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Double Dipping in Environmental Markets

Abstract: There is an increasing tendency to use markets to induce the provision of environmental services. As such markets increase in scope, potential market participants might sell multiple environmental services. The question we consider here is whether participants in such markets should be allowed to sell credits in more than one market simultaneously. Some have argued in favor of such “double dipping,” because it would make the provision of environmental services more profitable. In practice, however, most programs do not allow double-dipping. We show that if the optimal level of pollution abatement is sought, then double-dipping maximizes societal net benefits. However, if pollution policies are set in a piecemeal fashion, then the caps for each market are unlikely to be optimal and, in this second-best setting, a policy prohibiting double dipping can lead to greater social net benefits. We explore conditions under which a single-market policy is preferred, or equivalently, where piecemeal policies are likely to yield particularly inefficient outcomes.

Keywords: Environmental policy, tradable discharge permits, numerical methods
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I. Introduction

Suppose that a farmer adopts a conservation practice that both improves water quality and sequesters carbon. Should policy makers allow the farmer to sell credits generated by this single practice change in both a water quality credit market and a carbon market? That is, should the farmer be allowed to “double dip”? This is the question addressed in this paper.

Driven by the simple intuition that it makes sense to minimize the cost of pursuing environmental improvements, since the early 1990s a wide range of market-based programs have been developed to address environmental problems. In the U.S., air pollution trading programs include the national SO₂ trading program, California’s Reclaim program, and the multi-state Ozone Transport Commission. For water pollution, encouraged by the USEPA (USEPA 2004), over 50 watershed or statewide programs are in various stages of development and trading has taken place in 25 of these (USEPA 2007). A market-based approach is used in wetland mitigation banking (Shabman and Scodari 2005), in Habitat Conservation Plans to comply with the Endangered Species Act, as a tool in urban planning (McConnell et al. 2006), to encourage renewable energy (Berry 2002), and in the European climate change policy (Kruger et al. 2007). Virtually every new environmental policy in the U.S. includes a market element and the list of environmental goods and service covered by such programs continues to expand.

In recent years, market based programs have increasingly been making use of offsets or what Dewees (2001) calls Emission Reduction Credits. Offset provisions are used to create incentives for sources that are not included in the aggregate cap and have been used to reduce nonpoint source pollution (Woodward, Kaiser, and Wicks 2002), to offset wetland losses (Shabman and Scodari 2005), and is as a way to reduce CO₂ emissions. For example, as a way
to fulfill a portion of their obligations under the Kyoto Protocol, countries with binding obligations (Annex 1 countries) can sponsor emission reducing programs in developing nations under the Clean Development Mechanism. Such offsets reduce CO$_2$ emissions from developing countries while lowering the cost to the Annex 1 nations.

Together, the growing scope of market based programs along with the increasing use of offsets has greatly expanded the potential for interaction across markets. In particular, when offsets are generated through changes in land management, more than one environmental benefit will often result. A number of authors have called attention to this, highlighting the potential of multiple markets, what we will call double dipping (Kieser & Associates 2003; Davis 2006; von Hagen 2006; Greenhalgh 2008). If generators of environmental services can sell credits in many markets then the incentive to create these services will increase.

To an economist, allowing double dipping may at first glance appear to be as logical as allowing a cotton farmer to sell both the lint and the seed. Nonetheless, it is controversial. One reason for this is that most programs have strict provisions requiring that credits be “additional,” meaning that they “have arisen because of the new incentive of trading the permit or obligation on the market” (Haddad 1997). As a practical matter, if a source is selling credits in several markets it is more difficult to establish that all of the offsets are in fact additional. This paper explores whether there are economic reasons to allow or forbid double dipping.

Double dipping is possible when two or more pollutants are complements in the firm’s abatement cost function. At the firm level, complementarity means that abatement of two pollutants can be done at lower cost than the sum of the costs of abating each pollutant separately. At an economy-wide level, complementarity means that an increase in abatement of
one pollutant reduces the social marginal cost of the other related pollutants. As a result, complementarity leads to an increase in the socially optimal level of aggregate abatement.

As we show in section IV, a policy of allowing double dipping, which we will refer to as a multiple-markets (MM) policy, leads to the least-cost allocation of abatement. It follows, therefore, that if the caps are set optimally, the resulting equilibrium under an MM policy will lead to the social optimum. In order to achieve optimality, however, not only must policy makers have full knowledge of the cost and benefit functions, but in addition the policies for all the related pollutants must be coordinated. If policy makers do not take into account complementarities, then their estimate of the marginal cost will be too high and the resulting targets will be “second best,” falling short of the optimal level.

When abatement targets are below the social optimum, then it is possible that greater net benefits to society can be obtained from a policy of not allowing double dipping, what we will call a single market (SM) policy. The reason is that for a given program cap, an SM policy will actually lead to more total abatement since a source selling credits in one market will, because of complementarities, also abate other pollutants. As a result, although costs are higher under an SM policy, the social benefits are also greater.

Unfortunately, the question of whether an MM or SM policy yields greater net benefits is not clear cut. In sections V and VI we evaluate the conditions that tend to make each policy option preferred, looking at key parameters and the number of firms participating in the markets. To briefly summarize our findings, the SM policy option is likely to be most appealing when complementarity is significant, when the slope of the marginal benefit curves are relatively flat, and when there is greater heterogeneity in the pollution abating firms. An MM policy
becomes more attractive if these conditions do not hold or if the slopes of the marginal benefit curves for the various pollutants differ greatly. We should note that social net benefits can always be increased by moving the caps toward the optimum and adopting a cost-minimizing MM policy; if possible, that would be a better policy alternative. Hence, the conditions that tend to favor an SM policy can also be interpreted as indicators that of the importance of complementarities and the need to coordinate policies.

II. Literature Review

Dales (1968a, 1968b) and Crocker (1966) are credited with coming up with the idea of using tradable pollution permits to control pollution. The first formal treatment of this problem was provided by Montgomery (1972). While Montgomery’s model incorporated the general features of a multiple pollutant problem, he characterized it as a single pollutant with multiple receptor points.

In what appears to be the first direct treatment of tradable permits with multiple pollutants, Beavis and Walker (1979) established the optimality conditions for the control of multiple water pollutants when there are nonseparable interactions in both the damage function and in the cost functions of abating firms. They showed that in most situations, when there are multiple pollutants to be controlled, the optimal policy choice requires jointly choosing the level for all pollutants simultaneously. The case of multiple pollutants that cause tropospheric ozone was studied by von Ungern-Sternberg (1987), who showed the importance of considering costs when developing a policy. Michaelis (1992) took a similar approach to the case of climate

1 Throughout we will refer to a policy option as *preferred* if it yields greater net social benefits.
change where multiple pollutants (e.g., CO$_2$ and methane) lead to a single consequence, and develops relative prices for such pollutants.

Montero (2001) considers the question as to whether cross-pollutant trading should be allowed, i.e., whether a firm should be allowed to increase emissions of pollutant A by buying credits generated by reducing pollutant B. Such cross-pollutant trading can be economically efficient except if the pollutants enter the social benefit function in a Leontief or maximin manner. In a fashion akin to Weitzman (1974), Montero finds that the relative slopes of the marginal benefit and marginal cost curves prove critical to determining if cross-pollutant trading should be allowed or not. If the marginal damage curves are steep, then it is less efficient to allow cross pollutant trading.

Caplan and Silva (2005) and Caplan (2006) investigate a multipollutant problem in an international setting. A productive sector in each nation creates both a regional and global pollutant in a fixed proportion. They show that if policy makers respond optimally in a three stage decision process, optimal caps will be chosen and trading programs at the international and national level can lead to a Pareto efficient result. Caplan (2006) goes on to show that if taxes are used, as opposed to a cap-and-trade approach, then the resulting equilibrium in similar three-stage games is not socially efficient. As in the current paper, the inability of the decision makers to coordinate can lead to an inefficient outcome.

More closely related to the current paper, Horan et al. (2004) ask whether farmers can receive government subsidies to implement best management practices and then sell the credits generated by those practices in a transferable rights market. They show that efficiency gains occur under double dipping when two payments scheme are coordinated. But even in the uncoordinated or stand-alone setting, double dipping increases efficiency with well-targeted
payment incentives. If payment incentives are not well-targeted, then it is more efficient to restrict farmers to participating in either the trading program or the government program.

Finally, our analysis is closely related to the problem of adverse selection that arises when sources are paid to carry out an environmental action, even though they would have implemented the practice without a market incentive. Of particular note are the papers by Lewis (1996), who provides a general discussion of this problem in environmental problems, and Montero (2000), who shows how this arises when there is one set of firms that is regulated and another that has the opportunity to voluntarily opt-in to the program. The current paper explores how the presence of multiple pollution markets can lead to this phenomenon.

III. Basic graphical analysis of the multiple pollutants problem

A firm’s choice to abate two pollutants, $a_1$ and $a_2$, leads to costs, $g(a_1, a_2)$. In a fashion similar to Helfand (1991), in Figure 1 we present the iso-cost curves associated with differing levels of abatement of the two pollutants for a representative firm with costs increasing in the distance from the origin. The rays labeled $g_1=0$ and $g_2=0$ that traverse the iso-cost curves indicate the set of points along which the marginal cost of abatement of one pollutant is equal to zero. For example, at each point on the line labeled $g_2=0$, for a given level of $a_1$, the marginal cost to increase $a_2$ is equal to zero. These lines, therefore, are the reaction functions of the firm’s abatement of one pollutant to abatement of the other. In Figure 1a, reaction functions are horizontal and vertical, so that if a firm is required to abate pollutant 1, it will take no actions with regard to pollutant 2. In Figure 1b, the pollutants are complements – if the firm is obligated to abate a positive quantity of pollutant 1 then, without any policy intervention on pollutant 2, the firm’s cost minimizing choice will be to reduce its net emissions of pollutant 2 by following the $g_2=0$ ray. Finally, in Figure 1c the pollutants are substitutes – a requirement to abate pollutant 1
will lead the cost-minimizing firm to actually increase its emissions of pollutant 2 \((a_2<0)\). We will focus throughout the paper on the case in which the pollutants are complements (Figure 1b).

To introduce our policy problem, we start by considering a very simple case of an economy with \(n\) possible sources of offsets of two pollutants, P1 and P2. In Figure 2 we represent the case of P2, for which a market based program is being developed that will allow firms to be paid to reduce P2. The purchasers of these credits will be other sources that are subject to a cap on their emissions. The costs of the capped polluters are high enough that all required abatement will be provided by the \(n\) sources of offsets. Society has a known marginal benefits curve for aggregate reductions in this pollutant, \(MB_2\), and government has estimated the social marginal costs of abatement for these \(n\) firms, \(MC_2\). Attempting to maximize net social benefits, we assume that the P2 policy makers set the cap at \(\hat{A}_2\) where \(MB_2 = MC_2\).

However, unbeknownst to the P2 policy maker, simultaneously a market-based policy affecting P1 is being introduced. Furthermore, while first \(n-1\) firms are unaffected by the P1 policy, the \(n\)th firm can offset both P1 and P2 and its cost function is characterized by complementarity as in Figure 1b. For that firm, the P1 policy induces it to abate \(a_2\) units of P2
resulting to a rightward shift in the social marginal cost curve, from $MC_2$ to $MC^*$ in Figure 2. This shifted curve is the true social marginal cost curve contingent on the P1 policy since it reflects the fact that there is no additional cost to generate the $a_2$ units of abatement that arose due to the P1 policy. The socially optimal level of abatement of P2 is $A_2^*$, where the social marginal cost equals the social marginal benefit. Following standard tradable permits intuition (Baumol and Oates 1988), if a cap were set at $A_2^*$, the socially optimal level of abatement would be achieved at lowest possible cost.

![Figure 2: Marginal benefits and marginal costs of abating pollutant 2](image)

While $A_2^*$ would be the optimal cap, $\hat{A}_2$ is second-best in the sense that it is made based on the limited information available to the policy maker. Given the balkanized fashion in which most environmental policy is carried out, this second-best policy captures important features of the real world.\(^2\)

\(^2\) Our assumption that the policy maker has a clear picture of the marginal benefit curve, on the other hand, is admittedly unrealistic. We discuss below how errors in measurement of the MB curves can have effects that are similar to those in which the MC curve is mis-specified.
We can now ask whether an SM or MM policy is preferred in this second-best setting. Note that regardless of the policy choice, the $a_2$ units of abatement will occur and will generate benefits. The question is whether they should be counted in satisfying the P2 cap, $\hat{A}_2$. If an MM approach is taken, then the $a_2$ units can be used to satisfy the cap so that total abatement of P2 is exactly $\hat{A}_2$, less than $A_2^*$. If multiple markets are not allowed, then the $n^{th}$ firm cannot sell its $a_2$ credits and the other $n-1$ firms must supply $\hat{A}_2$ units of abatement so that under an SM policy total abatement of P2 is $\hat{A}_2 + a_2$, more than $A_2^*$.

Since neither $\hat{A}_2$ nor $\hat{A}_2 + a_2$ is socially optimal; the question about whether an MM or SM policy is preferred depends on which policy yields a smaller welfare cost. As seen in Figure 3, if an MM policy is adopted, then the equilibrium price paid in the pollution market will be $p$ and, a welfare cost will result, indicated by the triangle labeled MM. Under an SM policy, the market-clearing price would be $\bar{p}$, leading to inefficiently high level of total abatement and social costs labeled SM.

Figure 3: Effect on welfare costs under a second-best cap $\hat{A}_2$ with steep MB and steep MC
In this simple example, the question of which policy is more efficient depends on the slopes of the MC and MB curves. In the first panel of Figure 3 we present the base case when the welfare costs for the two policies are about equal. In the second panel we consider the case when the MB curve is steeper. As the curve becomes steeper, the SM triangle grows and the MM triangle shrinks – an MM policy yields greater social benefits. Intuitively, this makes sense; if the MB curve were vertical, the first- and second-best caps would be at the same point and the preferred policy would be that which minimizes the cost, which achieved by the MM policy. On the other hand, if the MB curve were horizontal, the key to avoiding social cost is to ensure that all abatement with a cost less than the MB were achieved, and that would follow from the SM policy.

In the third panel of Figure 3 we present the case when the MC curve is steep. In this case, the $a_2$ units of abatement achieved because of complementarity has a large relative effect on total costs. A steepening of the MC curve, therefore, increases the gap between $\hat{A}_2$ and $A^*$ so that the error associated with an SM policy shrinks. When the MC curve is flat, on the other hand, then the rightward shift from $MC_2$ to $MC^*$ would have little effect on the intersection with the MB curve. As a result, as the MC curve flattens, the welfare cost associated with the MM policy declines as this is the less expensive way to achieve a level of abatement.

Our simple graphical analysis shows that in a second-best setting it is not obvious whether allowing double dipping is economically efficient. The simple model gives some indication of the conditions when an MM policy would be preferred: when the MB curve is relatively steep or MC curve is relatively flat. These findings, however, stem from a very simplistic framework and are not generalizable. How will other parameters enter into the policy
question? What if there are many firms characterized by complementarity? In the remainder of this paper we answer these questions in more general settings.

IV. The multiple pollutants problem and policy alternatives

For the remainder of this paper we consider the policy problem in which there are \( j=1,\ldots,m \) pollutants, \( P_j \), and aggregate abatement of each pollutant, \( A_j \), yields additively separable benefits to society, \( B(A) = \sum_j B_j(A_j) \). An abatement cap, \( \hat{A}_j \), places a restriction on high-cost sources of \( P_j \), but these sources are allowed to fulfill their obligations by purchasing credits from \( n \) low-cost firms. We assume that the cost differential is such that any abatement cap will be satisfied entirely by the low-cost firms, hence explicit modeling of the high-cost firms is not necessary. The low-cost firms, which have no explicit limit on their emissions, can generate credits by reducing their net emissions relative to their initial level. The cost of abating pollution for these \( i=1,\ldots,n \) sources is a function of their vector of abatement activities, \( g_i(a_i) \), where \( a_i=(a_{i1}, a_{i2}, \ldots, a_{im}) \) and \( a_{ij} \) is the abatement of pollutant \( j \) by firm \( i \).

A. The planner’s problem

We will maintain throughout the following regularity assumptions: (1) the benefit and cost functions are all continuously differentiable; (2) each of the benefit functions is assumed to be strictly concave with \( B' > 0 \) and \( B'' < 0 \); (3) the cost functions \( g_i(\cdot) \) are strictly convex and, since we assume that each firm’s baseline level is the previous optimum, the marginal cost curves begin at the origin, i.e. \( \frac{\partial g_i(0)}{\partial a_{ij}} = 0 \) for all \( i, j \), where \( 0 \) is the null vector. In addition, because of our interest in pollutants that are complements, (4) we will assume that
\[ \frac{\partial}{\partial a_k} \left( \frac{\partial g_i}{\partial a_{ij}} \right) \leq 0 \] for all \( i \) and all \( j \neq k \), which we will refer to as strict complementarity if the inequality is strict. (5) At the market level we will assume that no supplier has market power so that any vector of prices is treated as exogenous by the individual firms.

For a vector of aggregate abatement levels, \( A = (A_1, A_2, \ldots, A_m) \), a cost effective allocation of abatement activities is found by solving the optimization problem:

\[ \min_{\{a_i\}} \sum_{i=1}^{n} g_i(a_i) \quad \text{s.t.} \sum_{i=1}^{n} a_{ij} \geq A_j \quad \text{for all} \ j. \]  

(1)

Since the \( g_i(\cdot) \) functions are convex by assumption, the total cost function is also convex and first-order conditions are necessary and sufficient for the solution of (1):

\[ \frac{\partial g_i}{\partial a_{ij}} = \lambda_j \left( \sum_{i=1}^{n} a_{ij} - A_j \right) = 0 \]  

(2)

where \( \lambda_j \) is the \( j^{th} \) Lagrange multiplier. Solving (2), a cost-minimizing allocation will be characterized by the equi-marginal conditions:

\[ \frac{\partial g_i(a_i)}{\partial a_{ij}} = \frac{\partial g_i(a_i)}{\partial a_{ij}} \]  

for all \( i, j, \) and \( l \). (3)

Note that because of complementarity, it can hold that for \( a_{ij} > 0 \), \( \frac{\partial g_i(a_i)}{\partial a_{ij}} = 0 \). Hence, even if \( A_j > 0 \), the \( j^{th} \) constraint may not bind.

Ideally, the planner would be concerned with not only minimizing costs, but in maximizing societal net benefits, i.e.,

\[ \max_{\{a_i\}} \sum_{j=1}^{m} B_j(A_j) - \sum_{i=1}^{n} g_i(a_i) \quad \text{s.t.} \ A_j = \sum_{i=1}^{n} a_{ij} \]  

(4)

As with (1), (4) is a convex programming problem so that the optimal vector of aggregate abatement, \( A^* \), is characterized by the first-order conditions,
\[ B'_i (A_j^*) = \frac{\partial g_i (a_i)}{\partial a_{ij}} \text{ for all } i,j. \] (5)

Hence, to achieve the social optimum it is required that the cost minimization is satisfied and the aggregate levels of abatement are set optimally.

**B. The MM policy**

In this paper we consider two decentralized policies to incentivize abatement effort. Under an MM policy, the \( n \) sources can generate valuable credits for all reductions in their net emissions. For a given vector of prices, \( p=(p_1,\ldots,p_m) \), the problem of these profit maximizing firms can be written

\[
\max_{a_i} \sum_j p_j a_{ij} - g_i (a_i)
\] (6)

with first-order conditions

\[ p_j = \frac{\partial g_i (a_i)}{\partial a_{ij}} \text{ for all } i,j. \] (7)

Inverting (7), a firm’s profit maximizing vector of abatement can be written \( a_i (p) \). By complementarity, we know that \( \partial a_{ij} (p) / \partial a_k \geq 0 \). We will occasionally assume what we call constant complementarity, \( \partial^2 a_{ij} (p) / \partial a_k^2 = 0 \), though this will only be required for sufficiency conditions.

A price vector, \( p^{MM} \), leads to a market equilibrium if, for a given vector of abatement caps, \( \hat{A} = (\hat{A}_1, \hat{A}_2, \ldots, \hat{A}_m) \), \( \sum_i a_{ij} (p^{MM}) \geq \hat{A}_j \) for all \( j \). Since the firms’ cost functions are convex and continuously differentiable, it follows that under the MM policy, aggregate abatement \( A_j^{MM} \) will exactly equal the cap in all markets if \( p_j^{MM} > 0 \) for all \( j \). On the other hand, it is possible for a
high cap on one pollutant to lead to complementary abatement in another pollutant so that for some \( j \) \( A_j^{\text{MM}} > \hat{A}_j \) and \( p_j^{\text{MM}} = 0 \).

Using the first order conditions, we obtain the following proposition.

**Proposition 1:** Under the regularity assumptions, the equilibrium under an MM policy will result in the cost effective allocation of abatement to achieve a level of abatement \( A \geq \hat{A} \). If the caps are chosen optimally, \( \hat{A} = A^* \), the equilibrium will also achieve the socially optimal level and allocation of abatement.

**Proof:** For a vector of caps \( \hat{A} \), the MM equilibrium conditions, (7), are equivalent to (3) when \( \hat{A} = A \). For optimality, if \( \hat{A} = A^* \) then (7) also ensures that

\[
p_j = B'_j = \frac{\partial g_i (a_i)}{\partial a_{ij}} \quad \text{for all } i, j \text{, which satisfies (5)}.
\]

It will be helpful to refer to the aggregate benefits, costs, and net benefits in an equilibrium under an MM policy as a function of a vector of abatement caps: \( B^{\text{MM}} (\hat{A}) \), \( C^{\text{MM}} (\hat{A}) \), and \( NB^{\text{MM}} (\hat{A}) \).

**Proposition 2:** Under the regularity assumptions, \( \frac{\partial B^{\text{MM}} (\hat{A})}{\partial \hat{A}_j} \geq 0 \) and

\[
\frac{\partial C^{\text{MM}} (\hat{A})}{\partial \hat{A}_j} \geq 0 . \quad \text{Furthermore, constant complementarity is sufficient for}
\]

\[
\frac{\partial^2 B^{\text{MM}} (\hat{A})}{\partial \hat{A}_j^2} \leq 0 \quad \text{and} \quad \frac{\partial^2 C^{\text{MM}} (\hat{A})}{\partial \hat{A}_j^2} \geq 0 .
\]

**Proof:** See Appendix A.
C. The SM policy

Under an SM policy, each polluter can sell credits for abating only one of the \( m \) pollutants. Hence, the firm optimization problem becomes

\[
\max_{j, a_i} p_j a_{ij} - g_i(a_i).
\]

Distinct from (6), under an SM policy, credit suppliers must choose both the market in which to participate and how much abatement to carry out. For a given price vector, the decision to participate in market \( j \) will be optimal if

\[
p_j a_{ij}^* - g_i(a_i^*) \geq p_k a_{ik}^* - g_i(a_k^*) \text{ for all } k \neq j,
\]

where the vector \( a_i^* \) is the vector of abatement levels that are optimal given that the firm is participating in market \( j \).

Because of the discrete nature of the choices made by firms under the SM policy, the equilibrium is not necessarily characterized by convenient first-order solutions. Consider the following example. Suppose that there are two pollutants with caps, \( \hat{A}_1 = \hat{A}_2 \). There are 101 identical firms that emit both pollutants with cost functions that satisfy the regularity assumptions and also have the characteristic that the marginal cost to abate the pollutants are equal in the sense that \( \partial g_i(a_i, a_{i2})/\partial a_i = \partial g_i(a_{i1}, a_{i2})/\partial a_{i2} \) for \( a_i = a_{i1} \) and \( a_{i2} = a_{i2}' \). Consider a potential equilibrium with 50 firms selling credits in market one and 51 in market two. Because of its greater number of firms, the slope of the aggregate marginal cost curve in market two would be slightly flatter than that for market one. Hence, using the supply functions that follow from the first order conditions, the price needed to reach \( \hat{A}_2 \) would be lower than the price needed to supply \( \hat{A}_1 \). However, such a price vector cannot be an equilibrium since if \( p_2 < p_1 \) there will be an incentive for the price-taking firms to shift from market two to market one. This
means that, the equilibrium price in market 2 cannot equal the marginal cost for the firms participating in that market and still exactly achieve the cap. In equilibrium, the price paid for credits in market two will be greater than the marginal cost at the equilibrium allocation.

When the equilibrium price is greater than the marginal cost, the regularity assumptions do not ensure that profit maximizing choices will lead to a cost-minimizing supply of credits – multiple equilibria are possible. We will assume that there are competitive forces that push firms to a cost minimizing allocation so that equimarginal conditions are satisfied, i.e.,

$$\frac{\partial g_i(a_i)}{\partial a_i} = \frac{\partial g_k(a_k)}{\partial a_k} \quad \text{for all } i, k.$$  

Hence, for firms $i$ and $l$ participating in the $j^{th}$ SM market, in addition to (9), the following first-order conditions must be satisfied at the equilibrium:

$$\frac{\partial g_i()}{\partial a_i} = \frac{\partial g_i()}{\partial a_j} \leq p_j, \quad \text{and} \quad \frac{\partial g_i()}{\partial a_k} = \frac{\partial g_i()}{\partial a_l} = 0 \quad \text{for all } k \neq j. \quad (10)$$

As above, we will define aggregate benefit, cost, and net benefit functions, $B^{SM}(\hat{A})$, $C^{SM}(\hat{A})$, and $NB^{SM}(\hat{A})$, that follow from the equilibrium responses when the SM policy is used. Under an SM policy, only a portion of all the potential abaters of a pollutant $j$ will be participating in the $j^{th}$ market to generate $\hat{A}_j$ credits. However, the firms participating in markets $k \neq j$ will still abate the $j^{th}$ pollutant due to complementarities. Hence, the total abatement of the $j^{th}$ pollutant, $A_j^{SM}$, will be greater than $\hat{A}_j$ and we can state the following proposition.

Proposition 3: Assuming the regularity conditions, under an SM policy, $A_j^{SM} \geq \hat{A}_j$ for all $j$, $B^{SM}(\hat{A}) \geq B^{MM}(\hat{A})$ and $C^{SM}(\hat{A}) \geq C^{MM}(\hat{A})$ with the inequalities being strict if strict complementarity holds.

Proof: See Appendix A.
Proposition 3 tells us that, for a given cap, the SM policy leads to benefits and costs that are greater than those generated by the MM policy. It does not, however, infer anything about the net benefits of two policy options. However, it turns out that in the neighborhood of the origin, $\hat{A} = 0$, the SM policy leads to greater net benefits.

Proposition 4: In the neighborhood of the origin where $\hat{A}_j > 0$ for at least two pollutants, $NB^{SM}(\hat{A}) \geq NB^{MM}(\hat{A})$ with a strict inequality holding if strict complementarity holds.

Proof: See Appendix A

**Figure 4: Example of how $NB^{SM}$ and $NB^{MM}$ intersect at $\hat{A}^c$**

Together, propositions 1 and 4 tell us that there exist caps where each option generates greater net benefits. If the caps are set at the optimal level, the MM policy is preferred. In the neighborhood of the origin, the SM policy does better. If the net benefit functions are both
strictly concave, then it will holds that the space of all possible caps can be divided into two parts, a lower portion where the SM policy dominates and an upper portion where the MM policy is preferred. This idea is shown in Figure 4 which presents the hypothetical net benefit surfaces for the two policy options for a range of cap levels. Somewhere between $0$ and $A^*$ the $NB^{SM}$ surface will cross the $NB^{MM}$ surface, an intersection which we refer to as $\hat{\theta}^c$.

If the $NB^{SM}$ and $NB^{MM}$ surfaces are smooth and concave, then $\hat{\theta}^c$ will be a single continuous line as presented Figure 4. Because of the discrete nature of choices under the SM policy, the $NB^{SM}$ may or may not be smooth and concave even if the regularity conditions are satisfied. However, the general pattern is more important and is established by propositions 1 and 4: there will tend to be a region of smaller caps where the SM policy will tend to be preferred, and another region close $A^*$ where the MM policy does better.

V. Double-dipping in a fully symmetric economy

In the remainder of the paper we seek to understand the conditions that would tend to favor one or the other of the two policies. To obtain our results we narrow our focus to a model with specific functional forms for the cost and benefit equations. The essential features of the abatement technology that we want to capture are heterogeneous costs across the $i=1,\ldots,n$ firms and the potential for complementarity in the cost function to abate the pollutants $P_1$ and $P_2$. A parsimonious cost function that satisfies these requirements is a quadratic abatement cost function of the form

$$g_i(a_{i1},a_{i2}) = \frac{\alpha_{i1}}{2}a_{i1}^2 + \frac{\alpha_{i2}}{2}a_{i2}^2 + \gamma_i a_{i1}a_{i2}$$

(11)

with $\alpha_i>0$. The interaction term, $\gamma_i$, is assumed to be non-positive so that the two pollutants are complements as in Figure 1b. If a firm faces prices $p_1$ and $p_2$, then it will profit from the supply
of abatement credits \( a_{i1} \) and \( a_{i2} \): \( \pi_i = p_1 a_{i1} + p_2 a_{i2} - g_i (\cdot) \). From the first order conditions we can obtain the profit maximizing supply functions:

\[
a_{i1} = \tilde{\alpha}_{i1} p_1 - \gamma_i p_2 \quad \text{and} \quad a_{i2} = \tilde{\alpha}_{i1} p_2 - \gamma_i p_1
\]

where \( \tilde{\alpha}_i = \alpha_i / (\alpha_{i1} \alpha_{i2} - \gamma_i^2) \) and \( \tilde{\gamma}_i = \gamma_i / (\alpha_{i1} \alpha_{i2} - \gamma_i^2) \). In order to ensure that the supply curves are upward sloping in the own price, we only consider case when \( (\alpha_i \alpha_{i2} - \gamma_i^2) > 0 \), which is sufficient to impose convexity of the cost function as assumed in the regularity conditions. The additively separable benefit function, \( B(A) = \sum_j B_j(A_j) \), is assumed to take the form:

\[
B_j(A_j) = \Omega_j A_j - \frac{\theta_j}{2} A_j^2 \quad \text{where} \quad \Omega_j > 0, \ \theta_j > 0, \ A_j = \sum_i a_{ij}, \ j = 1,2.
\]

In order to obtain clear analytical results, in this section we use a case of the model in which there are two pollutants and two representative firms. In this fully symmetric case we set \( \alpha_{i1} = \alpha_{22} = \alpha \in (0,1] \) and \( \alpha_{i2} = \alpha_{21} = 1/\alpha \) and let \( \gamma_1 = \gamma_2 = \gamma \).³ On the benefit side, we assume that the benefit functions for the two pollutants are identical: \( \Omega_1 = \Omega_2 = \Omega \) and \( \theta_1 = \theta_2 = \theta \).

The model is now reduced to four parameters and each has a clear intuitive meaning. Since \( \alpha_i = \alpha < 1 \) and \( \alpha_i = 1 \), firm 1 will tend to abate more of P1 while firm 2 will specialize on P2. A reduction in \( \alpha \), therefore, will be referred to as an increase in the degree of specialization of the firms. The \( \gamma \) parameter captures the degree to which there is complementarity in the economy so that for \( \gamma = 0 \) the goods are independent (Figure 1a) and as \( \gamma \) approaches \(-1\), the complementarity.

³ Complementarity, along with the restriction that \( (\alpha_{i1} \alpha_{i2} - \gamma_i^2) > 0 \) implies \( \gamma \in (-1,0] \).
degree of complementarity across the goods increases (Figure 1b). The final key variable is the slope parameter in the benefit function, $\theta \geq 0$. As $\theta$ increases in absolute value, the marginal benefit curves become steeper. The second parameter in the benefit function, $\Omega$, simply scales the marginal benefit functions.

At the first-best optimum, the marginal benefit of abatement would equal marginal cost, $\theta = \alpha_j a_{ij} + \gamma a_{ik}$, $i=1,2$, $j=1,2$, $k \neq j$. We know from Proposition 1 that the optimum is achieved through an MM policy. In this case the optimal cap can be written

$$
\hat{A}^* = \frac{\Omega \left( 2 \gamma \alpha - \alpha^2 - 1 \right)}{\left( \gamma^2 \alpha + 2 \gamma \theta \alpha - \alpha^2 \theta - \alpha - \theta \right)^4}
$$

To identify the optimal cap, therefore, the planner must have knowledge of all the parameters of the economy, including the interaction term, $\gamma$ which may be particularly difficult to observe or estimate.

We now consider the benefits and costs under the different policies at arbitrary cap levels, $\hat{A}_1 = \hat{A}_2 = \hat{A}$. Under an MM policy, firms can sell credits in both markets so that equilibrium is achieved where $a_{11}^{MM} + a_{21}^{MM} = a_{12}^{MM} + a_{22}^{MM} = \hat{A}$. Solving for the equilibrium prices and then using (12), in this case we have

$$
a_{11}^{MM} = a_{22}^{MM} = \frac{1 - \gamma \alpha}{1 + \alpha^2 - 2 \gamma \alpha} \hat{A} \quad \text{and} \quad a_{12}^{MM} = a_{21}^{MM} = \frac{\alpha^2 - \gamma \alpha}{1 + \alpha^2 - 2 \gamma \alpha} \hat{A}.
$$

\footnote{Despite the relatively parsimonious specification, the analytical solutions to this problem were quite cumbersome. Analytical results were derived using Matlab and confirmed manually or numerically. The Matlab code used for the in the paper will be made available online upon publication.}
In the SM case, firms can only sell credits in one market and, since \( \alpha_{11} < \alpha_{12} \) and \( \alpha_{22} < \alpha_{21} \),

\[
\alpha_{11}^{SM} = \alpha_{22}^{SM} = \hat{A} \quad \text{while} \quad \alpha_{12}^{SM} \quad \text{and} \quad \alpha_{21}^{SM} \quad \text{is the complementary abatement, i.e.,} \quad
\]

\[
\alpha_{11}^{SM} = \alpha_{22}^{SM} = \hat{A} \quad \text{and} \quad \alpha_{12}^{SM} = \alpha_{21}^{SM} = -\gamma \alpha \hat{A} .
\] (16)

Note that \( \alpha_{21}^{SM} \) and \( \alpha_{12}^{SM} \) are affected by both \( \alpha \) and \( \gamma \), so in this specification it is not possible to completely isolate the slope of the MC curve from the level of complementarity as was done in Figure 3. From (16) it follows that the SM policy results in abatement of each pollutant in excess of \( \hat{A} \), leading to greater social benefits but higher costs.

Substituting (15) and (16) into the benefit and cost functions and simplifying, the net benefits of policies given an arbitrary cap \( \hat{A} \) can be written

\[
NB^{MM} (\hat{A}) = \frac{-\hat{A} \left( \hat{A} \alpha \gamma + 2 \theta \hat{A} \gamma \alpha - 4 \Omega \gamma \alpha + 2 \alpha^2 \Omega - \theta \hat{A} \alpha \gamma^2 - \hat{A} \alpha + 2 \Omega - \theta \hat{A} \right)}{2 \gamma \alpha - 1 - \alpha^2} \] (17)

and

\[
NB^{SM} (\hat{A}) = -\left( 2 \Omega (-1 + \gamma \alpha) + \theta \hat{A} \left( 1 - 2 \gamma \alpha + \alpha^2 \gamma \right) + \hat{A} \alpha \left( 1 - \gamma \right) \right) \hat{A} .
\] (18)

If \( NB^{SM} (\hat{A})/NB^{MM} (\hat{A}) > 1 \) then at that cap level an SM policy would be preferred. In Appendix A we show \( \frac{\partial}{\partial \hat{A}} \left[ NB^{SM} (\hat{A})/NB^{MM} (\hat{A}) \right] < 0 \). By Proposition 4 we know that \( NB^{SM} (\hat{A})/NB^{MM} (\hat{A}) \) is greater than 1 in the neighborhood of the origin, and, by Proposition 1, the ratio is less than 1 when \( \hat{A} = A^* \). The monotonicity of the derivative, therefore, implies that there is a unique cap level \( \hat{A}^c \), where the two policies yield equivalent net benefits. In Appendix A we solve for this value:
\[
\hat{A}^c = 2\Omega \gamma \left[ \left( 2\theta \gamma - \theta \gamma^2 \alpha - 1 + \gamma^2 \right) - \frac{(\gamma^2 - 1)}{(\alpha^2 + 1 - 2\gamma \alpha)} \right]^{-1}.
\] (19)

The ratio of \( \hat{A}^c \) to \( \hat{A}^* \) tells us how far away from the first best cap the policy maker must be in order for the SM policy to actually deliver greater net benefits. For example, if \( \frac{\hat{A}^c}{\hat{A}^*} = 0.3 \), then as long as the cap that is set is at least 30% of the optimal level, it would be preferable to adopt the MM policy. Dividing (19) by (14) and simplifying we obtain:

\[
\frac{\hat{A}^c}{\hat{A}^*} = \frac{-2(\alpha \gamma^2 + 2\gamma \theta \alpha - \alpha - \theta - \alpha^2 \theta) \gamma}{-5\theta \gamma^2 \alpha + 2\gamma \theta + 2\gamma \theta \alpha^2 + 2\theta \gamma^3 \alpha^3 - \theta \gamma^2 \alpha^3 + 2\gamma \alpha - \alpha^2 - 2\gamma \alpha^2 + \alpha^2 \gamma}. \quad (20)
\]

Although this expression is quite complicated, its one obvious feature is that \( \Omega \) does not appear— the intercepts of the marginal benefit equations have no effect on the ratio. Instead, \( \Omega \) simply scales the net benefits under both policies.

The derivatives of (20) with respect to the parameters will provide insights into the circumstances where an SM policy is most likely to be appropriate. We present the derivatives of \( \frac{\hat{A}^c}{\hat{A}^*} \) with respect to \( \alpha, \gamma \), and \( \theta \) in Appendix A. The derivative with respect to \( \alpha \) is negative implying that as specialization increases (\( \alpha \) declines) the portion of the space where the SM policy dominates increase. We also show that \( \partial \left( \frac{\hat{A}^c}{\hat{A}^*} \right) / \partial \theta > 0 \), meaning that the switching point at which MM policy begins to dominate becomes closer to \( \hat{A}^* \) as the marginal benefit curve becomes steeper.

These patterns can be seen in figure 5, which presents level curves of \( \frac{\hat{A}^c}{\hat{A}^*} \) over the range of values of \( \alpha \) and \( \gamma \) for two values of \( \theta \). On the left we present the case where \( \theta = 0.5 \), i.e. the marginal benefit curves are relatively flat. As is seen in the figure, when \( \alpha \) is small, \( \hat{A}^c \) is
very close to $\hat{A}^\ast$. That is, when the firms have a strong tendency to specialize in one or the other of the pollutants, the SM policy is preferred for almost all cap levels and, interestingly, the complementarity level, $\gamma$ does not affect this very much. On the other hand, as $\alpha$ increases and the firms become more similar in terms of their marginal costs of abatement, the chosen cap on aggregate abatement would have to be much lower than the optimal level before an SM policy would be preferred. At $\alpha=0.6$, for example, for even high levels of complementarity, the caps would have to be set less at less than 80% of optimal for an SM policy to be preferred and, if low levels of complementarity prevail, say $\gamma=-0.1$, the MM policy would be preferred unless the caps were set at less than 40% of the optimum.

The right side of Figure 5 presents the same level curves, but for demand curves that are four times steeper. In this case the MM policy yields greater net benefits only if caps are set very nearly at their optimal levels. The ratio $\hat{A}^c/\hat{A}^\ast$ is greater than 80% for almost the entire parameter space. So when the marginal benefit curve is steep, if policy makers suspect that the caps they choose may be suboptimal, they can adopt an SM policy with a relatively high degree of confidence. This result makes intuitive sense since as $\theta$ increases, the consequences of abatement that is too low are greater; anything that would increase abatement, including an SM policy, becomes increasingly attractive.
We now compare the two policies when caps are set at their second-best level,

\[
\hat{A}^2 = \Omega \left( 1 + \alpha^2 \right) \left( \alpha + \theta + \alpha^2 \theta \right),
\]

found by setting \( \gamma = 0 \) in (14). In Figure 6 we present the ratio \( NB^{SM} / NB^{MM} \) when the second-best cap is chosen for different values of \( \gamma \) and \( \theta \) when \( \alpha = 0.2 \) and \( \alpha = 0.8 \). In both cases, the SM policy yields greater net benefits at \( \hat{A}^2 \) when complementarities are significant (\( \gamma \) is large in absolute value) and the MB curve is relatively flat (\( \theta \) is relatively small). However, when \( \theta \) is large, the MM policy yields greater net benefits even if complementarities are at their maximum level. This confirms what we found in Figure 3. On the other hand, contrary to what was suggested by Figure 3, if \( \gamma \) is close to zero then a flat marginal benefit curve, \( \theta = 0 \), does not ensure that the SM policy will be preferred at \( \hat{A}^2 \).
There are two important differences in Figure 6 between the case when $\alpha=0.2$ and when $\alpha=0.8$. First there is a matter of scale. When $\alpha=0.2$, the percent error is small so that at $\hat{A}^2$ choosing the incorrect policy results in an error of less than 10%. When there are high levels of specialization, the policy choice will have little effect on in actual outcomes. When $\alpha=0.8$, on the other hand, the supply curve is steep and the cost of making an incorrect policy choice can exceed 50%. Second, the steeper supply curve also substantially diminishes the portion of the parameter space where the SM policy is preferred. This is consistent with Figure 3; steep MC curves tend to favor an MM policy.

The fully symmetric model, though extremely restrictive, does give us some clues as to the conditions under which an SM policy might be chosen. As the firms become less specialized ($\alpha \to 1$), the cap level at which the MM policy begins to dominate falls. As the degree of complementarity increases ($\gamma \to -1$), under a second-best policy an SM policy becomes more attractive. Finally, the slope of the marginal benefit curve shows that for flat curves, second-best caps tend to yield greater net benefits under the SM policy than under MM policy. In the last
main section of this paper we relax the restrictions of the fully symmetric model and use numerical analysis to explore further the MM and SM policies.

**VI. Numerical analysis for more than two firms and multiple sources of heterogeneity**

In this section we study how our findings change when there are more than two firms. One can think of the two-firm case as representative of an economy in which there are many firms, but only two types of firms. Increasing the number of firms in the model, therefore, can be thought of as representative of adding more heterogeneity to the economy. Unfortunately, as we move beyond the two-firm model, the increasing number of parameters makes it difficult to obtain clean analytical results. Hence, in this section we use numerical methods.

The first question that we ask is how increasing the number of firms alters the point at which the preferred policy switches from SM to MM. In figures 7 and 8 we present the results of 1000 simulated markets. In each market the parameters for the benefit and cost functions are drawn independently from distributions as described in Appendix B. Once a set of parameters are chosen, we then vary the caps for the two pollutants, \( \hat{A}_1 \) and \( \hat{A}_2 \), from 6% to 125% of their optimal levels. At each level we solve for the equilibria under the MM and SM policies. In the case of the MM policy, there is no variation in the results; for any set of parameters a proportional change in the caps leads to the exact same change in net benefits relative to the first-best level. For the SM policy, on the other hand, the relative net benefits can vary substantially, and we present the 95% confidence bounds for this policy.

Comparing figures 7 and 8, we see that increasing the number of firms has three discernable effects. First, there is a noticeable upward shift in the net benefits that can be achieved by the SM policy. While the average NB curve for the SM case peaks at about 90% of the first-best optimum in the 2-firm case (figures 7), this curve peaks at 98% of the optimum
when 30 firms are simulated. This upward shift also moves to the right the point at which the net benefits under the SM policy will cross the MM curve. Finally, we see a tightening of the distributions around the mean, though this is attributable at least in part to the fact that the distribution of the average cost from 30 firms will be tighter than that for 2 firms.

In figures 9, 10 and 11 we present the results of Monte Carlo simulations for economies with thirty heterogeneous firms, while holding one of the parameters constant at a variety of
levels. In Figure 9 we vary the $\alpha$ parameter which determines the extent to which there is specialization in the firms’ cost functions. In each simulation, the 30 firms are randomly set into one of two groups, $\alpha_1=\alpha$ or $\alpha_2=\alpha$. While on average 15 firms would be in each group, the actual numbers vary across draws. When the firm are highly specialized ($\alpha=0.1$), the two policies yield very similar benefits for any cap. As noted above, this makes sense since when firms are specialized it also means that the complementary abatement falls and the policy decision regarding double dipping is less consequential. However, as specialization declines ($\alpha=0.7$), the two policies diverge. In such an economy, if the cap is set too low, an SM policy can generate about 40% more net benefits to society. On the other hand, if the cap is set at the first-best optimum or above, when $\alpha$ is high the cost of choosing an SM policy can be very great.

![Figure 9: Effect of specialization on the average net benefits for SM and MM policies as percent of optimum](image)

To make the figures easier to read, only the average results are presented in these figures. In all cases, the variation across draws was substantially reduced, making the confidence bounds much tighter than in figures 7 and 8.
In Figure 10 we vary the complementarity parameter, setting $\gamma$ at four levels, from $-0.1$, where there is only slight complementarity, to $\gamma = -0.9$. It is not surprising that the potential advantages of an SM policy only arise for greater levels of complementarity. While there is a shift in the point at which the two policies cross, this very slight for value of $\gamma \leq -0.3$.

Finally, in Figure 11 we present the results of simulation analysis in which we explore the role of $\theta$, the slope parameter of the marginal benefit curve. As distinct from our analysis of the fully symmetric case, here we set the slopes of the marginal benefit curves at different levels. It is clear from this figure that the relative slopes are an important factor in determining if an SM or MM policy is preferred. In the topmost curve, both slopes equal. As the difference between the slopes increases, the net benefits from an SM policy fall monotonically. We know that an SM policy can lead to total abatement in excess of the first best optimum in one market and

\[\text{In separate analysis, we found that for } \theta_1 = \theta_2, \text{ the actual value of the slope made little difference and curves with the value set from 0.1 to 2.0 were virtually indistinguishable.}\]
insufficient supply in the other. This tendency is exacerbated as the difference between $\theta_1$ and $\theta_2$ grows.

![Figure 11: Effect of the slope of marginal benefit curve on the average net benefits for SM and MM policies as percent of optimum](image)

VII. Discussion, policy implications, and conclusions

In this paper we have addressed an important policy issue – should participants in pollution trading programs be allowed to use a single practice change to generate credits in multiple markets? As mentioned in the introduction, this question is related to the issue of “additionality,” i.e., that credits should be granted only in response to the incentives created by the program. When there is complementarity in the firms’ cost functions and multiple environmental markets are present, identifying what reductions should be counted as additional can be complicated. Two different interpretations of additionality are frequently adopted (Wunder 2005). In the first case, a baseline is established at a particular point in time and any reductions in emissions from that baseline would be additional and creditable. Alternatively, baselines can be thought of as emissions that follow from a *business as usual* path and additionality is recognized when emissions fall relative to the baseline level.
Our analysis can be thought of as a stylized presentation of these two approaches to additionality when multiple markets are present. If double-dipping is allowed, then a seller of credits can be compensated for any reductions in emissions regardless of how or why those reductions arose – all abatement relative to the starting point is additional. If double-dipping is not allowed, then they must dedicate their credits to one or the other markets – the response to one policy becomes the baseline path and emission reductions that come about because of complementarities in the cost function are not additional.

In our model, it is clear that for any level of total abatement, an MM policy leads to a cost-effective allocation of effort and, if the caps are chosen correctly, will maximize aggregate net benefits. Based on what we see of policy in practice, however, we believe that such multiple pollutant optimization is rarely accomplished. Consider the fact that the agricultural sector might soon be allowed to sell carbon offsets to manufacturer, phosphorus offsets to a waste water treatment plant, and wetland mitigation credits to a developer. Yet it seems unlikely that the policies regarding these three markets will be coordinated.

Under what conditions might an SM policy might be desirable? We find that this tends to occur when there is evidence of substantial complementarities, when the firms’ costs curves are such that they do not tend to specialize, and when the MB curves tend to have similar slopes. If all these conditions are satisfied, then an SM policy may yield greater net benefits. Alternatively, and probably better, when these conditions are satisfied, a coordinated policy that takes into account market interactions is particularly important.

A step even more substantial than setting the caps correctly would be to bring all sources under the suite of cap and trade programs, thus eliminating the voluntary offset programs. This would provide positive incentives for abatement of any and all pollutants and negative incentives
for any increase in emissions. Of course there are numerous practical and political challenges that have pushed policy makers to use offsets, but any offsets policy is in itself a second-best solution.

There are several issues that make the “real world” substantially more complicated than the stylized model used here. First, there is the issue of sequencing. It is rarely the case that two or more environmental markets are created simultaneously. Instead, the programs are rolled out over time; the U.S. SO$_2$ market, predates the water quality markets, which predate markets for CO$_2$ and biodiversity. In this case, not only have polluters often committed to one market first, but that they may not have committed to the market in which they have comparative advantage. Inefficient sorting and inefficient practices can result. As new markets are created, policy makers should ask how they will interact with previous ones. Although our results may be helpful in guiding the policy decisions that must be made, they do not apply directly.

Second, we should point out that our stylized policy makers act as if they are completely ignorant of complementarities when setting the caps, but then admit this possibility when choosing between the SM and MM policy options. Such schizophrenic behavior is unlikely to hold completely true in reality. It seems more likely that the policymakers would be uncertain about the extent that complementarities are present and would hopefully take into account this possibility when setting the cap and when choosing between an SM and MM policy. In a simple case akin to that of Figure 3, the similarities to Weitzman (1974) become even more apparent: the SM policy behaves like a price rule and the MM policy like a quantity restriction. We leave the extension of our analysis along these lines for future work.

7 This recalls the results of Atkinson and Tietenberg (1991) for a single market.

8 We thank a reviewer for pointing out this extension of our model.
We have also narrowed most of our analysis to situations in which there are only two pollutants. Yet as just noted, markets across many environmental services are arising and relationships across three or more such markets are possible. In Section IV we show that generally, if caps are sufficiently low, an SM policy is preferred and, that if the caps are set optimally, the MM policy will yield greater net benefits. As far as the effect of other parameters goes, however, we can only speculate that implications of the two-firm model also apply when three or more pollutants are involved.

Finally, we should point out that we have ignored the issue of transaction costs and other regulations. As Stavins (1995) showed, when transaction costs occur the market equilibrium can be inefficient and the initial allocation of permits can matter. The issue of transaction costs is particularly important in the multiple markets problem since the transaction costs in an MM structure might actually be lower than in an SM structure (Greenhalgh 2008). It is also the case that firms face a variety of other environmental regulations, not all of which have market-based elements.

There is great interest in finding ways to make environmental markets work together and there is much merit to the idea that incentives should be created for projects that generate multiple environmental benefits. If complementarities exist in the production of environmental services, then a policy of allowing double-dipping will maximize such incentives. However, complementarities also lead to a reduction in the cost of providing these services so that a higher environmental standard will be socially efficient. In the development of environmental policy, these factors should be considered. Ideally, policy makers would take into account complementarities when setting caps and allow double dipping. But if coordinated policies are not possible, allowing double dipping may not lead to the greatest net benefits to society.
Appendix A: Propositions, Proofs and Derivations

Proof of Proposition 2

Let \( p^{MM} = (p_1^{MM}, \ldots, p_m^{MM}) \) be the MM equilibrium price vector given a cap, \( \hat{A} \). We consider three cases: \( p_j^{MM} > 0 \) for all \( j \), and \( p_j^{MM} = 0 \) for all \( j \), and \( p_j^{MM} > 0 \) with \( p_k^{MM} = 0 \) for some \( k \neq j \).

Case 1 (\( p_j^{MM} > 0 \) for all \( j \)): In this case, \( A_j^{MM} = \hat{A}_j \) for all \( j \) at MM equilibrium so that

\[
\frac{\partial A_j^{MM}}{\partial \hat{A}_j} = 1 \quad \text{and} \quad \frac{\partial A_k^{MM}}{\partial \hat{A}_j} = 0.
\]

Hence, \( B^{MM}(\hat{A}) = B(\hat{A}) \), and \( B(\cdot) \) is increasing and concave in its arguments. Further, given the assumptions on \( g_i(\cdot) \), an increase in \( \hat{A}_j \) will lead to an increase in \( a_{ij} \) for all \( j \). Since \( C^{MM}(\hat{A}) = C(A^{MM}) = \sum_i g_i(a_i^{MM}) \), and the sum of a monotonic convex function is monotonic and convex, it follows that

\[
\frac{\partial C^{MM}(\hat{A})}{\partial \hat{A}_j} \geq 0 \quad \text{and} \quad \frac{\partial^2 C^{MM}(\hat{A})}{\partial \hat{A}_j^2} \geq 0.
\]

Case 1 also applies in the case where \( p_j = 0 \) but \( \hat{A}_j = A_j^{MM} \).

Case 2 (\( p_j^{MM} = 0 \)): In this case, the aggregate supply of \( A_j^{MM} > \hat{A}_j \) so an increase in \( \hat{A}_j \) will not affect benefits or costs, i.e.,

\[
\frac{\partial B^{MM}(\hat{A})}{\partial \hat{A}_j} = \frac{\partial^2 B^{MM}(\hat{A})}{\partial \hat{A}_j^2} = \frac{\partial C^{MM}(\hat{A})}{\partial \hat{A}_j} = \frac{\partial^2 C^{MM}(\hat{A})}{\partial \hat{A}_j^2} = 0.
\]

Case 3 (and \( p_j^{MM} > 0 \) and \( p_k^{MM} = 0 \) for some \( k \neq j \)): In this case, the marginal change in \( \hat{A}_j \) will also cause a change in \( A_k^{MM} \). Hence,

\[
\frac{\partial B^{MM}(\hat{A})}{\partial \hat{A}_j} = \frac{\partial B_j}{\partial A_j^{MM}} + \frac{\partial B_k}{\partial A_k^{MM}} \frac{\partial A_k^{MM}}{\partial \hat{A}_j}.
\]

We can decompose the second term as

\[
\frac{\partial B_k}{\partial A_k^{MM}} \frac{\partial A_k^{MM}}{\partial \hat{A}_j} = \sum_i \frac{\partial B_k}{\partial a_{ik}} \frac{\partial a_{ik}}{\partial \hat{A}_j} \frac{\partial a_{ik}}{\partial A_j}.
\]

Under constant complementarity, the second
derivative of $B_k$ with respect to $a_{ij}$ vanishes, so concavity holds due to concavity of $B_j$. On the cost side, the monotonicity and convexity of $C^{MM}$ are as in case 1.

**Proof of Proposition 3.**

Let $I_j^{SM}$ be the set of firms participating in the $j^{th}$ market under the SM policy and let $N_j^{SM}$ be the number of firms in this set.

Suppose that $\hat{A}_j > 0$ while $\hat{A}_k = 0$ for all $k \neq j$. In this case $p_j^{SM} > 0$ and $p_k^{SM} = 0$ and $N_j^{SM} = n$. The SM equilibrium will be exactly the same as in the MM case, so that $B^{SM}(\hat{A}) = B^{MM}(\hat{A})$ and $C^{SM}(\hat{A}) = C^{MM}(\hat{A})$. In this case, $A_j = \hat{A}_j$ while for $k \neq j$ complementarity ensures that $A_k \geq 0$.

If $\hat{A}_j > 0$ and $\hat{A}_k > 0$ for at least one market $k \neq j$, then from (10), $p_j^{SM} > 0$ and $p_k^{SM} > 0$ and $N_j^{SM} < n$ for all $j$. Because of complementarity, the firms in $I_k^{SM}$ for $p_k^{SM} > 0$ will also abate $A_j$.

Hence, under the SM policy $A_j^{SM} \geq A_j^{MM} \geq \hat{A}_j$ for all $j$, implying that $B^{SM}(\hat{A}) \geq B^{MM}(\hat{A})$ with strict inequalities holding if strict complementarity holds. By Proposition 1, an MM policy is cost effective way to reach any level of abatement, including $A_j^{MM}$. By monotonicity of the cost functions, therefore, $C^{SM}(\hat{A}) \geq C^{MM}(\hat{A})$.

**Proof of Proposition 4.**

Consider a marginal increase in $\hat{A}_j$ and $\hat{A}_k$ at 0. This will induce firms to separate into markets such that $N_j^{SM} + N_k^{SM} = n$. In the equilibrium, the firms $I_j^{SM}$ abate $\hat{A}_j$ plus complementary reductions in $k$ while the firms $I_k^{SM}$ abate $\hat{A}_k$ plus complementary reductions in $j$. In contrast, in
the MM case, abatement in a market \( j \) can exceed \( \hat{A}_j \) only if \( p_j^{MM} = 0 \). Hence, under strict complementarity, either \( A_j^{SM} > A_j^{MM} \) or \( A_k^{SM} > A_k^{MM} \) or both. Hence, \( B^{SM} (\hat{A}) > B^{MM} (\hat{A}) \).

On the cost side, since we assume that all firms’ marginal costs start at the origin,
\[
\frac{\partial C^{MM}}{\partial \hat{A}_j} = \frac{\partial C^{MM}}{\partial \hat{A}_j} = 0.
\]
Hence, the marginal net benefit under the SM policy exceeds the marginal net benefit under MM so that in the neighborhood of the origin \( NB^{SM} (\hat{A}) > NB^{MM} (\hat{A}) \).

**Derivation of derivative of \( NB^{SM}/NB^{MM} \) with respect to \( \hat{A} \)**

Using (18) and (17),
\[
\frac{NB^{SM}}{NB^{MM}} = \frac{-2\Omega (-1 + \gamma \alpha) + \theta \hat{A} (1 - 2\gamma \alpha + \alpha^2 \gamma^2) + \hat{\lambda} \alpha (1 - \gamma^2) \hat{A}}{-\hat{A} (\hat{\lambda} \alpha \gamma^2 + 2\theta \hat{\lambda} \alpha - 4\Omega \gamma \alpha + 2\alpha^2 \Omega - \hat{\lambda} \alpha^2 - \hat{\lambda} \alpha + 2\Omega - \theta \hat{A})}. \tag{A1}
\]
This can be simplified as follows:
\[
\frac{NB^{SM}}{NB^{MM}} = \frac{(-2\Omega + 2\Omega \gamma \alpha + \theta \hat{A} - 2\theta \hat{\lambda} \alpha \gamma - \hat{\lambda} \alpha \gamma^2 + \hat{\lambda} \alpha - \hat{\lambda} \alpha \gamma^2)}{(\hat{\lambda} \alpha \gamma^2 + 2\theta \hat{\lambda} \alpha \gamma - 4\Omega \gamma \alpha + 2\alpha^2 \Omega - \hat{\lambda} \alpha^2 - \hat{\lambda} \alpha + 2\Omega - \theta \hat{A})} (-1 + 2\gamma \alpha - \alpha^2)
\]
\[
\frac{\partial NB^{SM}}{\partial \hat{A}} = -2 (-1 + 2\gamma \alpha - \alpha^2) \alpha \Omega (\alpha^2 \gamma^2 + \gamma \alpha - \alpha^2 - \gamma^2 \alpha - 3\theta \alpha \gamma^2 + \theta \alpha \gamma^2 + 2\theta \alpha \gamma^3 - \theta \alpha^2 \gamma^2 + \gamma \theta)
\]
\[
\frac{\partial NB^{SM}}{\partial \hat{A}} = \frac{(-1 + 2\gamma \alpha - \alpha^2) \alpha \Omega (\alpha^2 \gamma^2 + \gamma \alpha - \alpha^2 - \gamma^2 \alpha - 3\theta \alpha \gamma^2 + \theta \alpha \gamma^2 + 2\theta \alpha \gamma^3 - \theta \alpha^2 \gamma^2 + \gamma \theta)}{(\hat{\lambda} \alpha \gamma^2 + 2\theta \hat{\lambda} \alpha \gamma - 4\Omega \gamma \alpha + 2\alpha^2 \Omega - \hat{\lambda} \alpha^2 - \hat{\lambda} \alpha + 2\Omega - \theta \hat{A})^2}
\]
Since the denominator is positive, the sign of the derivative depends upon the sign of the numerator. By assumption, \( \alpha, \Omega \) are both positive. The first expression in parentheses, \( -2 (1 + 2\gamma \alpha - \alpha^2) \alpha \Omega \) is positive. Hence the product of the terms except the final expression in parentheses is positive. Rearranging the final term we have
\[
(\gamma \alpha - \gamma \alpha^2 + \alpha^2 \gamma^2 - \alpha^2 - 3\theta \alpha \gamma^2 + \theta \alpha \gamma^2 + 2\theta \alpha \gamma^3 - \theta \alpha^2 \gamma^2 + \gamma \theta), \text{ or}
\]
\[
(\gamma \alpha \left(1 - \gamma^2\right) - \alpha^2 \left(1 - \gamma^2\right) - 3 \theta \alpha \gamma^2 + \theta \alpha \gamma^2 + 2 \theta \alpha^2 \gamma^2 - \theta \alpha^3 \gamma^2 + \gamma \theta).
\]

It can easily be seen that for \(\theta > 0\), \(\alpha > 0\), \(\gamma < 0\) and \((1 - \gamma^2) > 0\), each term in this expression is negative. Hence, the numerator of the derivative is negative.

**Derivation of equation (19)**

By definition, \(NB^{SM}\left(\hat{A}^c\right) = NB^{MM}\left(\hat{A}^c\right)\). From (A1), this will hold where

\[
\left[(2\gamma \alpha - \alpha^2 - 1)(-2\Omega + 2\Omega \gamma \alpha) + (\theta - 2\theta \gamma \alpha + \theta \alpha^2 \gamma^2 + \alpha - \alpha \gamma^2)\hat{A}^c (2\gamma \alpha - \alpha^2 - 1)\right]
= \left(\hat{A}^c \left(\alpha \gamma^2 + 2\theta \gamma \alpha - \theta \alpha^2 - \alpha - \theta\right) - 4\Omega \gamma \alpha + 2\Omega \alpha^2 + 2\Omega\right)
\]

This can be simplified to

\[
\hat{A}^c \left[(2\gamma \alpha - \alpha^2 - 1)(\theta - 2\theta \gamma \alpha + \theta \alpha^2 \gamma^2 + \alpha - \alpha \gamma^2) - (\alpha \gamma^2 + 2\theta \gamma \alpha - \theta \alpha^2 - \alpha - \theta)\right]
= (-4\Omega \gamma \alpha + 2\Omega \alpha^2 + 2\Omega) + (2\gamma \alpha - \alpha^2 - 1)(-2\Omega + 2\Omega \gamma \alpha)
\]

so that

\[
\hat{A}^c = \frac{(-4\Omega \gamma \alpha + 2\Omega \alpha^2 + 2\Omega) - (2\gamma \alpha - \alpha^2 - 1)(-2\Omega + 2\Omega \gamma \alpha)}{(2\gamma \alpha - \alpha^2 - 1)(\theta - 2\theta \gamma \alpha + \theta \alpha^2 \gamma^2 + \alpha - \alpha \gamma^2) - (\alpha \gamma^2 + 2\theta \gamma \alpha - \theta \alpha^2 - \alpha - \theta)}.
\]

Simplifying

\[
\hat{A} = \frac{2\Omega \left((-2\gamma \alpha + \alpha^2 + 1) + \gamma \alpha \left(\alpha^2 + 1 - 2\gamma \alpha\right) - (\alpha^2 + 1 - 2\gamma \alpha)\right)}{(2\gamma \alpha - \alpha^2 - 1)(\theta - 2\theta \gamma \alpha + \theta \alpha^2 \gamma^2 + \alpha - \alpha \gamma^2) - (\alpha \gamma^2 + 2\theta \gamma \alpha - \theta \alpha^2 - \alpha - \theta)}
\]

\[
\hat{A} = \frac{2\gamma \alpha \left(1 - 2\gamma \alpha + \alpha^2\right)}{\left(-\left(1 - 2\gamma \alpha + \alpha^2\right)(\theta - 2\theta \gamma \alpha + \theta \alpha^2 \gamma^2 + \alpha - \alpha \gamma^2) - (\alpha \gamma^2 + 2\theta \gamma \alpha - \theta \alpha^2 - \alpha - \theta)\right)}
\]

\[
\hat{A}^c = \frac{2\Omega \gamma \alpha}{\left(-\left(\theta - 2\theta \gamma \alpha + \theta \alpha^2 \gamma^2 + \alpha - \alpha \gamma^2\right) - \frac{(\alpha \gamma^2 - \alpha + \theta \left(2\gamma \alpha - \alpha^2 - 1\right))}{(1 - 2\gamma \alpha + \alpha^2)}\right)}.
\]
Derivation of the derivatives of $\frac{\hat{A}}{\hat{A}^*}$ with respect to model parameters

The derivatives of $\frac{\hat{A}}{\hat{A}^*}$ with respect to the parameters were obtained using Matlab. The derivative with respect to $\gamma$ is

$$\frac{\partial \hat{A}}{\partial \gamma} = \frac{2\Omega' \gamma}{\left(2\theta' \gamma - \theta' \gamma^2 - 1 + \gamma^2\right) - \frac{(\gamma^2 - 1)}{(1 - 2\gamma' \alpha + \alpha^2)}}.$$

As seen in Figure 5, the sign of $\frac{\hat{A}}{\hat{A}^*}$ varies over the parameter space.

The derivative of $\frac{\hat{A}}{\hat{A}^*}$ with respect to $\alpha$ is

$$\frac{\partial \hat{A}}{\partial \alpha} = \frac{2\Omega' \gamma}{\left(2\theta' \gamma - \theta' \gamma^2 - 1 + \gamma^2\right) - \frac{(\gamma^2 - 1)}{(1 - 2\gamma' \alpha + \alpha^2)}}.$$

We can sign the derivative if the sign of the numerator can be established. Simplifying and collecting terms that can be signed, we can write
\[
\text{sign} \left( \frac{\partial \hat{A}^c}{\partial \alpha} \right) = \text{sign} \left( 2\gamma \left( \frac{2\theta \alpha^2 (\gamma^5 - \gamma) + 2\theta \alpha \gamma^2 (1 - \gamma) + 2\alpha (1 - \gamma^2) - 4\gamma \theta^2 \alpha}{(\gamma^2 + \alpha^2 + \alpha^2 \gamma^2 + 2 + \frac{\gamma + \alpha^2 \gamma - 2 + 2 \alpha^2 + 4 \gamma^2 \theta^2 + \gamma - 4 \gamma \theta^2 \alpha}{\gamma^2 + \alpha^2 + \alpha^2 \gamma^2 + 2} \right) \right) \]
\]

(A2)

where the + and - symbols indicate the signs of the respective terms. Hence, this derivative will be negative if \( \left( \frac{-2 + 2 \theta^2 + \alpha^2 \theta^2 + 4 \gamma \theta^2 + \gamma^2 - 4 \gamma \theta^2 \alpha}{\gamma^2 + \alpha^2 + \alpha^2 \gamma^2 + 2} \right) > 0 \). For any value of \( \gamma < 0 \) and \( \alpha > 0 \), this expression takes on its lowest value if \( \theta = 0 \). In that case, the last two terms in (A2) become

\[
\alpha^2 + \alpha^2 \gamma^2 \left( \frac{-2 + \gamma^2}{\gamma^2 + \alpha^2 + \alpha^2 \gamma^2 + 2} \right) = \alpha^2 \left( 1 + \gamma^2 \left( \frac{\gamma^2 - 2}{\gamma^2 + \alpha^2 + \alpha^2 \gamma^2 + 2} \right) \right).
\]

However, for all \( \gamma \in [-1,0] \), \( \left( 1 + \gamma^2 \left( \gamma^2 - 2 \right) \right) \geq 0 \).

Hence, \( \text{sign} \left( \frac{\partial \left( \hat{A}^c \right)}{\partial \alpha} \right) < 0 \).

Finally, the derivative with respect to \( \theta \) is

\[
\frac{\partial \hat{A}^c}{\partial \theta} = \frac{2 \left( \gamma^4 - 2 \gamma^2 + 1 \right) \alpha \left( 2 \gamma \alpha - 1 - \alpha^2 \right) \gamma}{\left( -5 \gamma^2 \alpha + 2 \gamma \theta + 2 \gamma \alpha \theta^2 + 2 \theta \gamma^2 \alpha^2 - \theta ^2 \gamma \alpha^3 + 2 \gamma \alpha - \alpha^2 - 2 \gamma \alpha + \alpha^2 \gamma^2 + \right)^3}.
\]

The sign of the derivative, therefore, will depend on the sign of the numerator. Most of the terms can be signed as follows:

\[
2 \left( \gamma^4 - 2 \gamma^2 + 1 \right) \alpha \left( 2 \gamma \alpha - 1 - \alpha^2 \right) \gamma.
\]

Hence, the sign of the derivative will depend on the sign of \( \left( \gamma^4 - 2 \gamma^2 + 1 \right) \), which is equal to \( \left( 1 + \gamma^2 \left( \gamma^2 - 2 \right) \right) \), which as indicated above is positive. Hence, \( \text{sign} \left( \frac{\partial \left( \hat{A}^c \right)}{\partial \theta} \right) > 0 \).
Appendix B: Details of numerical analysis solutions in first- and second-best policies
(We recommend that this Appendix and the Matlab code used in the paper be unpublished but
made available online upon publication)

A. Supply functions

The cost function for the \( i \)th firm in our model is

\[
g_i(a_i) = \frac{\alpha_{i1}}{2} a_{i1}^2 + \frac{\alpha_{i2}}{2} a_{i2}^2 + \gamma_i a_{i1} a_{i2}.
\]

Profits from abatement are

\[
\pi_i = p_1 a_{i1} + p_2 a_{i2} - g_i(a_i).
\]

For any price vector, therefore, the firm will maximize \( \pi_i \) yielding the first order conditions

\[
\alpha_{ij} a_{ij} + \gamma_j a_{ik} = p_j, \quad j=1,2.
\]

Solving this system of two equations with two unknowns leads to the supply functions

\[
a_{i2} = \hat{\alpha}_{i1} p_2 - \hat{\gamma}_i p_1 \quad \text{or} \quad a_{i1} = \hat{\alpha}_{i2} p_1 - \hat{\gamma}_i p_2
\]

(B1)

where \( \hat{\alpha}_i = \alpha_i / \left( \alpha_{i1} a_{i2} - \gamma_i^2 \right) \) and \( \hat{\gamma}_i = \gamma_i / \left( \alpha_{i1} a_{i2} - \gamma_i^2 \right) \). The supply functions in (B1) hold regardless of the policy choice that is made, though in an SM approach one of the prices would be zero. We restrict our analysis to parameters such that \( \alpha_{i1} a_{i2} - \gamma_i^2 > 0 \) in order to ensure that the cost function is convex and the supply curves are upward sloping.

Aggregate supply of the two pollutants under an MM policy is sum of the individual supplies:

\[
A_1 = p_1 \sum_i \hat{\alpha}_{i1} - p_2 \sum_i \hat{\gamma}_i \quad \text{and} \quad A_2 = p_2 \sum_i \hat{\alpha}_{i2} - p_1 \sum_i \hat{\gamma}_i.
\]

(B2)

The supply in an SM setting is more complicated since a firm participating in market 1 would face \( p_2 = 0 \).
B. Market equilibrium, the first best case

Social benefits \( TB = \sum_j B_j (A_j) \) where \( B_j = \Omega_j A_j - \frac{\theta_j}{2} A_j^2 \). Under first-best case the optimum is found taking into account the full degree to which prices will adjust. Hence, the optimum will be set where the price is equal to the marginal benefit, \( B'_j = p_j \), which will hold where

\[
A_j = \frac{\Omega_j - p_j}{\theta_j}.
\]  

(B3)

Equation (B3) can be thought of as the demand for aggregate abatement from the social planner’s perspective.

We can, therefore, solve for the market-clearing price in the social optimum by finding the prices that equate the aggregate supply (B2) and aggregate demand (B3) equations. Solving this system of two equations for \( p_1 \) and \( p_2 \), after some somewhat tedious algebra, we reach

\[
p_1 = \frac{\Omega_1}{\theta_1} + \left( \frac{\Omega_2}{1 + \theta_2 \sum_i \alpha_{1i}} \right) \sum_i \gamma_i \left( \frac{1}{\theta_2 + \sum_i \alpha_{1i}} \right) \quad \text{and} \quad p_2 = \frac{\Omega_2}{\theta_2} + p_1 \sum_i \gamma_i \left( \frac{1}{\theta_2 + \sum_i \alpha_{1i}} \right).
\]

Substituting these prices into (B1) yields the distribution of abatement activities across the firms in the market.
C. The second-best cap

If the agency ignores the interaction effects, then the social objective function can be written as additively separable social maximization problems

$$\max \Omega_j A_j - \frac{\theta_j}{2} A_j^2 - \sum_i \frac{a_{ij}}{2} a_{ij}^2$$

with first-order conditions,

$$\Omega_j - \theta_j A_j - \alpha_j a_{ij} = 0 \forall i .$$

The second-best caps can, therefore, be found as the solution to a system of linear equations:

$$\theta_j \sum_i a_{ij} + \alpha_j a_{ij} = \Omega_j \forall i$$

or, in matrix notation,

$$\begin{bmatrix}
\theta_j + \alpha_{1j} & \theta_j & \ldots & \theta_j \\
\theta_j & \theta_j + \alpha_{2j} & \ldots & \theta_j \\
\vdots & \vdots & \ddots & \vdots \\
\theta_j & \theta_j & \ldots & \theta_j + \alpha_{nj}
\end{bmatrix}
\begin{bmatrix}
a_{1j} \\
a_{2j} \\
\vdots \\
a_{nj}
\end{bmatrix}
= \begin{bmatrix}
\Omega_j \\
\Omega_j \\
\vdots \\
\Omega_j
\end{bmatrix}$$

which can be solved for $a_{im}$ using linear algebra and then be summed to yield $\hat{A}_j$, the second-best cap.

D. MM policy in the second-best setting

Firms facing a second-best cap will not ignore the $\gamma$ parameters. Hence, they will still supply following the (B1). The market clearing prices are then found by solving equations (B2) such that the second-best caps, $\hat{A}_j$, are supplied.

This is again a system of linear equations:
\[
\begin{bmatrix}
\sum_{i} \hat{\alpha}_{i2} & -\sum_{i} \hat{\gamma}_{i} \\
-\sum_{i} \hat{\gamma}_{i} & \sum_{i} \hat{\alpha}_{i1}
\end{bmatrix}
\begin{bmatrix}
p_{1} \\
p_{2}
\end{bmatrix}
= \begin{bmatrix}
\hat{A}_{1} \\
\hat{A}_{2}
\end{bmatrix}
\]

which can be solved to find \( p_{1} \) and \( p_{2} \) in the MM setting.

**E. Second-best supply – single market case**

The single-market case is somewhat more complicated because of the discrete nature of the choices. The firms will enter into only one of the two markets, choosing the market that generates the highest profits.

Firms continue to supply according to (B1), but in this case one of the prices will equal zero; they have to choose between \( \tilde{a}_{i1} = \hat{\alpha}_{i2} p_{1} \) and \( \tilde{a}_{i2} = -\hat{\gamma}_{i} p_{1} \) if they sell in market 1, and \( \tilde{a}_{i1} = -\hat{\gamma}_{i} p_{2} \) and \( \tilde{a}_{i2} = \hat{\alpha}_{i1} p_{2} \) if they sell in market 2. Accordingly, profits if they sell in market 1 will be

\[
\pi_{1} = p_{1} \tilde{a}_{i} - \left( \frac{\alpha_{i1}}{2} \tilde{a}_{i1}^{2} + \frac{\alpha_{i2}}{2} \tilde{a}_{i2}^{2} + \gamma_{i} \tilde{a}_{i1} \tilde{a}_{i2} \right)
\]

which can be simplified to

\[
\pi_{1} = p_{1}^{2} \left( \frac{\hat{\alpha}_{i2} - \frac{\alpha_{i1}^{2}}{\hat{\alpha}_{i1}}}{}^{2} - \frac{\alpha_{i2}^{2} + \gamma_{i}^{2} \alpha_{i1}^{2}}{2} + \gamma_{i} \hat{\gamma}_{i} \hat{\alpha}_{i2} \right)
\]

and, by symmetry, if they sell in market 2

\[
\pi_{2} = p_{2}^{2} \left( \frac{\hat{\alpha}_{i1} - \frac{\alpha_{i1}}{2} \hat{\gamma}_{i}^{2} - \frac{\alpha_{i2}}{2} \hat{\gamma}_{i} \hat{\alpha}_{i1}}{} + \gamma_{i} \hat{\gamma}_{i} \hat{\alpha}_{i1} \right)
\]

A firm will sell in market 1 if \( \pi_{1} > \pi_{2} \), i.e.,

\[
p_{1}^{2} \left( \frac{\hat{\alpha}_{i2} - \frac{\alpha_{i1}^{2}}{\hat{\alpha}_{i1}}}{}^{2} - \frac{\alpha_{i2}^{2} + \gamma_{i} \hat{\gamma}_{i} \hat{\alpha}_{i2}}{2} + \gamma_{i} \hat{\gamma}_{i} \hat{\alpha}_{i2} \right) > p_{2}^{2} \left( \frac{\hat{\alpha}_{i1} - \frac{\alpha_{i1}}{2} \hat{\gamma}_{i}^{2} - \frac{\alpha_{i2}}{2} \hat{\gamma}_{i} \hat{\alpha}_{i1}}{} + \gamma_{i} \hat{\gamma}_{i} \hat{\alpha}_{i1} \right).
\]

As a matter of algebra, it can be shown that a firm will participate in market 1 if
\[
\left( \frac{p_1}{p_2} \right)^2 > \frac{\alpha_i}{\alpha_{i2}} 
\]  \hspace{1cm} (B4)

and in market 2 otherwise.

For any given price ratio, therefore, the firms can be ordered based on the ratio \(\frac{\alpha_i}{\alpha_{i2}}\) and it will hold that firms with lower ratios will tend to supply to market 1 and those with higher ratios will supply to market 2.

However, for \(n\) firms, there are \(n-1\) possible divisions between the 2 markets, each of which is capable of generating \(\hat{A}_1\) and \(\hat{A}_2\) units of total abatement. We solve the problem in several steps. First we check each possible division of the \(n\) firms and then choose the division that minimizes total costs to achieve the caps. It is not always the case that there is a pair of prices within the set of price ratios defined by (B4) that will, using the supply functions (B1), yield exactly \(\hat{A}_1\) and \(\hat{A}_2\) and also support the cost-minimizing division of the firms. As explained in the text, when this occurs, we assume that firms within the market will settle on a price that will be more than sufficient to supply \(\hat{A}_1\) and \(\hat{A}_2\), but that the firms will actually only supply the capped levels, \(\hat{A}_1\) and \(\hat{A}_2\).

\(F.\) \hspace{0.5cm} \textit{Calculation of net benefits}

Once the first-best, MM, or SM allocation of abatement activities is found, the individual and aggregate supplies are substituted into the cost and benefit functions to calculate the net benefits under each setting.
G. Parameters used for Monte Carlo simulations

When conducting policy analysis, we draw 1000 sets of parameters for each from independent uniform distributions for each parameter. The range for these parameters is presented in Table B1. For $\alpha$ and $\gamma$, the parameters’ ranges are naturally bound. The $\theta$ parameter could go arbitrarily high, though an upper bound is chosen that yields results that are qualitatively interesting. The value of $\Omega$ is set so that it increases with the number of firms so that per-firm abatement will be similar regardless of the number of firms participating. Since we have shown that $\Omega$ does not affect the relative performance of the SM and MM policies, it is held constant for any given market size.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum</th>
<th>Maximum</th>
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<td>$\alpha$</td>
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<td>1.0</td>
</tr>
<tr>
<td>$\theta$</td>
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<td>1.5</td>
</tr>
<tr>
<td>$\gamma$</td>
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<td>0.0</td>
</tr>
<tr>
<td>$\Omega$</td>
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<td>$25\cdot n$</td>
</tr>
</tbody>
</table>

Table B1: Parameters used in most simulations
References


