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Abstract We extend the literature on endogenous lifetime and economic growth by Chakraborty (2004) and Bunzel and Qiao (2005) to endogenous fertility. It is shown that development traps due to under investments in health can never appear when fertility is an economic decision variable.

Keywords Endogenous fertility; Health; Life expectancy; OLG model

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1. Introduction

When endogenous mortality, based on public health investments, is introduced in the overlapping generations (OLG) model of neoclassical growth with exogenous fertility (Diamond, 1965), development traps can occur and, hence, long-run differences in output[1] and longevity across countries emerge when production is relatively capital oriented (Chakraborty, 2004) and the level of technological development is relatively low (Bunzel and Qiao, 2005). In contrast, we show that when both adult mortality and fertility are endogenously determined, the economy always approaches to a unique long-run outcome. Therefore, we argue that to the extent that during the stages of development individuals tend to acquire a more rational wisdom of the choice of the number of children to raise, as suggested by the home economics literature (e.g. Becker, 1960), development traps due to scarce health investments are avoided.

On the one hand, the importance of endogenous fertility on economic growth is well established at least starting from the seminal papers by Becker and Barro (1988), Barro and Becker (1989) and Becker et al. (1990). On the other hand, a recent literature on endogenous mortality and economic growth, that however abstracts from modelling fertility as an individual choice variable, is emerging (see, e.g., Chakraborty, 2004; Bhattacharya and Qiao, 2007; Leung and Wang, 2010). This paper contributes to these two strands of literature and the value added grounds in showing the importance of endogenous fertility as the main determinant of a dramatic change in the dynamical events with respect to an economy with exogenous fertility.[2]

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[1] As regards the literature on cross-country income and growth differentials see, e.g., Durlauf and Johnson (1995) and Fiaschi and Lavezzi (2003, 2007).

[2] Moreover, in an influential paper, Blackburn and Cipriani (2002) linked endogenous fertility and endogenous mortality within an OLG growth context, but assuming, however, human capital accumulation through education (rather than public health investments) as the main determinant of longevity.

The remainder of the paper is organised as follows. In Section 2 we present the OLG model à la Chakraborty (2004) extended with endogenous fertility. In Section 3 we study the dynamic path of capital accumulation and show that multiple equilibria can never appear. Section 5 concludes.

2. The model

Consider a general equilibrium OLG closed economy populated by identical individuals, identical firms and a government that runs a public health programme at a balanced budget. The lifetime of the typical agent is divided into childhood and adulthood, the latter being, in turn, divided into working time (youth) and retirement time (old age). As a child the individual does not make economic decisions. When adult she draws utility from material consumption and the number of children (see, Eckstein and Wolpin, 1985; Galor and Weil, 1996). Young individuals of generation t (N_t) are endowed with one unit of time inelastically supplied on the labour market, while receiving wage income at the rate w_t. It is assumed that the probability of surviving from youth to old age is endogenous and determined by the individual health level, augmented with the public provision of health care services (see Chakraborty, 2004). The survival probability at the end of youth of an individual started working at t, π_t, depends upon her health capital, h_t, and is given by a non-decreasing concave function $\pi_t = \pi(h_t)$, where $\pi(0) = 0$, $\lim_{h \to \infty} \pi(h) = \beta \leq 1$ and $\lim_{h \to 0} \pi'_h(h) = \gamma < \infty$.

We assume that the per worker health investment at t (h_t) is financed at a balanced budget with a (constant) wage income tax $0 < \tau < 1$ (see Chakraborty, 2004), that is:

$$h_t = \tau w_t . \tag{1}$$

Moreover, the costs of children are assumed to be fixed and given by $e > 0$ per child (see, e.g., van Groezen et al., 2003; van Groezen and Meijdam, 2008). Therefore, the budget constraint of an individual of the working-age (child-bearing) generation at t reads as:

$$c_{1,t} + s_t + e n_t = w_t(1-\tau), \tag{2}$$

i.e. wage income – net of contributions paid to finance health expenditure – is divided into material consumption when young, $c_{1,t}$, savings, s_t, and the cost of raising n descendants.

Old individuals are retired and live uniquely with the amount of resources saved when young plus the interest accrued from time t to time $t+1$ at the rate r_{t+1}. The existence of a perfect annuity market (where savings are intermediated through mutual funds) implies that old survivors will benefit not only from their own past saving plus interest, but also from the saving plus interest of those who have deceased. Hence, the budget constraint of an old retired individual started working at t can be expressed as

$$c_{2,t+1} = \frac{1+r}{\pi_t} s_t, \tag{3}$$

where $c_{2,t+1}$ is old-aged consumption.

The representative individual of generation t chooses savings and fertility to maximise the lifetime utility function

$$U_t = \ln(c_{1,t}) + \pi_t \ln(c_{2,t+1}) + \phi \ln(n_t), \tag{4}$$

subject to Eqs. (2) and (3), where $\phi > 0$ captures the parents' relative taste for children.

The constrained maximisation of Eq. (4) gives the demand for children and the saving rate, respectively:

$$n_t = \frac{\phi w_t(1-\tau)}{(1+\pi_t+\phi)e}, \tag{5.1}$$

$$s_t = \frac{\pi_t w_t(1-\tau)}{1+\pi_t+\phi}. \tag{5.2}$$

2.1. Production and equilibrium

Firms are identical and act competitively on the market. Aggregate production takes place according to the constant returns to scale Cobb-Douglas technology $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where Y_t, K_t and $L_t = N_t$ are output, capital and the labour input at time t respectively, $A > 0$ represents a scale parameter and $0 < \alpha < 1$ is the output elasticity of capital. Profit maximisation yields:[3]

$$r_t = \alpha A k_t^{\alpha-1} - 1, \tag{6}$$

$$w_t = (1-\alpha) A k_t^{\alpha}. \tag{7}$$

where $k_t := K_t / N_t$ is capital per worker.

Knowing that $N_{t+1} = n_t N_t$, market-clearing in goods and capital market implies $n_t k_{t+1} = s_t$, that is combined with Eqs. (5.1) and (5.2) to obtain:

$$k_{t+1} = \frac{e}{\phi} \pi(k_t), \tag{8}$$

where $\pi(k_t) = \pi\left[\tau(1-\alpha) A k_t^{\alpha}\right]$. Eq. (8) reveals that if longevity were exogenous there would be no transitional dynamics and the capital stock would therefore approach its steady state after one period only. In the case of endogenous longevity, however, things are different, as is shown below.

3. Dynamics

Analysis of Eq. (8) gives the following proposition:

Proposition 1. *The dynamic system described by Eq. (8) possesses two steady state* $\{0, \bar{k}\}$, *with* $\bar{k} > 0$ *(only the positive one being asymptotically stable).*

Proof. Let first the following lemma be established.

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[3] The price of final output is normalised to unity and capital totally depreciates at the end of each period.

Lemma 1. *Define the right-hand side of (8) as* $G(k)$. *Then, we have: (1.i)* $G(0)=0$, *(1.ii)* $G_k'(k)>0$ *for any* $k>0$, *(1.iii)* $\lim_{k\to+\infty}\frac{G(k)}{k}<1$, *(1.iv)* $\lim_{k\to 0^+} G_k'(k)=+\infty$.

From Eq. (8), (1.*i*) and (1.*ii*) follow immediately. Now, since $\lim_{h\to+\infty}\pi(k)=\beta$, then

$$\lim_{k\to+\infty}\frac{G(k)}{k}=\frac{e}{\phi}\lim_{k\to+\infty}\frac{\pi(k)}{k}=\frac{e\beta}{\phi}\lim_{k\to+\infty}\frac{1}{k}=0,$$

which proves (1.*iii*). Moreover,

$$\lim_{k\to 0^+} G_k'(k)=\frac{e}{\phi}\alpha\tau(1-\alpha)A\lim_{k\to 0^+}\frac{\pi_k'(k)}{k^{1-\alpha}}=\frac{e}{\phi}\alpha\tau(1-\alpha)A\gamma\lim_{k\to 0^+}\frac{1}{k^{1-\alpha}}=+\infty,$$

where we used $\lim_{k\to 0^+}\pi_k'(k)=\gamma$. This proves (1.*iv*).

Proposition 1 therefore follows. In fact, by properties (1.*i*) and (1.*iv*), zero is always an unstable steady state of Eq. (8). By (1.*ii*) and (1.*iii*), $G(k)$ is a monotonic increasing function of k and eventually falls below the 45° line. Since $\pi(k)$ is a non-decreasing concave function of k, then one and only one positive stable steady state exists for any $k>0$. **Q.E.D.**

Comparison of Proposition 1 above with the results of the existing literature gives the importance of our findings. In fact, in an OLG context with exogenous fertility, Chakraborty (2004, Proposition 1 (i), p. 126) showed that a necessary condition for the existence of multiple steady states is $\alpha>1/2$, while Bunzel and Qiao (2005) found that a large enough level of technological development (high values of A) represents a sufficient condition for the existence of at least one positive stable steady state when $\alpha>1/2$, otherwise an economy is permanently entrapped into poverty. Unlike previous findings, development traps due to scarce health investments can never appear when fertility is endogenous, i.e. the unique equilibrium scenario of the Diamond's growth models is restored.

4. Conclusions

In the past few years Chakraborty (2004) and Bunzel and Qiao (2005) have shown that if public investments on health are relatively scarce, development traps can appear in the Diamond overlapping generations model with exogenous fertility and endogenous lifetime. This may explain long-run differences in output and longevity across countries. In this paper we revisited the above-cited literature by extending it with endogenous fertility. To the extent that during the stages of development individuals rationally choose the desired family size by comparing benefits and costs of children, it is shown that development traps due to under investments in health are avoided, i.e. the economy always converges towards a unique long-run outcome.

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