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Abstract This article analyses how long-run pay-as-you-go public pensions react to a change in fertility in the basic overlapping generations model of neoclassical growth. While it would seem well established both in the academic and political debates that the decline in fertility represents a "demographic time bomb" for the sustainability of public pensions, it is shown that a falling birth rate need not necessarily cause long-run pension benefit to fall.

Keywords Fertility • PAYG pensions • OLG model

JEL Classification J26 • O41

1 Introduction

In recent decades population ageing and the decline in fertility experienced especially in Europe raised several serious concerns about the sustainability of existing social security programmes, notably the pay-as-you-go (PAYG) pension systems. The current debate between economists and politicians starts out with the common belief that the fertility drop represents a "demographic time bomb" and tries to the appropriate way of reforming public pensions for disarming it. Conclusions are therefore essentially for a reduction of the welfare state (see, e.g., Boeri et al., 2001).

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The current debate may be resumed in the words of some authoritative scholars. For instance, Bovenberg (2007, p. 23) believes that “Whereas both funded and PAYG pension systems are vulnerable to increased longevity, PAYG pension schemes are especially vulnerable to lower fertility because they rely on the human capital of the young to finance the pensions of older generations.” Cigno, (2007, p. 37), instead, observes that “The combined effect of fewer births, longer lives and sluggish retirement age is putting public pension systems, all essentially pay-as-you-go, under increasing strain. Several governments are responding to this by either raising contributions or cutting benefits.” Finally, Sinn (2007, p. 9), with reference to several western countries, claims that “Roughly speaking, these countries will experience a doubling in the number of elderly relative to the young within the next thirty years. Consequently, the implication for the pay-as-you-go system is straightforward. Either we double the contribution rate if we want to keep the pensions in line with wages, or we halve the pensions relative to wages.”

Though it is undoubted that the rapid change in economic and social environments may naturally give rise to the need of changing pension rules in developed countries, it is striking that the theoretical effects of both a low and falling fertility on PAYG pensions have not yet received a deep attention in the economic theoretical literature. The question we address, therefore, in this paper is simple: is the public PAYG system actually vulnerable to the falling birth rates? To answer this question we rely on the textbook overlapping generations model by Diamond (1965), which has been widely adopted to study several aspects of social security (see, e.g., Burbidge, 1983; van Groezen et al., 2003; Fanti and Gori, 2010; Fenge and von Weizsäcker, 2010). In particular, we use the double Cobb-Douglas economy model as we think the simplest case could aid researchers to better understand the key economic forces at work, while also representing a useful abstraction and a good starting point for future theoretical analyses. In line with the neoclassical growth model, we assume the rate of fertility is exogenously given (for instance, due to biological reasons, religious beliefs, unchecked sexuality and so on). However, whatever the reasons why a positive fertility rate exists, raising children is costly.[1] In such a context it is shown that the fertility drop may cause public pensions to raise in the long run. The policy implications of our findings are straightforward: (*i*) policies aim at recovering fertility could unexpectedly reduce to size of public pensions, and then (*ii*) pension reforms aim at tackling the fertility-drop-effect might not be necessary.

The remainder of the paper is organised as follows. Section 2 describes the model. In Section 3 analyses how a change in fertility affects long-run PAYG pensions. Section 4 concludes.

2 The model

Consider a general equilibrium overlapping generations closed economy with identical two-period lived individuals and an exogenous number of children, n. Life is divided into youth and old age. Individuals belonging to generation t draw utility (U_t) from young-aged ($c_{1,t}$) and old-aged ($c_{2,t+1}$) consumptions. Only young individuals (N_t) join the workforce. They are endowed with one unit of time inelastically supplied on the labour market, while receiving a unitary wage income at the rate w_t. Although the number of children is exogenously given, we assume that raising children is costly, and the amount of resources needed for parents to care about each child is given by a monetary cost $q\,w_t$, with $0<q<1$ being the percentage of child-rearing cost on working income (see, e.g., Wigger, 1999; Boldrin and Jones, 2002). Therefore, the budget constraint of the young at t is:

$$c_{1,t} + s_t + q\,w_t n = w_t(1-\theta), \tag{1}$$

that is, wage income – net of contributions paid to transfer resources from work time to retirement time, where $0<\theta<1$ – is used to consume, save (s_t) and take care of n descendants. When old, individuals are retired and live with the amount of resources saved when young plus the expected

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[1] "That children impose economic costs on their parents seems to be widely accepted." (Deaton and Muellbauer, 1986, pp. 720–721).

interest accrued at the rate r^e_{t+1}, and the expected pension benefit, p^e_{t+1}. Hence, the budget constraint of the old born at t reads as:

$$c_{2,t+1} = \left(1 + r^e_{t+1}\right)s_t + p^e_{t+1}. \tag{2}$$

The representative individual of generation t chooses how much to save out of her disposable income to maximise the following lifetime utility function:

$$U_t = \ln\left(c_{1,t}\right) + \beta \ln\left(c_{2,t+1}\right), \tag{3}$$

subject to Eqs. (1) and (2), where $0 < \beta < 1$. Maximisation of Eq. (3) gives:

$$s_t = \frac{\beta w_t (1 - \theta - qn)}{1 + \beta} - \frac{p^e_{t+1}}{(1 + \beta)\left(1 + r^e_{t+1}\right)}. \tag{4}$$

Firms are identical and act competitively on the market. The (aggregate) constant returns to scale Cobb-Douglas technology is $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where Y_t, K_t and $L_t = N_t$ are output, capital and the time-t labour input respectively, while $A > 0$ and $0 < \alpha < 1$ are a scale parameter and the distributive capital share. Profit maximisation therefore implies:[2]

$$r_t = \alpha A k_t^{\alpha - 1} - 1, \tag{5}$$

$$w_t = (1 - \alpha) A k_t^{\alpha}, \tag{6}$$

where $k_t := K_t / N_t$.

The government redistributes between generations through unfunded pensions financed with labour income taxes at the (constant) rate $0 < \theta < 1$. Therefore, the per capita pension expenditure at t (p_t) is constrained by:

$$p_t = \theta w_t n. \tag{7}$$

Inserting the one-period-forward pension accounting rule Eq. (7) into Eq. (4), the saving rate is:

$$s_t = \frac{\beta w_t (1 - \theta - qn)}{1 + \beta} - \frac{\theta w^e_{t+1} n}{(1 + \beta)\left(1 + r^e_{t+1}\right)}. \tag{8}$$

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[2] We assume that capital fully depreciates at the end of each period and output is sold at unit price.

Given Eq. (7) and knowing that $N_{t+1} = n N_t$, market-clearing in goods and capital markets is:

$$n k_{t+1} = s_t , \tag{9}$$

Combination of Eqs. (8) and (9) yields:

$$k_{t+1} = \frac{\beta w_t (1-\theta-qn)}{n(1+\beta)} - \frac{\theta}{1+\beta} \cdot \frac{w^e{}_{t+1}}{1+r^e{}_{t+1}} . \tag{10}$$

Exploiting Eqs. (5), (6) and (10) and assuming individuals are perfect foresighted the dynamics of capital is:

$$k_{t+1} = \frac{\beta\alpha(1-\alpha)A(1-\theta-qn)}{n[\alpha(1+\beta)+\theta(1-\alpha)]} k_t^{\alpha} . \tag{11}$$

Steady-state implies $k_{t+1} = k_t = k^*$. Therefore,

$$k^*(n) = \left\{ \frac{\beta\alpha(1-\alpha)A(1-\theta-qn)}{n[\alpha(1+\beta)+\theta(1-\alpha)]} \right\}^{\frac{1}{1-\alpha}} . \tag{12}$$

From Eq. (12) it can easily be seen that a rise in n negatively affects the per capita stock of capital. Of course, $n < (1-\theta)/q := \overline{n}$ must hold to ensure $k^*(n) > 0$.

3 Fertility and PAYG pensions in the long run

We now study how long-run public pensions react to a change in fertility. In particular, Proposition 1 below shows, contrary to the commonplace, that when the cost of children is realistically taken into account into the OLG model, a negative relationship between PAYG pensions and fertility may emerge in the long run.

From Eq. (7), the long-run pension benefit as a generic function of the number of children is

$$p^* = p^*\{n, w^*[k^*(n)]\}, \tag{13}$$

and the total derivative with respect to n gives:

$$\frac{dp^*}{dn} = \overbrace{\frac{\partial p^*}{\partial n}}^{+} + \underbrace{\overbrace{\frac{\partial p^*}{\partial w^*}}^{+} \cdot \overbrace{\frac{\partial w^*}{\partial k^*}}^{+} \cdot \overbrace{\frac{\partial k^*}{\partial n}}^{-}}_{-} . \tag{14}$$

Eq. (14) reveals that the final effect of a reduction in fertility on long-run public pensions is ambiguous and depends on two opposite forces: a positive direct effect, and a negative indirect general equilibrium feedback effect. The former effect causes PAYG pensions to shrinks simply because the number of young contributors is now lower. The latter instead implies a rise in the capital stock and, hence, in both the wage (tax base) and pension benefit. If the latter effect dominates, a fall in the birth rate increases pensions. We now combine Eqs. (6), (7) and (12) to obtain the following steady-state pension formula:

$$p^*(n) = \theta(1-\alpha)A\left[\frac{\beta\alpha(1-\alpha)A(1-\theta-qn)}{\alpha(1+\beta)+\theta(1-\alpha)}\right]^{\frac{\alpha}{1-\alpha}} \cdot n^{\frac{1-2\alpha}{1-\alpha}} . \tag{15}$$

From Eq. (15), the following proposition summarises the main result of the paper.

Proposition 1 *(1) Let* $0 < \alpha < 1/2$ *hold. Then* $p^*(n)$ *is inverted U-shaped with* $n = n_p$ *being the pension-maximising number of children. (2) Let* $1/2 < \alpha < 1$ *hold. Then a reduction in the number of children always cause PAYG pensions to raise in the long run.*

Proof The proof uses the following derivative:

$$\frac{\partial p^*(n)}{\partial n} = \frac{\theta A\left\{\frac{\beta\alpha(1-\alpha)A(1-\theta-qn)}{n[\alpha(1+\beta)+\theta(1-\alpha)]}\right\}^{\frac{\alpha}{1-\alpha}}[(1-\theta)(1-2\alpha)-q(1-\alpha)n]}{1-\theta-qn} . \tag{16}$$

If $0 < \alpha < 1/2$, $\frac{\partial p^*(n)}{\partial n} \underset{<}{\overset{>}{=}} 0$ if and only if $n \underset{>}{\overset{<}{=}} n_p$ where

$$n_p := \frac{(1-\theta)(1-2\alpha)}{q(1-\alpha)} , \tag{17}$$

being an interior global maximum and $n_p < \bar{n}$. If $1/2 < \alpha < 1$, $\frac{\partial p^*(n)}{\partial n} < 0$ for any $n < \bar{n}$. *Q.E.D.*

Eq. (17) shows that the higher the contribution rate (θ), the percentage of child rearing cost on working income (q) and the output elasticity of capital (α), the more likely a decline in fertility raises pensions. Moreover, simple numerical examples using the observed values of capital shares, children costs and contribution rates for several real economies also reveal that the current fertility drop may be beneficial to the size of public pensions: for instance, assuming $\theta = 0.15$[3] and $q \cong 0.3$,[4] when α is close to 0.33, a rise in n beyond 1.4 (the replacement fertility rate is almost 1.04) is harmful to the PAYG system. However, when α is about 0.4 a fall in the birth rate below the replacement fertility (i.e., until 0.96) increases the size of public pensions. The intuition behind our result is simple (see Eq. 17): (*i*) the higher the output elasticity of capital, the larger the rise in wages due to the increased capital stock caused by the lower number of children; (*ii*) the higher the cost of children, the lower both the saving rate and stock of capital; (*iii*) the higher the contribution rate, the higher the percentage reduction in the agents' disposable income (i.e. the wage net of both the contribution rate and cost of children). Intuitively, when the birth rate falls, there are fewer young workers to support pensioners, but also they need less income to support their children, and this causes PAYG pensions to raise.

4 Conclusions

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[3] The current average contribution rate is estimated to be around 0.16 in Europe and 0.11 in U.S. (Liikanen, 2007, p. 4).

[4] For instance Deaton and Muellbauer (1986, p. 720) found that "Sri Lankan and Indonesian data suggest that children cost their parents about 30–40 percent of what they spend on themselves," while also arguing that such estimates "would not be appropriate for developed countries where children bring heavy non-food expenditures." (p. 741) Therefore, $q \cong 0.3$ is reasonable for developed economies.

This paper deals with an important policy debate: the best way to redesign government pension programmes when fertility declines. For doing this, we use the Diamond overlapping generations model augmented with costs of children, and analyse how a change in fertility affects long-run public pensions. Our findings are clear-cut: the possibility that the fertility recovery over the replacement rate would lead to higher pensions is found to be the exception rather than the rule. This suggests a rather paradoxical policy implication: the fertility drop may cause the size of public pensions to raise in the long run, or, alternatively, by keeping unaltered it, it may leave room for a reduction in the contribution rate as well as in the retirement age.

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