A Simple Model of Capital Imports

Arif, Shawky

University of East Anglia, School of Economics

13 October 2010

A Simple Model of Capital Imports

Shawky Arif*

October 13, 2010

Abstract

Following Ramsey (1928) theoretical framework, this paper develops a dynamic model where a community is assumed to be importing two forms of foreign capital: external debt and foreign direct investment (FDI). The community is assumed to derive utility from consumption of goods and positive externalities of FDI, while deriving disutility from negative externalities of external borrowing. Results suggest that: first, a higher disutility of debt implies a higher shadow interest rate.[1] The higher the utility derived from FDI, however, the lower the shadow interest rate. Second, external borrowing will be attractive as long as the relevant interest rate is less or equal to the net marginal product of capital. Third, the study of the social optimum shows that the externalities that arise from foreign capital do not affect the steady state which is always a saddle point.

Keywords: FDI, External borrowing, Dynamic optimization.

1 Introduction

In 2008, statisticians of the United Nations reported that over the past decades, the stock of foreign direct investment (FDI) world wide has grown roughly 10 times from about $1.2 trillion to $12 trillion in 2006 UNCTAD (2008). According to the same report, various parts of the world witnessed continued growth in their inflows of FDI. Inflows to developing countries, for instance, reached a record level of $379 billion. In addition to its role as a source of capital, FDI is regarded as an important channel for transmission of spillover benefits and technology. Empirical findings suggest that these externalities have a positive impact, not only on the performance of domestic firms, but also on the community as a whole. FDI is "considered responsible for welfare increase in the host country due to advantages related to the introduction of new technologies and innovation, new managerial techniques and development of additional skills."[2]

As far as capital imports are concerned, models constructed along the lines of Ramsey (1928) have only considered the external borrowing dimension. In this case, the problem is that of a community that wishes to maximize the present value of utility subject to the foreign borrowing constraint. In the present paper, I extend the concept of capital imports and include FDI. The problem of the community, thus, is to maximize the discounted stream of utility subject to an external borrowing and FDI constraints. In order to take account of the externalities generated by imported capital, I assume that external debt generates, in addition to the monetary cost, a non-monetary cost from which the community derives disutility. In fact the dependency of the borrower on the lender[3] and

*Email address: c.arif@uea.ac.uk.

[1] Blanchard (1983) used the concept of shadow interest rate to mean the interest rate that takes into account of negative/positive externalities of foreign capital. In the present paper, shadow interest rate and relevant interest rate are interchangeably used.

[2] Botrić and Škuflić (2006), p. 362.

[3] Pranab (1967) argues that"it is a fact of life that many borrowing countries do derive some kind of disutility from their dependence on foreign capital." Blanchard (1983) also considers the disutility of external borrowing in order to capture the non-monetary cost of the debt.

the conditionalities that the foreign lender might impose on the borrower can be thought of as a source of discomfort and therefore disutility. Regarding FDI, I suppose that the community derives some utility from technological transfers and spillover effects caused by FDI. In order to keep the analysis simple, it is assumed that institutions are the sole "device" by which the community attracts FDI. So the better the institutions, the larger the inflows of FDI. Indeed, recent developments in the literature on FDI support the above assumption. Transparent legal framework and sound economic policies are fundamental and are as important as economic factors in attracting FDI. Busse and Hefeker (2007) find that political risk and institutional indicators such as government stability, internal and external conflicts, law and order and bureaucratic quality are important determinants of foreign investment flows.

The remainder of the paper is organized as follows. Section 2 formulates the problem. Section 3 derives the dynamics of external borrowing. Section 4 studies the social optimum in the presence of the externalities generated by foreign capital. Section 5 concludes.

2 Formulation of the Problem

Consider a community that imports two forms of foreign capital, external debt (D) and FDI (F), in order to produce an output Y. The production function $Q(K_t)$ satisfies the following conditions

$$Q(0) = 0, \quad Q'(K_t) > 0, \quad Q''(K_t) < 0$$

Our community borrows from abroad whenever output is less than investment and consumption. A larger current account deficit implies a larger external debt. I assume that the interest rate on external debt is an increasing function of the stock of debt. Regarding FDI, the community attracts foreign investors by offering a transparent legal framework and maintaining sound economic policies. The better the quality of the economic and political institutions, the larger the flows of FDI. For simplicity, I assume that institutions are the sole "device" by which the community attracts FDI.

Imported capital is assumed to generate externalities that affect the social welfare. While the community derives utility from FDI, it derives disutility from external borrowing. For simplicity, I consider an additive utility function[4] in consumption, FDI and external debt $u(C_t) + v(F_t) - z(D_t)$ with the following properties

$$u'(C_t) > 0, \quad u''(C_t) < 0 \tag{2.1}$$

$$\lim_{C \to 0} u'(C_t) = +\infty, \quad \lim_{C \to +\infty} u'(C_t) = 0 \tag{2.2}$$

$$v'(F_t) > 0, \quad v''(F_t) < 0 \tag{2.3}$$

$$\lim_{F \to 0} v'(F_t) = +\infty, \quad \lim_{F \to +\infty} v'(F_t) = 0 \tag{2.4}$$

$$z'(D_t) > 0, \quad z''(D_t) > 0 \tag{2.5}$$

$$\lim_{D \to 0} z'(D_t) = 0, \quad \lim_{D \to +\infty} z'(D_t) = +\infty \tag{2.6}$$

[4] A separable utility function is used because in the non-separable case it is difficult to give an economic interpretation to all the signs of the cross partial derivatives.

For given initial stocks of capital, external debt and FDI K_0, D_0 and F_0, respectively, and a positive discount rate, ψ, the problem of the community can be formulated as follows

$$\begin{array}{ll} \max & \int_0^\infty [u(C_t) + v(F_t) - z(D_t)]e^{-\psi t}dt \\ \text{subject to} & \dot{K} = I_t - \delta K_t \\ & \dot{D} = C_t + I_t + r(D_t)D_t - Q(K_t) - \lambda_t F_t \\ & \dot{F} = -\lambda_t F_t \\ \text{with} & K(0) = K_0, \quad D(0) = D_0, \quad F(0) = F_0 \end{array}$$

where I and K are, respectively, capital formation and the stock of capital. $0 < \delta < 1$ is the depreciation rate of the stock of capital. D is the stock of external debt and $r(D)D$ is the interest payment. The interest rate is assumed to be an increasing function of the stock of debt, that is $r'(D_t) > 0$. F is the stock of FDI, λ_t is a control variable reflecting the quality of economic and political institutions. For convenience, from here on subscripts t will be dropped except where their inclusion is instructive.

The current-valued Hamiltonian for the optimization problem is

$$H = [u(C) + v(F) - z(D)] + \omega[I - \delta K] + \mu[C + I + r(D)D - Q(K) - \lambda F] - \rho\lambda F$$

The first order conditions characterizing the equilibrium are[5]

$$\frac{\partial H}{\partial C} = 0 \quad \Rightarrow u'(C) = -\mu \tag{2.7}$$

$$\frac{\partial H}{\partial \lambda} = 0 \quad \Rightarrow -\mu = \rho \tag{2.8}$$

$$\frac{\partial H}{\partial I} = 0 \quad \Rightarrow -\mu = \omega \tag{2.9}$$

$$\dot{\omega} = \psi\omega - \frac{\partial H}{\partial K} \quad \Rightarrow \dot{\omega} = \omega(\psi + \delta) + \mu Q'(K) \tag{2.10}$$

$$\dot{\mu} = \psi\mu - \frac{\partial H}{\partial D} \quad \Rightarrow \dot{\mu} = \mu(\psi - r'(D)D - r(D)) + z'(D) \tag{2.11}$$

$$\dot{\rho} = \psi\rho - \frac{\partial H}{\partial F} \quad \Rightarrow \dot{\rho} = \rho(\psi + \lambda) + \mu\lambda - v'(F) \tag{2.12}$$

The following transversality conditions are imposed

$$\lim_{t\to\infty} e^{-\psi t}\omega K = 0 \qquad \lim_{t\to\infty} e^{-\psi t}\mu D = 0 \qquad \lim_{t\to\infty} e^{-\psi t}\rho F = 0$$

3 Consumption and the Dynamics of External Borrowing

The first order conditions derived above allow us to determine the growth rate of consumption in three different ways. The first way involves the disutility of external borrowing and goes as follows. Differentiate (2.7) with respect to time gives

$$\dot{\mu} = -u''(C)\dot{C} \tag{3.1}$$

Substituting for $\dot{\mu}$ and μ in (2.11) yields

$$-u''(C)\dot{C} = -u'(C)(\psi - r'(D)D - r(D)) + z'(D)$$

[5]Condition (2.7) reads that the marginal utility of consumption is negative, which seems to contradict the standard assumption $u'(C) > 0$. The negative sign appears because of the way the borrowing constraint is defined. If the current account was defined as $Q(K) + \lambda F - I - C - r(D)D$ the negative sign would disappear and the results would remain the same. Another way of avoiding such a sign is to write the shadow price of the debt with a negative sign in the Hamiltonian.

Divide both sides by $u'(C)$ gives

$$-\frac{u''(C)\dot{C}}{u'(C)} = -(\psi - r'(D)D - r(D)) + \frac{z'(D)}{u'(C)}$$

Similarly

$$-\frac{u''(C)\dot{C}C}{u'(C)C} = -(\psi - r'(D)D - r(D)) + \frac{z'(D)}{u'(C)} \tag{3.2}$$

Let $\theta = -u''(C)C/u'(C)$ be the elasticity of the marginal utility of consumption, then (3.2) becomes

$$\frac{\dot{C}}{C} = (r'(D)D + r(D) + \eta - \psi)\frac{1}{\theta}, \tag{3.3}$$

where $\eta = z'(D)/u'(C)$.

Now, from (2.9), (2.10) and (3.1) we get

$$u''(C)\dot{C} = u'(C)(\psi - Q'(K)) \tag{3.4}$$

Divide both sides by $u'(C)$ and rearrange terms gives

$$\frac{\dot{C}}{C} = (Q'(K) - \delta - \psi)\frac{1}{\theta}, \tag{3.5}$$

The third way of determining the growth rate of consumption takes into account FDI. Making use of (2.7), (2.12) and (3.1) yields

$$\frac{\dot{C}}{C} = (\epsilon - \psi)\frac{1}{\theta} \tag{3.6}$$

where $\epsilon = v'(F)/u'(C)$.

To sum up, it has been shown that

$$\frac{\dot{C}}{C} = (r'(D)D + r(D) + \eta - \psi)\frac{1}{\theta}; \tag{3.7}$$

$$= (Q'(K) - \delta - \psi)\frac{1}{\theta}; \tag{3.8}$$

$$= (\epsilon - \psi)\frac{1}{\theta}. \tag{3.9}$$

From (3.8) and (3.9) we get

$$u'(C) = \frac{v'(F)}{Q'(K) - \delta}$$

Now equate (3.7) and (3.8) gives

$$r'(D)D + r(D) + \frac{z'(D)}{u'(C)} = Q'(K) - \delta \tag{3.10}$$

Replace $u'(C)$ by its expression in (3.10) and rearrange yields

$$Q'(K) - \delta - \frac{r(D)}{1-\alpha} = \frac{r'(D)D}{1-\alpha} > 0 \tag{3.11}$$

where $\alpha = z'(D)/v'(F)$.

Expression (3.11) suggests that external borrowing will be attractive as long as the relevant interest rate at which the community borrows, $r(D)/(1-\alpha)$, is less than the marginal product of capital adjusted for depreciation, $Q'(K) - \delta$. This, roughly speaking, is also the condition for the solvency of the economy. Since the marginal product of capital is decreasing over time ($Q''(K) < 0$), the community is left with one option to ensure its solvency: reduce the relevant interest rate at which it borrows. The reduction of this interest rate can be achieved by: (1) borrowing less or (2) attracting more FDI. This last option requires the improvement of the quality of institutions.

If FDI has to be excluded from the analysis, (3.11) reduces to

$$Q'(K) - \delta - r(D) = \frac{z'(D)}{u'(C)} + r'(D)D > 0 \tag{3.12}$$

which agrees well with the result of Pranab (1967)[6]

$$f'(k) - r(k_f) = \frac{D'(k_f)}{U'(c)} + r'(k_f)k_f > 0 \tag{3.13}$$

where $f'(k)$ is the marginal product of capital, $r(k_f)$ is the interest rate as a function of per capita external debt (with $r'(k_f) > 0$), $D'(k_f)/U'(c)$ is the marginal disutility of per capita external debt over the marginal utility of per capita consumption and $r'(k_f)k_f$ is the marginal cost of external borrowing.

Pranab interprets (3.13) as follows: atomistic borrowers, who are not directly affected by the cost of external debt, tend to borrow excessively. Moreover, they fail to take into account the cost of borrowing and ignore the increase in the disutility for the community. A few remarks, however, concerning this interpretation are in order. Pranab's argument fails to take account of the fact that the community is a sum of individuals. Some of these individuals are borrowers. Therefore, if the community, as a whole, derives disutility from external debt, it is because single individuals, including the private borrowers, derive disutility from external borrowing. Furthermore, one might expect borrowers to be more sensitive to the cost of external debt, and therefore derive a higher disutility than the other members of the community. In fact failure to honor financial engagements will affect the borrower's credibility. This implies that borrowers are also concerned with the level of external borrowed capital, and, therefore, they cannot borrow excessively.

4 The Social Optimum

In the economy there are positive externalities generated by FDI and negative externalities generated by external borrowing. In order to determine which of these effects dominates, this section study the social optimum.

For convenience, denote $r(D)D$ by $\phi(D)$ (and therefore $r'(D)D + r(D)$ by $\phi'(D)$). The

[6] By comparing (3.12) and (3.13), the reader can easily notice that Pranab conducts his analysis in per capita terms and ignored the depreciation of capital. In (3.13) I use the same notation as Pranab.

steady state is characterized by the following relations

$$\begin{aligned}
I &= \delta K \\
0 &= C + \delta K + \phi(D) - \lambda F - Q(K) \\
0 &= -\lambda F \\
0 &= \omega(\psi + \delta) + \mu Q'(K) \\
0 &= \mu(\psi - \phi'(D)) + z'(D) \\
0 &= \rho(\psi + \lambda) + \mu\lambda - v'(F)
\end{aligned}$$

Linearizing around the steady state gives

$$\begin{bmatrix}
-\delta & 0 & 0 & 0 & 0 & 0 \\
\delta - Q'(K) & \phi'(D) & -\lambda & 0 & 0 & 0 \\
0 & 0 & -\lambda & 0 & 0 & 0 \\
\mu Q''(K) & 0 & 0 & \psi + \delta & Q'(K) & 0 \\
0 & -\mu\phi''(D) + z''(D) & 0 & 0 & \psi - \phi'(D) & 0 \\
0 & 0 & -v''(F) & 0 & \lambda & \psi + \lambda
\end{bmatrix}$$

The characteristic polynomial is

$$(\delta + x)(\phi' - x)(\lambda + x)(\psi + \delta - x)(\psi - \phi' - x)(\psi + \lambda - x) = 0 \tag{4.1}$$

Expanding and rearranging terms gives

$$(\delta(\delta + \psi) - (x^2 - x\psi))(\phi'(\phi' - \psi) - (x^2 - x\psi))(\lambda(\lambda + \psi) - (x^2 - x\psi)) = 0 \tag{4.2}$$

Setting $A = (\delta(\delta + \psi)$, $B = (\phi'(\phi' - \psi)$, $E = (\lambda(\lambda + \psi)$ and $X = (x^2 - x\psi)$, (4.2) reduces to

$$(A - X)(B - X)(E - X) = 0 \tag{4.3}$$

Provided that $\phi' > \psi$ (to insure that B is positive), equation (4.3) has three real positive roots and the matrix of the linearized system has six eigenvalues three positive and three negative. The steady state is therefore a saddle point.

Regarding externalities generated by FDI and external debt, it is straightforward to verify that (4.2) does not involve the utility function and therefore externalities generated by the imported capital do not affect the steady state equilibrium. This also implies that whatever the form the social welfare function might take (e.g. separable or non-separable) and whatever the externalities generated by foreign capital and how do they interact with each other and the way they affect social welfare (i.e. signs of the cross partial derivative in the non-separable case), the steady state is a saddle point.

5 Summary

The importance of FDI, both as a source of capital and as a channel for transmission of innovations and spillover benefits, has gained more importance in recent decades. Models adopting the framework of intertemporal decision making, however, have limited capital imports to its external borrowing dimension. This paper extends the concept of capital imports and includes the FDI dimension. It assumes a community that imports both forms of capital to produce some output. The community derives utility from consumption of goods and the positive externalities of FDI, while deriving disutility from external debt.

Theoretical results suggest that: (1) Externalities generated by foreign capital affect the relevant interest rate at which the community borrows. The higher the disutility of external debt the higher the shadow interest rate. The higher the utility derived from FDI, however, the lower the shadow interest rate. (2) The community will keep borrowing untill the relevant interest rate is equal to the net marginal product of capital. This can also be seen as the condition that guaranties the solvency of the economy. (3) The study of the social optimum shows that externalities that arise from foreign capital do not affect the steady state, which is always a saddle point. This implies that whatever the form the utility function might take, the steady state equilibrium is a saddle point.

To keep the analysis simple, this paper assumes that there is no link between external borrowing and FDI. In reality, however, countries can use external debt to build infrastructure, and improve their political and economic institutions. Future development can address this question by considering the relationship between FDI and external debt.

References

Blanchard, O. J. (1983). Debt and the current account deficit in brazil. In P. A. Armella et al. (Eds.), *Financial Policies and the World Capital Market: The Problem of Latin American Countries*, pp. 187–197. University of Chicago Press.

Botrić, V. and L. Škuflić (2006). Main determinants of foreign direct investment in the southeast european countries. *Transition Studies Review 13*, 359–377.

Busse, M. and C. Hefeker (2007). Political risk, institutions and foreign direct investment. *European Journal of Political Economy 23*(2), 397–415.

Pranab, B. K. (1967). Optimal foreign borrowing. In K. Shell (Ed.), *Essays on the Theory of Optimal Economic Growth*, Chapter 6, pp. 117–128. MIT Press.

Ramsey, F. P. (1928). A mathematical theory of saving. *The Economic Journal 38*(152), 543–559.

UNCTAD (2008). Development and globalization: Facts and figures. Technical report, The United Nations.