Preferences over Consumption and Status

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Abstract

In many models of interdependent preferences the payoffs have not only personal value but also enter the social part of the utility. This duality creates a problem of distinguishing what influences the choice more: consumption or social concerns. To identify what drives the behavior it is necessary to have a model of preferences that allows for unambiguous separation of personal and social components. I use the preferences for consumption and status as an example to show that the axioms in the paper describe the preferences that have unique expected utility representation with consumption and social utilities entering additively. This makes it possible to experimentally determine the nature of social preferences without ad hoc assumptions and to estimate whether consumption or social value is more important in economic decisions.

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1 Introduction

There is an abundance of studies in experimental economics that investigate how the behavior of subjects is influenced by the presence of others (Andreoni, 1995; Ball, Eckel, Grossman, and Zame, 2001; Costa-Gomes and Zauner, 2001; Fehr and Gächter, 2000). To explain this behavior many models of utility which incorporates the characteristics and possessions of other participants were proposed (Fehr and Schmidt, 1999; Bolton and Ockenfels, 2000; Levine, 1998). All these models follow the same logic. An assumption is made about the nature of interdependence in preferences, for example, inequality aversion or altruism, and then some parametric functional form is proposed with an idea to find the estimates of the parameters from the experimental data. There are several problems with this approach. First, in many experiments different assumptions on the nature of interdependence generate the same behavior. For example, proposing non-zero amount in the Ultimatum game can be explained by both inequality aversion and altruism. Second, specific functions for personal and social utility are postulated even though under the assumption of interdependent preferences it is not clear how to disentangle the two. In order to do it the social utility should be completely eliminated which requires that subject is observed in complete solitude which does not seem plausible.[1] Third, the models above do not clearly address how uncertainty in the payoffs enters interdependent preferences. The necessity for such specification becomes clear when analyzing mixed strategies in games.

The important feature of most interdependent preferences models is that the possessions of the subject that enter personal utility also play role in social part of utility. For example, higher income is enjoyed not only because it brings more consumption but also because it increases the social rank. This duality creates a problem: when an experimenter observes that subject prefers more payoff to less, is it because the subject just likes it for consumption value or just because he values social rank above all? The truth is probably somewhere in between, in which case the question is: How big a role do consumption and rank play in the choice?

The goal of this paper is to build a model of interdependent preferences that addresses the issues stated above. The purpose is to: 1) understand what kind of information about the revealed choice is needed to uniquely separate consumption part of the preferences from the social part; 2) derive the system of axioms that would allow for such separation; and 3) create the framework that can be conveniently used to deal with uncertainty in payoffs of all subjects when interdependent preferences are present.

The model can be used to separate any type of social preferences from personal ones. To illustrate how it works I consider agents who care about consumption and social rank or status. This choice was made for several reasons. At first, many authors including Smith (1759), Veblen (1899) and Frank (1985) considered the desire for status as the primal incentive for economic behavior once the subsistence level of consumption is reached. From their perspective, the behavior below the subsistence level is driven mostly by the desire for more consumption whereas above this level status plays the primary role. The model in this paper can help with experimental testing of this hypothesis.

[1]This problem is like the Heisenberg Uncertainty Principle in physics. To estimate the pure consumption part of the utility, the subject should be observed choosing alone, which is impossible given that experimenter himself can be considered one of the others.

At second, there is growing evidence that envy and the resulting desire for status is an evolutionary adaptation that exists not only in humans but also in other primates (Cummins, 2005). Recent behavioral and fMRI studies confirm this. In the fMRI experiment, Rustichini and Vostroknutov (2006b) find activation in Orbitofrontal Cortex and Nucleus Accumbens[2] when subjects compare their winnings with winnings of others after playing a game of skill. In a related study (Rustichini and Vostroknutov, 2006a) we find that subjects lost nearly half of total winnings by subtracting money from those who won more than they did after the game of skill.

I use Anscombe and Aumann (1963) framework to construct preferences that are represented by unique[3] expected utility function that is given by

$$U(x_0, x_1, ..., x_T) = f(x_0) + \sum_{i \in T} \pi_i u(x_0, x_i)$$

on the certain outcomes. Here x_0 is a measure of possessions of agent 0, whose preferences are studied. $(x_i)_{i \in T}$ are the same measures for other agents in subgroup T of some set S of all possible others. Agent 0 cares about two things. First, x_0 has some consumption value. Second, agent 0 derives social value from x_0 by comparing it to what others have. The consumption part of the utility is represented by $f(x_0)$ whereas the status part is the weighted sum over others. The function $u(x_0, x)$ describes the specific way agent 0 cares about his position relative to one other person and $(\pi_i)_{i \in T}$ are the weights that represent the importance or "closeness" of each other individual to agent 0. In order to obtain uniqueness of this representation it is necessary that the preferences of agent 0 are observed in different subgroups of others.

Somewhat related construction can be found in Ok and Koçkesen (2000). In this paper authors study the consequences of different assumptions about negatively interde-

[2]These areas are known to be involved in the representation of reward (Ernst, Nelson, Jazbec, McClure, Monk, Leibenluft, Blair, and Pine, 2005; Rolls, 2004; Schultz, 2004).

[3]Up to a positive affine transformation

pendent preferences. In their model agents have preferences only over certain outcomes and all other agents have the same "closeness" weight. In comparison, I do not assume that the interdependence is necessarily negative, preferences are constructed over lotteries and the "closeness" weights can be different. Ok and Koçkesen (2000) approach, however, gives results that are hard to obtain in my framework. Their axioms allow for the utility representation to depend on various *aggregate* possessions of others, like average income. In my model this possibility depends on the observable subgroups of others.

This paper is organized as follows. In part 2 I use the status example to talk about some conceptual problems with the separability of social and personal parts of the preferences. In particular, it is argued why the additive functional form above is the appropriate way to model interdependency. Part 3 starts with the description of the framework and the issue of how to model uncertainty. In parts 4 and 5 the axioms and representation theorems are given for the two different uncertainty models. Part 6 concludes. Proofs of the theorems and lemmata can be found in parts 7 and 8.

2 Separability of Status and Consumption

People choose to buy some goods purely for consumption purposes, for example cheap food. Other things are chosen for purely status reasons, for example the choice between going through some highly unpleasant initiation ritual in a fraternity and not doing so. However, most goods are chosen for both reasons at once. A good example is cars. People like cars because they are convenient. However, it can hardly be denied that certain cars are produced and bought for status reasons as well (Hummer limousines).

When one wants to model the preferences involving status, it is, thus, important to have consumption and status parts intertwined. How should these parts be represented? Naturally, the consumption part of the preferences should be independent of anything related to others. It should depend only on the possessions of an agent himself. Let us then denote consumption utility by $U_c(x_0)$. Status preferences should depend on what others have *as well as on the possessions of the agent himself.* This is an important point if we want to be consistent with the possible evolutionary explanations of status. People care about others' possessions only *relative* to their own. This implies that status utility should be represented by a function $U_s(x_0, others)$ which is not additively separable.[4] Now we can write the utility as

$$U(x_0, others) = U_c(x_0) + U_s(x_0, others).$$

I think of consumption and status as completely independent reasons that drive the behavior. That is why U_c and U_s are summed.

Here is a problem. Choose any function $g(x_0)$ and redefine the utility as

$$U(x_0, others) = g(x_0) + \bar{U}_s(x_0, others)$$

where $\bar{U}_s(x_0, others) = U_s(x_0, others) + U_c(x_0) - g(x_0)$. It is clear that $\bar{U}_s$ is still not additively separable. But then any function g can be the utility for consumption![5] This shows that, given intuitive restrictions on U_s and U_c, it is impossible *in principle* to separate status from consumption in a unique way if we observe preferences with an unchanging group of people. One way out of this is to assume that we can observe the choice of the agent when he cares separately about different subgroups of others. By comparing observations from different subgroups it is possible to disentangle consumption from status in a unique way.

To illustrate the intuition consider college students who choose whether or not to go through a fraternity initiation ritual (for example, staying without sleep for three days).

[4]For otherwise we are back to the case of non-relative status.

[5]This trick can be performed even when consumption and status are not additive.

When this choice is made, the students are among their peers and fraternity members. If the only thing we observed was that the ritual is chosen by many students, we would not be able to understand if students do it because they *like* it or because they care about the position in the fraternity. However, it is clear that they would prefer not to stay without sleep for so long while spending time with their families on Christmas. This shows that their preferences over going through the initiation are purely status related: their choice depends on the subgroup. To the contrary, if the students prefer, say, double cheeseburgers over Big Macs, they will choose them regardless of the current subgroup they are in (fraternity or family): their choice does not depend on the subgroup which means that the choice is driven by consumption. These stories are on the extremes, however, the same principle can be applied to any choice.

To separate consumption and status we need to observe preferences in more than one subgroup. However, there is another problem. Consider the example with the fraternity, the family and the ritual. Suppose that these two subgroups are disjoint. Then there are two possible explanations of the behavior. First situation: the student cares a lot about the fraternity members, who respect him for having no sleep, cares very little about the family members,[6] and dislikes having no sleep. In this case he will choose to go through the initiation ritual while in the fraternity and not do it while at home. Second situation: the student cares very little about the fraternity members, cares a lot about the family members and loves having no sleep. In addition, having more sleep increases his status among the family members. The behavior in this case will be exactly the same as in the first situation, however, the explanations of the behavior are the opposites of each other. One way to avoid this ambiguity is to assume monotonicity in personal utility by postulating that the student does not like to have no sleep. However, I find such assumptions undesirable as they prevent us from investigating other possibilities. Another way is to try to observe the behavior in subgroups which *intersect*. In our

[6] Status-wise.

example, this could be the requirement that the student has a brother who is in the fraternity. So, the brother belongs to both subgroups. If such observations are possible, then no additional assumptions on the personal utility are needed.

To summarize: the model constructed below give rise to the utility over consumption and status that enter additively. This is necessary assumption if one thinks about consumption and status as independent reasons for behavior. In order to uniquely identify the utility it is necessary to observe the behavior in different subgroups of others. Moreover, these groups should not be disjoint.

3 The Model

The world consists of agent 0 and a finite set S of other agents with $|S| > 1$. We are interested in modeling the preferences of agent 0. Agent 0 and any other agent $i \in S$ have the measures of social status $x_0, x^i \in X$. The measures can be some aggregates that are calculated from the possessions or some qualities of the agents, depending on the social group of interest. For example, it can be the money value of all the goods that the agents have. The crucial assumption is that x_0 plays dual role of bringing not only consumption but also status utility.

Think of S as a "big" set of all people that agent 0 can possibly care about. This can be, for example, people of the same profession, like all economists, or any other big social group. It is realistic to assume that at any given time agent 0 does not take into consideration everybody in S, but only some subset $T \subseteq S$. Agent 0 knows statuses of people in T, but not those in $S \backslash T$. Also, everybody in T have information about the status of agent 0. It is possible that at some point agent 0 will be considering different subset of agents, say $R \subseteq S$. This can happen, for example, if agent 0 moves to a different city, which makes the information about his choices unavailable to the old subgroup and information about the old subgroup unavailable to agent 0.

For the model to be testable, it is necessary to have as few constraints on the subgroups observed as possible. For example, the model that requires that the choices in any subgroup should be observed can be hardly tested (it is impossible to see the behavior of an economist in any imaginable subgroup of other economists). As it was pointed out in the section 2, in order to have unique description of the behavior it is necessary to have intersecting subgroups. This suggests the following definition.

Definition 1. *Say that the collection of observed subgroups $\mathcal{C} \subseteq 2^S$ is **connected** if $\{\emptyset\} \notin \mathcal{C}$, $|\mathcal{C}| > 1$, $\cup\mathcal{C} = S$, and for all $T, R \in \mathcal{C}$ there exist $C_1, ..., C_k \in \mathcal{C}$ such that*

$$T \cap C_1 \neq \emptyset, \quad C_i \cap C_{i+1} \neq \emptyset, \quad C_k \cap R \neq \emptyset \qquad \textit{where } i = 1..k-1.$$

The first three requirements say that 1) we do not observe the behavior of agent 0 in complete solitude (the presence of observer himself makes it impossible); 2) there is more than one subgroup (for otherwise we cannot uniquely separate status and consumption); 3) subgroups cover all other agents (if not, then remove unobserved agents from S) and 4) any two subgroups can be "connected" by the sequence of intersecting subgroups (otherwise we would have "disconnected" collections of subgroups again making unique identification impossible, see section 2).

3.1 Models of Uncertainty

Throughout the paper I assume that agent 0 has some unique way of caring about his status relative to the status of any other person. The intuition is the following. Fix some $T \in \mathcal{C}$. Agent 0 encounters people from the subgroup T all the time. On meeting $i \in T$, agent 0 observes some outcome $(x_0, x) \in X^2$ (or lottery), which represents what agent 0 and the person i have. Agent 0 does not have a prior over the probabilities of meeting others in T, all he knows is that he will meet somebody. This situation can be conveniently modeled in the "horse lotteries" framework of Anscombe and Aumann

(1963). The question however remains: What is the dependency between the outcome that agent 0 gets and the identities of other agents? There are several intuitive ways of defining it.

Let $\Delta(X^2)$ be the set of all simple lotteries[7] over the pairs of statuses $(x_0, x) \in X^2$. Following Anscombe and Aumann (1963), let

$$\mathcal{H}_T := \{h : T \to \Delta(X^2)\}$$

be the set of all horse lotteries. Here $h_i = h(i)$ is the assignment of a lottery over agent 0's and agent i's statuses. Notice that agent 0's status, measured by the marginal lottery over x_0 derived from any h_i, depends on the identity of the other agent. This interpretation can have a meaning in the situations where agent 0's possessions and status somehow depend on the characteristics of the person he meets.

It is natural, however, to think of agent 0's outcome as independent of others. This simply means that agent 0 compares some possessions or qualities of his to the possessions or qualities of others, who cannot change the possessions of agent 0. Let $\mu_0(h_i)$ denote the marginal distribution of x_0 for any lottery in $\Delta(X^2)$. Consider the set of horse lotteries

$$\mathcal{F}_T := \{h \in \mathcal{H}_T \ : \ \forall i, j \in T \quad \mu_0(h_i) = \mu_0(h_j)\}. \tag{3.1}$$

Each element of $\mathcal{F}_T$ is an assignment of the distribution over x_0 to agent 0 and some distributions of statuses to all other agents. The distribution over x_0 does not depend on the identity of others. Agent 0 gets a lottery $\mu_0(h_i)$, which is the same for all $i \in T$.

[7] Lotteries with finite support.

4 The Space $\mathcal{H}_T$

4.1 Axioms

Choose any connected collection $\mathcal{C} \subseteq 2^S$ of subgroups (see Definition 1) and let

$$\mathcal{A} := \bigcup_{T \in \mathcal{C}} \mathcal{H}_T$$

be the set of all lotteries in $\mathcal{H}_T$ in all available subsets of other agents. Consider preference relation $\succcurlyeq$ over $\mathcal{A}$ with $\sim$ and $\succ$ being its symmetric and asymmetric parts.

For $T \in \mathcal{C}$ and $h \in \mathcal{H}_T$ write $h = (h_R, h_{-R})_T$ to emphasize the lotteries corresponding to agents in $R \subseteq T$. For $\ell \in \Delta(X^2)$ and $x_0, x \in X$ write ℓ_T or $(x_0, x)_T$ for the horse lottery that assigns lottery ℓ (or (x_0, x)) to agent 0 and any other agent in T.

Define a mixture of two horse lotteries $h, z \in \mathcal{H}_T$ with the same domain T to be

$$\alpha h + (1-\alpha) z = (\alpha h_i + (1-\alpha) z_i)_{i \in T} \in \mathcal{H}_T. \tag{4.1}$$

This turns $\mathcal{H}_T$ into a mixture set as defined in Herstein and Milnor (1953).[8]

Suppose that the following axioms hold:

A1 $\succcurlyeq$ is reflexive, transitive, total.[9] It is also non-trivial: for any $T \in \mathcal{C}$ there are $x_0, x, x' \in X$ such that

$$(x_0, x)_T \succ (x_0, x')_T$$

A2 *Independence.* For all $T \in \mathcal{C}$, all $p, q, r \in \mathcal{H}_T$ and all $\alpha \in (0, 1)$

$$p \succ q \;\Rightarrow\; \alpha p + (1-\alpha) r \succ \alpha q + (1-\alpha) r$$

[8]See Lemma 1 for the proof.

[9]Totality: $a \neq b \Rightarrow [a \succcurlyeq b \vee b \succcurlyeq a]$.

A3 *Continuity.* For all $T \in \mathcal{C}$, all $p, q, r \in \mathcal{H}_T$ there exist $\alpha, \beta \in (0,1)$

$$p \succ q \succ r \;\Rightarrow\; \alpha p + (1-\alpha) r \succ q \succ \beta p + (1-\beta) r$$

A4 *Anonymity.* For all $T \in \mathcal{C}$, $h \in \mathcal{H}_T$, $i, j \in T$, and $\ell, m \in \Delta(X^2)$

$$(\ell, h_{-i})_T \succcurlyeq (m, h_{-i})_T \iff (\ell, h_{-j})_T \succcurlyeq (m, h_{-j})_T$$

A5 *Unimportance.* For all $x_0 \in X$ there exists $x^*(x_0) \in X$ such that for all intersecting $T, R \in \mathcal{C}$, all $Q \in T \cap R$ and all $x \in X$

$$((x_0, x^*(x_0))_{T \setminus Q}, (x_0, x)_Q) \sim ((x_0, x^*(x_0))_{R \setminus Q}, (x_0, x)_Q)$$

A6 *Group Disparity.* There exist $S_1, S_2 \in \mathcal{C}$ such that for all $x_0, x, x' \in X$ with $(x_0, x)_{S_1} \sim (x_0, x)_{S_2}$

$$\begin{aligned} (x_0, x')_{S_1} \succ (x_0, x)_{S_1} \quad &\Rightarrow \quad (x_0, x')_{S_1} \succ (x_0, x')_{S_2} \quad \text{and} \\ (x_0, x)_{S_1} \succ (x_0, x')_{S_1} \quad &\Rightarrow \quad (x_0, x')_{S_2} \succ (x_0, x')_{S_1} \end{aligned}$$

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Axioms A1-A3 are standard necessary conditions for existence of an expected utility representation for each $T \in \mathcal{C}$.

Axiom A4 says that agent 0 does not care about the names of the other agents. Given any fixed outcomes for all agents but i, if agent 0 prefers lottery ℓ to m then he will also prefer ℓ to m in a situation when he faces agent j instead of i with all other outcomes still being fixed. Together with the axioms above, A4 implies that in each restriction $\succcurlyeq_T$ agent 0 treats all other agents in T in the same way. The only difference comes from the weights he attaches to different agents. These weights describe the relative "social" closeness of others to agent 0, whereas being in subgroup T incorporates the idea of "topological" closeness.

A4 puts restrictions on what can happen inside each subgroup T. The rest of the axioms deal with what happens between different subgroups. Without A5-A6 any two restrictions $\succcurlyeq_T$ and $\succcurlyeq_R$ are completely unrelated. It is desirable, however, that agent 0 choose somewhat consistently in different subgroups.

For each level of status x_0 of agent 0, axiom A5 asks for the existence of special status level $x^*(x_0)$ of any agent i, such that agent 0, when facing the outcome $(x_0, x^*(x_0))$, does not care about i and chooses as if i does not exist. For example, agent 0 might not care about others as long as they have no status or possessions at all ($x^*(x_0) = 0$), but he starts taking them into account once they have more than that.

Axiom A6 requires that there exist two subgroups $T, R \in \mathcal{C}$ to which agent 0 attaches different total social weight. In particular, if for some (x_0, x) it so happens that $(x_0, x)_T \sim (x_0, x)_R$, then if agent 0 prefers having $(x_0, x')_T$ to $(x_0, x)_T$ then he prefers it also over $(x_0, x')_R$. This means that subgroup T is preferable to subgroup R only because agent 0 likes having agents T around more than agents R. A counterexample might be the situation when all subgroups in $\mathcal{C}$ have the same number of others and the same social weights are attached to all of them. In this case we will have $h_T \sim h_R$ for all $h \in \mathcal{H}_T$ and any $T, R \in \mathcal{C}$, which leads to the indeterminacy of status component of the preferences. Axiom A6 is necessary when there are no two subgroups in $\mathcal{C}$ such that one is the strict subset of the other. If such subgroups exist, then A6 can be dropped without consequences.

There is no axiom that explicitly describes the "statusness" of the preferences. This is so because such an axiom is not required for the derivation of the utility. Therefore, this construction of preferences can be used for any types of interdependent preferences. The status axiom might look like this:

Status Monotonicity. For all $T \in \mathcal{C}$, $h \in \mathcal{H}_T$, $i \in T$ and $x_0, x, x' \in X$

$$x' \geq x \;\Rightarrow\; ((x_0, x), h_{-i})_T \succcurlyeq ((x_0, x'), h_{-i})_T$$

It states that if we choose any agent $i \in T$ and fix the horse lottery outcomes for all other agents, then agent 0 weakly prefers having less of x for agent i. This reflects the observation that people dislike others with bigger status.

4.2 Utility Representation

Two theorems in this section state that $\succcurlyeq$ satisfies A1-A6 if and only if there is a utility representation of the form (4.2). The proofs are provided for the space $\mathcal{H}_T$. Notice that the restriction of this utility to the subspace $\mathcal{F}_T$ gives the desired consumption-status additive utility

$$U[(x_0, x^i)_T] = f(x_0) + \sum_{i \in T} \pi_i u(x_0, x^i)$$

Definition 2. *Call any $u : X^2 \to \mathbb{R}$ a **status function** if it is not constant and there is a function $x^* : X \to X$ such that $u(x_0, x^*(x_0)) \equiv 0$.*

Theorem 1. *Suppose that $\succcurlyeq$ satisfies A1-A6. Then there are positive numbers $(\pi_i)_{i \in S}$, a function $f : X \to \mathbb{R}$ and a status function $u : X^2 \to \mathbb{R}$ such that for any $T \in \mathcal{C}$ and any $h, z \in \mathcal{H}_T$*

$$h \succcurlyeq z \iff U_T[h] \geq U_T[z]$$

where

$$U_T[h] = \sum_{i \in T} \pi_i \int_{X^2} \frac{f(x_0)}{\sigma_T} + u(x_0, x) dh_i(x_0, x) \tag{4.2}$$

and $\sigma_T = \sum_{i \in T} \pi_i$.

Moreover, the function $U : \mathcal{A} \to \mathbb{R}$ defined as $U[h] = U_T[h]$ for all $T \in \mathcal{C}$ and $h \in \mathcal{H}_T$, *is the utility representation for $\succcurlyeq$, unique up to a positive affine transformation.*

Proof. See Section 7.

Theorem 2. *Fix any $\mathcal{C}$ as described in the Definition 1 and suppose that $f : X \to \mathbb{R}$ is any function, $u : X^2 \to \mathbb{R}$ is a status function, and $(\pi_i)_{i\in S}$ are positive numbers. For $T \in \mathcal{C}$ and $h \in \mathcal{H}_T$ let*

$$U_T[h] = \sum_{i\in T} \pi_i \int_{X^2} \frac{f(x_0)}{\sigma_T} + u(x_0, x) dh_i(x_0, x)$$

Define $U : \mathcal{A} \to \mathbb{R}$ to be $U[h] = U_T[h]$. If there are two subgroups $S_1, S_2 \in \mathcal{C}$ such that

$$\sum_{i\in S_1} \pi_i \neq \sum_{i\in S_2} \pi_i \tag{4.3}$$

then the preference relation $\succcurlyeq$ over $\mathcal{A}$ generated by U satisfies A1-A6.

Proof. See Section 7.

5 The Space $\mathcal{F}_T$

In this section I restrict the space of possible horse lotteries to $\mathcal{F}_T$ (see the equation (3.1) above). In subgroup T in each horse lottery agent 0 now always has the same marginal distribution over x_0 for all other agents in T. This represents the idea that the possessions of agent 0 are independent of those of other agents. The space $\mathcal{F}_T$ is much smaller than $\mathcal{H}_T$ but is still a mixture set. It turns out that it is impossible to construct a weight-additive utility of the form (8.1) for subgroup T using only the axioms like A1-A4. This happens because Anonymity Axiom (A4), reformulated for $\mathcal{F}_T$, cannot compare arbitrary simple lotteries from $\Delta(X^2)$ keeping the rest of agents fixed, as this restricts the comparison to only the lotteries with the same marginals over x_0.

The solution to this problem lies in finding an additional assumption that would make it possible to "compare" any two lotteries in some way.

5.1 Axioms

I will abuse notation and use the same symbol $\succcurlyeq$ for the preferences over new space in this section. This does not create any notational issues as all the results in this paper refer to only one type of space at a time.

Choose any connected collection of subgroups $\mathcal{C} \subseteq 2^S$ and let

$$\mathcal{A}_F := \bigcup_{T \in \mathcal{C}} \mathcal{F}_T$$

be all horse lotteries in $\mathcal{F}_T$ in all available subgroups. Consider preference relation $\succcurlyeq$ over $\mathcal{A}_F$ with $\sim$ and $\succ$ being its symmetric and asymmetric parts.

For $T \in \mathcal{C}$ and $\mu \in \Delta(X)$ write $h^\mu \in \mathcal{F}_T$ to emphasize that the marginal distribution over agent 0 possessions is the same for all $i \in T$ and is equal to μ: $\forall i \in T \quad \mu_0(h_i^\mu) = \mu$. Also write $\ell^\mu \in \Delta(X^2)$ to emphasize that $\mu_0(\ell^\mu) = \mu$.

In order to express additional axiom I need the following definition.

Definition 3. *Say that the correspondence $\Theta : \Delta(X^2) \rightrightarrows \Delta(X^2)$ is* ***ubiquitous*** *if for all $\ell, m \in \Delta(X^2)$ and $\alpha \in (0,1)$*

U1) $\ell \in \Theta(\ell)$

U2) $\mu_0[\Theta(\ell)] = \Delta(X)$,[10]

U3) $m \in \Theta(\ell) \;\Rightarrow\; \ell \in \Theta(m)$

U4) $\forall \ell^* \in \Theta(\ell)\ \forall m^* \in \Theta(m) \quad \alpha\ell^* + (1-\alpha)m^* \in \Theta(\alpha\ell + (1-\alpha)m)$

Suppose that the following axioms hold.

[10]For $A \subseteq \Delta(X^2)$, $\mu_0[A]$ is $\{\mu_0(a) : a \in A\}$.

AF1, AF2, AF3 The same as A1-A3 only with $\mathcal{F}_T$ in place of $\mathcal{H}_T$

AF4 *Anonymity.* For all $T \in \mathcal{C}$, $\mu \in \Delta(X)$, $h^\mu \in \mathcal{F}_T$, $i, j \in T$, and $\ell^\mu, m^\mu \in \Delta(X^2)$

$$(\ell^\mu, h^\mu_{-i})_T \succcurlyeq (m^\mu, h^\mu_{-i})_T \iff (\ell^\mu, h^\mu_{-j})_T \succcurlyeq (m^\mu, h^\mu_{-j})_T$$

AF5, AF6 The same as A5 and A6

AF7 *Complete Substitutability.* For all $T \in \mathcal{C}$ there exists a ubiquitous correspondence Θ_T such that for all $h \in \mathcal{F}_T$, $\mu \in \Delta(X)$, and all $z^\mu \in \times_{i \in T} \Theta_T(h_i)$

$$h \sim z^\mu$$

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There are two axioms that carry different meaning in comparison with the previous set of axioms for $\mathcal{H}_T$. First, Anonymity Axiom AF4 now states that $\succcurlyeq$ is agent independent only inside each given marginal distribution over x_0, but not across them. This makes the standard subjective probability arguments fail to prove that weighted-additive utility exists. Second, Complete Substitutability Axiom AF7 says that for any horse lottery h^ν with fixed amount of possessions $\nu \in \Delta(X)$ for the agent 0 and any other amount of his possessions, $\mu \in \Delta(X)$, it is possible to find the levels of statuses of others z^μ such that agent 0 is indifferent between h^ν and z^μ. Moreover, given any two components $i, j \in T$ of the horse lotteries, agent 0 indifference is the same componentwise (since the same correspondence Θ_T works for any i). So, agent 0 is always ready to substitute some wealth for some status in the same way for any component i and any change in possessions.

5.2 Utility Representation

The goal is to show the existence of the expected utility

$$U_T[h^\mu] = \int_X f(x_0)d\mu(x_0) + \sum_{i\in T} \pi_i \int_{X^2} u(x_0,x)dh_i^\mu(x_0,x).$$

with unique f, u, and $(\pi_i)_{i\in S}$ for any $T \in \mathcal{C}$. The main challenge with the space $\mathcal{F}_T$ is to show the existence of the weighted-additive expected utility

$$U_T[h] = \sum_{i\in T} \pi_T^i \int_{X^2} \bar{u}_T(x_0,x)dh_i(x_0,x)$$

with function $\bar{u}_T$ unique for each T. Once this is done the rest of the proof is the same as for the $\mathcal{H}_T$ case.

As I mentioned above, it is impossible to use standard subjective probability results for this case. Therefore, another way of constructing the utility should be used. Let us define some new notation. For any $\mu \in \Delta(X)$ let

$$X_\mu := \{h \in \mathcal{F}_T : \mu_0(h_1) = \mu\}.$$

X_μ is the set of all horse lotteries with marginal distribution over x_0 being μ. Notice that $\mathcal{F}_T = \cup_{\mu\in\Delta(X)} X_\mu$.

Complete Substitutability leads to the following result: for any μ and ν in $\Delta(X)$ the preference $\succcurlyeq$ generates "the same" order on X_μ and X_ν once "sameness" is appropriately defined. To be more specific, fix $T \in \mathcal{C}$ and consider any $X_\mu \subseteq \mathcal{F}_T$ with the order generated by $\succcurlyeq$. Debreu (1983) defines *natural topology* on X_μ as any topology in which both upper and lower contour sets are open for all $h \in X_\mu$.[11] Call the coarsest natural topology the *order topology*. The subbasis of the order topology consists only of all upper and lower contour sets (Eilenberg, 1941). Endow X_μ with the order topology τ_μ and

[11] Upper contour set for $h \in X_\mu$ is $\{z \in X_\mu : z \succ h\}$. Lower contour set is $\{z \in X_\mu : h \succ z\}$.

find the quotient space $X_\mu/\sim$. Let $\tau^\star_\mu$ be the quotient topology on $X_\mu/\sim$ derived from τ_μ. Now, we say that two orders on X_μ and X_ν are equivalent if it is possible to find bijective order and topology preserving map between spaces $(X_\mu/\sim, \tau^\star_\mu)$ and $(X_\nu/\sim, \tau^\star_\nu)$.

Theorem 3. *Suppose AF7 holds. Then for any $T \in \mathcal{C}$ and any $\mu, \nu \in \Delta(X)$ there exists an order preserving homeomorphism $(X_\mu/\sim, \tau^\star_\mu) \mapsto (X_\nu/\sim, \tau^\star_\nu)$.*

Proof. See Section 7.

The Theorem essentially says that given AF7 the preference $\succcurlyeq$ generates the same order on all $X_\mu \in \mathcal{F}_T$ regardless of μ. So we can choose one μ, say some degenerate distribution $x_0 \in X$, continue constructing utility on X_{x_0} and then extend the utility function to all other elements of $\mathcal{F}_T$.

Theorem 4. *Suppose AF1-AF4 and AF7 hold. Then for any $T \in \mathcal{C}$ there are positive numbers $(\pi^i_T)_{i\in T}$ and a function $\bar{u}_T : X^2 \to \mathbb{R}$ such that*

$$U_T[h] = \sum_{i\in T} \pi^i_T \int_{X^2} \bar{u}_T(x_0, x) dh_i(x_0, x)$$

is a utility representation for $\succcurlyeq_T$ on $\mathcal{F}_T$ unique up to a positive affine transformation.

Proof. See Section 7.

Now we are ready to give main representation theorems of this section. They are different from those for the case $\mathcal{H}_T$ as Complete Substitutability (AF7) imposes restrictions on the shape of the admittable utility functions.

Definition 4. *Say that a function $\bar{u} : X^2 \to \mathbb{R}$ is* ***equispread*** *if for all $x_0, y_0 \in X$* $\sup\{\bar{u}(x_0, X)\} = \sup\{\bar{u}(y_0, X)\}$ *and* $\inf\{\bar{u}(x_0, X)\} = \inf\{\bar{u}(y_0, X)\}$.

Theorem 5. *Suppose that $\succcurlyeq$ satisfies AF1-AF7. Then there are positive numbers $(\pi_i)_{i\in S}$, a function $f : X \to \mathbb{R}$ and a status function $u : X^2 \to \mathbb{R}$ such that for any $T \in \mathcal{C}$ and any $h, z \in \mathcal{F}_T$*

$$h \succcurlyeq z \iff U_T[h] \geq U_T[z]$$

where

$$U_T[h^\mu] = \int_X f(x_0)d\mu(x_0) + \sum_{i\in T} \pi_i \int_{X^2} u(x_0, x)dh_i^\mu(x_0, x) \tag{5.1}$$

and for all $T \in \mathcal{C}$ the function $f(x_0)/\sigma_T + u(x_0, x)$ is equispread where $\sigma_T = \sum_{i\in T} \pi_i$.

Moreover, the function $U : \mathcal{A} \to \mathbb{R}$ defined as $U[h] = U_T[h]$ for all $T \in \mathcal{C}$ and $h \in \mathfrak{F}_T$, is the expected utility representation for $\succcurlyeq$, unique up to a positive affine transformation.

Proof. See Section 7.

Theorem 6. *Fix any $\mathcal{C}$ as described in the Definition 1 and suppose that $f : X \to \mathbb{R}$ is any function, $u : X^2 \to \mathbb{R}$ is a status function, and $(\pi_i)_{i\in S}$ are positive numbers. For $T \in \mathcal{C}$ and $h^\mu \in \mathfrak{F}_T$ let*

$$U_T[h^\mu] = \int_X f(x_0)d\mu(x_0) + \sum_{i\in T} \pi_i \int_{X^2} u(x_0, x)dh_i^\mu(x_0, x).$$

Define $U : \mathcal{A} \to \mathbb{R}$ to be $U[h] = U_T[h]$. If all functions $f(x_0)/\sigma_T + u(x_0, x)$ are equispread and there are two subgroups $S_1, S_2 \in \mathcal{C}$ such that

$$\sum_{i\in S_1} \pi_i \neq \sum_{i\in S_2} \pi_i \tag{5.2}$$

then the preference relation $\succcurlyeq$ over $\mathcal{A}$ generated by U satisfies AF1-AF7.

Proof. The proof of Theorem 2 applies exactly with replacing all occurrences of axioms A# with AF#. Axiom AF7 follows from the Lemma 8. ■

6 Conclusion

The model of preferences constructed in this paper shows that it is possible to separate consumption preferences from social preferences. In order to do so one needs to observe the choices people make in different subgroups. This creates the possibility to experimentally find out what social preferences are without making ad hoc assumptions.

The next step in this research is to design an experiment or find the data which would help to understand the relative importance of social and personal components of preferences. The first step is to check whether the axioms proposed in this paper hold. The experiment in question is not beyond one's imagination. It is not hard to observe how behavior changes in different subgroups. To give the simplest example, think of how people behave differently being at work among colleagues or at home among relatives. It should be relatively easy to test the axioms experimentally by making various pieces of information on the outcomes available to different subjects during one treatment.

7 Proofs

Proof of Theorem 1. The idea of the proof is to 1) establish the existence of the weighted-additive utilities U_T for all $T \in \mathcal{C}$; 2) show that a unique function f can be constructed in a way that is consistent with each of the utility functions; 3) rescale the now redefined utility functions to show that all the utilities can have the specific form described in the Theorem.

Fix any $T \in \mathcal{C}$. Then A1-A3 and the fact that $\mathcal{H}_T$ is a mixture set imply existence of the expected utility $U_T : \mathcal{H}_T \to \mathbb{R}$, unique up to a positive affine transformation (Theorem 8.4 of Fishburn (1970)). Lemma 2 shows that U_T on the certain outcomes has the weighted-additive form

$$U_T[(x_0^i, x^i)_T] = \sum_{i \in T} \pi_T^i \bar{u}_T(x_0^i, x^i).$$

Now by Lemma 5 the assertion of the Theorem is true. ■

Proof of Theorem 2. A1 holds since u is a status function, which is assumed to be not constant. For any $T \in \mathcal{C}$ A2-A3 hold by the "only if" part of the Theorem 8.4 of Fishburn (1970). Additivity of U_T immediately implies A4. The assumption that u is a status function implies that for each x_0 there is $x^*(x_0)$ such that $u(x_0, x^*(x_0)) = 0$, so A5 follows. It is left to show that A6 holds. Without loss of generality assume that

$$\sum_{i \in S_1} \pi_i > \sum_{i \in S_2} \pi_i$$

where S_1 and S_2 are as in the description of this Theorem. Suppose for some $(x_0, x) \in X^2$ we have $U[(x_0, x)_{S_1}] = U[(x_0, x)_{S_2}]$. Then

$$\sum_{i \in S_1} \pi_i u(x_0, x) = \sum_{i \in S_2} \pi_i u(x_0, x)$$

can happen only when $u(x_0, x) = 0$ since we assume (4.3). Now, take any x' such that $U[(x_0, x')_{S_1}] > U[(x_0, x)_{S_1}]$. This implies that $u(x_0, x') > u(x_0, x) = 0$. But then

$$\sum_{i \in S_1} \pi_i u(x_0, x') > \sum_{i \in S_2} \pi_i u(x_0, x')$$

and therefore $U[(x_0, x')_{S_1}] > U[(x_0, x')_{S_2}]$. This is the first part of A6. Second part is proved by the exactly same argument. ■

Proof of Theorem 3. Consider a mapping $\Psi_{\mu\nu} : (X_\mu/\sim, \tau^\star_\mu) \to (X_\nu/\sim, \tau^\star_\nu)$ that takes equivalence class $[h]$ to equivalence class $[z]$ whenever $h \sim z$. Notice that $(X_\mu/\sim, \succ)$ and $(X_\nu/\sim, \succ)$ are linearly ordered sets. By Lemma 6, $\Psi_{\mu\nu}$ is the order preserving bijection, so $\Psi_{\mu\nu} = \Psi^{-1}_{\nu\mu}$. By Lemma 7, $\tau^\star_\mu$ and $\tau^\star_\nu$ are the order topologies, so $\Psi^{-1}_{\nu\mu}$ takes any upper (lower) contour set of $X_\mu/\sim$, which is open, to upper (lower) contour set of $X_\nu/\sim$, which is also open. Therefore, since $\Psi^{-1}_{\nu\mu}$ is a bijection, any open set in the basis of $\tau^\star_\mu$ goes to the open set in the basis of $\tau^\star_\nu$. This immediately implies that any open set in $\tau^\star_\mu$ is mapped by $\Psi^{-1}_{\nu\mu}$ to an open set in $\tau^\star_\nu$. Thus $\Psi_{\nu\mu} = \Psi^{-1}_{\mu\nu}$ is continuous.[12] By the same argument $\Psi_{\mu\nu} = \Psi^{-1}_{\nu\mu}$ is continuous. Thus $\Psi_{\mu\nu}$ is an order preserving homeomorphism.

■

Proof of Theorem 4. Fix any $T \in \mathcal{C}$ and $x_0 \in X$. AF1-AF4 imply that there is weighted-additive expected utility on X_{x_0} of the form

$$U_T[h^{x_0}] = \sum_{i \in T} \pi^i_T \bar{u}_T[h^{x_0}_i] \tag{7.1}$$

where (π^i_T) are positive weights and $\bar{u}_T[h^{x_0}_i]$ stands for an expectation of $\bar{u}_T$ with respect to $h^{x_0}_i$ (Theorem 13.2 of Fishburn (1970)).

[12]This argument is based on the fact that the image of a finite intersection (arbitrary union) of any sets is equal to the finite intersection (arbitrary union) of images under a bijective map.

Now, for any $\mu \in \Delta(X)$ extend U_T to X_μ by setting

$$U_T[h^\mu] = \sum_{i \in T} \pi_T^i \bar{u}_T[h_i^\mu] \tag{7.2}$$

where $\bar{u}_T[h_i^\mu] = \bar{u}_T[z_i^{x_0}]$ for any $z_i^{x_0} \in \Theta_T(h_i^\mu)$,which is non-empty by the ubiquitousness of Θ_T. The choice of $\bar{u}_T[h_i^\mu]$ here is unambiguous since AF7 guarantees that $\bar{u}_T[z_i^{x_0}] = \bar{u}_T[p_i^{x_0}]$ for all $p_i^{x_0} \in \Theta_T(z_i^{x_0})$.[13] Now, by Theorem 3 the order on X_{x_0} is homeomorphic to the order on X_μ for any $\mu \in \Delta(X)$, thus this procedure defines U_T for all elements of X_μ and since μ was arbitrary, for all elements of $\mathfrak{F}_T$.

The utility $U_T : \mathfrak{F}_T \to \mathbb{R}$ constructed in this way conforms with AF4 and AF7. The only thing left to check is that U_T indeed represents $\succcurlyeq_T$ and that it has expected utility form. These properties follow from the fact that U_T satisfies them by construction when restricted to X_{x_0}.

Indeed, fix any $h, z \in \mathfrak{F}_T$ with $h \succcurlyeq z$. Let $h^{x_0} \in \times_{i \in T} \Theta_T(h_i)$ and $z^{x_0} \in \times_{i \in T} \Theta_T(z_i)$ be horse lotteries in X_{x_0}. Then

$$h \succcurlyeq z \underset{AF7}{\iff} h^{x_0} \succcurlyeq z^{x_0} \underset{(7.1)}{\iff} U_T[h^{x_0}] \geq U_T[z^{x_0}] \underset{(7.2)}{\iff} U_T[h] \geq U_T[z]$$

The first equivalence works by definition of AF7 and Θ_T; second, because U_T restricted to X_{x_0} is a utility representation (7.1); third, by construction (7.2). Therefore U_T on $\mathfrak{F}_T$ is a utility function for $\succcurlyeq_T$.

It is still left to show that U_T on $\mathfrak{F}_T$ is *expected* utility function. To do that it is enough to show that for all $\ell, m \in \Delta(X^2)$ and $\alpha \in (0, 1)$

$$\bar{u}_T[\alpha\ell + (1 - \alpha)m] = \alpha\bar{u}_T[\ell] + (1 - \alpha)\bar{u}_T[m].$$

[13] In the definition of AF7 fix z^{x_0}, let $\mu = x_0$ and apply Θ_T to only ith component leaving the rest unchanged (which we can always do since $\ell \in \Theta_T(\ell)$ for any ℓ).

Fix any $(\alpha\ell + (1-\alpha)m)^{x_0} \in \Theta_T(\alpha\ell + (1-\alpha)m)$, $\ell^{x_0} \in \Theta_T(\ell)$, and $m^{x_0} \in \Theta_T(m)$. By property U4 of Definition 3

$$\alpha\ell^{x_0} + (1-\alpha)m^{x_0} \in \Theta_T(\alpha\ell + (1-\alpha)m)$$

which implies

$$\bar{u}_T[\alpha\ell^{x_0} + (1-\alpha)m^{x_0}] = \bar{u}_T[(\alpha\ell + (1-\alpha)m)^{x_0}]$$

But u_T on X_{x_0} does have expected utility property by construction (7.1). Therefore,

$$\alpha\bar{u}_T[\ell^{x_0}] + (1-\alpha)\bar{u}_T[m^{x_0}] = \bar{u}_T[(\alpha\ell + (1-\alpha)m)^{x_0}]$$

which by definition of utility (7.2) implies

$$\alpha\bar{u}_T[\ell] + (1-\alpha)\bar{u}_T[m] = \bar{u}_T[\alpha\ell + (1-\alpha)m].$$

This finishes the proof. ■

Proof of Theorem 5. By Theorem 4 $\succcurlyeq_T$ for each $T \in \mathcal{C}$ has the utility representation

$$U_T[h] = \sum_{i \in T} \pi_T^i \int_{X^2} \bar{u}_T(x_0, x) dh_i(x_0, x).$$

By Lemma 5 we can rewrite these utilities as

$$U_T[h] = \sum_{i \in T} \pi_i \int_{X^2} \frac{f(x_0)}{\sigma_T} + u(x_0, x) dh_i(x_0, x)$$

for all $T \in \mathcal{C}$ such that U as defined above is unique up to a positive affine transformation. By the nature of $\mathcal{F}_T$ (each horse lottery has the same marginal distributions on x_0) the above becomes

$$U_T[h^\mu] = \int_X f(x_0)d\mu(x_0) + \sum_{i\in T} \pi_i \int_{X^2} u(x_0,x)dh_i^\mu(x_0,x).$$

By Lemma 8 all functions $f(x_0)/\sigma_T + u(x_0,x)$ are equispread. ■

8 Lemmata

Lemma 1. *For all $T \in \mathcal{C}$ the sets $\mathcal{H}_T$ and $\mathcal{F}_T$ as defined in part 3.1 are mixture sets.*

Proof. Fix any T. For any two horse lotteries h and z from either $\mathcal{H}_T$ or $\mathcal{F}_T$ let

$$\alpha h + (1-\alpha)z = (\alpha h_i + (1-\alpha)z_i)_{i\in T}$$

for any $\alpha \in [0,1]$. First it is necessary to show that the mixture of h and z stays in the set from which they came from. Then we need to show that the definition of mixture satisfies conditions (1-3) of Herstein and Milnor (1953).

For $h, z \in \mathcal{H}_T$ the mixture is trivially in $\mathcal{H}_T$. For $\mathcal{F}_T$ this is true since the marginal distributions of T mixtures of pairs of lotteries with the same marginals on x_0 are still the same, thus by definition mixture is in $\mathcal{F}_T$. The conditions (1-3) of Herstein and Milnor (1953) for mixture set are trivially satisfied since we are mixing independent probability distributions. ■

Lemma 2. *Suppose that $\succcurlyeq$ satisfies A1-A4. Then for any $T \in \mathcal{C}$, the restricted preference relation $\succcurlyeq_T$, defined over $\mathcal{H}_T$, has expected utility representation of the form*

$$U_T[(x_0^i, x^i)_T] = \sum_{i\in T} \pi_T^i \bar{u}_T(x_0^i, x^i)$$

where $\pi_T^i > 0$ for all $i \in T$. Moreover, U_T is unique up to a positive affine transformation.

Proof. Fix any $T \subseteq \mathcal{C}$ and consider the restriction $\succcurlyeq_T$. A1-A3 still hold for $\succcurlyeq_T$.

Now notice that by condition (4.1) $\mathcal{H}_T$ is a mixture set. A1-A3 are exactly the requirements of Theorem 13.1 of Fishburn (1970).[14] The first condition of Theorem 13.2 follows from the non-triviality of $\succsim_T$ and A4. The second condition of the theorem is exactly A4. Therefore, expected utility representation U_T obtains. ■

Lemma 3. *Suppose A5-A6 (AF5-AF6) hold. Then for all $C_1, C_2 \in \mathcal{C}$, all $x_0 \in X$ and all $x^*(x_0) \in X$ satisfying A5 (AF5)*

$$(x_0, x^*(x_0))_{C_1} \sim (x_0, x^*(x_0))_{C_2}$$

Proof. Let us first assume that $C_1 \cap C_2 \neq \emptyset$. Then by putting $x = x^*(x_0)$ in the definition of A5 (AF5) we get the desired

$$(x_0, x^*(x_0))_{C_1} \sim (x_0, x^*(x_0))_{C_2}$$

Now, $\mathcal{C}$ is the connected collection of subsets (see Definition 1). Therefore, any two disjoint subgroups can be connected by the sequence of intersecting ones. Therefore, by transitivity of $\sim$ the result above holds for all $C_1, C_2 \in \mathcal{C}$. ■

Lemma 4. *Suppose A5-A6 (AF5-AF6) hold. Then for all $x_0 \in X$ there exists a non-empty set*

$$X^*_{x_0} = \{x \in X : \forall T, R \in \mathcal{C} \ \ (x_0, x)_T \sim (x_0, x)_R\}.$$

Moreover, for all $x_0 \in X$, $x, y \in X^*_{x_0}$ and all $T \in \mathcal{C}$

$$(x_0, x)_T \sim (x_0, y)_T$$

Proof. A5 (AF5) says that for all $x_0 \in X$ there is $x^*(x_0)$, which by Lemma 3 satisfies the condition for being a member of $X^*_{x_0}$. Therefore, we have shown that non-empty

[14] To verify that $\succ$ is a weak order see Proposition 2.4 of Kreps (1988).

$X^*_{x_0}$ exists for all x_0.

Now suppose that the second condition of the Lemma does not hold. In other words, there is x_0 and $x, y \in X^*_{x_0}$ such that for some $T \in \mathcal{C}$

$$(x_0, x)_T \succ (x_0, y)_T$$

Let $S_1, S_2 \in \mathcal{C}$ be the two subgroups satisfying A6 (AF6). Then, by definition of y

$$(x_0, y)_{S_1} \sim (x_0, y)_{S_2}$$

Moreover, the definitions of x and y and the assumption give

$$(x_0, x)_{S_1} \sim (x_0, x)_T \succ (x_0, y)_T \sim (x_0, y)_{S_1}$$

The two conditions above and A6 (AF6) imply that

$$(x_0, x)_{S_1} \succ (x_0, x)_{S_2}$$

which contradicts the fact that x is an element of $X^*_{x_0}$. ■

Lemma 5. *For the space $\mathcal{H}_T$ ($\mathcal{F}_T$) suppose that $\succcurlyeq_T$ admits an expected utility representation that on the certain outcomes is given by*

$$U_T[(x_0^i, x^i)_T] = \sum_{i \in T} \pi_T^i \bar{u}_T(x_0^i, x^i) \qquad \left(U_T[(x_0, x^i)_T] = \sum_{i \in T} \pi_T^i \bar{u}_T(x_0, x^i) \right) \tag{8.1}$$

for each $T \in \mathcal{C}$ and A5-A6 (AF5-AF6) hold for $\succcurlyeq$. Then the statement of Theorem 1 (Theorem 5) is true.

Proof.

1. Lemma 4 says that for any x_0 there exists a non-empty set $X^*_{x_0}$ which consists of

all the points $x \in X^*_{x_0}$ such that for all $T, R \in \mathcal{C}$ $(x_0, x)_T \sim (x_0, x)_R$. Moreover, for all $x, y \in X^*_{x_0}$ and all $T \in \mathcal{C}$ we have $(x_0, x)_T \sim (x_0, y)_T$.[15] The pairs in $X^*_{x_0}$ are perfect candidates for the representation of the pure consumption value of x_0: agent 0 does not care to which subgroup he belongs when choosing among pairs from sets $X^*_{x_0}$. Let

$$X^* := \bigcup_{x_0 \in X} \{(x_0, x) \in X^2 \ : \ x \in X^*_{x_0}\}.$$

Notice that the choice between any two pairs $(x_0, x), (y_0, y) \in X^*$ depends only on x_0 and y_0 and nothing else. In terms of the utilities defined on the previous step, we have

$$U_T[(x_0, x)_T] = U_T[(x_0, y)_T] = U_R[(x_0, x)_R]$$

for all $(x_0, x), (x_0, y) \in X^*$, all $T, R \in \mathcal{C}$.

Define $f : X \to \mathbb{R}$ to be

$$f(x_0) := U_T[(x_0, x)_T]$$

for any $x \in X^*_{x_0}$ and any $T \in \mathcal{C}$ and rewrite U_T as

$$U_T[(x_0^i, x^i)_T] = \sum_{i \in T} \pi_T^i \Big(\frac{f(x_0^i)}{\sigma_T} + u_T(x_0^i, x^i) \Big) \tag{8.2}$$

where $u_T(x_0, x) = \bar{u}_T(x_0, x) - f(x_0)/\sigma_T$, $\sigma_T = \sum_{i \in T} \pi_T^i$ and $u_T(x_0, x) = 0$ for all $(x_0, x) \in X^*$.[16]

2. Fix $i \in S$ and consider all subgroups $C_1, ..., C_k \in \mathcal{C}$ to which i belongs. Then A5

[15] Lemma 4 makes sure that there are no other points outside $X^*_{x_0}$ that satisfy these conditions. Thus, $X^*_{x_0}$ is the biggest "unique" set with these properties.

[16] Notice that if all x_0^i are the same, then U_T becomes $f(x_0) + \sum_{i \in T} \pi_T^i u_T(x_0, x^i)$.

(AF5) with $Q = \{i\}$ implies that for all $(x_0, x) \in X^2$

$$((x_0, x)_i, (x_0, x^*(x_0))_{-i})_{C_1} \sim ... \sim ((x_0, x)_i, (x_0, x^*(x_0))_{-i})_{C_k}$$

Therefore,

$$f(x_0) + \pi^i_{C_1} u_{C_1}(x_0, x) = ... = f(x_0) + \pi^i_{C_k} u_{C_k}(x_0, x)$$

implying

$$\pi^i_{C_1} u_{C_1}(x_0, x) = \pi^i_{C_2} u_{C_2}(x_0, x) = ... = \pi^i_{C_k} u_{C_k}(x_0, x) \tag{8.3}$$

for all $(x_0, x) \in X^2$.

Now fix some $T, R \in \mathcal{C}$ such that there are $i, j \in T \cap R$. Then by the above

$$\begin{aligned} \pi^i_T u_T(x_0, x) &= \pi^i_R u_R(x_0, x) & (8.4) \\ \pi^j_T u_T(x_0, x) &= \pi^j_R u_R(x_0, x) & (8.5) \end{aligned}$$

By A1 (AF1) the preferences $\succcurlyeq$ are non-trivial on all subgroups. So there is $(y_0, y) \in X^2$ such that $u_T(y_0, y) \neq 0$. The connectedness of $\mathcal{C}$, positiveness of (π^i_T) and (8.3) implies then that $u_C(y_0, y) \neq 0$ for all $C \in \mathcal{C}$.

The equations (8.4-8.5) hold for (y_0, y). So, by dividing them we obtain

$$\frac{\pi^i_T}{\pi^i_R} = \frac{\pi^j_T}{\pi^j_R} =: L_{T,R} \tag{8.6}$$

for all intersecting T, R and all $i, j \in T \cap R$. If $T \cap R$ has only one element i, then set

$$\frac{\pi^i_T}{\pi^i_R} =: L_{T,R}$$

Notice as well that for $L > 0$[17]

$$\sum_{i\in T} \pi_T^i \Big(\frac{f(x_0^i)}{\sigma_T} + u_T(x_0^i, x^i) \Big) = \sum_{i\in T} \frac{\pi_T^i}{L} \Big(\frac{f(x_0^i)}{\sigma_T/L} + L u_T(x_0^i, x^i) \Big) \tag{8.7}$$

For intersecting T, R we can rescale all the weights (π_T^i) and u_T using $L_{T,R}$ in place of L in (8.7). This makes the weights for all $i \in T \cap R$ equal in both subgroups. Also rescaled u_T becomes equal to u_R. Denote this rescaled U_T by $L_{T,R}(U_T)$.

3. $\mathcal{C}$ can be represented as a graph. Let all elements of $\mathcal{C}$ be nodes. Two nodes C_1, C_2 are connected by an edge if $C_1 \cap C_2 \neq \emptyset$. By definition of $\mathcal{C}$ the resulting finite graph G is connected.[18]

   For each node $C \in G$ there corresponds a collection of weights (π_C^i) and a status function u_C. Call $\langle G, \{(\pi_C^i), u_C\}_{C\in G}\rangle$ a graph structure.

   Choose any nodes $(T, (\pi_T^i), u_T)$ and $(R, (\pi_R^i), u_R)$ connected by an edge. Rescale U_T to $L_{T,R}(U_T)$ and contract the two nodes into one node $(T \cup R, (\pi_{T\cup R}^i), u_{T\cup R})$, where $u_{T\cup R} = u_R$.

   This turns the structure $\langle G, \{(\pi_C^i), u_C\}_{C\in G}\rangle$ into the structure

$$\langle G_1, \{((\pi_{T\cup R}^i), u_{T\cup R}), ((\pi_C^i), u_C)\}_{C\in G\setminus\{T,R\}}\rangle$$

   where G_1 is a minor of G obtained by the contraction of an edge between T and R.

   Continue contracting edges until there are none left. The sequence of graph structures thus obtained is finite and its last element $\langle G_N, (\pi_S^i), u_S\rangle$ has one node and no edges. By construction, for any agent $i \in S$ the weight π_S^i is the same in all subgroups i belongs to. The status function u_S is also same in all subgroups. Let

[17] For the case of the space $\mathcal{F}_T$ remove indexes i from all occurences of x_0^i.

[18] See the definitions of all graph theoretic terms in Diestel (2000).

$\pi_i = \pi_S^i$ and $u = u_S$, then we obtain desired utility $U : \mathcal{A} \to \mathbb{R}$ defined on each $\mathcal{H}_T$ or $\mathcal{F}_T$ as

$$U_T[h] = \sum_{i \in T} \pi_i \int_{X^2} \frac{f(x_0)}{\sigma_T} + u(x_0, x) dh_i(x_0, x)$$

Each U_T is unique up to a positive affine transformation. In addition, all functions U_T are restricted by A5 (AF5) to have the same weights and status functions. Thus the whole U is unique up to a positive affine transformation. ■

Lemma 6. *Suppose AF7 holds. Then for any $T \in \mathcal{C}$, all $\mu, \nu \in \Delta(X)$ and all $h \in X_\mu$ there is $z \in X_\nu$ such that $h \sim_T z$.*

Proof. By AF7 the correspondence Θ_T is ubiquitous (see Definition 3). This means that for any $h \in \mathcal{F}_T$ and any $i \in T$ the set $\Theta_T(h_i) \subseteq \Delta(X^2)$ contains some lotteries whose marginal distribution over x_0 is any ν. Therefore, the set

$$\{z \in \times_{i \in T} \Theta_T(h_i) : \forall i \in T \quad \mu_0(\Theta_T(h_i)) = \nu\} \subseteq X_\nu$$

is not empty. By AF7 any element of this set is indifferent to h. ■

Lemma 7. *For any $T \in \mathcal{C}$ and any $\mu \in \Delta(X)$ the quotient topology $\tau_\mu^\star$ is the order topology on $X_\mu/\sim$.*

Proof. The quotient map $[\cdot] : (X_\mu, \tau_\mu) \to (X_\mu/\sim, \tau_\mu^\star)$ takes open sets to open. Therefore, $[\cdot]$ maps any upper contour set to the open upper contour set in $X_\mu/\sim$. The same is true for lower contour sets. But then these contour sets form the subbasis of the order topology on $X_\mu/\sim$. Therefore, the quotient topology $\tau_\mu^\star$ is no coarser than the order topology on $X_\mu/\sim$. Suppose that it is actually strictly finer. Then, there exists an open set $A \in \tau_\mu^\star$ that is not a finite intersection and/or arbitrary union of the images of upper and lower contour sets under $[\cdot]$. But $[\cdot]$ is continuous, thus there should exist an open inverse image of A in X_μ, which is also not a finite intersection and/or arbitrary union of upper and lower contour sets. This contradicts the assumption that τ_μ is the order topology. ■

Lemma 8. *Suppose that for $T \in \mathcal{C}$ there is an expected utility function that represents $\succcurlyeq_T$ and on the certain outcomes it is given by $U_T[(x_0, x^i)_T] = \sum_{i \in T} \pi_T^i \bar{u}_T(x_0, x^i)$. Then $\bar{u}_T$ is equispread (Definition 4) if and only if AF7 holds.*

Proof. [AF7 $\Rightarrow$ equispread]. Suppose that AF7 holds for $\succcurlyeq_T$ on $T \in \mathcal{C}$ and there is expected utility U_T with $\bar{u}_T$ being not equispread. Then there are $x_0, y_0 \in X$ such that either $\sup\{\bar{u}_T(x_0, X)\} > \sup\{\bar{u}_T(y_0, X)\}$ or $\inf\{\bar{u}_T(x_0, X)\} < \inf\{\bar{u}_T(y_0, X)\}$. In case of supremum take any $x \in X$ with $\bar{u}_T(x_0, x) > \sup\{\bar{u}_T(y_0, X)\}$. It is clear that for no $\ell^{y_0} \in \Delta(X^2)$ is it true that $\bar{u}_T[\ell^{y_0}] = \bar{u}_T(x_0, x)$. This contradicts AF7 and the fact that U_T is utility function as Lemma 6 states that there should exist ℓ^{y_0} with $\bar{u}_T[\ell^{y_0}] = \bar{u}_T(x_0, x)$. The case of infimum is treated similarly.

[Equispread $\Rightarrow$ AF7]. Suppose $\bar{u}_T$ is equispread and fix a real number r such that $\inf\{\bar{u}_T(x_0, X)\} \leq r \leq \sup\{\bar{u}_T(x_0, X)\}$.[19] Take any x_0 and find some $x, y \in X$ such that $\bar{u}_T(x_0, x) \leq r \leq \bar{u}_T(x_0, y)$. Then it is possible to find $\alpha \in [0, 1]$ such that $\alpha \bar{u}_T(x_0, x) + (1 - \alpha)\bar{u}_T(x_0, y) = r$. Thus we found a lottery which has utility r. This procedure can be performed for arbitrary x_0 and any r. It is also possible to find a lottery with utility r for any marginal distribution $[\alpha_k, x_0^k] \in \Delta(X)$ on x_0 by just taking the lotteries with marginal distributions x_0^k which have utility r and combining them with appropriate probabilities α_k. Thus for any r between infimum and supremum and any $\mu \in \Delta(X)$ we can find a lottery h^μ with $\bar{u}_T[h^\mu] = r$. Construct Θ_T by mapping each point in $\Delta(X^2)$ to the subset of $\Delta(X^2)$ of points with the same utility. It is trivial to check that Θ_T is ubiquitous and that AF7 holds. ■

[19] Any x_0 here is fine by assumption of equispreadness.

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