Outsourcing and R&D Investment with Costly Patent Protection

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Abstract

We analyze decisions of firms on outsourcing of intermediate goods and R&D investment. If firms choose in-house production, a high profit discount is incurred due to the inefficiency of producing the intermediate goods, whereas if firms search for and outsource to specialists, the production costs decrease, but an imitation risk arises by specialists, who may become competitors in the final-good market. Accordingly, patents are used to mitigate this possibility, which are costly. We show that in outsourcing, all firms outsource to the same specialist to minimize the possibility of successful imitation in equilibrium. Moreover, firms still invest in R&D activities and outsource their intermediate goods with some patent protection even though the selected specialist put effort into imitation.

1 Introduction

Outsourcing has been widely used by firms in many industries as a prevalent way to reduce production costs and increase profits. By outsourcing the intermediate-good production (even the whole production process) to intermediate-good specialized producers (henceforth specialists), firms can lower production costs and spend more on R&D activities, which consequently increases their profits. At the same time, outsourcing is also socially preferred, as it organizes production of both intermediate and final goods more efficiently and thus generates higher productivity with lower social costs.

However, a vast amount of literature illustrates different issues on outsourcing. Specifically, information leakage has been addressed recently as one of the most important aspects of outsourcing.

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In outsourcing, before intermediate good production, firms may have to disclose the whole or partial details of the product to specialists, in particular, technology and assembling requirements. In this case, information leakage, which is not only related to the core technology of the product, but also the design and other aspects of the product, becomes unavoidable. Once the details of the product have been delivered, it becomes difficult for firms to monitor how specialists use the information. Even if it is possible to monitor, a high cost would be incurred by firms.

Information leakage may give an opportunity for intermediate-good specialized producers to learn and imitate the firms’ products (core technology and product designs), and this is difficult for firms to verify. Even if firms keep details of the product in secrecy, it is still relatively easy for specialists to do reverse engineering when they obtain partially disclosed information. Successful imitation not only provides specialists an opportunity for “free riding”, but also a chance to become rivals to compete with firms in the final-product market. A typical example of imitation by specialists can be seen in the consumer electronic industry. During the 1960s and 1970s, outsourcing of intermediate goods was ubiquitous among American electronic companies as one of the cost reduction strategies. Firms such as Emerson Radio, RCA, Zenith, and Magnavox outsourced their intermediate-good production to some Japanese companies, in order to gain the benefit of low labor costs in production. However, after obtaining technological competency, the Japanese companies launched their own R&D to imitate the products, and then market competition between the American and Japanese firms started when these Japanese firms successfully imitated and sold their final products back to the U.S. market. By the end of the 1960s, the number of Japanese companies significantly outweighed the U.S ones in this industry.

In this article, we model the tradeoff for firms between outsourcing intermediate goods and protecting the secrecy of the product information. In a monopolistically competitive market, all firms invest in cost-cutting technology and choose to either outsource or produce in house their intermediate goods. On the one hand, if firms choose in-house production, a relatively high profit discount is incurred to produce the intermediate goods by themselves due to the inefficiency of intermediate-good production, which lowers firms’ profits in the market. On the other hand, if firms search for and outsource to intermediate-good specialists, they can reduce the costs in intermediate-good production, but have to face the risk of imitation by specialists who may accordingly become competitors in the final-good market. For simplicity, we assume that if specialists imitate successfully, then they can produce similar products without infringement, and the new similar products and the original products of firms are substitutes to consumers in the final-good market. Consequently, each firm has to manufacture the intermediate inputs by itself, which surely reduces firms’ market shares and profits. After the intermediate goods are produced, no matter whether by specialists or in-house production, each firm assembles them with final parts produced by itself, and sells the

1Besides this, a mushrooming amount of literature related to solving the “hold-up problem” has also been created.  
As discussed above, the possibility of imitation by specialists is a crucial concern when firms make the decision on outsourcing. It may be argued that this imitation would potentially enforce technology diffusion and competition in the final-good market, from which consumers would benefit. However, this possibility decreases the *ex ante* incentive for firms to outsource and invest in R&D, or even causes them to choose production in house which increases the social cost of production. Therefore, it becomes necessary for firms to protect their production information in order to reduce the possibility of imitation in outsourcing.

In our model, we introduce the standard property right protection method — patents — to protect the final products before outsourcing intermediate goods. Throughout this article, patent protection is a broader concept than is commonly used. Patents are used not only for technology protection, but also other aspects which are particularly related to the product, such as model design and appearance design. By rewarding this exclusive property right, obviously, even though patents cannot entirely eliminate the possibility of successful imitation in outsourcing, they can increase the cost and difficulty of imitation, and hence R&D activities are encouraged in outsourcing. This effect discourages specialists from putting effort into imitation, and then mitigates the corresponding possibility. However, patenting is not an easy task and the cost associated with this activity has to be paid by patent applicants. This cost includes money and time in investigating the complexity of the product, and the patenting process. Lemley (2001) lists specific application fees and estimating fees for patent protection, and shows that the patenting cost is not trivial for patent applicants. Thus, when firms choose patents to protect their outsourced production in outsourcing, the patenting cost is an important variable that cannot be ignored.

Given that costly patent protection can be used in outsourcing, we separately study firms’ strategies in patent protection and R&D investment, and the imitation strategy of specialists. We show that there exists a unique solution in equilibrium such that each firm still invests in R&D activities and outsources its intermediate goods with a certain degree of patent protection, even though specialists will put some effort into imitation and the possibility of successful imitation in outsourcing is positive.

Furthermore, we also analyze the searching strategy of firms in the outsourcing market. Rather than matching different specialists, our model gives a more intuitive prediction that firms who compete in a final-good market often coordinately hire the same intermediate-good specialist. This is because by using the same specialist, firms can minimize information leakage, the number of potential imitators and the probability of successful imitation. Moreover, this prediction is also consistent with the observations in outsourcing markets. One typical example is described in Dean 3

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3In fact, firms can also outsource their sales and marketing to specialists, but it is beyond the scope of our article. Thus, in the model we still assume that firms are involved in market competition of final goods.

4In the reality of patenting, the relevant costs are increasing as the patented innovations become more complicated. See [http://www.uspto.gov/web/offices/ac/qs/ope/fee2009september15.htm](http://www.uspto.gov/web/offices/ac/qs/ope/fee2009september15.htm) for more details.

5Baccara (2007) also shows this concentration in outsourcing markets. However, our interpretation differs from
(2007)'s report: Foxconn Technology Group, one of the biggest Original Equipment Manufacturers (OEM) in the manufacture of electronics and computer components, does outsourcing of desktop PCs/parts for Dell and Hewlett-Packard, who are competitors in the personal computer market; Iphone for Apple, and cellphones/parts for Motorolla and Nokia, who are competitors in the mobile market; and lastly PlayStation2 video game consoles and PSP handheld game units for Sony, and Wii videogame consoles and DS game units for Nintendo, who are competitors in the video game market.

Looking backward to the stage of “outsourcing or in house production”, we discuss the variables that determine firms' decision on outsourcing. As can be seen above, reducing production cost is one of the main reasons to conduct outsourcing. We show that the magnitude of the profit discount (inefficiency) of intermediate-good production for firms plays an important role in the outsourcing decision. If the degree of inefficiency in producing intermediate goods in house is high enough for firms, then even if there exists a positive probability of successful imitation by the specialist, firms are still willing to take the risk and choose to outsource their intermediate goods.

Lastly, we analyze the effect of the patenting cost on the outsourcing market and policy implications. In fact, besides the essential social cost related to patenting, charging a fee (as part of the patenting cost) as an important method to encourage R&D activities has also been utilized by policymakers. Hunt (1999), U.S. Federal Trade Commission (2003) and Hunt (2006) show that adding a patent fee/tax or raising the requirements for award of patents can increase the difficulty and costs for firms of obtaining patent protection, and thus change the incentive for innovation. Our model suggests that a reasonable level of patenting cost may need to be imposed to maximize social welfare.

□ Related literature. The “make-or-buy” decision as the boundary of a firm was originally studied by Coase (1937), and beginning with Williamson (1975, 1985), a mushrooming amount of literature has been created to solve this decision-making problem.\textsuperscript{6} This literature mainly focuses on the bilateral relationship, specifically, on the “hold-up problem”, which is caused by specific rights over assets and incomplete contracts. Seminal articles on this issue include Klein, Crawford, and Alchian (1978), Grossman and Hart (1986), Hart and Moore (1990) and Aghion and Tirole (1994). Focusing on the same issue but using a different modeling approach, Grossman and Helpman (2002) construct a general equilibrium model to explore the “make-or-buy” decision in an industrial structure. Their model provides an intuitive method to study the interdependence among firms and industrial structure when outsourcing is conducted. However, according to Holmström and Roberts (1998), the current explanation for the boundary of a firm may be too limited for a general case.

\textsuperscript{6}A similar issue about whether to conduct international outsourcing or domestic production in the literature of international trade has also been discussed. More details can be seen in Antràs (2003), Antràs and Helpman (2004), Antràs (2005) and Grossman and Helpman (2005).
Information leakage has been considered recently as another important aspect of the decision on “outsourcing or in-house production” for intermediate goods. Baccara (2007) considers the implications of information leakage in an outsourcing market when firms outsource their professional tasks to external contractors. She claims that information leakage concentrates the outsourcing market to a single contractor, which induces a monopoly information market. Lai, Riezman, and Wang (2009) study the outsourcing of innovation from firms to research subcontractors. They show that information leakage reduces the incentive for R&D outsourcing, and also suggest that the incentive for R&D outsourcing and economic efficiency can be increased by using stronger protection of trade secrecy.

Although the modeling framework and the discussed points are not the same, our article shares the common view of these articles that when outsourcing is conducted, specialists may make unverified profits from involuntary information leakage. Besides this, we consider a further step that, by using information leakage from firms, specialists may imitate the final product and become market competitors against firms. In addition, we investigate how firms make the ex ante decision on “outsourcing or in-house production”, and if outsourcing is conducted, how much R&D investment and how strong patent protection should be implemented by firms to mitigate the possibility of imitation. Furthermore, we also discuss the policy implication of patent protection in outsourcing.

The remainder of the article is organized as follows. Section 2 introduces the setup of the model. In Section 3, we show the unique solution of R&D investment, number of patents of firms and imitation effort of specialists, when outsourcing is conducted, and the decision making on “outsourcing or in-house production” is analyzed. Section 4 discusses the policy implications of the patenting cost. Section 5 concludes, and the Appendix including the proofs of propositions follows.

2 The model

■ Firms. Consider a monopolistically competitive market with \( m \) firms, where \( m \) is a finite number, each of whom faces a downward sloping inverse demand curve \( y(p) \). For convenience, we assume that the functional form of the demand curve has the property of constant elasticity: \( y(i) = A_m p(i)^{-\epsilon} \), where \( i \) is the identity of the firms, \( \epsilon \in (1, \infty) \) represents the absolute price elasticity of the demand for goods, and \( A_m \) is the scaling constant which represents the market size captured by firm \( i \). \( A_m \) decreases when there are more firms in the final-good market (\( m \) increases). Furthermore, we assume that the monopolistically competitive firms produce final goods with a constant marginal cost \( \mu \). Therefore, the profit of firm \( i \) can be written as \( \pi(i) = y(i)[p(i) - \mu] = A_m p(i)^{-\epsilon}[p(i) - \mu] \).

Denoting that \( \alpha \equiv 1 - 1/\epsilon \) and after maximizing \( \pi(i) \) with respect to \( p(i) \), the equilibrium price and quantity for the goods are \( p(i)^* = \mu/\alpha \) and \( y(i)^* = A_m (\mu/\alpha)^{-1/\alpha} \) respectively, implying that the maximized market profit of firm \( i \) is \( \pi(i)^* = R_m \mu^{-\alpha} \), where \( R_m \equiv A_m (1 - \alpha)(1/\alpha)^{-1/\alpha} \).

Moreover, we consider a cost-cutting technology of producing final goods regarding to this market profit function. Let \( \lambda \) denote the proportion of reduction in the marginal cost \( \mu \), where
\(\lambda \in (0, 1]\). In other words, \(\lambda\) represents the measurement of progress in the cost-cutting technology of final-good production. Then, we can rewrite the market profit of firm \(i\) with technological progress in final goods as follows: 

\[\pi(i)^*(\lambda) = R_m(\lambda \mu)^{-\frac{\alpha}{1-\alpha}}.\]

For computational simplicity, we define 

\[k(i) = (\lambda \mu)^{-\frac{\alpha}{1-\alpha}},\]

and call it the R&D-activity level. Obviously, as \(\lambda\) decreases, \(k(i)\) and \(\pi(i)^*\) increase. In the market, \(\lambda\) is one of the variables that firm \(i\) has to endogenously decide in its strategy profile. Moreover, \(\lambda\) is one-to-one mapping to the R&D-activity level \(k(i)\). Thus, in this article, we only need to focus on the R&D-activity level \(k\), and assume that to achieve \(k\), an increasingly convex cost \(f(k)\) has to be incurred, where \(f'(k) > 0, f''(k) > 0\) and \(f(\mu^{-\frac{\alpha}{1-\alpha}}) = 0.7\).

To complete and sell the products in the market, two compulsory components should be produced: intermediate and final goods, which are combined according to a fixed ratio. At the beginning, only firms have capability to produce final goods. For convenience, we assume that firms assemble the two components (intermediate and final goods) with zero cost. Firms have two options to organize the production of intermediate goods: either to produce them in house or outsource them to specialists.

If firm \(i\) chooses in-house production, a proportion of its market profit has to be paid for intermediate-good production due to the inefficiency of in-house production compared to using specialists, namely \((1 - \delta)R_m k(i)\), where \(\delta\) is the fixed discounting factor and \(\delta \in [0, 1]\). Thus, in this case, the market profit of firm \(i\) is given by \(\delta R_m k(i)\).

If outsourcing occurs, firm \(i\), denoted as \(H\), has to search for and outsource its intermediate goods to a specialist. In the outsourcing market, information leakage cannot be avoided. Therefore, before technology delivery, patents will be used to protect the outsourced products. Let \(n_i\) denote the number of patents and each patent costs \(c > 0\), where \(n_i \in [0, \infty)\). Then the total patenting costs are \(n_i c\). Note that this patent protection system includes all aspects of the products, namely, it includes not only the cost reduction technology innovated by the R&D investment of firms, but also other aspects of the products, such as appearance and model designs. To simplify the model, we also assume that there does not exist patent overlapping problem among firms.\(^8\)

\(\Box\) **Specialists.** The outsourcing market consists of \(q\) identical specialized producers, where \(q\) is a finite number. We normalize the production cost of intermediate goods of the specialists to be zero. Let \(O\) denote the intermediate-good specialist \(j\) selected by \(H\) (firm \(i\)). When outsourcing is conducted, \(H\) discloses the product requirements and (partial) details to the specialist \(O\), but there are no ex ante enforceable contracts between them.\(^9\) After obtaining the (partial) product details, and before producing the intermediate goods, \(O\) can put effort \(\theta\) into imitating the outsourced

\(^7\)Note that each firm may or may not have an identical technology in cost reduction, but the technologies that they adopt generate the same effect to their marginal cost, reflected by the setup that they have the same \(\mu\) and innovation cost function \(f(k)\).

\(^8\)Hunt (2006) considers a case where R&D investment among firms may decrease when there exists patent overlapping.

\(^9\)The setting of no ex ante enforceable contracts in outsourcing and its explanation can be seen in Antràs (2003), Antràs and Helpman (2004), Antràs (2005) and Grossman and Helpman (2005).
products. This imitation is associated with cost $g(\theta_j)$, which is an increasingly convex function: 

$$g(\theta_j) = \frac{1}{2}c_\theta \theta_j^2,$$

where $c_\theta > 0$.\(^{10}\)

The probability of successful imitation, denoted as $P$, is endogenously determined by two factors: the number of patents $n_i$ from $H$ (firm $i$) and the imitation effort $\theta_j$ from $O$ (specialist $j$). On the one hand, the more patents ($n_i$) a firm applies, the lower probability ($P$) the imitation can succeed. On the other hand, if the specialist contributes more $\theta$ to imitation, this probability ($P$) should increase. Therefore, in order to capture both effects, we assume that the probability of successful imitation is governed by the function:

$$P(n_i, \theta_j) = b\theta_j e^{-\gamma n_i}, \forall n_i, \theta_j,$$

where $b$ and $\gamma$ are scaling constants, so that $P \in [0, 1]$. With probability $P$, $O$ successfully imitates $H$’s products and then it enters the final-good market and produces substitutes against $H$’s without infringement. In this case, $O$ becomes a competitor in the final-good market and $H$ has to produce the intermediate goods by itself. With probability $1 - P$, $O$ fails in imitation and has to follow the outsourcing instruction to manufacture the intermediate goods. After completing and selling the products in the market, $H$ and $O$ split the market profits by fractions $\beta$ and $1 - \beta$ in a Nash bargaining game, where $\beta \in (0, 1)$.

Furthermore, in order to solve the model, we make the following two assumptions:

**Assumption 1.** $(1 - \beta)mR_m < R_{m+1}$.

**Assumption 2.** $\beta > \delta$.

The first assumption ensures that $O$ has sufficient incentive to invest in imitation. Otherwise, $O$ puts $\theta_j = 0$ and $H$ does not do patenting ($n_i = 0$), and thus outsourcing will be the equilibrium outcome because there is no possibility of imitation. The second assumption guarantees that outsourcing is always preferred to production in house by $H$ if there is no information leakage and imitation. Moreover, in this case, patent protection becomes unnecessary.

\( \Box \) **Timing of the game.** The timing of the game is illustrated in Figure 1. At the first stage, each firm $i$ ($H$) decides to choose either outsourcing or in-house production for its intermediate goods. If $H$ chooses production in house, then it decides the R&D-activity level $k_i$ and incurs the proportion $1 - \delta$ of the market profit to produce the intermediate goods. By combining the self-produced goods, $H$ produces the final goods and obtains the payoff in the $m$-firm market (a final product market with $m$ competitors). If $H$ chooses outsourcing, it firstly searches for a specialist $j$ ($O$), and then decides the R&D-activity level $k_i$ and the number of patents $n_i$. Next, this firm transfers the technology, the blue prints and the requirements of intermediate goods to the selected specialist.

\(^{10}\)Imitation is not entirely free for specialists, and imitation costs, which could incur a large amount of expenditure, have to be paid, including expert training fees, imitation experiment fees, and so on. A related article can be seen in Gallini (1992), who studies costly imitation in patented technology.
At the second stage, $O$ invests $\theta_j$ to imitate the whole products of $H$. If imitation is successful, there will have one more competitor engaged in the final-good market competition. In this case, $H$ needs to produce the intermediate goods by itself. If imitation fails, $H$ and $O$ operate a Nash bargaining game with fractions $\beta$ and $1 - \beta$, respectively, over the market profit, after all intermediate goods are produced and the final goods are completed and sold.

3 The equilibrium

Search, R&D investment, patents and imitation in outsourcing. In this section, we use backward induction to solve the game. Before we obtain the equilibrium values of $k_i$, $n_i$ and $\theta_j$ in outsourcing, it is important to investigate how a firm searches for a specialist in the outsourcing market. Moreover, we suppress the identity subscripts $i$ and $j$ when they do not cause notational confusion, as they specifically represent a firm ($H$) and a specialist ($O$). Therefore, along the equilibrium path of outsourcing, we claim that:

Proposition 1. No matter which search procedure a firm uses to match a specialist, if patent protection is costly, in equilibrium it is optimal for all firms to outsource their intermediate goods to a single specialist in the outsourcing market.\(^{11}\)

\(^{11}\)To simplify the formation of the outsourcing market, we assume that the specialists who are not searched for by
The proof is straightforward. Given a non-trivial patenting cost \(c\), the probability of successful imitation is greater than zero, \(P > 0\). Suppose that \(q\) specialists have been selected by all firms in outsourcing market, and then the probability of all \(q\) specialists failing in imitation is \((1 - P)^q\). Therefore, the probability that at least one of the specialists has successful imitation and thus becomes a competitor in the final-good market is \(1 - (1 - P)^q\). However, if all firms outsource to a single specialist, then the probability for this specialist being successful in imitation is only \(P\). Obviously, \(1 - (1 - P)^q > P\). Therefore, it is optimal for all firms to outsource to the same specialist.

This result shares the same prediction in Baccara (2007), but the implications are different. In her model, the main incentive for a firm to choose the same specialist is to gain information leakage (technology) from other firms. However, we show that costly patent protection is another reason why we observe concentration in an outsourcing market. When patenting costs have to be paid, the number of patents a firm can use to protect the outsourced products becomes limited. Then, in order to minimize the possibility of successful imitation by specialists, prevent new entry in the final-good market and maintain the market profits as large as possible, all firms have to coordinately select the same intermediate-good specialist, as in the example discussed in the Introduction. Furthermore, it is also obvious to see that if the patenting cost is zero, we have the following strategy where all firms will play in the outsourcing market:

**Corollary 1.** If \(c = 0\), firms apply for an infinite number of patents and outsourcing of their intermediate goods is always preferred. Moreover, there is no “concentrated” specialist in the outsourcing market.

If \(c = 0\), \(n\) approaches infinity. Then \(\theta = 0\) and \(P = 0\). Therefore, the probability that there exists at least one specialist who successfully imitates the firms’ technology is given by \(1 - (1 - P)^q = 0\), which is equivalent to the probability of successful imitation when only a single specialist is selected. Thus, if outsourcing is chosen in this case, the firms are indifferent in outsourcing to one or more than one specialists.

Now we turn to investigate the strategy profile \((k_i, n_i, \theta_j)\) for the firms and the specialist when outsourcing is conducted. Given \(H\)’s searching strategy, when outsourcing occurs, the payoff function for \(H\) under the patent scheme is:

\[
\max_{k,n} H = k[P\delta R_{m+1} + (1 - P)\beta R_m] - nc - f(k),
\]

where \(c\) is the unit cost for patenting, \(R_{m+1}\) denotes the scaling constant of the market profit for each final-good competitor when \(O\) imitates successfully, and there are \(m + 1\) firms in the final-good the firms leave the outsourcing market.
market. In contrast, the payoff function for $O$ under patent scheme is presented as:

$$\max_\theta O = k[PR_{m+1} + (1 - P)(1 - \beta)mR_m] - g(\theta),$$ (3)

recalling that $P = b\theta e^{-\gamma n}$, $g(\theta)$ is the imitation cost incurred by $O$, and $O$ simultaneously does outsourcing of intermediate goods for $m$ firms if its imitation fails. Therefore, solving the game by backward induction yields:

**Proposition 2.** If outsourcing is conducted in equilibrium, all firms outsource their intermediate goods to the same specialist, and there exists a unique solution such that each firm conducts R&D-activity level $k^*$ and chooses $n^*$ patents, and the specialist puts $\theta^*$ into imitation. In this equilibrium, the probability of successful imitation is positive, namely, $P^* = b\theta^* e^{-\gamma n^*} > 0$, where $n^*$, $k^*$ and $\theta^*$ are uniquely determined by the following equation system:

$$\frac{\theta^*}{be^{-\gamma n^*}k^*[R_{m+1} - (1 - \beta)mR_m]} = c\theta,$$

$$-2\gamma e^{-2\gamma n^*}(k^*)^2 \tau = c,$$

and

$$2e^{-2\gamma n^*}k^* \tau + \beta R_m = f'(k^*),$$

where $\tau = \frac{b^2(R_{m+1} - (1 - \beta)mR_m)(\delta R_{m+1} - \beta R_m)}{c\theta}.$

*Proof.* See the Appendix.

Proposition 2 shows the behaviors of all firms and the selected specialist in the outsourcing market. In this equilibrium, even though imitation from the specialist cannot be completely forbidden, R&D activities can still be conducted in outsourcing. Of course, patent protection becomes necessary for the firms to implement R&D activities.

As we have seen above, in outsourcing, the patenting cost plays an important role in not only the search strategy of the firms, but also their level of patent protection in R&D activities. Thus, in the following, we check the effects of the patenting cost $c$ on $\theta$, $k$ and $P$, when outsourcing is chosen in equilibrium. equations (4), (5) and (6) yield the following proposition:

**Proposition 3.** In equilibrium where outsourcing is conducted, an increase in the patenting cost $c$ induces: (a) a decrease in the R&D-activity level $k$; (b) an increase in the imitation effort $\theta$; and (c) an increase in probability $P$.

*Proof.* See the Appendix.
It is easy to understand the intuition in Proposition 3. When the patenting cost increases, it becomes more costly for the firms to obtain patent protection, which makes the R&D activities more insecure from imitation and thus reduces the incentive of the firms to invest in R&D. However, at the same time, the more costly patent protection also encourages the specialist to conduct more imitation. Hence, the effects of both variables induce an increase in the probability of successful imitation.

Furthermore, we investigate the variables that would have influence on $H$’s R&D-activity level. From the above equilibrium equation system, we can obtain:

**Proposition 4.** In equilibrium where outsourcing is conducted, the R&D-activity level $k^*$ is independent of $\delta R_{m+1}$.

This result is because $k^*[\beta R_m - f'(k^*)] = c$ by substituting equation (5) into (6), which shows that: in the monopolistically competitive market, keeping an advantage in R&D activities is important for a firm to survive; in other words, when a firm decides the investment in R&D activities, even though there could exist a potential new entrant after outsourcing, the main consideration of a firm is competition with other firms in the final-good market. Thus, it seems that firms’ investment in R&D activities is likely to be engaged in a way as Bertrand competition, where the firm who owns the most sophisticated technology (with highest level of $k$) possesses the entire final product market, whereas other firms with lower level of technology are forced to leave the market because they do not earn positive profits. In equilibrium where outsourcing is conducted, therefore, all firms invest in the same R&D-activity level in order to survive in the market. This implies that market competition with incumbents overweights the potential threat from new market entrants, which may explain why $k^*$ is independent of $\delta R_{m+1}$.

□ Outsourcing versus in-house production. Now backward to the first stage, $H$ has to make a decision between in-house production and outsourcing. First, if $H$ chooses to produce in house, it does not need to pay the patenting costs, but a relatively high discount of market profit will be incurred. Then the $H$’s revenue function, when in-house production is chosen, is given by:

$$\max_k H^{IH} = \delta R_m k - f(k).$$

Differentiating the above equation with respect to $k$, the optimal level of R&D activity is $f'^{-1}(\delta R_m)$, and the maximized profit is such that $H^{IH*} = f'^{-1}(\delta R_m)\delta R_m - f(f'^{-1}(\delta R_m))$.

If $H$ does outsourcing, its expected profit can be obtained by substituting equations (4) - (6) into equation (2), such as:

$$H^* = k^*[P^*\delta R_{m+1} + (1 - P^*)\beta R_m] - n^*c - f(k^*).$$

Furthermore, we check the effect of the degree of inefficiency of intermediate-good production on the choice of outsourcing for the firms. Given that $k^*$ is independent of $\delta$ in Proposition 3, we
obtain the following result:

**Proposition 5.** Given $\beta$ and $c$, there exists a unique $\delta^*$ such that, if $\delta \in [0, \delta^*)$, outsourcing of the intermediate goods is conducted in equilibrium; otherwise, in-house production is preferred by the firms.

*Proof.* See the Appendix.

The result in Proposition 5 is intuitive: when $\delta$ is very small, $H$ (firm $i$) needs to pay a relatively high profit discount for intermediate-good production, but by outsourcing $H$ can avoid this cost and raise its revenue, even if the imitation risk by $O$ cannot be eliminated. In other words, the marginal benefit from outsourcing is larger than the marginal risk of imitation. However, when $\delta$ increases, the benefit from outsourcing gradually disappears. Critically, up to $\delta^*$, the benefit from avoiding the inefficiency of production cannot outweigh the risk of imitation, and then $H$ switches to production in house. This finding of the importance of inefficiency in intermediate-good production is consistent with the findings of Aghion and Tirole (1997), Grossman and Helpman (2002), and Acemoglu, Aghion, and Zilibotti (2003), who show that the cost of governance plays the main role in the decision on outsourcing for firms.

## 4 Social welfare

From the above sections, we see that by introducing patent protection, even though it is costly, information leakage and imitation possibility can be mitigated significantly. Patent protection increases the incentive of the firms to outsource their intermediate goods and invest in R&D activities. In this case, by the firms outsourcing their intermediate goods, production can be organized in a more efficient way with lower social cost. In the following, we discuss the policy implication.

The social planner can have impact on the willingness of the firms to outsource intermediate goods and invest in R&D activity level via partially controlling the patenting cost. More specifically, the patenting cost $c$ can be classified into two parts: $c = \chi + C$, where $\chi$ is the social cost associated with money and time of patenting (which has to be paid by patent applicants), and $C$ represents a control variable charged by the social planner. Let $C.S_m$ and $C.S_{m+1}$ represent the consumer surplus of the demand faced by each firm in the markets with $m$ and $m + 1$ final-good producers, respectively, and $W$ denote the social welfare. Then we have that:

$$W = P(m + 1)C.S_{m+1} + (1 - P)mC.S_m + mH + O,$$

where the first two terms on the right of the equation represent the expected consumer surplus, depending on the success of imitation by the specialist $O$, and the last two terms are the producer surplus.
To focus on the policy implication, we normalize $\chi = 0$ and check the effect of the patenting cost on social welfare. The result shows that:

**Proposition 6.** When outsourcing is conducted, a reasonable level of patenting cost needs to be imposed by the social planner so as to maximize total social welfare.

The proposition can be illustrated in details as follows. We omit the identity of firm $i$ and the asterisk * for the equilibrium levels of $n, k$ and $P$ to avoid notational confusion. Given the demand function ($y = A_m p^{-\epsilon}$), the equilibrium price and quantity of the final products are equal to $p = \lambda \mu / \alpha$ and $y = A_m (\lambda \mu / \alpha)^{-\frac{1}{1-\alpha}}$ respectively, which yield $C.S_m$ as follows:

$$C.S_m = \int_0^{A_m (\lambda \mu / \alpha)^{-\frac{1}{1-\alpha}}} A_m^{-\alpha} y^{-(1-\alpha)} dy - A_m (\lambda \mu / \alpha)^{-\frac{\alpha}{1-\alpha}}$$

$$= A_m (\lambda \mu / \alpha)^{-\frac{\alpha}{1-\alpha}} \left( \frac{1}{\alpha} - 1 \right) = \frac{1}{\alpha} R_m k,$$

recalling that $R_m \equiv A_m (1 - \alpha)(1/\alpha)^{-\frac{\alpha}{1-\alpha}}$ and $k \equiv (\lambda \mu)^{-\frac{\alpha}{1-\alpha}}$. Similarly, $C.S_{m+1} = \frac{1}{\alpha} R_{m+1} k$.

Obviously, we see that consumers are better off when the firms invest more in R&D activity (cost-cutting technology), namely, $\partial C.S / \partial k > 0$. This also implies that consumer surplus decreases when $c$ increases, i.e., $C.S'(c) = \frac{\partial C.S}{\partial k} < 0$.

Then by differentiating equation (9) with respect to $c$, we obtain the first order condition of $W$ with respect to $c$:

$$\frac{\partial W}{\partial c} = (m+1) \left( \frac{1}{\alpha} k R_{m+1} \frac{\partial P}{\partial c} + \frac{1}{\alpha} P R_{m+1} \frac{\partial k}{\partial c} + \frac{1}{\alpha} (1-P) R_m \frac{\partial k}{\partial c} - m \frac{1}{\alpha} k R_m \frac{\partial P}{\partial c} + \frac{m}{\alpha} P \frac{\partial H}{\partial c} + \frac{\partial O}{\partial c} \right) = 0.$$

After simplifying the algebra and substitution, we obtain that:

$$\frac{\partial W}{\partial c} = m \left( A \frac{\partial P}{\partial c} + B \frac{\partial k}{\partial c} - n \right) = 0, \quad (10)$$

where

$$A \equiv \left( \frac{1}{m} + \delta + \frac{m + 1 + 1}{m} \alpha \right) k R_{m+1} - \left\{ f'(k) + c \left[ \frac{2 e^{-2\gamma n \tau} - f''(k)}{4 e^{-2\gamma n k \tau}} \right] \right\} \frac{2 [\delta R_{m+1} - \beta R_m]}{f''(k)} > 0,$$

and

$$B \equiv \left( \delta + \frac{m + 1 + 1}{m} \alpha \right) P R_{m+1} + \left[ \frac{1}{\alpha} (1-P) + 1 - P \beta \right] R_m + \frac{1}{m} P (R_{m+1} - (1-\beta) m R_m) \left[ \frac{2 e^{-2\gamma n \tau} - f''(k)}{4 e^{-2\gamma n k \tau}} \right] > 0.$$

The derivation of equation (10) is present in the Appendix. Given that $A$, $B$, and $\partial P/\partial c$ are
positive, and $\partial k/\partial c$ is negative, equation (10) shows that when the social welfare is maximized, a positive level of $c$ needs to be taxed by the social planner.

The interpretation of Proposition 6 can be stated as follows: the social planner faces a trade-off when she adjusts $c$. When $c$ increases, on the one hand, $H$ has to decrease its number of patents in protection, which increases the probability of successful imitation for $O$. In this case, the final-good market is likely to switch to an $m + 1$-firm market, and consumers are better off from the increase in quantity competition of the products. In the meantime, technological diffusion is achieved. On the other hand, an increase in $c$ may discourage $H$ to invest in R&D activities, which consequently reduces the consumer surplus. Thus, there should exist a upper bound $\bar{c}$ for $c$. In contrast, when $c$ decreases, the above argument reverses, then there should exist a lower bound $\underline{c}$ for $c$. Therefore, depending on these effects of $c$, the social planner consequently has to decide a socially optimal value $c$ within the interval $[\underline{c}, \bar{c}]$ if it exists.

5 Conclusions

■ Concluding remarks. In this article, the aim is to investigate how firms make the decisions on “outsourcing and in-house production” and R&D investment, when unavoidable information leakage may induce imitation from intermediate-good specialists. We show that by using patent protection, even if it is costly, outsourcing of the intermediate goods can still be conducted. In equilibrium where outsourcing is conducted, all firms, in order to minimize the probability of successful imitation and avoid severe potential competition, tend to outsource their intermediate goods to a single specialist. In addition, each firm still invests in R&D activities and outsources its intermediate goods with a certain degree of patent protection, even though the specialist will put some effort into imitation and the possibility of successful imitation is positive in outsourcing. Furthermore, we show that in this equilibrium, the firms’ investment level of innovation only depends on the patenting cost, their bargaining power, and the $m$-firm profit.

Moreover, we have also examined the role of inefficiency of producing intermediate goods when the firms make the decision on “outsourcing or in-house production”. We show that given a bargaining power and the patenting cost, the firms choose to outsource their intermediate goods if the discounting factor is small enough, namely, the degree of inefficiency of producing the intermediate goods is sufficiently large. Lastly, we discuss social welfare and the policy implication of outsourcing. In the outsourcing market, the social planner faces the tradeoff between technology diffusion and R&D investment. Strong patent protection (charging a low patenting cost) encourages R&D investment but reduces the possibility of technology diffusion and competition in the

\footnote{In fact, an intuitive assumption which has been made in this model is that $(m + 1)C.S_{m+1} > mC.S_m$. This implies that the more firms there are in the market, the higher consumer surplus will be created. However, if $(m + 1)C.S_{m+1} \leq mC.S_m$, the maximized social welfare may be achieved with $c = 0$ and $n = \infty$. This is because in this case, technological progress ($k$) plays a more important role in the increase in social welfare.}
final-good market, which consequently decreases consumer surplus, and vice versa. We show that to maximize social welfare, a reasonable level of patenting cost may need to be charged when the firms choose to outsource their intermediate goods.

□ Further research. The model presented in this article may be extended in several theoretical directions for future research. Firstly, further work could consider imitation from the specialist in a dynamic environment. In that setting, if the specialist fails in the current imitation, it may try again in the following stages. Or, even if imitation is successful, the specialist still needs more periods to enter the final-good market. Then, it would be interesting to check whether our results still hold in this dynamic setting. Another interesting extension may be to consider the research capability of the specialist after successful imitation. If the specialist cannot undertake further innovations, it is still difficult for it to survive in the final-good competition, given that firms in the final product market may engage in a head-to-head Bertrand competition for R&D activities as the case in Proposition 4. Under this circumstance, the specialist may have less incentive to put effort into imitation at the beginning of the game.

Appendix

□ Proofs for propositions and derivation of equation (10).

Proof of Proposition 2. We solve the model by backward induction. In the second stage, the specialist decides how much effect (θ) should be put into the non-infringement imitation, namely, the best response of the specialist. Taking the first-order condition (F.O.C) with respect to θ so as to maximize O’s profit, we have:

\[
\frac{\partial O}{\partial \theta} = 0, \quad \text{and thus} \quad \theta^* = \frac{be^{-\gamma n}k[R_{m+1} - (1 - \beta)mR_m]}{c_\theta}.
\] (11)

Given Assumption 1, equation (11) shows that \( \theta^* \) is positive, which means that the specialist always invests in imitation, but the imitation investment \( \theta^* \) lowers as the number of patents \( n \) increases.\(^{13}\)

Given the imitation investment \( \theta^* \) by the specialist \( O \), in the first stage firm \( i \)'s expected payoff function in outsourcing is given by:

\[
H(n, k, \theta^*) = k[b\theta^* e^{-\gamma n} \delta R_{m+1} + (1 - b\theta^* e^{-\gamma n}) \beta R_m] - nc - f(k),
\] (12)

where the number of patents \( n \) and the R&D-activity level \( k \) are chosen by \( H \). We take F.O.C for maximizing the profit of \( H \) with respect to \( n \) and \( k \) separately:

\[
\frac{\partial H}{\partial n} = 0 \quad \text{and} \quad \frac{\partial H}{\partial k} = 0.
\] (13)

\(^{13}\)The second order condition is satisfied because \( \frac{\partial^2 O}{\partial \theta^2} = -c_\theta < 0. \)
Thus, the best response \((n^*, k^*)\) is determined by the following equation system:\(^{14}\)

\[
-2\gamma e^{-2\gamma n^* (k^*)^2 \tau} = c, \quad \text{and} \quad 2e^{-2\gamma n^* k^* \tau + \beta R_m - f'(k^*)} = 0, 
\]

where \(\tau = \frac{b^2 (R_{m+1} - (1-\beta)mR_m)(\delta R_{m+1} - \beta R_m)}{c^2} < 0.\)

Apparently, the existence and uniqueness of the solution are also satisfied. Thus, we can conclude that the strategy profile \((n^*, k^*, \theta^*)\) consists of a unique solution if outsourcing is conducted. This completes the proof. Q.E.D.

**Proof of Proposition 3.** Differentiating equations (14) and (15) with respect to \(c\), and with the asterisk \(\ast\) of the variables omitted for convenience, we have the partial differential equation system as follows:

\[
4e^{-2\gamma n k^2 \gamma^2 \tau} \frac{\partial n}{\partial c} - 4e^{-2\gamma n k \gamma \tau} \frac{\partial k}{\partial c} = 1, \quad \text{and} \quad (16)
\]

\[
-4e^{-2\gamma n k \gamma \tau} \frac{\partial n}{\partial c} + (2e^{-2\gamma n \tau - f''(k)}) \frac{\partial k}{\partial c} = 0. \quad (17)
\]

Solving equations (17) and (16), we obtain that:

\[
\frac{\partial n}{\partial c} = \frac{2e^{-2\gamma n \tau} - f''(k)}{-4e^{-2\gamma n k^2 \gamma^2 \tau} [f''(k) + 2e^{-2\gamma n \tau}]} \quad \text{and} \quad (18)
\]

\[
\frac{\partial k}{\partial c} = \frac{-1}{k \gamma [f''(k) + 2e^{-2\gamma n \tau}]} \quad (19)
\]

Intuitively, when \(c\) increases, it becomes more costly for a firm to apply for patent protection in outsourcing, and then the number of patents \(n\) will decrease. This implies that equation (18) should be less than zero, namely, \(\frac{\partial n}{\partial c} < 0\), which also yields that:

\[
[f''(k) + 2e^{-2\gamma n \tau}] > 0. \quad (20)
\]

Obviously, from equation (20), the sign of equation (19) can be determined as \(\frac{\partial k}{\partial c} < 0\). The R&D-activity level decreases as the patenting cost increases.

In the following, we check the signs of change in \(\theta\) and \(P\), when \(c\) increases. First, differentiating

\(^{14}\)The second order conditions are satisfied as \(\frac{\partial^2 H}{\partial n^2} = 4\gamma^2 e^{-2\gamma n k^2 \tau} < 0\), and \(\frac{\partial^2 H}{\partial k^2} = 2e^{-2\gamma n \tau} - f''(k) < 0.\)
\( \theta \) with respect to \( c \) and substituting \( \frac{\partial n}{\partial c} \) and \( \frac{\partial k}{\partial c} \) into the differential equation give that:

\[
\frac{\partial \theta}{\partial c} = \left[ \frac{\partial k}{\partial c} - k\gamma \frac{\partial n}{\partial c} \right] e^{-\gamma n} b(\frac{R_{m+1} - (1 - \beta)mR_m}{c_\theta}) \left[ -4e^{-2\gamma n} k\gamma \tau (f''(k) + 2e^{-2\gamma n} \tau) \right] - \frac{1}{k\gamma (f''(k) + 2e^{-2\gamma n} \tau)} e^{-\gamma n} b(\frac{R_{m+1} - (1 - \beta)mR_m}{c_\theta}) > 0.
\]

(21)

Furthermore, differentiating \( P \) with respect to \( c \) and substituting \( \frac{\partial n}{\partial c} \) and \( \frac{\partial \theta}{\partial c} \) into the differential equation yield:

\[
\frac{\partial P}{\partial c} = \left( \frac{\partial \theta}{\partial c} - \frac{\partial n}{\partial c} \frac{\partial \theta}{\partial \gamma} \right) e^{-\gamma n} b \\
= \frac{b^2(R_{m+1} - (1 - \beta)mR_m)}{-4k\gamma \tau c_\theta} \left[ 1 - \frac{2e^{-2\gamma n} \tau \gamma k\gamma \tau}{f''(k)} \right] \\
= \frac{b^2 f''(k)(R_{m+1} - (1 - \beta)mR_m)}{-2k\gamma \tau c_\theta (f''(k) + 2e^{-2\gamma n} \tau)} > 0.
\]

(22)

Thus, when the patenting cost increases, the effort of imitation by the specialist and the corresponding probability of successful imitation increase. The proof is complete. Q.E.D.

Proof of Proposition 5. We prove the effect of \( \delta \) in this proposition by (i) investigating the monotonicity of \( H^*(\delta) \) and \( H^{1H*}(\delta) \), and (ii) checking their boundaries at \( \delta = 0, 1 \). Again, the asterisk * of the variables is dropped for convenience.

Part (i): First of all, \( k^* \) is independent of \( \delta \) according to Proposition 4. Then, by differentiating equations (14) - (15) with respect to \( \delta \), we have:

\[
4e^{-2\gamma n}k^2\gamma^2 \tau \frac{\partial n}{\partial \delta} - 2e^{-2\gamma n}k^2\gamma \frac{\partial \tau}{\partial \delta} = 0 \quad \text{and},
\]

\[
-4e^{-2\gamma n} k\gamma \tau \frac{\partial n}{\partial \delta} + 2e^{-2\gamma n} k \frac{\partial \tau}{\partial \delta} = 0.
\]

(23)

Given that \( \partial \tau/\partial \delta = \frac{b^2(R_{m+1} - (1 - \beta)mR_m)}{c_\theta} R_{m+1} > 0 \) and \( \tau < 0 \), equations (23) and (24) immediately imply the following:

\[
\frac{\partial n}{\partial \delta} = \frac{1}{2\gamma \tau} \frac{\partial \tau}{\partial \delta} = \frac{b^2 R_{m+1} (R_{m+1} - (1 - \beta)mR_m)}{2\tau c_\theta} < 0.
\]

(25)
Furthermore, replicating the above method for $\theta$ and $P$, we obtain:

\[
\frac{\partial \theta}{\partial \delta} = -k\gamma \left( \frac{\partial n}{\partial \delta} \right) \frac{e^{-\gamma n}b(R_{m+1} - (1 - \beta)mR_m)}{c_\theta} = -\frac{k\beta^2 e^{-\gamma n}R_{m+1}(R_{m+1} - (1 - \beta)mR_m)^2}{2\tau c_\theta^2} > 0 \quad \text{and}
\]

\[
\frac{\partial P}{\partial \delta} = \left( \frac{\partial \theta}{\partial \delta} - \frac{\partial n}{\partial \delta} \theta \right) e^{-\gamma n}b = -\frac{k\beta^2 e^{-\gamma n}R_{m+1}(R_{m+1} - (1 - \beta)mR_m)^2}{2\tau c_\theta^2} > 0.
\]

The effect of $\delta$ on $H^*$ is:

\[
\frac{\partial H^*}{\partial \delta} = k \left[ \frac{\partial P^*}{\partial \delta} \delta R_{m+1} + P^* R_{m+1} - \frac{\partial P^*}{\partial \delta} \beta R_m \right] - \frac{\partial n^*}{\partial \delta}.
\]

Then, substituting equations (26) and (27) into equation (28) yields that $\frac{\partial H^*}{\partial \delta} = -\frac{\partial n^*}{\partial \delta} > 0$, namely, $H^*$ is increasing in $\delta$.

When the firm chooses in-house production, the maximized expected payoff is such that $H^{IH^*} = f^{f^{-1}(\delta R_m)}(\delta R_m) - f(f^{-1}(\delta R_m))$, and taking its derivative with respect to $\delta$ gives: $\partial H^{IH^*}/\partial \delta = \frac{\partial}{\partial \delta} \left[ \frac{R_m}{\delta} + f''(\delta R_m) \left( f^{-1}(\delta R_m) - 1 \right) \right] > 0$. Thus, $H^{IH^*}$ is increasing in $\delta$.

Part (ii): It is now to check the boundaries of $H^*$ and $H^{IH^*}$: When $\delta = 0$, $k > 1$ gives that $H^{IH^*}(0) = -f(f^{-1}(0)) \leq 0$, whereas from equation (25), $H^*(0) = k^*[1-P^*] \beta R_m - n^* c - f(k^*) > 0$. Thus $H^*(0) > H^{IH^*}(0)$. In contrast, when $\delta = 1$, from equations (2) and (3), $R_{m+1} < R_m$ and $\beta < 1$ yield:

\[
H(1) = k[PR_{m+1} + (1 - P)\beta R_m] - nc - f(k) < k[PR_{m+1} + (1 - P)R_m] - nc - f(k) = H^{IH}(1).
\]

Therefore,

\[
H^*(1) = \max_{k,n} H(1) < \max_k H^{IH}(1) = H^{IH^*}(1).
\]

To summarize, given that both $H^*$ and $H^{IH^*}$ are increasing in $\delta$, and $H^*(0) > H^{IH^*}(0)$ and $H^*(1) < H^{IH^*}(1)$, the mean value theorem implies that there exists a unique threshold $\delta^*$ so that $H^*(\delta^*) = H^{IH^*}(\delta^*)$ and outsourcing is conducted if $\delta \in [0, \delta^*)$. The proof is complete.\(^{15}\) Q.E.D.

\(^{15}\)A short version of this proof can be done by using the envelop theorem. Differentiating $H^{IH}$ and $H$ with respect to $\delta$, we have $\frac{\partial H^{IH}}{\partial \delta} > 0$ and $\frac{\partial H}{\partial \delta} > 0$. When $\delta = 0$, $H^{IH} < H$; when $\delta = 1$, $H^{IH} > H$. Thus, there also exists a unique $\delta^*$ such that $H^{IH}(\delta^*) = H(\delta^*)$, and outsourcing is conducted if $\delta \in [0, \delta^*)$.  

Derivation of $\partial W/\partial c$. we substitute the following expressions into the first order condition
of equation (10) with respect to $c$:

\[
\frac{\partial P}{\partial c} = \frac{f''(k)}{2[\delta R_{m+1} - \beta R_m]} \frac{\partial k}{\partial c},
\]

\[
\frac{\partial H}{\partial c} = \frac{\partial k}{\partial c} [P\delta R_{m+1} + (1 - P)\beta R_m] + k \left[ \frac{\partial P}{\partial c} \delta R_{m+1} - \frac{\partial P}{\partial c} \beta R_m \right] - c \frac{\partial n}{\partial c} - f'(k) \frac{\partial k}{\partial c} - n, \quad \text{and}
\]

\[
\frac{\partial O}{\partial c} = \frac{\partial k}{\partial c} [P\delta R_{m+1} + (1 - P)(1 - \beta)m R_m] + k \left[ \frac{\partial P}{\partial c} \delta R_{m+1} - \frac{\partial P}{\partial c} (1 - \beta)m R_m \right] - c \theta \theta \frac{\partial \theta}{\partial c}.
\]

Consequently, by using that $\frac{\partial \theta}{\partial c} = \left[ \frac{\partial k}{\partial c} - k \gamma \frac{\partial n}{\partial c} \right] e^{-\gamma \frac{n}{\theta}} - \frac{f''(k)}{c \theta}$, $\partial W/\partial c$ can be calculated as:

\[
\frac{\partial W}{\partial c} = \left[ \left( \frac{1}{m} + \delta + \frac{m + 1}{m} \right) \delta R_{m+1} - \frac{1}{\alpha + 1} \right] m \frac{\partial P}{\partial c} + \left[ \frac{1}{m} + \frac{1}{\alpha + 1} \right] P \delta R_{m+1} + \left( \frac{1}{\alpha + 1} \right) (1 - P) R_m + P(1 - \beta) R_m - f'(k) \right] m \frac{\partial k}{\partial c}.
\]

We then restrict our focus on the effects of $c$ on $P$ and $k$. From the above Appendix, we also can write $\frac{\partial n}{\partial c} = \frac{\partial n}{\partial c} = \left[ 2e^{-2\gamma n} \tau - f''(k) \right] / \left[ 4e^{-2\gamma n} k \gamma \tau \right] (\partial k/\partial c)$. Substituting this relation into equation (29) yields $\frac{\partial W}{\partial c} = m \left( A \frac{\partial P}{\partial c} + B \frac{\partial k}{\partial c} - n \right) = 0$. Q.E.D.

References


