Pricing, Advertising, and Market Structure with Frictions

Gomis-Porqueras, Pedro and Julien, Benoit and Wang, Chengsi

Research School of Economics, Australian National University, School of Economics CAMA, University of New SouthWales, School of Economics, University of New SouthWales

16 October 2010

Pricing, Advertising, and Market Structure with Frictions*

Pedro Gomis-Porqueras†
Research School of Economics
Australian National University

Benoit Julien‡
School of Economics & CAMA
University of New South Wales

Chengsi Wang§
School of Economics
University of New South Wales

October 16, 2010

Abstract

This paper develops a model of pricing and advertising in a matching environment with capacity constrained sellers and uncoordinated buyers. Sellers' search intensity attracts buyers only probabilistically through costly informative advertisement. Equilibrium prices and profit maximizing advertising levels are derived and their properties analyzed. The model generates an inverted U-shape relationship between individual advertisement and market tightness which is robust to alternative advertising technologies. The well known empirical fact in the IO literature reflects the trade-off between price and market tightness-matching effects. Finally, in this environment we can alleviate the discontinuity problem, allowing for unique symmetric equilibrium price to be derived.

JEL Classification: L11, L13, M37.
Keywords: Directed searching, Advertising, Pricing, Market structure

1 Introduction

Recent research has focused on models where trade is neither Walrasian nor random matching. In a Walrasian world, communication between buyers and sellers is costless. In contrast, random matching models have very large costs mainly due to a pairwise matching restriction. Less severe informational restrictions are considered in models with directed search. In these environments, sellers face capacity constraints and buyers are uncoordinated when selecting a particular seller, but sellers communicate with buyers to influence their choice of trading partner. Thus, a seller might be selected by more than one buyer, in which case they may face rationing. Central to this literature is the information flow from sellers to buyers.[1]

*We would like to thank Carlos Pimienta, Bill Schworm and seminar participants at the workshops at UNSW and University of Essex for helpful comments and suggestions. The usual disclaimer applies.

†Email: pedro.gomis@anu.edu.au

‡Email: benoit.julien@unsw.edu.au

§Email: chengsi.wang@unsw.edu.au

[1] See Peters(1991), Burdett, Shi and Wright(2001) and Julien, Kennes, and King (2000,2005), among others, for more on this literature.

The idea of introducing informative advertisements sent by sellers has a long tradition in economics. In his seminal work, Butters(1977) considers an exogenous market structure with many buyers and sellers who set the price for a homogeneous good. These buyers are transient, buy one unit each in a single purchase and the only possible information flow between sellers and buyers is through costly advertising. More recently, Stegemen (1991 extends Butters (1977) by considering heterogeneous reservation values for buyers. Stahl (1994), on the other hand, considers finite number of buyers and sellers, allowing also for downward sloping individual demand curves while considering a general advertising technology. All these models exhibit the standard non-existence of pure price equilibrium strategies. This is the case because sellers' payoffs are discontinuous when prices are equal. A common assumption in these models is that their environments do not consider sellers with capacity constraints, thus reducing the value of the signal. Moreover, these papers do not deliver a non linear relationship between market structure and advertising.

A substantial body of empirical work has been devoted to study the regularities between the information sent by sellers, as measured by advertising, and market tightness, the ratio of buyers to sellers. Schmalensee (1989) reports that among consumer goods industries, advertising intensity increases with concentration at low levels of concentration; the relation may vanish or change sign at high levels of concentration. For instance, Buxton, Davies and Lyon(1984) find an inverted U-shape pattern between market structure and advertising in the U.S. manufacturing for the period 1963 to 1977 industries. Similarly, Uri(1987) using data for 1977 for the United States for 301 four-digit SIC industries and advertising-sales data from Business Survey (1978) show a similar non-linear pattern. For the Korean manufacturing industries, Lee (2002) also finds an inverted U-shaped relationship between concentration and advertising intensity.[2]

The equilibrium existence problems found in the literature and the empirical regularities between advertisement and market structure motivate our work. The objective of this paper is to provide a formal framework that explores how the incentives of sending probabilistic signals from sellers to buyers changes as different market structures are considered. In particular, we characterize the equilibrium relationships between advertising, pricing and market tightness in an environment with coordination frictions, capacity constraints and probabilistic signals. The paper closest in spirit to our work is that of Lester(2010) where he allows an exogenous number of informed and uninformed buyers to trade when

[2] For an excellent survey of the advertising literature, we refer to Bagwell(2007).

signals are costless and there is no role for advertising. He finds that having more informed buyers can lead to a decrease in prices, have no effect at all, or even lead to an increase in prices in finite markets.

In our framework sellers have access to an advertisement technology. This technology allows buyers to know that sellers exist. Each add, which we refer to as a signal, contains seller's location, capacity and price information. These costly signals sent by sellers are only observed probabilistically by buyers.[3] Without the advertising technology and buyer search, sellers would not be able to sell and the market would not exist as emphasized by Butters(1977). In other words, when sellers send signals only a fraction of buyers observe all prices with a certain probability. Thus returns on advertising are probabilistic.

In this paper we find that higher prices make costly advertising more worthwhile for sellers. The magnitude of this benefit critically depends on the market structure and the type of advertising technology. Environments with substantially fewer sellers than buyers (concentrated markets) have a larger price effect on advertising than markets with substantially more sellers than buyers (competitive markets). Moreover, the probability with which a seller is visited by one buyer is lower whenever there are lots of them. Since numerous sellers imply a very low probability that a buyer will select a particular seller, this reduces the returns to advertising. On the other hand, when there are too few sellers, the probability with which a buyer will select a particular seller is higher, so that each seller is more likely to be visited by several buyers. Hence, no need to advertise much. Thus we are able to show that the profit maximizing advertisement level is *inverted U-shape* in market tightness, and reaches an interior maximum whenever there is a sufficiently small marginal cost of advertisement. Otherwise, for large marginal costs, there is no advertisement. This finding is robust to alternative advertising technologies.

The rest of the paper is organized as follows: Section 2 presents the model and characterizes the buyers and sellers' choices while considering Butters(1977) advertising technology and characterizes the relationship between advertising and market structure. Section 3 considers a more general advertising technology and characterizes its implications for equilibrium advertising levels. Finally, Section 4 concludes.

[3]This paper then proposes a model of directly informative advertising.

2 Butters' Advertising Technology

In this paper we examine market outcomes when firms produce a product, such as an agricultural commodity, which is sufficiently homogenous that advertising expenditures by one firm enhances the demand facing all firms.[4] On the supply side there is a well-established tradition in economics of treating advertising as a quantity of homogeneous messages, which can be purchased by the firm in a competitive market at marginal cost. The assumption that the advertising industry is competitive is reasonable in so far as there are many advertising agencies competing for clients.

The market considered here consists of a large number of M identical sellers each carrying only one unit to sell and a large number of N identical potential buyers with unitary demand. We further assume that the number of buyers and sellers are common knowledge.

In this environment direct communication between buyers and sellers does not exist. Each seller i must advertise in order to be known in the market. Sellers send signals, denoted by a_i, that provide the seller's location, capacity and price information. Each buyer observes all the sellers' signals with a certain probability γ. [5]

Each buyer can only purchase from a particular seller if and only if she has observed the signal from that seller. This assumption is consistent with the costly search for uninformed buyers and the purchase of durable goods. To simplify exposition we further assume that the product characteristics are common knowledge; thus focusing on search goods and directly informative advertising. The sequence of events in our environment are as follows:

1. Each seller i decides to advertise or not. If so, each seller i chooses *simultaneously* advertising $a_i \in \mathbb{R}_+$ and a price $p_i \in [0,1]$ to maximize expected profits taking as given the choices of other sellers.

2. Buyers observe the entire vector of prices $\mathbf{p} = \{p_1, p_2, \cdots\cdots, p_N\}$ with probability γ.

3. Upon observing all prices, each buyer selects one and only one particular seller with which to trade.

[4]The potential free-rider problem that arises in such commodity markets has led to the creation of a large number of government-sponsored generic advertising campaigns financed by compulsory contributions from industry members. For example, in the U.S. we observed these two campaigns: "Beef: what is for dinner?" and "Got milk". See Norman, Pepall and Richards (2008).

[5]Tremblay and Tremblay(1995) find that the firm's own advertising has a positive and significant effect on its output price. In addition, rivals' advertising has a significant positive though relatively small effect on another firm's price.

4. Matches are formed, trade occurs at the advertised prices, and payoffs unfold.

This sequence of events defines a game between buyers and sellers. Throughout the rest of the paper, we focus on symmetric equilibria so that all sellers choose the same price and advertisement, and buyers select over sellers using mixed strategies in equilibrium.

From the buyer's perspective, in order to select seller i with positive probability, the buyer must assess the probability that the other buyers select seller i. This particular structure implies a conflicted interest between sellers and buyers. Buyers attempt to minimize competition for any given seller, but sellers try to maximize it. In other words, buyers select over sellers trading off a price and a probability of trade as in all standard directed search environments.

2.1 Advertising, Pricing and Payoffs

A strategy for a seller i is a combination of prices and signals $(p_i, a_i) \in \mathbb{R}^2_+$. Each seller, then, chooses its price and signal simultaneously to maximize expected profits, taking as given other sellers' choices, and *expected* buyers' behavior.[6]

In order to be active on the market each seller must send a signal a_i, and incur a fixed cost $F \in [0,1]$, which we interpret as the cost of setting up a location.[7] Each seller then faces a variable cost of advertising denoted by $c(a_i)$ with constant marginal cost; i.e., $c'(a_i) > 0$ and $c''(a_i) = 0$. The marginal cost of advertising $c(a_i) = \beta a_i$, where β is a positive constant.

In the spirit of Butters (1977), we consider a probabilistic advertising technology where with probability $\gamma(A) = 1 - e^{-\frac{A}{N}}$ all buyers observe all prices, where $A = \sum_{i=1}^{M} a_i$.[8] We note that Butters advertising aims at getting more consumers informed of all locations and prices. Thus it does not have the business-stealing effect whereby an increase in firm's advertising will decrease consumers' awareness of their competitors products. Moreover, Butters' technology does not consider the effect of competing for attention or crowding-out effect. We quote Anderson (2005):

"Advertising Attributes may enhance consumer valuation of characteristics. However, when a firm advertises a characteristic of its product, this also raises the perceived quality of other rival products that have the characteristic. There are thus positive externality in advertising (a "Raise-all-boats" effect)..."

[6] For alternative timings we refer to McAfee(1994) who considers a two-stage model where firms choose advertising first and prices in the second stage and Robert and Stahl(1993) who consider simultaneous choices.

[7] Advertisements are costly to set up, or more generally a marketing campaign needs preparation.

[8] Note that $\gamma(A)$ is increasing in M and decreasing in N.

Although products are homogeneous, one can interpret our environment in which advertising is about prices only instead of characteristics. Hence, when one seller advertises its price, this also raises the existence of other rival prices and locations, as well as the more traditional competition effect that comes from advertising a different price.

The expected profit for seller i can be summarized as follows:

$$\Pi_i\left(\mathbf{p},\mathbf{a};M,N\right)=\gamma(A)\,p_i q_i(\mathbf{p})-c(a_i)-F, \tag{1}$$

where $\gamma(A)q_i(\mathbf{p})$ represents the probability of sale (the probability to be selected by at least one buyer) and $\mathbf{p}$ is the relevant price vector for the entire economy.[9]

The strategy for a buyer is a selection over sellers from which they have observed prices. Assuming that each buyer extracts utility from consuming the good, which we normalize to 1, the buyer surplus from selecting seller i is $(1-p_i)$. Since each seller has only one unit of the good, the rationing rule is such that when several buyers select the same seller, each one gets the good with equal probability. Buyers' selection strategies and this rationing rule translate into a probability Λ_i^j for buyer j to be served by seller i. Notice that this probability depends on the number of other buyers also selecting seller i. The probability of facing other buyers at seller i only depends on the vector of prices, $\mathbf{p}$, since buyer j knows all other buyers are informed when herself is informed. Hence, buyer j's expected utility from selecting seller i is given by:

$$U_i^j(\mathbf{p})=(1-p_i)\,\Lambda_i^j(\mathbf{p}). \tag{2}$$

To derive the probabilities $\Lambda_i^j(\mathbf{p})$ and $q_i(\mathbf{p})$, we need to evaluate the probability that any other particular buyer k selects firm i, which we denote by $\theta_i^k(\mathbf{p})$. The general characterization of $\theta_i^k(\mathbf{p})$ represents the mixed strategies of buyers. Buyers and sellers use this probability to evaluate the payoffs associated with their strategies. In particular, a buyer evaluates the probability that other buyers select seller i, and hence, the probability of being served. A seller, on the other hand, tries to determine the probability of sale; that is, the probability to be selected by at least one buyer. Finally, since we focus on symmetric equilibrium with all sellers setting the same price and signals while buyers select over sellers with identical probabilities. Under these assumptions, for any arbitrary vector of prices and signals set by sellers,

[9] Although we could introduce costly buyers search by allowing them to sample more communication media or simply searching for prices as in Robert and Stahl(1993), we focus on search intensity on the sellers' side. Also, all results derived in this paper are robust to introduction of outside option for buyers and sellers which would occur with probability $(1-\gamma(A))$.

we have $\theta_i^j(\mathbf{p}) = \theta_i(\mathbf{p})$, $\forall j$. Taking other buyers' selection strategy as given, a typical buyer maximizes her expected payoff. Thus we must have that:

$$\theta_i(\mathbf{p}) > 0 \tag{3}$$

$$\Rightarrow U_i(\mathbf{p}) = \max_k U_k(\mathbf{P}). \tag{4}$$

The first order conditions yields the mixed-strategy equilibrium selection for a buyer, so that she is indifferent between selecting any two observed sellers. In other words, $\theta_i(\mathbf{p})$ satisfies:

$$U_i^j(\mathbf{p}) = U_\ell^j(\mathbf{p}) \quad \forall i, \ell \in M. \tag{5}$$

If a particular buyer is informed, her probability of getting served by seller i, Λ_i, must satisfy the following local market clearing condition:

$$N\theta_i(\mathbf{p})\Lambda_i(\mathbf{p}) = 1 - \left(1 - \theta_i(\mathbf{p})\right)^N. \tag{6}$$

Notice that the left hand side of equation (6) is the expected number of buyers who visit seller i and are served, while the right hand side is the expected number of sales. Therefore, the probability of being served by seller i is given by:

$$\Lambda_i(\mathbf{p}) = \frac{1 - \left(1 - \theta_i(\mathbf{p})\right)^N}{N\theta_i(\mathbf{p})},$$

which is the probability that at least one buyer selects seller i, $\left(1 - (1 - \theta_i(\mathbf{p})\right)^N$, divided by the expected number of buyers visiting seller i $N\theta_i(\mathbf{p})$.

Suppose that all sellers have the same advertising level a, and that one seller deviates setting a price $\hat{p}$ when all other sellers set price p. From (4) it has to be the case that $\hat{\theta} + (M-1)\theta = 1$, where $\hat{\theta}$ represents the probability of buyers selecting the deviating seller. Thus, the probability with which a buyer selects a non-deviating sellers is:

$$\theta = \frac{1 - \hat{\theta}}{(M-1)}.$$

As a result, the expected utility from selecting a non deviating seller is given by:

$$U(\hat{p},\mathbf{p}_{-1}) = (1-p)\frac{\left[1-\left(1-\frac{1-\hat{\theta}}{(M-1)}\right)^N\right]}{N\left(\frac{1-\hat{\theta}}{M-1}\right)}, \tag{7}$$

while the expected utility from selecting the deviating seller is given by:

$$\hat{U}(\hat{p},\mathbf{p}_{-1}) = (1-\hat{p})\frac{\left[1-(1-\hat{\theta})^N\right]}{N\hat{\theta}}. \tag{8}$$

Equating (7) and (8) yield an implicit solution for $\hat{\theta}$ which is given by:

$$\frac{(1-p)}{(1-\hat{p})} = \frac{(1-\hat{\theta})\left[1-(1-\hat{\theta})^N\right]}{(M-1)\hat{\theta}\left[1-\left(1-(\frac{1-\hat{\theta}}{M-1})\right)^N\right]} \equiv \Psi(\hat{\theta}). \tag{9}$$

Fortunately, the explicit solution is not needed to derive the equilibrium.[10] All we need are the conditions under which there exists a unique mixed-strategy equilibrium selection between selecting a deviating and a non deviating seller.

Equation (9) describes the price wedge between a deviating and a non-deviating seller. It is easy to check that this price wedge, $\Psi(\hat{\theta})$, is strictly decreasing in $\hat{\theta}$ and has the following properties:

$$\lim_{\hat{\theta}\to 0}\Psi(\hat{\theta}) = \frac{N/(M-1)}{1-\left(1-\left(\frac{1}{M-1}\right)\right)^N} \geq 1,$$
$$\lim_{\hat{\theta}\to 1}\Psi(\hat{\theta}) = \frac{1}{N}.$$

It is possible to have $\Psi(1)<(1-p)/(1-\hat{p})<\Psi(0)$. Thus, for an appropriate $(1-p)/(1-\hat{p})$, there is a unique $\hat{\theta} = \hat{\theta}(\hat{p},p)\in(0,1)$ that makes buyers indifferent between sellers posting p and $\hat{p}$.[11] The equilibrium selection strategy is then given by:

$$\hat{\theta} = \begin{cases} 0 & \text{if } \frac{(1-p)}{(1-\hat{p})} > \Psi(0), \\ 1 & \text{if } \frac{(1-p)}{(1-\hat{p})} < \Psi(1), \\ \hat{\theta}(\hat{p},p) & \text{otherwise.} \end{cases} \tag{10}$$

[10] This property of equilibrium $\hat{\theta}$ is particular to the use of a posted price mechanism. When using auctions or ex post bidding, these probabilities have closed form solutions, see Julien, Kennes, and King (2000,2005) for more details.

[11] See Burdett, Shi and Wright(2001) for a similar demonstration of this condition.

When all sellers set the same price and signal, the unique symmetric mixed-strategy equilibrium for buyers is $\hat{\theta} = \theta^* = \theta = \frac{1}{M}$. Using the selection strategy defined by (10), when all buyers select a particular seller i with the same (but arbitrary) probability θ_i, the probability that at least one buyer selects seller i is given by:

$$q_i(\mathbf{p}) = 1 - \left[1 - \theta_i(\mathbf{p})\right]^N .$$

2.2 Equilibrium Prices and Advertising

In this section we analyze the equilibrium properties of this economy with uncoordinated buyers, capacity constrained sellers and probabilistic signals.

Definition *An equilibrium for this environment is:*

1. *A vector of prices and signals* $(\mathbf{p};\mathbf{a}) = (p_1,...,p_M; a_1,...,a_M)$ *maximizing sellers' expected profits as best response to each other, and taking as given the buyers' mixed strategies.*
2. *Symmetric mixed strategies for each buyer* $\theta_i, i = 1,...,M,$ *maximizing expected utility taking as given* (p,a).
3. *The best responses of buyers and sellers are consistent with each other.*

Assume that all sellers but one set a price p and signal a, while a deviating seller sets price $\hat{p}$ and signal $\widehat{a}$. The objective of a deviating seller is given by:

$$\max_{\hat{a},\hat{p}} \left\{\gamma(\hat{A})\ \hat{p}\widehat{q}(\hat{p},\mathbf{p}_{-1}) - c(\widehat{a}) - F\right\}, \tag{11}$$

where $q\left(\hat{p},\mathbf{p}_{-1}\right) = 1-(1-\hat{\theta})^N$ is the probability of sale for the deviating seller. To simplify notation we define $\hat{\theta} = \theta\left(\hat{p},\mathbf{p}_{-1}\right)$. The profit maximizing deviation satisfies the following first-order conditions:

$$\gamma'(\hat{A})\,\hat{p}\left[1-(1-\hat{\theta})^N\right] - c'(\widehat{a}) = 0, \tag{12}$$

$$\gamma(\hat{A})\left[1-(1-\hat{\theta})^N + \hat{p}\ N(1-\hat{\theta})^{N-1}\frac{\partial\hat{\theta}}{\partial\hat{p}}\right] = 0. \tag{13}$$

where $\gamma(\hat{A}) = 1 - e^{-\frac{\hat{A}}{N}}$.

Assuming that $\hat{\theta} \in (0,1)$, differentiating (9) with respect to $\hat{p}$ and imposing the symmetric equilibrium conditions $\hat{p} = p$, $\widehat{a} = a$ for all sellers, and $\hat{\theta} = \theta = 1/M$ for all buyers, we obtain the price change effect on buyers selection probability which is given by:

$$\frac{\partial \hat{\theta}}{\partial \hat{p}} = \frac{1}{(1-\hat{p})\Psi'(\hat{\theta})}, \tag{14}$$

or

$$\frac{\partial \hat{\theta}}{\partial \hat{p}} = \frac{-(M-1)^2 \left[1 - \left(\frac{M-1}{M}\right)^N\right]}{M^2 (1-p) \left[M - 1 - \left(\frac{M-1}{M}\right)^N (M+N-1)\right]} < 0, \tag{15}$$

which implies that a higher price leads to lower probability for a seller to be selected.

From equation (12), the individual equilibrium advertisement is given by:

$$A^*(M,N) = N \ln \left(\frac{1 - \frac{N}{M}\left(1 - \frac{1}{M}\right)^{N-1} - \left(1 - \frac{1}{M}\right)^N}{\beta N} \right). \tag{16}$$

Substituting (15) into (13), yields the following equilibrium price:

$$p^*(M,N) = \frac{M - M\left(\frac{M+N-1}{M-1}\right)\left(\frac{M-1}{M}\right)^N}{M - \left(\frac{M^2-M+N}{M-1}\right)\left(\frac{M-1}{M}\right)^N}, \tag{17}$$

with $p_M < 0$ and $p_N > 0$ representing the partial derivatives with respect to M and N respectively.

Let $\phi = N/M$ represent the market tightness (the buyers to sellers ratio). In large markets, we fix $M = \bar{M}$ and assume $\bar{M}$ and N are large enough so that $(1 - \frac{1}{M})^N \approx e^{\phi}$, the equilibrium advertising and price can be approximated by:[12]

$$A^*(\phi) = \bar{M}\phi \ln\left(\frac{1 - e^{-\phi} - \phi e^{-\phi}}{\beta \bar{M} \phi}\right), \tag{18}$$

$$p^*(\phi) = 1 - \frac{\phi}{e^{\phi} - 1}. \tag{19}$$

Proposition 1 *In arbitrarily large (but finite) market with small marginal advertising cost β, an advertis-*

[12] The limit price is the same as in Burdett, Shi and Wright(2001) and advertising does not affect the equilibrium price for a given market tightness.

ing equilibrium exists, i.e., there exists a pair $(\bar{\phi}, \underline{\phi})$ *with* $0 < \underline{\phi} < \bar{\phi} < \infty$ *such that* $A^*(\underline{\phi}) = A^*(\overline{\phi}) = 0$, *and* $\forall \phi \in (\underline{\phi}, \overline{\phi})$, $A^*(\phi) > 0$. *Moreover, the industry advertising level is inverted U-shape in the market tightness* ϕ.

Proof. See the Appendix. ■

Note that if advertising is costless, that is $\beta = 0$, then $a^*(\phi) \to \infty$ with $\gamma(A) \to 1$ and this is the standard directed search model with price posting (see Burdett, Shi and Wright(2001)). Otherwise, for competitive markets, small ϕ, an increase in market structure yields a higher probability to face only *one* or no buyer by a seller relative to the probability to face many. But the probability to make a sale is driven by the probability to face at least one buyer. As ϕ increases, it is worthwhile for a seller to increase advertising to maximize expected profits. For large ϕ, an increase in market structure yields a lower probability of facing only *one* relative to the probability of facing several buyers, there is eventually less need for advertising and advertising intensity decreases in ϕ.

When competition is less severe, larger ϕ, the probability to not make a sale as the number of seller decreases becomes important. This is the case because there are already a relatively small number of sellers. Since the decrease in probability of not making a sale becomes less important, and the probability to be visited by several buyers is higher, there is less need for each seller to advertise and lower advertising is observed. Thus, there is a trade-off between the incentives to increase advertising driven by equilibrium price increase, and the incentive to advertise less since there are less sellers and the probability of no sale decreases. For low ϕ, the price effect is more important. Sellers have an incentive to advertise more to compensate the marginal reduction in probability of no sale; advertising increases in market tightness. See Figure 1 for a specific example where the inverted U-shape between advertising and market tightness is observed.

With a large buyer to seller ratio, an exit by one seller will not have much of an impact on the equilibrium price. However, when such a ratio is small, an exit by one seller creates a bigger impact on the equilibrium price. This effect is consistent with standard models yielding a continuous relationship between market structure and price/quantity such as the Cournot model.

As the market becomes more competitive ($\phi \to 0$), the equilibrium price converges to zero. The model's predictions converge to a perfectly competitive outcome with equilibrium price equals marginal cost of production which is zero in the model. On the other hand, as the market becomes a monopoly

($\phi \to \infty$), the price tends to 1. Since advertising is essential for prices to be known by buyers and for the market to exist, for all $\phi \notin (\underline{\phi}, \overline{\phi})$, we find $a^*(\phi) = p^*(\phi) = 0$. Furthermore, since $\overline{\phi}$ is increasing in β, for small enough β it can be shown that $p^*(\overline{\phi})$ is very close to 1, however, generally we find, $p^*\left(\underline{\phi}\right) > 0$. This suggest, naturally, that a minimal price required for the market to exist. The large market result is also valid in finite markets.

Lemma 1 *Given $\bar{\beta} > 0$ such that $\forall\, 0 < \beta < \bar{\beta}$ and any $\bar{M} < \infty$, there exists a pair $0 < \underline{N} < \bar{N} < \infty$ such that $a^*(\bar{M}, \underline{N}) = a^*(\bar{M}, \bar{N}) = 0$, and $\forall N \in (\underline{N}, \bar{N})$, $a^*(\bar{M}, N) > 0$ with $a^*(\bar{M}, N)$ being inverted U-shape in N.*

Proof. See the Appendix. ■

3 Alternative Advertising Technology

In the previous section we assumed that $\gamma(A)$ to be the probability that all buyers know all prices. Now we assume a more general advertising technology which we denote by $\delta(A)$ which represents the probability that **one** buyer observes all prices. As a consequence, the probabilities of being informed across buyers are independent, thus we must consider $N+1$ possible events. These are given by:[13]

0. No buyer is informed, thus the market does not exist. This event occurs with probability $C_N^0[1-\delta(A)]^N$;

1. Only one buyer is informed of all prices, while the rest of buyers are not. The probability of this event is given by $C_N^1\delta(A)[1-\delta(A)]^{N-1}$;

2. Only two buyers are informed of all prices, while the other buyers do not know any prices. This event occurs with probability $C_N^2\delta(A)^2[1-\delta(A)]^{N-2}$;

N. All buyers are informed of all prices. The probability of such event is given by $C_N^N[\delta(A)]^N$.

Note that what seller i is concerned about is the probability that at least one other buyer shows up at her store. The complementary event that no buyer shows up at seller i is given by:

$$\begin{aligned}\mathbf{Pr}(n_i = 0) =& C_N^0[1-\delta(A)]^N + C_N^1\delta(A)[1-\delta(A)]^{N-1}[1-\theta_i(\mathbf{p},\mathbf{a})] + \\ & C_N^2\delta(A)^2[1-\delta(A)]^{N-2}[1-\theta_i(\mathbf{p},\mathbf{a})]^2 + \cdots\cdots + C_N^N[\delta(A)]^N[1-\theta_i(\mathbf{p},\mathbf{a})]^N\end{aligned}$$

[13] C_N^0 denotes the combinatorial operation corresponding to N buyers and 0 zero been informed.

where the buyers' strategy is given by:

$$\theta(\mathbf{p},\mathbf{a}) = \{\theta_1(\mathbf{p},\mathbf{a}), \cdots, \theta_i(\mathbf{p},\mathbf{a}), \cdots, \theta_M(\mathbf{p},\mathbf{a})\}.$$

By the Binomial theorem we can rewrite the complementary probability that no buyer shows up at seller i as follows:

$$\mathbf{Pr}(n_i = 0) = \left\{(1-\delta(A)) + \delta(A)[1-\theta_i(\mathbf{p},\mathbf{a})]\right\}^N = \left\{1-\delta(A)\theta_i(\mathbf{p},\mathbf{a})\right\}^N.$$

Thus the probability that seller i gets at least one buyer is $1-\left[1-\delta(A)\theta_i(\mathbf{p},\mathbf{a})\right]^N$. Then the seller i's profit maximization problem can then be written as follows:

$$\max_{a_i,p_i} \Pi(\mathbf{p},\mathbf{a}) = \left\{1-\left[1-\delta(A)\theta_i(\mathbf{p},\mathbf{a})\right]^N\right\} p_i - C(a_i) - F. \tag{20}$$

As in the previous section, if a particular buyer is informed, her probability of getting served by seller i, Λ_i, must satisfy the following local market clearing condition:

$$N\delta(A)\theta_i(\mathbf{p},\mathbf{a})\Lambda_i = 1-\left[1-\delta(A)\theta_i(\mathbf{p},\mathbf{a})\right]^N. \tag{21}$$

Notice that the left hand side of equation (20) is the expected number of buyers who visit seller i and are served, while the right hand side is the expected number of sales. Therefore, the probability of being served by seller i is given by:

$$\Lambda_i = \frac{1-\left[1-\delta(A)\theta_i(\mathbf{p},\mathbf{a})\right]^N}{N\delta(A)\theta_i(\mathbf{p},\mathbf{a})}.$$

Since we focus on symmetric equilibrium, we take seller i as the deviator by letting her choose $(\hat{a},\hat{p})$, while the rest of sellers follow the symmetric strategy denoted by (a,p). Correspondingly, we denote $\hat{\theta}$ as the probability by which buyers choose seller i, and θ as the probability of buyers choosing other sellers.

Given the probability of being served, the expected utility of a buyer from selecting seller i is then given by:

$$\hat{U}(\mathbf{p},\mathbf{a}) = (1-\hat{p})\hat{\Lambda} = (1-\hat{p})\frac{1-\left[1-\delta(A)\hat{\theta}(\mathbf{p},\mathbf{a})\right]^N}{N\delta(A)\hat{\theta}(\mathbf{p},\mathbf{a})}, \tag{22}$$

while the expected utility of selecting a seller other than seller i can be written as follows:

$$U(\mathbf{p},\mathbf{a}) = (1-p)\frac{1-\left[1-\delta(A)\frac{1-\hat{\theta}(\mathbf{p},\mathbf{a})}{M-1}\right]^N}{N\delta(A)\frac{1-\hat{\theta}(\mathbf{p},\mathbf{a})}{M-1}}, \tag{23}$$

where we have imposed the fact that $1-\hat{\theta} = (M-1)\theta$.

The mixed strategy used by buyers requires that $\hat{U}(\mathbf{p},\mathbf{a}) = U(\mathbf{p},\mathbf{a})$, the buyer's indifference condition, which is equivalent to:

$$\frac{1-p}{1-p_i} = \frac{(1-\hat{\theta})1-[1-\delta(A)\hat{\theta}]^N}{(M-1)\hat{\theta}\{1-\left[1-\delta(A)\frac{1-\hat{\theta}}{M-1}\right]^N\}}. \tag{24}$$

If we now consider seller i's problem, the first-order conditions corresponding to her profit maximization problem are as follows:

$$1-\left[1-\delta(A)\hat{\theta}\right]^N + N\hat{p}\delta(A)\left[1-\delta(A)\hat{\theta}\right]^{N-1}\frac{\partial\hat{\theta}}{\partial\hat{p}} = 0, \tag{w.r.t $\hat{p}$}$$

$$N\hat{p}\left[1-\delta(A)\hat{\theta}\right]^{N-1}\left[\frac{d\delta(A)}{d\hat{a}}\hat{\theta} + \frac{\partial\hat{\theta}}{\partial\hat{a}}\delta(A)\right] - C'(\hat{a}) = 0. \tag{w.r.t $\hat{a}$}$$

In order to determine $\frac{\partial\hat{\theta}}{\partial\hat{p}}$ and $\frac{\partial\hat{\theta}}{\partial\hat{a}}$, we need to totally differentiate equation (23) with respect to $\hat{a}$ while setting $\hat{a} = a$, $\hat{p} = p$ and $\hat{\theta} = \frac{1}{M}$. We then have that:

$$\left\{1-\left(1-\frac{\delta(A)}{M}\right)^N - N\frac{\delta(A)}{M}\left(1-\frac{\delta(A)}{M}\right)^{N-1}\right\}\frac{\partial\theta}{\partial a} = 0, \tag{25}$$

which implicitly assumes that sellers are able to see the aggregate advertising level as well as the individual one.

In a symmetric equilibrium, equation (24) implies either the term in the braces equals zero or $\frac{\partial\theta}{\partial a}$ equals zero. Notice that from the point view of the seller, the term in the braces is the probability that at least two buyers show up to her store.

Similarly, by totally differentiating (23) with respect to $\hat{p}$ and imposing $\hat{a} = a$, $\hat{p} = p$ and $\hat{\theta} = \frac{1}{M}$, we have that in the equilibrium the mixed strategy must satisfy:

$$\frac{\partial \theta}{\partial p} = \frac{(M-1)\left[1-(1-\frac{\delta(A)}{M})^N\right]}{M(1-p)\left\{N\delta(A)(1-\frac{\delta(A)}{M})^{N-1} - M\left[1-(1-\frac{\delta(A)}{M})^N\right]\right\}}. \tag{26}$$

Given these conditions, if we substitute $\frac{\partial \theta}{\partial p}$ into the first-order condition with respect to $\hat{p}$, we have that the equilibrium price is given by:

$$p = \frac{\left[1-\left(1-\frac{\delta(A)}{M}\right)^N\right]\left\{N\delta(A)\left(1-\frac{\delta(A)}{M}\right)^{N-1} - M[1-\left(1-\frac{\delta(A)}{M}\right)^N]\right\}}{N\delta(A)\left(1-\frac{\delta(A)}{M}\right)^{N-1}\left[1-\left(1-\frac{\delta(A)}{M}\right)^N\right] - M^2\left[1-\left(1-\frac{\delta(A)}{M}\right)^N\right]^2}. \tag{27}$$

To determine the large market counterpart, let $\phi = \frac{N}{M}$ and we fix $M = \bar{M}$ and assume $\bar{M}$ and N are large enough so that $(1-\frac{1}{M})^N \approx e^{\phi}$. Then the equilibrium price p for large markets is as follows:

$$p \to \frac{1-e^{-\phi\delta(A)} - \phi\delta(A)e^{-\phi\delta(A)}}{1-e^{-\phi\delta(A)}}.$$

The equilibrium price is similar to the previous section, now because of the different advertising technology, it depends on the expected market tightness, $\phi\delta(A)$. It is easy to observe that if $\delta(A) = 1$, the two equilibrium prices are equivalent. Notice that for a large market, the price tends to zero if $1-e^{-\phi\delta(A)} - \phi\delta(A)e^{-\phi\delta(A)} = 0$. Note that the term in the braces of equation (25), converges to $1-e^{-\phi\delta(A)} - \phi\delta(A)e^{-\phi\delta(A)}$ as the market becomes large. Therefore this leads to an equilibrium where the market shuts down. This is the case because if sellers price at $p = 0$, then they have no incentive to invest in costly advertising, i.e., $\delta(A) = 0$. However, this is consistent with $1-e^{-\phi\delta(A)} - \phi\delta(A)e^{-\phi\delta(A)} = 0$. If a seller deviates and spends $a > 0$, she would not be able to price at $p > 0$ since buyers will have a positive probability to learn that other sellers are charging a zero price.

Proposition 2 *There exist two types of equilibria in the model with general advertising technology, when the probabilities of being informed across different consumers are independent:*

(i) There always exists a non-advertising equilibrium such that $A^* = 0$ and $p^* = 0$. Thus the market collapses.

(ii) There always exists an advertising equilibrium such that $\frac{d\theta}{da}|_{A=A^*} = 0$ *with* $p^* > 0$*. Furthermore, the aggregate advertising level is inverted U-shape in market tightness for any concave enough advertising technology.*

Proof. See the Appendix. ■

Proposition 2 shows that the inverted U-shape pattern of advertising and market tightness is an equilibrium outcome of a matching environment with capacity constrained sellers, uncoordinated buyers while sellers face a general costly informative advertisement technology.

4 Conclusion

This paper provides a formal framework to explore the incentives of sending signals from sellers to buyers changes as different market structures and advertising technologies are considered. The environment under study has sellers that are capacity constrained and buyers are uncoordinated when selecting a particular seller. When sellers send signals only a fraction of buyers observe all prices. Thus returns on advertising are probabilistic. This modeling strategy allows a more detailed analysis of the relationship between advertising and market prices.

The costly directed search environment presented in this paper alleviates the discontinuity problem of pure price equilibrium strategies when prices are equal that is typically observed in the literature. Here we are able to characterize the unique symmetric equilibrium price and advertisement.

Finally, we find that the relationship between market concentration and advertising is unimodal (inverted U-shape) while being robust to different advertising technologies. For relatively low concentration industries, increases in market price due to higher concentration create a positive association between advertising and the market price. This yields more advertisements per seller, and more informative signals in the industry. On the other hand, for relatively high concentration industries, an increase in concentration leads to a negative association. These predictions are consistent with the empirical literature as documented in Bagwell(2007).

References

[1] Anderson, S. (2005): "Marketing Characteristics", http://oscar.virginia.edu/asp/PublicAward.asp?AwardID=97064.

[2] Bagwell, K. (2007): "The Economic Analysis of Advertising", *Handbook of Industrial Organization*, 3, 1701-1844, North-Holland Amsterdam.

[3] Buxton, A. J., Davies, S. W. and B. R. Lyons (1984): "Concentration and Advertising in buyer and Producer Markets," *Journal of Industrial Economics*, 32, 451-64.

[4] Burdett, K., S. Shi and R. Wright (2001): "Pricing and Matching with Frictions", *Journal of Political Economy*, 109, 1060-1085.

[5] Butters, G. (1977): "Equilibrium Distribution of Sales and Advertising Prices", *Review of Economic Studies*, 44, 465-491.

[6] Deneckere, R. and J. Peck (1995): "Competition Over Price and Service Rate When Demand is Stochastic: A Strategic Analysis", *Rand Journal of Economics*, 26, 148-162.

[7] Julien, B., J. Kennes and I. King (2000): "Bidding for Labor", *Review of Economic Dynamics,* 3, 619-649.

[8] Julien, B., J. Kennes and I. King (2005): "Efficient Bidding and Equilibrium Unemployment", *Canadian Journal of Economics,* 38, 174-193.

[9] Lee, C. (2002) "Adverting, Its Determinants and Market Structure", *Review of Industrial Organization* 21, 89-101.

[10] Lester, B. (2010): "Information and Prices with Capacity Constraints ", *American Economic Review* Forthcoming.

[11] McAfee, R. P. (1994): "Endogenous Availability, Cartels, and Merger in an Equilibrium Price Dispersion," *Journal of Economic Theory*, 62, 24-47.

[12] Norman, G., L. Pepall and D. Richards (2008): "Generic Product Advertsing, Spillovers and Market Concentration," *American Journal of Agricultural Economics*, 90, 719-732.

[13] Peters, M. (1991): “Ex Ante Price Offers in Matching Games Non-Steady States”, *Econometrica*, 59, 1425-1454.

[14] Robert, J. and D. O. Stahl (1993): “Informative Price Advertising in a Sequential Search Model,” *Econometrica*, 61, 657-86.

[15] Schmalensee, R. (1989) "Inter-industry Studies of Structure and Performance", in: R. Schmalensee and R. Willig, eds., *Handbook of Industrial Economics* (North-Holland, Amsterdam) 952-1009.

[16] Stegemen, M. (1991): “Advertising in Competitive Markets”, *American Economic Review*, 81, 210-223.

[17] Stahl, D. O., (1994): “Oligopolistic pricing and Advertising”, *Journal of Economic Theory*, 64, 162-177.

[18] Tremblay, C. and V. Tremblay (1995) “Advertising, Price, and Welfare: Evidence from the U. S. Brewing Industry ”, *Southern Economic Journal*, 62, 367-381.

[19] Uri, N. D. (1987), “A Re-examination of the Advertising and Industrial Concentration Relationship,” *Applied Economics*, 19, 427-35.

5 Appendix

Proof of Proposition 1. Fix $M = \bar{M}$, we have $A^* = \bar{M}\phi \ln \frac{1-e^{\phi}-\phi e^{\phi}}{\bar{M}\phi\beta}$. Notice the fraction in the logarithm function tends to 0 as ϕ goes to 0. It is easy to see that $\frac{1-e^{\phi}-\phi e^{\phi}}{\bar{M}\phi\beta}$ is hump shape. If β is small enough so that $\bar{M}\beta$ is not too large and κ=1 where κ=$\frac{1-e^{-\phi}-\phi e^{-\phi}}{\beta\bar{M}\phi}$, then we have that $A^*(\underline{\phi}) = A^*(\overline{\phi}) = 0$, and $\forall \phi \in (\underline{\phi}, \overline{\phi})$, $A^*(\phi) > 0$.

Let us now differentiate $A^*(\phi) = \bar{M}\phi \ln\left(\frac{1-e^{-\phi}-\phi e^{-\phi}}{\beta\bar{M}\phi}\right)$ w.r.t ϕ, so that:

$$\frac{dA}{d\phi} = \bar{M}\left[\ln\left(\frac{1-e^{-\phi}-\phi e^{-\phi}}{\beta\bar{M}\phi}\right) + \frac{\phi^2 e^{-\phi}+e^{-\phi}+\phi e^{-\phi}-1}{1-e^{-\phi}-\phi e^{-\phi}}\right].$$

We know that the first part in the fraction is positive between $\underline{\phi}$ and $\bar{\phi}$ but eventually goes to $-\infty$. The second part can be re-written as $\frac{\phi^2 e^{-\phi}}{1-e^{-\phi}-\phi e^{-\phi}} - 1$ where $\frac{\phi^2 e^{-\phi}}{1-e^{-\phi}-\phi e^{-\phi}}$ decreases from $+\infty$ to 0. Thus $\frac{\phi^2 e^{-\phi}}{1-e^{-\phi}-\phi e^{-\phi}} - 1$ has positive positive values when ϕ is small and decreases eventually negative values as ϕ increases. In sum, there exist a unique maximizer $\phi \in [\underline{\phi}, \overline{\phi}]$ for $a^*(\phi)$, and this function must be inverted U-shape. ■

Proof of Lemma 1. Follows from Proposition 1 using (15) and for any finite N, set $\underline{\phi} = \underline{N}/\bar{M}$ and $\overline{\phi} = \bar{N}/\bar{M}$. ■

Proof of Proposition 2. The first part in proposition 2 is immediate following from the previous explanation. Hereafter we prove the second part.

Existence. Notice that

$$F(\delta, A) = \frac{d\delta(A)}{dA} = \frac{d\delta(A)}{da} = \frac{\beta(1-e^{-\phi\delta(A)})}{\phi\delta(A)e^{-\phi\delta(A)}[1-e^{-\phi\delta(A)})-\phi\delta(A)e^{-\phi\delta(A)})]}. \tag{28}$$

is an ordinary differential equation. We can set the initial condition as $\delta(A_{\min}) = \hat{\delta}$ with $A_{\min} > 0$ sufficiently close to 0. It is easily to show that $F(\delta, A)$ is decreasing in both arguments. Therefore $F(\delta, A)$ is bounded by $F(\hat{\delta}, A_{\min})$ when $A_{\min} \leq A \leq +\infty$ and $\hat{\delta} \leq \delta \leq 1$. Clearly, $F(\delta, A)$ is continuous in all its region. By applying standard sufficient condition of existence, the solution to $\frac{d\delta A}{dA} = F(\delta, A)$ always exists when $A \geq A_{\min}$.

Inverted U-shape. Taking logarithms on both sides of equation (27), we have that

$$\ln(\delta'(A)) = \ln(\beta) + \ln(1 - e^{-\phi\delta(A)}) - \ln(\phi) - \ln(\delta(A)) + \phi\delta(A) - \ln(1 - e^{-\phi\delta(A)} - \phi\delta(A)e^{-\phi\delta(A)}). \tag{29}$$

Totally differentiating the previous expression, we have that the aggregate advertising has the following property with respect to market tightness:

$$\frac{dA}{d\phi} = \frac{\delta(A)^2}{\phi\delta'(A)}\frac{\tau_t}{\tau_b}; \tag{30}$$

where

$$\begin{aligned}
\tau_t &= 1 + \frac{e^{-\phi\delta(A)}}{1 - e^{-\phi\delta(A)}} - \frac{1}{\phi\delta(A)} - \frac{\phi\delta(A)e^{-\phi\delta(A)}}{1 - e^{-\phi\delta(a)} - \phi\delta(A)e^{-\phi\delta(a)}}, \\
\tau_b &= \frac{\delta''(A)}{\delta(A)^2} - \frac{\phi e^{-\phi\delta(A)}}{1 - e^{-\phi\delta(A)}} + \frac{1}{\delta(A)} - \phi + \frac{\phi^2\delta(A)e^{-\phi\delta(A)}}{1 - e^{-\phi\delta(a)} - \phi\delta(A)e^{-\phi\delta(a)}}.
\end{aligned}$$

By rearranging, we have

$$\frac{dA}{d\phi} = \frac{\delta(A)^2}{\phi\delta'(A)}\frac{\tau'_t}{\tau'_b}; \tag{31}$$

where

$$\begin{aligned}
\tau'_t &= \left[1 - e^{-\phi\delta(A)}\right]\left[\phi\delta(A) + e^{-\phi\delta(A)} - \phi^2\delta^2 e^{-\phi\delta(A)} - 1\right] \\
&\quad - \phi\delta(A)e^{-\phi\delta(A)}\left[\phi\delta(A) + e^{-\phi\delta(A)} - 1\right] \\
\tau'_b &= \left[\delta''(A) + \delta(A)\right]\left[1 - e^{-\phi\delta(A)}\right]\left[1 - e^{-\phi\delta(A)} - \phi\delta(A)e^{-\phi\delta(A)}\right] \\
&\quad - \left[\phi^2\delta(A)^3 e^{-\phi\delta(A)}\right]\left[1 - e^{-\phi\delta(A)} - 2\phi\delta(A)e^{-\phi\delta(A)} + \phi\delta(A)e^{-2\phi\delta(A)}\right]
\end{aligned}$$

Let's simply assume that the concave advertising technology has constant second derivative with $\delta'' = k < 0$ small enough. It's easy to see τ'_b is always strictly negative when $\phi > 0$ and δ'' is negative enough. This is because the second term in τ'_b

$$\left[\phi^2\delta(A)^3 e^{-\phi\delta(A)}\right]\left[1 - e^{-\phi\delta(A)} - 2\phi\delta(A)e^{-\phi\delta(A)} + \phi\delta(A)e^{-2\phi\delta(A)}\right]$$

is bounded away around -0.1 from below and eventually goes to $+\infty$.. In fact, the first derivative of

$\left[1-e^{-\phi\delta(A)}-2\phi\delta(A)e^{-\phi\delta(A)}+\phi\delta(A)e^{-2\phi\delta(A)}\right]$ is

$$\delta^2\left[1-e^{-\phi\delta(A)}-\phi\delta(A)e^{-\phi\delta(A)}+2\phi\delta(e^{-\phi\delta}-e^{-2\phi\delta})(\phi\delta-1)\right]$$

which can be negative only if $\phi\delta<1$. This condition implies that the absolute value of the second part in τ_b' is very small when it is negative. Thus, given that the first part in τ_b' is negative enough when δ is concave enough, τ_b' is always negative.

Notice that $\phi\delta(A)+e^{-\phi\delta(A)}-\phi^2\delta^2e^{-\phi\delta(A)}-1$ equals $\delta(A)-1<0$ when $\phi=0$ and goes from negative values to positive values. However, $\phi\delta(A)+e^{-\phi\delta(A)}-1$ is always positive when $\phi>0$. Thus, τ_t' is negative when ϕ is small. It goes to $+\infty$ when ϕ gets large, which can be seen easily by re-writing τ_t' as

$$\left[1-e^{-\phi\delta(A)}-\phi\delta(A)e^{-\phi\delta(A)}\right]\left[\phi\delta(A)+e^{-\phi\delta(A)}-1\right]-\phi^2\delta(A)^2e^{-\phi\delta(A)}\left[1-e^{-\phi\delta(A)}\right].$$

In sum, the first derivative of A w.r.t ϕ changes from positive values to negative ones. Therefore, the industry advertising level A must be inverted U-shape.

Figure 2 shows the property of $\frac{dA}{d\phi}$. The blue section of the three dimensional graph represents the values of (δ,ϕ) when $\frac{dA}{d\phi}\le 0$. Clearly, for high values of $\delta(A)$, $\frac{dA}{d\phi}$ peaks at low values of ϕ, and vice versa for low values of $\delta(A)$. It also shows that, for ϕ close to zero, $\frac{dA}{d\phi}$ tends to infinity, a property similar to Inada condition. ■

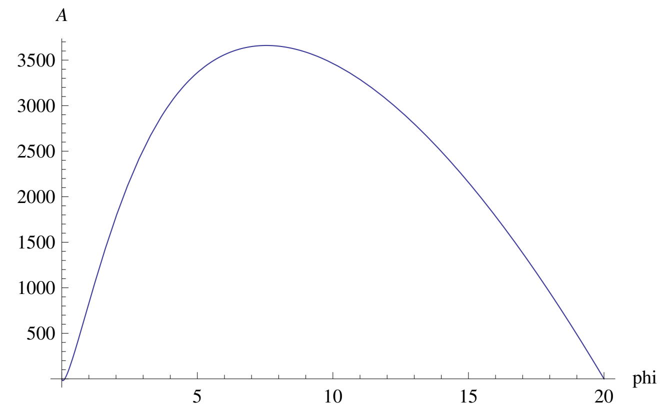

Figure 1: Advertising intensity for $\beta = 0.0001, M = 500$.

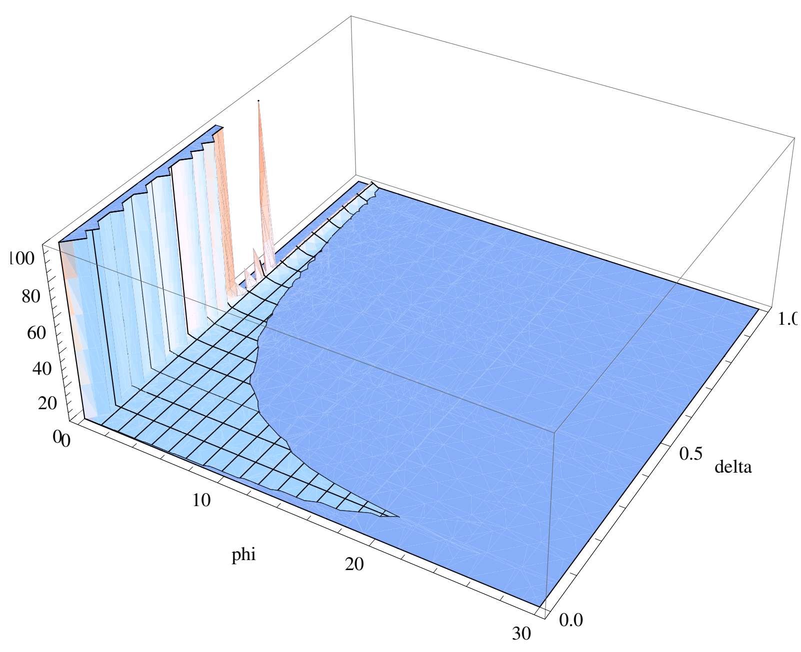

Figure 2: $\frac{dA}{d\phi}$ with general advertising technology.