# Asset Pricing with Durable Goods and Nonhomothetic Preferences

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# Asset Pricing with Durable Goods and Non-Homothetic Preferences*

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**Abstract**

I present a consumption-based asset pricing model that is capable of matching the empirically observed Sharpe ratios of the aggregate market portfolio as well as the Fama-French value-minus-growth portfolio. The model also matches the level of the risk-free rate and the equity premium with a plausible aversion to wealth bets. In empirical analysis, the model performs well in explaining the cross section of average returns of the 25 Fama-French portfolios. The model features a novel non-diversifiable macroeconomic source of risk: the distortion of the variety of the consumption portfolio. In the model, investors derive utility from two consumption goods - nondurables and durables - which are perfect complements. The novel consumption risk of the stock market stems from the inability to sell durables in recessions in order to restore the optimal variety of the consumption basket.

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# 1 Introduction

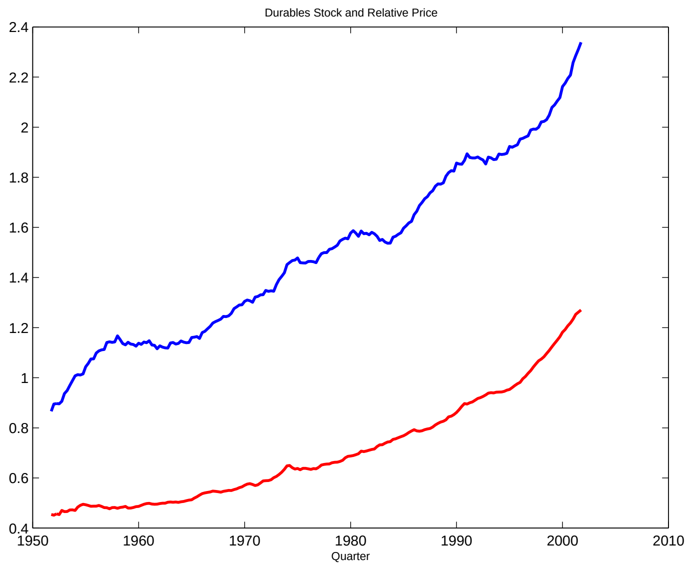
"... many nondurables are used in almost fixed proportions with durables,..."
Grossman and Laroque (1990), p.47

The canonical Consumption-Based CAPM [Breeden (1979), Lucas (1978)] tells us that stocks are risky because they co-vary with consumption growth. Stocks that pay off badly in recessions, defined as times of low consumption growth, are not as desirable and consumers demand larger risk premia to hold them. When aggregate nondurable consumption is used as a measure of consumption, the implied magnitudes of the risk premia do not match the data well [Hansen and Singleton (1982), (1983), Mehra and Prescott (1985)] and the stochastic discount factor is not volatile enough to pass Hansen-Jagannathan bounds [Hansen and Jagannathan (1997)]. One interpretation of this seeming failure is that a model with a single consumption good, nondurables, does not do a good job capturing the actual welfare loss from consumption variation in bad times[1]. Alternative measures of consumption and their effect on marginal utility may do a better job of capturing the risks that agents face. Suppose, for instance, that there is another good that enters consumer preferences. Then, the variation in nondurable consumption may become more costly because it also forces households to adjust on an additional margin. Moreover, if the two goods are complements, a fall in nondurable consumption in recessions will be more costly for consumers than it is in a single-good economy. In equilibrium, assets that pay badly in such periods must offer higher risk premia. This intuition tells us that consumption complementarity could be instrumental in addressing quantitative asset pricing puzzles.

There is a clear secular rise in the consumption of service flow from durables over non-durables (see Figure 1). In fact, the ratio of durables over nondurables more than doubled in the last 50 years. In light of this, it seems to be important to consider service flow from durables in the preference specification. Furthermore, as Grossman and Laroque (1990) point out many nondurables are used in nearly fixed proportions with durables. It is the focus of this paper to explore the quantitative asset-pricing implications of *complementarity* between nondurable consumption, $C_t$, and the service flow from consumer durables, $D_t$, in combination with *non-homotheticity*. A compact summary is that *we need high complementarity between nondurables and service flow to obtain reasonable magnitudes of risk premia with a low coefficient of risk aversion and non-homotheticity to obtain a reasonable relative demand for consumer durables.*

[1] See the influential study of Lucas (1987) for estimates of the welfare cost.

Figure 1: Time Series of Durables Stock and the Relative Price




NOTE - The plot portrays the quarterly time-series of the ratio of durables / nondurables (blue line) and the relative price of nondurables in terms of durables (red line).

Formally, I proceed in a fashion similar to Eichenbaum and Hansen (1990), Heaton (1995) and Ogaki and Reinhart (1998). I introduce another good, the flow of services from a stock of durable household assets $D_t$. The per-period utility is an iso-elastic function of a particular modification of the constant-elasticity-of-substitution (CES) consumption index. This relaxes the implicit restriction on the elasticity of substitution embedded in the Cobb-Douglas consumption index. Using asset pricing and consumption data, I find that the complementarity between nondurables and durables is high and thus the elasticity of intra-temporal substitution is low. When it is difficult to substitute between the two goods the significant change in consumption shares affects the marginal utility of nondurable consumption dramatically. Therefore, stocks that pay off badly in recessions have to offer higher equilibrium risk premia[2].

The specification that I consider also admits non-homotheticity in preferences. It allows the relative demands of the two consumption goods to depend on household income in addition to the relative price of the two goods. It is true that the ratio of the service flow of durables to nondurable consumption has been trending upward over time. The typical interpretation is that this is due to a substitution between nondurables and durables caused by the downward trend in the relative price of durables. The empirical results in the paper indicate that the substitution effects are actually very small and thus the income effects must be very impor-

[2]Cochrane (2001) stresses the role of a recession factor for asset pricing.

tant to get a realistic relative demand function. In this sense, non-homotheticity is dictated by the data rather than an appetite to obtain another degree of freedom. Consumers have been buying more durables because the fall in the relative price of durables increased their real income. In fact, a one percent rise in the real income induced more than one percent rise in the consumption of the services flow, and hence durables are luxury goods. Furthermore, durables are unique in sense that they do not have an easy substitute and thus the Hicksian price effects should be small. Most studies have assumed homothetic preferences [3]. Some imposed subsistence levels but it is not clear a priori how general non-homotheticity such an assumption allows for. Models with homothetic preferences underemphasize income effects and tend to ascribe all changes in the relative demand to a pure substitution effect.

The model features a novel non-diversifiable macroeconomic source of risk: the distortion of the variety of the consumption portfolio. This risk has most dramatic impact on asset prices when durables and nondurables are strong complements. Consumption complementarity has the potential to translate the seemingly small variation in nondurable consumption into large risk premia. One reason this occurs is that in recessions the nondurables consumption and durables investment both fall but the stock of durables, and hence the service flow, does not[4]. In other words, the ratio of the stock of durables over the nondurables is counter-cyclical. This leads to a distortion of the structure and variety of the consumption basket and is costly for the consumer, depending on the complementarity between the nondurables and the service flow from consumer durables. The model implies that the stochastic discount factor is the marginal rate of substitution between non-durable consumption at two consecutive time periods. I find that high consumption complementarity between nondurables and durables dramatically increases the volatility of the stochastic discount factor. Furthermore, the empirical results show that this happens with relatively low concavity of the marginal utility of wealth. In summary, *the model seems to explain the equity premium with time-separable preferences and a relatively low coefficient of risk aversion toward atemporal gambles.*

To better understand how strong consumption complementarity allows to explain asset returns with a low coefficient of risk aversion, recall the first order condition that the marginal utility of wealth equals the marginal utility of nondurable consumption

$$J_W = u_C \tag{1}$$

which states that the consumer is indifferent whether he consumes or saves an additional dollar.

[3]See Ait-Sahalia, Parker and Yogo (2003) for an exciting exception.

[4]Unless, of course, the utilization of the durable assets changes, which does not seem to be the case at least by introspection. In fact, consumers may actually want to increase the durable assets utilization to recuperate the utility losses from the decline in nondurables, which only strengthens the effect.

This condition implies that the variance of the marginal utility of wealth equals the variance of the marginal utility of nondurables

$$var\,(J_W) \;=\; var\,(u_C) \tag{2}$$

Because the variation in financial wealth is a lot larger than the variation in nondurable consumption, this equation implies that a good model should deliver very concave marginal utility of nondurables, $u_{CC}$ high, and little concave marginal utility of wealth, $J_{WW}$ small. I show how high complementarity between the goods achieves exactly this.

In addition, the framework developed in this paper also offers new insights into the behavior of the relative demand function for consumer durables. It shows how to decompose the demand function for consumer durables into substitution and income effect, using rigorous microeconomic analysis. The result may be of independent interest to macroeconomists.

Interestingly, the model exhibits similar properties to habit formation economies, such as the seminal works of Constantinides (1990), Abel (1990) or Campbell and Cochrane (1999), considered widely to be the most successful representative-agent asset pricing model. In detail, both the habit $X_t$ and the stock of durables, due to the its *durability*, are slowly moving variables. Furthermore, the nondurable consumption risk is measured not only by the variation in nondurables $C_t$ but, more importantly, by the departure of nondurables from either the habit level or the durables. In a two-good economy with consumer durables, the consumer cares not only about the nondurable consumption growth $C_{t+1}\,/\,C_t$ but also about the structure and the variety of the consumption portfolio. In fact, the distortion in the consumption variety becomes additional and most significant source of risk. In habit formation economies, the new consumption risk is measured either as $C_t\;/\;X_t$, or as a ratio $C_t\;-\;X_t$, which, in addition, delivers a time variation in the risk aversion coefficient. For comparison, in the model investigated in this article, the distortion in the consumption variety may be gauged, for example, as a ratio of durables over nondurables $D_t\,/\,C_t$ or as $\min\,(C_t, D_t)$. Therefore, the proposed model identifies *periods of economic distress* as periods of a significantly larger decline in nondurable consumption relative to service flow.

Secondly, the felicity function in habit formation economies usually takes the form $u(C_t, X_t) = u(C_t - X_t)$, $u''(\bullet) < 0$ and thus the two goods - habit $X$ and nondurables $C$ - are complements[5]. An increase in the habit level raises the marginal utility of nondurables $u_{CX} \;>\; 0$. It is this consumption complementarity that helps to fit the market price of risk. Motivated by this insight I introduce a CES consumption index that allows for high complementarity between

[5]Note that $u_{CX}(C_t, X_t) \;=\; -u''(C_t, X_t) \;>\; 0$ due to the strict concavity of the felicity function $u(C_t, X_t)$.

nondurables and service flow and show that nondurables and durables become complements, $u_{CD}(C_t, D_t) > 0$. The advantage of durable goods is that they are arguably more tightly linked to macroeconomic data.

## 2 Related Literature

There is an exciting related literature. Specifically, Eichenbaum and Hansen (1990) and Ogaki and Reinhart (1998) introduce the service flow from durables in a general-equilibrium model. Both impose homotheticity. Dunn and Singleton (1986) assume that the preferences over nondurables and durables are Cobb-Douglas and investigate the term-structure implications of the durability. In addition to homotheticity they restrict the elasticity of substitution to be one. In a similar setup, Pakoš (2000) investigates the role of durable goods in resolving the equity premium puzzle. Grossman and Laroque (1990) introduce transaction costs into an only-durable-good economy and investigate the optimal portfolio choice. Piazzesi, Schneider and Tuzel (2003) and Lustig and Van Nieuwerburgh (2002) focus on housing instead of durable goods. Using nonhomothetic preference specification Ait-Sahalia, Parker and Yogo (2003) use luxury goods to address the equity premium puzzle. Cochrane (1991, 1996) looks at the production side of the economy and explores the cross-sectional implications of real investment growth. Secondly, durability of goods is also a function of the frequency at which we look at the consumption data. In fact, at short horizons many goods are durable. Constantinides and Ferson (1991) and Heaton (1995) develop this idea further to investigate its asset pricing implications.

In a related paper, Yogo (2003) extends Hansen and Eichenbaum (1990) and investigates the cross-sectional implications of durable goods. His setup does not allow for homotheticity and therefore yields upwardly biased estimate of the elasticity of substitution between nondurables and durables. In his model, durables and nondurables are easy substitutes which decreases the consumption risk of the stock market compared to the canonical CCAPM. In fact, his view of the stock market and, in particular, recessions is that investors are not afraid of fluctuations in nondurable consumption as they can easily hedge by substituting into durables. This leads to a worsening of the equity premium puzzle as reflected in the extremely high estimate of the coefficient of risk aversion.

# 3 Preferences and Asset Prices

## 3.1 Preferences and Technology

I specify a representative agent utility function defined over nondurable consumption $C$ and services flow from durables $X$

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \, u \left( \Omega \left[ C_t, X_t \right] \right) \right] \tag{3}$$

where $u(\Omega) \; = (1-\gamma)^{-1} \, \Omega^{1-\gamma}$ is an iso-elastic felicity function and the consumption index $\Omega$ is a generalization of the constant-elasticity-of-substitution (CES) function

$$\Omega \left( C, X \right) = \left\{ \left( a \, C \right)^{1-\frac{1}{\theta}} + \, X^{1-\frac{\eta}{\theta}} \right\}^{\frac{\theta}{\theta-1}} \tag{4}$$

and displays *non-homotheticity.* As it will become clear later on, non-homotheticity is dictated by the data to fit the relative demand for durables, rather than an appetite for an additional degree of freedom. I interpret $\eta$ as the ratio of within-period expenditure elasticities of nondurables and services flow. Empirically, I find $\eta \; < \; 1$ and therefore durables are luxury goods and nondurables necessary goods. Homothetic preferences[6] correspond to $\eta \; = \; 1$.

Furthermore, for the parameter choice $\theta \; < \; \frac{1}{\gamma}$, nondurables and durables are complements. An increase in the stock of durables raises the marginal utility of nondurables $u_{CD} \left( C_t, D_t \right) \; > \; 0$. The limiting case is $\theta \; = \; 0$ when the consumption index is Leontief and the two goods are perfect complements.

Consumers accumulate the stock of durables $D$ according to

$$D_t \; = \; (1-\delta) \; D_{t-1} \; + \; I_t \tag{5}$$

and the durables investment is irreversible

$$I_t \; \geq \; 0 \tag{6}$$

The flow of services $X$ is produced by a linear household production function [Stigler and Becker (1977)], which is time- and state-independent

$$X \; = \; k \, D \tag{7}$$

I normalize $k \; = \; 1$ and use the services flow $X$ and the stock of durables $D$ interchangeably.

[6] For example, Dunn and Singleton (1986) assume that the consumption index $\Omega(C, X)$ is Cobb-Douglas and their implied $\theta \; = \; 1$ and $\eta \; = \; 1$. Eichenbaum and Hansen (1990) and Ogaki and Reinhart (1998) relax the restriction $\theta \; = \; 1$ but still keep the homotheticity assumption $\eta \; = \; 1$.

## 3.2 Interpretation of the Parameters

The preference parameters $\theta$ and $\eta$ are most easily interpreted in a deterministic setup. Let us think of consumers as renting the durables in a perfect rental market, with the rental cost given by the right-hand side of equation (27),

$$R_t \;=\; Q_t \;-\; (1-\delta)\, E_t \left\{ M_{t+1}\, Q_{t+1} \right\} \tag{8}$$

In a deterministic setup $R_t^f = M_{t+1}^{-1}$ and $R_t = Q_t - (1-\delta)\,(R_t^f)^{-1}\, Q_{t+1}$. Viewed this way, the preferences over nondurables and durables are weakly separable. Weak separability is necessary and sufficient condition for the second-stage of two-stage budgeting to hold [Deaton and Muellbauer (1980)]. Intuitively, suppose the consumer has already chosen his optimal consumption of nondurables $C_t$ and services flow $D_t$, with $E_t$ denoting the within-period $t$ expenditure,

$$C_t \;+\; R_t\, D_t \;=\; E_t \tag{9}$$

If the second-stage optimization

$$\max_{\{C_t,\, D_t\}} \;\; u\left[\Omega\left(C_t,\, D_t\right)\right] \tag{10}$$

subject to the budget constraint (9) does not hold, the consumer can alter his consumption plans $C_t$ and $D_t$, and increase his per-period $t$ utility. Because the expenditures in other periods are unaffected, he thereby increases his lifetime well-being, which contradicts the assumed optimality of the consumption plans.

The first-order condition associated with the second stage is helpful to interpret the preference parameters. The Marshallian demands are functions of the relative price $R_t$ and the expenditure $E_t$,

$$C_t \;=\; C_t\left(R_t, E_t\right) \tag{11}$$
$$D_t \;=\; D_t\left(R_t, E_t\right) \tag{12}$$

Log-differentiating yields

$$d\,log\, C_t \;=\; \varepsilon_{12}\, d\,log\, R_t \;+\; \eta_1\, d\,log\, E_t \tag{13}$$
$$d\,log\, D_t \;=\; \varepsilon_{22}\, d\,log\, R_t \;+\; \eta_2\, d\,log\, E_t \tag{14}$$

The parameters $\eta_1$ and $\eta_2$ are the expenditure elasticities associated with the within-period $t$ expenditure level $E_t$. They tell us how much the demands $C_t$ and $D_t$ change (in percentage terms) in response to a 1% rise in the expenditure $E_t$, *ceteris paribus*. The budget constraint

(9) implies that the weighted average of these elasticities must be one. The parameters $\varepsilon_{12}$ and $\varepsilon_{22}$ are Marshallian price elasticities. Subtracting one equation from the other, using Slutsky equation $\varepsilon_{ij} = \varepsilon^*_{ij} - \eta_i\, s_j$, where $s_j$ is the share of good $j \in \{C, D\}$ in expenditures $E_t$, implies that the relative demand function satisfies (see Appendix for derivation)

$$d\,log\,(C_t\,/\,D_t) \;=\; \underbrace{ES\;\,d\,log\,R_t}_{substitution\;effect} \;+\; \underbrace{(\eta_1 - \eta_2)\,d\,log\,\hat{E}_t}_{income\;effect} \tag{15}$$

The parameter $\hat{E}_t$ is the *real* expenditure on both consumption goods and it is defined implicitly as $d\,log\,\hat{E}_t \;=\; d\,log\,E_t - s_2\,d\,log\,R_t$. The elasticity of substitution is defined as a percentage change in the relative Hicksian demand in response to a percentage change in the relative price, $ES \;=\; \partial\,log(C^*_t/D^*_t)\,/\,\partial\,log\,R_t \;=\; \varepsilon^*_{12} - \varepsilon^*_{22}$ and it is a measure of the concavity of the indifference curves. For instance, $ES \,=\, 0$ for Leontief preferences and thus the indifference curves are extremely concave.

The equation (15) shows that the relative demand changes either due to a substitution effect or due to an income effect. It offers a framework to understand the secular rise in the consumption of durables relative to nondurables. The typical interpretation is that consumers substituted (in the sense of Hicks) to durables in response to their falling relative price. This corresponds to the case where elasticity of substitution $ES$ is large and the preferences are homothetic, which automatically kills the income effect. Formally,

$$d\,log\,(C_t/D_t) \;=\; \theta\,d\,log\,R_t \tag{16}$$

The interpretation advanced in this paper is the exact opposite. As the empirical results in later sections indicate, the substitutability between the services flow and nondurables $ES$ is very small and equation (15) dictates significant non-homotheticity. Formally,

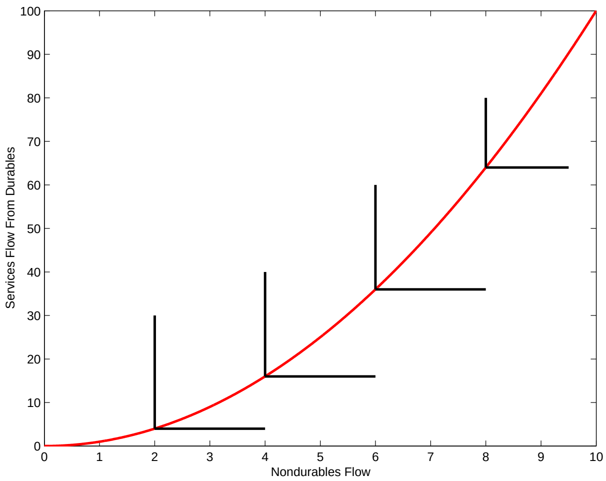
$$d\,log\,(C_t/D_t) \;\approx\; (\eta_1 - \eta_2)\,d\,log\,\hat{E}_t \tag{17}$$

The real income in the U.S. economy has been rising steadily and thus the previous equation implies that $\eta_1 - \eta_2 \;<\; 0$. Because the average income elasticity must be one, we get the plausible result that durables are luxury goods and nondurables are necessary goods, i.e. $\eta_1 \;<\; 1 \;<\; \eta_2$. Empirically, I estimate $\eta$ less than one. This result is also consistent with the Engel's law [Ogaki(1992)] that the budget share of food declines with income.

The special case of $\theta \;\rightarrow\; 0$ delivers the non-homothetic case of Leontief sub-utility function, so-called Prais-Houthakker model, first proposed by Prais-Houthakker (1955)

$$\Omega(C, D) = min\,\{a\,C,\; D^{\eta}\} \tag{18}$$

Figure 2: Indifference Curves and the Income Expansion Path: Prais-Houthakker (1955) model




NOTE - The graph is plotted for the parameter $\eta = 0.5$.

with the income expansion path $D = D(C)$ defined implicitly by

$$a\,C = D^{\eta} \tag{19}$$

I plot the indifference curves and the income expansion path in Figure 4. The preference specification has the feature that both goods are *normal* and services flow is a *luxury* good (i.e. income elasticity $\eta_2$ is greater than one) and non-durable consumption is a *necessary* good with income elasticity $\eta_1$ less than one. Consumers cannot substitute from non-durables to services flow (in the sense of Hicks) but as their real income rises they choose to consume more services flow from the stock of durables.

Homotheticity of the felicity function $u\,(C_t,\,D_t)$ eliminates income effects in that the relative demand $C_t\,/\,D_t$ depends only on the relative price and hence the Engel curves are straight lines. It ascribes all changes in the relative demand for durables to the pure substitution effect. That, however, biases upward the estimate of the elasticity of substitution $ES$. *We need high complementarity between services flow from durables and non-durables because then a small change in consumption variety translates into a dramatic variation in the marginal utility and that amplifies risk premia. Non-homotheticity is then dictated by equation (15) to fit the relative demand for durables.*

The typical way to impose non-homotheticity in macroeconomics has been to consider subsistence levels. However, it is not clear how general non-homotheticity subsistence levels actually allow for. The advantage of the preference specification in this paper is that it allows to specify the ratio of the expenditure elasticities $\eta$ explicitly.

There is ample additional evidence in favor of non-homotheticity. The income elasticities vary across categories of goods and they probably also depend on income and prices themselves as suggested by their time variation. Houthakker (1957) and Houthakker and Taylor (1970), and Ogaki (1992), using cross-sectional and time-series data, respectively, find empirical support for the Engel's law that the budget share of food declines with the level of wealth. Costa (2001) estimates the income elasticities for food at home 0.47 in 1960-94, 0.62 in 1917-35. Those for total food are 0.65 in 1960-94 and 0.68 in 1917-35 and in 1888-1917. Those for recreation are 1.37 in 1972-94, 1.41 in 1917-35, and 1.82 in 1888-1917.

Appendix shows that the preference parameter $\theta$ and the elasticity of substitution $ES$ are related $\theta - ES = \epsilon^*_{22}(1-\eta)$. Empirically, the ratio of income elasticities $\eta$ is estimated below 1 and $\epsilon^*_{22}$ is by definition negative[7]. Thus, the parameter $\theta$ *underestimates* the true elasticity of substitution. However, I interpret it as a yardstick of substitutability. For $\theta$ small we get that $|ES - \theta|$ is small. In fact, they are exactly equal for $\theta = 0$ in which case there is no substitutability between the goods, $ES = 0$. Furthermore, the preference parameter $\eta$ is equal to the ratio of expenditure elasticities $\eta_1$ and $\eta_2$, $\eta = \eta_1 / \eta_2$. Homotheticity corresponds to $\eta = 1$.

### 3.3 Marginal Utility

Marginal utility of nondurable consumption (see Figure 3) is

$$u_C(t) = a^{\frac{\theta-1}{\theta}} C_t^{-\gamma} F_t^{-\frac{\theta\gamma-1}{\theta-1}} \tag{20}$$

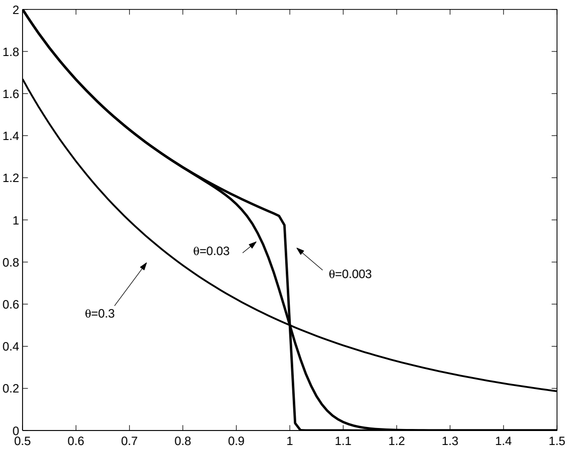
where $F$, defined as

$$F_t = 1 + \frac{D_t^{1-\frac{\eta}{\theta}}}{(a\, C_t)^{1-\frac{1}{\theta}}} \tag{21}$$

is a measure of the *consumption portfolio distortion*. The reason why it enters the marginal utility is as follows. A fall in nondurable consumption $C$ has two effects. Firstly, it increases the marginal utility $u_C$ directly. Secondly, it forces the agent to consume more of service flow $D$ relative to nondurables $C$, *ceteris paribus*, and that decreases the *consumption variety*. Because the preferences are convex, it indirectly raises the marginal utility of nondurable consumption $u_C$. The limiting case are Leontief preferences where consumers want to keep nondurables and durables in a fixed proportion, and the marginal utility $u_C$ responds most to the distortion of the consumption variety (Figure 3). As can be seen, the welfare cost of business cycle

[7]This follows because the substitution matrix is negative semi-definite.

Figure 3: Marginal Utility of Non-Durable Consumption $u_C$ as a Function of the Nondurable Consumption $C$.




NOTE - The graph portrays the marginal utility of nondurable consumption $u_C$ as a function of the nondurable consumption $C$, holding the stock of durables constant at $D = 1$. The preference parameters are $\eta = 0.7$ (close to the estimate, see the empirical section), $\gamma = 1$, $\beta = 0.95$ and $a = 1$. I vary the measure of the substitutability $\theta = 0.3$, 0.03 and 0.003. The parameter $\theta = 0.3$ is closest to the point estimate from the empirical section.

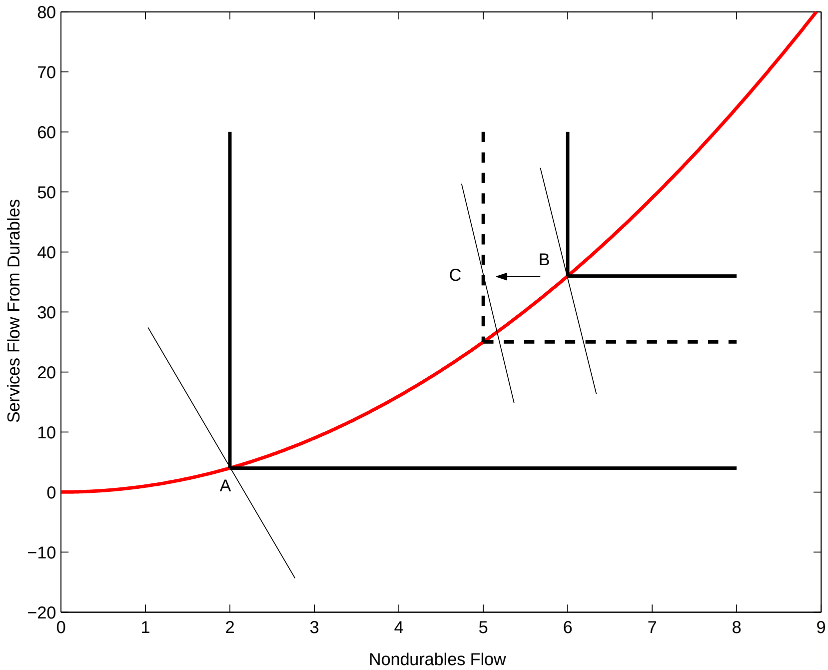
fluctuations is driven not only by the time-variation in nondurables but also by how distorted the consumption portfolio gets.

The stochastic discount factor is given by

$$M_{t+1} = \beta \, \frac{u_C(t+1)}{u_C(t)} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{F_{t+1}}{F_t}\right)^{-\frac{\theta\,\gamma-1}{\theta-1}} = M_{t+1}^{LB} \left(\frac{F_{t+1}}{F_t}\right)^{-\frac{\theta\,\gamma-1}{\theta-1}} \tag{22}$$

Consumption portfolio distortion $F$ becomes an additional risk factor and it enriches the asset pricing implications of the Lucas-Breeden stochastic discount factor $M_{t+1}^{LB} = \beta \, (C_{t+1} \,/\, C_t)^{-\gamma}$. In a Lucas-Breeden economy, stocks are risky because they tend to pay off badly in times of low nondurable consumption growth rate. This type of risk is too low to explain the observed market price of risk for a plausible RRA coefficient [Hansen and Singleton (1982, 1983), Mehra and Prescott (1985)]. I propose an economy with consumer durables where bad times are identified as periods of not only low nondurable consumption growth but also of distorted consumption variety as captured by the factor $F$. There, stocks are risky because they pay off badly in times when economic agents consume relatively more service flow and their marginal utility of nondurable consumption is particularly high due to the preference for consumption variety.

Figure 4: Novel Interpretation of Economic Downturn




The intertemporal first-order condition is the standard Euler equation

$$1 \;=\; E_t \left[ M_{t+1}\, R_{t+1} \right]; \quad R_{t+1} \;=\; \frac{P_{t+1} + DIV_{t+1}}{P_t} \tag{23}$$

Hansen and Jagannathan re-interpret this equation as a restriction on the maximum Sharpe ratio, where the upper bound is the volatility of the discount factor,

$$\max_{\{all\ assets\}} \frac{E_t\left[R - R^f\right]}{\sigma_t\left[R - R^f\right]} \;\leq\; \frac{\sigma_t[M]}{E_t[M]} \tag{24}$$

These bounds could be tighter if the correlation between the discount factor $M$ and excess return $R - R^f$ is less than one in absolute value. As it is well-known, the volatility of the Lucas-Breeden discount factor $M_{t+1}^{LB}$ is small and the bounds are violated. Consumption complementarity between nondurables and services flow introduces a new recession risk factor $F$ - the distortion of the consumption portfolio - and thus magnifies the variation in the discount factor $M$. The factor $F$ becomes dominant in the case of Leontief preferences where consumers want to keep durables and nondurables in a fixed proportion.

The distortion of the consumption portfolio $F$

$$F_t \;=\; 1 \,+\, \frac{D_t^{1-\frac{\eta}{\theta}}}{(a\, C_t)^{1-\frac{1}{\theta}}} \tag{25}$$

is a novel non-diversifiable macroeconomic source of risk. Its role in asset pricing is most dramatic if the two consumption goods are strong complements. Suppose the economy is in boom. The declining relative price of durables raises the real income of investors and the ratio of durables over nondurables raises. In Figure 4, we move from point $A$ to point $B$. Suppose a recession hits the economy. Consumption expenditures fall and because they are normal goods, nondurables and durables investment fall. In Figure 4, we move from point $B$ to point $C$. Clearly, given the strong complementarity of the consumption goods, consumers want to sell their durables to restore their optimal variety of their consumption basket. The irreversibility of durables investment however prevents them from doing so. This is why the relative price of durables "cuts" the indifference curve in equilibrium. As a result, recessions are costly because nondurable consumption declined but also because the consumption basket is highly skewed toward service flow from durables. Observe that the model predicts that risk factor $F$ is *procyclical* and the marginal utility of nondurables $u_C$ *countercyclical*.

If the durables investment is reversible, the intratemporal first-order condition states that the marginal rate of substitution between durables and nondurables equals the rental cost

$$\frac{u_D(C_t, D_t)}{u_C(C_t, D_t)} \;=\; Q_t \;-\; (1-\delta)\, E_t\left\{M_{t+1}\, Q_{t+1}\right\} \tag{26}$$

Intuitively, suppose we rent one unit of durables at price $Q_t$, which after one period depreciates to $1-\delta$. We can sell it for $(1-\delta)\, Q_{t+1}$. The rental cost is the net present value of this transaction, which is $Q_t \;-\; (1-\delta)\, E_t\left\{M_{t+1}\, Q_{t+1}\right\}$.

The irreversibility of the durables investment drives a wedge between the marginal rate of substitution and the rental cost of durables in those states of the world where the constraint $I_t \,\geq\, 0$ binds. Formally, let us introduce the Lagrange multiplier $\nu_t$ on this constraint. Then, the intra-temporal first-order condition takes the form

$$\frac{u_D(C_t, D_t)}{u_C(C_t, D_t)} \;=\; Q_t \;-\; (1-\delta)\, E_t\left\{M_{t+1}\, Q_{t+1}\right\} \;-\; \nu_t \;+\; (1-\delta)\,\beta\, E_t\left[\nu_{t+1}\right] \tag{27}$$

See the Appendix for formal derivation.

### 3.4 Consumption Complementarity and the RRA Coefficient

Dynamic setting allows to define the RRA coefficient in terms of either temporal or atemporal gamble. It turns out that in a Lucas-Breeden economy with one nondurable consumption good and power felicity function $u(C) \,=\, \frac{1}{1-\gamma}\, C^{1-\gamma}$, this distinction is irrelevant and the RRA coefficient of risk aversion coincides with the power $\gamma$ regardless of how the gamble is defined. This irrelevance is one of the main culprits of the poor performance of the canonical CCAPM and gives rise to many quantitative asset pricing puzzles. This is best seen from the dynamic programming point of view. The first-order condition for the optimum states that the marginal utility of wealth equals the marginal utility of nondurables,

$$J_W \;=\; u_C \tag{28}$$

On the margin, the consumer is indifferent whether he saves or consumes one dollar. The asset pricing implications are interesting if we interpret $W_t$ as the aggregate financial wealth. It is well-known that $var\,(W_t)$ is much larger than the $var(C_t)$[8]. However, the previous equation implies that

$$var\,(J_W) \;=\; var\,(u_C) \tag{29}$$

This is only possible if the concavity of the marginal utility $u_C$ is a lot larger than the concavity of the value function $J_W$. In other words, if the risk aversion toward atemporal gambles $RRA \;=\; -W\,J_{WW}\,/\,J_W$ is smaller than the risk aversion toward temporal, nondurable consumption, gambles $-C\,u_{CC}\,/\,u_C$.

Constantinides (1990), section C, analyzes how habit formation drives a wedge between the RRA defined over temporal and atemporal gambles. Consumption complementarity achieves a similar thing. For simplicity, I focus on the case of Leontief consumption index $\Omega(C,D) \;=\; \min(C,D)$ and iso-elastic felicity function $u\,[\Omega(C,D)] \;=\; \frac{1}{1-\gamma}\,[\min(C,D)]^{1-\gamma}$. Fair temporal gamble over nondurables is more costly than atemporal one. In a temporal gamble, the consumer cannot adjust the stock of durables[9]. The cost is high because nondurables and durables are perfect complements. In fact, the willingness to pay to avoid this gamble is dictated by the non-linearity of the marginal utility of nondurables $u_C$ and is a *decreasing* function of the elasticity of substitution $\theta$. See Figure 3 to observe this functional dependence. In an atemporal gamble the consumer can re-plan all future consumption streams including durables. I show hereafter that RRA over temporal gamble is related to the preference parameter $\gamma$ but not the elasticity of subsititution $\theta$.

Let us investigate this issue from the dynamic programming point of view. For the purpose of clarity and simplicity I focus on the deterministic setup. Define the value function $J(W,D)$ as a function of the financial wealth and the stock of durables. Then, it must be true that

$$J(W_t, D_t) \;=\; \max_{\{C_t,\,I_t\}}\;\{u\,(\Omega(C_t,\,D_{t+1}))\;+\;\beta\,J(W_{t+1}, D_{t+1})\} \tag{30}$$

subject to 2 laws of motion

$$D_{t+1} \;=\; (1-\delta)\,D_t \;+\; I_t \tag{31}$$

[8]Better in the growth rates, as the variables are non-stationary.

[9]He doesn't want to sell them because $u_D \;>\; 0$.

and

$$W_{t+1} \;=\; R\,(W_t \;-\; C_t \;-\; Q_t\, I_t) \tag{32}$$

where $R$ is the gross risk-free rate.

**Proposition 1** *Suppose the consumption index is Leontief*

$$\Omega\,(C_t, D_t) \;=\; \min\,(C_t, D_t)$$

*the gross interest rate $R \;=\; 1$, the relative price of durables $Q_t \;=\; 1$ and the depreciation rate $\delta \;=\; 1$. Then the value function that solves the dynamic program is given by*
$J\,(W_t + D_t) \;=\; \frac{A}{1-\gamma}\,(W + D)^{1-\gamma},\;\; A \in R^+$

*Proof*
If the consumption index is Leontief, then the consumer chooses $C_t \;=\; D_t \,+\, I_t$. As there are no non-negativity constraints on $I_t$, the dynamic program takes the form

$$J(W_t, D_t) \;=\; \max_{\{0.5(W+D)\,\geq\, C_t\,\geq 0\}} \left\{ \frac{1}{1-\gamma}\, C_t^{1-\gamma} \;+\; \beta\, J\,[W_t \;-\; 2C_t + D_t, C_t] \right\}$$

Because the state variables $W$ and $D$ enter in an additive manner as $W{+}D$, we may seek $J(W,D) \;=\; V(W+D)$ for some function $V$. Then,

$$V(W_t \;+\; D_t) \;=\; \max_{\{0.5(W+D)\,\geq\, C_t\,\geq 0\}} \left\{ \frac{1}{1-\gamma}\, C_t^{1-\gamma} \;+\; \beta\, V\,[W_t \;+ D_t \;-\; C_t] \right\}$$

It is easy to verify that such a dynamic program has the value function $V(W{+}D) \,=\, A\,(W{+}D)^{1-\gamma}\,/\,(1-\gamma)$, $A \in R^+$. $\square$

**Proposition 2** *Suppose the consumption index is of constant elasticity of substitution form*

$$\Omega\,(C_t, D_t) \;=\; \left( C_t^{\frac{1-\theta}{\theta}} \;+\; D_t^{\frac{1-\theta}{\theta}} \right)^{\frac{\theta}{1-\theta}}$$

*and the elasticity of substitution $\theta$ is small. Then, the value function $J(W,D)$ is approximately*
$J(W,D) \;\approx\; A\,(W+D)^{1-\gamma}\,/\,(1-\gamma),\; A \in R^+$

*Proof*
This follows by invoking continuity arguments. $\square$

I follow Constantinides (1990) and Campbell and Cochrane (1999), among others, and define the RRA coefficient in terms of an atemporal gamble. The economy investigated in this paper has two consumption goods, nondurables and durables, and thus a new issue arises. Because the stock of durables is a part of the total wealth of the household, the gamble may be defined either over the *financial wealth* $W$ or the *total wealth* $W \,+\, D$. In the first case,

$$RRA \;=\; \frac{W\, J_{WW}(W_t, D_t)}{J_W(W_t, D_t)} \;=\; \frac{\gamma}{1 + D_t\,/W_t}$$

This is very similar to the formula (28) in Constantinides (1990). If we define the gamble over the total wealth, including the stock of durables, the formula reduces to

$$RRA \;=\; \frac{(W+D)\, J_{W+D,W+D}(W_t, D_t)}{J_{W+D}(W_t, D_t)} \;=\; \gamma$$

In the empirical section, I interpret the preference parameter $\gamma$ as an upper bound on the RRA coefficient.

# 4 Empirical Section

## 4.1 Evidence in Favor of the Intra-Temporal First-Order Condition

I test the null hypothesis that the series $c_t$, $d_t$ and $q_t$ are difference stationary against the alternative of trend stationarity. Small letters are in logs. Using Phillips-Perron test and including a constant and a linear time trend I cannot reject the hypothesis that the data are difference stationary. Table 1 summarizes the results. Therefore, the marginal rate of substitution $M_{t+1}$ and the ratio $Q_{t+1}\,/\,Q_t$ are stationary, and hence the conditional expectation $E_t\left\{M_{t+1}\,\frac{Q_{t+1}}{Q_t}\right\}$ is stationary as well.

First, I assume a perfect rental market in that the irreversibility constraint (6) does not bind in all states of the world (i.e. $\nu_t \;=\; 0$) and divide the intra-temporal first-order condition (27) by $Q_t$

$$\frac{u_D(C_t, D_t)}{Q_t\, u_C(C_t, D_t)} \;=\; 1 \;-\; (1-\delta)\, E_t\left\{M_{t+1}\,\frac{Q_{t+1}}{Q_t}\right\} \tag{33}$$

Doing the algebra and imposing the stationarity of the right-hand side I obtain

$$c_t \;=\; const \;+\; \theta\, q_t \;+\; \eta\, d_t \;+\; \varepsilon_t \tag{34}$$

If the variables $c_t$, $d_t$ and $q_t$ are co-integrated, I can estimate the preference parameters $\theta$ and $\eta$ super-consistently (under the assumption of a perfect rental market) as a cointegrating vector by running a regression in levels. This follows Ogaki and Reinhart (1998) who estimate the elasticity of intratemporal substitution. They focus on the homothetic case $\eta \;=\; 1$ and their regression is

$$c_t \;-\; d_t \;=\; const \;+\; \theta\, q_t \;+\; \varepsilon_t \tag{35}$$

The presence of significant nonhomotheticity may bias the estimate of the elasticity of substitution $\theta$. In fact, imposing the homotheticity assumption in the case of durable goods *biases upward* the parameter $\theta$. Intuitively, the relative demand may change either due to income effect or due to substitution effect. Homotheticity dictates that it was Hicksian substitution in response to a secular change in the relative price that led consumers to purchase more durable

Table 1: Phillips-Perron Test for the Null of Difference Stationarity

| | Quarterly | | Annual | |
|---|---|---|---|---|
| | $z_\rho$ | $z_t$ | $z_\rho$ | $z_t$ |
| $c_t$ | -4.5648 | -1.3831 | -4.4052 | -1.3408 |
| $d_t$ | -9.6081 | -2.7024 | -15.1129 | -3.2770 |
| $q_t$ | 0.4265 | 0.1965 | 1.2249 | 0.6094 |

NOTE - Critical value for $z_\rho$ (quarterly data) is $-20.7$ (5% level) and $-17.5$ (10% level), $z_t$ (quarterly data) is $-3.45$ (5% level) and $-3.15$ (10% level). Critical value for $z_\rho$ (annual data) is $-19.8$ (5% level) and $-16.8$ (10% level), $z_t$ (annual data) is $-3.5$ (5% level) and $-3.18$ (10% level). The number of lags in Newey-West estimator (Bartlett weight) is 4. Lower-case letters denote logs. Sample period is 1951,1-2001,4.

goods. In general, considering income effects may be important in particular for goods with large expenditure shares and/or no substitutes, such as housing or durables, as Slutsky equation says that the income effects are proportional to the expenditure shares.

I test for co-integration using likelihood ratio test[10] [Johansen (1989, 1991]. The likelihood ratio test of the null hypothesis of no cointegration versus the alternative of one cointegrating vector $LR = 40.05 > 21.28$, the 5% critical value. The likelihood ratio test of the null hypothesis of no cointegration versus the alternative of three cointegrating vectors $LR = 55.5 > 31.27$, the 5% critical value. I reject the hypothesis of no cointegration at 5% significance level.

With 3 variables there can be $N = 3$ cointegrating vectors. I therefore test the null hypothesis that there is only one co-integrating vector by likelihood ratio test. Firstly, I test $H_0 : N = 1$ vs. $H_1 : N = 2$. The likelihood ratio $LR = 12.16 < 14.6$, the 5% critical value. Secondly, I test $H_0 : N = 1$ vs. $H_1 : N = 3$. The likelihood ratio $LR = 15.45 < 17.84$, the 5% critical value. In both cases, I cannot reject the null hypothesis of one co-integrating vector.

Stock and Watson (1993) and Wooldridge (1991) suggest to augment the regression (34) with leads and lags of the right hand side variables to correct for the correlation between the innovations in $d_t$ and $q_t$ and the cointegrating residual $\epsilon_t$. This is important for the construction of confidence intervals and hypothesis testing. I therefore estimate

$$c_t = const + \theta q_t + \eta d_t + \sum_{s=-p}^{p} b_{d,s} \Delta d_{t-s} + \sum_{s=-p}^{p} b_{q,s} \Delta q_{t-s} + \varepsilon_t \tag{36}$$

where $p = 4$ is the number of leads/lags.

[10] I assume 4th order VAR for likelihood ratio test and AR(2) for the cointegrating residual to create confidence intervals and t-stats.

Table 2: Co-integrating Vector

| *const* | $\theta$ | $\eta$ |
|---|---|---|
| | Quarterly | |
| -0.1312 | 0.2553 | 0.7056 |
| (0.02090 | (0.0446) | (0.0226) |
| | Annually | |
| -0.1689 | 0.3004 | 0.7211 |
| (0.0406) | (0.0795) | (0.0555) |

NOTE - The table reports the estimated co-integrated vector at quarterly and annual frequency, sample period 1951,1-2001,4. There are 4 lags in the augmented co-integrating regression [Stock and Watson (1993), Wooldridge (1991)]. Standard errors are in parentheses.

Table 2 reports the estimates of the elasticity of substitution $\theta$ and the ratio of income elasticities $\eta = \eta_1 / \eta_2$. I estimate the elasticity $\theta = 0.26$ and the ratio of expenditure elasticities $\eta = 0.7056$. I test the null hypothesis of homotheticity $H_0 : \eta = 1$. The t-statistics $t = -13.0175$ and *I thus reject the null hypothesis of homothetic preferences in favor of non-homotheticity at* 1% *significance level.* In fact, $\eta$ is estimated less than one which confirms the evidence found in the literature (see the theoretical section) that nondurables are necessary goods and durables luxury goods [11]. In addition, in case the preferences were homothetic, the variables $c_t - d_t$ and $q_t$ would be co-integrated. However, that is not the case[12]. *I interpret the lack of co-integration between* $c_t - d_t$ *and* $q_t$ *as an additional evidence in favor of significant non-homotheticity.*

Furthermore, I test the hypothesis of zero substitutability between nondurables and services flow $H_0 : \theta = 0$. The t-statistics $t = 5.728$ and I thus reject the hypothetis that the consumption index $\Omega(C_t, D_t)$ is Leontief at 5% significance level, *under the assumption of a perfect rental market.* The parameter $\theta$ is estimated lower than in the related literature and the reason for this is discussed in the next paragraph. The estimates at quarterly and annual frequency are not statistically different from each other but the annual point estimate is larger. Although I do not model adjustment costs it is true than over longer horizons people can adjust on more margins and hence the demand is more elastic in the long run. This may explain the higher magnitude of the elasticity of substitution at annual frequency.

Using homothetic CES index for nondurables and durables, Ogaki and Reinhart (1998) cannot reject the null hypothesis $H_0 : \theta > 1$. Their result does not conform to the estimates in Table 2. The reason is twofold. Firstly, Ogaki and Reinhart use Gordon's data which arguably

[11] Ait-Sahalia, Parker and Yogo (2003) address the equity premium puzzle using luxury goods.

[12] The results are available from the author.

better adjust for the quality. That series, unfortunately, ends in the early 80s. Secondly, they do not allow for nonhomotheticity and hence the rising real income in the post-war U.S. economy cannot affect the relative demand. However, as can be seen in Table 2, the parameter $\eta_1 / \eta_2 = \eta < 1$ and therefore the expenditure elasticity of durables $\eta_2$ is greater than one[13]. Durables are luxury goods and not surprisingly, part of the increase in the consumption of durables is due to the income effect. It turns out that imposing homotheticity *ex ante* biases upward the estimate of the elasticity of substitution. In a related paper, Mayo (1981) estimates the elasticity of substitution $\theta$ using housing data and obtains magnitudes between 0.3 and 0.9, surprisingly similar to my estimates as reported in Table 2.

In conclusion, I estimate the elasticity of the intra-temporal substitution $\theta$ significantly lower than 1, which is already a novel result. I argue that market imperfections, such as irreversibility constraints, may drive this parameter even lower. To anticipate upcoming sections, low substitutability, and thus high complementarity, between nondurables and durables is crucial for understanding the behavior of asset prices. Non-homotheticity introduces income effects into the relative demand function for durables and partly reconciles the estimate from the intra-temporal first-order condition with the GMM estimate of $\theta$ from the Euler equation.

### 4.2 Evidence in Favor of the Inter-Temporal First-Order Condition

Prices and returns of financial assets have to satisfy the Euler equation, namely,

$$1 \;=\; E_t\left[M_{t+1}\, R_{t+1}\right] \tag{37}$$

and

$$P_t \;=\; E_t\left[M_{t+1}\; (P_{t+1} + DIV_{t+1})\right] \tag{38}$$

Hansen and Jagannathan (1991) reinterpret these Euler equations as restrictions on the volatility of the discount factor $M_{t+1}$

$$\max_{\{all\ assets\}} \frac{E(R-R^f)}{\sigma(R-R^f)} \;\leq\; \frac{\sigma(M)}{E(M)} \tag{39}$$

The mean of the discount factor has to be slightly above to 1 to fit the unconditional risk free rate and therefore the volatility of the discount factor is bounded from below by the maximum unconditional Sharpe ratio. Furthermore, asset returns are not *i.i.d.* and hence Sharpe ratios increase with the horizon, giving rise to the long-horizon equity premia puzzle. Hansen and Jagannathan showed us an interesting way to visualize this restriction. They constructed a cup-shaped region in the $(E(M),\, std(M))$ space, defined by

$$std(M) \;\geq\; \sqrt{(1-E(M)E(R))'\; var^{-1}(R)\; (1-E(M)E(R))} \tag{40}$$

[13]Recall that the average expenditure elasticity must be one.

The long-horizon equity premium puzzle manifests itself in shifting the whole region up.

It is an empirical fact that the unconditional Sharpe ratios on the value-weighted market return are hard to reconcile within the consumption-based asset pricing paradigm. The problem is even worse when the asset universe consists of 25 Fama-French portfolios. These large Sharpe ratios dictate that we need a huge volatility of the marginal rate of substitution.

Furthermore, let us rewrite the Euler equation (37) in a form commonly used in the cross-sectional tests of asset pricing models,

$$E(R^e_{t+1}) \;=\; \frac{cov(M, R^e)}{var(M)} \left(-\frac{var(M)}{E(M)}\right) \;=\; \beta\,\lambda \tag{41}$$

where $R^e$ is a return in excess of the risk-free rate. The coefficient[14] $\beta$ defined

$$\beta \;=\; \frac{cov(M, R^e)}{var(M)} \tag{42}$$

is a *measure of the riskiness of an asset* and $\lambda$, defined as

$$\lambda \;=\; -\frac{var(M)}{E(M)}, \tag{43}$$

is the *market price of risk*. Therefore, the ability to satisfy the Hansen-Jagannathan bounds can be restated as an ability to fit the market price of risk $\lambda$, an output of the second stage of Fama-MacBeth regression.

Many exciting papers such as Lustig and Nieuwerburgh (2003) or Yogo (2003), among others, ask the important question whether the dispersion in betas with the discount factor, or risk factors in general, helps account for the cross-sectional variation in average returns. Many times, unfortunately, the model-implied market price of risk is hard to reconcile with its second stage estimate from Fama-MacBeth regression (see Lewellen and Nagel (2004) for more on this point).

#### 4.2.1 Discount Factor and Volatility Bounds: Calibration with the Intra-temporal First-Order Condition Estimates

In this section I empirically investigate the ability to fit the market price of risk $\lambda$ when the discount factor $M_{t+1}$ is calibrated with the parameter estimates $\hat{\theta}$ and $\hat{\eta}$ obtained from the intra-temporal first-order condition. Recall that these estimates were obtained under the assumption that the irreversibility constraint does not bind. However, because the crucial

[14] Not to be confused with the subjective discount factor.

ingredient of the model is that this constraint binds in recessions, we expect that the discount factor will have a hard time pricing financial assets.

Figure 6, top panel, portrays the volatility of the discount factor $var(M_{t+1})$ at quarterly frequency for two preference parameter choices, $a = 1$ and $a = 5000$. As can be seen, for both choices of the preference parameter $a$, the standard deviation of the marginal rate of substitution is an increasing function of the curvature parameter $\gamma$. This suggests that it may be possible to pass the diagnostic test of Hansen and Jagannathan (1991). To test this hypothesis, I construct the volatility bounds for both the universe of value-weighted market return and the risk-free rate (thin line), and the universe of the 25 Fama-French portfolios (thick line). Not surprisingly, given the difficulty pricing these latter portfolios, their bounds are tighter (Figure 6, bottom panel). It may be observed that as we raise the curvature coefficient $\gamma$, we indeed raise the volatility of the discount factor $M_{t+1}$. However, its mean $E(M_{t+1})$ is a declining function of the curvature $\gamma$. Although it is true that the market price of risk $\lambda = -\frac{var(M_{t+1})}{E(M_{t+1})}$ rises in magnitude, this comes at the expense of the ability to price the unconditional risk free rate. Of course, there exists a sufficiently large value of $\gamma$ such that precautionary savings motive begins to dominate and we may get inside HJ bounds with the unconditional risk-free rate priced correctly. That, however, occurs for extreme magnitudes of $\gamma$, i.e. $\gamma > 200$. As a result, I conclude that the calibration of the discount factor $M_{t+1}$ based on the estimates from the intra-temporal first-order condition fails to pass the diagnostic test of Hansen and Jagannathan. Although it is true that it may be possible to fit the market price of risk, it is so only because we significantly mis-price the unconditional risk free rate.

Figure 7 portrays the same results at annual frequency. This is motivated by Marshall and Daniel (1997) and Parker and Julliard (2003) who argue that consumption-based models should work better at lower frequencies. However, a brief look at the HJ bounds suggests that the bounds are tighter at annual frequency, which is a sign of *long-horizon equity premium puzzle.* It is that much more difficult to satisfy these bounds at lower frequencies. The empirical results are very similar to their quarterly counterparts. Given the estimates of the non-homotheticity $\eta$ and substitutability $\theta$, obtained from the intra-temporal first-order condition, we are unable to satisfy the volatility bounds and hence account neither for the average excess market return in postwar U.S. economy nor the average returns on 25 FF portfolios. In contrast to Marshall and Daniel and Parker and Julliard, the ability to fit the market price of risk is not helped by lower frequencies. In fact, due to the long-horizon equity premium puzzle the task is actually harder. However, it is true that the cross-sectional results improve at yearly frequency compared to quarterly one as may be seen in Figure 8. In conclusion, it seems that looking at data at lower frequencies helps especially in the cross-section of expected returns literature but fitting the market price of risk is harder due to the long-horizon equity premium puzzle.

Another way to see the same problem is to take the discount factor $M_{t+1}$ and run Fama-MacBeth regression. As the equation (41) shows, the output of the second stage of Fama-MacBeth regression is the estimated market price of risk $\hat{\lambda}$. To price the unconditional risk-free rate, we must have $E(M_{t+1}) \approx 1$ and the market price of risk $\lambda \approx var(M_{t+1})$. Therefore, we can then try to reconcile the estimate $\hat{\lambda}$ with its approximate model-implied counterpart $var(M_{t+1})$. Figure 8 shows the cross-sectional fit when the risk factor is the marginal rate of substitution $M_{t+1}$. The parameters $\theta$ and $\eta$ are calibrated from the intra-temporal first-order condition, the subjective discount rate is $\beta = 0.95$, the preference weight $a = 1$ and the curvature parameter $\gamma = 1$. As can be seen, the cross-sectional $R^2$s are quite high, both at quarterly and annual frequency. The estimated market prices of risk at quarterly frequency is $\lambda_q = 0.0116$ and has the wrong sign. The one at annual frequency $\lambda_a = -0.0307$. As the disccussion in the previous paragraph suggests, one does not pass the diagnostic test of Hansen and Jagannathan with the volatility of the discount factor on the order of 0.03. This analysis points to the risk of having quite good cross-sectional fit and totally mis-fitting the market price of risk.

#### 4.2.2 Discount Factor and Volatility Bounds: Comparative Statics

The failure of the discount factor $M_{t+1}$, calibrated with the estimates from the intra-temporal first-order condition $\theta$ and $\eta$, to satisfy Hansen-Jagannathan bounds highlights the crucial role of the irreversibility constraints. As discussed in the theoretical section, they bind in recessions when the consumption basket is skewed toward the service flow from durables. I argue that neglecting these binding constraints (i.e. $\nu_t > 0$ in some states of the world) biases upward the parameter estimate $\hat{\theta}$ obtained in the previous section. I proceed by performing a comparative statics on the parameter vector $(\theta, \eta, \gamma)$. I plot the volatility surfaces of $var(M_{t+1})$ for various permutations of $(\theta, \eta, \gamma)$ and discover that raising the complementarity between nondurables and service flow dramatically raises the volatility of the discount factor.

Figure 9, top left panel displays the volatility of the discount factor $var(M_{t+1})$ as a function of the preference parameters $\eta$ and $\theta$. I calibrate the rest of the parameter vectors as follows. I set $a = 1$, the subjective discount factor $\beta = 0.95$ and the curvature coefficient $\gamma = 1$. It is interesting to see how lower substitutability $\theta$ between nondurables and service flow pushes the volatility $var(M_{t+1})$ up. In fact, the effect of $\theta$ is a lot more pronounced that that of $\eta$ - nonhomotheticity parameter. This indicates that indeed the *crucial ingredient to explain the magnitude of the average excess return on the value-weighted market return, or, in general, any asset, is to decrease $\theta$ and thus increase the complementarity between the goods,* *ceteris paribus.* Recall that we needed nonhomotheticity to fit the relative demand function for durables which is trending. There are two possible effects that may drive this demand function - income effect or substitution effect. In detail, suppose we assume homothetic preferences. Then, we implicitly argue that the secular decline in the rental cost of durables led consumers to *substitute* into durable goods. Or, secondly, we assume non-homotheticity and Leontief consumption index, and then the declining rental cost of durables led to a *rise in real income* and hence it was the income effect that led consumers to buy more durables. Thirdly, the life is most likely a combination of both the substitution and income effects. Notice however, that it is not possible to decrease the substitutability, or raise the complementarity, if you like, have homothetic preferences and still satisfy the intra-temporal first order condition. The only hope to decrease the parameter $\theta$ and thus raise the complementarity between nondurables and service flow, and still fit the relative demand function for durables, is to introduce income effects into the relative demand function for durables, that is, non-homothetic preferences.

The top right panel portrays the same surface of the volatility of the discount factor $var(M_{t+1})$, with the only difference that the curvature parameter $\gamma$ is set to 50. Notice how this pushes the whole surface $var(M_{t+1})$ up. We have therefore another channel to raise the standard deviation of the marginal rate of substitution $M_{t+1}$; crank up the curvature parameter $\gamma$. This has two unfortunate effects. Firstly, recall that the concavity of the value function is related to the curvature parameter $\gamma$. As we raise $\gamma$, we are implicitly raising the RRA coefficient defined in terms of an atemporal gamble. Secondly, as suggested by the results in the previous section, as we raise $\gamma$ we are likely to mis-price the unconditional risk-free rate unless $\gamma$ is so large that the precautionary saving starts to dominate. These two effects reduce the attractiveness of the curvature coefficient $\gamma$ to deliver the volatility of the discount factor $M_{t+1}$ and thus explain the high observed Sharpe ratios.

The bottom left panel portrays the volatility surface as a function of the parameters $\gamma$ and $\eta$. As before, the nonhomotheticity parameter $\eta$ does not appear to be crucial. In fact, it is the curvature parameter that dictates the volatility of the pricing kernel $var(M_{t+1})$.

The final graph, bottom right one, shows the volatility surface as a function of the parameters $\gamma$ and $\theta$. Clearly, higher $\gamma$ and lower $\theta$ both deliver more volatile pricing kernel $M_{t+1}$. Notice that we can achieve the same volatility of the discount factor $M_{t+1}$ with either $\gamma$ small and $\theta$ small, or, with $\gamma$ large and $\theta$ large. This has significant asset pricing ramifications. Recall that the interpretation of the curvature parameter $\gamma$ is as a coefficient of the relative risk aversion toward atemporal gambles. As we raise $\gamma$ we raise the risk aversion toward atemporal gambles. However, to the first-order, $\theta$ does not affect the risk aversion. Therefore, this last graph indicates that *with sufficiently small substitutability between nondurables and service flow $\theta$ we may be able to satisfy Hansen-Jagannathan bounds for a reasonable $\gamma$ and thus a reasonable risk aversion toward atemporal gambles.* Furthermore, it seems to be nearly impossible to explain the *magnitude* of the average excess return on the value-weighted market return with $\theta$ greater than one and $\gamma$ small. This may be one explanation why Yogo (2003) estimated $\gamma$ on the order of 500. He assumed homothetic preferences and implicitly set $\theta = 1$. Nonhomotheticity allows to keep $\gamma$ down by decreasing the parameter $\theta$ and thus increasing the complementarity between nondurables and durables. Figure 12 portrays the same surfaces at annual frequency. The results have analogous interpretation. Both $\theta$ and $\gamma$ are crucial ingredients to deliver a volatile discount factor $M_{t+1}$.

As a final general remark, because the RRA coefficient, defined in terms of atemporal gamble, is related to the parameter $\gamma$, a preferred way to raise $var(M_{t+1})$ is to decrease $\theta$ and thus *raise the complementarity between nondurables and durables, and keep the curvature coefficient $\gamma$ small.* This is a preferred parameter choice as *it may actually lead to a new resolution of the equity premium puzzle.*

#### 4.2.3 Discount Factor and Volatility Bounds: Simulation Results

The results of the comparative statics from the previous section suggest that there are two ways to raise the volatility of the discount factor and thus fit the market price of risk. We either decrease the elasticity of substitution $\theta$ between nondurables and durables, or increase the curvature coefficient $\gamma$. In this section, I perform the first exercise and then run Fama-MacBeth regression with the factor being the marginal rate of substitution.

Figure 14 top panel portrays the volatility bounds for the universe of the value-weighted market return and the risk-free rate (thin line), and 25 Fama-French portfolios (thick line). I calibrate the marginal rate of substitution as follows. I set the subjective rate of time preference $\beta = 0.95$, the curvature coefficient $\gamma = 1$ and the preference weight $a = 1$. Furthermore, I set $\eta$ equal to the estimate from the intratemporal first-order condition (27), specifically, $\eta = 0.7056$ in quarterly data and $\eta = 0.7211$ in annual data. I allow the parameter $\theta$ to take values from the set $\left\{\hat{\theta}, 0.05, 0.02, 0.017, 0.015\right\}$, where $\hat{\theta}$ is the estimate from the intra-temporal first-order condition.

In summary, as we decrease the substitutability between the two goods we clearly move toward the HJ bounds (Figure 14). Interestingly, it seems that we may satisfy these bounds with a low magnitude of the preference parameter $\gamma$. As the theoretical section suggests, $\gamma$ is an upper bound on the RRA coefficient defined in terms of the atemporal gamble over the wealth. In fact, this proposition was proved for $\theta$ small, which is the case. This indicates that consumption complementarity may lead to a new exciting resolution of the equity premium puzzle. Furthermore, the bottom panel shows the results of the Fama-MacBeth regression with just one factor, namely, the marginal rate of substitution $M_{t+1}$.

In detail, at quarterly frequency the elasticity of intra-temporal substitution between durables and nondurables $\theta$ must be around 0.02 to get close to the volatility bounds. I use so calibrated marginal rate of substitution to run Fama-MacBeth regression. This single factor explains around 61% of cross-sectional variation in average excess returns but the market price of risk is estimated positive counter to theory (estimate not reported). This indicates that there is something going wrong at quarterly frequency despite interesting $R^2$. Results at annual frequency are more encouraging. A bit higher elasticity of substitution, around 0.04, is sufficient to get close to the bounds. The associated cross-sectional regression delivers $R^2$s of the order of 77%. Importantly, the market price of risk comes out negative, consistent with the theory. It seems that a nonhomothetic marginal rate of substitution with low substitutability between durables and nondurables manages to satisfy the volatility bounds for the value-weighted market return and the risk-free rate. However, the complementarity required is substantially lower than the estimate obtained from the intra-temporal condition under the assumption of a perfect rental market. This highlights the crucial role of a binding irreversibility constraint in recessions.

### 4.2.4 Consumption Complementarity and Sharpe Ratios on Equity and Value Premia

The previous sections uncovered that the consumption complementarity plays a crucial role in determining the volatility of the discount factor and thus the magnitude of the market price of risk. This section analyzes whether consumption complementarity delivers the right sign and magnitude of the Sharpe ratios on the excess value-weighted market return and the HML portfolio.

Figure 11 displays the Sharpe ratios on the equity premium and value premium (HML portfolio), both quarterly and annually, as a function of the elasticity of substitution $\theta$ and the curvature coefficient $\gamma$. As may be seen, consumption complementarity yields the correct sign of the expected excess return on the equity premium. However, the sign the expected value premium has the opposite sign at quarterly frequency. This may be one reason why the estimate of the market price of risk of the discount factor $M_{t+1}$ from the Fama-MacBeth regression comes out positive, counter to the economic theory.

In terms of magnitudes, the sample Sharpe ratios on the value-weighted market return in quarterly data is 0.18 and on the value premium 0.15 (see Table 6). As Figure 11, top left box, indicates sufficiently low parameter $\theta$ and thus high complementarity between nondurables and durables is capable to match both the sign and magnitude of the Sharpe ratio on the equity premium. In annual data, the sample Sharpe ratios on the value-weighted market return is 0.36 and on the value premium 0.38 (see Table 6). As in quarterly data, there seems to be a hope of matching both the sign and magnitude of the Sharpe ratio on the equity premium. Furthermore, consumption complementarity succeeds in matching the sign and partly the magnitude of the Sharpe ratio on the HML portfolio

#### 4.2.5 GMM Estimation of Euler Equations: Risk-Free Rate and Value-Weighted Market Return

The primary testable asset pricing implications of the model are the set of Euler equations. Motivated by the diagnostic tests of Hansen and Jagannathan (1991) and following Hansen and Singleton (1982, 1983) I test these restrictions by estimating the Euler equations. I first estimate the preference parameter vector $p = (\theta, \gamma, \beta, a)$ off asset prices. In addition, I set the non-homotheticity parameter $\eta$ equal to the estimate from the intra-temporal first-order condition. I then test the conditional asset pricing model and attempt to reconcile the estimate of the elasticity of substitution $\theta$ with its counterpart from the intra-temporal first-order condition. The scaled risk-free rate and the value-weighted market return are the universe of the assets.

Formally, I test the conditional asset pricing model by conditioning down using a vector of instruments $z_t$. I use two important scaling variables. Firstly, Lettau and Ludvigson (2001) find that the cointegrating residual $cay_t$ is a good predictor of the return on the value-weighted market return at business cycle frequencies. It is therefore a good scaling variable. Secondly, Campbell and Shiller (1988) and Fama and French (1988), among others, discover that the price-to-dividend ratio $P_t \,/\, D_t$ predicts the excess returns on the market portfolio as well. As a result, I scale the universe of assets with the following instrument

$$z_t = (1,\, cay_t,\, P_t \,/\, D_t)$$

The model is conditioned down as

$$E\,[z_t] = E\,[M_{t+1}\, R_{t+1} \otimes z_t]$$

which delivers six moment conditions. Because the spectral density matrix $S$ at frequency zero is singular, I run the first-stage GMM only[15]. Table 3 summarizes the estimates when the moment conditions are the scaled risk-free rate and the scaled value-weighted market return. The estimated parameters are the yardstick of substitutability, $\theta$, the curvature coefficient, $\gamma$, the subjective discount factor, $\beta$ and the preference weight, $a$. The non-homotheticity parameter $\eta$ is set equal to the super-consistent estimate obtained from the log-linear intratemporal first-order condition (34). Consistent with the results in the previous sections, the yardstick of substitutability $\theta$ is estimated small and nondurables and durables are strong complements. Furthermore, the magnitude of $\theta$ is smaller than the estimate obtained from the intra-temporal first-order. I reconcile these two estimates by introducing binding irreversibility constraints. Recall that economic downturns are periods when the relative price of durables "cuts" through the indifference curve as investors are unable to sell durable to restore the optimal variety of their consumption basket. Econometric analysis of the intra-temporal first-order condition did not consider these constraints and yielded upward biased estimate of the parameter $\theta$. Furthermore, some may worry that because a low estimate of the parameter $\theta$ gives rise to a highly curved indifference curves it also generates a high short-term volatility in the rental cost of durables and thus in the relative durables price. However, if the felicity function is Leontief (i.e. $\theta = 0$), the marginal rate of substitution does not equal the rental cost as the indifference curve is non-differentiable. In fact, the rental cost is determined in the production sector and equals the marginal rate of transformation.

The theoretical section interprets the curvature parameter $\gamma$ as an *upper bound* on the coefficient of relative risk aversion defined in terms of an atemporal gamble. The point estimate at quarterly frequency is $\hat{\gamma} = 44$ and $\hat{\gamma} = 36$ at annual frequency. Although these estimates may seem high, the confidence intervals are quite large and include the economically plausible magnitudes of $\gamma$ less than 5. In addition, the service flow from the stock of durables is a smooth consumption series because it corresponds to the geometrically-weighted sum of the durables purchases, and it is actually smoother than nondurable consumption series. Adding a consumption series smoother than nondurables worsens the equity premium puzzle. In fact,

[15] The sample pricing errors are computed as

$$g_T(p) \;=\; \frac{1}{T}\sum_{t=1}^{T} M_{t+1}\,(p)\; R_{t+1} \,\otimes\, z_t \;-\; z_t$$

and I numerically minimize the GMM objective function

$$\hat{p} \;=\; \arg\min_{p}\; g_T'\,(p)\; W^{-1}\, g_T\,(p)$$

where $W$ is a weighting matrix. Specifically, $W = diag(cov(R))^{-1}$. The spectral density matrix is estimated singular and that prevents me from applying the efficient GMM.

Table 3: First-Stage GMM Estimates of the Unknown Preference Parameters

| | quarterly | | annually | |
|---|---|---|---|---|
| | parameter | s.e. | parameter | s.e. |
| $\theta$ | 0.0075 | (0.0015) | 0.012 | (0.0062) |
| $\gamma$ | 44.063 | (25.334) | 35.591 | (13.524) |
| $\beta$ | 0.752 | (0.212) | 1.200 | (0.833) |
| $a$ | 0.9734 | (1.423) | 1.127 | (3.445) |
| | value | p-value | value | p-value |
| $J_T$ | 11.536 | (0.003) | 34.09 | (0.000) |

NOTE - The table reports the estimated preference parameters using the first-stage GMM with the weighing matrix $W = diag(cov(R))^{-1}$. The parameter $\eta$ is set equal to the estimate from the intra-temporal first-order condition. Standard errors are in parenthesis. Multiplicity of local minima were encountered.

Yogo (2003) imposes homotheticity *ex ante* and estimates $\gamma$ on the order of 500. It is the high consumption complementarity that helps resolve this equity premium puzzle worsening but does not fully solve the puzzle itself. The role of non-homotheticity is important as it allows to push the yardstick of consumption complementarity $\theta$ down and still fit the long-term trend in the durables demand curve.

I calibrate the marginal rate of substitution $M_{t+1}$ using the estimated preference parameter vector. Figures 15 and 16 portray the volatility bounds of Hansen and Jagannathan (1991) at quarterly and annual frequencies. The thin line corresponds to the universe of assets that includes the risk-free rate and the value-weighted market return. The thick line is for the universe of assets of scaled risk-free rate and scaled value-weighted market return. The graphs indicate that high consumption complementarity as implied by the first-stage GMM estimates (Table 3, delivers volatile enough discount factor $M_{t+1}$ and appear capable to account for the high Sharpe ratios observed in the U.S. financial markets. Unfortunately, as the $J_T$ statistics and its $p$-value indicate, the model is statistically rejected at conventional significance levels, both at quarterly and annual frequencies.

Overall, the output of the Generalized Method of Moments supports the idea that strong consumption complementarity between nondurables and another consumption good, in particular, services flow from household durables, is helpful in accounting for the risk-free rate and the equity premium with a plausible aversion to wealth bets.

Table 4: Annual Frequency GMM Estimates: Fama-French Portfolios

| Model | $\theta$ | $\eta$ | $\gamma$ | $\beta$ | $a$ | $J_T$ |
|---|---|---|---|---|---|---|
| | Moments: $R^f$ and all 25 FF portfolios | | | | | |
| C-CAPM with Durables | 0.0022 | 0.565 | 95.677 | 1.211 | 0.956 | |
| | (0.0018) | (0.014) | (145.185) | (1.518) | (4.020) | |
| | Moments: $R^f$ and 8 FF portfolios (11,12,13,14,15 and 54,55) | | | | | |
| C-CAPM with Durables | 0.0015 | 0.561 | 138.912 | 0.784 | 0.861 | 8.838 |
| | (0.0028) | (0.0202) | (492.2957) | (3.3523) | (12.4345) | (0.0315) |

NOTE - The table reports first-stage GMM results (annual data, $W = inv(diag(cov(R)))$) obtained by minimizing the quadratic form of pricing errors over the parameter space $R^5_+$. Standard errors are in parentheses. Spectral density matrix is estimated using Bartlett weight (with 3 lag). In $J_T$ column, p-value is in parentheses.

#### 4.2.6 GMM Estimation of Euler Equations: Risk-Free Rate and 25 Fama-French Portfolios

In this section I investigate whether the novel consumption risk of the stock market - the distortion of the consumption variety - helps account for the cross-section of 25 Fama-French portfolios. Specifically, I follow Hansen and Singleton (1992, 1993) and Eichenbaum and Hansen (1990), and estimate the Euler equation where the universe of the assets is the risk-free rate and 25 Fama-French portfolios. We learned from the calibration exercises in the previous sections that the model has a difficulty of matching the sign of the Sharpe ratio on the Fama-French value-minus-growth portfolio at the quarterly frequency but not at the annual frequency. As a result, the test of the cross-sectional predictability is done using annual returns.

Table 4 reports the estimated preference parameters $\theta$, $\beta$, $\gamma$ and $a$ where the measure of the non-homotheticity $\eta$ is set equal to the estimate obtained from the intra-temporal first-order condition. It is not possible to test the over-identifying restriction when all 25 Fama-French portfolios are used as there are too many moment conditions compared to the sample size. I therefore focus on a subset of portfolios (see Table 4). The statistics $J_T = 8.8$ and the model is not rejected at 1% level. Figure 17 plots the realized vs. average returns on 25 Fama-French portfolios.

The parameter estimates provide further evidence in favor of strong consumption complementarity between nondurables and durables. Indeed, the parameter $\theta$ is estimated on the order of 0.002 and thus the consumption index is close to Leontief. However, the Sharpe ratios of 25 Fama-French portfolios are very high and as a result the curvature coefficient $\gamma$

is estimated on the order of 100, higher than in the previous section where the universe of assets is the risk-free rate and the value-weighted market return. This suggests that although consumption complementarity goes a long way toward explaining the average returns of 25 Fama-French portfolios, other sources of risk may also be important.

# 5 Conclusion

I present a consumption-based asset pricing model that in a unified framework matches the risk-free rate, matches the sign and magnitude of the Sharpe ratio on the aggregate market portfolio and the Fama-French value-minus-growth portfolio and accounts for the cross-section of 25 Fama-French portfolios. The model is furthermore capable to match the equity premium with a plausible aversion to wealth bets. The model succeeds by identifying a novel non-diversifiable macroeconomic source of risk, the distortion of the variety of the consumption portfolio. The new view of the recession is not only as a period of a decline in nondurable consumption but also as a period of highly skewed consumption variety toward service flow. The novel consumption risk of the stock market stems from the inability to sell durables in recessions in order to restore the optimal variety of the consumption portfolio.

There is a plethora of avenues for future research. In fact, in a follow-up paper [Pakoš (2004)] I use the framework developed in this paper and show that the expected risk premia on 25 Fama-French portfolios fluctuate in response to a change in the consumption variety during recessions. As the present model implies nondurables, durables and the relative price are co-integrated and deviations from this shared trend are a strong univariate forecasting variable, especially for small and value stocks, where the time series $R^2$s reach levels above 30%.

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# 6 Data Description

*Nondurable Consumption:* Real and nominal purchases of nondurables plus services $C_t$ from the U.S. National Income and Product Accounts. Quarterly from 1947,1-2001,4. Corrected for SAAR by dividing by 4 and converted to per-capita basis by dividing by population at the end of the quarter. Converted from quarterly to annual by summing for all 4 quarters.

*Durables:* Real and nominal purchases of durables $I_t$ from the U.S. National Income and Product Accounts. Quarterly from 1947,1-2001,4. Corrected for SAAR by dividing by 4 and converted to per-capita basis by dividing by population at the end of the quarter. Converted from quarterly to annual by summing for all 4 quarters. Stock of durables $D_t$ constructed as a weighted average of past purchases $D_t = \sum_{i=1947}^{t} (1-\delta)^{t-i} I_i$ with the depreciation rate $\delta = 6\%$ per quarter. This corresponds to about 21.9% per annum. This is consistent with Wykoff (1970) estimates of a depreciation rate from resale values of automobiles. Ogaki and Reinhart (1998) use value of 22 percent. I start in the year 1952 but perform tests since 1964 because of two reasons. One is that the initial stock of durables as of 1952 is unknown but may be safely assumed to have depreciated away by 1964. The other is that the portfolios used tend to be rather thin before 1964.

*Population:* Both quarterly and annual consumption series are converted to a per capita basis using the number of people in the U.S. at the end of the year. Annually 1947-2001.

*Risk-Free Return:* Annualized three-month Treasury bill. Converted to ex-post real returns by the implicit price deflator for the total consumption. Obtained from Ken French's web site. Yearly 1964-2001.

*Asset Returns:* Returns on 25 Fama-French portfolios formed according to the same criteria as in Fama and French (1992,1993). These data are value-weighted returns for the intersections of five size portfolios and five book-to-market equity (BE/ME) portfolios on the New York Stock Exchange, the American Stock Exchange, and NASDAQ stocks in Compustat. The portfolios are constructed at the end of June, and market equity is market capitalization at the end of June. The ratio BE/ME is book equity at the last fiscal year end of the prior calendar year divided by market equity at the end of December of the prior year. All asset returns are converted to ex-post real rates by the implicit price deflator for the total consumption. Obtained from Professor Kenneth French's web site. Quarterly and Yearly 1964-2001.

# 7 Technical Details of the Estimation

Note that in the utility function the parameter $\theta$ enters in the power as $\frac{\theta-1}{\theta}$ and $\frac{\theta}{\theta-1}$ so in a sense they offset each other. However, when $\theta$ is close to zero by the time we "undo" the first power computer has already truncated the decimal points and we get zeros. For this reason, it is important to rescale the series by dividing $C_t$ with $mean(C_t)$, and $D_t$ with $mean(D_t)$

$$\Omega(C_t, D_t) = \left\{ (C_t/mean(C))^{1-\frac{1}{\theta}} + a\ (D_t/mean(D))^{1-\frac{\eta}{\theta}} \right\}^{\frac{\theta}{\theta-1}} \tag{44}$$

and keep only the preference weight $a$ on the services flow. Notice that I can do it even if the series trend as it is just a convenient normalization of the series. I am not arguing that $mean(C)$ converges in probability to $E(C)$, it doesn't.

## 8 First-Order Conditions

I introduce uncertainty through the shocks $\omega_t$ that follow a first-order Markov process, with the transition matrix $\Pi$. I denote $\omega^t = (\omega_0, \omega_1, ..., \omega_t)$ the history of shocks up to time $t$. The transition matrix generates the conditional probabilities $\pi(\omega^s|\omega_t)$ on all histories $\omega^t \preceq \omega^s$. I denote $\pi(\omega^t)$ the unconditional probability of that particular history.

I allow the consumer to trade in $J$ different types of Lucas (1978) trees, with each tree $j$ yielding dividends $DIV^j(\omega^t)$, $j \in J$. The number of outstanding shares of each type is normalized to one. All trades are made at ex-dividend prices $P^j(\omega^t)$. The consumer's problem is to take financial asset price sequences $\left(P^1(\omega^t), ..., P^J(\omega^t)\right)$ and durables price sequences $q(\omega^t)$ as given and maximize

$$\sum_{t=0}^{\infty} \sum_{\omega^t} \beta^t \, u\left(\Omega\left[C(\omega^t), D(\omega^t)\right]\right) \, \pi(\omega^t) \tag{45}$$

subject to the sequence of budget constraints

$$C(\omega^t) \; + \; Q(\omega^t)\, I(\omega^t) + \sum_{j \in J} P^j(\omega^t)\, a^j(\omega^t) \; = \; \sum_{j \in J} a^j(\omega^{t-1}) \, \left[P^j(\omega^t) + DIV^j(\omega^t)\right] \tag{46}$$

that have to hold for each $t \geq 0$ and for each history $\omega^t$. The additional constraints in the consumer's program are the law of motion for the stock of durables

$$D(\omega^t) \; = \; (1-\delta) \; D(\omega^{t-1}) \; + I(\omega^t) \tag{47}$$

with $\delta$ denoting the depreciation rate of durables and the irreversibility constraint on the consumer durables

$$I(\omega^t) \; \geq \; 0 \tag{48}$$

Let us put Lagrange multipliers $\lambda(\omega^t)$ on the budget constraint, $\mu(\omega^t)$ on the law of motion for the stock of durables and $\kappa(\omega^t)$ on the non-negativity constraint. The Lagrangian is

$$\begin{aligned} L \; = \; \sum_{t=0}^{\infty} \sum_{\omega^t} \beta^t \, u\left(\Omega\left[C(\omega^t), D(\omega^t)\right]\right) \, \pi(\omega^t) \; + \; \beta^t \, \mu(\omega^t) \; \left[(1-\delta) \; D(\omega^{t-1}) \; + I(\omega^t) - D(\omega^t)\right] \\ + \beta^t \, \lambda(\omega^t) \left(\sum_{j \in J} a^j(\omega^{t-1}) \, \left[P^j(\omega^t) \; + \; DIV^j(\omega^t)\right] - C(\omega^t) \; - \; Q(\omega^t)\, I(\omega^t) - \sum_{j \in J} P^j(\omega^t)\, a^j(\omega^t)\right) \\ + \beta^t \, \kappa(\omega^t)\, \pi(\omega^t)\, I(\omega^t) \end{aligned}$$

Kuhn-Tucker theorem yields the following first-order conditions

$$u_C\left(\Omega\left[C(\omega^t), D(\omega^t)\right]\right) \, \pi(\omega^t) \; = \; \lambda(\omega^t) \tag{49}$$

$$u_D\left(\Omega\left[C(\omega^t), D(\omega^t)\right]\right) \, \pi(\omega^t) \; = \; \mu(\omega^t) \; - \; (1-\delta)\, \beta \sum_{\omega^{t+1}} \mu(\omega^{t+1}) \tag{50}$$

$$\mu(\omega^t) \; + \; \kappa(\omega^t)\, \pi(\omega^t) \; = \; \lambda(\omega^t)\, Q(\omega^t) \tag{51}$$

$$P^j(\omega^t)\, \lambda(\omega^t) \; = \; \beta \sum_{\omega^{t+1}} \lambda(\omega^{t+1}) \, \left[P^j(\omega^{t+1}) \; + \; DIV^j(\omega^{t+1})\right] \tag{52}$$

plus the budget constraint, the law of motion for durables and the complementarity slackness condition

$$\begin{aligned}\kappa(\omega^t) &\geq 0 \\ I(\omega^t) &\geq 0 \\ \kappa(\omega^t)\, I(\omega^t) &= 0\end{aligned}$$

Rearranging the terms, denoting $\sum_{\omega^{t+1}} \pi(\omega_{t+1}|\omega_t)$ as the conditional expectation operator $E_t$ and suppressing the dependence on the history of shocks $\omega^t$ by using the subscript $t$ yields the Euler equation

$$P_t = E_t \left\{M_{t+1} \left(P_{t+1} + DIV_{t+1}\right)\right\} \tag{53}$$

and the intratemporal first-order condition

$$\frac{u_D(C_t, D_t)}{u_C(C_t, D_t)} = Q_t - (1-\delta)\, E_t \left\{M_{t+1}\, Q_{t+1}\right\} - \nu_t + (1-\delta)\, \beta\, E_t \left[\nu_{t+1}\right] \tag{54}$$

where I define

$$\nu(\omega^t) = \frac{\kappa(\omega^t)}{u_C(C(\omega^t), D(\omega^t))} \tag{55}$$

and the stochastic discount factor $M_{t+1}$ is defined as

$$M_{t+1} = \beta\, \frac{u_C(C_{t+1}, D_{t+1})}{u_C(C_t, D_t)} \tag{56}$$

The marginal utility of non-durable consumption is

$$u_C(C_t,\, D_t) = a^{\frac{\theta-1}{\theta}}\, C_t^{-\frac{1}{\theta}} \left\{(a\, C_t)^{1-\frac{1}{\theta}} + ((1-a)\, D_t)^{1-\frac{\eta}{\theta}}\right\}^{\frac{1-\theta\,\gamma}{\theta-1}} \tag{57}$$

Asset market clearing dictates that

$$\forall\, j \in J\, \forall\, t \geq 0\, \forall\, \omega^t \,:\, a^j(\omega^t) = 1 \tag{58}$$

To clear the goods market we need that

$$\forall\, t \geq 0\, \forall\, \omega^t \,:\, C(\omega^t) = \sum_{j \in J_1} DIV^j(\omega^t) \tag{59}$$

and

$$\forall\, t \geq 0\, \forall\, \omega^t \,:\, I(\omega^t) = \sum_{j \in J_2} DIV^j(\omega^t) \tag{60}$$

both hold. The subsets $J_1$ and $J_2$, $J_1 \cup J_2 = J$, contain stocks that produce nondurables and durables.

# 9 Interpretation of Parameters

The Marshallian demands $C_t$ and $D_t$ satisfy, after log-differentiation,

$$d\,log\, C_t = \varepsilon_{12}\, d\,log\, R_t + \eta_1\, d\,log\, E_t \tag{61}$$

$$d\,log\, D_t = \varepsilon_{22}\, d\,log\, R_t + \eta_2\, d\,log\, E_t \tag{62}$$

The Slutsky equation relates the Marshallian price elasticities $\varepsilon_{ij}$, Hicksian price elasticities $\varepsilon_{ij}^*$, expenditure shares $s_j$ and expenditure elasticities $\eta_i$,

$$\varepsilon_{ij} = \varepsilon_{ij}^* - \eta_i \, s_j \tag{63}$$

where $i$ refers to the good under consideration ($C \equiv 1$ and $D \equiv 2$), and $j$ refers to the price of a good (i.e. $\varepsilon_{12}$ refers to the elasticity of good 1, which is nondurables $C$, to a change in the price of good 2, which is services flow $D$. Imposing the Slutsky equation

$$d\,log\,C_t = (\varepsilon_{12}^* - \eta_1 \, s_2)\; d\,log\,R_t + \eta_1 \, d\,log\,E_t \tag{64}$$

$$d\,log\,D_t = (\varepsilon_{22}^* - \eta_2 \, s_2)\; d\,log\,R_t + \eta_2 \, d\,log\,E_t \tag{65}$$

and substracting

$$d\,log\;(C_t \,/\, D_t) = (\varepsilon_{12}^* - \varepsilon_{22}^*)\; d\,log\,R_t + (\eta_1 - \eta_2)\;(d\,log\,E_t - s_2 \, d\,log\,R_t) \tag{66}$$

Define the real expenditure $\hat{E}_t$ as $d\,log\,\hat{E}_t = d\,log\,E_t - s_2 \, d\,log\,R_t$ and the elasticity of substitution $ES$ is defined as $ES = \partial\,log(C_t^*/D_t^*) \,/\, \partial\,log\,R_t = \varepsilon_{12}^* - \varepsilon_{22}^*$. Then we get

$$d\,log\;(C_t \,/\, D_t) = ES\;\, d\,log\,R_t + (\eta_1 - \eta_2)\; d\,log\,\hat{E}_t \tag{67}$$

which is the equation (15) in the text.

I now derive the deterministic equivalent of the intratemporal f.o.c. (27). Eliminate the expenditure $E_t$ in the system (64) to get

$$d\,\log\,C_t = (\varepsilon_{12}^* - \varepsilon_{22}^*\,\eta)\; d\,\log\,R_t + \frac{\eta_1}{\eta_2}\, d\,\log\,D_t \tag{68}$$

Define

$$\theta = (\varepsilon_{12}^* - \varepsilon_{22}^*\,\eta) \tag{69}$$

and

$$\eta = \eta_1 \,/\, \eta_2 \tag{70}$$

Hence,

$$d\,\log\,C_t = \theta\, d\,\log\,R_t + \eta\, d\,\log\,D_t \tag{71}$$

One may integrate the equation - this is perfectly consistent with the model which is log-linear (see the intra-temporal condition above) - and hence the parameters are constant, to obtain

$$\log\,C_t = const + \theta\,\log\,R_t + \eta\,\log\,D_t \tag{72}$$

Let us link the rental cost of durables $R_t$ to the price of durables $Q_t$

$$\log\,R_t = \log\,Q_t + \log\left(1 - E_t\left\{M_{t+1}\,\frac{Q_{t+1}}{Q_t}\right\}\right) \tag{73}$$

where the second term on the right hand side is stationary to obtain the intra-temporal first-order condition

$$\log\,C_t = const + \theta\,\log\,Q_t + \eta\,\log\,D_t + error \tag{74}$$

If stochastic setting is considered explicitly, all stochastic stationary terms go into the error term.

Table 5: Summary Statistics of Consumption Series

| | $\Delta\, C_t$ | $\Delta\, D_t$ | $\Delta\, Q_t$ |
|---|---|---|---|
| Univariate Summary Statistics | | | |
| Mean (in %) | 0.55 | 1.02 | -0.61 |
| Standard Deviation (in %) | 0.67 | 0.53 | 0.56 |
| Correlation Matrix | | | |
| $\Delta\, C_t$ | 1 | 0.34 | 0.07 |
| $\Delta\, D_t$ | | 1 | -0.21 |
| $\Delta\, Q_t$ | | | 1 |

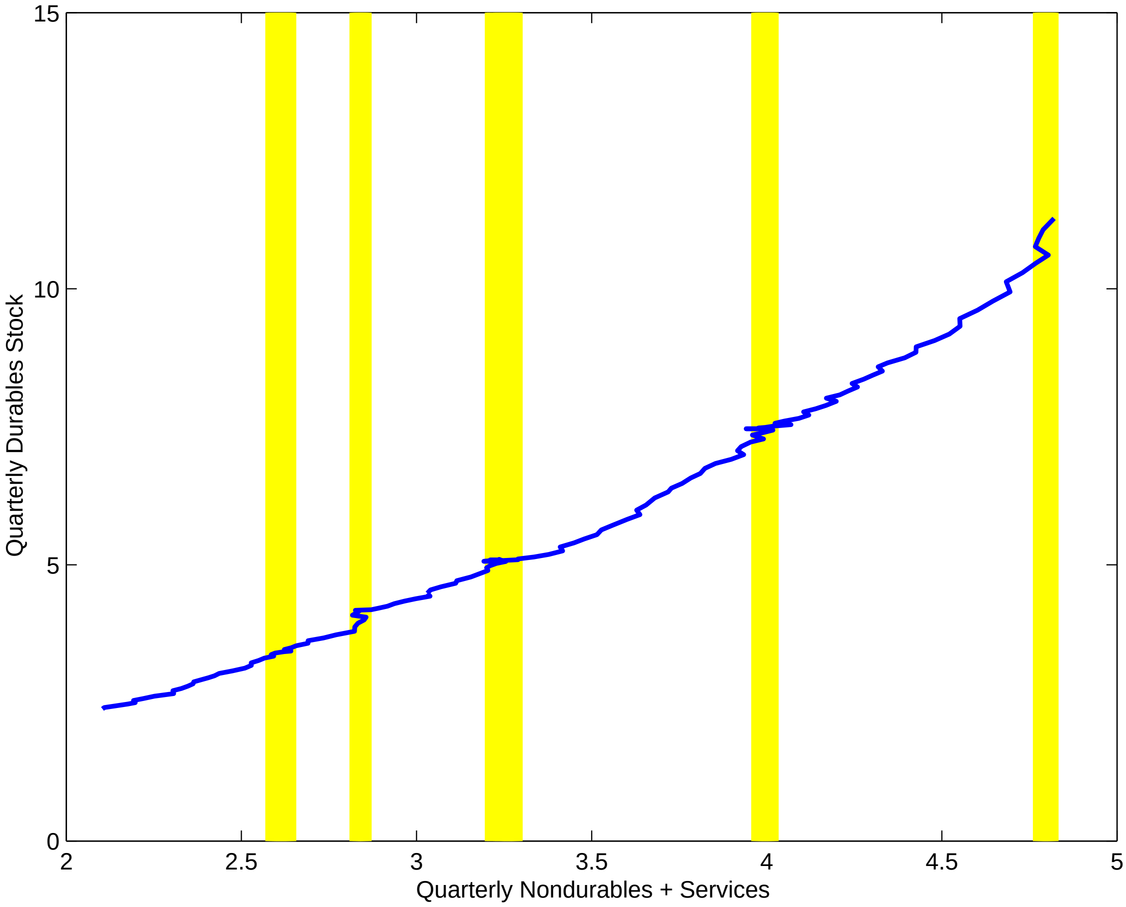
NOTE - All consumption series are quarterly, sample period 1964,1-2001,4.

Table 6: Summary Statistics of Asset Returns

| | | | |
|---|---|---|---|
| Quarterly | | | |
| | $R^f$ | *Equity Premium* | *Value Premium* |
| Mean (in %) | 0.38 | 1.47 | 0.95 |
| Standard Deviation (in %) | 0.71 | 8.52 | 6.37 |
| Sharpe Ratio | 0.54 | 0.18 | 0.15 |
| Correlation Matrix | | | |
| $R^f$ | 1 | 0.16 | -0.04 |
| *Equity Premium* | | 1 | -0.44 |
| *Value Premium* | | | 1 |
| Annually | | | |
| | $R^f$ | *Equity Premium* | *Value Premium* |
| Mean (in %) | 1.54 | 5.83 | 5.78 |
| Standard Deviation (in %) | 2.32 | 16.02 | 15.08 |
| Sharpe Ratio | 0.66 | 0.36 | 0.38 |
| Correlation Matrix | | | |
| $R^f$ | 1 | 0.22 | 0.16 |
| *Equity Premium* | | 1 | -0.34 |
| *Value Premium* | | | 1 |

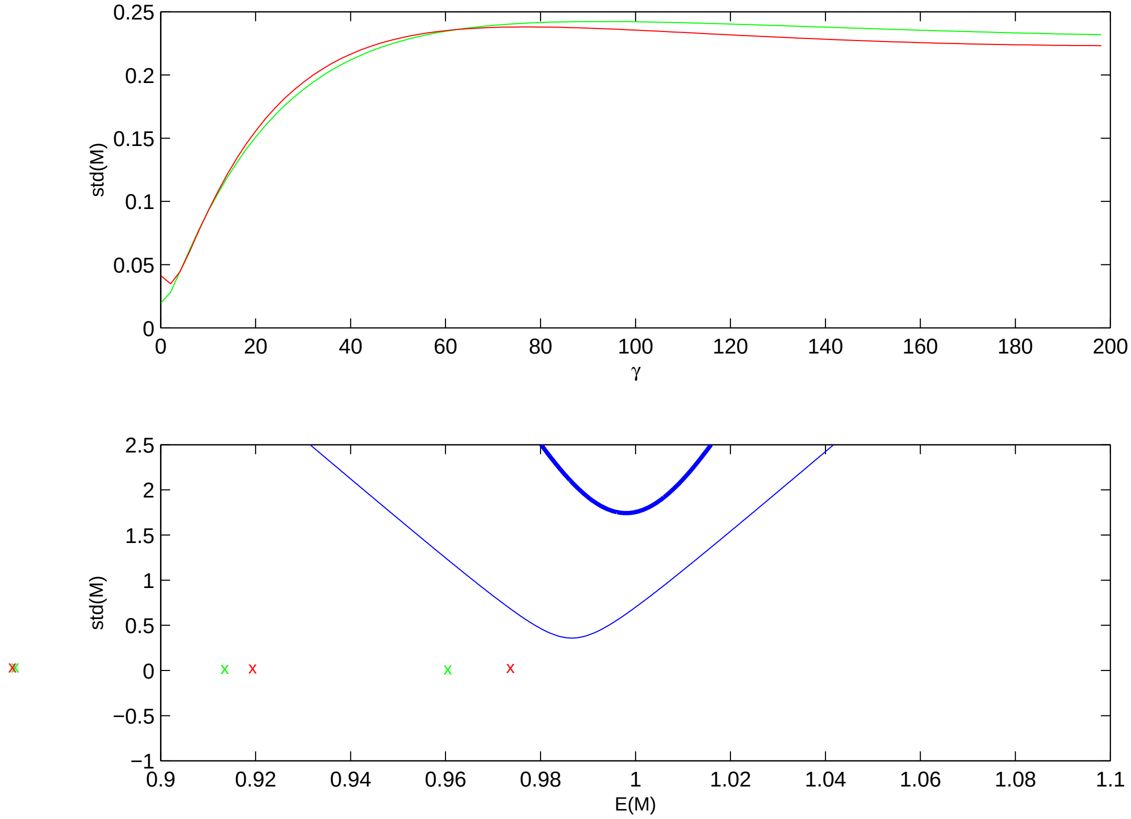
NOTE - All financial time series are quarterly and annually, sample period 1964-2001.

Figure 5: Income Expansion Path, Quarterly



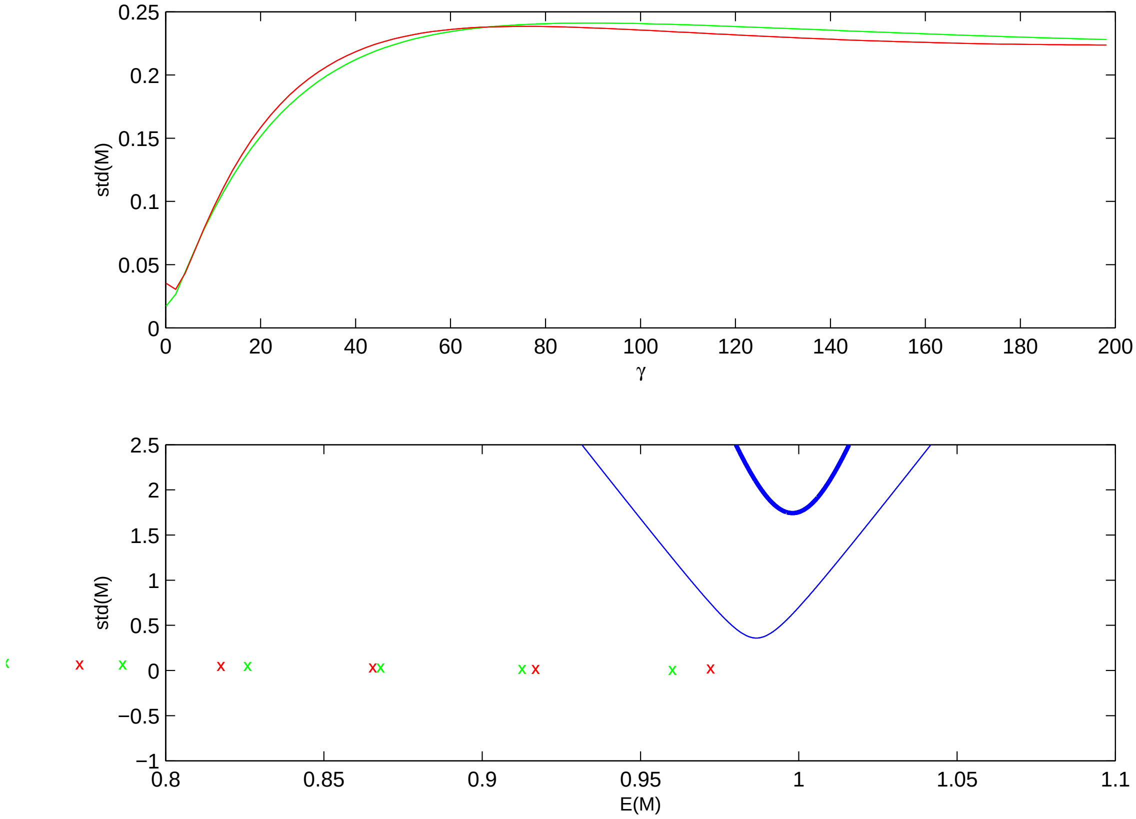

NOTE - The plot displays the quarterly nondurable consumption against the quarterly stock of durables. Sample period 1964,1-2001,4.

Figure 6: Volatility of the Discount Factor and Volatility Bounds as a Function of the Curvature Parameter $\gamma$ and the Preference Weight $a$, Quarterly.



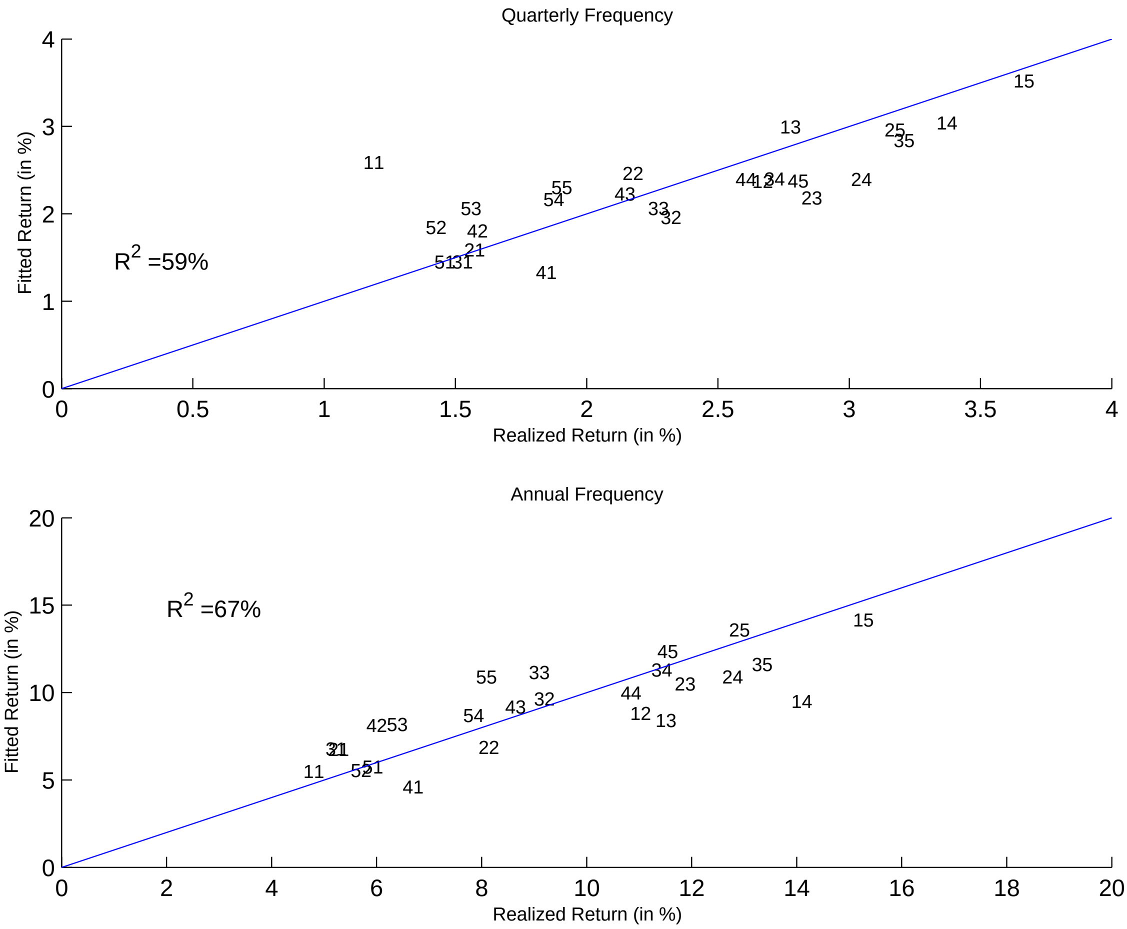

NOTE - The discount factor $M_{t+1}$ used is the marginal rate of substitution $M_{t+1} = \beta \frac{u_C(C_{t+1},D_{t+1})}{u_C(C_t,D_t)}$. I parametrize $\theta$ and $\eta$ with the super-consistent estimates from the intra-temporal first-order condition. Furthermore, I set $\beta = 0.95$. I vary the preference weight $a \in \{1, 5000\}$ and the curvature coefficient $\gamma \in [0.1, 200]$.

Figure 7: Volatility of the Discount Factor and Volatility Bounds as a Function of the Curvature Parameter $\gamma$ and the Preference Weight $a$, Annually.



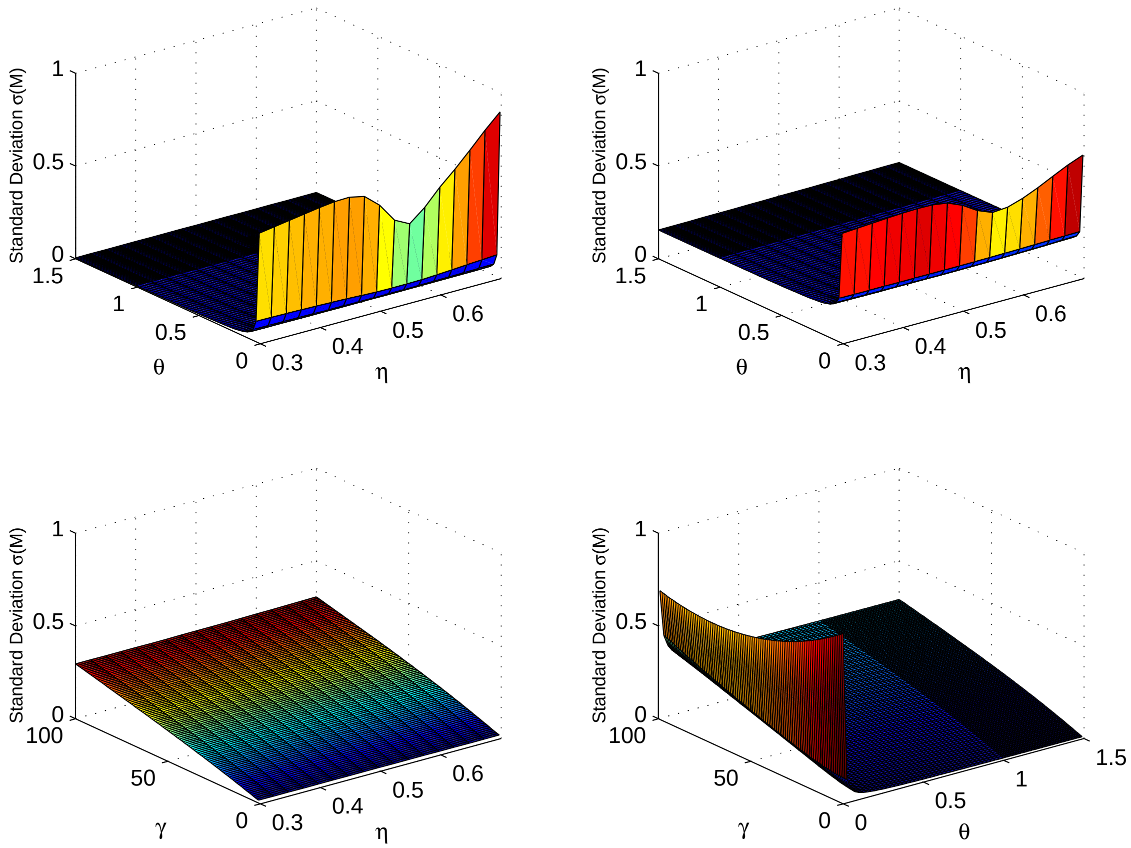

NOTE - The discount factor $M_{t+1}$ used is the marginal rate of substitution $M_{t+1} = \beta \frac{u_C(C_{t+1}, D_{t+1})}{u_C(C_t, D_t)}$. I parametrize $\theta$ and $\eta$ with the super-consistent estimates from the intra-temporal first-order condition. Furthermore, I set $\beta = 0.95$. I vary the preference weight $a \in \{1, 5000\}$ and the curvature coefficient $\gamma \in [0.1, 200]$.

Figure 8: Average versus Fitted Excess Returns on 25 Fama-French Portfolios



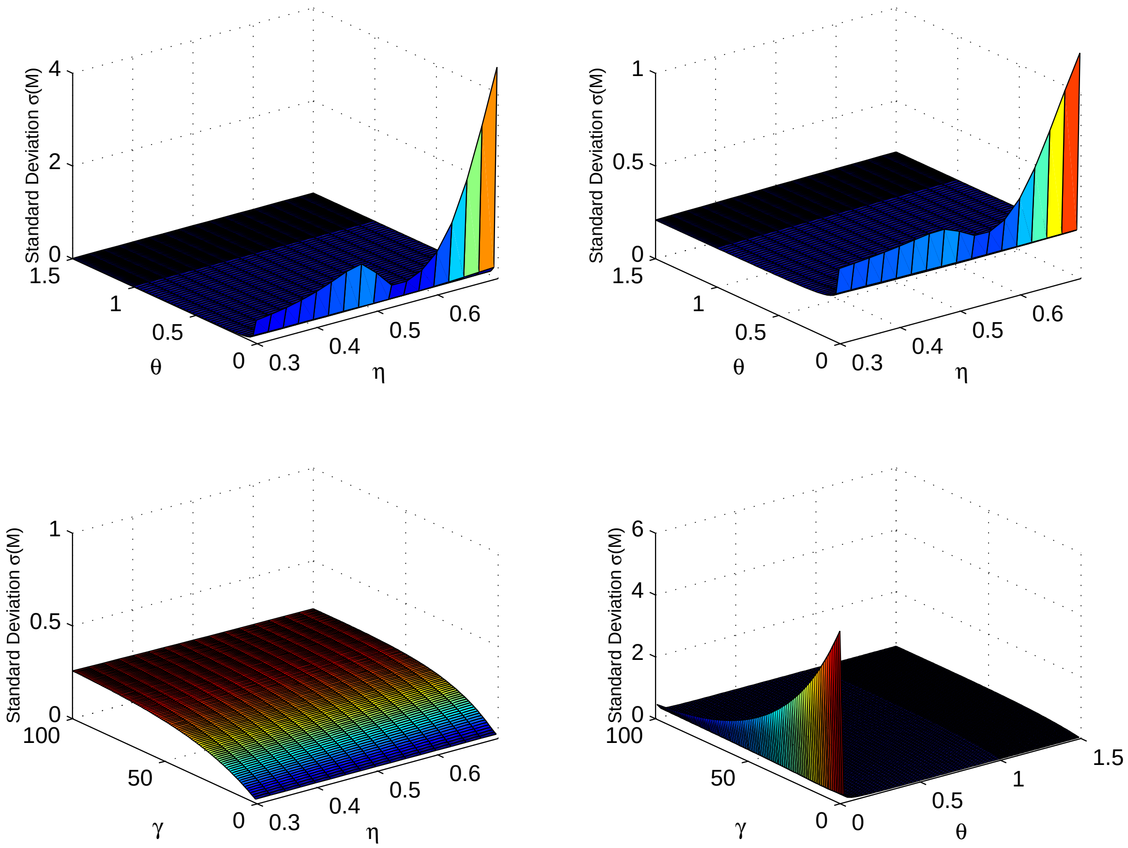

NOTE - The figure shows the cross-sectional plots of average fitted vs. average realized excess returns on 25 Fama-French portfolios where the risk factor in the Fama-MacBeth regression is the marginal rate of substitution $M_{t+1} = \beta \frac{u_C(C_{t+1}, D_{t+1})}{u_C(C_t, D_t)}$. The preference parameters $\theta$ and $\eta$ are calibrated from the intra-temporal first-order condition. The other parameters are calibrated as $\gamma = 1$, $\beta = 0.95$ and $a = 1$. Quarterly and Annually.

Figure 9: Volatility of the Discount Factor as a Function of the Preference Parameters, Quarterly



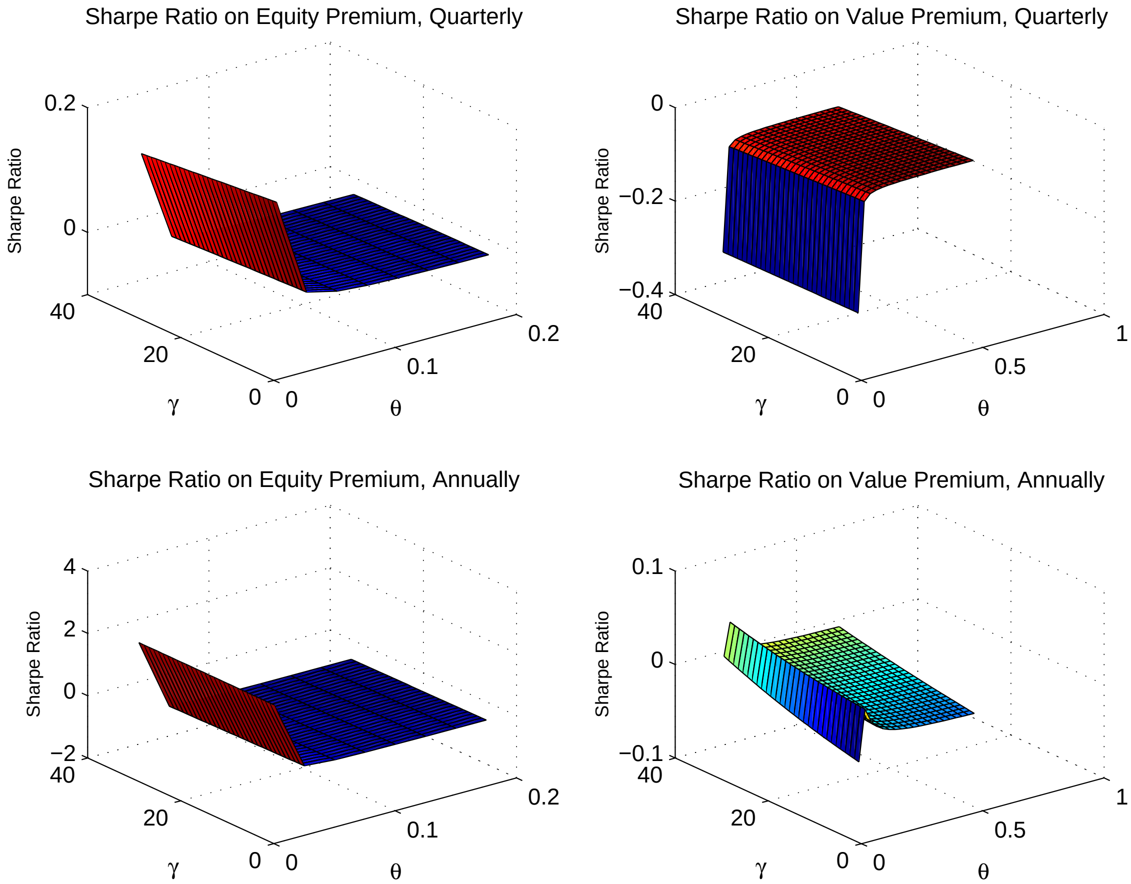

NOTE - The discount factor $M_{t+1}$ is the marginal rate of substitution $M_{t+1} = \beta \frac{u_C(C_{t+1}, D_{t+1})}{u_C(C_t, D_t)}$. The top left box portrays the volatility of the discount factor $\sigma(M_{t+1})$ as a function of the elasticity of substitution $\theta$ and the non-homotheticity parameter $\eta$. I calibrate the rest of the parameter vector as $\gamma = 1$, $\beta = 0.95$ and $a = 1$. The top right box differs in that I just calibrate $\gamma = 50$, all else the same. The bottom left box portrays the volatility of the discount factor $\sigma(M_{t+1})$ as a function of the non-homotheticity parameter $\eta$ and the curvature parameter $\gamma$. I furthermore calibrate $\theta$ from the intra-temporal first-order condition and set $\beta = 0.95$ and $a = 1$. The bottom right box portrays the volatility of the discount factor $\sigma(M_{t+1})$ as a function of the elasticity of substitution $\theta$ and the curvature parameter $\gamma$. I furthermore calibrate $\eta$ from the intra-temporal first-order condition and set $\beta = 0.95$ and $a = 1$.

Figure 10: Volatility of the Discount Factor as a Function of the Preference Parameters, Annually




NOTE - The discount factor $M_{t+1}$ is the marginal rate of substitution $M_{t+1} = \beta \frac{u_C(C_{t+1}, D_{t+1})}{u_C(C_t, D_t)}$. The top left box portrays the volatility of the discount factor $\sigma(M_{t+1})$ as a function of the elasticity of substitution $\theta$ and the non-homotheticity parameter $\eta$. I calibrate the rest of the parameter vector as $\gamma = 1$, $\beta = 0.95$ and $a = 1$. The top right box differs in that I just calibrate $\gamma = 50$, all else the same. The bottom left box portrays the volatility of the discount factor $\sigma(M_{t+1})$ as a function of the non-homotheticity parameter $\eta$ and the curvature parameter $\gamma$. I furthermore calibrate $\theta$ from the intra-temporal first-order condition and set $\beta = 0.95$ and $a = 1$. The bottom right box portrays the volatility of the discount factor $\sigma(M_{t+1})$ as a function of the elasticity of substitution $\theta$ and the curvature parameter $\gamma$. I furthermore calibrate $\eta$ from the intra-temporal first-order condition and set $\beta = 0.95$ and $a = 1$.

Figure 11: Sharpe Ratios on Equity and Value Premia Implied by the CCAPM with Durables.

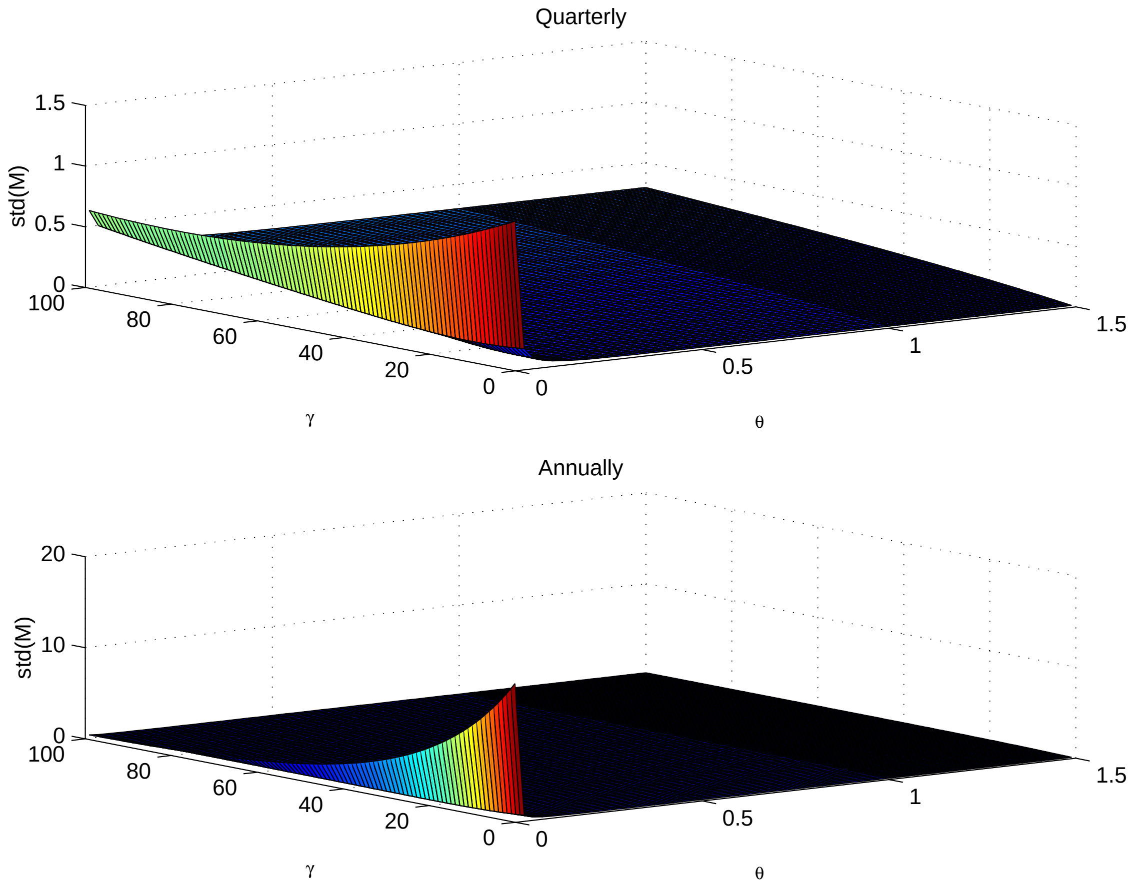



NOTE - The picture portrays Sharpe ratios on both the equity and value premia as a function of the curvature coefficient $\gamma$ and the elasticity of substitution $\theta$. The discount factor $M_{t+1}$ is the marginal rate of substitution $M_{t+1} = \beta \frac{u_C(C_{t+1},D_{t+1})}{u_C(C_t,D_t)}$. I calibrate the parameter $\eta$ from the intra-temporal first-order condition. I furthermore set $\beta = 0.95$ and $a = 1$. Sharpe ratios are computed using the formula

$$Sharpe\ Ratio \;=\; -correl(M_{t+1}, R^e_{t+1})\,\frac{\sigma(M_{t+1})}{E(M_{t+1})}$$

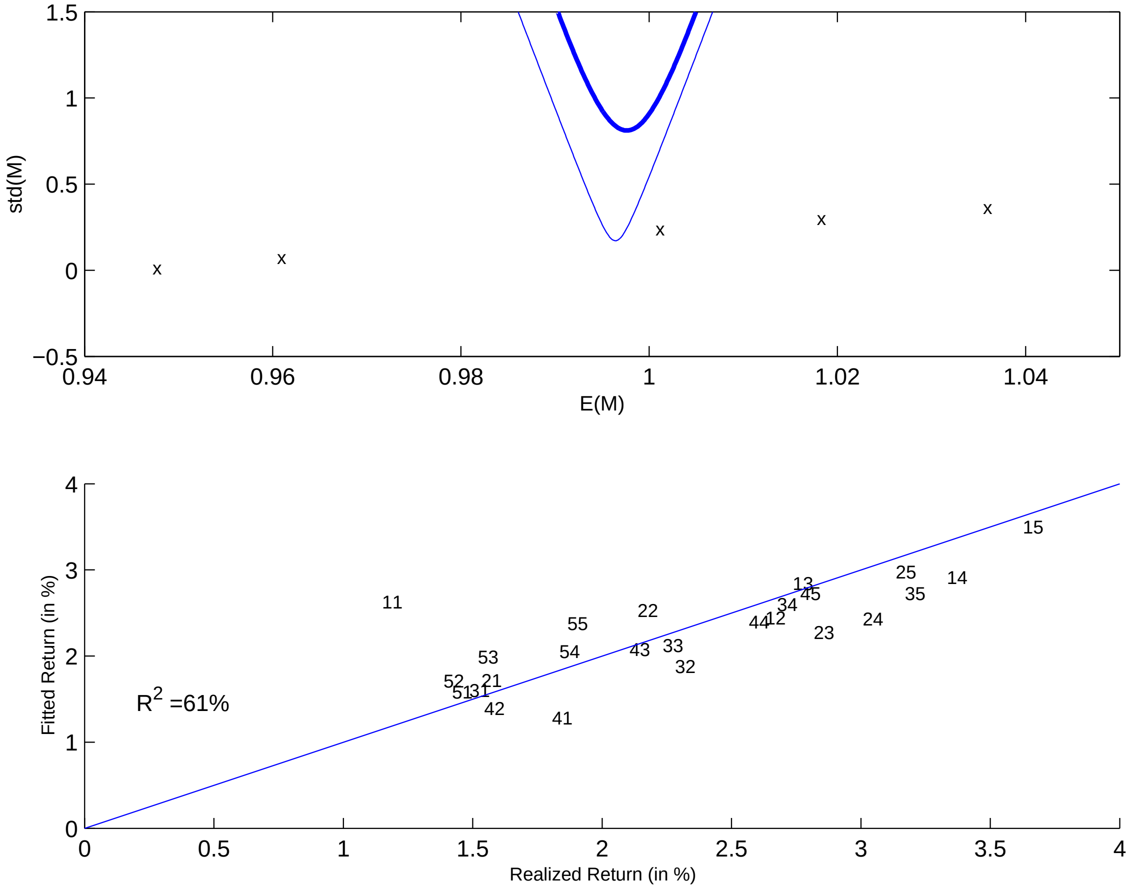
where the excess return $R^e$ is either the equity or the value premium.

Figure 12: Volatility of the Discount Factor as a Function of the Preference Parameters when Homotheticity is Imposed



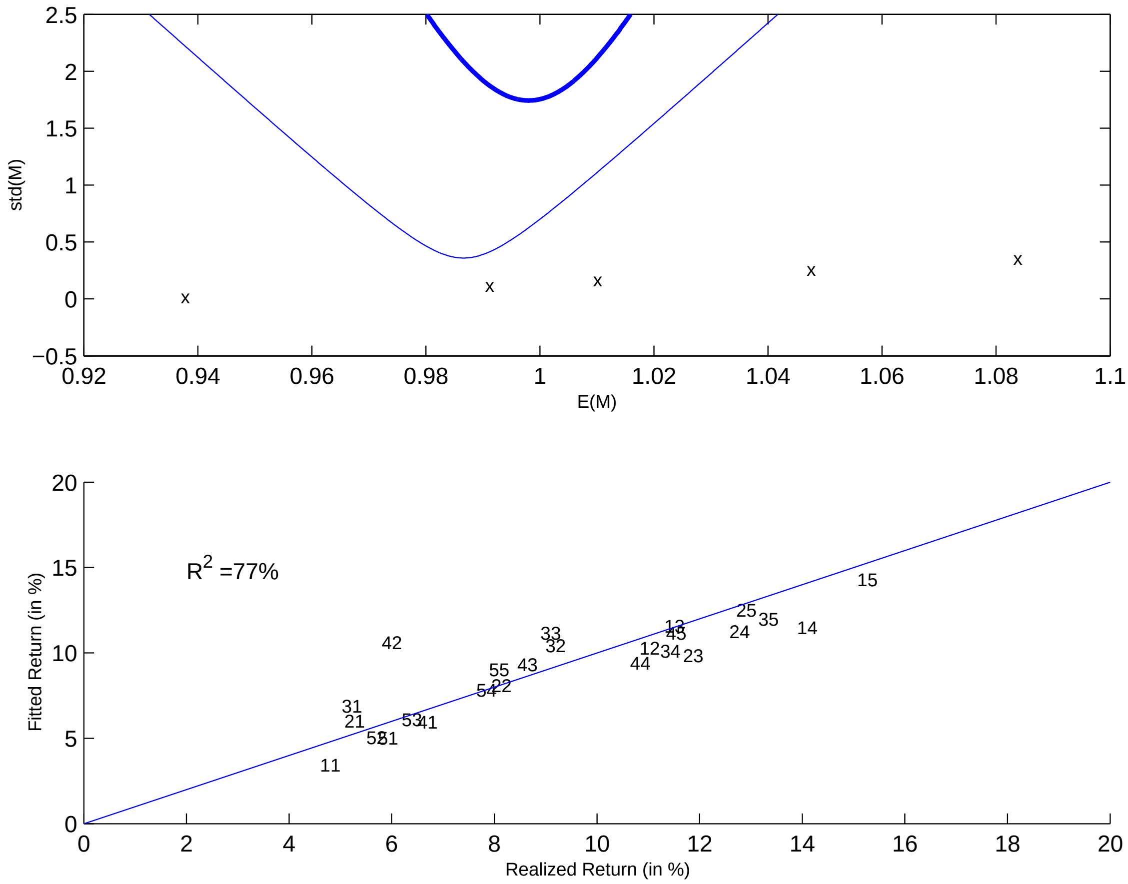

NOTE - The discount factor $M_{t+1}$ is the marginal rate of substitution $M_{t+1} = \beta \frac{u_C(C_{t+1}, D_{t+1})}{u_C(C_t, D_t)}$. I impose homotheticity and set $\eta = 1$. Furthermore, I calibrate $\beta = 0.95$ and $a = 1$. I vary the curvature parameter $\gamma$ and the elasticity of substitution $\theta$.

Figure 13: Volatility Bounds: Comparative Statics, Quarterly



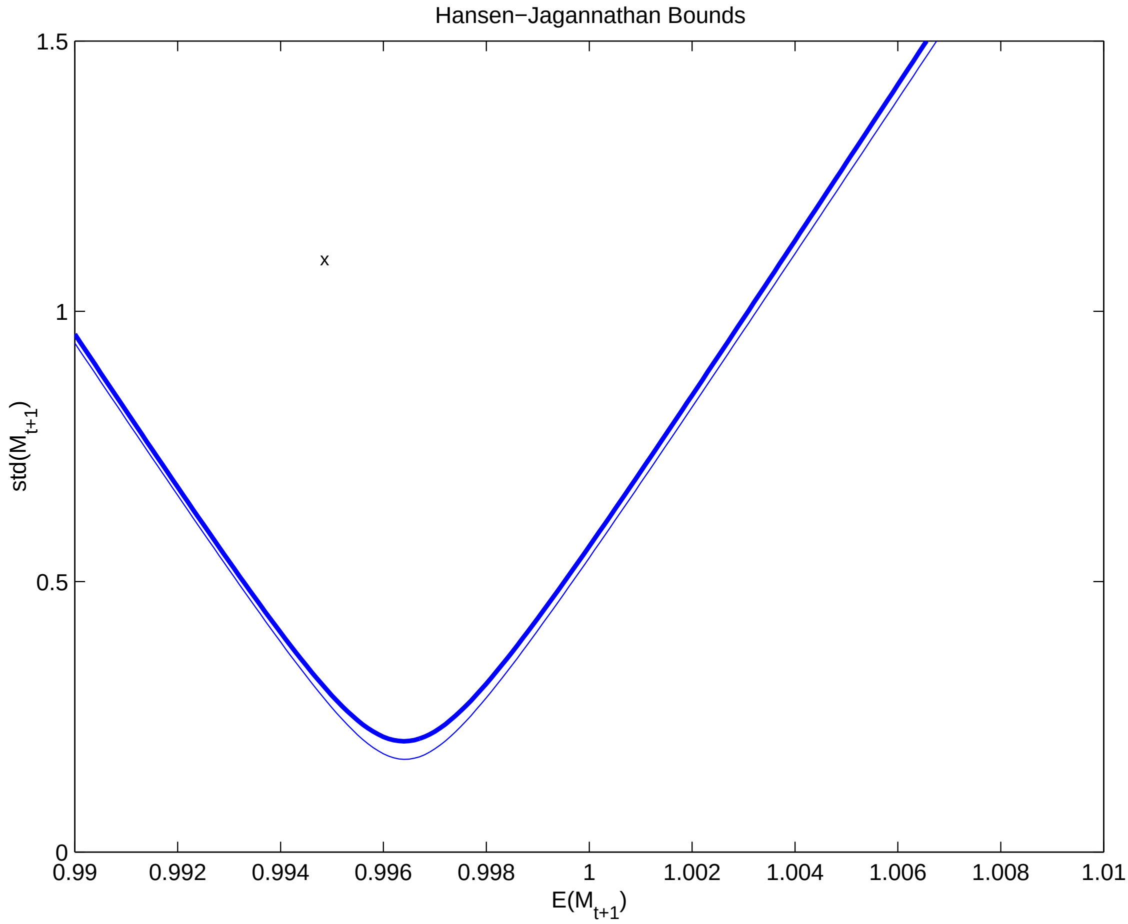

NOTE - The top box in the picture portrays Hansen-Jagannathan bounds for the market return and the risk-free rate (thin line), and for 25 Fama-French portfolios and the risk-free rate (thick line). Quarterly 1964,1-2001,4. The **x** in the picture corresponds to $\theta = 0.2553$, 0.05, 0.02, 0.017 and 0.015, from left to right. Other parameters are as $\eta = 0.7056$ (the super-consistent estimate), $\gamma = 1$, $\beta = 0.95$ and $a = 1$. The lower box show the cross-sectional plots of average fitted vs. average realized excess returns on 25 Fama-French portfolios where the factor is the marginal rate of substitution $M_{t+1} = \beta \frac{u_C(C_{t+1}, D_{t+1})}{u_C(C_t, D_t)}$. I choose $\eta = 0.7056$ (the super-consistent estimate), $\theta = 0.02$ (calibrated to bring me closest to the volatility bounds), $\gamma = 1$, $\beta = 0.95$ and $a = 1$.

Figure 14: Volatility Bounds: Comparative Statics, Annually



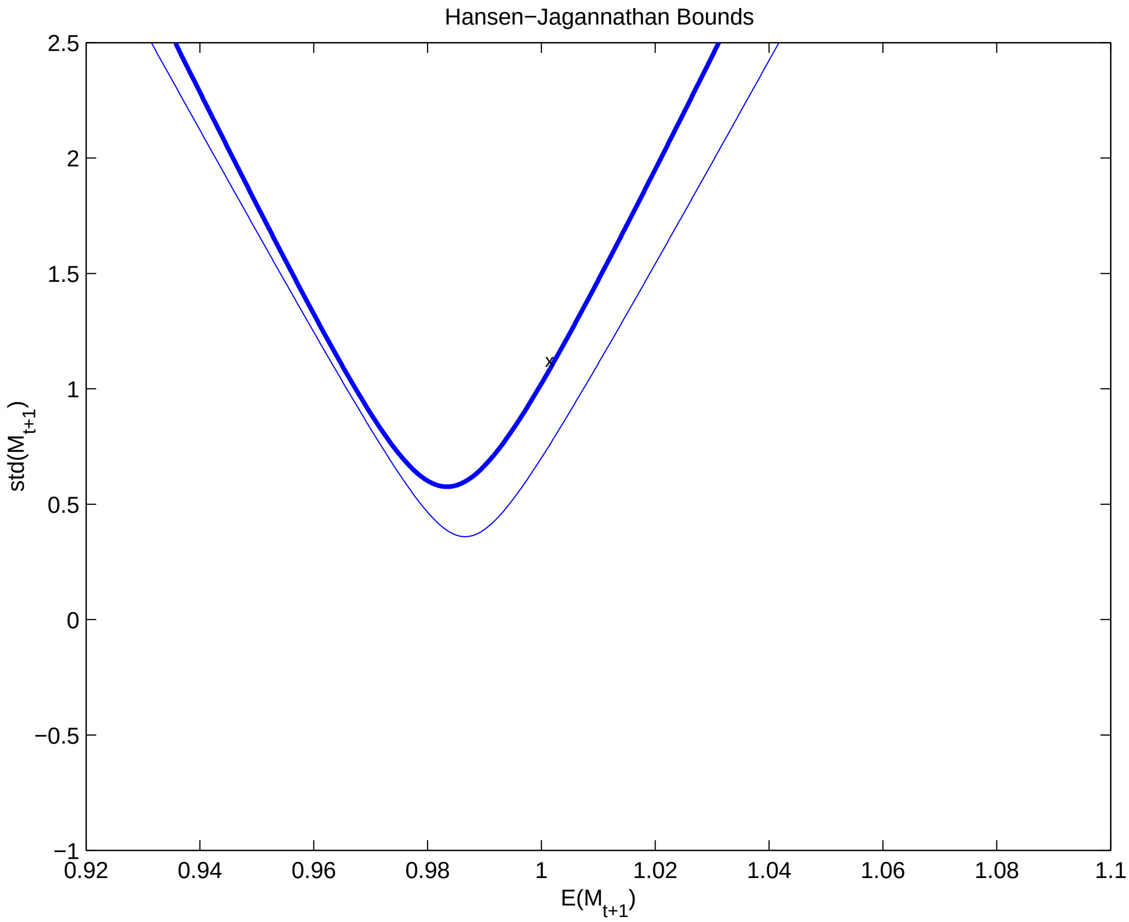

NOTE - The top box in the picture portrays Hansen-Jagannathan bounds for the market return and the risk-free rate (thin line), and for 25 Fama-French portfolios and the risk-free rate (thick line). Annually 1964-2001. The **x** in the picture corresponds to $\theta = 0.3004$, 0.05, 0.04, 0.03 and 0.025, from left to right. Other parameters are as $\eta = 0.7211$ (the super-consistent estimate), $\gamma = 1$, $\beta = 0.95$ and $a = 1$. The lower box show the cross-sectional plots of average fitted vs. average realized excess returns on 25 Fama-French portfolios where the factor is the marginal rate of substitution $M_{t+1} = \beta \frac{u_C(C_{t+1}, D_{t+1})}{u_C(C_t, D_t)}$. I choose $\eta = 0.7211$ (the super-consistent estimate), $\theta = 0.04$ (calibrated to bring me closest to the volatility bounds), $\gamma = 1$, $\beta = 0.95$ and $a = 1$.

Figure 15: Volatility Bounds when the Marginal Rate of Substitution is Calibrated with the First-Stage GMM Estimates, Quarterly



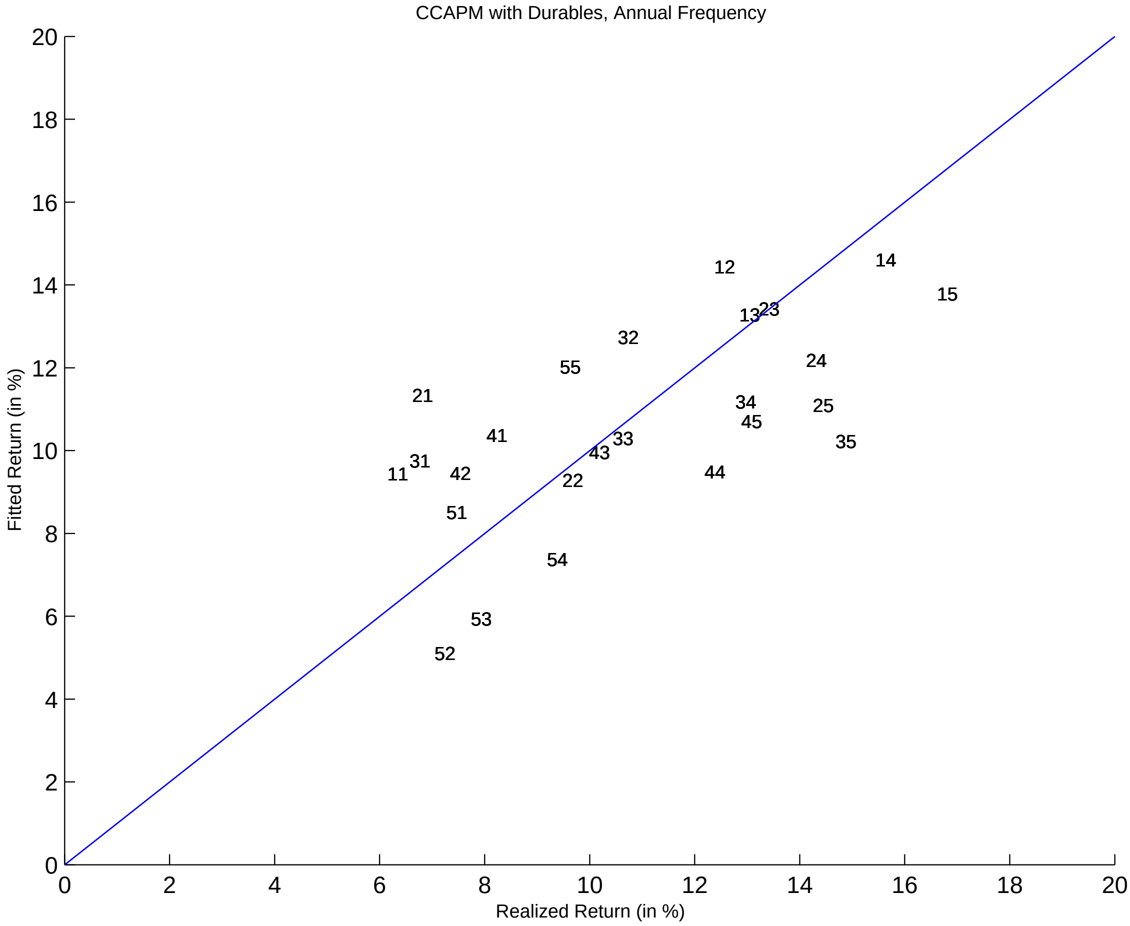

NOTE - The picture portrays Hansen-Jagannathan bounds for the value-weighted market return and the risk-free rate (thin line), and for the value-weighted market return and the risk-free rate scaled with the co-integrating residual $cay_t$, suggested by Lettau and Ludvigson (2001), and price-to-dividend ratio $P_t$ / $D_t$ (thick line), quarterly frequency. The symbol $\mathbf{x}$ in the picture corresponds to the marginal rate of substitution $M_{t+1}$ calibrated with the first-stage GMM estimates.

Figure 16: Volatility Bounds when the Marginal Rate of Substitution is Calibrated with the First-Stage GMM Estimates, Annually




NOTE - The picture portrays Hansen-Jagannathan bounds for the value-weighted market return and the risk-free rate (thin line), and for the value-weighted market return and the risk-free rate scaled with the co-integrating residual $cay_t$, suggested by Lettau and Ludvigson (2001), and price-to-dividend ratio $P_t$ / $D_t$ (thick line), annual frequency. The symbol **x** in the picture corresponds to the marginal rate of substitution $M_{t+1}$ calibrated with the first-stage GMM estimates.

Figure 17: GMM Results for the Cross-Section of 25 Fama-French Portfolios




NOTE - The graphs display realized vs. fitted average returns for the C-CAPM with durables at annual frequency. The stochastic discount factor is the marginal rate of substitution $m_{t+1} = \beta \frac{u_c(t+1)}{u_c(t)}$. The parameters are estimated using first-stage GMM with the weighting matrix $W = inv(diag(cov(R)))$.