Stock Index Volatility: the case of IPSA

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This paper introduces alternative measurements that use additional information of prices during the day: opening, minimum, maximum, and closing prices. Using the binomial model as the distribution of the stock price we prove that these alternative measurements are more efficient than the traditional ones that rely only in closing price. Following Garman and Klass (1980) we compute the relative efficiency of these measurements showing that are 3 to 4 times more efficient than using closing prices. Using daily data of the Chilean stock market index we show that a discrete-time approximation of the stock price seems to be more accurate than the continuous-time model. Also, we prove that there is a high correlation between intraday volatility measurements and implied ones obtained from options market (VIX). For that we propose the use of intraday information to estimate volatility for the cases where the stock markets do not have an associated option market.

JEL: C22, G11, G12

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I. INTRODUCTION

During the last years, financial markets have been affected by volatile episodes which have increased the movements on the asset prices. Knowing and understanding the volatility measures for key financial assets is the most important task for market participants and for supervisors as well.

For developed markets, the use of derivatives helps to estimate the volatility of the underlying. That is the case of VIX which is a well-known measurement of the stock index volatility. Demeterfi et al. (1999) provides a formal proof for the VIX showing that it is a general version of the implied volatility, which is obtained by assuming the Black-Scholes formula. In the case of countries without derivative markets the standard approach is to rely on statistical models in which a dynamic for the second moment of the return is added. Those are the cases of ARCH, GARCH, or EGARCH models which are estimated by non-linear methods (Wilmott, 2006). Those models rely on the asymptotic of the estimation methods for that a large sample is needed. Indeed, a rule of thumb for estimating a GARCH model, by maximum likelihood, is to use at least 500 observations.

In this paper we study the use of intraday information for estimating the volatility of stock indexes. Following the works of Parkinson (1980), Garman and Klass (1980), and Rogers and Satchell (1991) we rely on four statistics of the stock price during the day: open, minimum, maximum, and close. This set of information is usually collected by several trading companies such as Bloomberg or Reuters. Assuming that the stock index can be characterized by the binomial distribution we show that intraday measurements of volatility are about 3 to 4 times more efficient than the estimate obtained by using only closing prices. These findings provide accurate estimates of the continuous-time results of Garman and Klass (1980), and Rogers and Satchell (1991). Also, we show that the bias of intraday measures depends on the number of steps of the tree. This explains why the Parkinson's estimate is usually downward biases toward zero.

We apply the intraday measurements for the case of Chilean stock market finding that the empirical bias-correction is lower than the continuous-time value. We interpret this result as the continuous-time model is a good approximation for countries where thousands of transactions are made daily, meanwhile the binomial distribution could be used for stock index in smaller financial markets. Finally, we apply the intraday measurements to the S&P 500 index finding that those are highly correlated with the VIX. This evidence supports the use of intraday information in equity markets without derivates.

The paper is organized as follows: in the Section II we discuss the intraday volatility measurements; in Section III we provide an application of these measurements to the Chilean Stock Index, IPSA; Section IV compares the effective and the implied volatility measures for an international stock index; and conclusions are given in Section V, then Appendix A covers the technical details.

II. Intraday Measurements

Traditional volatility measurements use only closing prices and exclude important information which is generated through the day. In this section, opening, closing, minimum and maximum prices are incorporated in the analysis. This information has been used previously by Parkinson (1980), Garman and Klass (1980) and Rogers and Satchell (1991).

However, these authors have based their studies in the assumption of a Brownian motion for the asset price. Such assumption is valid for developed market where there are a high number of daily transactions. In the case of a less liquid market, a binomial model developed by Cox, Ross and Rubinstein (1979) is proposed for the dynamic of the price.

In this model, the asset price in the next time could be described by two possible scenarios, which are determined by the volatility of the asset (σ):

i) Up scenario: the asset price is increased by the factor $u = \exp\left(\sigma\sqrt{1/N}\right)$ and

ii) Down scenario: the asset price is decreased by $d = 1/u$,

where N is the number of steps of the binomial tree. It is important to note that if N tends to infinity then the binomial model converges to the continuous-time Brownian motion. Thus, the number of steps of the binomial model represents the "depth" of the financial market and the binomial model is the most convenient model for the Chilean market.

The probability of each scenario depends on the volatility and the asset return and it can be represented by (Wilmott, 2006):

$$p=\frac{1}{2}+\frac{\mu}{2\sigma}\sqrt{\frac{1}{N}}$$

As a simplification a zero return asset has been assumed $(\mu=0)$, thus the probability of each scenario is equal to 0.5.

Hereafter, observable intraday prices are assumed and the following definitions are applied: o is the logarithm of the opening price, which is assumed equal to 1 for simplicity; c is the logarithm of the closing price; h is the logarithm of the maximum price; and l is the logarithm of the minimum price. Then, the following volatility measures can be calculated: $\sigma_{CC}^2 \equiv (c-o)^2$ and $\sigma_{HL}^2 \equiv (h-l)^2$, where CC is the traditional volatility measure based on closing prices and HL is the new estimator which uses maximum and minimum prices that are observed during the day, as it is suggested by Parkinson (1980).

As an example, Table 1 considers a binomial tree with two steps (*N*=2). The results in the table correspond to the logarithm of the prices. Thus the closing price for the Up scenario is $c=\log(u)=\sigma\sqrt{1/1}=\sigma$ which coincides with the maximum price; and the minimum price is the logarithm of the normalized opening price. It can be seen that only the Up-Up and Down-Down scenarios give information for the calculation of CC whereas HL uses the available information in all scenarios for the volatility estimation.

Table 1: Results for a two-steps model

Scenarios	c	h	l
$Up-Up$	$\sigma\sqrt{2}$	$\sigma\sqrt{2}$	0
$Up-Down$	0	$\sigma\sqrt{2}/2$	0
$Down-Up$	0	0	$-\sigma\sqrt{2}/2$
$Down-Down$	$-\sigma\sqrt{2}$	0	$-\sigma\sqrt{2}$

Source: Authors' calculations.

The expected values for CC and HL may be calculated as following:

$$E\left(\sigma_{CC}^2\right) = \frac{1}{4}\left(\sigma\sqrt{2}\right)^2 + \frac{1}{4}\left(-\sigma\sqrt{2}\right)^2 = \sigma^2$$

$$E\left(\sigma_{HL}^2\right) = \frac{1}{4}\left(\sigma\sqrt{2}-0\right)^2 + \frac{1}{4}\left(\frac{\sigma\sqrt{2}}{2}-0\right)^2 + \frac{1}{4}\left(0+\frac{\sigma\sqrt{2}}{2}\right)^2 + \frac{1}{4}\left(0+\sigma\sqrt{2}\right)^2 = 1.25\sigma^2$$

In a model with two steps, only CC is unbiased whereas HL overestimates the true variance in 0.25 times. The bias is originated for using discrete data of maximum and minimum prices in a two-steps model. In practice, the calculation of the HL volatility depends on the strong assumption of a significant number of scenarios for the asset price.

Parkinson (1980) demonstrates that if the asset price follows a Brownian motion without trend and it can be observed continuously then $E\left(\sigma_{HL}^2\right) = 4\log(2)\sigma^2 \approx 2.773\sigma^2$. This means that HL overestimates the true variance and the bias is conditional to the "depth" of the financial market.

Although HL is biased, it is more efficient than CC in terms of Mean Squared Error (MSE). In particular, for N=2 the formulas included in the appendix of this study show that $ECM\left(\sigma_{CC}^2\right)=\sigma^4$, and $ECM\left(\sigma_{HL}^2\right)=0.625\sigma^4$.

An intuitive explanation is that the maximum and minimum prices contain more information about the volatility than the opening and closing prices, because the former obtain their values during the transaction time while the latter are only instant pictures of the process.

Defining a_N as the bias factor of HL, then for $N=2$ this factor is equal to $a_N=1.25$. As a consequence, an unbiased estimator (HL2) can be obtained from HL as follows:

$$\sigma_{HL2}^2 \equiv \frac{1}{a_N}\sigma_{HL}^2 = 0.8\sigma_{HL}^2 \rightarrow E\left(\sigma_{HL2}^2\right)=\sigma^2 \text{, and } ECM\left(\sigma_{HL2}^2\right)=0.36\sigma^4$$

The new estimator HL2 is unbiased as CC, but it has lower variance. Another unbiased and efficient estimator was proposed by Garman and Klass (1980) and its adaptation to the binomial model is such that:

$$\sigma_{GK}^2 = \frac{1}{2}(h-l)^2 - \left(\frac{a_N}{2}-1\right)(c-o)^2$$

For construction, GK is unbiased and for the case of $N=2$, $ECM\left(\sigma_{GK}^2\right)=0.5625\sigma^4$. However, GK is more efficient than HL when $N>12$ as it can be seen in the Table 2.

Previous results could be misleading if the asset price presents any trend during the day, this is $\mu \neq 0$. For instance, if the price increases then $h = c$, and $l = o$, and HL and HL2 would be the results of spurious variances. As a way of controlling for the trend of the asset, the opening and closing prices could be included in the variance formula as it is suggested by Rogers and Satchell (1991) for a continuous time scenario:

$$\sigma_{RS}^2 = (h - o)(h - c) + (l - o)(l - c).$$

This measurement uses the maximum and minimum prices and incorporates the opening and closing prices in order to subtract the intraday trend of the price. The basic statistics for RS assuming a binomial tree with two steps are the following:

$$E(\sigma_{RS}^2) = \frac{1}{4}\left(\frac{\sigma\sqrt{2}}{2}\right)^2 + \frac{1}{4}\left(\frac{-\sigma\sqrt{2}}{2}\right)^2 = 0.25\sigma^2 \text{, and } ECM(\sigma_{RS}^2) = 0.625\sigma^4$$

RS is also a biased estimator and its bias is bigger than HL. Defining b_N as the bias factor for RS, it is possible to calculate an unbiased estimator of RS as follows:

$$\sigma_{RS2}^2 = \frac{1}{0.25}\sigma_{RS}^2 = 4\sigma_{RS}^2 \to E(\sigma_{RS2}^2) = \sigma^2 \text{, and } ECM(\sigma_{RS2}^2) = \sigma^4$$

It is important to note that if the asset price can be characterized by a binomial tree with two steps, then RS2 presents the same properties than CC. However RS2 does not present trend. In their original paper, Rogers and Satchell (1991) demonstrate that RS is unbiased in a continuous time. In the model presented in this paper, the bias factor for RS converges to unity when N is big enough.

Up to now, the analysis has included the calculation for the bias and the bias factor for the measurements HL and RS for a two-steps model and for assets without trend. An extension for a binomial model with a greater number of steps is exhibited in Table 2[1]/.

In general, the advantages of the classical estimator of volatility CC are its simplicity and its unbiasness. However, its main disadvantage is that it ignores available information which could be important for the efficiency of the estimator. In this way, Garman and Klass (1980) state that the CC would be a benchmark through which other estimators would be measured, and a Relative Efficiency (RE) ratio could be calculated as follows:

$$RE\left(\sigma_A^2\right) = \frac{Var(\sigma_{CC}^2)}{Var(\sigma_A^2)}$$

Where $Var(\sigma_{CC}^2)$ is the variance of CC and $Var(\sigma_A^2)$ corresponds to the variance of an unbiased estimator (σ_A^2) for which the RE ratio is being calculated.

[1]/ It is important to note that CC and GK are unbiased for all number of steps.

Garman and Klass (1980) and Rogers and Satchell (1991) calculate RE for the estimators of variance in a continuous time, whereas the theoretical values of RE in binomial trees with steps lower than 24 are presented in Table 2[2]/.

Table 2: Correction factors and Relative Efficiency

	Correction factors		Relative efficiency to CC (*)		
N	PK	RS	PK	RS	GK
1	1.000	0.000			
2	1.250	0.250	2.78	1.00	1.78
3	1.500	0.333	3.28	1.33	2.37
4	1.594	0.406	3.09	1.77	2.43
5	1.700	0.450	3.28	2.06	2.73
6	1.760	0.490	3.27	2.28	2.83
7	1.826	0.518	3.39	2.47	3.04
8	1.868	0.544	3.40	2.63	3.14
9	1.914	0.564	3.49	2.77	3.31
10	1.946	0.583	3.52	2.89	3.40
11	1.980	0.599	3.58	3.00	3.53
12	2.005	0.613	3.59	3.09	3.61
13	2.033	0.626	3.65	3.18	3.72
14	2.053	0.638	3.67	3.26	3.79
15	2.075	0.648	3.71	3.33	3.89
16	2.092	0.657	3.73	3.40	3.95
17	2.111	0.666	3.76	3.45	4.04
18	2.125	0.674	3.78	3.51	4.09
19	2.141	0.682	3.81	3.57	4.16
20	2.154	0.689	3.82	3.61	4.21
21	2.168	0.695	3.85	3.66	4.28
22	2.179	0.701	3.86	3.70	4.33
23	2.191	0.707	3.89	3.74	4.39
24	2.201	0.713	3.90	3.79	4.44
Infinite	2.773	1.000	5.20	6.04	7.41

(*) Adjusted by correction factors.
Sources: Garman and Klass (1980), Rogers and Satchell (1991) and authors' calculation.

[2]/ For steps equal to N, the number of possible scenarios is 2^N. Due to computational limitations, Table 2 presents results only until 24 steps.

III. RESULTS FOR A CHILEAN STOCK INDEX

In order to calculate the intraday volatilities for the IPSA[3]/, the correction factors of the Chilean market must be known. Using daily data for the prices of the IPSA from January 2, 1996 to January 18, 2008, the historical values of volatility are estimated. The ratio between the non-corrected volatility measure of Parkinson and the daily standard deviation brings a median value of $1.54\,(=\sqrt{2.37})$. This means that the correction factor for the Chilean equity market is lower than the value suggested by Parkinson (1980) for a continuous-time process (2.773), but it is greater than the correction factor for a 24 steps of the binomial tree (2.201). With this estimated value of the correction factor for the IPSA, the unbiased volatility measure in annual terms is estimated as follows[4]/:

$$\hat{\sigma}_{HL2,t} = \sqrt{250}\,\frac{\hat{\sigma}_{HL,t}}{1.54} = \sqrt{\frac{250}{2.37}}\sqrt{(h_t - l_t)^2} \approx 10\cdot\left(\frac{H_t}{L_t} - 1\right),$$

where H_t and L_t are the maximum and minimum prices for a specific day, respectively.

Likewise, the median value for the ratio between the Rogers-Satchell volatility and the daily standard deviation is 0.88[5]/, and the unbiased volatility measure in annual terms for the IPSA in this case is calculated as:

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[3]/ IPSA is the Selective Price Index of the Santiago Stock Exchange and it considers the 40th most traded stocks of the stock exchange. In international comparison is widely used as the main stock index for Chile.

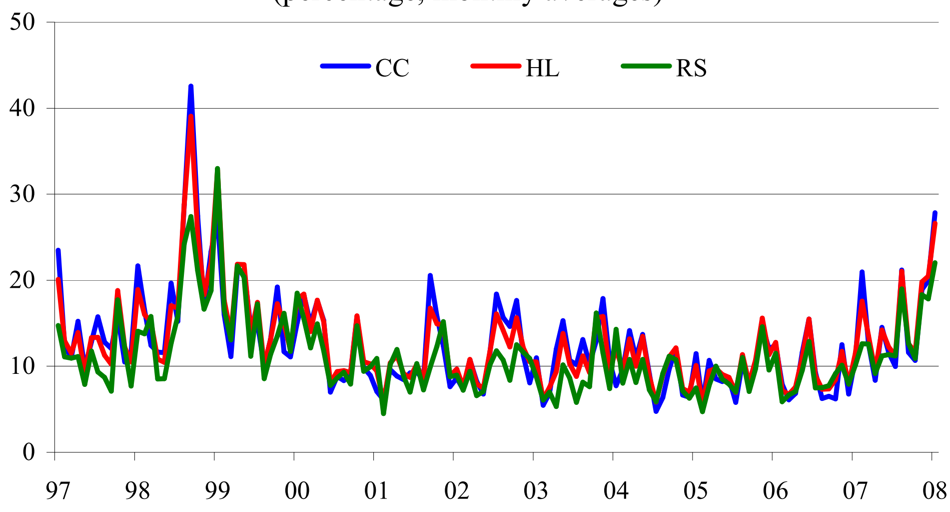
[4]/ Recall that HL incorporates the daily trend of the price of the asset. Yang and Zhang (2000) argue that such trend should be small for intraday information as the investors are not expecting significant movements of the asset.

[5]/ The 95% confidence interval for both correction factors implies the following estimation ranges (1.50-1.58) and (0.84-0.92), respectively.

$$\hat{\sigma}_{RS2,t} = \sqrt{250}\,\frac{\hat{\sigma}_{RS,t}}{0.88} \approx 18\sqrt{\left[\left(\frac{H_t}{C_t}-1\right)\cdot\left(\frac{H_t}{O_t}-1\right)+\left(\frac{L_t}{C_t}-1\right)\cdot\left(\frac{L_t}{O_t}-1\right)\right]},$$

where H_t, L_t, O_t and C_t are the maximum, minimum, opening and closing prices, respectively. Graph 1 shows $\hat{\sigma}_{HL2,t}, \hat{\sigma}_{RS2,t}$, and the standard deviation (CC) for a 21-days moving average.

Graph 1: Volatility measurements
(percentage, monthly averages)



Sources: Bloomberg and authors' calculation.

From the graph, it is observed that all the volatility measurements are positively correlated with the turbulent episodes. In particular, through this time HL2 and CC (0.95) are the most correlated measurements. This implies that an intraday volatility measure, as HL2, is able to include the same information than a traditional one, as CC, but HL2 would do it in a more efficient way. In other words, we could have the same standard error in HL2 than CC, but using less days in the computation of the fist one.

IV. A COMPARISON OF EFFECTIVE AND IMPLIED VOLATILITY

In order to validate the volatility measurements obtained from the previous section, these measures are calculated for equity markets which present available information of equity options. In particular, a comparison of the effective volatility measurements and the volatility index calculated with options (VIX) for the S&P 500 index is performed.

Using daily data for the 2004-2008 periods the CC, HL and RS for the S&P 500 are calculated. As it is the case of a developed equity market, the standard correction factors suggested by the literature are applied.

Results show that HL and RS exhibit lower standard deviation than CC (Table 3). This means that the estimated value of the volatility is more efficient if the maximum and minimum prices are included. Moreover, RS presents a lower average than HL and CC because it does not consider the trend of the asset.

Table 3: Volatility of S&P 500 index
(percentage)

	Mean	Median	Standard deviation	Percentile 5%	Percentile 95%
CC	9.47	7.05	8.91	0.73	25.78
HL	9.80	8.44	5.62	3.91	20.22
RS	8.62	7.50	5.78	2.08	18.42
VIX	15.24	13.97	4.45	10.74	25.54

Sources: Bloomberg and authors' calculation.

On the other hand, the VIX is greater than the effective volatility of the S&P 500 Index (CC, HL or RS) while its standard deviation is lower. The first issue arises because VIX contains a risk premium associated with the uncertainty of the derivates, and the lower standard deviation of the VIX emerges from the fact that it is calculated as the average volatility for the next 30 days. Finally, the effective volatility measurements and the VIX are correlated in 0.5 for daily data and 0.9 for moving averages (Table 4). This implies that HL and RS capture properly the movements of the VIX and they could be good substitutes in the case of equity markets without derivates.

Table 4: Correlation between VIX and volatility measurements for the S&P 500
(percentage)

	Daily measures	Moving average (1)
CC	0.45	0.89
HL	0.67	0.90
RS	0.52	0.86

(1) Exponentially weighted using a factor of 0.94.
Sources: Bloomberg and authors' calculation.

V. CONCLUSIONS

In this study we introduced the use of intraday information to improve the efficiency in the estimation of volatility for stock prices. From the theoretical analysis, it is possible to conclude that those estimates proposed are more efficient than relying only in closing prices. This is proved under the assumption that the stock price follows a binomial distribution. The result is close to the one presented in the literature by Parkinson (1980) or Rogers and Satchell (1991); however, we provide an extension that shows that the relative efficiency of those estimates is 3 to 4 times bigger compared with the use of close prices. Previous research provides higher level of efficiency because they assume that the stock prices follow a continuous-time Brownian motion. Moreover, the empirical analyses have showed that financial series move away from their historical averages in volatile episodes, and as a consequence, in these periods only the most recent information is relevant.

Based on the Chilean equity market, two volatility measurements that use intraday information have been proposed. The first one is an adaptation of the volatility index suggested by Parkinson (1980) and the second one corresponds to an adaptation of the Rogers and Satchell (1991) volatility estimator. Both are bias-corrected for which we note that the correction factors imply that there is a finite number of steps that accommodates the binomial distribution of the stock market index. We confirm the use of these intraday measurements by showing the high level of correlation between those applied to the S&P 500 index and the VIX, which is defined as an implied volatility from options over the same index. With that evidence we conclude that intraday volatility measurements are appropriate for equity markets without options markets.

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APPENDIX A

For calculating the mean squared error it is necessary to know the fourth moment of the variances, whose computation is presented here:

$$E\left(\sigma_{CC}^4\right)=\frac{1}{4}\left(\sigma\sqrt{2}\right)^4+\frac{1}{4}\left(-\sigma\sqrt{2}\right)^4=2\sigma^4,\; E\left(\sigma_{HL}^4\right)=2\left[\frac{1}{4}\left(\sigma\sqrt{2}\right)^4+\frac{1}{4}\left(\frac{\sigma\sqrt{2}}{2}\right)^4\right]=2.125\sigma^4,$$

$$E\left(\sigma_{RS}^4\right)=\left[\frac{1}{4}\left(\frac{\sigma\sqrt{2}}{2}\right)^4+\frac{1}{4}\left(\frac{-\sigma\sqrt{2}}{2}\right)^4\right]=0.125\sigma^4,$$

$$\begin{aligned}E\left(\sigma_{GK}^4\right)&=\frac{1}{4}E\left(\sigma_{PK}^4\right)-2*\frac{1}{2}\left(\frac{1{,}25}{2}-1\right)E\left[(h-l)^2c^2\right]+\left(\frac{1{,}25}{2}-1\right)^2E\left(\sigma_{CC}^4\right),\\&=0.25\cdot 2.125\sigma^4+0.375\cdot 2\sigma^4+0.375^2\cdot 2\sigma^4=1.5625\sigma^4\end{aligned}$$

$$E\left[(h-l)^2c^2\right]=\frac{1}{4}\left(\sigma\sqrt{2}\right)^2\left(\sigma\sqrt{2}\right)^2+\frac{1}{4}\left(\sigma\sqrt{2}\right)^2\left(-\sigma\sqrt{2}\right)^2=2\sigma^4.$$