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Abstract

The impact that competition exerts on the incentives of firms to pass through reductions in their marginal costs is an important consideration in assessing the performance of alternate market structures. This paper examines the role of product differentiation on firm-specific and industry-wide pass-through rates. Relying on Shubik's (1980) model of differentiated Cournot competition with linear demand, we show that there exists an initial critical range over which the firm-specific cost pass-through rate decreases in the number of firms. Beyond this range the rate continually increases – approaching 50 percent as the number of firms goes to infinity. This contrasts with a model of differentiated Bertrand competition in which cost pass-through monotonically decreases in the number of firms. The disparate effects across the Cournot and Bertrand models are shown to stem from the influence of competition and product differentiation on the respective firm reaction functions. Suggestions for future empirical work based upon the models' predictions and implications for antitrust policy are also discussed.

Keywords: Competitive effects; Oligopoly; Merger; Pass-through; Product differentiation

JEL Classification: L13; L40

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1 Introduction

The impact that competition exerts on the incentives of firms to pass through reductions in their costs, including possible efficiencies realized through mergers, is an important consideration in assessing the performance of alternate market structures. Bulow and Pfleiderer (1983), hereinafter B&P, were the first to formally consider this issue. They show that a monopolist facing a linear demand curve and constant marginal cost will always pass through 50 percent of a change in the level of marginal cost. B&P also show that an individual perfectly competitive firm will not pass through any change in its marginal cost onto consumers. Further, B&P demonstrate—in the context of their specific model—that the only instance in which the pass-through rate is greater under perfect competition than under monopoly is where the cost change in the former is industry-wide. When all of the competitive firms realize the same change in marginal cost, the industry-specific competitive pass-through rate is always 100 percent (*i.e.*, complete pass-through).

The failure to recognize the difference between the *firm-specific* and *industry-wide* pass-through rates in the case of perfect competition has lead some to mistakenly conclude that a monopolist will have a lower incentive to pass through a marginal cost decrease than will an individual competitive firm or, more generally, that a firm's cost pass-through rate is decreasing in its own market share.[1] Froeb *et al.* (2005), in the context of merger review, demonstrate just the opposite. They show that there is direct relationship between the gross price effect of a merger and the pass-through rate as both are influenced by the second derivatives of the demand curve. Further, for some functional forms of demand the monopoly pass-through rate can equal or even *exceed* the *industry-wide* competitive pass-through rate (Vita and Yde, 2005, p. 3).[2]

[1] Yde and Vita (1996) provide a comprehensive overview of these and related issues. Also see ten Kate and Niels (2005) for additional discussion.
[2] This result is also true for oligopoly vs. perfect competition; *see* Anderson *et al*. (2001). An "overshifting" of industry-wide cost changes can occur, *e.g.*, when demand is of the constant elasticity variety.

Recent research has extended the B&P analysis to consider the impact of competition on the incentives for cost pass-through in oligopolies. In particular, ten Kate and Niels (2005) consider the question in a model of Cournot competition with homogeneous output. Their analysis confirms a result suggested by B&P's study that, in general, there is an inverse relationship between the level of competition and the extent to which an individual oligopolist realizing a marginal cost change will pass through that change to consumers. Specifically, in an n firm Cournot model with linear demand and constant, firm-specific marginal costs, ten Kate and Niels show that the firm-specific pass-through rate is given by $\frac{1}{n+1}$, which is clearly decreasing in the number of firms. Like the pass-through rates derived by B&P, this rate is independent of the original level of marginal cost and any key demand parameters (*e.g.*, the slope of the market demand curve or the elasticity of demand). Further, the pass-through rate is independent of the market share of the particular firm realizing the cost change.[3]

Another important insight of ten Kate and Niels concerns the influence that firm interdependency plays in affecting oligopoly cost pass-through rates for homogeneous goods. When one individual firm realizes a reduction in its marginal cost, all other firms must absorb the resultant price decrease caused by the pass-through, and as such, will also have to lower their prices in order to maximize profits. This fact leads the authors to conclude that, "it might therefore also be of relevance to compare the price decrease with the *average* cost savings across all firms rather than with the cost savings realized by an individual firm" (p. 329, emphasis in original). Using a simple average of cost savings across all firms, the pass-through rate of average cost savings to price is $\frac{n}{n+1}$, which is never less than the pass-through rate of a monopolist and approaches complete pass-through as $n \to \infty$.

[3] Note that this is consistent with Bergstrom and Varian (1985) who demonstrate that the Cournot equilibrium is independent of the agent characteristics.

This paper extends the literature on cost pass through in oligopolies by considering the influence of *product differentiation* on pass-through rates where homogeneous output is special case of the more general model. To the best of our knowledge, a systematic consideration of the influence of product differentiation on pass through in oligopoly markets has not been considered in the previous literature. Unlike the homogeneous goods case, it is shown that in a Cournot model with linear demand, constant and symmetric marginal costs, and differentiated output that the equilibrium marginal cost pass-through rate is increasing in the level of competition. That is, as the degree of market power faced by a firm decreases, its incentive to pass through a cost change increases.

In order to explore the robustness of the results, a model of differentiated Cournot competition with constant but asymmetric (*i.e.*, firm-specific) marginal costs is also considered. It is shown that the positive effect of competition on the firm-specific pass-through rate is largely retained in the more general case with the results of homogeneous goods models obtained as limiting cases. The asymmetric case is also examined in a model of Bertrand competition. It is shown that when firms compete by choosing price, competition has a strictly negative effect on the firm-specific pass-through rate as in the homogeneous Cournot model of ten Kate and Niels. Finally, the paper concludes by considering the implications for competition policy and by suggesting further avenues of research.

2 Differentiated Cournot competition with symmetric firms

As is done by ten Kate and Niels, we begin with a simple model of linear demand with symmetric firms but we allow products to be differentiated. It is assumed that a market is served by $n > 1$ firms, indexed by i, who compete non-cooperatively in differentiated quantities and produce with positive, constant, and symmetric marginal costs denoted by c.[4] The inverse market demand facing firm i is given by

$$p_i(\mathbf{q}) = \alpha - q_i - \phi \sum_{j=1}^{n-1} q_j,\ i \neq j, \tag{1}$$

where $\mathbf{q}$ is the vector of quantities for all firms. The parameter $\phi \in [0,1]$ measures the degree of product differentiation, or substitutability, across firms. When $\phi = 0$ the output of each firm is unrelated and when $\phi = 1$ the outputs are perfect substitutes (*i.e.*, homogeneous output). Thus, higher values of ϕ are associated with increasingly homogeneous output.

The profit-maximization problem of firm i is,

$$\max_{q_i} \pi_i = (p_i(\mathbf{q}) - c)q_i, \tag{2}$$

$i \neq j$, such that $\alpha - c > 0$. The first-order condition from this problem yields the following reaction function for firm i,

$$R_i(\mathbf{q_j}) = \frac{1}{2}\left(\alpha - c - \phi \sum_{j=1}^{n-1} q_j\right),\ i \neq j, \tag{3}$$

where $\mathbf{q_j}$ is the vector of outputs excluding the output of firm i.[5] The reaction function confirms that firm outputs are strategic substitutes. Solving the system of reaction functions, under the assumption of symmetry, yields the equilibrium output and price for firm i's variety, given by,

$$q_i^{SC} = \frac{\alpha - c}{2 + \phi(n-1)}, \tag{4}$$

and

$$p_i^{SC} = \frac{\alpha + (1 + \phi(n-1))\,c}{2 + \phi(n-1)}, \tag{5}$$

[4] The latter assumption would seem to apply, for example, to many retail goods markets, where competing firms have access to the same production technology but where product differentiation is determined primarily by differences in branding and/or advertising.
[5] Both the second-order condition and the stability condition, which ensures that the reaction curves intersect, are satisfied for the range of parameters under consideration.

respectively. The SC superscript denotes Cournot solutions with symmetric costs. The optimal output and price of firm i are decreasing both in the number of firms and in the degree of substitutability between firm outputs.

Partially differentiating equation (5) with respect to c gives the expression for the cost pass-through rate,

$$\frac{\partial p_i^{SC}}{\partial c} = \frac{1+\phi(n-1)}{2+\phi(n-1)} > 0\,.\text{[6]} \qquad (6)$$

A positive cost pass-through rate means that some portion of a marginal cost change will be passed through to price regardless of the level of competition. In order to determine the effect of competition on the intensity of pass-through we partially differentiate equation (6) with respect to n, which gives,

$$\frac{\partial^2 p_i^{SC}}{\partial c \partial n} = \frac{\phi}{\left(2+\phi(n-1)\right)^2} > 0\,. \qquad (7)$$

Thus, an increase in n corresponds to a higher pass-through rate. For example, if $\phi = 1/2$ an increase in the number of competitors from four to five implies that the cost pass-through rate will increase from approximately 71 to 75 percent. In the case of monopoly ($n = 1$), the cost pass-through rate is 50 percent while in the case of perfect competition ($n \to \infty$), the pass-through of a marginal cost change is complete. These results correspond to those first reported by B&P and accord to *a priori* expectations as the cost-change is industry-wide.

Holding the number of firms constant, equation (6) demonstrates that for perfectly differentiated output ($\phi = 0$) the pass-through rate is also 50 percent. If consumers do not view the output of individual firms to be substitutable, then each firm is able to price as a monopolist. In this extreme case the market is actually segmented along n individual monopolists, each of whom makes its output decision independently of all other firms. At the other extreme, when

[6] Following previous studies, it is assumed that all marginal cost changes are output-independent.

output is homogeneous ($\phi = 1$) the pass-through rate is $\dfrac{n}{n+1}$. Interestingly, as shown by ten Kate and Niels (2005), this value corresponds to the average industry-wide cost pass-through rate in a Cournot oligopoly with homogeneous output but firm-specific constant marginal costs. Note that this pass-through rate is never less than the monopoly pass-through rate.

Equation (7) demonstrates that the pass-through rate is monotonically increasing in the number of competitors. That is, firms will have a progressively higher incentive to pass-through marginal cost changes to consumers as the number of firms in the market increases. Under a more general model, however, as shown in the following section, the firm-specific pass-through rate is not monotonic and can be increasing or decreasing in the number of competitors as determined by a critical level of competition.

3 Differentiated Cournot competition with asymmetric firms

The results of the previous section are based upon a model of Cournot competition that assumes identical technologies and a symmetric equilibrium. As noted above, while this particular model might serve as a reasonable abstraction of market conditions in at least some industries, it is unlikely to do so in other cases. Further, recall that some of the confusion regarding the relationship between cost pass-through rates and competition apparently stems from failing to distinguish between firm-specific and industry-wide cost changes. With common marginal costs across all firms, it is arguable that this distinction becomes somewhat obfuscated. Therefore, determining whether the previous results hold under a more general model is of interest.

In order to introduce asymmetry, we follow Wang and Zhao (2007) who consider Shubik's (1980) model of Cournot oligopoly in differentiated products in order to examine the welfare effects of cost changes.[7] The same model considered by Wang and Zhao is employed herein to explore whether the positive relationship between the firm-specific cost pass-through rate and the number of competitors holds more generally. The inverse firm-specific demand functions are given by

$$p_i(\mathbf{q}) = \alpha - q_i + \frac{\phi}{1+\phi}(q_i - \frac{1}{n}\sum_{i=1}^{n} q_i)\,, \tag{8}$$

where $\mathbf{q}$ is the vector of firm outputs. Marginal cost, denoted by c_i, is positive and constant as in the symmetric case but now is *firm-specific*. Asymmetric marginal costs allow for differing equilibrium prices and market shares across firms. We retain the notation and parameter restrictions of the previous section except that the differentiation parameter (ϕ) now varies over the interval $[0, \infty)$. Again, higher values of the differentiation parameter correspond to less-differentiated products being offered across firms.[8]

The profit-maximization problem of firm i is

$$\max_{q_i} \pi_i = (p_i(\mathbf{q}) - c_i)q_i \tag{9}$$

for all $i = 1, \ldots, n$. The first-order condition for this problem is given by,

$$\frac{\partial \pi_i}{\partial q_i} = \alpha - c_i - \frac{1}{1+\phi}\left(2\left(1+\frac{\phi}{n}\right)q_i + \frac{\phi}{n}\sum_{j=1}^{n-1} q_j\right) = 0\,, \tag{10}$$

which yields the following reaction function for firm i,

$$q_i(\mathbf{q_j}) = \frac{n(1+\phi)}{2(n+\phi)}\left(\alpha - c_i - \frac{\phi}{n(1+\phi)}\sum_{j=1}^{n-1} q_j\right). \tag{11}$$

Wang and Zhao (2007, p. 178) show that the Cournot-Nash equilibrium quantity and price for firm i are given by,

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[7] Wang and Zhao (2007), however, do not otherwise consider the relationship between *market structure* (*i.e.*, number of competitors) and average or firm-specific cost *pass-through* rates.

[8] The model of Wang and Zhao (2007) considers an alternative interpretation of ϕ that was unnecessary to the analysis of the symmetric case and which limits direct comparison with the previous literature. Their model, however, produces largely similar results when symmetric costs are assumed.

$$q_i^{AC} = n(1+\phi)\left(\frac{\alpha}{2n+(n+1)\phi} + \frac{n\phi\overline{c}}{(2n+\phi)(2n+(n+1)\phi)} - \frac{c_i}{2n+\phi}\right), \tag{12}$$

and,

$$p_i^{AC} = \frac{(n+\phi)\alpha}{2n+(n+1)\phi} + \frac{n\phi(n+\phi)\overline{c}}{(2n+\phi)(2n+(n+1)\phi)} + \frac{nc_i}{2n+\phi}, \tag{13}$$

respectively, where $\overline{c} = \frac{1}{n}\sum_{i=1}^{n} c_i$ and the superscript AC denotes Cournot solutions with asymmetric costs.

3.1 Own pass-through rate

The rate at which firm i passes through a change in its marginal cost to its own price is,

$$\frac{\partial p_i^{AC}}{\partial c_i} = \frac{2n^2 + n(n+2)\phi + \phi^2}{(2n+\phi)(2n+(n+1)\phi)} > 0. \tag{14}$$

Observation 1. *(i)* $\lim_{\phi \to 0}\left\{\frac{\partial p_i^{AC}}{\partial c_i}\right\} = \frac{1}{2}$ *and (ii)* $\lim_{\phi \to \infty}\left\{\frac{\partial p_i^{AC}}{\partial c_i}\right\} = \frac{1}{1+n}$.

The effects demonstrated in Observation 1 conform to the results of the previous section as well as the relevant literature. Part (i) demonstrates that as outputs become unrelated, a firm's pass-through rate approaches the monopoly rate of 50 percent. Part (ii) demonstrates that as outputs become more homogeneous, a firm's pass-through rate approaches the rate when outputs are indeed homogeneous but costs are firm-specific. Thus, the central results of B&P and ten Kate and Niels (2005) can be seen as special cases of the more general model presented here.

Partially differentiating equation (14) with respect to n, holding the degree of product differentiation fixed, determines the extent to which competition affects the incentive of a firm realizing a cost change to pass through this change to its own price. This effect is given by

$$\frac{\partial^2 p_i^{AC}}{\partial c_i \partial n} = \frac{\phi^2(2n(n-2) + (n^2 - 2n - 2)\phi - \phi^2)}{(2n+\phi)^2(2n+(n+1)\phi)^2}. \tag{15}$$

Consideration of the above expression leads to the following proposition.

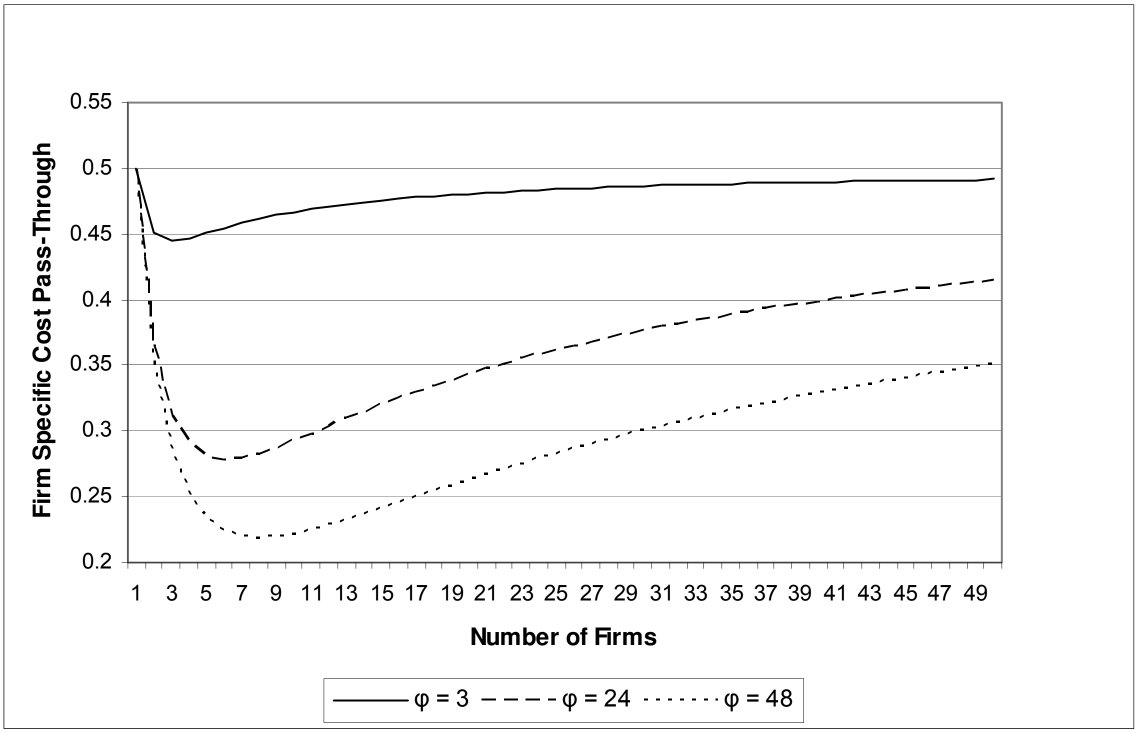
Proposition 1. *Given* $\phi \geq 0$, $\exists n^*$ *such that* $\frac{\partial^2 p_i^{AC}}{\partial c_i \partial n} \leq 0$ *for* $n \in (0, n^*)$ *and* $\frac{\partial^2 p_i^{AC}}{\partial c_i \partial n} \geq 0$ *for* $n \in (n^*, \infty)$ *and* $n^* = 1 + \sqrt{1+\phi}$.

Proof: Holding ϕ fixed, equation (15) has stationary points only at $n = 1 \pm \sqrt{1+\phi}$.
Evaluating $\frac{\partial\left(\frac{\partial^2 p_i^{AC}}{\partial c_i \partial n}\right)}{\partial n}$ at the non-negative stationary point yields,
$\frac{2\phi^2 (1+\phi)(2+\phi)\left(\phi^2 + 8\phi + 8 + 4(2+\phi)\sqrt{1+\phi}\right)}{\left(2+\phi+2\sqrt{1+\phi}\right)^3 \left(2(1+\phi)+(2+\phi)\sqrt{1+\phi}\right)^3} > 0$. Therefore, $n^* = 1 + \sqrt{1+\phi}$ is the only local minimum point for equation (15) over the relevant range of n. The rest of the proposition follows from the properties of local minima. ***Q.E.D***

Proposition 1 implies that the *lower* the degree of product differentiation, the *greater* the range of n over which the firm-specific pass-through rate will be falling, as demonstrated in Figure 1. Figure 1 plots the firm specific pass-through function from equation (15) for various values of the differentiation parameter (ϕ). Again, for $n = 1$ the pass-through rate is always 50 percent. For $n \in (0, n^*)$ the firm-specific pass-through function monotonically decreases, while for $n \in (n^*, \infty)$ it monotonically increases, approaching 50 percent in the limit.

In the absence of product differentiation, the firm-specific own cost pass-through rate is monotonically decreasing in n as demonstrated by ten Kate and Niels (2005). As the number of firms in the market increases, an individual firm behaves less like a monopolist and more like a price-taker. Therefore, when goods are homogeneous, an individual firm has less of an incentive to pass-through cost changes as the number of firms in the market increases.

Figure 1: Firm-Specific Own Cost Pass-Through with Asymmetric Costs



The presence of product differentiation has the opposite effect on an individual firm's incentive to pass-through cost changes. The slope of the reaction function for firm i from equation (11) is increasing in the differentiation parameter (ϕ). This means that as goods become more differentiated, the slope of the reaction function decreases and each firm's response to changes in rival output is smaller. That is, each firm behaves more like a monopolist in the presence of product differentiation. This effect is stronger the greater the degree of product differentiation, as demonstrated in Figure 1. At lower levels of ϕ a greater proportion of cost changes are passed through than at higher levels of ϕ. Beyond the critical point (n^*) the pass-through function begins to rise. The slope of the *reaction* function in the presence of product differentiation is decreasing in n whereas in the absence of product differentiation the slope is constant.[9] The impact of product differentiation strengthens as the number of firms in the market increases; consequently, each firm passes through a greater proportion of a cost change for higher levels of n.

In summary, there are two effects of competition on cost pass-through in the presence of product differentiation. The first effect is that an increase in the number of firms leads each individual firm to act more like a price-taker and pass-through declines. This first effect is present when goods are homogenous as well. The second effect is that product differentiation results in a flattening of the reaction function which, in Cournot competition, leads each individual firm to act less like a price-taker when the number of firms increases. This second effect causes pass-through to increase. The positive effect of the number of firms on cost pass-through rates swamps the negative effect of the number of firms at the critical value n^* and the pass-through rate switches from decreasing in n to increasing in n. The positive effect of the number of firms is stronger when product differentiation is greater. Therefore, the critical value n^* is increasing in the differentiation parameter (ϕ).

3.2 Cross pass-through rate

Recall that a change in the marginal cost of firm i will also affect the equilibrium prices charged by all other firms in the market, as emphasized by ten Kate and Niels (2005) and Werden *et al.* (2005). For an arbitrary firm j ($j \neq i$), this "cross" pass-through effect is,

$$\frac{\partial p_j^{AC}}{\partial c_i} = \frac{\phi(n+\phi)}{(2n+\phi)(2n+(n+1)\phi)} > 0. \tag{16}$$

While the own pass-through rate and the cross pass-through rate are both positive, indicating a firm will pass through a change in its own and in its rival's marginal cost, the rate of pass-through for own marginal cost changes exceeds the rate of pass-through for rival marginal cost changes.

[9] The slope when output is homogeneous, which can be found by taking the limit of the reaction function from equation (11) as $\phi \to \infty$, is -1/2.

Partially differentiating equation (16) with respect to $n,$ holding the degree of product differentiation fixed, determines the extent to which competition affects the incentive of a firm to pass through a change in its rival's marginal cost. This effect is given by,

$$\frac{\partial^2 p_j^{AC}}{\partial c_i \partial n} = -\frac{\phi(\phi^3 + (4n+3)\phi^2 + 2n(n+4)\phi + 4n^2)}{(2n+\phi)^2(2n+(n+1)\phi)^2} < 0. \tag{17}$$

Equation (17) demonstrates that the rate at which rival cost changes are passed through is always decreasing in the number of firms. The differing effects of competition on the own and cross pass-through rates make it necessary to examine some average measure of pass-through in order to understand how the overall level of prices in an industry is affected by a change in an individual firm's marginal cost.

3.3 Average pass-through rate

Unlike the homogeneous Cournot model, a single market demand curve from which price can be determined does not exist in the asymmetric model. Rather, the presence of product differentiation implies that there are n individual demand curves and prices which correspond to each Cournot competitor. It is, therefore, necessary to define a measure of the average equilibrium market price in the heterogeneous goods case. Wang and Zhao (2007, p. 174), define the average Nash-Cournot market equilibrium price as,

$$\overline{P}^{AC} = \frac{1}{n}\sum_{i=1}^{n} p_i^{AC} \ . \tag{18}$$

The pass-through of an individual firm's marginal cost change to the average market price is given by,

$$\frac{\partial \overline{P}^{AC}}{\partial c_i} = \frac{\phi^2 + (2n+1)\phi + 2n}{(2n+\phi)(2n+(n+1)\phi)} > 0\,. \tag{19}$$

Partially differentiating equation (19) with respect to $n,$ holding the degree of product differentiation fixed, determines the extent to which competition affects the pass-through of a change in an individual firm's marginal cost to the average market price. This effect is given by,

$$\frac{\partial^2 \overline{P}^{AC}}{\partial c_i \partial n} = -\frac{(1+\phi)(2+\phi)}{(2n+(n+1)\phi)^2} < 0. \tag{20}$$

In this simple average the own pass-through effect is weighted by $\frac{1}{n}$ whereas the cross pass-through effects are weighted collectively by $\frac{n-1}{n}$. As n increases, the proportion of the average pass-through that is due to the own pass-through effect decreases. Therefore, the own pass-through effect, despite being positive for sufficiently large n, is dominated by the cross pass-through effects and the pass-through to the average price is decreasing in the number of firms.

Instead of using a simple average as was done in Wang and Zhao (2007), it may be more appropriate to use the weighted average,

$$\overline{P}^{AC'} = \frac{1}{n}\sum_{i=1}^{n}\frac{q_i^{AC}}{Q^{AC}}\, p_i^{AC} \tag{21}$$

where $Q^{AC} = \sum_{i=1}^{n} q_i^{AC}$ and the individual prices are weighted by each firm's market share. The pass-through of an individual firm's marginal cost change to the weighted average market price is given by,

$$\frac{\partial \overline{P}^{AC'}}{\partial c_i} = \frac{-1}{n^3(\alpha-\overline{c})^2(2n+\phi)^2(2n+(n+1)\phi)} * \left(\begin{array}{l} n(2n+(n+1)\phi)^2\left(c_i^2+\sum_{j=1}^{n}c_j^2\right) \\ +(2n+(n+1)\phi)^2\left(\left(2n^2+\phi\right)\alpha+2n^2\overline{c}\right)c_i \\ +\phi(2n+(n+1)\phi)^2\,\alpha\sum_{j=1}^{n-1}c_j \\ -n((2n+\phi)\alpha+n\phi\overline{c})\left(\begin{array}{l}\left(n\phi^2+\left(2n^2-n+2\right)\phi+2n^2\right)\alpha \\ +\phi(\phi-n(n-2))\overline{c}\end{array}\right) \end{array} \right). \tag{22}$$

We cannot sign the pass-through to the weighted average market price without assuming particular values for the parameters, but given that the own and cross pass-through rates are positive we would expect the pass-through rate to the weighted average price to be positive as well. The derivative of equation (22) with respect to n, which is not reported here, also cannot be signed. We can get a sense of the sign by examining the relative values of the weights used in calculating the weighted average price. The weights only vary by each firm's individual output and, therefore, by each firm's marginal cost. If the firm experiencing the cost change has especially low marginal cost relative to its rivals, then that firm will have significantly higher output and market share. As a consequence, we would expect the effect of competition on pass-through to weighted average price to be positive as it is for the own pass-through rate for sufficiently large n. If the firm experiencing the cost change is not the low cost firm in the industry, then we would expect the cross pass-through effects to dominate, and the effect of competition on pass-through to weighted average price would be negative.

4 Differentiated Bertrand competition with asymmetric firms

The results of the previous sections are based upon models of Cournot competition. We further analyze the robustness of these results by examining the case of asymmetric firms in a model of Bertrand competition. In this section we employ the Shubik (1980) model of Bertrand oligopoly in differentiated products also considered in Wang and Zhao (2007) to explore whether the relationship between the firm-specific cost pass-through rate and the number of competitors found in the Cournot model holds in a Bertrand model as well. The firm-specific demand functions are given by

$$q_i(\mathbf{p}) = \alpha - p_i - \phi(p_i - \frac{1}{n}\sum_{i=1}^{n} p_i), \tag{23}$$

where $\mathbf{p}$ is the vector of firm prices. All of the notation and parameter restrictions of the previous section are maintained, including the assumption that marginal costs are firm-specific.

The profit-maximization problem of firm i is

$$\max_{p_i} \pi_i = (p_i - c_i) q_i(\mathbf{p}) \tag{24}$$

for all $i = 1, \ldots, n$. The first-order condition for this problem is given by,

$$\frac{\partial \pi_i}{\partial p_i} = n\alpha + (n + (n-1)\phi) c_i + \phi \sum_{j}^{n-1} p_j - 2(n + (n-1)\phi) p_i = 0,\ j \neq i, \tag{25}$$

which yields the following reaction function for firm i,

$$p_i(\mathbf{p_j}) = \frac{n\alpha + (n + (n-1)\phi) c_i + \phi \sum_{j=1}^{n-1} p_j}{2(n + (n-1)\phi)}, \tag{26}$$

where $\mathbf{p_j}$ is the vector of prices excluding the price of firm i. The slope of the reaction function is positive, confirming that prices are strategic complements. Wang and Zhao (2007, p. 175) show that the Bertrand-Nash equilibrium price and quantity for firm i are given by,

$$p_i^{AB} = \frac{n(\alpha - \overline{c})}{2n + (n-1)\phi} + \frac{n(1+\phi)\overline{c} + (n + (n-1)\phi) c_i}{2n + (2n-1)\phi}, \tag{27}$$

and,

$$q_i^{AB} = (n + (n-1)\phi)\left(\frac{\alpha - \overline{c}}{2n + (n-1)\phi} - \frac{(1+\phi)(c_i - \overline{c})}{2n + (2n-1)\phi}\right), \tag{28}$$

respectively, where $\overline{c} = \frac{1}{n}\sum_{i=1}^{n} c_i$ and the superscript AB denotes Bertrand solutions with asymmetric costs.

4.1 Own pass-through rate

The rate at which firm i passes through a change in its marginal cost to its own price is,

$$\frac{\partial p_i^{AB}}{\partial c_i} = \frac{n(2+\phi)(n + (n-1)\phi)}{(2n + (2n-1)\phi)(2n + (n-1)\phi)} > 0. \tag{29}$$

Observation 2. *(i)* $\lim_{\phi \to 0} \left\{ \frac{\partial p_i^{AB}}{\partial c_i} \right\} = \frac{1}{2}$ *and (ii)* $\lim_{\phi \to \infty} \left\{ \frac{\partial p_i^{AB}}{\partial c_i} \right\} = \frac{n}{2n-1}$.

As in the case of Cournot competition, part (i) demonstrates that as outputs become unrelated, a firm's pass-through rate approaches the monopoly rate of 50 percent. A comparison of part (ii) from Observation 2 with part (ii) from Observation 1 demonstrates that when goods are homogeneous, the proportion of a marginal cost change that a firm passes through to its own price is greater when firms compete by choosing prices instead of quantities. This is true in the general case as well. A firm passes through more of a marginal cost change to its own price for any degree of production differentiation in the Bertrand model than in the Cournot model. Furthermore, part (ii) of Observation 2 suggests that when goods are homogeneous, a monopolist will pass through an entire marginal cost change and the own pass-through rate is never less than the pass-through rate when goods are unrelated.

Partially differentiating equation (29) with respect to $n,$ holding the degree of product differentiation fixed, determines the extent to which competition affects the incentive of a firm realizing a cost change to pass-through this change to its *own* price. This effect is given by

$$\frac{\partial^2 p_i^{AB}}{\partial c_i \partial n} = \frac{-\phi^2 (2+\phi)((n-1)^2 \phi + n(n-2))}{(2n+(2n-1)\phi)^2 (2n+(n-1)\phi)^2} < 0 \text{ for } n \geq 2. \tag{30}$$

Unlike in the Cournot model, pass-through monotonically decreases in n. The two influences competition has on pass-through observed in the Cournot model, however, are present in the Bertrand model as well. First, in the absence of product differentiation, the firm-specific own cost pass-through rate continues to be monotonically decreasing in n. To see this, differentiate the limit in part (ii) of Observation 2 and note that this derivative is always negative. Therefore, as in the Cournot model, as the number of firms in the market increases, an individual firm behaves less like a monopolist and more like a price-taker. That is, when goods are homogeneous, an individual firm has less of an incentive to pass through cost changes as the number of firms in the market increases.

Unlike in the Cournot model, however, the presence of product differentiation in the Bertrand model reinforces the effect of competition on pass-through when goods are homogenous. The slope of the reaction function for firm i from equation (26) is increasing in the differentiation parameter (ϕ). This means that as goods become more differentiated, the slope of the reaction function decreases. But in the Bertrand model, a decrease in the slope of the reaction function implies that each firm's response to changes in rival output is larger. That is, each firm behaves more like a price-taker in the presence of product differentiation. As in the Cournot model, the impact of product differentiation strengthens as the number of firms in the market increases as the slope of the reaction function in the presence of product differentiation is decreasing in n. Therefore, for a given level of product differentiation, each firm behaves more like a price taker as the number of firms in the market increases and passes through a smaller proportion of a cost change.

In summary, as in the Cournot model, there are two effects of competition on cost pass-through in the presence of product differentiation. The first effect, which is also present in the homogenous goods case, is that an increase in the number of firms leads each individual firm to act more like a price-taker and pass-through declines. The second effect is that product differentiation results in a flattening of the reaction function which, in Bertrand competition, leads each individual firm to act more like a price-taker when the number of firms increases. This second effect also causes pass-through to decrease. Therefore, in the Bertrand model, the two effects work in the same direction and the pass-through rate in monotonically decreasing in the number of firms.

4.2 Cross pass-through rate

[11] The key role of functional form in demand for determining pass-through rates (whether for industry-wide or firm-specific cost changes) is well-established in the literature. *See*, *e.g.*, Bulow and Pfleiderer (1981), Crooke *et al*. (1999), Anderson *et al*. (2001), and Froeb *et al.* (2005). Froeb *et al.* show that the simulated post-merger pass-through rate of a firm can be over seven times greater with isoelastic demand relative to linear demand, an effect that is attributable to the second derivatives (curvature) of the demand curves.

Recall that the cross pass-through effect measures how a change in the marginal cost of firm i will also affect the equilibrium prices charged by all other firms in the market. For an arbitrary firm j ($j \neq i$), the cross pass-through effect is,

$$\frac{\partial p_j^{AB}}{\partial c_i} = \frac{\phi(n+(n-1)\phi)}{(2n+(2n-1)\phi)(2n+(n-1)\phi)} > 0. \tag{31}$$

As in the Cournot model, both the own and cross pass-through rates are positive and the own pass-through rate exceeds the cross pass-through rate. The cross pass-through rate in the Cournot model, however, exceeds the cross pass-through rate in the Bertrand model.

In order to determine how the cross pass-through rate is affected by competition, equation (31) is partially differentiated with respect to n, holding the degree of product differentiation fixed. This effect is given by,

$$\frac{\partial^2 p_j^{AB}}{\partial c_i \partial n} = -\frac{\phi\begin{pmatrix}2(n-1)^2\phi^3 + (8n^2-12n+3)\phi^2 \\ +2n(5n-4)\phi + 4n^2\end{pmatrix}}{(2n+(2n-1)\phi)^2(2n+(n-1)\phi)^2} < 0. \tag{32}$$

4.3 Average pass-through rate

Using a simple average of equilibrium prices from the Bertrand model, we have that the pass-through of a change in the marginal cost of firm i to the average price is,

$$\frac{\partial \overline{P}^{AB}}{\partial c_i} = \frac{n+(2n-1)\phi}{n(2n+(2n-1)\phi)} > 0\,. \tag{33}$$

The rate of pass-through to average price in the Bertrand model exceeds that of the Cournot model despite the fact that cross pass-through is greater in the Cournot model. This indicates that the own pass-through rate in the Bertrand model is significantly larger than in the Cournot model. Partially differentiating equation (33) with respect to n reveals how the pass through to the average price varies with the level of competition. This differentiation yields,

$$\frac{\partial^2 \overline{P}^{AB}}{\partial c_i \partial n} = -\frac{(2n-1)^2\phi^2 + n(7n-4)\phi + 2n^2}{n^2(2n+(2n-1)\phi)^2} < 0. \tag{34}$$

The effect of competition on pass-through to average price is to be expected as competition has a negative effect on both own and cross pass-through rates.

Using an average of equilibrium prices from the Bertrand model weighted by market shares, we have that the pass through to the average price of a marginal cost change is,

$$\frac{\partial \overline{P}^{AB'}}{\partial c_i} = \frac{-1}{n^3(\alpha-\overline{c})^2(2n+(2n-1)\phi)^2(2n+(n-1)\phi)} * \begin{pmatrix} (1+\phi)(n+(n-1)\phi)(2n+(n-1)\phi)^2\left(c_i^2+\sum_{j=1}^{n-1}c_j^2\right) \\ +(1+\phi)(2n+(n-1)\phi)^2\begin{pmatrix}(2n^2+(2n^2-2n+1)\phi)\alpha \\ -2n(n+(n-1)\phi)\overline{c}\end{pmatrix}c_i \\ +\phi(1+\phi)(2n+(n-1)\phi)^2\alpha\sum_{j=1}^{n-1}c_j \\ -n\begin{pmatrix}(2n+(2n-1)\phi)\alpha \\ +\phi(n+(n-1)\phi)\overline{c}\end{pmatrix}\begin{pmatrix}\begin{pmatrix}(2n^2-n-1)\phi^2 \\ +(4n^2-5n+2)\phi+2n^2\end{pmatrix}\alpha \\ +\phi((n-1)^2\phi-n(n-2))\overline{c}\end{pmatrix} \end{pmatrix} \tag{35}$$

As in the Cournot model, we cannot sign the pass-through to the weighted average market price without assuming particular values for the parameters, but given that the own and cross pass-through rates are positive we would expect the pass-through rate to the weighted average price to be positive as well. The derivative of equation (22) with respect to n, which is not reported here, also cannot be signed. But as with the simple average we would expect the he effect of competition on pass-through to the weighted average price is to be negative as competition has a negative effect on both own and cross pass-through rates.

5 Concluding remarks

This paper examines the effect of competition, as measured by the number of firms in a given market, on firm-specific and average marginal cost pass-through rates in differentiated oligopolies with linear demand. For differentiated Cournot competition with symmetric costs, the firm-specific pass-through rate is increasing in the number of competitors. With asymmetric costs, the pass-through rate to own price is initially decreasing in n but rises after hitting a critical minimum value. In the case of differentiated Bertrand competition with asymmetric costs, the firm-specific pass-through rate is monotonically decreasing in n. In both the asymmetric Cournot and Bertrand models, the uniformly weighted market price is decreasing in the level of competition.

These results have implications for the evaluation of efficiency claims that arise during the review by antitrust authorities of proposed mergers involving differentiated products. Assuming that the various criteria for crediting any claimed cost efficiency can be satisfied (*e.g.*, that they are cognizable and of sufficient magnitude to offset any potential post-merger price increase), determining what the incentives are for the merged firm to pass-through any marginal cost reduction depend critically on the nature and scale of competition. In particular, the notion that a firm would have a stronger incentive to pass-through any cost savings absent the merger may not be valid for the reasons discussed above.

In some cases, such as the differentiated Cournot model with asymmetric costs, for a market comprised of relatively few firms a merger will tend to increase the incentive of the merged firm to pass-through any cost savings. However, for "sufficiently high" levels of competition the merger will tend to reduce the extent of pass-through. On the other hand, with the assumption of symmetric marginal costs a merger will always lead to a lower extent of firm-specific pass-through. Clearly, the results suggest that aside from issues of cognizance antitrust authorities must also consider how firms compete, the extent to which the products are substitutable across suppliers, the nature of the cost structure characterizing the industry, and the current and expected future levels of competition when evaluating efficiency claims put forward by firms proposing to merge.

The disparate results that obtain from the models considered in this analysis also suggest further theoretical and empirical work to test the predictions. The results obtained herein are derived solely in the context of linear demand models, which may be a strong restriction. Future work would benefit from considering different functional forms of demand and their implications for past-through incentives in the same general framework.[11] There is a paucity of research that has attempted to examine cost pass-through rates, and their relationship with competition, in a differentiated products context. Donghun and Cotterill (2008) is a notable exception.[12] An attempt to estimate changes in cost pass-through rates across industries that have experienced substantial episodes of entry and exit over time would be an interesting empirical extension that would be a welcome complement to the theoretical results derived here.[13]

Finally, it is important to consider what objective antitrust authorities have in mind when evaluating mergers. If, for example, such authorities are concerned with determining the relative effects of firm-specific cost changes on consumer vs. producer welfare (*e.g.*, treating "total welfare" as the relevant criteria on which to evaluate transactions),[14] then it might be of primary interest to consider the direct effect of pass-through rates on these constructs as opposed to some aggregate measure of average market price. In this regard, the important results of Wang and Zhou (2007), who show that, taking the number of firms as *given*, small firm-specific cost reductions always increase consumer surplus while potentially decreasing producer surplus (and thus total surplus as well) if output shares are beneath particular critical levels, are likely to be particularly relevant.

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[12] Donghun and Cotterill (2008) estimate cost pass-through rates in the processed cheese market and conclude that the extent of cost pass-through will be greater under Bertrand-Nash behavior relative to collusion. While this result is seemingly in contrast to those obtained from the differentiated Bertrand model considered here, there are a number of differences between it and the theoretical structure assumed by Donghun and Cotterill that likely explain the result. For instance, those authors consider the case of Bertrand competitors who compete in different (localized) markets where costs are assumed to be symmetric within markets but may differ across markets. Further, the authors assume that demand is of the logit or mixed logit variety, while the results obtained here are from linear demand specifications.

[13] It is not clear to what extent estimating *firm-specific* pass-through rates may be feasible as there may be little or no data that would allow the researcher to assess the higher-order properties of demand curves, which, as mentioned above, can be an important determinant of the magnitude of pass-through rates. Werden (1996) and Froeb and Werden (1998) offer an alternative approach that relies on estimating "compensating marginal cost reductions" (*i.e.*, the magnitude of the cost reduction that would keep post-merger prices from rising). This approach requires much less information that would be required for a direct estimate of pass-through rates (Yde and Vita 2006).

[14] Heyer (2006) and Carlton (2007) discuss various welfare standards employed by antitrust agencies while advocating for the adoption of a total welfare standard in evaluating proposed mergers.

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