Value chain structure and flexible production technologies*

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Abstract

Industries are transformed by the adoption of flexible production technologies and complementary changes in firms’ organization. Some of the results of this transformation include companies extending their product lines and reshaping their relationships with outside partners. In this paper I analyze how the structure of the upstream industry influences upstream manufacturers’ decisions regarding the choice of production technologies that enable them to extend product variety. The results of a theoretical model with two pairs of supply chains in which producers procure inputs from either two or a single supplier reveal that the benefits of new technologies to manufacturers might be eroded. In particular, an increased intra-brand competition and the introduction of inter-brand competition have adverse effect on producers’ payoffs. Eventually, the choice made by downstream manufacturers departs from the socially optimal outcome.

Keywords: flexible production technology, merger, supply chain, vertical relations.

JEL-Classification: L12, L14, L25

1 Introduction

In this paper I analyze the interplay between the value chain organization, the adoption of flexible production technologies (FPT) leading to more product variety and welfare implications

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of such actions. In order to link the characteristics of FPT with the firm scope and supplier relations, I develop a theoretical model that allows for an analysis of suppliers' incentives to merge and manufacturers' decisions regarding the choice of production technologies that are specific to inputs produced by suppliers. Choosing FPT over dedicated production technology (DPT) allows a manufacturer to gain access to inputs necessary to extend its product variety. Two questions are of major importance here: First, how does the structure of the upstream industry, market size and the degree of product differentiation affect producers' incentives to adopt FPT? Second, what are the welfare implications of the decisions regarding investments in the production technologies under different structures of the upstream industry?

The motivation for this paper is the technological shock that, over the last two decades, has lead to a reorganization of value chain structure and that moves industries away from mass production to mass customization. Flexible machines and multitasking production equipment replace specialized and single-purpose equipment. Because FPT can be reprogrammed quickly, they can be seen as an ability to produce several products in a single plant or on a single assembly line at a low cost, relatively to the specialized equipment designed for mass production of homogeneous products (Milgrom and Roberts (1990)). Consequently, modern manufacturing is being transformed by the adoption of FPT and complementary changes in firms’ strategy and organization. In particular, FPT enable firms to change their strategies and the way they organize their activities within and between organizations. New strategies include broadening product lines, frequent product introductions and improvements. Such behavior is consistent with the argument that, as firms seek to escape competition, implementing technologies that allow for extending product line and increasing product variety is a major point of emphasis in the quest for additional profits (Lancaster (1990)). Regarding organizational changes, a firm's processes, internal structure, and the relationships with outside partners are strongly influenced by its product strategy and technology in place (Brynjolfsson et al. (2002)). Consequently, value chain characteristics and production technology have a considerable impact on the competition in the input and product markets and, eventually, on welfare.

To model the interplay between value chain organization, technology and increasing product variety, regarding the technology characteristics, I follow the assumptions made by Röller and Tombak (1990). In a linear framework, they model FPT as a technology that enables firms
to produce parallel two products instead of only one. According to their results, investing in FPT leads to more competition and subsequently reduces prices and profits of both firms. Their conclusion is that flexibility is detrimental to firms’ profits and companies are better-off when they remain one-product monopolists. Furthermore, although more flexibility leads to the transfer of surplus from producers to consumers, parallel production of both goods is desirable from the social point of view only when products are enough differentiated (Gupta (1998)).

An alternative treatment of flexible production technologies can be found in Eaton and Schmitt (1994). They use a Hotelling model in which they describe the effects of new production system on firms’ ability to lower the cost of product customization and the cost of switching the production process from one variant to another. The focus of their analysis is, however, different from the one of the current work. In their work, they study the implications of flexible manufacturing systems for market structure and find that they promote concentration through preemption and mergers. By using a similar approach, Norman and Thisse (1999) arrive to a conclusion that the monopoly preemption is still feasible, but it will lead to excessive product variety.

Concerning the value chain dimension, I follow Horn and Wolinsky (1988) who model a duopoly in which producers buy inputs through bilateral monopoly relations with suppliers. This setting reflects the vertical relations modelled in this paper in a state when both downstream firms use dedicated technologies. Analyzing horizontal mergers, they find that the distribution of bargaining power might have important implications for merger incentives. For example, in contrast to the finding of this paper, they argue that under some conditions duopoly structure might be profit maximizing for the upstream industry. Following this line of analysis, Milliou and Petrakis (2005) modify the bargaining setting and find that under some conditions suppliers prefer to act independently as well.

To my best knowledge, the problem of flexible production technologies in the context of vertical relations has been not analyzed yet. Available literature on vertical relations and technology focuses on supplier-buyer specific investments and the impact of vertical merger on such investments. This approach assumes that technology used by vertically separated firms influences input price, not product variety. For example, Kranton and Minehart (2004) use a framework in which there are two upstream and two downstream firms. Downstream firms compete for
one indivisible input unit produced by each supplier. Buyers’ valuation of input depends on their investments into supplier-specific assets. Focusing on vertical merger and its effect on downstream investment decisions into assets that are specific to supplier-buyer relations, they find that vertical merger might distort investments into technologies reducing production costs of the remaining firms. Similarly, regarding efficiency-increasing investments, Inderst and Wey (2003) analyze the effect of upstream and downstream market structure on suppliers’ innovation investments. They consider technological flexibility in terms of production volume and technology choice that determines the level of production costs. Another work by Inderst and Wey (2007) follows this line and analyzes the question of how the distribution of bargaining power in value chain affects suppliers’ incentives to make technological improvements and reduce marginal production costs.

The model developed in this paper exhibits characteristics of successive monopolies and foreclosure. Regarding the issue of successive monopolies, due to the type of relations between upstream and downstream firms, we can observe a well-known problem of double marginalization (Spengler (1950)). Although designing a remedy to this problem is beyond the scope of this paper, it is worthwhile to mention that the use of flexible production technologies intensifies competition in the input and consumer market and reduces the harmful effect of double marginalization. Regarding the foreclosure concept, provided that there are cost or technological barriers to procure inputs necessary to broaden a firm’s product line, flexible technologies can be seen as a device to bypass foreclosure. Consequently, this links the current paper to the discussion of vertical foreclosure and incentives to invest (see for example Hart et al. (1990), Baake et al. (2003) and Fumagalli et al. (2006)).

The current model includes some elements of exclusive dealing and vertical integration as well. These issues are analyzed, for example, by Bonanno and Vickers (1988). By using a two-part tariff contract as a mechanism to set prices, they model two single-product manufacturers that sell their products to independent retailers and analyze what are producers’ incentives to vertically integrate or to sell their products through independent retailers. Similar approach can be found, for example, in Rey and Tirole (1986) and Rey and Stiglitz (1988).

Other papers that are closely related to the current one include Lin (1990) and Ziss (1995).

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1For a literature overview on vertical foreclosure see Rey and Tirole (2006).
The former one models two retail chains and argues that upstream firms choose an exclusive dealing set-up in order to relax intra-brand competition. In the context of the current model, flexible technologies might have pro-competitive effects, as they remove the exclusive dealing constraints, intensify inter-brand rivalry and introduce intra-brand competition. Ziss (1995) explicitly includes the issue of vertical mergers at both levels. His main finding is that both types of merger have anti-competitive effects.

Despite drawing on some already analyzed concepts, the design of the current paper differs from previous contributions. First, using the framework of complementarity between technology, strategy and organization, I formalize the idea of the adoption of flexible vs. specialized production equipment and link it to firms’ strategies regarding broadening product line and their impact on competition at the upstream and downstream level. Second, I discuss the concept of technology as a means to bypass foreclosure and its pro-competitive effects. This casts some new light on how technological progress removes barriers to competition and, consequently, influences social welfare. Lastly, I analyze welfare implications of different technology states under both competitive and monopolistic structure of the upstream industry.

Besides confirming existing findings, the contributions of the paper are manyfold. First, running counter to the intuition, the results reveal that an increased competition due to the diffusion of FPT might erode any benefits of such technologies. Although the use of FPT might be profitable from a point of view of an individual firm, when all firms in the industry adopt such technologies they collectively forsake profits. Consequently, as there is a coordination problem, firms end up in a Prisoners’ Dilemma situation. To a large extent, this confirms the results of Röller and Tombak (1990). Considering the effect of the structure of the upstream industry, I show that under some conditions, i.e. when products are complements, manufacturers are more likely to adopt FPT when there is a multi-product monopolistic supplier of both inputs, as compared to a state with two independent suppliers. The reverse is true when products are substitutes. Second, I show that the introduction of FPT by downstream producers has two effects on the payoff to the upstream industry. On the one hand, selling to both firms increases intra-brand competition and reduces suppliers’ profits. On the other hand, suppliers benefit from the pro-competitive effects of FPT on the final market. It seems that the latter effect dominates the former one. Furthermore, regarding suppliers, unlike in Lin (1990), under
the current setting suppliers prefer to supply both downstream firms to exclusive interaction with only one of them. Third, the adoption of FPT is always beneficial from the consumers and suppliers point of view. The fact that new technologies have always positive effect on consumer surplus is different from the findings of Röller and Tombak (1990) who concluded that such investments by downstream firms generate benefits to consumers only for sufficiently differentiated products. Furthermore, considering the structure of the upstream industry, I find that a supplier merger increases consumer surplus when products are complements. In total, the diffusion of FPT has a positive impact on total welfare, provided that products are not close substitutes, and for some intermediate values of market size, companies’ equilibrium decisions lead to socially inefficient outcomes.

Empirical research confirms that information and communication technologies (ICT) enable firms to expand product variety and to deal with a following raise in the complexity and sophistication of technological and business processes (e.g. Bakos, et al. (1986), Jaikumar, (1986), Holland et al. (1997)). For example, a combination of new computer-based flexible machinery with new work practices allowed a Johnson&Johnson factory to significantly increase the variety of products it could manufacture and reduce costs (Brynjolfsson et al. (2002)). Similar trends can be observed in the electronics industry where the choice of products and their functionalities have been dramatically increasing over the last decades (Petkova (2003)). Dell Computer is a quintessential New Economy company known for its flexibility, leanness, and a variety of products cut to customers’ needs. Because of massive investments in ICT, Dell extended the reach and scope of its business at a relatively low cost (Kraemer, et al. (2001)). Sophisticated technologies allowed it to automate business and production functions and to coordinate a network of suppliers and business partners who carry out most of the tasks involved in developing, building and distributing of personal computers.

The impact of new technologies on product variety is not limited to manufacturing industries only. Studies of the financial intermediation, airline, hotel and rental car industries report that systems linking organizations in value chain, i.e. inter-organizational systems or IOS, allow an agent to access quickly a wide range of products offered by different providers and to bundle them in order to create a combination cut to individual customer needs (Hess et al. (1994) or Johnston et al. (1988)). In a study of the causes of Wal-Mart’s growth, Basker (2007) cites bar code and
RFID as major sources of the retailer’s success. The availability of a technology that reduces the inventory tracking costs and improves the overall efficiency of the supply chain increases the incentives to add product lines led directly to the creation of supercenters that sell a full line of groceries in addition to general merchandise. The rapid and ubiquitous spread of ICT and flexible production systems have implications for suppliers of firms implementing them as well. Dewan et al. (1998) analyzes the link between the scale and scope of a firm and its ICT investments with an emphasis of the role of ICT in coordination with suppliers. The results suggest that ICT investments are positively related to the degree of firm diversification. Similarly, Hempell et al. (2005) study how ICT drives product and process innovations by enhancing organizational flexibility. They conclude that by facilitating communication and access to information, ICT favours the use of easily programmable machines and improves the coordination with suppliers. Moreover, ICT increases the organizational flexibility as it allows for a quick reaction to changes in consumer preferences. This additionally increases the incentives to expand the product line.

However, the process of strategy and organizational changes driven by the diffusion of new technologies has consequences that go far beyond the boundaries of firms that adopt them. For example, as firms broadening product line remove the boundaries between separate markets, suppliers of such firms are exposed to stronger competition. Furthermore, an increase in the number of downstream firms demanding inputs might raise supplier’s minimum efficient size of operations necessary to satisfy new demand. Thus, it can be expected that facing stronger pressure to take over a number of responsibilities and deal with an increased demand for intermediary products, suppliers might need to either preempt of response to these challenges. One way to deal with them is to increase the scale of operations or to reduce the intensity of upstream competition. Both effects can be achieved through a consolidation process. Indeed, it has been observed that firms are going through intensive shake-outs and waves of mergers during periods of especially high demand (Bernile, et al. (2007)). Consequently, if profitable at first glance, the adoption of new production system might not bring the expected payoff once preempted by other partners in the value chain.

The automotive industry provides an interesting example of the relationship between technology, product variety and vertical organization. The consolidation within the automotive suppliers’ network is a response to constantly increasing quality demands, taking over more
operations in design and production, on the one hand, and to reduce price, on the other hand. Consequently, manufacturers are becoming more dependent on suppliers and suppliers are becoming more involved into development and manufacture of a greater number of products. As a result, the number of automotive suppliers worldwide is expected to shrink by half by 2015 (VDA (2004)). At the same time, new technologies deployed in product design, manufacturing and interactions with suppliers allow car manufacturers to steadily increase the number of car models (Dicken (2003)). In 2002 there were over 1000 car models offered for sale in the United States, double as much as in 1980 (van Biesebroeck (2006)). The answer to the question regarding profitability of such changes is more complex than it seems. For example, since early '90s, the BMW product line has expanded from 5 to 10 lines (PWC, mimeo). At the same time, the production volume reached over 1.3 million in 2005 from 0.5 million units in 1990. Although, between 2001 and 2005, the revenues increased by over 30% to nearly 46 billion Euro, gross margin has in fact declined from 8.3% to 6.5% in the same period. Thus, the changes in vertical structure and the impacts of the diffusion of FPT might go far beyond the increased product variety offered by firms.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 analyses outcomes that emerge in the current structure and Section 4 identifies equilibria. Section 5 considers welfare implications of both technology choice and the market structure of the upstream industry. Section 6 concludes and suggests some potential extensions to the current analysis. Appendix includes proofs.

2 The Model

I consider two supply chains in which downstream producers buy inputs from either one or two suppliers. There are two products in the market, X and Y. In order to produce each product, downstream firm needs technology that is specific to the input for a particular product. Figure 1 illustrates all possible technology states. If downstream producer $P_1$ ($P_2$) uses dedicated technology (D) it can purchase input from upstream supplier $U_1$ ($U_2$) and supply final good X (Y). This is case (D,D). If downstream firm uses flexible technology (F) they can be active in both parts of the market (case (F,F)). Mixed outcomes, i.e. (F,D) and (D,F), in which downstream firms can use different technologies are allowed. An identical situation is possible.
when there is only one supplier of both inputs (dashed circles).

The game has four stages:

- **Stage one:** Suppliers decide on whether to merge or stay independent.

- **Stage two:** Producers choose between dedicated and flexible technology.

- **Stage three:** Suppliers set input prices. I assume that each supplier maximizes his profit with respect to input price, given demand of downstream firms and the strategy of the other supplier. If there is only one multi-product supplier, he maximizes its profit with respect to both input prices.

- **Stage four:** Producers play a Cournot game on the consumer market.

In contrast to Horn and Wolinsky (1988), I assume that suppliers have all the bargaining power and make take-it-or-leave-it offers to manufacturers. Within this framework, it is very likely that in equilibrium suppliers choose to merge over acting independently. The motivation behind this assumption is the question of how technology choices of downstream firms differ with respect to the market structure of the upstream industry.

In order to describe the market demand for product X and Y, I follow Matutes et al. (1989) and assume some degree of product differentiation. For each of the two goods consumers maxi-
mize a utility function that is separable in the numeraire good $M$:

$$V[X, Y, M] = U(X, Y) + M$$  \hfill (1)

where $X$ and $Y$ are the total quantities of both products. Let the quadratic utility function be given by:

$$U = \alpha (X + Y) - \frac{1}{2} (X^2 + Y^2 + 2\gamma XY),$$  \hfill (2)

then the first order conditions of the consumer maximization problem yield the following demand functions:

$$p^X = \alpha - \gamma Y - X \text{ and } p^Y = \alpha - \gamma X - Y$$  \hfill (3)

where $p^X$ and $p^Y$ are the prices for product $X$ and $Y$. $X = x_1 + x_2$ and $Y = y_1 + y_2$ are the total quantities of product $X$ and $Y$ produced by downstream firm $i = 1, 2$. Parameter $\gamma$ can be interpreted as a determinant of product differentiation. Products are complements when $-1 \leq \gamma < 0$ and substitutes when $0 < \gamma \leq 1$. If $\gamma = 0$, products are not related. These conditions guarantee that the own price effect is always stronger than the cross-price effect. The value of $\alpha$ can be interpreted as the potential size of the market.

Regarding the cost structure, I assume that suppliers do not incur any costs and that the cost of inputs is the only marginal cost of producers. Thus, given the above defined demand system and cost structure, the payoff function of each downstream firm $i = 1, 2$ is:

$$\pi^{P_k}_{i,j} = (p^X_{j} - w^X_{j})x^k_{ij} + (p^Y_{j} - w^Y_{j})y^k_{ij} - f_t$$  \hfill (4)

where $P$ denotes downstream firm and $k = I, M$ represents the structure of the upstream industry where $I$ denotes independent suppliers and $M$ a multi-product monopolistic supplier. Technology choice of downstream producers is denoted by $j = 1, \ldots, 4$ where $j = 1$ indicates that both firms invest in dedicated technologies $(D, D)$, $j = 2$ when firm 1 invests in technology $F$ and firm 2 in technology $D$, $(F, D)$. In this case, firm 2 procures input only from supplier 2 and firm 1 from both of them. The reverse is true in $j = 3$ when firm 2 chooses technology $F$ and firm 1 technology $D$, $(D, F)$. The last case, $j = 4$, is when both firms invest in technology $F$, $(F, F)$. This means that both downstream firms can procure inputs for both products. Input prices for product $X$ and $Y$ are given by $w^X_{j}$ and $w^Y_{j}$ respectively. $f_t$ is the fixed cost of the production technology $t = D, F$ that downstream firms need to incur. Let the cost of dedicated technology
be \( f_D = 1 \) and the cost of flexible technology \( f_F = 1 + s \). To make it realistic, I assume that technology F is more costly than technology D, i.e. \( s > 0 \). Both production technologies exhibit constant returns to scale, i.e. out of one unit of input downstream firm produces one unit of output. As tie-breaking rules, if downstream producer is indifferent between technologies it will choose flexible one.

The payoff function of independent upstream firms \( i = 1, 2 \) is given by:

\[
\pi_{1,j}^U = w^X_j X_j^k \quad \text{and} \quad \pi_{2,j}^U = w^Y_j Y_j^k
\]

(5)

where \( U \) denotes upstream firm. Supplier 1 produces input for final product X and supplier 2 for final product Y. A single supplier faces the following maximization problem

\[
\pi_{M,j}^U = w^X_j X_j^k + w^Y_j Y_j^k
\]

(6)

where \( M \) denotes a multiproduct monopolistic supplier. Whenever suppliers are indifferent between merging or remaining independent they will merge. Table 1 exhibits the payoffs to downstream and upstream firms in all possible settings.

<table>
<thead>
<tr>
<th>Table 1: Technology choice and firms’ payoffs</th>
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<tbody>
<tr>
<td><strong>Firm 1</strong></td>
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<tr>
<td><strong>D</strong></td>
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<tr>
<td><strong>F</strong></td>
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3 **Input prices and product quantities**

Except for the whole game, there are two sub-games. One with independent suppliers of each intermediary product and one with a multi-product upstream monopoly. Depending on the technology choice, there are four possible outcomes at the downstream level. By using backward induction, in the following section I solve the game for the final quantities and input prices that arise in each technology state under both structures of the upstream industry. Then, I use these results to find a sub-game perfect equilibrium of the merger and technology game.
3.1 Independent suppliers

Technology outcome (D,D): In the first case, (D, D), both manufacturers use dedicated technologies and supply only one product. Firm 1 produces good X and firm 2 produces good Y. In this case, firm 1 interacts with supplier 1 and firm 2 with supplier 2. Both downstream firms have symmetric profit functions of the form:

\[ \pi_{11} = (p_1^X - w_1^X) x_{11} - 1 \quad \text{and} \quad \pi_{21} = (p_1^Y - w_1^Y) y_{21} - 1. \] (7)

Solving for Cournot equilibrium outcomes, yields symmetric quantities:

\[ x_{11}^* = \frac{\alpha(\gamma - 2) - \gamma w_1^X + 2w_1^X}{\gamma^2 - 4} \quad \text{and} \quad y_{21}^* = \frac{\alpha(\gamma - 2) - \gamma w_1^Y + 2w_1^Y}{\gamma^2 - 4}. \] (8)

As one piece of input is used up to produce one piece of output, quantities produced by manufacturers are at the same time the demanded quantities for inputs. Thus, by maximizing their profit functions with respect to \( w_1^X \) and \( w_1^Y \), suppliers set symmetric prices for product X and Y:

\[ w_1^{X*} = w_1^{Y*} = \frac{\alpha(\gamma - 2)}{\gamma - 4}. \] (9)

Substituting input prices into the reaction functions of downstream firms leads to the following expression:

\[ x_{11}^* = y_{21}^* = \frac{2\alpha}{(\gamma + 2)(4 - \gamma)}. \] (10)

Technology outcome (F,D) or (D,F): In the second and third case, asymmetric outcomes arise, i.e. when one downstream firm uses dedicated and the other one flexible technology. Let us consider the (F,D) outcome when firm 1 uses technology F and firm 2 technology D. In this case, manufacturer 1 is able to interact with supplier 1 and 2. As a result, it produces X and Y. Manufacturer 2, in contrast, procures only from supplier 2 and produces product Y only. Thus, the payoff functions of manufacturers are asymmetric, i.e.

\[ \pi_{12} = (p_2^X - w_2^X) x_{12} + (p_2^Y - w_2^Y) y_{12} - (1 + s) \] (11)

\[ \text{and} \quad \pi_{22} = (p_2^Y - w_2^Y) y_{22} - 1. \] (12)

By maximizing the above expression with respect to \( x_{12}^* \), \( y_{12}^* \), and \( y_{22}^* \), one obtains the following reaction functions:

\[ x_{12}^* = \frac{\alpha(\gamma - 1) - \gamma w_2^Y + w_2^X}{2(\gamma^2 - 1)} \] (13)
and \( y_{12}^* = \frac{\alpha(\gamma^2 - 3\gamma + 2) - \gamma^2 w_2^Y + 3\gamma w_2^X - 2w_2^Y}{6(1 - \gamma^2)} \), \( y_{22}^* = \frac{\alpha - w_2^Y}{3} \). (14)

The above quantities enter the profit functions of upstream firms, which maximize them with respect to input prices \( w_2^X \) and \( w_2^Y \). As a result, asymmetric prices for X and Y arise:

\[
\begin{align*}
  w_2^X &= \frac{\alpha(\gamma^3 - 5\gamma^2 - 4\gamma + 8)}{16 - 7\gamma^2} \\
  w_2^Y &= \frac{\alpha(5\gamma^2 + 3\gamma - 8)}{7\gamma^2 - 16}.
\end{align*}
\] (15)

By substituting input prices into the reaction functions of downstream firms, one obtains the following equilibrium quantities:

\[
\begin{align*}
  x_{12}^* &= \frac{\alpha(\gamma^2 - 4\gamma - 8)}{2(\gamma + 1)(7\gamma^2 - 16)}, \\
  y_{12}^* &= \frac{\alpha(\gamma^3 + 10\gamma^2 + 2\gamma - 16)}{6(\gamma + 1)(7\gamma^2 - 16)}, \quad y_{22}^* = \frac{\alpha(2\gamma^2 - 3\gamma - 8)}{3(7\gamma^2 - 16)}.
\end{align*}
\] (16)(17)

It is straightforward, that the demand for input Y is higher than for input X. Consequently, supplier 2 earns a higher profit because it sells inputs to both downstream firms.

In the third case, (D, F), the reverse of the (F, D) state is true. That is, \( \pi_{13}^{P1} = \pi_{22}^{P1} \) and \( \pi_{23}^{P1} = \pi_{12}^{P1} \) at the downstream level, and \( \pi_{13}^U = \pi_{22}^U \) and \( \pi_{23}^U = \pi_{12}^U \) at the upstream level.

Technology outcome (F, F): In the fourth technology case, (F, F), both manufacturers invest in F technology and each of them procures both inputs and produces X and Y. Consequently, manufacturers maximize symmetric profit functions with respect to \( x_{i4}^I \) and \( y_{i4}^I \):

\[
\begin{align*}
  \pi_{14}^{P1} &= (p_4^X - w_4^X) x_{14}^I + (p_4^Y - w_4^Y) y_{14}^I - (1 + s) \\
  \pi_{24}^{P1} &= (p_4^X - w_4^X) x_{24}^I + (p_4^Y - w_4^Y) y_{24}^I - (1 + s).
\end{align*}
\] (18)(19)

This gives symmetric quantities of \( x_{i4}^I \) and \( y_{i4}^I \) manufactured by downstream firms, i.e.:

\[
\begin{align*}
  x_{14}^I &= x_{24}^I = \frac{\alpha(\gamma - 1) - \gamma w_4^Y + w_4^X}{3(\gamma^2 - 1)} \\
  y_{14}^I &= y_{24}^I = \frac{\alpha(\gamma - 1) - \gamma w_4^X + w_4^Y}{3(\gamma^2 - 1)}.
\end{align*}
\] (20)(21)

In contrast to the (D, D) outcome, suppliers deliver their inputs to both downstream firms and their profit functions can be expressed as

\[
\begin{align*}
  \pi_{14}^U &= w_4^X (x_{14}^I + x_{24}^I) \\
  \pi_{24}^U &= w_4^Y (y_{14}^I + y_{24}^I).
\end{align*}
\] (22)(23)
Again, this leads to symmetric input prices for X and Y:

\[ w^X_{I^*} = w^Y_{I^*} = \frac{\alpha(\gamma - 1)}{\gamma - 2} \]  

and eventually symmetric quantities sold on the final market are:

\[ x^*_{I1} = x^*_{I2} = y^*_{I1} = y^*_{I2} = \frac{\alpha}{3(2 - \gamma)(\gamma + 1)} . \]  

3.2 A monopolistic multiproduct supplier

**Technology outcome (D,D):** As in the case with independent suppliers, downstream firms have symmetric profit functions (7) and, consequently, symmetric reaction functions (8). A multiproduct single supplier sets both input prices at the highest possible level, i.e. \( w^X_{M1} = w^Y_{M1} = \frac{\alpha}{2} \). The resulting equilibrium quantities are

\[ x^*_{M11} = y^*_{M21} = \frac{2\alpha}{2(\gamma + 2)} . \]  

**Technology outcome (F,D) or (D,F):** Again, in the asymmetric case, a monopolistic supplier takes the demand of manufacturers given by (13) and (14) and maximizes its profit by setting monopolistic prices for both inputs. Thus, substituting input prices into the reaction functions of downstream firms leads to the following expression:

\[ x^*_{M12} = \frac{\alpha}{4(\gamma + 1)}, \]  

and

\[ y^*_{M12} = \frac{\alpha(2 - \gamma)}{12(\gamma + 1)}, \quad y^*_{M22} = \frac{\alpha}{6} . \]  

In the (D, F) case, the reverse of the second state is true, i.e. \( x^*_{M13} = y^*_{M22} \), \( x^*_{M23} = y^*_{M12} \), \( y^*_{M23} = x^*_{M12} \), \( \pi^P_{13} = \pi^P_{22} \) and \( \pi^P_{23} = \pi^P_{12} \) at the downstream level, and \( \pi^U_{M3} = \pi^U_{M2} \) at the upstream level.

**Technology outcome (F,F):** Similarly as above, the monopolistic supplier maximizes its total profit by setting monopolistic input prices for both products. The final outcome of the Cournot competition by downstream firms can be expressed as

\[ x^*_{M14} = x^*_{M24} = y^*_{M14} = y^*_{M24} = \frac{\alpha}{6(\gamma + 1)} . \]  

Table 1A and 2A (Appendix) present equilibrium expressions for all possible technology states when suppliers stay independent or when they merge.
4 Equilibrium analysis

By comparing equilibrium profits in each market structure and technology outcome, I look for a sub-game perfect equilibrium. In addition, this section discusses the impact of technology cost, market size and the degree of product differentiation on the choice of upstream market structure and the adoption of production technology.

4.1 Independent suppliers

This section deals with the sub-game with a competitive structure of the upstream industry. After suppliers chose to stay independent in the first stage, producers choose between FPT and DPT in the second stage. Subsequently, each supplier maximizes its profit with respect to input price, given quantities ordered by downstream firms and the strategy of the other supplier. In the last stage, downstream producers compete in a Cournot game.

Independent suppliers and (D, D) equilibrium: The state with two independent suppliers and downstream firms choosing dedicated technologies is an equilibrium when two conditions are satisfied. First, from table 1 we see that both downstream firms choose D technology when

\[ \pi_{12}^{PI} - \pi_{11}^{PI} < 0 \quad \text{and} \quad \pi_{23}^{PI} - \pi_{21}^{PI} < 0. \]  

This can be expressed as:

\[ f_{DD}^I(\gamma) = \frac{s}{\alpha^2} < 0 \]  

where

\[ f_{DD}^I(\gamma) = \frac{(\gamma^7 + 3\gamma^6 - 68\gamma^5 + 8\gamma^4 - 176\gamma^3 - 832\gamma^2 - 1152\gamma - 1024)}{36(\gamma + 1)(\gamma - 4)^2(\gamma + 2)(7\gamma^2 - 16)} \]  

Second, for the equilibrium to exist, profits of independent suppliers must be higher than this of a monopolistic supplier. This is true when

\[ \pi_{11}^U + \pi_{21}^U > \pi_{M1}^U. \]  

This condition is satisfied if

\[ \frac{a^2\gamma^2}{2(\gamma + 2)(\gamma - 4)^2} < 0. \]  

Regarding the choice of technology, expression (31) suggests that both buyers remain using D technologies when \( \alpha \) is small and \( s \) is high. In other words, small market size discourages firms...
from entering the part of the market dominated by the other firm. Each firm prefers to stay a one-product monopolist rather than to get involved into direct competition with the other firm. Large values of \( s \) additionally discourage firms investments into flexible production systems.

Turning to the second condition \((34)\), it is known from Salant (1983) that merger to monopoly is always profitable. This result holds when all the firms collude, so that there are no outsiders. In this case, there are only two firms and it can be seen that whenever products \( X \) and \( Y \) are complements or substitutes, i.e. \( \gamma \in [-1, 1] \), \((34)\) is always positive and suppliers choose to merge.

**Independent suppliers and \((F, F)\) equilibrium:** The state with two independent suppliers and both downstream firms choosing flexible technology is an equilibrium when two conditions are satisfied. First, from table 1 we see that both downstream firms choose \( F \) technology when

\[
\pi_{14}^{PI} \geq \pi_{12}^{PI} \quad \text{and} \quad \pi_{24}^{PI} \geq \pi_{23}^{PI}.
\]

This condition is satisfied if

\[
f_{FF}^{I}(\gamma) - \frac{8}{\alpha^2} \geq 0
\]

where

\[
f_{FF}^{I}(\gamma) = \frac{(256 - 192\gamma - 164\gamma^2 + 128\gamma^3 - 35\gamma^4 - 13\gamma^5 + 24\gamma^6 - 4\gamma^7)}{9(\gamma + 1)(\gamma - 2)^2(7\gamma^2 - 16)^2}.
\]

Second, profits of independent suppliers must be higher than a monopolist’s payoff. This is true when

\[
\pi_{14}^{U} + \pi_{24}^{U} > \pi_{M4}^{U}
\]

which can be expressed as:

\[
\frac{\alpha^2\gamma^2}{3(\gamma - 2)^2(\gamma + 1)} < 0.
\]

In line with the above case, both producers will switch to technology \( F \) when \( \alpha \) is large. In other words, large market size gives firms a strong incentive to produce both products and to get involved into direct competition with the other firm. The revenue generated from the sales of the additional product compensate the losses from more intense competition and lower prices. Of course, high technology costs reduces the gains from an increased product variety.
Regarding the upstream market structure, as in the previous case condition (39) is fulfilled for all values of \( \gamma \). Thus, suppliers always choose to merge, given that they are price setters in the input market.

**Independent suppliers and mixed equilibria:** In \((F, D)\) or \((D, F)\) equilibrium, producer 1 uses technology \(F\) and producer 2 technology \(D\) or the other way around. Such mixed equilibria exist when two conditions are satisfied. First, from table 1 we know that \((F, D)\) or \((D, F)\) is an equilibrium when

\[
\pi_{12}^{PI} - \pi_{11}^{PI} \geq 0 \text{ and } \pi_{24}^{PI} - \pi_{22}^{PI} < 0
\]

and

\[
\pi_{14}^{PI} - \pi_{13}^{PI} < 0 \text{ and } \pi_{23}^{PI} - \pi_{21}^{PI} \geq 0.
\]

This can be expressed as:

\[
f_{DD}^{I}(\gamma) - \frac{s}{\alpha^2} \geq 0
\]

and

\[
f_{FF}^{I}(\gamma) - \frac{s}{\alpha^2} < 0.
\]

Second, profits of independent suppliers must be higher than this of a monopolistic supplier. This is true when

\[
\pi_{12}^{U} + \pi_{22}^{U} > \pi_{M2}^{U} \text{ and } \pi_{13}^{U} + \pi_{23}^{U} > \pi_{M3}^{U}.
\]

This condition is satisfied if

\[
\frac{\alpha^2\gamma^2(112 + 112\gamma + 5\gamma^2 - 13\gamma^3)}{8(\gamma + 1)(7\gamma^2 - 16)^2} < 0.
\]

Regarding the technology choice of downstream companies, these sub-game equilibria would result in the region between condition (31) and (36). However, because the pay-off to the multi-product monopolistic supplier is always higher than to the independent suppliers and upstream firms choose to merge. Consequently, expression (45) is positive for all values of \( \alpha \) and \( \gamma \) and there is no equilibrium with a competitive structure of the upstream industry. In conclusion, the analysis of the above equilibrium conditions can be summarized in the following Lemma:

**Lemma 1:** For all values of \( \alpha, \gamma \) and \( s \), suppliers choose to merge and there is no equilibrium with a competitive upstream market structure.
Proof: The proof emerges from the suppliers’ profit conditions, i.e. condition (34), (39) and (45) are never satisfied. In other words, the profit of a multiproduct monopolistic supplier is never smaller than the cumulative profit of independent suppliers. The same result can be found in Salant (1983). Consequently, there is no outcome with a competitive upstream market structure in the equilibrium.

4.2 A single supplier

This section analyses candidate equilibria that emerge given the upstream industry was monopolized in the first stage of the game. Proceeding as above, the game is solved by backward induction. After suppliers decided to merge in the first stage, downstream firms choose production technology in the second stage. Subsequently, a monopolist supplier maximizes its profit function with respect to input prices given quantities ordered by downstream firms. In the last stage manufacturers set the quantities of the final goods in a Cournot game.

A single supplier and (D, D) equilibrium The state with one supplier and downstream firms choosing dedicated technology is an equilibrium when two conditions are satisfied. First, both downstream firms choose D technology when the following condition holds:

\[ f_{DD}^M(\gamma) - \frac{8}{\alpha^2} < 0 \]  

(46)

where

\[ f_{DD}^M(\gamma) = \frac{(16 - 4\gamma - 7\gamma^2 - 5\gamma^3)}{144(\gamma + 1)(\gamma + 2)^2}. \]  

(47)

Second, suppliers merge if the monopolistic profit is higher than the profits of the two sellers. This condition is fulfilled if

\[ \frac{\alpha^2\gamma^2}{2(\gamma + 2)(\gamma - 4)^2} \geq 0. \]  

(48)

From the above analysis we know that condition (46) depends on the degree of product differentiation, market size and the cost of F technology. The smaller the market and the higher the cost of F technology relatively to D technology, the more are downstream firms inclined to stay with D technology. Regarding the upstream firms, profit of a single supplier is always higher than the profits of separate firms and, consequently, suppliers choose to merge.
A single supplier and (F, F) equilibrium: The state with one supplier and both downstream firms choosing F technology is an equilibrium when two conditions are satisfied. First, downstream firms choose F technology when

$$f_{FF}^M(\gamma) - \frac{s}{\alpha^2} \geq 0$$

(49)

where

$$f_{FF}^M(\gamma) = \frac{(1 - \gamma)}{36(\gamma + 1)}.$$  

(50)

Second, profit of a monopolistic supplier is higher than profits of independent suppliers when

$$\frac{\alpha^2\gamma^2}{3(\gamma - 2)^2(\gamma + 1)} \geq 0.$$  

(51)

Considering condition (49), the choice of F technology is again dependent on the market size, the degree of product differentiation, and the cost of flexible technology. However, although a larger market size encourages both firm to switch to flexible technology and to produce both products, the incentive to expand is reduced by the cost of the new technology. The effect of $\gamma$ is equally important for the technology choice. It is easy to see that the expansion strongly depends on the degree of product differentiation. Regarding the merger incentive, we know that (51) is positive for all values of $\gamma$.

A single supplier and mixed equilibria: The state with one supplier and downstream firms choosing different technologies is an equilibrium when two conditions are satisfied. First, from table 1 we see that (F, D) or (D, F) are selected when

$$f_{DD}^M(\gamma) - \frac{s}{\alpha^2} \geq 0$$

(52)

and

$$f_{FF}^M(\gamma) - \frac{s}{\alpha^2} < 0.$$  

(53)

Second, the profit of a monopolistic supplier is higher than the profits of independent suppliers when

$$\frac{\alpha^2\gamma^2(112 + 112\gamma + 5\gamma^2 - 13\gamma^3)}{8(\gamma + 1)(7\gamma^2 - 16)^2} \geq 0.$$  

(54)

Considering that (54) is positive for all $\gamma$ and, consequently, that suppliers always merge, mixed equilibria would result in the region between condition (53) and (54). Both conditions are
satisfied when firms sell complementary products. Thus, unlike in Röller and Tombak (1990), for some values of \( \alpha \) and \( \gamma \), mixed equilibria can emerge.

The analysis of the above equilibrium conditions can be summarized in the following lemma:

**Lemma 2:** Suppliers always choose to merge. The technology choice of downstream firms depends on \( \alpha, \gamma \) and \( s \).

Proof: In the previous sub-section, I proved that conditions (34), (39) and (45) are never satisfied and, therefore, suppliers always prefer to merge to act independently. Regarding the choice of technology by downstream firms, it can be seen from condition (46) and (49) that the incentives to choose particular technology vary with \( \alpha, \gamma \) and \( s \). Concluding, Lemma 1 and Lemma 2 can be summarized in the following proposition:

**Proposition 1** In equilibrium, suppliers choose to merge. The technology choice of downstream firms depends on the size of the market, the degree of product differentiation and the technology cost in the following way:

- \( (D,D) \) is an equilibrium if \( \alpha < \sqrt{\frac{s}{F_{DD}(\gamma)}} \),
- \( (F,F) \) is an equilibrium if \( \alpha \geq \sqrt{\frac{s}{F_{FF}(\gamma)}} \),
- and \( (F,D) \) or \( (D,F) \) arise in equilibrium when \( \sqrt{\frac{s}{F_{DD}(\gamma)}} \leq \alpha < \sqrt{\frac{s}{F_{FF}(\gamma)}} \).

Figure 2 illustrates equilibrium regions for all technology choices in \( \alpha \) and \( \gamma \) for a value of \( s = 0.5 \). For illustrative purposes, I include equilibrium outcomes that would emerge under the competitive structure of the upstream industry. Let us consider first the sub-game with two independent suppliers. The region below the \( (D,D)^i \) curve includes the technology state in which both downstream firms remain with dedicated technology. The surface above the \( (F,F)^i \) curve shows the technology state in which both firms switch to flexible technology. The area between \( (D,D)^i \) and \( (F,F)^i \) includes mixed technology choices. It can be seen that, if there were two suppliers, all possible combinations of technology usage patterns could emerge. For example,

---

2 See Kim et al. (1992) and Gupta (1993), that in the corrected Röller and Tombak (1990) model mixed equilibria cannot emerge.

3 To check the robustness of the results, the figure was plotted for various parameters of \( s \). The results are not sensitive to the changes of parameter values.
when products X and Y are complements mixed outcomes would exist for $\gamma \in [-1, -0.41]$. That is, one firm would invest into FPT whereas the other one would remain with specialized equipment. Similarly, mixed outcomes could emerge for substitutable products. Then, both companies would invest into flexible production technologies only when market was sufficiently large. Otherwise either one or both firms would remain with specialized equipment. For $\gamma \in [-0.41, 0]$ both equilibria could exist, i.e. (D,D) and (F,F).

Investment decision look slightly differently when there is a multi-product monopolistic supplier. Similar as above, the region below the (D,D)$^m$ curve includes the technology state in which both downstream firms remain with dedicated technology. The surface above the (F,F)$^m$ curve shows the technology state in which both firms use flexible technology. The area between (D,D)$^m$ and (F,F)$^m$ includes mixed technology outcomes. Again, we can observe that all technology choices can emerge in equilibrium. For example, mixed equilibria exist only when products are complements. For substitutable products only (F,F) or (D,D) emerge in equilibrium. Furthermore, for $\gamma \in [0, 1]$ two equilibria exist and companies face a coordination problem for some values of $\alpha$.

As it emerges from the above analysis, it is quite striking how the degree of product differentiation influences the decision to adopt flexible technologies under different structures of the
upstream industry. Compared to a situation when there are two independent suppliers, when manufacturers produce complementary products they are more likely to adopt F technology in small markets under monopolistic upstream market structure. This is, however, reversed when products are substitutes. Then FPT would be adopted in much smaller markets if supplier acted independently.

In conclusion, taking into account all possible outcomes, in equilibrium there is a monopolistic supplier of both products. The final outcome regarding the choice of production is less obvious and depends on the values of $\gamma$, $\alpha$ and $s$, i.e. it is a function of market size, the degree of product differentiation and the cost difference between dedicated and flexible production technology.

## 5 Welfare implications

### 5.1 Consumer surplus

Considering that there are linear demand functions for product X and Y, consumer surplus is given by the difference between consumers’ utility and the total expenses for purchased goods. Thus, the combined consumer surplus for a given technology state $j$ is given by:

$$CS_j^k = \frac{1}{2}((X_j^k)^2 + (Y_j^k)^2) + \gamma X_j^k Y_j^k \quad (55)$$

where $X_j^k$ and $Y_j^k$ are the equilibrium total quantities of product X and Y under upstream market structure $k = I, M$ and technology state $j$. Equilibrium values of consumer surplus are given in Table 2.

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<th>Table 2: Consumer surplus</th>
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<td><strong>Independent suppliers</strong></td>
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<td>(D,D)</td>
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<td>Mixed equilibrium</td>
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<td>(F,F)</td>
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An analysis of the consumer surplus equilibrium values leads to the following lemma:

**Lemma 3:** If $CS_j^k$ is the total consumer surplus in technology state $j$ under upstream market structure $k$, then:

a) $CS_4^I \geq CS_2^I = CS_3^I \geq CS_1^I$;

b) $CS_4^M \geq CS_2^M = CS_3^M \geq CS_1^M$;

c) $CS_4^I \geq CS_4^M$ if $\gamma \in [0, 1]$;

d) $CS_4^I \leq CS_4^M$ if $\gamma \in [-1, 0]$.

Proof. See appendix.

Results (a) and (b) imply that consumer surplus is always maximized when downstream firms extend their product range. In other words, consumers benefit from an increased competition that results from firms’ decisions to serve both parts of the market. If there were two independent input suppliers, the introduction of flexible production systems would introduce intra-brand competition and intensify inter-brand rivalry as well. When the upstream industry is monopolized the main trigger of consumer welfare increase is the intensified competition in the final product market. Consequently, the structure of the upstream industry does not have any effect on consumer surplus when only the technology states are considered. It should be noted that this is true for all types of products.

Although intuitively straightforward, this finding is slightly different from the result in Röller et al. (1990) and Gupta (1998).\(^4\) They show that consumer surplus is maximized in (F,F) technology state only for $0 < \gamma \leq 0.80$. In other words, the use of FPT decreases consumer surplus when product X and Y are close substitutes. Under the current setting, however, consumer surplus is the highest when both producers use FPT over the entire range of $\gamma$. Consequently, I show that it is always desirable to adopt FPT from the consumers’ point of view. This is due to the fact that this type of technologies enable firms to cross the boundaries of the originally separated parts of the market, which in turn leads to more competition among firm that were previously isolated from each other and thereby lower prices. This result holds even when the upstream industry is monopolized as well.

\(^4\)See Gupta (1993) that (F, F) equilibrium is the most optimal from consumers’ point of view, except when products are strong substitutes.
This finding is different from the conclusion made by Norman and Thisse (1999). According to them, consumers might not benefit from tougher competition. Within the current setting, however, one can conclude that consumers always get the benefit of an increased competition between manufacturers. The source of this discrepancy is the difference in model setting and in the scope of analysis. In particular, they analyze the impact of the introduction of flexible technologies on entry, an issue not covered here, and find that this type of manufacturing deters entry. This, in turn, reduces the pro-competitive effect of flexibility.

In conclusion, the results regarding the use of technology can be summarized in the following proposition:

**Proposition 2** The use of FPT always maximizes consumer surplus.

Proof: The proof of this proposition follows directly from the results described above and summarized in Lemma 3.

Considering the effect of the upstream industry structure on consumer surplus, the above discussion suggests that consumer surplus should be maximized under competitive conditions. However, this depends on the type of products. Results (c) and (d) imply that, on the one hand, when products are substitutes, a competitive market structure of the input market would lead to the maximization of the consumer surplus. On the other hand, however, when products are complements consumer surplus is maximized when there is a single supplier of both inputs. This indicates that upstream merger might be desirable from the consumers’ point of view when products are complements. The source of this positive effect of the monopolization of the input industry is that, when products are complements, monopolistic input price is always lower than the price set by independent suppliers. To see this, one needs to consider the difference between input prices under both upstream market structures. For the (D,D) technology state and for the (F,F) technology state the following conditions are always fulfilled for complementary products: $w_1^{X_1} - \frac{\alpha}{2} \geq 0 \iff \frac{\alpha \gamma}{2(\gamma-4)} \geq 0$ and $w_4^{Y_1} - \frac{\alpha}{2} \geq 0 \iff \frac{\alpha \gamma}{2(\gamma-2)} \geq 0$. In a mixed case, this effect exists only for the input for which there is a demand from two suppliers, i.e. in the (F,D) technology state $w_2^{X_1} - \frac{\alpha}{2} \leq 0 \iff \frac{\alpha(32+8\gamma+3\gamma^2-2\gamma^3)}{2(\gamma^2-16)} \leq 0$ and, as above, $w_2^{Y_1} - \frac{\alpha}{2} \geq 0 \iff \frac{3\alpha \gamma (2+\gamma)}{2(\gamma^2-16)} \geq 0$. 

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5.2 Producer surplus

Although it might be intuitively justified for downstream firms to extend their product range, the profitability of such a move needs to be verified in light of adverse impacts they might have on the competition in the final product market and in the input market. Thus, the following section analyses the payoffs to the upstream and downstream industries in different equilibria and, in addition, compares them with the outcomes under a competitive structure of the upstream industry. I first look at the payoffs of upstream firms and then turn to the surplus of downstream companies. The total profits of the upstream and downstream industry are presented in Appendix (see Table 1A and Table 2A).

5.2.1 Upstream firms

The introduction of FPT by downstream firms should increase the demand for intermediary products. However, when downstream firm switches from a dedicated to a flexible technology it begins to procure both inputs and, as a result, imposes some externality on its original supplier. The type of this externality depends on the degree of product differentiation. For example, let us consider a situation in which only producer 1 adopts flexible technology and producer 2 remains with dedicated production system. On the one hand, when products are complements higher demand for input Y should increase the demand for input X. Thus, the decision of producer 1 to enter the other part of the market creates positive externality for supplier 1. On the other hand, however, when products are substitutes, the demand for input Y increases at the expense of input X. This, in turn, imposes negative externality on supplier 1 which increases in $\gamma$. To see this, it is enough to show that

\[ US_1^I - US_2^I \geq 0 \quad \text{and} \quad US_4^I - US_3^I \geq 0 \]  

(56)

and

\[ US_4^I - US_1^I \geq 0 \]  

(57)

where $US_j^k$ represents total payoffs of suppliers given technology state $j$ and sub-game equilibrium $k = I, M$ where $I$ stands for independent suppliers and $M$ for a single supplier. Condition
(56) is given by

\[
\frac{\alpha^2(128 - 256\gamma - 40\gamma^2 + 192\gamma^3 - 10\gamma^4 - 6\gamma^5 + 3\gamma^6 - 11\gamma^7)}{3(\gamma - 2)^2(1 + \gamma)(16 - 7\gamma^2)^2} \geq 0 \tag{58}
\]

and condition (57) can be expressed as

\[
\frac{8\alpha^2(\gamma^4 - 4\gamma^3 + 6\gamma^2 - 10\gamma + 4)}{3(\gamma + 1)(\gamma + 2)(\gamma - 4)^2(\gamma - 2)^2} \geq 0. \tag{59}
\]

The effect of the degree of product differentiation on the payoffs of upstream firms can be expressed graphically. Figure 4 depicts suppliers’ total surplus in all technology states as a function of \(\gamma\). The surplus of upstream firms is at first the highest in \((F,F)\) technology state and decreases in \(\gamma\). When products become close substitutes, however, the negative externality posed by downstream firms increases the rivalry in the input market to such extent, that the payoff in \((F,F)\) state becomes smaller than in \((D,D)\) state. Consequently, (58) and (59) hold only for some values of \(\gamma\). In this particular case, i.e. when \(\alpha = 10\), (58) holds for \(\gamma \in [-1, 0.6]\) and (59) is true for \(\gamma \in [-1, 0.51]\). Then, the surplus of upstream firms is maximized when both downstream firms use D technology. In other words, suppliers prefer to maintain exclusive relations with producers.

Figure 3: Suppliers’ surplus (independent suppliers, \(\alpha=10\))

![Figure 3: Suppliers’ surplus (independent suppliers, \(\alpha=10\))](image)

Figure 4:
Let us now consider the case when there is one multi-product supplier. From the previous analysis we know that the monopolistic supplier is always able to set prices that maximize monopolist’s profit in all technology cases, i.e. \( w^{XM}_j = w^{YM}_j = \frac{a}{2} \). The possibility to set monopolistic prices for both inputs offsets the negative effects of increased intra-brand competition. Consequently, by selling both inputs to both downstream firms, a single supplier always benefits from a higher demand stimulated by the use of FPT. This leads to the following proposition:

**Proposition 3** The profit of a multi-product monopolistic supplier is the highest when both manufacturers use FPT.

Proof. See appendix.

Figure 5: Suppliers’ surplus (a single supplier, \( \alpha=10 \))

Because the choice of technology made by downstream firms determines the final quantities of both products and, as a result, the demand for intermediate inputs, it has also an impact on the seller’s final payoff. The total payoff of the upstream industry is however less sensitive to the degree of product differentiation and the technology choice when there is only a monopolistic supplier of both inputs, compared to the structure with two independent suppliers. Again, this can be illustrated graphically. Figure 5 shows that, although product substitutability is negatively related to the total payoff, a multi-product monopoly always maximizes its profit when each manufacturer is active in both parts of the market.
The above findings differ from the result obtained by Lin (1990). According to him, upstream firms prefer to choose exclusive dealing relations with downstream firms, which allow them to earn greater profits. In the current setting, however, provided that products are either complements or moderate substitutes, independent suppliers would benefit from downstream competition that has a direct impact on the quantities ordered by downstream firms. The total payoff of a monopolistic multiproduct supplier is always maximized when both manufacturers serve the two parts of the market.

5.2.2 Downstream firms

Intuitively, downstream companies forsake some profits when they are active in only one part of the market. By using D technology, each firm excludes itself from the other part of the market and reduces its final payoff. However, according to Röller et al. (1990), when both firms invest into flexible production technologies they reduce their profits due to increased rivalry in both parts of the market. Only one firm can benefit from producing both goods provided that the other firm supplies only one part of the market. Consequently, because producers collectively forsake profits when both of them adopt F technology, the technology choice takes a form of a Prisoners’ Dilemma. Thus, the answer to the question of what technology downstream firm should choose to maximize its payoff might not be straightforward. Therefore, proceeding as before, in the following section I analyze the payoffs to the downstream manufacturers in all technology states and under both structures of the upstream industry. The main question here is which technology maximizes producers’ profits.

First, I would like to discuss this question under the assumption that upstream industry has a competitive structure. Downstream firms might be inclined to adopt FPT in order to increase the product range. This, however, might have adverse impact on their final payoffs. Similarly as in the case of the input market, the competition and profits in the final product market will be influenced by the type of dependency between both products. To see this, let us turn to a graphical representation of producers’ profits in all technology states. Figure 6 illustrates producers’ payoffs for \( \alpha = 10 \) and \( s = 0.5 \). It can be seen that producers’ payoff is maximized in \((F,F)\) technology state for a very limited range of \( \gamma \). For most of the values of \( \gamma \) producers are better off when they remain with specialized equipment or when only one of them invests in
FPT. Thus, an analysis of the manufacturers’ surplus leads to the following Lemma:

**Lemma 4:** If $PS^I_j = \pi^{PI}_{1,j} + \pi^{PI}_{2,j}$ is the total profit of the downstream industry in technology state $j$, given a competitive structure of the upstream industry $I$, and when $\alpha = 10$ and $s = 0.5$ then:

a) $PS^I_1 \geq PS^I_2 = PS^I_3$ if $\gamma \in [-0.65, 0.24]$;

b) $PS^I_1 \geq PS^I_4$ if $\gamma \in [-0.75, 0.32]$;

c) $PS^I_1 \geq PS^I_2 = PS^I_3$ if $\gamma \in [0.54, 0.95]$;

Proof. See appendix.

Consequently, as it emerges from the above discussion, it is not always optimal from producers’ stand point when both firms invest in flexible technologies.

Proceeding as above, let us now turn to the equilibrium outcomes that emerge under a monopolized upstream industry. Figure 7 illustrates the surplus of downstream firms for $\alpha = 10$ and $s = 0.5$. It can be seen that the negative effect of FPT on producers’ profits is even stronger when inputs are procured from a multi-product monopolist. Nearly over the entire range of $\gamma$ downstream firms maximize the total industry profit when they remain active in separated markets. An analysis of producers’ payoffs leads to the following following lemma:

**Lemma 5:** If $PS^M_j = \pi^{PM}_{1,j} + \pi^{PM}_{2,j}$ is the total profit of the downstream industry in technology state $j$, given a monopolistic structure of the upstream industry $M$, and when $\alpha = 10$ and $s = 0.5$
then:

a) \( PS_1^M \geq PS_2^M = PS_3^M \) if \( \gamma \in [-0.39, 1] \);

b) \( PS_1^M \geq PS_4^M \) if \( \gamma \in [-0.55, 1] \);

c) \( PS_4^M \leq PS_2^M = PS_3^M \) if \( \gamma \in [-1, 1] \).

Proof: See appendix.

Lemma 5 implies that the payoffs of downstream firms decrease in \( \gamma \) and as the level of product complementarity decreases or products become stronger substitutes, companies forsake profits when they use FPT. This counters the intuition that firms can increase their payoffs by diversification. This leads to the following proposition:

**Proposition 4** In equilibrium outcomes, the cumulative profit of downstream firms depends on the market size and the degree of product differentiation. The use of FPT by both producers never maximizes the total payoff of the downstream industry.

Proof of the above proposition follows from the proof of Lemma 5 (see Appendix).

Figure 7: Producers’ surplus (a single supplier, \( \alpha=10, s=0.5 \))

To a large extent these results are consistent with those by Röller et al. (1990). In particular, they confirm that, in most of the cases, an extension of the product range is detrimental to the producers’ total payoff. Although the ability to produce many products leads to a significant
increase in the profits of the producer switching from DPT to FPT, it happens at the expense of the other producer. Once both producers adopt FPT, the result is an immediate increase of intra-brand competition and the introduction of inter-brand rivalry. Eventually, both firms are worse off when they parallel choose flexible production technology. Obviously, this leads to a Prisoners’ Dilemma.

5.3 Total welfare

This section discusses the effects of technology choice by downstream firms and consolidation of the upstream industry on total welfare. As usual, total welfare includes consumer surplus and total profits of both industries. Table 3 gives expressions for total welfare in all technology states and under both structures of the upstream industry.

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</tbody>
</table>

Let us first consider the effect of technology choice by producers when there are two independent suppliers. Intuitively, FPT has a competition-increasing effect. Consequently, total welfare should be maximized when downstream firms choose F technology and supply both products. Then, from the social point of view, total welfare should be increased as the supply in the final market rises and prices decrease. To see this, let us turn to a graphical illustration. Figure 8 shows that the use of F technology by both firms is socially desirable indeed, but only to some extent. Surprisingly, the presence of both firms in both parts of the market is desirable from the social welfare perspective only if \(\gamma \leq 0.95\). When products are strong substitutes, asymmetric solution, in which one firm uses D technology and the other F technology, is preferred. In other words, if products are strong substitutes investing in FPT by both firms might be socially not
efficient. This coincides with the previous conclusion regarding the possible technology outcomes under a competitive market structure. As illustrated in Figure 2, mixed equilibria could emerge if there were two independent suppliers in the region of $\gamma$ close to 1. Thus, theoretically, there would be no concern of overinvestment in FPT. However, in such a case, downstream firms face a Prisoners’ Dilemma, because, when both firms invest into flexible production technologies they reduce their profits due to increased rivalry in both parts of the market. Only one firm can benefit from producing both goods provided that the other firm supplies only one part of the market.

Figure 8: Total surplus (independent suppliers, $\alpha=10$, $\sigma=0.5$)

Let us now turn to a sub-game with a multi-product monopolistic supplier. As illustrated in figure (9), the negative effect of FPT on producers’ payoff is even stronger when there is a monopolistic supplier of both inputs. Consequently, this indicates that when products are substitutable, investment into FPT might not be desired from the social welfare standpoint. The benefits of increased competition between closely substitutable products do not justify large investments into new production technologies. This result holds for both types of the upstream industry structure. Consequently, the following can be stated:
**Proposition 5** When final products are sufficiently and $\alpha=10, s=0.5$, the investment in FPT by both companies maximizes total welfare.

Proof. See appendix.

![Figure 9: Total surplus (a single supplier, $\alpha=10, s=0.5$)](image)

5.4 Total welfare and equilibrium technology choice

Taking stock of the implications of the technology choice and the structure of the upstream industry on the total surplus, it is worthwhile to ask to what extent equilibrium technology choice of downstream firms overlap with what is optimal from the total welfare point of view. Figure (10) illustrates all technology equilibria that emerge in a sub-game with merged suppliers together with the socially optimal outcomes in $\alpha$ and $\gamma$ for a value of $s = 0.5$.\(^5\) Regarding the equilibrium technology choice, the region below the red $(D,D)^m$ curve includes the technology state in which both downstream firms remain with dedicated technology. The surface above the blue $(F,F)^m$ curve shows the technology state in which both firms switch to flexible technology. The area between $(D,D)^m$ and $(F,F)^m$ includes mixed technology choices.

\(^5\)To check the robustness of the results, the figure was plotted for various parameters of $s$. The results are not sensitive to the changes of parameter values.
Regarding the socially optimal outcomes, the following curves illustrate the differences between the total welfare values under various technology states. In particular, the green line indicates the borderline for the condition that total welfare in a mixed outcome is greater than total welfare in (D,D) state, i.e. $W_{mixed}^m - W_1^m \geq 0$. Similarly, the violet and orange lines show for what values of $\alpha$ and $\gamma$ total welfare in (F,F) state is greater than in the (D,D) and the mixed state respectively, i.e. $W_4^m - W_1^m \geq 0$ and $W_4^m - W_{Mixed}^m \geq 0$.

Figure 10: Total surplus and technology equilibria (a single supplier, s=0.5)

An analysis of the equilibrium and the socially optimal welfare regions leads to the conclusion that, for some range of $\alpha$, the choice of manufacturers does not lead to a socially efficient outcome. In general, producers invest in flexible production technologies only when market is sufficiently large. For example, when products are complements, the socially optimal move from the (D,D) state to the mixed state lies above the green curve. In contrast, manufacturers choose the mixed equilibrium for the region of $\alpha$ and $\gamma$ that lies between the red and blue curves. Similarly, the equilibrium move from a mixed case to the (F,F) outcome lies above the blue curve, although the socially optimal choice of FPT by both manufacturers would be for much smaller value of $\alpha$, i.e. above the orange curve. A similar situation emerges when products are substitutes.\footnote{When there is a single multiproduct supplier of both inputs, mixed technology equilibria do not exist when...}
can be seen that firms invest into FPT for much larger values of $\alpha$ (above the blue line) than socially optimal (above the violet curve). In short, the socially inefficient outcomes emerge for values of $\alpha$ that lie:

- between the green and red lines when products are complements, and
- between the violet and blue lines for all types of products.

An additional analysis of the manufacturers’ choice and the socially optimal outcome under a competitive structure of the upstream industry revealed that the type of the upstream industry structure does not move producers to an outcome, which would be desirable from the total welfare point of view. Consequently, for some intermediate size of market there is a divergence between the manufacturers’ incentives and socially optimal outcome. The above results can be summarized in the following proposition:

**Proposition 6** There is a set of $\alpha$ for which the manufacturers’ choice does not lead to a socially efficient outcome with respect to the technology choice. In general, manufacturers choose to invest in FPT in much larger markets than it would be desirable from the social point of view.

Proof. See appendix.

6 Conclusions

The starting point of the discussion presented in this paper was the argument of Milgrom and Roberts (1990) that the adoption of FPT accompanied by complementary changes in firms’ strategy and organization change modern manufacturing and service industries. The current work aimed at illustrating these transformations and analyzing manufacturers’ decisions regarding the choice of production technologies in the context of vertical relations. By using a model of two supply chains, I show that the effects of the adoption of flexible production equipment on firms’ profits and welfare is far from straightforward. First, although profitable for an individual producer, the adoption of technologies increasing product variety across the entire industry
erodes producers’ payoffs. Although in some cases a mixed technology state in which one firm uses flexible and the other dedicated technology can be justified, downstream firms are better off when they remain with specialized equipment and produce only one product. In particular, when products are close substitutes, any benefits stemming from an increased product variety do not justify investments into flexible technology by all firms in the industry. As a result, producers end up in a Prisoners’ Dilemma. Second, the introduction of flexible technologies on the payoff of the upstream industry is not straightforward as well. Contrary to some previous findings, I show that, regardless of the upstream market structure, the upstream industry is always better off when both downstream firms use FPT. In particular, a multiproduct monopolistic suppliers benefits from an increased demand for both inputs. However, these benefits diminish as products become close substitutes. Third, regarding the structure of the upstream industry, suppliers maximize their profits when they merge in all cases, i.e. irrespective of the degree of product differentiation. Consequently, along consumers, a multiproduct monopolistic supplier always benefits from manufacturers’ move towards new technologies and greater product variety. Finally, the decisions made by producers with respect to the technology choice are not always efficient from the social welfare point of view. In particular, firms invest into new technologies only when markets are sufficiently large.

One of the limitations of the current work is that the design of the model influences the results. Consequently, it might be worthwhile to check their robustness under different frameworks. One way to extend the current structure would be an introduction of a bargaining game between upstream and downstream firms over two-part contracts, instead of a price maximization and Cournot competition. Another way is to relax the assumption with respect to the number of firms. Nevertheless, despite its simple framework, the model can be applied to a number of realistic situations. Furthermore, the results obtained here seem to reflect anecdotal and empirical evidence on the technology-driven impacts on vertical relations and firms’ strategies regarding product variety as well.

Concluding, the impacts of value chain transformation and the diffusion of FPT might go far beyond the increased product variety offered by firms. Other changes that can be triggered by the technological transition might include intensified competition in product and input markets. Eventually, the changes might have adverse effect on firms that seek to escape competition and
increase profitability by the adoption of new technologies.

7 Appendix

Table 1A: Equilibrium expressions for all technology states (independent suppliers)

<table>
<thead>
<tr>
<th>Upstream</th>
<th>(D,D)</th>
<th>Mixed equilibrium (e.g. (F,D))(^7)</th>
<th>(F,F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w_{X*})</td>
<td>(\alpha(\gamma-2)/(\gamma-4))</td>
<td>(\alpha(\gamma^3-5\gamma^2-4\gamma-8)/(16-7\gamma^2))</td>
<td>(\alpha(\gamma-1)/(\gamma-2))</td>
</tr>
<tr>
<td>(w_{Y*})</td>
<td>(\alpha(\gamma-2)/(\gamma-4))</td>
<td>(\alpha(5\gamma^2+3\gamma-8)/(7\gamma^2-16))</td>
<td>(\alpha(\gamma-1)/(\gamma-2))</td>
</tr>
<tr>
<td>(\pi_{1,j})</td>
<td>(2\alpha(2-\gamma)/(\gamma+2)(\gamma-4)^2)</td>
<td>(-\alpha(\gamma^5-9\gamma^4+8\gamma^3+64\gamma^2-64)/(2(\gamma+1)(7\gamma^2-16)^2))</td>
<td>(2\alpha^2/(3(\gamma-2)^2(\gamma+1)))</td>
</tr>
<tr>
<td>(\pi_{2,j})</td>
<td>(2\alpha(2-\gamma)/(\gamma+2)(\gamma-4)^2)</td>
<td>(\alpha(25\gamma^5+65\gamma^4-116\gamma^3-284\gamma^2+64\gamma+256)/(6(\gamma+1)(7\gamma^2-16)^2))</td>
<td>(2\alpha^2/(3(\gamma-2)^2(\gamma+1)))</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Downstream</th>
<th>(x_{1,j}, x_{2,j})</th>
<th>(y_{1,j}, y_{2,j})</th>
<th>(p_{X*})</th>
<th>(p_{Y*})</th>
<th>(\pi_{1,j})</th>
<th>(\pi_{2,j})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{2\alpha}{(\gamma+2)(4-\gamma)})</td>
<td>(0)</td>
<td>(\frac{\alpha(\gamma^3-4\gamma^2-8)}{2(\gamma+1)(7\gamma^2-16)})</td>
<td>(0)</td>
<td>(\frac{\alpha}{3(2-\gamma)(\gamma+1)})</td>
<td>(\frac{\alpha}{3(2-\gamma)(\gamma+1)})</td>
<td></td>
</tr>
<tr>
<td>(\frac{2\alpha}{(\gamma+2)(4-\gamma)})</td>
<td>(\frac{\alpha(\gamma^3-10\gamma^2+2\gamma-16)}{6(16-7\gamma^2)})</td>
<td>(\frac{\alpha(2\gamma^2-3\gamma-8)}{3(7\gamma^2-16)})</td>
<td>(\frac{\alpha(\gamma-4)}{3(\gamma-2)})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\frac{\alpha(\gamma^2-6)}{(\gamma-4)(\gamma+2)})</td>
<td>(\frac{\alpha(5\gamma^2-10\gamma^2+2\gamma+72)}{6(16-7\gamma^2)})</td>
<td>(\frac{\alpha(\gamma^2+4\gamma^2-32)}{3(7\gamma^2-16)})</td>
<td>(\frac{\alpha(\gamma-4)}{3(\gamma-2)})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\frac{\alpha(\gamma^2-6)}{(\gamma-4)(\gamma+2)})</td>
<td>(\frac{\alpha(17\gamma^2+6\gamma-32)}{3(7\gamma^2-16)})</td>
<td>(\frac{\alpha(\gamma^2+4\gamma^2-32)}{3(7\gamma^2-16)})</td>
<td>(\frac{\alpha(\gamma-4)}{3(\gamma-2)})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\frac{\alpha(\gamma^2-6)}{(\gamma-4)(\gamma+2)})</td>
<td>(\frac{\alpha(\gamma^2+4\gamma^2-32)}{3(7\gamma^2-16)})</td>
<td>(\frac{\alpha(\gamma^2+4\gamma^2-32)}{3(7\gamma^2-16)})</td>
<td>(\frac{\alpha(\gamma-4)}{3(\gamma-2)})</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^7\)In (D,F) equilibrium: \(w_{X*}^{X*} = w_2^{Y*}, w_3^{Y*} = w_2^{X*}, \pi_{13}^{U*} = \pi_{22}^{U*}, \pi_{23}^{U*} = \pi_{12}^{U*}\), in the upstream industry and \(x_{13}^{Y*} = y_{23}^{Y*} = y_{12}^{Y*} = y_{22}^{Y*} = x_{12}^{X*}, p_{3}^{Y*} = p_{2}^{Y*}, p_{3}^{Y*} = p_{2}^{X*}, \pi_{13}^{D*} = \pi_{22}^{D*}, \pi_{23}^{D*} = \pi_{12}^{D*}\) in the downstream industry.
Table 2A: Equilibrium expressions for all technology states (a single supplier)

<table>
<thead>
<tr>
<th>Upstream</th>
<th>(D,D)</th>
<th>Mixed equilibrium (e.g. (F,D))</th>
<th>(F,F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{jX}^M$, $w_{jY}^M$</td>
<td>$\frac{\alpha}{2}$</td>
<td>$\frac{\alpha}{2}$</td>
<td>$\frac{\alpha}{2}$</td>
</tr>
<tr>
<td>$\pi_{jX}^M$</td>
<td>$\frac{\alpha^2}{2(\gamma+2)}$</td>
<td>$\frac{\alpha^2(\gamma+7)}{24(\gamma+1)}$</td>
<td>$\frac{\alpha^2}{3(\gamma+1)}$</td>
</tr>
<tr>
<td>Downstream</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_{1,jX}^M$, $x_{2,jX}^M$</td>
<td>$\frac{2\alpha}{2(\gamma+2)}$, 0</td>
<td>$\frac{\alpha}{4(\gamma+1)}$, 0</td>
<td>$\frac{\alpha}{6(\gamma+1)}$, $\frac{\alpha}{6(\gamma+1)}$</td>
</tr>
<tr>
<td>$y_{1,jX}^M$, $y_{2,jX}^M$</td>
<td>0, $\frac{2\alpha}{2(\gamma+2)}$</td>
<td>$\frac{\alpha(2-\gamma)}{12(\gamma+1)}$, $\frac{\alpha}{6}$</td>
<td>$\frac{\alpha}{6(\gamma+1)}$, $\frac{\alpha}{6(\gamma+1)}$</td>
</tr>
<tr>
<td>$p_{jX}^M$</td>
<td>$\frac{\alpha(\gamma+3)}{2(\gamma+2)}$</td>
<td>$\frac{\alpha(9-\gamma)}{12}$</td>
<td>$\frac{2\alpha}{3}$</td>
</tr>
<tr>
<td>$p_{jY}^M$</td>
<td>$\frac{\alpha(\gamma+3)}{2(\gamma+2)}$</td>
<td>$\frac{2\alpha}{3}$</td>
<td>$\frac{2\alpha}{3}$</td>
</tr>
<tr>
<td>$\pi_{1,jX}^M$</td>
<td>$\frac{\alpha^2}{4(\gamma+2)^2} - 1$</td>
<td>$\frac{\alpha^2(13-5\gamma)}{144(\gamma+1)^2} - 1 - s$</td>
<td>$\frac{\alpha^2}{18(\gamma+1)} - 1 - s$</td>
</tr>
<tr>
<td>$\pi_{2,jX}^M$</td>
<td>$\frac{\alpha^2}{4(\gamma+2)^2} - 1$</td>
<td>$\frac{\alpha^2}{36} - 1$</td>
<td>$\frac{\alpha^2}{18(\gamma+1)} - 1 - s$</td>
</tr>
</tbody>
</table>

Proof of Lemma 3:

a) Providing that suppliers are independent, consumer surplus is maximized in (F,F) technology state when the following conditions are satisfied. First, consumer surplus in mixed technology state must be lower than consumer surplus in (F,F) technology outcome. This is true when

$$CS_4^I - CS_2^I \geq 0 \text{ and } CS_4^I - CS_3^I \geq 0$$

(60)

Second, consumer surplus in mixed equilibria must be higher than consumer surplus in (D,D) technology state. This is true when

$$CS_4^I - CS_1^I \geq 0 \text{ and } CS_3^I - CS_1^I \geq 0$$

(61)

Condition (60) can be expressed in the following way:

$$\frac{\alpha^2(1792 - 768\gamma - 896\gamma^2 + 32\gamma^3 - 596\gamma^4 + 224\gamma^5 + 267\gamma^6 - 55\gamma^7)}{72(\gamma + 1)(\gamma - 2)^2(7\gamma^2 - 16)^2} \geq 0$$

(62)

Similarly, condition (61) can be written as

$$\frac{\alpha^2(28672 + 18432\gamma + 17664\gamma^2 + 55040\gamma^3 + 23712\gamma^4 - 16128\gamma^5 - 9436\gamma^6 - 1104\gamma^7 - 267\gamma^8 + 55\gamma^9)}{72(\gamma + 1)(\gamma - 4)^2(7\gamma^2 - 16)^2} \geq 0$$

(63)

8In (D,F) equilibrium: $w_{3X}^M = w_{2X}^M$, $w_{3Y}^M = w_{2Y}^M$, $\pi_{3X}^M = \pi_{2X}^M$, in the upstream industry and $x_{13}^M = y_{23}^M$, $x_{23}^M = y_{13}^M$, $x_{22}^M = x_{12}^M, p_{3X}^M = p_{2X}^M, p_{3Y}^M = p_{2Y}^M, \pi_{13}^{DM} = \pi_{22}^{DM}, \pi_{23}^{DM} = \pi_{12}^{DM}$ in the downstream industry.
It can be shown graphically that both conditions are fulfilled for the relevant range of $\gamma \in [-1, 1]$ and $\alpha > 0$.

Q.E.D.

b) Proceeding as above, it is enough to show that two conditions are satisfied. First, consumer surplus in mixed equilibria must be lower than consumer surplus in (F,F) equilibrium. This is true when

$$ CS_4^M - CS_2^M \geq 0 \text{ and } CS_4^M - CS_3^M \geq 0 $$

(64)

Second, consumer surplus in mixed equilibria must be higher than consumer surplus in (D,D) equilibrium. This is true when

$$ CS_2^M - CS_1^M \geq 0 \text{ and } CS_3^M - CS_1^M \geq 0 $$

(65)

Condition (64) can be expressed as:

$$ \frac{7\alpha^2(1 - \gamma)}{288(\gamma + 1)} \geq 0 $$

(66)

Similarly, condition (65) can be written as

$$ \frac{\alpha^2(7\gamma^3 - 19\gamma^2 - 16\gamma + 28)}{288(\gamma + 1)(\gamma + 2)^2} \geq 0 $$

(67)

Both expressions, (66) and (67), are fulfilled for all values of $\gamma$.

Q.E.D.

c) and d) These results hold when the difference between consumer surplus in (F,F) with independent suppliers and consumer surplus in the same technology state but with a monopolistic input market is positive (negative) if products are substitutes (complements). This holds when

$$ CS_4^I - CS_4^M \geq 0 $$

(68)

or

$$ \frac{\alpha^2\gamma(4 - \gamma)}{9(\gamma - 2)^2(1 + \gamma)} \geq 0 $$

(69)

for $0 \leq \gamma \leq 1$ and

$$ CS_4^I - CS_4^M < 0 $$

(70)

or

$$ \frac{\alpha^2\gamma(4 - \gamma)}{9(\gamma - 2)^2(1 + \gamma)} < 0 $$

(71)
for $0 \leq \gamma \leq 1$. It is straightforward to see that the sign of both inequalities depends on the sign of $\gamma$ and that both conditions are satisfied for relevant types of products.

Q.E.D.

**Proof of Lemma 4:**

a) To prove this result, it needs be to shown that

$$PS^I_1 - PS^I_2 \geq 0 \text{ and } PS^I_1 - PS^I_3 \geq 0$$

(72)

hold for some value of $\gamma$. $PS^I_j$ represents total payoffs of downstream producers given technology state $j$ and upstream market structure $I$. Condition (72) is equivalent to

$$s - \frac{\alpha^2(23\gamma^9 - 75\gamma^8 - 696\gamma^7 + 1564\gamma^6 - 5472\gamma^5 - 22272\gamma^4 + 14848\gamma^3 + 56064\gamma^2 + 18432\gamma - 4096)}{36(\gamma + 1)(\gamma - 4)^2(\gamma + 2)^2(7\gamma^2 - 16)^2} \geq 0$$

(73)

and it can be shown graphically that it holds for some value of $\alpha$ and $s$. For example, if we set $\alpha = 10$ and $s = 0.5$ then $PS^I_1 \geq PS^I_2 = PS^I_3$ if $\gamma \in [-0.65, 0.24].$

Q.E.D.

b) To prove this result, it needs be to shown that

$$PS^I_1 - PS^I_4 \geq 0.$$

(74)

This can be expressed as

$$2s - \frac{4\alpha^2(\gamma^4 - 22\gamma^3 + 42\gamma^2 + 32\gamma - 8)}{9(\gamma + 1)(\gamma - 2)^2(\gamma^2 - 2\gamma - 8)^2} \geq 0$$

(75)

Again, it can be shown that if we set $\alpha = 10$ and $s = 0.5$, (75) holds for $\gamma \in [-0.75, 0.32].$

Q.E.D.

c) To prove this result, it needs be to shown that

$$PS^I_4 - PS^I_2 \geq 0 \text{ and } PS^I_4 - PS^I_3 \geq 0$$

(76)

hold for some $\gamma$. This can be expressed as

$$- \frac{\alpha^2(23\gamma^7 - 75\gamma^6 - 328\gamma^5 + 316\gamma^4 + 992\gamma^3 - 416\gamma^2 - 768\gamma + 256)}{36(1 + \gamma)(\gamma - 2)^2(7\gamma^2 - 16)^2} - s \geq 0$$

(77)

Again, by setting $\alpha = 10$ and $s = 0.5$ it can be shown graphically that it holds for $\gamma \in [0.54, 0.95].$

Q.E.D.
Proof of Lemma 5:

a) This result is true when

\[ PS_1^M - PS_2^M \geq 0 \text{ and } PS_1^M - PS_3^M \geq 0 \]  \hspace{1cm} (78)

hold for some values of \( \gamma \). Inequality (78) can be written as

\[ \frac{\alpha^2(\gamma^3 - 13\gamma^2 + 8\gamma + 4)}{144(1 + \gamma)(\gamma - 2)^2} + s \geq 0 \]  \hspace{1cm} (79)

This holds for positive \( \alpha \) and \( s \neq 0 \). For example, if \( \alpha = 10 \) and \( s = 0.5 \) the above expression is fulfilled when \( \gamma \in [-0.39, 1] \).

Q.E.D.

b) This result is true when

\[ PS_1^M - PS_4^M \geq 0 \]  \hspace{1cm} (80)

is true for some positive values of \( \gamma \). Condition (80) can be written as

\[ 2s - \frac{\alpha^2(2\gamma^2 - \gamma - 1)}{18(\gamma + 1)(\gamma + 2)^2} \geq 0 \]  \hspace{1cm} (81)

Again, it can be shown that for positive \( \alpha \) and \( s \neq 0 \), the above condition is fulfilled when \( \gamma \in [-0.55, 1] \).

Q.E.D.

c) This result is true when

\[ PS_4^M - PS_2^M \leq 0 \text{ and } PS_4^M - PS_3^M \leq 0 \]  \hspace{1cm} (82)

is true for some positive values of \( \gamma \). Condition (80) can be written as

\[ \frac{\alpha^2(\gamma - 1)}{144(\gamma + 1)} - s \leq 0 \]  \hspace{1cm} (83)

Again, it can be shown graphically that for positive \( \alpha \) and \( s \neq 0 \), the above condition is fulfilled for all relevant values of \( \gamma \).

Q.E.D.

Proof of Proposition 3. In order to prove this proposition, it is enough to show that

\[ US_4^M - US_2^M \geq 0 \text{ and } US_4^M - US_3^M \geq 0 \]  \hspace{1cm} (84)
and

\[ US^M_4 - US^M_1 \geq 0 \quad (85) \]

Inequality (84) is given by

\[ \frac{\alpha^2(1 - \gamma)}{24(1 + \gamma)} \geq 0 \quad (86) \]

and condition (85) can be written as

\[ \frac{\alpha^2(1 - \gamma)}{6(\gamma^2 + 3\gamma + 2)} \geq 0 \quad (87) \]

It is straightforward that both are positive for all values of \( \gamma \).

Q.E.D.

Proof of Proposition 5. In order to prove this proposition, it is necessary to show that for some values of \( \gamma \) close to 1 total welfare is not maximized when both producers are active in both markets. Let us first start with the case in which there are two suppliers. Then the following conditions must be met:

\[ W^I_4 - W^I_2 \geq 0 \quad \text{and} \quad W^I_4 - W^I_3 \geq 0 \quad (88) \]

and

\[ W^I_4 - W^I_1 \geq 0 \quad (89) \]

Where \( W^k_j \) represents total welfare given technology state \( j \) and sub-game equilibrium \( k = I, M \) where \( I \) stands for independent suppliers and \( M \) for a single supplier. Condition (88) can is given by

\[ \frac{\alpha^2(4352 - 5376\gamma - 1024\gamma^2 + 2656\gamma^3 - 1468\gamma^4 + 736\gamma^5 + 489\gamma^6 - 365\gamma^7)}{72(1 + \gamma)(32 - 16\gamma - 14\gamma^2 + 7\gamma^3)^2} - s \geq 0 \quad (90) \]

and (89) can be expressed as

\[ \frac{4\alpha^2(2168 - 2208\gamma - 978\gamma^2 + 238\gamma^3 - 273\gamma^4 + 246\gamma^5 - 67\gamma^6\gamma^7)}{9(\gamma - 4)^2(1 + \gamma)(\gamma^2 - 4)^2} - 2s \geq 0 \quad (91) \]

It can be shown graphically that for positive \( \alpha \) and \( s \neq 0 \), both conditions are fulfilled. For example, if \( \alpha = 10 \) and \( s = 0.5 \) condition (90) is satisfied for \( \gamma \in [-1, 0.95] \), i.e. except when products are strong substitutes. Condition (91) holds for all values of \( \gamma \).
Second, considering the case in which there is a multi-product monopolistic supplier, it is necessary to show that
\[ W_4^M - W_2^M \leq 0 \quad \text{and} \quad W_4^M - W_3^M \leq 0 \] (92)
and
\[ W_4^M - W_1^M \leq 0 \] (93)
hold for some values of \( \gamma \). Condition (92) can be expressed as
\[
\frac{17\alpha^2 (1 - \gamma)}{288(1 + \gamma)} - s \leq 0
\] (94)
Similarly, condition (93) is equivalent to
\[
\frac{\alpha^2 (17 - 10\gamma - 7\gamma^2)}{36(\gamma + 2)^2(1 + \gamma)} - 2s \leq 0
\] (95)
Proceeding as above, it can be illustrated that both expressions are fulfilled for positive \( \alpha \) and \( s \neq 0 \). For example, if \( \alpha = 10 \) and \( s = 0.5 \), (94) holds for \( \gamma \in [0.84, 1] \) and (95) holds when \( \gamma \in [0.78, 1] \).
Q.E.D.

**Proof of Proposition 6.** To prove this proposition, it is necessary to show that, for some values of \( \alpha \), the equilibrium regions of technology choice under a monopolized upstream market do not overlap with the socially efficient outcomes. Thus, let us start with complementary products and consider the case of the move from the (D,D) to the mixed technology state. It can be shown that the condition for a socially efficient adoption of FPT by one producer needs to satisfy the following inequality:
\[ W_2^M - W_1^M \geq 0 \quad \text{and} \quad W_3^M - W_1^M \geq 0, \] (96)
which is equivalent to
\[ \alpha \geq \sqrt{\frac{s}{f_{DD2Mix}^M(\gamma)}}, \] (97)
where
\[ f_{DD2Mix}^M(\gamma) = \frac{(68 - 80\gamma - 5\gamma^2 + 17\gamma^3)}{288(\gamma + 2)^2(1 + \gamma)}. \] (98)
From Proposition 1 in section 3, we know that there is a mixed equilibrium for complementary products when \( \sqrt{\frac{s}{f_{DD}^M(\gamma)}} \leq \alpha < \sqrt{\frac{s}{f_{PP}^M(\gamma)}} \). Thus, to show that the choice of manufacturers does
not lead to a socially optimal outcome it needs to be proved that the following condition is met:

\[ f_{DD2Mix}^M(\gamma) - f_{DD}^M(\gamma) \geq 0, \]  

(99)

which can be expressed as

\[ \frac{(3\gamma^2 - 5\gamma + 2)}{32(\gamma + 2)(1 + \gamma)} \geq 0. \]  

(100)

It can be seen that for complementary products the above condition is always satisfied. In other words, there are some values of \( \alpha \) in which mixed equilibria would be socially efficient, but they do not exist, as companies consider such markets as too small.

Second, considering further the case of complementary products, it needs to be shown that, for some values of \( \alpha \), companies do not move from mixed technology equilibria to the \( (F,F) \) technology state, although it would be socially desirable. The adoption of FPT by both companies versus only one is socially efficient when:

\[ W_4^M - W_2^M \geq 0 \text{ and } W_4^M - W_3^M \geq 0, \]  

(101)

which is equivalent to

\[ \alpha \geq \sqrt{\frac{s}{f_{Mix2FF}^M(\gamma)}}, \]  

(102)

where

\[ f_{Mix2FF}^M(\gamma) = \frac{17(1 - \gamma)}{288(1 + \gamma)}. \]  

(103)

From Proposition 1 in section 3, we know that there is a \( (F,F) \) equilibrium for complementary products when \( \alpha \geq \sqrt{\frac{s}{f_{FF}^M(\gamma)}} \). Thus, if the choice of manufacturers does not lead to a socially optimal outcome the following condition must hold:

\[ f_{Mix2FF}^M(\gamma) - f_{FF}^M(\gamma) \geq 0, \]  

(104)

which can be expressed as

\[ \frac{1 - \gamma}{32(1 + \gamma)} \geq 0. \]  

(105)

Again, it is straightforward that, for complementary products, the above condition is always satisfied.

Lastly, it needs to be shown that, for some values of \( \alpha \), companies do not move from the \( (D,D) \) equilibrium to the \( (F,F) \) technology state, although it would be socially desirable. The
adoption of FPT by both companies versus the (D,D) state is socially efficient when:

\[ W^M_4 - W^M_1 \geq 0, \quad (106) \]

which is equivalent to

\[ \alpha \geq \sqrt{\frac{s}{f^M_{DD2FF}(\gamma)}}, \quad (107) \]

where

\[ f^M_{DD2FF}(\gamma) = \frac{17 - 10\gamma - 7\gamma^2}{36(1 + \gamma)(2 + \gamma)^2}. \quad (108) \]

From Proposition 1 in section 3, we know that both companies choose to invest into FPT when

\[ \alpha \geq \sqrt{\frac{s}{f^M_{FF}(\gamma)}}. \]

Thus, to show that the choice of manufacturers does not lead to a socially optimal outcome it needs to be proved that the following condition is met:

\[ f^M_{DD2FF}(\gamma) - f^M_{FF}(\gamma) \geq 0, \quad (109) \]

which can be expressed as

\[ \frac{\gamma^3 - 4\gamma^2 - 10\gamma + 13}{36(1 + \gamma)(2 + \gamma)^2} \geq 0. \quad (110) \]

Again, it is easy to see that the above condition is always satisfied. Concluding, taking into account that conditions (100), (105) and (110) are satisfied for the relevant range of \( \gamma \), it has been proved that for some intermediary value of \( \alpha \), the decisions of downstream firms deliver socially inefficient outcomes.

Q.E.D.

References


