Identification of Interaction Effects in Survey Expectations: A Cautionary Note

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Identification of Interaction Effects in Survey Expectations: A Cautionary Note*

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Abstract

A growing body of literature reports evidence of social interaction effects in survey expectations. In this note, we argue that evidence in favor of social interaction effects should be treated with caution, or could even be spurious. Utilizing a parsimonious stochastic model of expectation formation and dynamics, we show that the existing sample sizes of survey expectations are about two orders of magnitude too small to reasonably distinguish between noise and interaction effects. Moreover, we argue that the problem is compounded by the fact that highly correlated responses among agents might not be caused by interaction effects at all, but instead by model-consistent beliefs. Ultimately, these results suggest that existing survey data cannot facilitate our understanding of the *process* of expectations formation.

Keywords: Survey expectations; model-consistent beliefs; social interaction; networks.

JEL codes: D84, D85, C83.

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1 Introduction

Expectations play a central role in economic theory, yet we know rather little about the actual process of expectation formation. A growing body of literature emphasizes the importance of social interactions in the process of expectation formation, and mostly finds empirical support for interaction effects in reported survey data. These survey expectations typically consist of several hundred monthly responses by several hundred agents. Here we consider a generic stochastic model of expectation dynamics that contains both a social interaction component and an exogenous signal that represents model-consistent beliefs. The purpose of this note is to show that it is essentially not possible to disentangle the two effects in survey data, and that even if social interactions were present, the required sample size to identify interaction effects is about two orders of magnitude larger than existing sample sizes. Even if we are willing to make strong assumptions about the structure of multidimensional responses, existing survey data will probably remain a very fragile source for the identification of interaction effects or model-consistent beliefs.

Modern macroeconomics assumes that agents know the 'true' model underlying the macroeconomic laws of motion, and that their predictions of the future are on average correct. In their extensive review, Pesaran and Weale (2006) find little if any evidence that survey expectations are model-consistent in this strong sense, which is hardly surprising given the complexity of our macroscopic environment. Weaker forms of macroeconomic rationality acknowledge that agents face model uncertainty and instead focus on learning (see, e.g., Evans and Honkapohja, 2001; Milani, 2010), informational rigidities (see, e.g., Mankiw and Reis, 2002; Mankiw et al., 2004; Coibion and Gorodnichenko, 2008), imperfect information (see, e.g., Woodford, 2001; Del Negro and Eusepi, 2009), and 'rational inattention' (see, e.g., Sims, 2003). While details of the forward-looking behavior of agents are crucial for the qualitative differences among these approaches, neither of them considers the actual process of expectations formation.

Recent econometric approaches are discussing the existence of heterogeneity in the updating behavior of forecasters (see, e.g., Clements, 2010), and laboratory experiments equally indicate heterogeneity in expectations (see, e.g., Hommes, 2010). The focus on heterogeneity intersects with another strand of research that emphasizes the importance of *social interactions* in the process of expectations formation. Empirical work on social interactions has traditionally employed discrete choice frameworks that allow for social spillovers in agents' utility (see, e.g., Brock and Durlauf, 2001), but this approach has been rather static in the sense that cross-sectional configurations are viewed as self-consistent equilibria. The discrete choice framework has also been investigated in the context of macroeconomic expectations formation, for instance by positing that agents choose between forming extrapolative expectations and (costly) rational expectations (see, e.g., Lines and Westerhoff, 2010), which can lead to endogenous fluctuations in macroeconomic variables.

Carroll (2003) suggests an alternative route to social interactions, hypothesizing that the diffusion of news from professional forecasters to the rest of the public leads to 'stickyness' in aggregate expectations. The diffusion of expectations is also a defining characteristic in several recent contributions that place greater emphasis on social interactions than on individual concepts of rationality in their study of (survey) expectations. These probabilistic approaches by and large aim for *positive* models of expectations formation, but yield mixed results so far. Bowden and McDonald (2008) study the diffusion of information in various network structures and find a trade-off between volatility in aggregate expectations and the speed at which agents learn the correct state of the world. Secondly, they argue that certain network structures can lead to information cascades. This would be consistent with the empirical results of Flieth and Foster (2002), who find that survey expectations are characterized with protracted periods of inertia punctuated by occasional switches from aggregate optimism to pessimism or vice versa. They also calibrate a model of 'interactive expectations' with multiple probabilistic equilibria from the data, which indicates that social interactions would have become less important over time. Lux (2009) confirms the empirical quality of survey expectations, with their pronounced swings in aggregate opinions, but he claims evidence in favor of strong interaction effects. Since both consider German survey expectations and utilize similar probabilistic formalizations of the expectations process, the question why they find conflicting results on the importance of interaction effects warrants some attention.

The source of the different findings might, at least in part, be due to the details of the probabilistic processes that the authors employ to model expectations formation. Both approaches formalize changes in expectations through transition probabilities that additively combine an autonomous and an interactive element. Flieth and Foster (2002) use a three-state model that can only be solved numerically, while Lux (2009) uses a two-state model that exploits well-known results in Markov chain theory and allows for closed-form solutions not only of the limiting distribution but, in principle, of the entire time evolution of the expectations process. Yet irrespective of a model's probabilistic details, we want to argue here that these differences are likely to originate from size limitations of existing surveys, because even if we knew the details of the interaction mechanism, including the exact parameterization of the expectations process and the network structure among agents, we would still not be capable of distinguishing between interaction effects and essentially random correlations in survey responses, nor would we be able to distinguish model-consistent beliefs from social interactions.

We place a premium on analytical tractability and thus conduct our investigation in the probabilistic tradition employed by Lux (2009). A number of results are known in this parsimonious modeling tradition, including (statistical) equilibrium properties for a wide range of model parameters and the time evolution of the probability density of beliefs. Understanding how the qualitative nature of the model changes with the parameters permits us to isolate the behavioral details of the expectations process from the question whether it is feasible to detect interaction and network effects from existing survey data.

2 Stochastic Model of Expectation Dynamics

The model utilized by Lux (2009) traces back to earlier contributions by Weidlich and Haag (1983) and Weidlich (2006), and is very similar, both formally and qualitatively, to the herding model of Kirman (1991, 1993). A prototypical setup in this tradition considers a population of agents of size N that is divided into two groups, say, X and Y of sizes n and $N - n$, respectively. In the context of survey expectations, the two groups would correspond to agents who have optimistic or pessimistic beliefs regarding the future state of an economic or financial indicator.

The basic idea is that agents change state (i) because they follow an exogenous signal, corresponding for instance to model-consistent beliefs, or (ii) because of the social interaction with their *neighbors*, i.e. agents they are communicating with during a given time period. The transition rate for an agent i to switch from state X to state Y is

$$\rho_i(X \to Y) = a_i + \lambda_i \sum_{j \neq i} D_Y(i,j), \tag{1}$$

where a_i governs the possibility of self-conversion caused by model-consistent beliefs, and the sum captures the influence of the neighbors. The parameter λ_i governs the interaction strength between i and its neighbors, indexed by j, while $D_Y(i,j)$ is an indicator function serving to count the number of i's neighbors that are in state Y,

$$D_Y(i,j) = \begin{cases} 1 & \text{if j is a Y-neighbor of i,} \\ 0 & \text{otherwise.} \end{cases}$$

Analogously the transition rates in the opposite direction, from a pessimistic to an optimistic state, are given by

$$\rho_i(Y \to X) = a_i + \lambda_i \sum_{j \neq i} D_X(i,j) \ . \tag{2}$$

Defining $n_Y(i, \mathbf{J}) = \sum_{j \neq i} D_Y(i,j)$ and $n_X(i, \mathbf{J}) = \sum_{j \neq i} D_X(i,j)$, where $\mathbf{J}$

denotes particular configurations of the neighbors, and using shorthands $\pi_i^- = \rho_i(X \to Y)$ and $\pi_i^+ = \rho_i(Y \to X)$, equations (1) and (2) can be written more compactly as

$$\pi_i^- = a_i + \lambda_i n_Y(i, \mathbf{J}), \tag{3}$$

$$\pi_i^+ = a_i + \lambda_i n_X(i, \mathbf{J}). \tag{4}$$

As a consequence of the interactions between neighboring agents, the rates $\pi_i^\pm$ still depend on the particular configurations of neighbors $\mathbf{J}$, making it difficult to handle (3) and (4) analytically, but we can employ a mean-field approximation in order to simplify the problem from a many-agent system to one with a sum of agents who are independently acting in an "external field" (see, e.g., Chap. 5 in Aoki, 1998) created by the opinions of other agents. In other words, we assume that individual agents are influenced by the average opinion of their neighbors, and that their behavioral parameters can be aggregated by averaging over all agents.

On the individual level, the instantaneous probability for agent i to switch from X to Y is given by (3). When the attitudes of i's neighbors fluctuate, π_i^- fluctuates around its mean

$$\left\langle \pi_i^- \right\rangle = a_i + \lambda_i \left\langle n_Y(i) \right\rangle, \tag{5}$$

where the dependence on $\mathbf{J}$ gets lost if we assume that inhomogeneities among the different configurations of neighbors are solely due to the fluctuations. Then we can replace the number of Y-neighbors around each agent i with the average number of neighbors that agents are linked to, say, D and $\langle n_Y(i) \rangle = D\, P_Y$, with P_Y being the probability that an i-neighbor is in state Y, which we can approximate with the unconditional fraction $(N-n)/N$ of agents in state Y, yielding

$$\left\langle \pi_i^- \right\rangle = a_i + \lambda_i D\, \frac{N-n}{N}, \tag{6}$$

and the quantity $\left\langle \pi_i^- \right\rangle$ becomes independent of the particular configuration of neighbors. Symmetrically, the expression for agents currently in state Y

becomes

$$\left\langle \pi_i^+ \right\rangle = a_i + \lambda_i D \, \frac{n}{N}. \tag{7}$$

Basically, the mean-field approximation reduces a complex system of heterogeneous interacting agents to a collection of independent agents who are acting "in the field" that is created by other agents' beliefs and their average behavior.

On the aggregate level, we are interested in the probability of observing a single switch on the system-wide level during some time interval Δt, hence we have to sum (7) over all agents in state Y in order to find the aggregate probability that an agent is switching from state Y to state X during Δt, assuming that Δt is small enough to constrain the switch to a single agent. Summing (7), which is permissible since the agents are now independent, we obtain

$$\pi^+ = (N-n) \, \left(a + \frac{\lambda D}{N} n \right), \tag{8}$$

for a switch from Y to X, and

$$\pi^- = n \, \left(a + \frac{\lambda D}{N} (N-n) \right), \tag{9}$$

for the reverse switch, where a, b are the mean values of a_i, b_i averaged over all agents. It turns out that replacement of behavioral parameters by their ensemble averages is only sensible if the network structure observes some regularity conditions and if the fraction of agents with strictly positive b_i is very large, i.e. as long as the fraction of isolated nodes in the agent network is very small (see Alfarano and Milaković, 2009, for details). We will return to the implications of this point in the final scenario of Section 3.

For notational convenience, we set

$$b \equiv \lambda D / N, \tag{10}$$

while setting $c \equiv \lambda D$ would recover the original formulation of Kirman's ant model.[1] The equilibrium concept associated with the generic transition

[1]It is well-known that the original formulation of the ant model suffers from the problem

rates (8) and (9) is a *statistical equilibrium* outcome: at any time, the *state of the system* refers to the concentration of agents in one of the two states. We define the state of the system through the concentration $z = n/N$ of agents that are in state X. For large N, the concentration can be treated as a continuous variable. Notice that none of the possible states of $z \in [0, 1]$ is an equilibrium in itself, nor are there multiple equilibria in the usual economic meaning of the term.

The notion of equilibrium instead refers to a statistical distribution that describes the proportion of time the system spends in each state. Utilizing the Fokker-Planck equation, we can show that for large N the equilibrium distribution of z is a beta distribution (see Alfarano et al., 2008, for details)

$$p_e(z) = \frac{1}{B(\epsilon, \epsilon)} z^{\epsilon-1}(1-z)^{\epsilon-1}, \tag{11}$$

where $B(\epsilon, \epsilon) = \Gamma(\epsilon)^2/\Gamma(2\epsilon)$ is Euler's beta function, while the shape parameter of the distribution is given by

$$\epsilon = a/b = aN/\lambda D. \tag{12}$$

Since ϵ is a ratio of quantities that depend (i) on the time scale at which the process operates ($1/a$ and $1/\lambda$), and (ii) on the spatial characteristics of the underlying network (D and N), the parameter of the equilibrium distribution is a well-defined dimensionless quantity. If $\epsilon < 1$ the distribution is bimodal, with probability mass having maxima at $z = 0$ and $z = 1$. Conversely, if $\epsilon > 1$ the distribution is unimodal, and in the "knife-edge" scenario $\epsilon = 1$ the distribution becomes uniform. The mean value of z, $E[z] = 1/2$, is independent of ϵ, and intuitively follows from the difference of the transition rates (8) and (9), $a(N - 2n)$, showing that in equilibrium the system approaches $n = N/2$.

Notice, nevertheless, that the system exhibits very different characteristics depending on the modality of the distribution. In the bimodal case, the

of N-dependence (or 'self-averaging' in the jargon of Aoki), i.e. the fact that the system converges to a unimodal equilibrium when the number of agents is enlarged while keeping the number of neighbors constant.

system spends least of the time around the mean, mostly exhibiting very pronounced herding in either of the extreme states, while mild fluctuations around the mean characterize the unimodal case. The bimodal case is apparently in line with the empirical finding of protracted periods of inertia with sudden switches in aggregate opinion. Since in that case $\epsilon < 1$ implies $b > a$, the model would seem to suggest that social interactions on average carry greater weight than idiosyncratic factors in the expectations formation of agents. The model can also be extended to account for asymmetries in the average aggregate state with the following transition rates

$$\pi^{+} = (N - n)(a_1 + bn) \quad \text{and} \quad \pi^{-} = n(a_2 + b(N - n)), \tag{13}$$

where the constants a_1 and a_2 now allow agents to have a 'bias' towards either state, for instance if $a_1 > a_2$ they will exhibit more optimistic than pessimistic beliefs on average. In this case (see Alfarano et al., 2005, for details), the corresponding equilibrium distribution is the beta distribution

$$q_e(z) = \frac{1}{B(\epsilon_1, \epsilon_2)}\, z^{\epsilon_1 - 1}\, (1 - z)^{\epsilon_2 - 1}\,, \tag{14}$$

where $B(\epsilon_1, \epsilon_2) = \Gamma(\epsilon_1)\Gamma(\epsilon_2)/\Gamma(\epsilon_1 + \epsilon_2)$ is again the beta function, while the shape parameters are now given by

$$\epsilon_1 = a_1/b \quad \text{and} \quad \epsilon_2 = a_2/b. \tag{15}$$

Figure 1 illustrates the flexibility in the shape of the beta distribution, with unimodal and bimodal cases similar to the symmetric case (11) in the top panel (a,b), but also including monotonically increasing or decreasing cases in situations where agents have a strong idiosyncratic signal in one direction, yet still exhibit a relatively pronounced herding tendency relative to the other state, i.e. $a_1/b < 1 < a_2/b$ or vice versa, as shown in the bottom panel (c,d).

In summary, the model provides quite a generic description of a stochastic expectations formation process that contains only a few behavioral parameters a_1, a_2, and b, yet allows for a large degree of agent heterogeneity. Despite

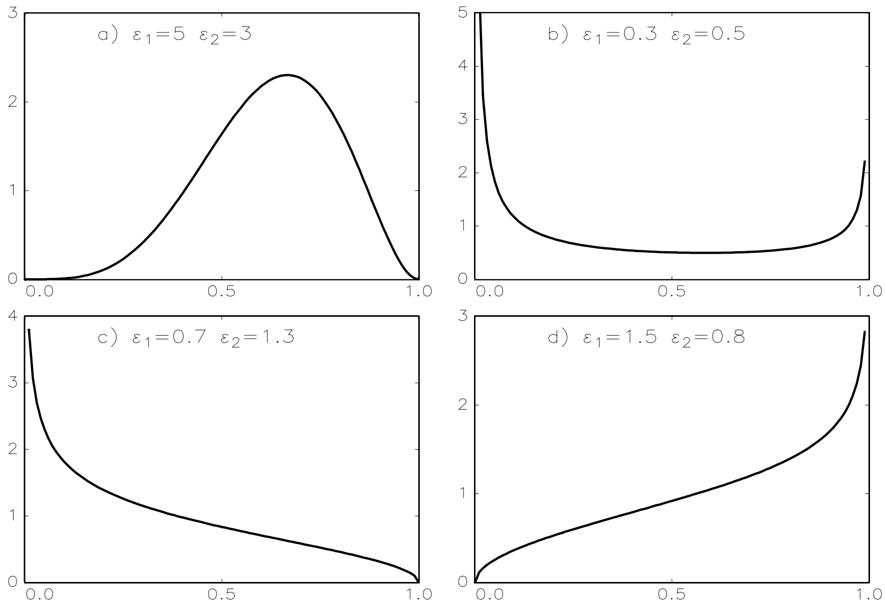

Figure 1: Beta distribution for various parameter combinations.

its parsimony, the model produces a wide range of qualitatively different statistical equilibria, including endogenous cycles in expectations caused by herding or imitation, but also equilibria where the vast majority of agents 'learns a correct state' in spite of being surrounded by noise that is created through the social interaction with neighboring agents. The qualitative features of the process are also parsimoniously summarized by the ratios (12) or (15), putting us in a position to isolate behavioral aspects from the question whether it is feasible to identify the communication or network structure among agents from survey data.

In the next section we argue that an 'omniscient modeler', endowed with perfect knowledge of the behavioral parameters and network structure among agents, would not be able to reliably recover this network structure based merely on the correlations in survey responses. Perhaps more troubling, if our knowledge is confined to the time evolution of survey responses, we will not even be able to reliably detect whether survey correlations originate from social interactions or model-consistent beliefs.

3 Random Benchmark and Simulation

We start with a thought experiment, putting ourselves in the position of an omniscient modeler (OM) who chooses a particular behavioral setup and network structure for the model in Section 2. Utilizing the individual transition rates (1) and (2), the OM simulates and records the time evolution of beliefs for all N agents in the system. Afterwards, the OM presents us with data on the individual histories of agents' beliefs (or *output* for short), from which we have to determine the network structure among agents based on correlations in the time evolution of their beliefs.

In actual data on survey expectations, with typically two to three hundred agents reporting monthly beliefs over roughly two hundred periods, we have no intrinsic knowledge of the network structure whatsoever. So to make life easier for us, the OM even informs us of the exact number D_i of neighbors for each agent $i = 1, \ldots, N$. We then compute the D_i highest correlations for each agent from the output and report it back to the OM as our best guess of the network structure in the output. In return, the OM checks our guesses against the actual identity of neighbors and reveals the fraction of correctly identified neighbors to us. The central question is: how many of the D_i neighbors do we expect to guess correctly by pure chance, i.e. irrespective of the correlation among responses? This establishes a random benchmark against which we have to judge the success of correlation-based procedures.[2]

In order to explain the random benchmark, it is instructive to consider a simple urn model. Let us draw d (read: D_i) colored balls without replacement from an urn containing a total of N balls, m of which are white (read: the true neighbors of agent i). The probability of drawing $k \leq m$ white balls in

[2]Instead of considering the time t correlation, we have conducted the subsequent analysis with various sums of leads and lags in the autocorrelations of responses, yet the results remain virtually unchanged.

d draws from a total of N balls is given by the hypergeometric distribution

$$P[k] = \frac{\binom{m}{k}\binom{N-m}{d-k}}{\binom{N}{d}} \,, \tag{16}$$

where the notation on the right hand side refers to binomial coefficients. In other words, (16) characterizes the distribution of the number of white balls drawn from the urn in d extractions. The mean value of the hypergeometric distribution is

$$E[k] = \frac{dm}{N}, \tag{17}$$

from which we can compute the random benchmark since $d = m$ in our OM setup.[3] The standard deviation of the hypergeometric distribution is

$$\sigma[k] = \sqrt{\frac{dm(N-m)(N-d)}{(N-1)N^2}}. \tag{18}$$

To keep our simulations in line with available survey data (for instance from the ZEW for German 'financial experts', or from the FRB Philadelphia for US 'professional forecasters'), we set the number of agents to $N = 250$; the available length of periods for individual agent IDs is on average between one and two hundred, while the number of questions per survey is typically between thirty and sixty. It will be a sobering experience to recall these figures when we present the simulation results.

3.1 Simulation setup

Regarding the network structure in our simulations, we consider three prototypical setups: random graphs in the Erdös-Renyi tradition, scale-free networks in the Barabasi-Albert tradition, and regular lattice structures.[4] To keep matters simple, we set the number of neighbors equal to twenty in the

[3]Notice that choosing a different number of extractions does not change any of the qualitative features in the following results, yet the approach immediately translates into quantitative prescriptions for measuring different benchmarks.

[4]The review article by Newman (2003) provides the historical background and a comprehensive summary of the many mathematical details of these graphs.

lattice, and tune the parameters of the scale-free and random networks such that we obtain adjacency matrices with an average number of twenty neighbors as well.[5] Given these numbers and (17) and (18), it is straightforward to compute that the fraction of correct answers we would expect purely by chance corresponds to $E[k] = 1.6$ with $\sigma[k] = 1.16$, or normalized with respect to the number of extractions $E[k]/d = 0.08$ and $\sigma[k]/d = 0.058$.

In the subsequent figures, we use the mean plus one standard deviation, $E[k] + \sigma[k] = 2.76$, to illustrate the statistical significance of the OM experiment. We can compute the probability of such an event from the cumulative hypergeometric distribution: since the hypergeometric distribution is defined for positive integer values of k, we have to consider $P(k \leq 2) = 0.79$ and $P(k \leq 3) = 0.94$. Hence the range $0 < E[k] + \sigma[k] < 3$ delivers a rather conservative confidence interval in accord with the usual econometric standards.

In line with (1), (2) and (13), the OM implements the transition probabilities ϕ_i for each agent i as

$$\phi_i^{(\pm)} = \rho_i^{(\pm)}\,\Delta t = [a_{(1,2)} + bD_i^{(\mp)}]\,\Delta_t \quad \text{with} \quad \Delta_t = 1/(a_{max} + bN), \tag{19}$$

where the notation $D_i^{(\mp)}$ refers to the number of i-neighbors that are in the opposite state, and $a_{max} = \max\{a1, a2\}$. The choice of Δt ensures both that $0 < \phi_i \leq 1$ and that all agents act on the same time scale.

The OM then confronts us with the output of the N time series of agents' beliefs, from which we compute $N(N-1)/2$ correlation coefficients. For every i, the OM also informs us of the actual number D_i of neighbors, and in turn we extract the D_i highest correlation coefficients from the output. Intuitively assuming that the highest correlation coefficients correspond to the neighbors of agent i, we construct the adjacency matrix of the agent network and report it to the OM who compares it with the actual adjacency matrix, and informs us of the fraction of correctly identified neighbors for each i. To aggregate and visualize the results for each of the following three scenarios, we average

[5]It turns out that changing the number of neighbors in the OM setup has virtually no influence on the subsequent results. We chose twenty neighbors because this figure does not appear to be entirely unrealistic. If anything, the communication with twenty neighbors already takes considerable time and effort in most professions.

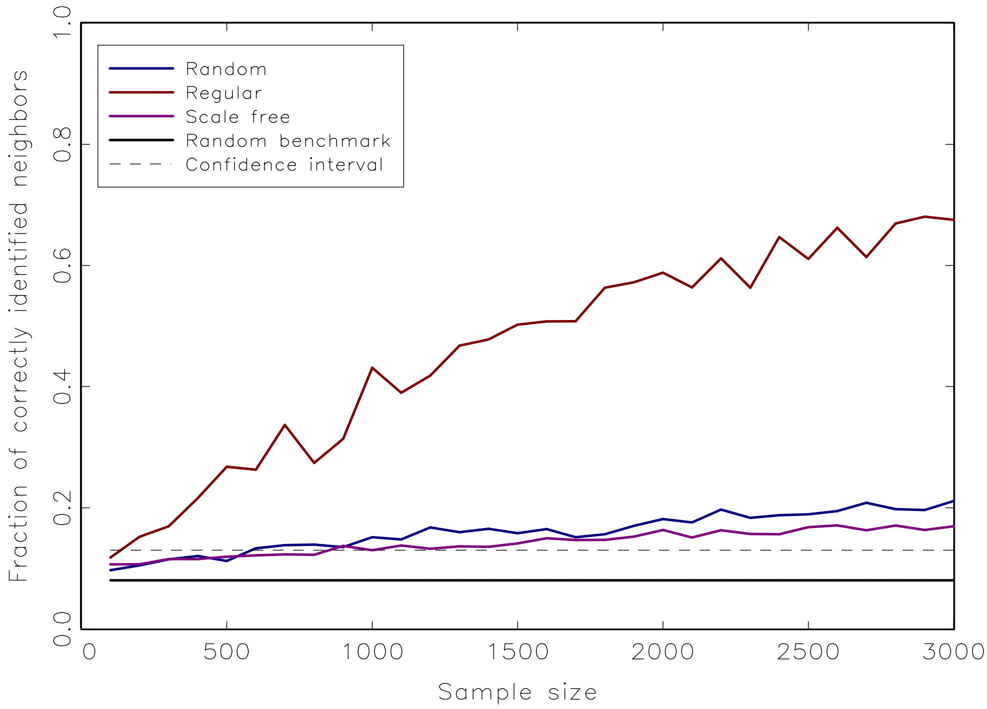

Figure 2: Average fraction of correctly identified neighbors vs length of individual agents' time series for a single question. We chose a bimodal simulation setup with parameters $\varepsilon_1 = \varepsilon_2 = .5$ and $b = 1$, i.e. a strong behavioral component relative to symmetric exogenous signals in either direction.

the correctly identified percentages for each agent over the entire pool of agents.

The following scenarios basically consider in how far we can recover the correct network structure (i) depending on the length of agent histories and (ii) depending on the number of simultaneous survey answers per agent, i.e. the volume of survey coverage. The final scenario (iii) takes up a more fundamental issue and examines what happens when correlation clusters are caused by model-consistent beliefs instead of social interactions. Put differently, is a correlation-based approach capable of distinguishing between clusters that are caused by either behavioral extreme?

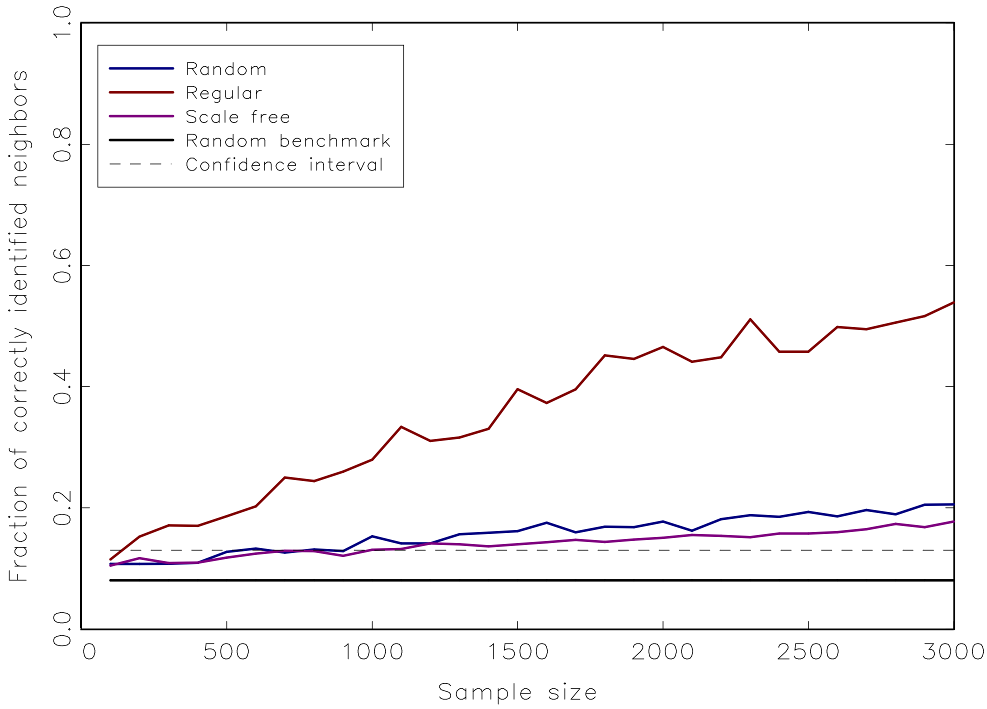

Figure 3: Average fraction of correctly identified neighbors vs length of individual agents' time series for a single question. Here we chose a unimodal simulation setup with parameters $\varepsilon_1 = \varepsilon_2 = 2$ and $b = 1$, i.e. a relatively strong exogenous signal compared to the behavioral component.

3.2 Scenario I: Single indicator histories

Suppose when agents answer questions regarding rather distant areas of expertise (e.g. international equity indices vs bonds vs GDP growth vs inflation etc.), they utilize different networks to form their expectations. So if we use histories for a single indicator in the OM experiment, what is the required number of observations per agent (or sample size for short) that is necessary to discriminate between some genuine network structure and random noise?

We consider both a bi- and a unimodal setup to control for behavioral biases, and display summary results under different network structures in Figures 2 and 3. The figures illustrate that there is little difference between random and scale-free setups, while it is easier to identify neighbors when they are all arranged in a regular lattice. Regular networks, however, are the least suitable representation of observed social networks, which tend to interpolate between random and scale-free structures (see, e.g., Newman, 2003, and the references therein). This is also the reason why we focus our attention on random graphs in the coming scenarios.

As one would intuitively suspect, the identification of interaction effects is somewhat facilitated in the bimodal case, i.e. when herding or imitation dominate the expectations formation process. According to Figures 2 and 3, however, this aspect has merely second-order effects. Up to a sample size of around one thousand periods, we are not able to distinguish between noise and network effects if our knowledge is restricted to the time evolution of univariate histories. Hence this also implies that we do not have a sufficient number of empirical survey data at our disposal to reliably identify the social interaction component. Viewed from this perspective, any cluster we identify based on the cross-correlations of answers is essentially pure noise. If we consider the confidence interval in Figures 2 and 3, the first scenario suggests that it is entirely unrealistic to identify even a rudimentary communication structure unless we increase the frequency of survey responses by one order of magnitude, i.e. from monthly to roughly twice per week. In addition, if we are indeed facing irregular network structures, the length of single indicator histories that is necessary to correctly identify about half the neighbors turns out to be two orders of magnitude larger than empirical sample sizes.[6]

3.3 Scenario II: Multiple indicator histories

Can we improve the identification of communication structures if we make the strong assumption that behavioral parameters in the expectations formation process of agents do not change across multiple questions, and that their network structure remains unchanged as well? And how many questions would be necessary in that case? To tackle this issue, we keep the parameterizations of the previous scenario and simulate the expectations formation process on a random network, fixing the length of single question histories

[6]Simulation results upon request.

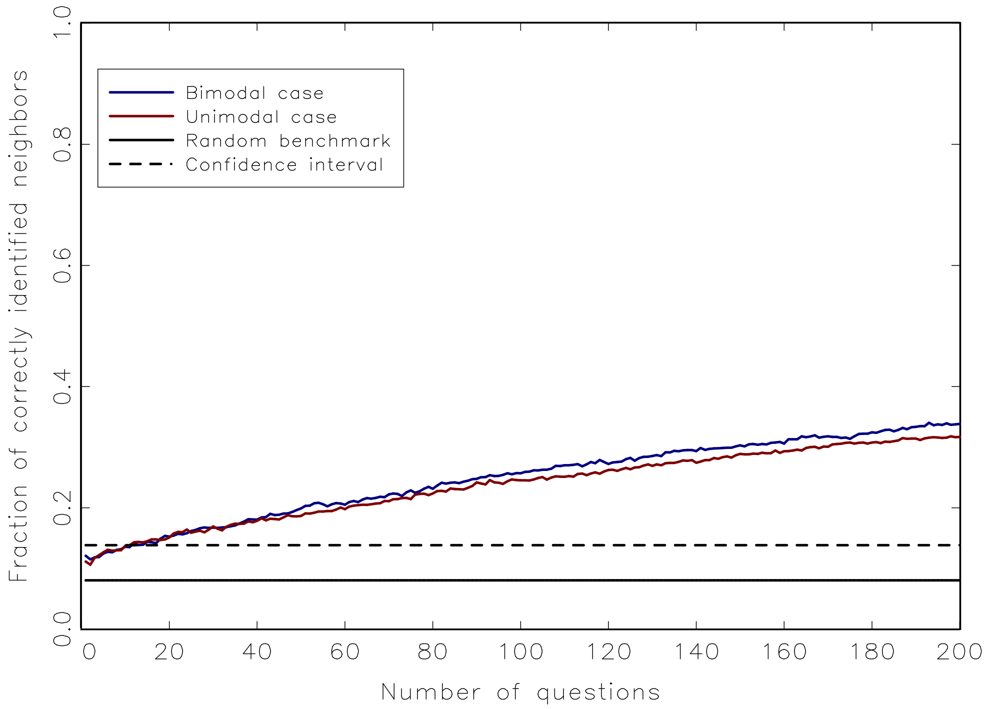

Figure 4: Increasing the number of questions and averaging over them improves the identification of interaction effects compared to the previous univariate scenario. The model parameterizations remain the same as before, and we utilize a random network whose structure remains fixed as well. The underlying univariate responses have a length of two hundred periods.

to two hundred while successively increasing the number of questions. Essentially, this means that the correlation coefficients are averaged both over agents and over questions. To operationalize this procedure, we fix the parameterization and underlying network structure of social interactions and run K independent simulations of the model for two hundred periods. For each single run of length two hundred, we perform the estimation procedure outlined in the previous scenario, and then average over the K questions.

The results both for uni- and bimodal setups, along with the random benchmark, are displayed in Figure 4 and show that a multivariate correlation-based procedure performs better than in the previous univariate scenario. As expected, a bimodal environment with strong interactions again somewhat facilitates the identification of the network structure, but the more appealing feature of this scenario is that the rate at which we discover actual links is markedly higher than in the univariate case. On the downside, however, the overall accuracy of the correlation-based procedure remains low. Keeping in mind that the empirical volume of survey coverage includes roughly thirty to sixty questions, correlation-based estimates of the interaction structure are almost not significantly different from pure noise, and certainly very low to begin with: we recover merely twenty percent of the actual network structure, and the fraction of correctly identified neighbors increases very slowly with the number of questions.

3.4 Scenario III: Exogenously switching signal

In both of the preceding scenarios we have assumed that all agents have a strictly positive interaction parameter, which we conveniently set to $b = 1$. But what happens if some agents are not socially interacting at all ($b = 0$) and instead form model-consistent beliefs from exogenous signals a_1, a_2 that we can think of as transmitting the correct state of the world? In principle, these 'rational' agents should exhibit highly correlated responses over time if the exogenous signal is sufficiently strong relative to the interaction parameter. If the state of the world does not change over time, the rational agents will all converge to the correct state, making it almost trivial to identify them from correlation-based procedures. In order to maintain an empirically more relevant scenario, we thus assume that the correct state of the world changes every now and then, i.e. the parameters a_1, a_2 are no longer constant but change over time.[7]

In this scenario, we keep the total number of agents at $N = 250$ in our simulations, and the underlying network remains a random graph with an average degree of twenty neighbors. The values of $\tilde{a_1} = \tilde{a_2} = \tilde{b} = 1$ are constant over time for the majority $\tilde{N} = 200$ of agents, while a smaller group of fifty 'rational' agents exhibits time-varying idiosyncratic coefficients, say $a_1(t)$ and $a_2(t)$, which essentially measure the speed at which rational agents

[7]From a mathematical point of view, this would correspond to a so-called *switching diffusion* process.

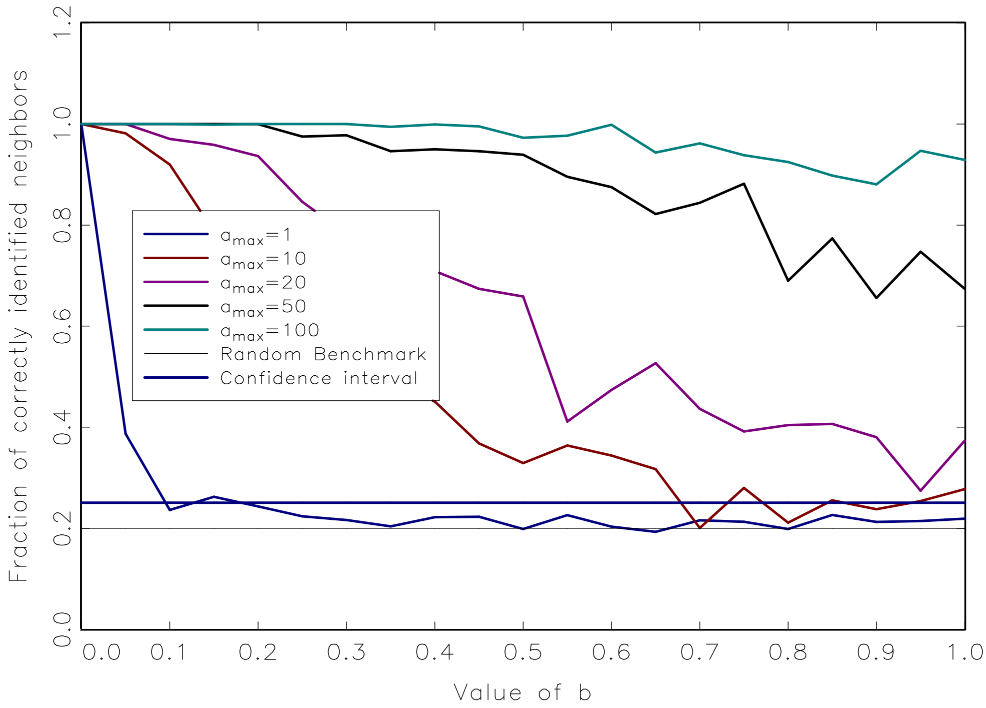

Figure 5: Fraction of correctly identified rational agents who follow a time-varying exogenous signal that is increasingly biased in either direction (denoted by an increasing value of a_{max}) vs their interaction strength b. At low values of b, the identification generally performs very well, while higher values of b might prevent a reliable detection, depending on the relative value of a_{max}.

learn the true state of the world. The time-varying coefficients take on values in the set $\{1, a_{max}\}$, where $a_{max} = \max\{a_1(t), a_2(t)\}$. Suppose for instance that $a_{max} = 10$ and that the currently 'true' state is such that $a_1(0) = 1$ and $a_2(0) = 10$, i.e. we are in an optimistic regime today. When the true state changes to pessimism, say in period τ, the parameters change to $a_1(\tau) = 10$ and $a_2(\tau) = 1$. As far as the switching probability in our simulations is concerned, we assume that the probability to switch is five percent, drawn randomly from a uniform distribution. In other words, an exogenous switch in the signal occurs on average every twenty months in our simulations.

The matter in question now concerns the fraction of rational agents that we can correctly identify if the true state of the world changes over time, as it certainly does in reality. (Notice that we have to adapt the error band since now $d = m = 50$.) None the less, we would expect the value of b to also have an influence on our ability to identify the rational agents: when b increases, the noise generated through the social interactions with the other agents should make it more difficult to identify rational agents correctly. On the other hand, when rational agents are not part of the social network ($b = 0$), and thus do not take possibly non-rational opinons into account, it should become easier to correctly identify them with correlation-based procedures.

Figure 5 plots the fraction of correctly identified rational agents for a given a_{max} when the interaction parameter b takes on values in $[0, 1]$. The different plots in Figure 5 refer to increasing values of a_{max} in the simulations. As expected, our ability to correctly identify the group of rational agents depends inversely on their interaction parameter b, possibly approaching the noise level as b approaches the value common to the other $\tilde{N}$ agents. On the other hand, when b approaches zero, we are in an increasingly comfortable position regarding the identification of rational agents. Finally, the faster the signal processing ability a_{max}, the easier it becomes to correctly identify the group of rational agents, asymptotically reaching the value of one hundred percent independently of b.

4 Discussion and Conclusions

All computations in the preceding scenarios have been performed under the assumption that the OM informs us of the actual number D_i of neighbors for each agent. Clearly, this is a most unrealistic assumption in the context of empirical applications to survey data, where we simply have no way of knowing whether agents interact socially in the first place, much less to whom they are linked to in case they do. Viewed from this perspective, our results are if anything overly optimistic to begin with.

Yet the third scenario delivers maybe the most fatal blow to any hopes that survey data could settle the question whether interaction effects are present in the expectations formation process of respondents or not. Our preferred way to read Figure 5 is that we can achieve any desired accuracy in the identification of network structure through an appropriate combination of b and a time-varying exogenous signal a_{max}. The other side of that coin is that we have no way of distinguishing between interaction effects and model-consistent beliefs, even if we identify relatively strong patterns in the correlations of a subset of agents.

Ultimately, these results suggest that existing survey data cannot facilitate our understanding of the *process* of expectations formation, which is particularly troubling in light of its central importance for modern macroeconomic theory. To end on a more constructive note, we would like to point out once more that our thought experiment presumed that we merely have data on the time evolution of agents' beliefs. In order to investigate whether interaction effects are indeed present in the data, it would be enormously helpful if surveys contained questions that refer directly to the presence of interaction effects.

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