An Asymptotically Non-Scale Endogenous Growth Model

Harashima, Taiji

20 October 2010

An Asymptotically Non-Scale Endogenous Growth Model[*]

Taiji Harashima[†]

October 2010

Abstract

This paper presents an endogenous growth model in which the economy grows without either scale effects or population growth. The key mechanism is substitution between investments in capital and technology when firms face increasing uncompensated knowledge spillovers. The model indicates that, as population increases, firms invest more in capital than in technology because there are more uncompensated knowledge spillovers as a result of both Marshall-Arrow-Romer and Jacobs externalities. Consequently, scale effects asymptotically diminish as population increases and disappear at a sufficiently large population while the economy can grow without population growth. In present-day industrialized economies, therefore, both scale effects and population growth have little influence over economic growth.

---

1. INTRODUCTION

Scale effects have been a central issue in the study of endogenous growth. Early endogenous growth models (e.g. Romer, 1986, 1987; Lucas, 1988) commonly included scale effects. However, the existence of scale effects in present-day economies is not supported by empirical evidence (Jones, 1995a). The source of scale effects lies in the assumption of a linear relation between capital and technology. Given a Harrod-neutral production function such that $y_t = A_t^\alpha k_t^{1-\alpha}$, the familiar optimal growth path is

$$\frac{\dot{c}_t}{c_t} = \varepsilon^{-1}\left[(1-\alpha)\left(\frac{A_t}{k_t}\right)^\alpha - n_t - \theta\right] \quad,$$

where $y_t = \frac{Y_t}{L_t}$, $k_t = \frac{K_t}{L_t}$, and $c_t = \frac{C_t}{L_t}$; Y_t (≥ 0) is output, K_t (≥ 0) is capital input, L_t (≥ 0) is labor input, A_t is technology, C_t (≥ 0) is consumption, and $n_t\left(= \frac{\dot{L}_t}{L_t}\right)$ is the population growth rate in period t. In addition, θ is the rate of time preference, ε is the coefficient of relative risk aversion, and α is a constant. Hence, if $\frac{A_t}{k_t}\left(= \frac{A_t L_t}{K_t}\right)$ and n_t are both constant, the growth rate of consumption is constant; that is, the economy can proceed on a balanced growth path.

To make $\frac{A_t}{k_t}$ constant, it is necessary that

$$\frac{\dot{K}_t}{K_t} - \frac{\dot{L}_t}{L_t} = \varphi_1 \frac{\dot{A}_t}{A_t} \quad,$$

where φ_1 is a constant. The simplest solution to construct a model that satisfies $\frac{\dot{K}_t}{K_t} - \frac{\dot{L}_t}{L_t} = \varphi_1 \frac{\dot{A}_t}{A_t}$ is to assume that there is a linear relation between K_t and A_t and that $\frac{\dot{L}_t}{L_t} = 0$. Early endogenous growth models such as the familiar "AK" model adopted this strategy (e.g., Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992).[1] Assuming a linear relation between A_t and $K_t \left(= k_t L_t\right)$ means that

$$\frac{A_t}{k_t} = \frac{\varphi_2 K_t}{k_t} = \varphi_2 L_t \quad,$$

where φ_2 is a constant. Hence, L_t plays an important role for growth because, as L_t increases, $\frac{\dot{c}_t}{c_t}$ also increases. This relationship is known as scale effects.

Jones (1995b) adopts a completely different strategy (see also Kortum, 1997; Segerstrom, 1998; Eicher and Turnovsky, 1999), which focuses on the relation between L_t and

---

[1] Early human-capital-based endogenous growth models also belong to this category of models.

A_t instead of that between K_t and A_t. A linear relation between $\dfrac{\dot{A}_t}{A_t}$ and $\dfrac{\dot{L}_t}{L_t}$ is assumed, such

that $\dfrac{\dot{A}_t}{A_t} = \varphi_3 \dfrac{\dot{L}_t}{L_t}$, where φ_3 is a constant. Only the case in which

$$\frac{\dot{K}_t}{K_t} = (1 + \varphi_1 \varphi_3)\frac{\dot{L}_t}{L_t} = \frac{\dot{Y}_t}{Y_t} \qquad (1)$$

is selected to be relevant because only this case simultaneously satisfies the relation $\dfrac{\dot{K}_t}{K_t} - \dfrac{\dot{L}_t}{L_t}$

$= \varphi_1 \dfrac{\dot{A}_t}{A_t}$ and achieves a balanced growth path. This model can eliminate scale effects because

there is no linear relation between K_t and A_t. Instead, the population growth rate $\dfrac{\dot{L}_t}{L_t}$ plays a

crucial role, as equation (1) clearly exhibits. In this sense, Jones's (1995b) model still does not appear to be successful as a model of endogenous growth.

To eliminate the influence of population growth, Young (1998), Peretto (1998), Aghion and Howitt (1998), and Dinopoulos and Thompson (1998) propose a third approach. They assume a relation between $\dfrac{\dot{A}_t}{A_t}$ and L_t such that $\dfrac{\dot{A}_t}{A_t} = \varphi_4 L_t^{1-\varphi_5}$, where φ_4 and φ_5 are

constants. Hence,

$$\frac{\dot{K}_t}{K_t} = \frac{\dot{L}_t}{L_t} + \varphi_1 \varphi_4 L_t^{1-\varphi_5} = \frac{\dot{Y}_t}{Y_t}$$

if the relation $\dfrac{\dot{K}_t}{K_t} - \dfrac{\dot{L}_t}{L_t} = \varphi_1 \dfrac{\dot{A}_t}{A_t}$ holds and the economy is on a balanced growth path. Therefore,

if $\varphi_5 = 1$, the economy grows at the constant rate $\varphi_1 \varphi_4$ even if $\dfrac{\dot{L}_t}{L_t} = 0$; that is, the influence of

population growth and scale effects can both be eliminated. However, Jones (1999) shows that this model crucially depends on a very special assumption, that $\varphi_5 = 1$.

Peretto and Smulders (2002) take a fourth approach. They assume that $A_t L_t$ and K_t are positively linked instead of A_t and K_t, and

$$\lim_{L_t \to \infty} A_t L_t = \varphi_6 K_t \quad,$$

where φ_6 is a constant. Hence, $\lim\limits_{L_t \to \infty} \dfrac{A_t}{k_t} = \lim\limits_{L_t \to \infty} \dfrac{A_t L_t}{K_t} = \varphi_6$, and the scale effects asymptotically

vanish. In addition, population growth is unnecessary for economic growth unlike in the non-scale model developed by Jones (1995b).

The model developed in this paper employs this fourth approach, but the mechanism through which the relation $\lim\limits_{L_t \to \infty} A_t L_t = \varphi_6 K_t$ emerges is fundamentally different from that of

Peretto and Smulders (2002). The key assumption in Peretto and Smulders (2002) is that uncompensated knowledge spillovers diminish as the number of firms (and thus the population)

increases. However, this assumption is problematic because the concepts of Marshall-Arrow-Romer (MAR) externalities (Marshall, 1890; Arrow, 1962; Romer, 1986) and Jacobs externalities (Jacobs, 1969) both predict that, if the number of firms increases, uncompensated knowledge spillovers will also increase. Hence, the key assumption of Peretto and Smulders (2002) contradicts the theory of knowledge spillover. This problem arises primarily because they neglect Jacobs externalities and focus only on the negative side of MAR externalities; that is, as the number of sectors increases, knowledge spillovers will work less effectively. Many empirical studies support the existence of Jacobs externalities (e.g., Glaeser et al., 1992; Chen, 2002; Stel and Nieuwenhuijsen, 2002), and neglecting them will heavily bias the structure of model.

The model in this paper, in contrast to that of Peretto and Smulders (2002), is consistent with knowledge spillover theory because uncompensated knowledge spillovers are assumed to increase when the number of firms increases. This reversed direction of the effect of knowledge spillovers could potentially make scale effects much worse, but it does not because of substitution between investments in capital and technology. Firms will invest more in capital than in technology if returns on investing in technology become less attractive because of an increase in uncompensated knowledge spillovers arising from an increased number of firms. Decisions about whether to invest in capital or technology are made by firms that compare returns on investing in capital ($\frac{\partial Y_t}{\partial K_t}$) with those in technology ($\psi \frac{\partial Y_t}{\partial A_t}$), where ψ is a variable that indicates the degree of uncompensated knowledge spillovers, that is, how much a firm that invests in technology can obtain from $\frac{\partial Y_t}{\partial A_t}$ as the return on generating a new technology. By arbitrage, both returns are equalized such that $\frac{\partial Y_t}{\partial K_t} = \psi \frac{\partial Y_t}{\partial A_t}$. If ψ is constant, familiar scale effects emerge. However, as mentioned above, the theory of knowledge spillover predicts that uncompensated knowledge spillovers and the number of firms are positively correlated. Hence, the theory indicates that ψ should not be constant but a function of the number of firms, and it decreases as the number of firms increases. The model in this paper explicitly incorporates this concept and exhibits the relation $\lim_{L_t \to \infty} A_t L_t = \varphi_6 K_t$. As a result, the model can eliminate both scale effects and the influence of population growth.

The paper is organized as follows. In Section 2, the production of technology and uncompensated knowledge spillovers are examined, and an endogenous growth model that incorporates substitution between investments in capital and technology is constructed. Section 3 shows that scale effects asymptotically diminish as population increases and shows that population growth is unnecessary for economic growth in the model. Concluding remarks are offered in Section 4.

2. THE MODEL

2.1 *Production of technologies*

Outputs Y_t are the sum of consumption C_t, the increase in capital $\dot{K}_t$, and the increase in technology $\dot{A}_t$ such that

$$Y_t = C_t + \dot{K}_t + v\dot{A}_t \quad .$$

Thus,

$$\dot{k}_t = y_t - c_t - \frac{v\dot{A}_t}{L_t} - n_t k_t \quad ,$$

where $v(>0)$ is a constant, and a unit of K_t and v^{-1} of a unit of A_t are equivalent; that is, they are produced using the same quantities of inputs (capital, labor, and technology). This means that technologies are produced with capital, labor, and technology in the same way as consumer goods and services and capital. Unlike most idea-based growth models, no special mechanism is required for the production of technology because endogenous balanced growth (i.e., constant $\frac{A_t}{k_t}$) is not materialized by any special property of the production function of technology but by uncompensated knowledge spillovers and arbitrage between investments in capital and technology.

      Because balanced growth paths are the focal point of this paper, Harrod-neutral technical progress is assumed.[2] Hence, the production function is $Y_t = K_t^{1-\alpha}(A_t L_t)^\alpha$; thus,

$$y_t = A_t^\alpha k_t^{1-\alpha} \quad .$$

Population has an upper boundary and $n_t = 0$ after some future period; thus, $\lim_{t\to\infty} n_t = 0$ and $\lim_{t\to\infty} L_t = \bar{L}$, where $\bar{L}$ is a positive constant.

2.2   *Substitution between investments in K_t and A_t*

For any period,

$$m = \frac{M_t}{L_t} \quad , \tag{2}$$

where M_t is the number of firms (which are assumed to be identical) and $m\ (> 0)$ is a constant. Equation (2) presents a natural assumption that the population and number of firms are positively correlated. In addition, for any period,

$$\frac{\partial Y_t}{\partial K_t} = \frac{\varpi}{M_t^{1-\rho}} \frac{\partial Y_t}{\partial(vA_t)} \quad ; \tag{3}$$

thus,

$$\frac{\partial y_t}{\partial k_t} = \frac{\varpi L_t^\rho}{m^{1-\rho} v} \frac{\partial y_t}{\partial A_t} \tag{4}$$

is always kept, where $\varpi(>1)$ and $\rho(0 \le \rho < 1)$ are constants. The parameter ρ describes the effect of uncompensated knowledge spillovers, and the parameter ϖ indicates the effect of patent protection. With patents, incomes are distributed not only to capital and labor but also to

---

[2] As is well known, only Harrod-neutral technological progress matches the stylized facts presented by Kaldor (1961). As Barro and Sala-i-Martin (1995) argue, technological progress must take the labor-augmenting form in the production function if the models are to display a steady state.

technology. For simplicity, the patent period is assumed to be indefinite, and no capital depreciation is assumed. An extended model with a finite patent period and capital depreciation is examined in Section 3.5.

Equations (3) and (4) indicate that returns on investing in capital and technology for the investing firm are kept equal. The driving force behind the equations is that firms exploit all opportunities and select the most profitable investments at all times. Through arbitrage, this behavior leads to equal returns on investments in capital and technology. With substitution between investments in capital and technology, the model exhibits endogenous balanced growth.

Because $\dfrac{\varpi}{mv}\dfrac{\partial y_t}{\partial A_t} = \dfrac{\partial y_t}{\partial k_t} \Leftrightarrow \dfrac{\varpi L_t^\rho \alpha}{m^{1-\rho}v}A_t^{\alpha-1}k_t^{1-\alpha} = (1-\alpha)A_t^\alpha k_t^{-\alpha}$, $A_t = \dfrac{\varpi L_t^\rho \alpha}{m^{1-\rho}v(1-\alpha)}k_t$ by equations (2)

and (3), which lucidly indicates that $\dfrac{A_t}{k_t}$ = constant, and the model can therefore show balanced

endogenous growth.

2.3 *Uncompensated knowledge spillovers*

Equations (3) and (4) also indicate that the investing firm cannot obtain all the returns on its investment in technology. That is, although investment in technology increases Y_t, the investing

firm's returns are only a fraction of the increase of Y_t, such that $\dfrac{\varpi}{M_t^{1-\rho}}\dfrac{\partial Y_t}{\partial(vA_t)}$, because

knowledge spills over to other firms without compensation and other firms possess complementary technologies.

Broadly speaking, there are two types of uncompensated knowledge spillovers: intra-sectoral knowledge spillovers (MAR externalities; Marshall, 1890; Arrow, 1962; Romer, 1986) and inter-sectoral knowledge spillovers (Jacobs externalities; Jacobs, 1969). MAR theory assumes that knowledge spillovers between homogenous firms are the most effective and that spillovers will primarily emerge within sectors. As a result, uncompensated knowledge spillovers will be more active if the number of firms within a sector is larger. On the other hand, Jacobs (1969) argues that knowledge spillovers are most effective among firms that practice different activities and that diversification (i.e., a variety of sectors) is more important in influencing spillovers. As a result, uncompensated knowledge spillovers will be more active if the number of sectors in the economy is larger. If all sectors have the same number of firms, an increase in the number of firms in the economy results in more knowledge spillovers in any case, as a result of either MAR or Jacobs externalities.

As uncompensated knowledge spillovers increase, the investing firm's returns on

investment in technology decrease. $\dfrac{\partial Y_t}{\partial A_t}$ indicates the total increase in Y_t in the economy by an

increase in A_t, which consists of increases in both outputs of the firm that invested in the new technologies and outputs of other firms that utilize the newly invented technologies, regardless of whether the firms obtained the technologies by compensating the originating firm or through uncompensated knowledge spillovers. If the number of firms increases and uncompensated

knowledge spillovers increase, the compensated fraction in $\dfrac{\partial Y_t}{\partial A_t}$ that the investing firm can

obtain becomes smaller, as do its returns on the investment in technology. The parameter ρ describes the magnitude of this effect. If $\rho = 0$, the investing firm's returns are reduced at the same rate as the increase of the number of firms. $0 < \rho < 1$ indicates that the investing firm's returns diminish as the number of firms increase but not to the same extent as when $\rho = 0$.

Both types of externalities predict that uncompensated knowledge spillovers will increase as the number of firms increases, and scale effects have not actually been observed

(Jones, 1995a), which implies that scale effects are almost canceled out by the effects of MAR and Jacobs externalities. Thus, the value of ρ is quite likely to be very small. From the point of view of a firm's behavior, a very small ρ appears to be quite natural. Because firms intrinsically seek profit opportunities, newly established firms work as hard as existing firms to profit from knowledge spillovers. An increase in the number of firms therefore indicates that more firms are trying to obtain the investing firm's technologies. In addition, a larger number of firms indicates that firms are more specialized. More specialized and formerly neglected technologies may become valuable to the larger number of specialized firms. Hence, knowledge spillovers will increase. As a whole, competition over technologies will increase as the number of firms increases. Through more fierce competition, uncompensated knowledge spillovers will also increase, eventually to the point that they increase at the same rate as the increase in the number of firms. The investing firm's fraction will thereby also be reduced at the same rate as the increase of the number of firms, which means that ρ will naturally decrease to zero as a result of firms' profit-seeking behavior.

Complementary technologies also reduce the fraction of $\dfrac{\partial Y_t}{\partial A_t}$ that the investing firm can obtain. If a new technology is effective only if it is combined with other technologies, the returns on investment in the new technology will belong not only to the investing firm but also to the firms that possess the other technologies. For example, an innovation in computer software technology generated by a software company increases the sales and profits of computer hardware companies. The economy's productivity increases because of the innovation but the increased incomes are attributed not only to the firm that generated the innovation but also to the firms that possess complementary technologies. A part of $\dfrac{\partial Y_t}{\partial A_t}$ leaks to these firms, and the leaked income is a kind of rent revenue that unexpectedly became obtainable because of the original firm's innovation. Most new technologies will have complementary technologies. Because of both complementary technologies and uncompensated knowledge spillovers, the fraction of $\dfrac{\partial Y_t}{\partial A_t}$ that an investing firm can obtain on average will be very small; that is, ϖ will be far smaller than M_t except when M_t is very small.[3]

2.4 The optimization problem

Because $\quad A_t = \dfrac{\varpi L_t^\rho \alpha}{m^{1-\rho} v (1-\alpha)} k_t$, then $\quad y_t = \left(\dfrac{\varpi L_t^\rho \alpha}{m^{1-\rho} v}\right)^\alpha (1-\alpha)^{-\alpha} k_t \quad$ and $\quad \dot{A_t} = \dfrac{\varpi L_t^\rho}{m^{1-\rho} v} \dot{k_t}\left(\dfrac{\alpha}{1-\alpha}\right)$. Hence,

$$\dot{k_t} = y_t - c_t - \frac{v\dot{A_t}}{L_t} - n_t k_t = \left(\frac{\varpi L_t^\rho \alpha}{m^{1-\rho} v}\right)^\alpha (1-\alpha)^{-\alpha} k_t - c_t - \frac{\varpi L_t^\rho}{m^{1-\rho} L_t}\dot{k_t}\left(\frac{\alpha}{1-\alpha}\right) - n_t k_t \quad \text{and}$$

$$\dot{k_t} = \frac{m^{1-\rho} L_t (1-\alpha)}{m^{1-\rho} L_t (1-\alpha) + \varpi L_t^\rho \alpha}\left[\left(\frac{\varpi L_t^\rho \alpha}{m^{1-\rho} v}\right)^\alpha (1-\alpha)^{-\alpha} k_t - c_t - n_t k_t\right] . \tag{5}$$

---

[3] If M_t is very small, the value of ϖ will be far smaller than that for sufficiently large M_t because the number of firms that can benefit from an innovation is constrained owing to the very small M_t. The very small number of firms indicates that the economy is not sufficiently sophisticated, and thereby the benefit of an innovation cannot be fully realized. This constraint can be modeled as $\varpi = \widetilde{\varpi}\left[1 - \left(1 - \widetilde{\varpi}^{-1}\right)^{M_t}\right]$, where $\widetilde{\varpi}(\geq 1)$ is a constant. Nevertheless, for sufficiently large M_t (i.e., in sufficiently sophisticated economies), the constraint is removed such that $\lim\limits_{M_t \to \infty} \widetilde{\varpi}\left[1 - \left(1 - \widetilde{\varpi}^{-1}\right)^{M_t}\right] = \widetilde{\varpi} = \varpi$.

As a whole, the optimization problem of the representative household is to maximize the expected utility

$$E \int_0^\infty u(c_t) \exp(-\theta t) dt$$

subject to equation (5) where $u(\bullet)$ is a constant relative risk aversion (CRRA) utility function and E is the expectation operator.

3. AN ASYMPTOTICALLY NON-SCALE BALANCED GROWTH PATH

3.1 Growth rate and transversality condition

Let Hamiltonian H be

$$H = u(c_t)\exp(-\theta t) + \lambda_t \frac{m^{1-\rho} L_t(1-\alpha)}{m^{1-\rho} L_t(1-\alpha) + \varpi L_t^\rho \alpha} \left[\left(\frac{\varpi L_t^\rho \alpha}{m^{1-\rho} v} \right)^\alpha (1-\alpha)^{-\alpha} k_t - c_t - n_t k_t \right],$$

where λ_t is a costate variable. The optimality conditions for the optimization problem shown in the previous section are

$$\frac{\partial u(c_t)}{\partial c_t}\exp(-\theta t) = \lambda_t \frac{m^{1-\rho} L_t(1-\alpha)}{m^{1-\rho} L_t(1-\alpha) + \varpi L_t^\rho \alpha} \tag{6}$$

$$\dot{\lambda}_t = -\frac{\partial H}{\partial k_t} \tag{7}$$

$$\dot{k}_t = \frac{m^{1-\rho} L_t(1-\alpha)}{m^{1-\rho} L_t(1-\alpha) + \varpi L_t^\rho \alpha} \left[\left(\frac{\varpi L_t^\rho \alpha}{m^{1-\rho} v} \right)^\alpha (1-\alpha)^{-\alpha} k_t - c_t - n_t k_t \right] \tag{8}$$

$$\lim_{t \to \infty} \lambda_t k_t = 0 \quad . \tag{9}$$

By equation (7),

$$\dot{\lambda}_t = -\lambda_t \frac{m^{1-\rho} L_t(1-\alpha)}{m^{1-\rho} L_t(1-\alpha) + \varpi L_t^\rho \alpha} \left[\left(\frac{\varpi L_t^\rho \alpha}{m^{1-\rho} v} \right)^\alpha (1-\alpha)^{-\alpha} - n_t \right] \quad . \tag{10}$$

Hence, by equations (6) and (10), the growth rate of consumption is

$$\frac{\dot{c}_t}{c_t} = \varepsilon^{-1} \left\{ \frac{m^{1-\rho} L_t(1-\alpha)}{m^{1-\rho} L_t(1-\alpha) + \varpi L_t^\rho \alpha} \left[\left(\frac{\varpi L_t^\rho \alpha}{m^{1-\rho} v} \right)^\alpha (1-\alpha)^{-\alpha} - n_t \right] - \theta \right\},$$

where $\varepsilon = -\dfrac{c_t u''}{u'}$. Note that usually $\left(\dfrac{\varpi L_t^\rho \alpha}{m^{1-\rho}v}\right)^\alpha (1-\alpha)^{-\alpha} - n_t > 0$, so this is the case examined in this paper.

By equation (8), $\dfrac{\dot{k}_t}{k_t} = \dfrac{m^{1-\rho}L_t(1-\alpha)}{m^{1-\rho}L_t(1-\alpha)+\varpi L_t^\rho \alpha}\left[\left(\dfrac{\varpi L_t^\rho \alpha}{m^{1-\rho}v}\right)^\alpha (1-\alpha)^{-\alpha} - n_t - \dfrac{c_t}{k_t}\right]$, and by

equation (10), $\dfrac{\dot{\lambda}_t}{\lambda_t} = -\dfrac{m^{1-\rho}L_t(1-\alpha)}{m^{1-\rho}L_t(1-\alpha)+\varpi L_t^\rho \alpha}\left[\left(\dfrac{\varpi L_t^\rho \alpha}{m^{1-\rho}v}\right)^\alpha (1-\alpha)^{-\alpha} - n_t\right]$. Hence,

$$\frac{\dot{\lambda}_t}{\lambda_t} + \frac{\dot{k}_t}{k_t} = -\frac{m^{1-\rho}L_t(1-\alpha)}{m^{1-\rho}L_t(1-\alpha)+\varpi L_t^\rho \alpha}\left(\frac{c_t}{k_t}\right) \quad .$$

Therefore, if $\dfrac{c_t}{k_t} > 0$ for any period, then $\dfrac{\dot{\lambda}_t}{\lambda_t} + \dfrac{\dot{k}_t}{k_t} < 0$ because $0 < L_t \le \overline{L}$, and transversality condition (9) is satisfied. Conversely, if $\dfrac{c_t}{k_t} = 0$ for any period after a certain period, the transversality condition is not satisfied.

3.2 *Balanced growth path*

There is a balanced growth path on which all the optimality conditions are satisfied.

Lemma: If and only if $\lim\limits_{t\to\infty}\dfrac{\dot{c}_t}{c_t} = \lim\limits_{t\to\infty}\dfrac{\dot{k}_t}{k_t}$, all the conditions (equations [5]–[8]) are satisfied.

Proof: (Step 1) $\lim\limits_{t\to\infty}\dfrac{\dot{c}_t}{c_t} = \varepsilon^{-1}\left\{\dfrac{m^{1-\rho}\lim\limits_{t\to\infty}L_t(1-\alpha)}{m^{1-\rho}\lim\limits_{t\to\infty}L_t(1-\alpha)+\varpi\lim\limits_{t\to\infty}L_t^\rho \alpha}\left[\left(\dfrac{\varpi\lim\limits_{t\to\infty}L_t^\rho \alpha}{m^{1-\rho}v}\right)^\alpha (1-\alpha)^{-\alpha}\right] - \theta\right\}$.

Because population has an upper boundary and $\lim\limits_{t\to\infty}L_t = \overline{L}$, $\lim\limits_{t\to\infty}\dfrac{\dot{c}_t}{c_t} =$ constant. On the other

hand, $\lim\limits_{t\to\infty}\dfrac{\dot{k}_t}{k_t} = \dfrac{m^{1-\rho}\lim\limits_{t\to\infty}L_t(1-\alpha)}{m^{1-\rho}\lim\limits_{t\to\infty}L_t(1-\alpha)+\varpi\lim\limits_{t\to\infty}L_t^\rho \alpha}\left[\left(\dfrac{\varpi\lim\limits_{t\to\infty}L_t^\rho \alpha}{m^{1-\rho}v}\right)^\alpha (1-\alpha)^{-\alpha} - \lim\limits_{t\to\infty}\dfrac{c_t}{k_t}\right]$.

(Step 2) If $\lim\limits_{t\to\infty}\dfrac{\dot{k}_t}{k_t} > \lim\limits_{t\to\infty}\dfrac{\dot{c}_t}{c_t}$, then $\dfrac{c_t}{k_t}$ diminishes as time passes because $\lim\limits_{t\to\infty}\dfrac{\dot{c}_t}{c_t} =$ constant by

(Step 1) while $\lim\limits_{t\to\infty}\dfrac{\dot{k}_t}{k_t}$ increases by (Step 1). Thus, eventually $\dfrac{c_t}{k_t}$ diminishes to zero, and as

shown in Section 3.1, transversality condition (9) is not satisfied.

If $\lim\limits_{t\to\infty}\dfrac{\dot{k}_t}{k_t} < \lim\limits_{t\to\infty}\dfrac{\dot{c}_t}{c_t}$, then $\dfrac{c_t}{k_t}$ increases indefinitely as time passes because $\lim\limits_{t\to\infty}\dfrac{\dot{c}_t}{c_t} =$

constant by (Step 1) while $\lim\limits_{t\to\infty}\dfrac{\dot{k}_t}{k_t}$ diminishes and eventually becomes negative by (Step 1).

Hence, k_t decreases and eventually equation (8) is violated because $k_t \ge 0$.

On the other hand, if $\lim\limits_{t\to\infty}\dfrac{\dot{c}_t}{c_t}=\lim\limits_{t\to\infty}\dfrac{\dot{k}_t}{k_t}$, then $\lim\limits_{t\to\infty}\dfrac{c_t}{k_t}$ is constant; thus, $\lim\limits_{t\to\infty}\dfrac{\dot{k}_t}{k_t}$ and $\lim\limits_{t\to\infty}\dfrac{\dot{c}_t}{c_t}$ are identical and constant because $\lim\limits_{t\to\infty}\dfrac{\dot{c}_t}{c_t}=$ constant by (Step 1). ■

Rational households will set an initial consumption that leads to the growth path that satisfies all the conditions. The Lemma therefore indicates that, given an initial A_0 and k_0, rational households will set the initial consumption c_0 so as to achieve the growth path that satisfies $\lim\limits_{t\to\infty}\dfrac{\dot{c}_t}{c_t}=\lim\limits_{t\to\infty}\dfrac{\dot{k}_t}{k_t}$, while firms will adjust k_t so as to achieve $\dfrac{\partial Y_t}{\partial K_t}=\dfrac{\varpi}{M_t^{1-\rho}}\dfrac{\partial Y_t}{\partial(vA_t)}$.[4] With this household behavior, the growth rates of technology, per capita output, consumption, and capital converge at the same rate.

Proposition: If all of the optimality conditions (equations [5]–[8]) are satisfied,

$$\lim\limits_{t\to\infty}\dfrac{\dot{y}_t}{y_t}=\lim\limits_{t\to\infty}\dfrac{\dot{A}_t}{A_t}=\lim\limits_{t\to\infty}\dfrac{\dot{c}_t}{c_t}=\lim\limits_{t\to\infty}\dfrac{\dot{k}_t}{k_t}\ .$$

Proof: Because $y_t=A_t^\alpha k_t^{1-\alpha}$, $\dot{y}_t=\left(\dfrac{A_t}{k_t}\right)^\alpha\left[(1-\alpha)\dot{k}_t+\alpha\dfrac{k_t}{A_t}\dot{A}_t\right]$. Since $\dot{A}_t=\dfrac{\varpi L_t^\rho}{m^{1-\rho}v}\dot{k}_t\left(\dfrac{\alpha}{1-\alpha}\right)$,

then $\dot{y}_t=\dot{k}_t\left(\dfrac{A_t}{k_t}\right)^\alpha\left[(1-\alpha)+\dfrac{\varpi L_t^\rho\alpha^2}{m^{1-\rho}v(1-\alpha)}\dfrac{k_t}{A_t}\right]$; thus, $\dfrac{\dot{y}_t}{y_t}=\dfrac{\dot{k}_t}{k_t}\left[(1-\alpha)+\dfrac{\varpi L_t^\rho\alpha^2}{m^{1-\rho}v(1-\alpha)}\dfrac{k_t}{A_t}\right]$.

Since $A_t=\dfrac{\varpi L_t^\rho\alpha}{m^{1-\rho}v(1-\alpha)}k_t$, $\dfrac{\dot{y}_t}{y_t}=\dfrac{\dot{k}_t}{k_t}[(1-\alpha)+\alpha]=\dfrac{\dot{k}_t}{k_t}$. Therefore, $\lim\limits_{t\to\infty}\dfrac{\dot{y}_t}{y_t}=\lim\limits_{t\to\infty}\dfrac{\dot{c}_t}{c_t}=\lim\limits_{t\to\infty}\dfrac{\dot{k}_t}{k_t}$. In

addition, since $\dot{y}_t=\dot{A}_t\left(\dfrac{A_t}{k_t}\right)^\alpha\left[\dfrac{m^{1-\rho}v(1-\alpha)^2}{\varpi L_t^\rho\alpha}+\alpha\dfrac{k_t}{A_t}\right]$ by $y_t=A_t^\alpha k_t^{1-\alpha}$ and $\dot{A}_t=\dfrac{\varpi L_t^\rho\alpha}{m^{1-\rho}v(1-\alpha)}\dot{k}_t$,

then $\dfrac{\dot{y}_t}{y_t}=\dfrac{\dot{A}_t}{k_t}\dfrac{m^{1-\rho}v(1-\alpha)^2}{\varpi L_t^\rho\alpha}+\alpha\dfrac{\dot{A}_t}{A_t}$. Because $\dot{A}_t=\dfrac{\varpi L_t^\rho\alpha}{m^{1-\rho}v(1-\alpha)}\dot{k}_t$, then $\dfrac{\dot{y}_t}{y_t}=(1-\alpha)\dfrac{\dot{k}_t}{k_t}+\alpha\dfrac{\dot{A}_t}{A_t}$.

Thereby, $\dfrac{\dot{y}_t}{y_t}=\dfrac{\dot{k}_t}{k_t}=(1-\alpha)\dfrac{\dot{k}_t}{k_t}+\alpha\dfrac{\dot{A}_t}{A_t}$ and $\lim\limits_{t\to\infty}\dfrac{\dot{k}_t}{k_t}=\lim\limits_{t\to\infty}\dfrac{\dot{A}_t}{A_t}=\lim\limits_{t\to\infty}\dfrac{\dot{A}_t}{A_t}=$ constant. Hence, by the

Lemma, $\lim\limits_{t\to\infty}\dfrac{\dot{y}_t}{y_t}=\lim\limits_{t\to\infty}\dfrac{\dot{A}_t}{A_t}=\lim\limits_{t\to\infty}\dfrac{\dot{c}_t}{c_t}=\lim\limits_{t\to\infty}\dfrac{\dot{k}_t}{k_t}$ if all the optimality conditions are satisfied. ■

This balanced growth path can be seen as a natural extension of the steady state in the conventional Ramsey growth model with exogenous technology.

---

[4] Arbitrage conditions (3) and (4) indicate that until $A_t=\dfrac{\varpi L_t^\rho\alpha}{m^{1-\rho}v(1-\alpha)}k_t$ is achieved, no investment is made in technology if $A_0>\dfrac{\varpi L_t^\rho\alpha}{m^{1-\rho}v(1-\alpha)}k_0$ and in capital if $A_0<\dfrac{\varpi L_t^\rho\alpha}{m^{1-\rho}v(1-\alpha)}k_0$.

3.3   Growth without population increase

The model indicates that population growth is not necessary for economic growth. If $n_t = 0$ for any period,

$$\frac{\dot{c}_t}{c_t} = \varepsilon^{-1}\left\{\frac{m^{1-\rho}L_t(1-\alpha)}{m^{1-\rho}L_t(1-\alpha)+\varpi L_t^\rho \alpha}\left[\left(\frac{\varpi L_t^\rho \alpha}{m^{1-\rho}v}\right)^\alpha(1-\alpha)^{-\alpha}\right]-\theta\right\} = \text{constant}$$

because L_t = constant. Clearly, $\frac{\dot{c}_t}{c_t}$ is irrelevant to n_t and is positive even though $n_t = 0.^5$ This result is important because it indicates that the economy can grow endogenously and indefinitely at a constant rate without population growth, which contrasts with the non-scale model shown in Jones (1995b).

3.4   Asymptotically diminishing scale effects

The model also indicates that scale effects asymptotically diminish as population increases. Scale effects are measured by

$$S(L_t) = \varepsilon\frac{\dot{c}_t}{c_t}+\theta \quad ;$$

that is, by the population related part of $\frac{\dot{c}_t}{c_t}$. In the model,

$$S(L_t) = \frac{m^{1-\rho}L_t(1-\alpha)}{m^{1-\rho}L_t(1-\alpha)+\varpi L_t^\rho \alpha}\left[\left(\frac{\varpi L_t^\rho \alpha}{m^{1-\rho}v}\right)^\alpha(1-\alpha)^{-\alpha}-n_t\right] \quad . \tag{11}$$

If $\frac{dS(L_t)}{dL_t} > 0$, scale effects exist, and if $\frac{dS(L_t)}{dL_t} = 0$, no scale effect exists. Equation (11) indicates that scale effects depend on the value of $\rho(0 \le \rho < 1)$ and the population.

   If $\rho = 0$, scale effects vanish asymptotically as population increases. When the population is small,

$$S(L_t) = \frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\varpi L_t\alpha}\left[\left(\frac{\varpi \alpha}{m^{1-\rho}v}\right)^\alpha(1-\alpha)^{-\alpha}-n_t\right] \quad .$$

Hence $\frac{dS(L_t)}{dL_t} > 0$ and scale effects exist. However,

$$\lim_{\bar{L}\to\infty}\lim_{L_t\to\bar{L}}S(L_t) = \left(\frac{\varpi \alpha}{m^{1-\rho}v}\right)^\alpha(1-\alpha)^{-\alpha} \quad , \tag{12}$$

---

[5] As mentioned above, usually $\left(\frac{\varpi L_t^\rho \alpha}{m^{1-\rho}v}\right)^\alpha(1-\alpha)^{-\alpha}-n_t > 0$, so this is the only case examined in this paper.

because $\displaystyle\lim_{\overline{L} \to \infty} \lim_{L_t \to \overline{L}} \frac{m^{1-\rho} L_t (1-\alpha)}{m^{1-\rho} L_t (1-\alpha) + \varpi L_t^\rho \alpha} = 1$ and $\displaystyle\lim_{L_t \to \overline{L}} n_t = 0$. Hence,

$$\lim_{\overline{L} \to \infty} \lim_{L_t \to \overline{L}} \frac{dS(L_t)}{dL_t} = 0 \quad.$$

As the population increases, scale effects asymptotically disappear. An economy with a sufficiently large population therefore can grow without scale effects.

Nevertheless, if $\rho \neq 0$, scale effects continue to influence growth to some extent even though the population is sufficiently large because, if $0 < \rho < 1$,

$$\lim_{\overline{L} \to \infty} \lim_{L_t \to \overline{L}} S(L_t) = \left(\frac{\varpi \overline{L}^\rho \alpha}{m^{1-\rho} v} \right)^\alpha (1-\alpha)^{-\alpha} \quad;$$

thus, $\displaystyle\lim_{\overline{L} \to \infty} \lim_{L_t \to \overline{L}} \frac{dS(L_t)}{dL_t} > 0$. As ρ becomes closer to unity, the effect of scale becomes more clearly observable. However, as argued in Section 2.3, the value of ρ is probably very small and scale effects are not actually observed (Jones, 1995a) because they are almost canceled out by the effects of MAR and Jacobs externalities. Because of firms' profit-seeking behavior, ρ will naturally decrease to zero. Therefore, as equation (12) shows, scale effects will become asymptotically negligible as the population becomes sufficiently large.

Equations (11) and (12) indicate that scale effects are economically important if the size of population is very small (i.e., the number of firms is very small), which implies that scale effects played a crucial role in early human history. Conversely, in present-day industrialized economies, scale effects have been observed to have no influence on growth (Jones, 1995a) because these economies are integrated with the world economy and have a large total population.

3.5   *Extension to a finite patent period*

In the previous sections, for simplicity, an indefinite patent period was assumed and capital depreciation was not taken into consideration. In this section, these assumptions are relaxed. Let $\chi(>0)$ be the length of the patent period and $\delta(>0)$ be the rate of capital depreciation. After the patent period of a technology ends, the price of the technology is zero and the returns on investment in that technology are also zero indefinitely. Thereby, after the end of patent period, the increased income generated by use of the technology is only distributed to owners of capital and labor, not to the owner of the technology. Hence, the total return on investment in technology to the investing firm during the patent period is

$$\chi \frac{\varpi L_t^\rho}{m^{1-\rho} v} \frac{\partial y_t}{\partial A_t} \quad.$$

Next, because capital depreciates by δ every period, the total return on investment in capital to the investing firm during the entire period is

$$\frac{\partial y_t}{\partial k_t} \int_0^\infty e^{-\delta s} ds = \delta^{-1} \frac{\partial y_t}{\partial k_t} \quad.$$

12

Through the arbitrage between investments in capital and technology,

$$\delta^{-1}\frac{\partial y_t}{\partial k_t} = \chi \frac{\varpi L_t^\rho}{m^{1-\rho}v}\frac{\partial y_t}{\partial A_t} \quad ;$$

thus,

$$\frac{\partial y_t}{\partial k_t} = \delta \chi \frac{\varpi L_t^\rho}{m^{1-\rho}v}\frac{\partial y_t}{\partial A_t} \quad . \tag{13}$$

Therefore, in an economy with a finite patent period and capital depreciation, equation (4) is replaced with equation (13). Equation (13) clearly shows that the original model's conclusion still holds with a finite patent period and capital depreciation. In many countries, the patent period is 20 or more years (i.e., $\chi \geq 20$), and the useful life of capital is usually about 20 years (a depreciation rate of about 0.05). For $\chi = 20$ and $\delta = 0.05$, $\delta\chi = 1$, which means that equation (4) and (13) will be practically identical for reasonable patent periods and depreciation rates. In this situation, it appears reasonable to assume for simplicity that the patent period is indefinite and the rate of capital depreciation is zero.

Although 20 years have been used as the patent period in many countries, there may be other possibilities. Because

$$\lim_{\overline{L}\to\infty}\lim_{L_t\to\overline{L}}\frac{\dot{c}_t}{c_t} = \varepsilon^{-1}\left[\left(\delta\chi\frac{\varpi L_t^\rho \alpha}{m^{1-\rho}v}\right)^\alpha (1-\alpha)^{-\alpha} - \delta - \theta\right] \quad ,$$

as the patent period χ increases, the growth rate of consumption increases if the population is sufficiently large. This result suggests that the patent period should be indefinite. However, this is not the case because

$$\lim_{\chi\to\infty}\frac{\dot{c}_t}{c_t} = \lim_{\chi\to\infty}\varepsilon^{-1}\left\{\frac{m^{1-\rho}L_t(1-\alpha)}{m^{1-\rho}L_t(1-\alpha)+\delta\chi\varpi L_t^\rho \alpha}\left[\left(\delta\chi\frac{\varpi L_t^\rho \alpha}{m^{1-\rho}v}\right)^\alpha (1-\alpha)^{-\alpha} - \delta - n_t\right] - \theta\right\} = -\frac{\theta}{\varepsilon}$$

for $\overline{L} < \infty$. If the patent period is very long, the growth rate of consumption becomes negative because firms will restrain their accumulation of capital because investments in technology will be much more lucrative as compared with those in capital. Equation (13) indicates that, as $\chi \to \infty$, firms will become extremely tempted to invest in technology rather than in capital and eventually no investment in capital will be made. Therefore, the patent period should be finite to achieve high growth rates.

The optimal length of the patent period depends on the parameter values. In addition, technological obsolescence may also have to be considered because, in many industrial countries, a technology is often replaced with other technologies or demands shift to other goods and services that use other technologies in a period that is shorter than the patent period. If we also consider obsolescence, equation (13) can be replaced with

$$\delta^{-1}\frac{\partial y_t}{\partial k_t} = \frac{\varpi}{mv}\frac{\partial y_t}{\partial A_t}\int_0^\chi \exp(-\mu t)dt \quad ;$$

thus

$$\frac{\partial y_t}{\partial k_t} = \frac{\delta}{\mu}\left[1 - \exp(-\mu\chi)\right]\frac{\varpi}{mv}\frac{\partial y_t}{\partial A_t} \quad,$$

where μ is the obsolescence rate. For example, if $\mu = 0.1$, then $\mu^{-1}\left[1 - \exp(-\mu\chi)\right] = 8.65$ for $\chi = 20$, 9.50 for $\chi = 30$ and 9.82 for $\chi = 40$. If $\mu = 0.15$, then $\mu^{-1}\left[1 - \exp(-\mu\chi)\right] = 9.50$ for $\chi = 20$, 9.89 for $\chi = 30$ and 9.98 for $\chi = 40$. Hence, the value of $\mu^{-1}\left[1 - \exp(-\mu\chi)\right]$ is almost identical if $\chi > 20$, which implies that an approximate 20-year patent period is sufficiently long and practically reasonable.

4.   CONCLUDING REMARKS

Early endogenous growth models (e.g. Romer, 1986, 1987; Lucas, 1988) employed scale effects. Jones (1995b) presents a different type of endogenous growth model that eliminates scale effects, but the population growth rate plays a crucial role for economic growth. Models developed by Young (1998), Peretto (1998), Aghion and Howitt (1998), and Dinopoulos and Thompson (1998) eliminate the influence of population growth as well as scale effects, but Jones (1999) argues that those models crucially depend on a very special assumption. Using a fourth approach, Peretto and Smulders (2002) assume that A_tL_t (instead of A_t) and K_t are positively linked and $\lim_{L_t \to \infty} A_tL_t = \varphi_6 K_t$; thus, scale effects asymptotically vanish.

The model developed in this paper employs this fourth approach, but the mechanism through which the relation $\lim_{L_t \to \infty} A_tL_t = \varphi_6 K_t$ emerges is fundamentally different from that of Peretto and Smulders (2002). The concepts of MAR and Jacobs externalities both predict uncompensated knowledge spillovers will increase as the number of firms increases, and to be consistent with the theory of knowledge spillover, uncompensated knowledge spillovers increase when the number of firms increases in the model presented in this paper. Even though the direction of the effect of knowledge spillovers is reversed, scale effects still asymptotically diminish as they do in Peretto and Smulders' (2002) model because of substitution between investments in capital and technology. Firms invest more in capital than in technology when returns on investment in technology become less attractive because of an increase in uncompensated knowledge spillovers. As a result, the relation $\lim_{L_t \to \infty} A_tL_t = \varphi_6 K_t$ also emerges.

By combining the theory of knowledge spillover and substitution between investments in capital and technology, a new asymptotically non-scale endogenous growth model that can eliminate both scale effects and the influence of population growth was constructed.

Asymptotically diminishing scale effects indicate that, if a population is very small, scale effects greatly influence growth, but if it is sufficiently large, scale effects vanish. This result suggests that scale effects were a crucial factor for economic growth in the early history of civilizations, but they are no longer important in present-day industrialized economies.

References

Aghion, Philippe and Peter Howitt. (1992). "A Model of Growth through Creative Destruction," *Econometrica*, Vol. 60, pp. 323–351.

Aghion, Philippe and Peter Howitt. (1998). *Endogenous Growth Theory*, Cambridge, MA, MIT Press.

Arrow, Kenneth J. (1962). "The Economic Implications of Learning by Doing," *Review of Economic Studies*, Vol. 29, pp. 155–173.

Barro, Robert J. and Xavier Sala-i-Martin. (1995) *Economic Growth*, McGraw-Hill, New York.

Chen, Hsin-Ping. (2002). "Urban Externalities and City Growth in Taiwan," *The Annals of Regional Science*, Vol. 36, pp. 531–550.

Dinopoulos, Elias and Peter Thompson. (1998). "Schumpeterian Growth without Scale Effects," *Journal of Economic Growth*, Vol. 3, pp. 313–335.

Eicher, Theo S. and Stephen J. Turnovsky. (1999). "Non-Scale Models of Economic Growth," *The Economic Journal*, Vol. 109, pp. 394–415.

Glaeser, Edward L., Hedi D. Kallal, Jose A. Scheinkman, and Andrei Shleifer. (1992). "Growth in Cities," *Journal of Political Economy*, Vol. 100, pp. 1126–1152.

Grossman, Gene M. and Elhanan Helpman. (1991). *Innovation and Growth in the Global Economy*, Cambridge, MA, MIT Press.

Harashima, Taiji. (2004). "A New Asymptotically Non-Scale Endogenous Growth Model," *EconWPA Working Papers*, ewp-dev/0412009.

Jacobs, Jane. (1969). *The Economy of Cities,* Random House, New York.

Jones, Charles I. (1995a). "Time Series Test of Endogenous Growth Models," *Quarterly Journal of Economics*, Vol. 110, pp. 495–525.

Jones, Charles I. (1995b). "R&D-Based Models of Economic Growth," *Journal of Political Economy*, Vol. 103, pp. 759–784.

Jones, Charles I. (1999). "Growth: With or Without Scale Effects?" *American Economic Review Papers and Proceedings*, Vol. 89, No.2, pp. 139–144.

Kortum, Samuel S. (1997). "Research, Patenting, and Technological Change," *Econometrica*, Vol. 65, pp. 1389–1419.

Lucas, Robert E. (1988). "On the Mechanics of Economic Development," *Journal of Monetary Economics*, Vol. 22, pp. 3–42.

Kaldor, Nicholas. (1961). "Capital Accumulation and Economic Growth," Chap. 10 of A. Lutz and D. C. Hague (eds.), *The Theory of Capital*, St. Martin's Press, New York.

Marshall, Alfred. (1890). *Principles of Economics,* Macmillan, London.

Peretto, Pietro. (1998). "Technological Change and Population Growth," *Journal of Economic Growth*, Vol. 3, pp. 283–311.

Peretto, Pietro and Sjak Smulders. (2002). "Technological Distance, Growth and Scale Effects," *The Economic Journal*, Vol. 112, pp. 603–624.

Romer, Paul M. (1986). "Increasing Returns and Long-Run Growth," *Journal of Political Economy*, Vol. 94, pp. 1,002–1037.

Romer, Paul M. (1987). "Growth Based on Increasing Returns Due to Specialization," *American Economic Review Papers and Proceedings*, Vol. 77, pp. 56–72.

Romer, Paul M. (1990). "Endogenous Technological Change," *Journal of Political Economy*, Vol. 98, pp. S71–S102.

Segerstrom, Paul. (1998). "Endogenous Growth without Scale Effects," *American Economic Review*, Vol. 88, pp. 1290–1310.

Stel, Andre and Henry Nieuwenhuijsen. (2002). "Knowledge Spillovers and Economic Growth," *Tinbergen Institute Discussion Paper*, TI 2002-051/3.

Young, Alwyn. (1998). "Growth without Scale Effects," *Journal of Political Economy*, Vol. 106, pp. 41–63.