solving these three equations, we have the Bertrand-Nash outcome of the parallel alliance case shown in Table 1 (Case-P). The superscript “P” stands for the corresponding equilibrium values.

It should be noted that in our model, the two allied firms continue to offer component A at a reasonable price \(p_2\), so that consumers can still choose the composite good 23. It may be considered that once firm 2 allies with firm 1, it will refuse to offer component A (i.e., to force composite good 23 out of the market), so as to ensure the monopoly position for good 1. However, this is true only in a situation where the market demand for the composite goods is fixed. In our model where consumers love variety (one major feature of the utility function), it is always relatively more profitable to continue offering component A, to obtain the profit from composite good 23.

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\(^{10}\)Deriving the Nash bargaining solution follows the Split-The-Difference Rule of a Nash bargaining problem (See Nash (1950) and Muthoo (1999)). It can be derived by solving the following maximization problem: \(\max_{(\pi_2, \pi_3) \in \Theta} (\pi_2 - \pi_I^2)(\pi_3 - \pi_I^3)\), where \(\Theta \equiv \{ (\pi_2, \pi_3) : \pi_2 + \pi_3 \leq \pi_{23}^C, \pi_2 \geq \pi_I^2, \pi_3 \geq \pi_I^3 \}\) is the set of possible profit pairs obtainable through agreement.

\(^{11}\)It is assumed that the relative degree of impatience and risk aversion of each firm is identical, implying that the allied firms equally split the gains from alliance. Thus, the difference in the bargaining position between the allied firms comes only from the location of the disagreement point. This assumption applies to the parallel alliance case. Section 3.3 considers the asymmetric bargaining power.

\(^{12}\)Given the symmetry between firms 2 and 3, the other parallel alliance case (i.e., an alliance between firms 1 and 3) need not be considered explicitly.
Now, we solve the Nash bargaining problem for Case-P. The disagreement point of the bargaining problem in this case is \((\pi_I^1, \pi_I^2)\). Firm 1’s share \(\pi_P^1\) and firm 2’s share \(\pi_P^2\) of the joint profits (Nash bargaining solution) can be derived and shown as follows:\(^{13}\)

\[
\pi_P^1 = \pi_C^1 + \frac{1}{2} \left[ \pi_{12}^P - (\pi_I^1 + \pi_I^2) \right] = \frac{a^2(3 - b)(54 + 42b - 27b^2 - 28b^3 - 5b^4)}{72(1 + b)(3 - b^2)^2},
\]

\[
\pi_P^2 = \pi_C^2 + \frac{1}{2} \left[ \pi_{12}^P - (\pi_I^1 + \pi_I^2) \right] = \frac{a^2(72 + 18b - 33b^2 - 3b^3 + 13b^4 + 5b^5)}{72(1 + b)(3 - b^2)^2}.
\]

Note that firms 1 and 2 obtain an unequal share of the joint profits through Nash bargaining. This is due to the asymmetry of the profits at the disagreement point between firms 1 and 2 (i.e., \(\pi_I^1 \neq \pi_I^2\)), and it plays a critical role in leading to comparison results in the next subsection.\(^{14}\)

### 2.4 Effects of the Two Types of Alliances

This section examines the effects of the two different types of alliances. These analyses are useful for an understanding of our main argument of the firms’ preferences for an alliance partner in Section 3.

The effects of the complementary alliance on market performance can be shown by comparing the outcomes of Case-C with Case-I in Table 1. In the table, we have the following comparison result:

\[
\pi_{23}^C - (\pi_I^1 + \pi_I^2) = \frac{a^2(1 - b)(2 - 4b^2 + b^4)}{2(1 + b)(2 - b)^2(3 - b^2)^2} \geq 0.
\]

(9)

It can be shown that (9) is positive (negative) when \(b < \bar{b}\) (\(b > \bar{b}\)), where \(\bar{b} \equiv \sqrt{2 - \sqrt{2}} \approx 0.77\).

All the other comparisons have also been done by the authors, and only the comparison results are shown in Table 2 for brevity. According to Table 2, we have the following lemma.

**Lemma 1** The complementary alliance between firms 2 and 3

(a) reduces the price of composite good 1. It also reduces the total price of composite good 23. Corresponding to the relatively greater decrease in the price of composite good 23, the equilibrium output of composite good 1 decreases, while the equilibrium output of composite good 23 increases.

(b) reduces the profits of the unallied firm. It increases (reduces) the joint profits of the two allied firms when \(b < \bar{b} \approx 0.77\) (\(b > \bar{b}\)).

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\(^{13}\) Similar to Case-C, the Nash bargaining solution can be derived by solving the following maximization problem:

\[
\max_{(\pi_1, \pi_2) \in \Gamma} (\pi_1 - \pi_I^1)(\pi_2 - \pi_I^2),
\]

where \(\Gamma \equiv \{(\pi_1, \pi_2) : \pi_1 + \pi_2 \leq \pi_{12}^P, \pi_1 \geq \pi_I^1, \pi_2 \geq \pi_I^2\}\).

\(^{14}\) It is possible to expect that the outside firm blocks an alliance, for example, by promising to give an allied (inside) firm a larger share of joint profits if the allied firm dissolves the partnership and allies with the outside firm. This possibility is excluded in our study by the assumption of one-time-only negotiation.
Table 2: Effects of Complementary/Parallel Alliances

<table>
<thead>
<tr>
<th>Prices</th>
<th>Demands</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comparison between equilibria of Case-C and Case-I</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_C^1 &lt; p_I^1$</td>
<td>$q_C^1 &lt; q_I^1$</td>
<td>$\pi_C^1 &lt; \pi_I^1$</td>
</tr>
<tr>
<td>$p_{23}^C &lt; (p_2^I + p_3^I)$</td>
<td>$q_{23}^C &gt; q_{23}^I$</td>
<td>$\pi_{23}^C &gt; (\pi_2^I + \pi_3^I)$</td>
</tr>
<tr>
<td>Comparison between equilibria of Case-P and Case-I</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_P^1 &gt; p_I^1$, $p_P^2 &gt; p_2^I$, $p_P^3 &lt; p_3^I$</td>
<td>$q_P^1 &lt; q_I^1$</td>
<td>$\pi_{12}^P &gt; (\pi_1^I + \pi_2^I)$</td>
</tr>
<tr>
<td>$(p_2^P + p_3^P) &gt; (p_2^I + p_3^I)$</td>
<td>$q_{23}^P &lt; q_{23}^I$</td>
<td>$\pi_3^P &lt; \pi_3^I$</td>
</tr>
</tbody>
</table>

The intuition behind the lemma is as follows: Due to the internalization of the vertical externality, the complementary alliance reduces the total price of good 23, and thus forces their rival (firm 1) to lower its composite good’s price. In other words, the complementary alliance enhances the price competition between two composite goods, and this causes a decrease in the allied firms’ joint profits. When the price competition is not substantial (i.e., parameter $b$ is small), the gains accrued by the internalization outweigh the losses due to the price competition, and hence, the allied firms’ joint profits increase. However, when the price competition is substantial (i.e., parameter $b$ is sufficiently large), the losses outweigh the gains, and hence, the joint profits of the allied firms decrease. This is in contrast to the usual result that complementary alliance generally increases the allied firms’ joint profits (Spengler (1950), Tirole (1988, Ch.4)).

Similarly, we compare the outcomes of Case-P with Case-I in Table 1, to investigate the effects of the parallel alliance. In Table 1, we have the following comparison result:

$$\pi_{12}^P - (\pi_1^I + \pi_2^I) = \frac{a^2 b (18 + 21 b + 15 b^2 + 13 b^3 + 5 b^4)}{36 (1 + b) (3 - b^2)^2} > 0, \quad \forall b \in (0, 1).$$

Note that the other comparison results are listed in Table 2. Here, we have the following lemma.

**Lemma 2** The parallel alliance between firms 1 and 2

(a) increases the price of the composite good 1. It also increases the price of firm 2’s component and reduces the price of the unallied firm 3’s component. Corresponding to the price changes, both the equilibrium outputs of the two composite goods decrease.

(b) increases the joint profits of the two allied firms, while it reduces the profits of the unallied firm.

In contrast to the result that the complementary alliance may reduce the allied firms’ joint profits (Lemma 1-(b)), the parallel alliance always increases the allied firms’ joint profits. This is because
the horizontal externality, which exists between firms 1 and 2, is internalized by cooperative pricing under the parallel alliance, and the allied firms now have room to raise their prices \( p_1 \) and \( p_2 \). On the other hand, this strategic pricing forces the unallied firm (firm 3) to lower its price. Thus price changes like those in Lemma 2 emerge, and the corresponding changes in outputs and profits can be obtained.

Finally, we further examine the relationship between the allied firms’ joint profits in the two different alliances. From Table 1, we have

\[
\pi_{12}^P - \pi_{23}^C = \frac{a^2(16 - 12b^2 + 5b^3)}{36(2-b)^2} > 0.
\]

**Lemma 3** The allied firms’ joint profits from the parallel alliance are always larger than those from the complementary alliance.

Lemma 3 simply indicates that the joint profits which can be shared between the parallel alliance partners (i.e., firms 1 and 2) are always larger than the joint profits which can be shared between the complementary alliance partners (i.e., firms 2 and 3).

### 3 Partner Choice and Welfare

#### 3.1 Preference for Alliance Partners

We discuss each firm’s preference for an alliance partner. According to (7) and Table 1, we have the relationship for firm 1’s profits as \( \pi_1^P > \pi_1^I \), which implies that firm 1 certainly prefers allying with firm 2 (or equivalently with firm 3) to acting independently.

Now, we discuss firm 2’s preference for an alliance partner. From (5), (8) and Table 1, we have the following relationships of firm 2’s profits:

\[
\pi_2^C - \pi_2^I = \frac{a^2(1-b)(2-4b^2+b^4)}{4(1+b)(2-b)^2(3-b^2)^2} \leq 0, \tag{11}
\]

\[
\pi_2^P - \pi_2^I = \frac{a^2b(18 + 21b + 15b^2 + 13b^3 + 5b^4)}{72(1+b)(3-b^2)^2} > 0, \tag{12}
\]

\[
\pi_2^C - \pi_2^P = \frac{a^2(36 - 144b + 60b^2 + 18b^3 - 13b^4 + 12b^5 - 5b^6)}{72(2-b)^2(3-b^2)^2} \geq 0. \tag{13}
\]

Eq. (11) has the same sign condition as (9), which is determined by the defined value \( \hat{b} \). While (13) cannot be unambiguously signed, numerical calculations show that it is positive (negative) when \( b < \hat{b} \) (\( b > \hat{b} \)), where \( \hat{b} \approx 0.29 \). Here, we can present a figure to demonstrate the preference for an alliance partner for firm 2.

In Figure 1, the value of parameter \( b \) is shown on the horizontal axis. The values of (5) and (8) are shown by the downward line noted by \( \pi_2^C \) and the U-shaped line noted by \( \pi_2^P \), respectively. According to Figure 1, we have the following lemma.
Lemma 4  (a) $\pi_C^2$ is decreasing in parameter $b$, (b) $\pi_P^2$ is decreasing (increasing) in parameter $b$ when $b$ is small (large).

Proof: From (5) and (8), we have

\[
\frac{\partial \pi_C^2}{\partial b} = -\frac{a^2[1+(1-b)(1-b^2)]}{(1+b^2)(2-b^2)^2} < 0, \quad (14)
\]

\[
\frac{\partial \pi_P^2}{\partial b} = \frac{a^2(-81+45b+144b^2+72b^3+45b^4+27b^5+4b^6)}{36(1+b)^2(3-b^2)^2}. \quad (15)
\]

It is obvious that (15) is negative (positive) when $b$ is small (large). Q.E.D.

Lemma 4-(a) simply holds because a larger $b$ implies that price competition is substantial and this leads to a decrease in the joint profits which can be shared between firms 2 and 3. To understand the intuition of Lemma 4-(b), we derive

\[
\frac{\partial (\pi_I^1 - \pi_I^2)}{\partial b} = -\frac{a^2[3-b+3b^2(1+b)]}{2(3-b^2)^2} < 0, \quad (16)
\]

\[
\frac{\partial \pi_{I2}^P}{\partial b} = -\frac{2a^2}{9(1+b)^2} < 0. \quad (17)
\]

Eq. (16) describes that the difference in bargaining position (threat point) between firms 1 and 2, which coincides with the difference between $\pi_I^1$ and $\pi_I^2$, becomes smaller, if $b$ becomes larger. Thus for firm 2, the disadvantage of its bargaining position can be weakened, as $b$ becomes larger (i.e., the degree of product differentiation becomes smaller). This turns into a positive effect on firm 2’s profits. On the other hand, (17) indicates that the joint profits, which should be shared between firms 1 and 2, become smaller, if price competition is substantial, and this causes a negative effect on firm 2’s profits. Then, Lemma 4-(b) holds due to the relationship between the positive and negative effects.

From the above lemmas, we have the following proposition.
Proposition 1

In a market where one single composite goods firm competes with two components producers (each provides the component that is necessary for forming a substitutive composite good),

(a) when $0 < b < \hat{b} \approx 0.29$, a components producer prefers the alliance with its complementary partner, to the alliance with its rival (the composite-goods firm).

(b) when $\hat{b} < b < 1$, a components producer prefers the alliance with its rival, to the alliance with its complementary partner.

Although the parallel alliance yields larger joint profits (Lemma 3), the components producers might still prefer a complementary alliance. For firm 2, larger (smaller) joint profits in the parallel (complementary) alliance do not necessarily guarantee that it will obtain comparatively larger (smaller) gains through bargaining with its alliance partner. This is because the bargaining position of its partner is varies, depending on which firm it allies with and on the market conditions. In particular, although the joint profits in the complementary alliance are comparatively small, firm 2 can equally share it with its partner (firm 3). In contrast to the complementary alliance, although the joint profits of the parallel alliance are comparatively large, firm 2 cannot equally share it with its partner (firm 1), which has a superior bargaining position.

Figure 2 may help us understand Propositions 1-(a) and -(b). In the figure, the disagreement point for the complementary (parallel) alliance is noted by $d_C$ ($d_P$). Corresponding to $\pi_{12}^P > \pi_{23}^C$, the frontier of the joint profits of the parallel alliance (noted by the line $\pi_{12}^P$) is located farther from the original point than that of the complementary alliance (noted by the line $\pi_{23}^C$). In addition, as parameter $b$ becomes larger, the disagreement point is located closer to the original point, and the
difference between $\pi_{I2}^I$ and $\pi_{I3}^C$ becomes larger.\textsuperscript{15} Thus, Figure 2-a (Figure 2-b) corresponds to the case where parameter $b$ is sufficiently small (large). Now, by comparing the Nash bargaining solution for the complementary alliance case (noted by point $N_C$) with that for the parallel alliance (noted by $N_P$) in Figure 2-a, the result of Proposition 1-(a) holds. Similar statements apply to the other case in Figure 2-b, and the result of Proposition 1-(b) holds as well.

### 3.2 Welfare and Policies for Alliance Formations

Finally, we compare the social welfare of the three cases, and then discuss the policy implications for alliance formation. Social welfare is defined as the sum of consumers’ and producers’ surplus. According to Table 1, social welfare in each case can be derived and written as

\[
W^I = \frac{a^2(47 + 21b - 24b^2 - 12b^3)}{8(1 + b)(3 - b^2)^2}, \quad W^C = \frac{a^2(3 - 2b)}{(1 + b)(2 - b)^2}, \quad W^P = \frac{a^2(47 + b)}{72(1 + b)}.
\]

Here, we have $W^C > W^I > W^P$ for all $b \in (0, 1)$, which can be shown in Figure 3-a. The welfare ranking is intuitive because the prices of both composite goods in Case-C are lower, while the prices in Case-P are higher than those in Case-I. In other words, the complementary alliance is welfare-enhancing, while the parallel alliance is welfare-reducing, compared to the welfare under no-alliance. However, as shown in Figure 3-b, the firms do not always prefer to form the welfare-enhancing complementary alliance.

We consider two types of policy intervention: prohibiting and subsidizing some types of alliance formation. We assume that the subsidy payments are financed by non-distortionary consumer taxation (such as lump-sum taxes on consumers) and the enforcement costs of both policies can be neglected. Combining the welfare rankings and the firms’ preference for alliance formation yields the following proposition.

**Proposition 2**

In the market,

(a) when $0 < b < \hat{b} \approx 0.29$, $W^C$ can be achieved under Laissez-Faire.

(b) when $\hat{b} < b < \bar{b} \approx 0.77$, $W^C$ can be achieved by prohibiting the parallel alliance.

(c) when $\bar{b} < b < 1$, $W^C$ can be achieved by prohibiting the parallel alliance and subsidizing the complementary alliance.

Proposition 2 has interesting policy implications for alliance formation in a composite goods market. When the degree of product differentiation is sufficiently large (region I in Figure 3), government interventions are not necessary because a welfare-enhancing complementary alliance will be spontaneously formed. When the degree of product differentiation is intermediate (region II), it

\textsuperscript{15}It can be confirmed that $(\pi_{I1}^I, \pi_{I2}^I = \pi_{I3}^I)$ is decreasing in parameter $b$, and $(\pi_{I2}^P - \pi_{I3}^C)$ is increasing in parameter $b$. 

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is socially desirable to prohibit a welfare-reducing parallel alliance. More significantly, this regulation of the parallel alliance can encourage the two component firms to form a welfare-enhancing complementary alliance. When the degree of product differentiation is sufficiently small (region I II), prohibiting a parallel alliance can prevent welfare reduction, but that alone cannot effectively encourage the two component firms to form a complementary alliance because it is not profitable for them (see Lemma 1-(b)). Thus, in this case, an additional policy (e.g., a subsidy policy financed by consumer taxation) for implementing the complementary alliance can also be socially desirable.

3.3 Asymmetric Bargaining Power

Propositions 1 and 2 are critically dependent on the process of Nash bargaining. In the case of the complementary alliance, a symmetry argument could justify the equal sharing. However, in the case of the parallel alliance, there may be asymmetric bargaining power (as well as asymmetric bargaining position) between firms 1 and 2.

In the case of asymmetric bargaining power, the Nash bargaining process of forming a parallel alliance can be described by

\[
\max_{(\pi_1, \pi_2) \in \Gamma} (\pi_1 - \pi'_1)^{\sigma} (\pi_2 - \pi'_2)^{1-\sigma},
\]

where \(\Gamma \equiv \{(\pi_1, \pi_2) : \pi_1 + \pi_2 \leq \pi'_{12}, \pi_1 \geq \pi'_1, \pi_2 \geq \pi'_2\}\). In this bargaining problem, \(\sigma \in [0, 1]\) rep-
represents the bargaining power of firm 1. This modification affects the value of \( \hat{b} \) and thus Proposition 1. The trivial calculation shows that when \( \sigma \) approaches unity, firm 2’s share of joint profit in the parallel alliance approaches its profit in the independent case (\( \pi_2^I \)), and thus \( \hat{b} \) also approaches to \( \bar{b} \). Conversely, the smaller the value of \( \sigma \), the smaller \( \hat{b} \) and firm 2 is more likely to prefer the parallel alliance.

The difference of the bargaining power in the parallel alliance also affects the welfare implication of policy interventions (Proposition 2). The derived value of social welfare is independent of the value of \( \sigma \) because \( \sigma \) does not affect the price of goods or components, but only affects the profit sharing between firms 1 and 2. Therefore, we can state that as the composite goods firm (firm 1) has greater bargaining power against its rival components producer, the welfare-enhancing complementary alliance is more likely to emerge under Laissez-Faire.

### 4 Concluding Remarks

This study investigated the issue of alliance partner choice in a market with vertical and horizontal externalities. In the market, a particular firm has two partners to choose for forming an alliance: one is its complementary partner who produces a complementary component; the other is its rival who produces substitutive (composite) goods. We first found that the corresponding joint profits obtained by the latter alliance are always larger than those obtained by the former alliance. Through the Nash bargaining approach, we further found that it is always profitable for the composite goods firm to form an alliance with a component producer. On the other hand, when the degree of product differentiation for the two composite goods is sufficiently large, although the parallel alliance yields larger joint profits, the components producers might still prefer a complementary alliance. These findings provide clear and useful managerial implications for those firms that consider allying with other firms in a composite goods market.

We showed that the complementary alliance is welfare-enhancing, whereas the parallel alliance is welfare-reducing. To encourage a welfare-enhancing complementary alliance, a government can implement the following policies: If the two composite goods are (i) highly differentiated, there is no need for government intervention, (ii) intermediately differentiated, prohibiting a parallel alliance can encourage firms to form a welfare-enhancing complementary alliance, and (iii) slightly differentiated, the prohibition of a parallel alliance may not be sufficient and a subsidy policy for implementing a complementary alliance can be socially desirable.

Finally, we discuss the limitations of this study and suggest some directions for future research. First, cost advantages which may be generated by alliance formations are not considered in our study. We recognize that there are arguments that merged/allied firms may enjoy a cost advantage.\(^{16}\) Considering the effect of cost-saving through alliance formation remains a subject for future research.

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\(^{16}\)See, for example, Perry and Porter (1985), Heywood and McGinty (2008) for merger studies.
research. Second, full compatibility between the two components has been assumed in this study. On the other hand, there is discussion on the issue of compatibility (for example, Economides (1989), Matutes and Regibeau (1988)). An extension for taking the compatibility issue into account remains for future work. Third, the present network structure is exogenously given and the number of composite goods producers (components producers) is restricted to one (two). It is meaningful to consider various types of market structure with a larger number of firms.

References


