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Piergallini, Alessandro and Rodano, Giorgio

University of Rome "Tor Vergata"

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Public Debt, Distortionary Taxation, and Monetary Policy*

Alessandro Piergallini†
University of Rome "Tor Vergata"

Giorgio Rodano‡
University of Rome "La Sapienza"

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†Corresponding author: Department of Economics, University of Rome "Tor Vergata", Via Columbia 2, 00133 Roma, Italy. E-mail: alessandro.piergallini@uniroma2.it. Phone: +390672595431. Fax: +39062020500.

‡Department of Economic Theory, University of Rome "La Sapienza", P.le Aldo Moro 5, 00185 Roma, Italy. E-mail: giorgio.rodano@uniroma1.it. Phone: +390649910690. Fax: +39064453870.

Abstract

Since Leeper's (1991, *Journal of Monetary Economics* 27, 129-147) seminal paper, an extensive literature has argued that if fiscal policy is *passive*, i.e., guarantees public debt stabilization irrespectively of the inflation path, monetary policy can independently be committed to inflation targeting. This can be pursued by following the Taylor principle, i.e., responding to upward perturbations in inflation with a more than one-for-one increase in the nominal interest rate. This paper analyzes an optimizing framework in which the government can only finance public expenditures by levying distortionary taxes. It is demonstrated that households' market participation constraints and Laffer-type effects can render passive fiscal policies unfeasible. For any given target inflation rate, there exists a *threshold* level of public debt beyond which monetary policy independence is no longer possible. In such circumstances, the dynamics of public debt can be controlled only by means of higher inflation tax revenues: inflation dynamics in line with the fiscal theory of the price level *must* take place in order for macroeconomic stability to be guaranteed. Otherwise, to preserve inflation control around the steady state by following the Taylor principle, monetary policy *must* target a higher inflation rate.

JEL Classification: E63; H31; H63.

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1 Introduction

The interaction between fiscal and monetary rules is one of the most controversial issues for policy design. Since Leeper's (1991) seminal contribution, modern theory has argued that if fiscal policy is *passive*, that is, guarantees public debt stabilization irrespectively of the inflation path, monetary policy can independently be committed to inflation targeting, for example, by managing the nominal interest rate on the basis of a Taylor-type rule (Taylor, 1993). Notably, a Taylor-type rule prescribes to implement an active monetary policy, responding to increases in inflation with a more than one-for-one increase in the nominal interest rate (the so-called Taylor principle). Conversely, if fiscal policy is *active*, that is, does not guarantee public debt stabilization for each dynamic path of inflation, monetary policy should be passive, responding to increases in inflation with a less than one-for-one increase in the nominal interest rate in order to rule out explosive dynamics for public debt. These results are known as Leeper's active/passive *dichotomy*, and have been proved to hold in economies with either flexible or sticky prices (Woodford, 2003).

The type of fiscal feedback rules commonly used in the literature to model government's policy involves the adoption of lump-sum taxes. This paper demonstrates that in the realistic case in which lump-sum taxes are unavailable, there are circumstances where it can be unfeasible to implement passive fiscal policies.

This result comes from two relevant implications of distortionary taxes when agents optimize: (*i*) the emergence of households' market participation constraints; (*ii*) the occurrence of two Laffer-type effects generated by *both* tax and interest-rate feedback rules.

We then prove that, for any given target inflation rate, there exists a *threshold* level of public debt beyond which monetary policy independence is no longer possible. Under these circumstances, the dynamics of public debt can be controlled only by means of higher inflation tax revenues.

Schmitt-Grohé and Uribe (1997) and Leith and von Thadden (2008) are the first to find the existence of bifurcations associated with the fiscal revenue maximizing tax rate. In this paper we extend the issue by investigating the interactions of Laffer effects on fiscal revenues with Laffer effects on inflation tax revenues. Hence we are able to derive the implications of excessive debt levels in terms of monetary policy design.

Specifically, we demonstrate the occurrence of two possible alternative scenarios: if the central bank is intended to preserve inflation control *around* the steady state by adopting the Taylor principle, it *must* fix a sufficiently higher target inflation rate; otherwise, inflation dynamics of the type studied by the "fiscal theory of the price level" (eg., Leeper, 1991; Sims, 1994; Woodford, 1994, 1995, 2003; Cochrane, 1998, 2005; Leeper and Yun, 2006) *must* occur in order for macroeconomic stability to be ensured.

Fiscal and monetary policy design in the aftermath of the financial crisis erupted in 2007 is currently one the most debated issues in macroeconomics. As government debt and deficits have sharply increased in several economies in the attempt to offset the Great Recession, the possible negative consequences of fiscal expansions on monetary policy independence are now a pressing issue. The analytical results derived in this paper give theoretical support to the argument recently advanced by Cochrane (2010) and Davig, Leeper and Walker (2010) that the large fiscal deficits decided by governments to offset the crisis can lead to the "Laffer limit" beyond which inflation must endogenously jump up according to the fiscal theory of the price level.

The paper proceeds as follows. Section 2 sets up a continuous-time general equilibrium optimizing framework with lump-sum taxation and discusses the central features of Leeper's dichotomy. The recourse by the government to lump-sum taxes as an operating instrument to implement passive fiscal policies is then removed. Section 3 concentrates on asset taxation. Section 4 concentrates on income taxation. Section 5 presents the conclusions.

2 A Baseline Monetary Model

In this Section, we set up a baseline continuous-time optimizing framework with lump-sum taxation.[1] In this context, we reconsider Leeper's dichotomy. In the subsequent Sections, we shall employ this model as a benchmark to study the consequences of distortionary taxation.

Consider an endowment economy with a private sector and a public sector. The private sector consists of a continuum of identical infinitely lived households.[2] The representative household has preferences given by the following lifetime utility function:

$$U = \int_0^{\infty} e^{-\rho t} \left[u(c, m) + f\left(g\right) \right] dt, \tag{1}$$

where c is real private consumption, m are real money balances, and g is real government consumption expenditure. The instantaneous utility function satisfies the following conditions: $u_c, u_m, f' > 0$ and $u_{cc}, u_{mm}, f'' < 0$. Consumption and real balances are Edgeworth complements, so that $u_{cm} > 0$.

[1] A continuous-time setup proves to be more convenient for the arguments developed in the present paper. A discrete-time setup would not alter the essence of our analysis, but would complicate economic intuitions, due to issues pertaining to timing conventions.

[2] Several issues on monetary and fiscal policy design in non-Ricardian economies in which new generations are born over time are studied by Benassy (2007).

The household's instant budget constraint in real terms is given by

$$c + \dot{a} + \dot{m} = (i - \pi)\, a + y - \tau + \tau_h - \pi m, \tag{2}$$

where a is the stock of interest-bearing assets, i is the nominal interest rate paid on assets, π is the inflation rate, y is a constant endowment of perishable goods, τ are lump-sum taxes, and τ_h are government transfers.[3] The right-hand-side of (2) represents disposable income; the left-hand-side shows the uses of disposable income: consumption and saving; the latter takes the form of increases in the stock of real assets and real balances. The household is prevented from engaging in Ponzi's games.

The public sector's budget constraint in real terms is given by

$$\dot{b} + \dot{m} = g + \tau_h + (i - \pi)\, b - \tau - \pi m, \tag{3}$$

where b is the stock of real public debt. Now, the right-hand-side of (3) represents government deficit net of inflation tax revenues; the left-hand-side shows how the public sector can finance its deficit: by issuing interest-bearing bonds and printing money.

The private sector chooses paths for private consumption, real balances, and bonds so as to maximize (1) subject to the budget constraint (2) and the transversality conditions, given the constant stream of the endowment y, and the initial conditions $m(0) = m_0$ and $a(0) = a_0$. Optimization yields

$$u_c(c, m) = \lambda, \tag{4}$$

$$u_m(c, m) = \lambda i, \tag{5}$$

$$\dot{\lambda} = \lambda\, (\rho + \pi - i)\,. \tag{6}$$

Consistently with Leeper (1991), the choices of the public sector are described by two rules, one pertaining to monetary policy, the other to fiscal policy.

The monetary authority fixes the nominal interest rate i in order to control the inflation rate π around the target inflation rate π^*. To facilitate the analysis, and without loss of generality, we assume $\pi^* > 0$. We summarize such a feedback rule as

$$i = \phi(\pi)\,, \tag{7}$$

where $\phi(\pi)$ is continuous, non-decreasing, and strictly positive. Monetary policy is defined

[3] The budget constraint in real terms (2) is derived dividing by the price level the budget constraint in nominal terms,

$$C + \dot{A} + \dot{M} = iA + Y - T + T_h,$$

where upper-case letters represent the corresponding nominal variables. Standard algebra leads to (2).

as *active* when the monetary authority reacts more than proportionally to changes in inflation, $di/d\pi = \phi' > 1$, according to the so-called Taylor principle. Monetary policy is defined as *passive* when the opposite occurs, $\phi' < 1$.

Let now consider fiscal policy. Public consumption g and transfers τ_h are assumed to be exogenous and constant. Taxes are described by the feedback rule

$$\tau = \bar{\alpha} + \alpha b, \tag{8}$$

where $\bar{\alpha}$ is a constant parameter and $\alpha \geq 0$ captures the degree of reactiveness of taxes to public debt. Fiscal policy is defined as *passive* when rule (8) guarantees stability of public debt around the steady state for each dynamic path of inflation. That is, a passive fiscal policy must respect the condition $\partial\dot{b}/\partial b\Big|_{(\pi^*,\, b^*)} < 0$. Conversely, fiscal policy is defined as *active* when the fiscal rule (8) is such that $\partial\dot{b}/\partial b\Big|_{(\pi^*,\, b^*)} > 0$.

2.1 Equilibrium

Combining the two constraints (2) and (3), one obtains the goods' market equilibrium condition, $y = c + g$, and the assets' market equilibrium condition, $b = a$. Since y and g are both exogenous and constant, it follows that $\dot{c} = 0$. Thus, from (4) and (5), we can derive the relationships between m and i, and between λ and i:[4]

$$m = m(i), \tag{9}$$

with $m' < 0$, and

$$\lambda = \lambda(i), \tag{10}$$

with $\lambda' < 0$.

We can now derive the equilibrium equation describing inflation dynamics. Time differentiating (10), using the costate equation (6) and the monetary policy rule (7), we obtain

$$\dot{\pi} = H(\pi)\left[\phi(\pi) - \pi - \rho\right], \tag{11}$$

where $H(\pi) = -\lambda/\lambda'\phi' > 0$.

We next derive the equilibrium equation describing public debt dynamics. We start from money money demand (9); using the monetary policy rule (7), differentiating with respect to time, using the inflation dynamics equation (11), substituting into the budget

[4] For analytical details, see Appendix A.

constraint (3), and taking into account the fiscal policy rule (8), we obtain

$$\dot{b} = [\phi(\pi) - \pi - \alpha] b + g + \tau_h - \bar{\alpha} + K(\pi)[\phi(\pi) - \pi - \rho] - \pi m[\phi(\pi)], \tag{12}$$

where $K(\pi) = \lambda m'/\lambda' > 0$.

The dynamics of the economy is described by the system of differential equations (11) and (12) in the variables (π, b). Since the monetary authority controls the nominal interest rate, money supply is endogenous, and adjusts to demand. Money demand turns out to depend on the inflation rate according to the function $m[\phi(\pi)]$. The inflation rate π results to be "chosen" indirectly by the private sector, thus being a jump variable. The level of public debt b is instead the state variable in the system. We can then define a perfect-foresight equilibrium as a pair of functions $\{\pi(t), b(t)\}$ that satisfy (11)-(12), given the initial condition $b(0) = b_0$ and the transversality conditions.

The system is in the steady state when $\dot{b} = 0$ and $\dot{\pi} = 0$. From (11), the steady-state value of inflation π^* is implicitly defined by

$$\phi(\pi^*) = \rho + \pi^*. \tag{13}$$

Using (13) into (12) yields the steady-state value of debt b^*:

$$b^* = \frac{\bar{\alpha} - g - \tau_h + \pi^* m(\rho + \pi^*)}{\rho - \alpha}. \tag{14}$$

As in Leeper (1991), the parameter $\bar{\alpha}$ is chosen to make b^* positive, and can be interpreted as a "scale" parameter.

The system (11)-(12) and its steady-state solution (13)-(14) enable us to specify when fiscal policy is passive and when it is active. We must compute the partial derivative of $\dot{b}$ with respect to b, evaluated at the steady state (π^*, b^*). If the value of this derivative is negative, fiscal policy is passive, and viceversa. We have

$$\left.\frac{\partial \dot{b}}{\partial b}\right|_{(\pi^*,\, b^*)} = \rho - \alpha, \tag{15}$$

Therefore, fiscal policy is passive if $\alpha > \rho$. Note the economic meaning of this condition: the *implicit* marginal tax rate on assets must be greater than the return on assets.

2.2 Dynamics

To study the dynamics of the system (11)-(12), let linearize it around the steady state (π^*, b^*):

$$\begin{pmatrix} \dot{\pi} \\ \dot{b} \end{pmatrix} = J \begin{pmatrix} \pi - \pi^* \\ b - b^* \end{pmatrix}. \tag{16}$$

The Jacobian J is

$$J = \begin{bmatrix} H^* (\phi' - 1) & 0 \\ A_{21} & \rho - \alpha \end{bmatrix}, \tag{17}$$

where $A_{21} = (b^* + K^*)(\phi' - 1) - m^* \left(1 - \eta^*_{m/\pi}\right)$, with $\eta^*_{m/\pi} = |(\pi^*/m^*)\, m'\phi'|$ denoting the elasticity of money demand with respect to inflation, evaluated at (π^*, b^*).

Since b is a state variable and π a jump variable, we have a saddle path if the following condition holds:

$$\det J = H^* (\phi' - 1)(\rho - \alpha) < 0. \tag{18}$$

Condition (18) is satisfied *either* if

$$\alpha > \rho \qquad \text{and} \qquad \phi' > 1$$

(i.e., under passive fiscal policy and active monetary policy) *or* if

$$\alpha < \rho \qquad \text{and} \qquad \phi' < 1$$

(i.e., under active fiscal policy and passive monetary policy). These are the two cases that specify Leeper's dichotomy. To see it at work, suppose to start from a value $b_0 \neq b^*$.

When $\alpha > \rho$, the solution of the system (16) is given by

$$b = b^* + (b_0 - b^*)\, e^{-(\alpha - \rho)t}, \tag{19}$$

$$\pi = \pi^*. \tag{20}$$

Since, by assumption, fiscal policy is passive, the monetary authority is perfectly able to control inflation according to the Taylor principle ($\phi' > 1$). The phase diagram of the system (16) is presented in Figure 1. The slope of the locus $\dot{b} = 0$, given by $(\rho - \alpha)/A_{21}$, depends on the sign of A_{21} which can be positive or negative. This is because inflation has two opposite effects on the level of public debt: on the one hand, it increases interest payments by the government, since, by assumption, $\phi' > 1$; on the other hand, it increases the inflation tax. In Figure 1, we have drawn the locus $\dot{b} = 0$ with positive slope, as it is more likely to occur when inflation is relatively low. Nevertheless, this slope has no relevance for the system dynamics, since in this case the saddle path coincides with the locus $\dot{\pi} = 0$. Note that equation (19) implies that the velocity through which debt converges to the steady state is an increasing function of α.

When $\alpha < \rho$, we must have $\phi' < 1$ for saddle-path stability to occur. The solution of the system (16) becomes

$$b = b^* + (b_0 - b^*)\, e^{H^*(\phi'-1)t}, \tag{21}$$

$$\pi = \pi^* + S(\phi', \alpha)(b - b^*), \tag{22}$$

where $S(\phi', \alpha) = -\mathrm{Tr}J/A_{21} > 0$ measures the slope of the saddle path, which is greater than the slope of the locus $\dot{b} = 0$. Since now fiscal policy is active ($\alpha < \rho$), the monetary authority cannot follow the Taylor principle. Assuming $b_0 > b^*$, the jump in inflation above the target π^* allows the real public debt to decrease gradually and converge to the steady state. This is because

$$\left.\frac{\partial \dot{b}}{\partial \pi}\right|_{(\pi^*,\, b^*)} = A_{21} < 0.$$

The intuition is as follows. Inflation decreases the real interest rate $\phi(\pi) - \pi$, increases the inflation tax, and hence increases the monetary financing of deficit.[5] The associated phase diagram is illustrated in Figure 2. The jump in inflation needed to ensure stability of real public debt is in accordance with the so-called "fiscal theory of the price level".[6] In synthesis, when fiscal policy is active, inflation dynamics depends on fiscal variables.

To summarize, Leeper's dichotomy establishes that monetary policy is able to control inflation consistently with the target level π^*, provided that fiscal policy takes the burden of controlling public debt. In the opposite case, it is monetary policy that must take the burden of bringing public debt back to the level b^*. Monetary policy can obtain this result only by allowing inflation to jump above the level π^* when $b(t) > b^*$.

Thus, Leeper's dichotomy states that a necessary condition for monetary policy *independence* in the presence of public debt is that fiscal policy is *passive*. In the next Sections, we explore the constraints that the fiscal authority can face in implementing a passive policy, as soon as we relax the simplified case of lump-sum taxation.

3 The Model with Asset Taxation

So far we have emphasized that fiscal policy is passive when its primary objective is public debt stabilization. To obtain this result, the *implicit* marginal tax rate α must be

[5] The term A_{21} can be decomposed in two parts. The first part is negative only when $\phi' < 1$. The second part is negative if the economy is on the upward-sloping side of the Laffer curve for seignorage, as it is efficient.

[6] See Woodford (2003, pp. 311-319) for a discussion.

greater than ρ, the steady-state real return on assets. Nevertheless, as long as taxation is lump sum and equilibrium is competitive, so that the single household is atomistic, the exogenous parameter α does not appear in the solution of the private agents' maximizing problem. This is because the representative household is not able to internalize taxation into its optimal choice. However, the condition for a passive fiscal policy, $\alpha > \rho$, will feature a feasibility problem when we remove the assumption of lump-sum taxation, thus enabling optimizing households to take into account the interaction between their choices and the level of taxation. Intuitively, the consumer will never demand an asset when she observes an after-tax negative return.[7]

Let us analyze the argument. Suppose that fiscal policy obtains its revenues by setting the interest-bearing nominal assets as tax base,[8] with a marginal tax rate equal to α. Tax revenues in real terms are thus equal to $\tau = \bar{\alpha} + \alpha a$. The households' participation constraint in the asset market imposes $\alpha < i$. We shall now show that, by setting up the model with such a participation constraint, fiscal policy cannot be passive.

The representative household's instant budget constraint in real terms is now

$$c + \dot{a} + \dot{m} = (i - \pi - \alpha)\, a + y - \bar{\alpha} + \tau_h - \pi m. \tag{23}$$

Performing optimization yields

$$u_c(c, m) = \lambda, \tag{24}$$

$$u_m(c, m) = \lambda\, (i - \alpha)\,, \tag{25}$$

$$\dot{\lambda} = \lambda\, (\rho + \pi + \alpha - i)\,. \tag{26}$$

The government's budget constraint is now

$$\dot{b} + \dot{m} = g + \tau_h + (i - \pi)\, b - \bar{\alpha} - \alpha a - \pi m. \tag{27}$$

In equilibrium, optimality conditions (24) and (25) can be written in implicit form as follows:

$$m = m\,(i - \alpha)\,, \tag{28}$$

$m' < 0$, and

$$\lambda = \lambda\,(i - \alpha)\,, \tag{29}$$

[7]Money is the only exception. There can be a positive demand for money also in the presence of a negative return due to inflation, since money has a positive marginal utility.

[8]It can be shown that the same argument applies by assuming taxation on nominal interest payments. For an analysis of macroeconomic stability under Taylor rules in a New Keynesian framework with nominal interest taxation, see Edge and Rudd (2007).

with $\lambda' < 0$.

The closed-form differential-equation system in the variables (π, b) is then given by

$$\dot{\pi} = H(\pi)\left[\phi(\pi) - \pi - \alpha - \rho\right], \tag{30}$$

$$\dot{b} = \left[\phi(\pi) - \pi - \alpha\right] b + g + \tau_h + K(\pi)\left[\phi(\pi) - \pi - \alpha - \rho\right] - \pi m\left[\phi(\pi)\right]. \tag{31}$$

The steady-state solutions are given by

$$\phi(\pi^*) = \alpha + \rho + \pi^*, \tag{32}$$

$$b^* = \frac{\bar{\alpha} - g - \tau_h + \pi^* m(\alpha + \rho + \pi^*)}{\rho}. \tag{33}$$

Using (31) and (32), it now follows that

$$\left.\frac{\partial \dot{b}}{\partial b}\right|_{(\pi^*,\, b^*)} = \rho. \tag{34}$$

This proves that fiscal policy cannot be passive, for households internalize asset taxation into their optimal decisions.

The implications for monetary policy are the following. Now, the Jacobian is

$$J = \begin{bmatrix} H^*(\phi' - 1) & 0 \\ A_{21} & \rho \end{bmatrix}. \tag{35}$$

The emergence of a saddle path requires $\phi' < 1$, that is, a passive monetary policy. The monetary authority is no longer able to control inflation.

To conclude, a passive fiscal policy cannot rely on asset taxation only. There are two alternatives to ensure macroeconomic stability. The first is to combine asset taxation with an inflationary path brought about by a passive monetary policy, along the lines depicted by the fiscal theory of the price level. But in this case, the monetary authority cannot be independent, i.e., cannot adopt a Taylor-type rule with $\phi' > 1$, in order to set inflation equal to the target level π^*. The second alternative is to raise revenues from another tax base.

4 The Model with Income Taxation

Let us focus on the implications of using income taxes as instrument of a passive fiscal policy.[9] Let $\tau_y < 1$ be the tax rate on income. The household's budget constraint is given by

$$c + \dot{a} + \dot{m} = (i - \pi) a + (1 - \tau_y) y + \tau_h - \pi m. \tag{36}$$

Since y is exogenous, the optimality conditions are exactly the same as in Section 2.

The government's budget constraint is given by

$$\dot{b} + \dot{m} = g + \tau_h + (i - \pi) b - \tau_y y - \pi m. \tag{37}$$

Fiscal policy is now described in terms of a feedback rule in which income taxation reacts to public debt:

$$\tau_y y = \bar{\alpha} + \alpha b. \tag{38}$$

The differential-equation system is the same as in Section 2. Hence, using income, as opposed of debt, as tax base allows to reestablish Leeper's dichotomy, so that a passive fiscal policy allows monetary policy independence. However, this result is subject to the following remark.

The steady-state marginal tax rate, τ_y^*, depends on the target inflation rate π^*, independently set by the monetary authority, and on the steady-state level of public debt b^*:

$$\tau_y^* = \rho \frac{b^*}{y} + \frac{g + \tau_h}{y} - \frac{\pi^* m (\rho + \pi^*)}{y}. \tag{39}$$

From (39), the fiscal rule may violate the participation constraint, which imposes $\tau_y < 1$. Because $\partial \tau_y^* / \partial b^* > 0$, it emerges a limit on the level of steady-state public debt. Let b_y^{M} be the threshold value of public debt beyond which the participation constraint is violated. From (39), it follows that

$$b_y^{\mathrm{M}} = \frac{y - g - \tau_h + \pi^* m (\rho + \pi^*)}{\rho}. \tag{40}$$

If $b_0 > b_y^{\mathrm{M}}$, it is not feasible to implement a passive fiscal policy, for $\tau_y (0) = (\bar{\alpha} + \alpha b_0) / y > 1$, which violates the constraint $\tau_y < 1$. A central bank intended to follow the Taylor principle has to accept a higher steady-state inflation rate in order to raise the monetary financing, thereby ensuring $b_0 \leq b_y^{\mathrm{M}}$.

The foregoing remark, it can be argued, is purely theoretical. The condition $b_0 > b_y^{\mathrm{M}}$

[9] Linnemann (2006) studies the dynamic effects of alternative fiscal rules in a New Keynesian model with income taxation.

could, in fact, result to be empirically implausible, at least for industrialized economies. However, recall that thus far we have assumed an endowment economy. Households' optimal decisions for consumption and saving do not affect the level of y, thereby not influencing fiscal revenues. Such an independence between households' optimal decisions and fiscal revenues no longer holds in a production economy. We shall examine the consequences in what follows.

4.1 Laffer Effects and Monetary Policy Independence

Suppose now the economy is populated by a continuum of identical household-firms. The production technology of the representative household-firm is given by

$$y = l, \tag{41}$$

where l represents labor supply. The household's lifetime utility function takes the following form:

$$U = \int_0^\infty e^{-\rho t} \left[u(c,m) + f(g) - v(l) \right] dt, \tag{42}$$

where $u(c,m)$ is linearly homogeneous, so that $u_{cc}u_{mm} - u_{cm}^2 = 0$, and $v', v'' > 0$.

Using (41), the household-firm's flow budget constraint is given by (36), and the optimality conditions associated with the maximization problem become

$$u_c(c,m) = \lambda, \tag{43}$$

$$u_m(c,m) = \lambda i, \tag{44}$$

$$v'(y) = \lambda (1 - \tau_y), \tag{45}$$

$$\dot{\lambda} = \lambda (\rho + \pi - i). \tag{46}$$

The government's budget constraint is given by (37). Fiscal policy is described by rule (38).

In equilibrium, conditions (43)-(44) can be expressed in implicit form as[10]

$$y = y(i, \tau_y), \tag{47}$$

with $y_i < 0$, $y_{\tau_y} < 0$,

$$m = m(i, \tau_y), \tag{48}$$

[10] For analytical details, see Appendix B.

with $m_i < 0$, $m_{\tau_y} < 0$, and

$$\lambda = \lambda(i), \tag{49}$$

with $\lambda' < 0$.

Using (47), the fiscal policy rule takes the following form:

$$\tau_y y(i, \tau_y) = \bar{\alpha} + \alpha b. \tag{50}$$

Differentiating with respect to time yields

$$\dot{\tau}_y = \frac{\alpha}{y\left(1 - \eta_{y/\tau_y}\right)}\dot{b} - \frac{\tau_y y_i}{y\left(1 - \eta_{y/\tau_y}\right)}\dot{i}, \tag{51}$$

where $\eta_{y/\tau_y} = \left|(\tau_y/y)\, y_{\tau_y}\right|$ denotes the elasticity of output with respect to the marginal rate. We assume $\eta_{y/\tau_y} < 1$, i.e., that the economy is on the upward-sloping side of the Laffer curve, for it results to be efficient. Therefore, we can write

$$\tau_y = \tau(b, i), \tag{52}$$

with $\tau_b > 0$ and $\tau_i > 0$.

The equilibrium dynamics can then be expressed in terms of the following differential-equation system:

$$\dot{\pi} = H(\pi)\left[\phi(\pi) - \pi - \rho\right], \tag{53}$$

$$\dot{b} = \frac{[\phi(\pi) - \pi - \alpha]\, b + g + \tau_h - \bar{\alpha} + K(\pi, b)[\phi(\pi) - \pi - \rho] - \pi m\left\{\phi(\pi), \tau\left[b, \phi(\pi)\right]\right\}}{1 + m_{\tau_y}\tau_b}, \tag{54}$$

where $K(\pi, b) = \lambda\left(m_i + m_{\tau_y}\tau_i\right)/\lambda' > 0$.

The steady-state solutions are given by

$$\phi(\pi^*) = \rho + \pi^*, \tag{55}$$

$$b^* = \frac{\bar{\alpha} - g - \tau_h + \pi^* m\left[\rho + \pi^*, \tau(b^*, \rho + \pi^*)\right]}{\rho - \alpha}. \tag{56}$$

It follows that

$$\begin{aligned} \left.\frac{\partial \dot{b}}{\partial b}\right|_{(\pi^*,\, b^*)} &= \rho - \alpha - \pi^* m_{\tau_y}\tau_b \\ &= \rho - \alpha\left[1 + \frac{\pi^* m_{\tau_y}}{y^*\left(1 - \eta^*_{y/\tau_y}\right)}\right], \end{aligned} \tag{57}$$

where we have used the fact that (52) evaluated at the steady state yields

$$\tau_b = \alpha / y^* \left(1 - \eta^*_{y/\tau_y}\right). \tag{58}$$

To facilitate our discussion on dynamic stability, and make the present analysis easily comparable with the results that apply in the benchmark model of Section 2, let us restrict attention to the case in which

$$y^* \left(1 - \eta^*_{y/\tau_y}\right) > \left|\pi^* m_{\tau_y}\right|. \tag{59}$$

This condition says that the increase in fiscal revenues generated by an increase in the tax rate is greater than the decrease in the inflation tax brought about by the associated fall in money demand. Therefore, *total* revenues, i.e., fiscal revenues plus the inflation tax, are assumed to raise following an increase in the tax rate. If condition (59) holds, then a passive fiscal policy requires

$$\alpha > \frac{\rho}{1 + \pi^* m_{\tau_y} / y^* \left(1 - \eta^*_{y/\tau_y}\right)}. \tag{60}$$

Since $\pi^* m_{\tau_y} / y^* \left(1 - \eta^*_{y/\tau_y}\right) < 0$, the feedback parameter α must be greater than in the endowment-economy case. The reason is clear. An increase in public debt causes the tax rate to raise via the fiscal policy feedback rule. The increase in the tax rate brings about a decrease in output and hence in money demand. This crowds out the inflation tax, thereby requiring a more aggressive reaction by the fiscal authority. The foregoing mechanism implies that the higher the elasticity of output with respect to the tax rate, the higher parameter α ensuring a passive fiscal policy, as it is apparent from (60).

The Jacobian is given by

$$J = \begin{bmatrix} H^* \left(\phi' - 1\right) & 0 \\ B_{21} & \rho - \alpha \left[1 + \frac{\pi^* m_{\tau_y}}{y^* \left(1 - \eta^*_{y/\tau_y}\right)}\right] \end{bmatrix}, \tag{61}$$

where

$$B_{21} = \frac{\left(b^* + K^*\right)\left(\phi' - 1\right) - m^* \left(1 - \eta^*_{m/\pi}\right) - \pi^* m_{\tau_y} \tau_i \phi'}{1 + m_{\tau_y} \tau_b}$$

does not affect the two eigenvalues of the matrix and hence the conditions for saddle-path stability. The latter occurs if the following condition applies:

$$\det J = H^* \left(\phi' - 1\right) - \rho - \alpha \left[1 + \frac{\pi^* m_{\tau_y}}{y^* \left(1 - \eta^*_{y/\tau_y}\right)}\right] < 0. \tag{62}$$

Condition (62) is verified either if

$$\alpha > \frac{\rho}{1 + \pi^* m_{\tau_y}/y^* \left(1 - \eta^*_{y/\tau_y}\right)} \qquad \text{and} \qquad \phi' > 1$$

or if

$$\alpha < \frac{\rho}{1 + \pi^* m_{\tau_y}/y^* \left(1 - \eta^*_{y/\tau_y}\right)} \qquad \text{and} \qquad \phi' < 1.$$

If fiscal policy is passive, i.e., $\alpha > \rho/\left[1 + \pi^* m_{\tau_y}/y^* \left(1 - \eta^*_{y/\tau_y}\right)\right]$, monetary policy independence is ensured.

However, for a given target inflation rate independently set by the monetary authority, the occurrence of Laffer-type effects does pose a limit on the level of steady-state public debt. Let indicate it by b_l^{M}. We shall demonstrate that beyond such a limit, a passive fiscal policy becomes unfeasible.

To prove this result, first notice that in the steady state it must be that

$$\tau_y^* y \left(\rho + \pi^*, \tau_y^*\right) + \pi^* m \left(\rho + \pi^*, \tau_y^*\right) = \rho b^* + g + \tau_h. \tag{63}$$

It follows that

$$b_l^{\mathrm{M}} = \frac{\max\limits_{\tau_y^*} \left[\tau_y^* y \left(\rho + \pi^*, \tau_y^*\right) + \pi^* m \left(\rho + \pi^*, \tau_y^*\right)\right] - g - \tau_h}{\rho}. \tag{64}$$

Maximization of total revenues with respect to the tax rate occurs when

$$y^* \left(1 - \eta^*_{y/\tau_y}\right) = -\pi^* m_{\tau_y}. \tag{65}$$

Since $m_{\tau_y} < 0$, total revenues are maximized on the left-hand-side of the Laffer curve. This is precisely because, for a given target inflation rate, higher tax rates generate a negative spillover on the inflation tax.

Now, substituting (65) into (57) yields

$$\left.\frac{\partial \dot{b}}{\partial b}\right|_{\left(\pi^*,\, b_l^{\mathrm{M}}\right)} = \rho. \tag{66}$$

This proves that if $b_0 > b_l^{\mathrm{M}}$, fiscal policy cannot be passive, for total revenues cannot be sufficient to reduce public debt over time.

Remarkably, the presence of Laffer effects on tax revenues causes the threshold level of public debt to be lower with respect to the endowment-economy case. That is, we have

$$b_l^{\mathrm{M}} < b_y^{\mathrm{M}}.$$

A central policy implication emerges. If $b_0 > b_l^{\mathrm{M}}$, the dynamics of public debt can be controlled *only* by means of inflation tax revenues. Monetary policy independence is no longer possible.

4.2 Maximum Debt, Inflation Targeting, and the Fiscal Theory of the Price Level

From (64), b_l^{M} is a function of the target inflation rate π^*, $b_l^{\mathrm{M}} = b_l^{\mathrm{M}}(\pi^*)$. To study this function, we apply the envelop theorem. We have

$$\begin{aligned} \frac{db_l^{\mathrm{M}}}{d\pi^*} &= \frac{\tau^{\mathrm{M}} y_i + m^* + \pi^* m_i}{\rho} \\ &= \frac{m^*}{\rho}\left(1 - \eta^*_{m/\pi} - \frac{\tau^{\mathrm{M}} y^*}{\pi^* m^*}\eta^*_{y/\pi}\right), \end{aligned} \tag{67}$$

where

$$\tau^{\mathrm{M}} = \arg\max_{\tau_y^*}\left[\tau_y^* y\left(\rho + \pi^*, \tau_y^*\right) + \pi^* m\left(\rho + \pi^*, \tau_y^*\right)\right].$$

From (67), $db_l^{\mathrm{M}}/d\pi^* > 0$ as long as $\eta^*_{m/\pi} + \left(\tau^{\mathrm{M}} y^*/\pi^* m^*\right)\eta^*_{y/\pi} < 1$. We let π^{M} be the value of the inflation rate such that $\eta^*_{m/\pi} + \left(\tau^{\mathrm{M}} y^*/\pi^* m^*\right)\eta^*_{y/\pi} = 1$, that is, $db_l^{\mathrm{M}}/d\pi^* = 0$.

Function $b_l^{\mathrm{M}}(\pi^*)$ is illustrated in Figure 3, and has the following interpretation. For $\pi^* = 0$, we have $\eta^*_{m/\pi} = \eta^*_{y/\pi} = 0$, so that $db_l^{\mathrm{M}}/d\pi^* = m^*/\rho > 0$. As long as π^* raises, both elasticities $\eta^*_{m/\pi}$ and $\eta^*_{y/\pi}$ increase. This is because the increase in inflation causes the nominal interest rate to raise, leading to a fall in both money demand and output. As a result, total revenues, that is, fiscal revenues plus the inflation tax, increase as long as $\pi^* < \pi^{\mathrm{M}}$, reach a maximum at $\pi^* = \pi^{\mathrm{M}}$, and decrease as long as $\pi^* > \pi^{\mathrm{M}}$. Two implications for the design of monetary policy rules arise.

First, if the monetary authority is intended to adopt the Taylor principle in order to maintain inflation control around the steady state, and at the same time avoid explosive paths in public debt, it *must* set an inflation target such that $\pi_0^* \leq \pi^* \leq \pi^{\mathrm{M}}$, thereby ensuring $b_0 \leq b_l^{\mathrm{M}}$. It should be noted that such a scenario resembles the classic "unpleasant monetarist arithmetic" example (Sargent and Wallace, 1981). This is because a sufficiently high target inflation rate must force the monetary authority to ensure a sufficiently high steady-state money growth – which is endogenous when the policy instrument is the nominal interest rate – in order to rule out fiscal insolvency.

Second, if the monetary authority sets a target inflation rate such that $\pi^* < \pi_0^*$, then we have $b_0 > b_l^{\mathrm{M}}$, and macroeconomic stability is guaranteed *only* by inflation dynamics along the lines of the fiscal theory of the price level. In fact, the Jacobian evaluated at $\left(\pi^*, b_l^{\mathrm{M}}\right)$ is given by

$$J_{\left(\pi^*,\, b_l^{\mathrm{M}}\right)} = \begin{bmatrix} H^*\left(\phi' - 1\right) & 0 \\ B_{21} & \rho \end{bmatrix}. \tag{68}$$

Saddle-path stability requires that monetary policy is passive, $\phi' < 1$. Violating the Taylor principle allows the inflation rate to jump up in order to rule out explosive dynamics in public debt. Nevertheless, in this second case the monetary authority clearly loses inflation control around the steady state.

5 Concluding Remarks

This paper reconsiders the issue of monetary policy independence in the presence of public debt in an environment in which the government *cannot* use lump-sum taxes to rule out potentially explosive debt dynamics.

The question of monetary policy independence in the presence of public debt is notably an old topic in macroeconomic theory, which dates back at least to the famous "unpleasant monetarist arithmetic" of Sargent and Wallace (1983). But it does have important implications for the recent macroeconomic situation, which sees huge increases in public deficits around the world to offset the Great Recession started in 2007 and, as a result, a drammatic accumulation of goverments' debt: can monetary policies be independent when an "exit strategy" for fiscal policies will be needed to preserve governments' solvency?

The paper's novel contributions to the literature are the following.

Our model can be read as a generalization – to an environment in which the fiscal authority can only finance expenditures by levying distortionary taxes – of the seminal contribution by Leeper (1991) on the interaction between active and passive monetary and fiscal policies.

It is first demonstrated that when lump-sum taxes are not available, an households' participation constraint emerges, which may render unfeasible to implement passive fiscal policies, i.e., policies ensuring public debt stabilization. This rules out the possibility to employ consumers' assets as the only tax base, because a passive fiscal policy requires a negative after-tax interest rate, which violates the households' participation constraint in the asset market. Thus, in the paper we explore the issue of monetary policy independence when the government raises revenues from an income tax base.

Using income as tax base to implement a passive fiscal policy leads to the emergence of Laffer-type effects. A novelty of this paper is to show that there exists an interaction between Laffer effects on fiscal revenues and Laffer effects on inflation tax revenues. Specifically, negative spillovers between the two Laffer curves arise. Because of such negative spillovers, the total revenues maximizing fiscal-tax rate occurs on the upward-sloping side of the Laffer curve. Most importantly, it is demonstrated that for any given target inflation rate independently chosen by the central bank, there always exists a threshold level of public debt beyond which monetary policy independence vanishes.

In addition, we analyze the implications of such results for monetary policy design. From this perspective, we show that the two popular theories that emphasize the threats of public debt for monetary policy independence, namely the "unpleasant monetarist arithmetic" of Sargent and Wallace (1981) and the "fiscal theory of the price level" of Leeper (1991), Sims (1994), Woodford (1994, 1995, 2003), and Cochrane (1998, 2005) can be considered in an unified framework.

Specifically, we construct a frontier of the maximum level of public debt as a function of the target inflation rate, and show the emergence of two possible policy scenarios.

First, if the monetary authority aims to control inflation around the steady state, using the Taylor principle, it *must* increase the target inflation rate. This scenario resembles the classic "unpleasant monetarist arithmetic", in the sense that a higher target inflation rate must imply a higher steady-state money growth (in the present framework the central bank controls the nominal interest rate so that money supply is endogenous), and then a higher debt monetization. However, it is shown that also the returns from the inflation tax are decreasing, because of both a Laffer-type effect on seignorage and the spillovers between fiscal and inflation tax revenues. As a result, there exists an absolute maximum for total revenues at which the gains from the inflation tax are completely crowded out by the fall in fiscal revenues.

Second, if monetary authority aims to maintain the target inflation rate independently fixed, it *must* give up the Taylor principle, so that the system falls in the environment in which the fiscal theory of the price level holds.

In both the alternatives considered, the monetary authority loses its independence.

Of course, the analysis presented in this paper is based on a number of simplifying assumptions needed to render the argument as transparent as possible. In particular, in order to make the analysis directly comparable with the standard literature, and highlight the implications of market participation constraints and of Laffer-type effects for monetary policy design, we worked under the assumption that government spending was exogenously set by fiscal authorities. However, should the economy embark on unsustainable levels of public debt, governments do have the option to decrease the level of public expenditure to rule out pressures on price-stability-oriented central banks, especially in the economies with very high tax rates. This resembles, for instance, the type of “exit strategy” that Euro Area Member States are currently trying to implement.

The theoretical analysis of such an additional scenario would lead away from the interrelations between distortionary taxation and monetary policy, the subject of this paper. This scenario is consistent, however, with the paper’s general point: the presence of distortionary taxation *per se* might pose restrictions for the adoption of aggressive monetary policy feedback rules of Taylor’s type capable of preserving price stability.

Appendix A

Consider the two optimality conditions (4) and (5). Differentiating with respect to time, recalling that $\dot{c} = 0$, we can write the results in matrix notation:

$$\begin{pmatrix} u_{cm} & -1 \\ u_{mm} & -i \end{pmatrix} \begin{pmatrix} \dot{m} \\ \dot{\lambda} \end{pmatrix} = \lambda \begin{pmatrix} 0 \\ \dot{i} \end{pmatrix}. \tag{A.1}$$

Let $\Delta = u_{mm} - u_{cm} i < 0$. Then we have

$$\dot{m} = \frac{\lambda \begin{vmatrix} 0 & -1 \\ \dot{i} & -i \end{vmatrix}}{\Delta} = \frac{\lambda}{\Delta} \dot{i}, \tag{A.2}$$

$$\dot{\lambda} = \frac{\lambda \begin{vmatrix} u_{cm} & 0 \\ u_{mm} & \dot{i} \end{vmatrix}}{\Delta} = \frac{\lambda u_{cm}}{\Delta} \dot{i}. \tag{A.3}$$

We can thus write (9)-(10).

Appendix B

Consider the three optimality conditions (43)-(45). Differentiating with respect to time and imposing the goods' market equilibrium condition, we can express the results as

$$\begin{pmatrix} u_{cc} & u_{cm} & -1 \\ u_{cm} & u_{mm} & -i \\ v'' & 0 & -(1-\tau_y) \end{pmatrix} \begin{pmatrix} \dot{y} \\ \dot{m} \\ \dot{\lambda} \end{pmatrix} = \lambda \begin{pmatrix} 0 \\ \dot{i} \\ -\dot{\tau}_y \end{pmatrix}. \tag{B.1}$$

Let $\Psi = v'' (u_{mm} - u_{cm} i) < 0$. Hence, we have

$$\begin{aligned} \dot{y} &= \frac{\lambda \begin{vmatrix} 0 & u_{cm} & -1 \\ \dot{i} & u_{mm} & -i \\ -\dot{\tau}_y & 0 & -(1-\tau_y) \end{vmatrix}}{\Psi} \\ &= \frac{\lambda (1-\tau_y) u_{cm}}{\Psi} \dot{i} - \frac{\lambda}{v''} \dot{\tau}_y, \end{aligned} \tag{B.2}$$

$$\dot{m} = \frac{\lambda \begin{vmatrix} u_{cc} & 0 & -1 \\ u_{cm} & \dot{i} & -i \\ v'' & -\dot{\tau}_y & -(1-\tau_y) \end{vmatrix}}{\Psi} = \frac{\lambda \left[v'' - (1-\tau_y) u_{cc}\right]}{\Psi}\dot{i} + \frac{\lambda \left(u_{cm} - u_{cc} i\right)}{\Psi}\dot{\tau}_y, \tag{B.3}$$

$$\dot{\lambda} = \frac{\lambda \begin{vmatrix} u_{cc} & u_{cm} & 0 \\ u_{cm} & u_{mm} & \dot{i} \\ v'' & 0 & -\dot{\tau}_y \end{vmatrix}}{\Psi} = \frac{\lambda v'' u_{cm}}{\Psi}\dot{i}. \tag{B.4}$$

We can thus write (47)-(49).

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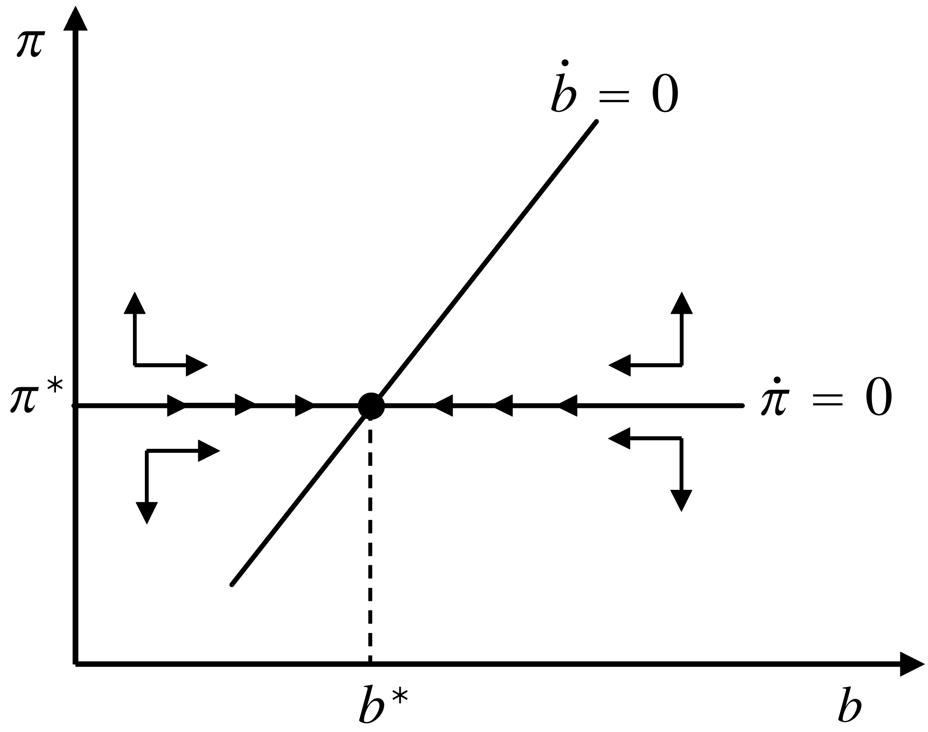

Figure 1: Passive Fiscal Policy and Active Monetary Policy

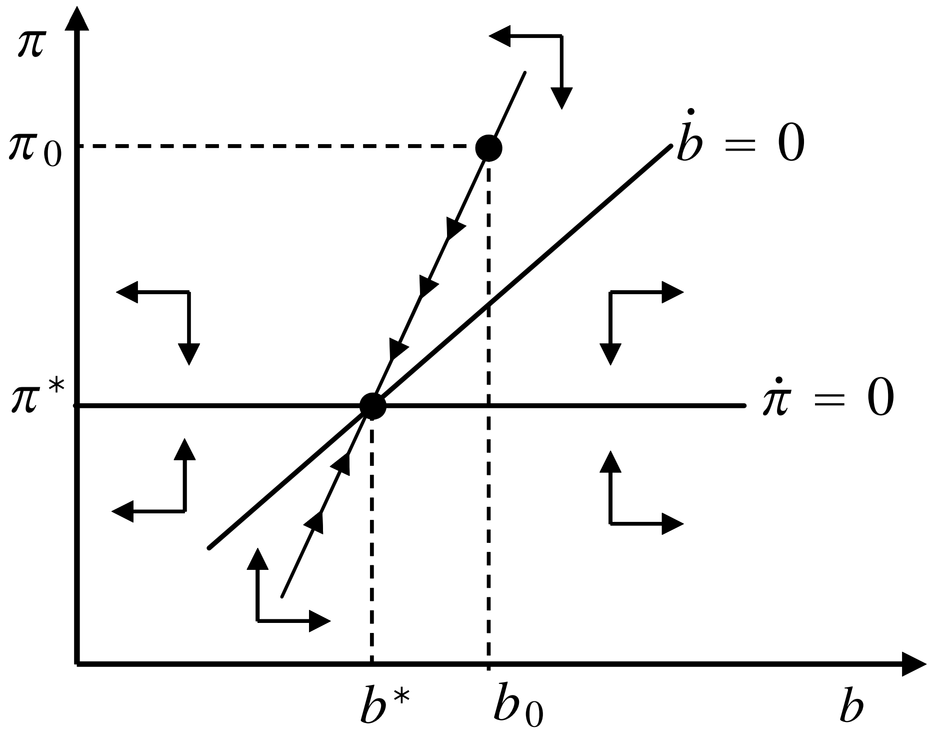

Figure 2: Active Fiscal Policy and Passive Monetary Policy

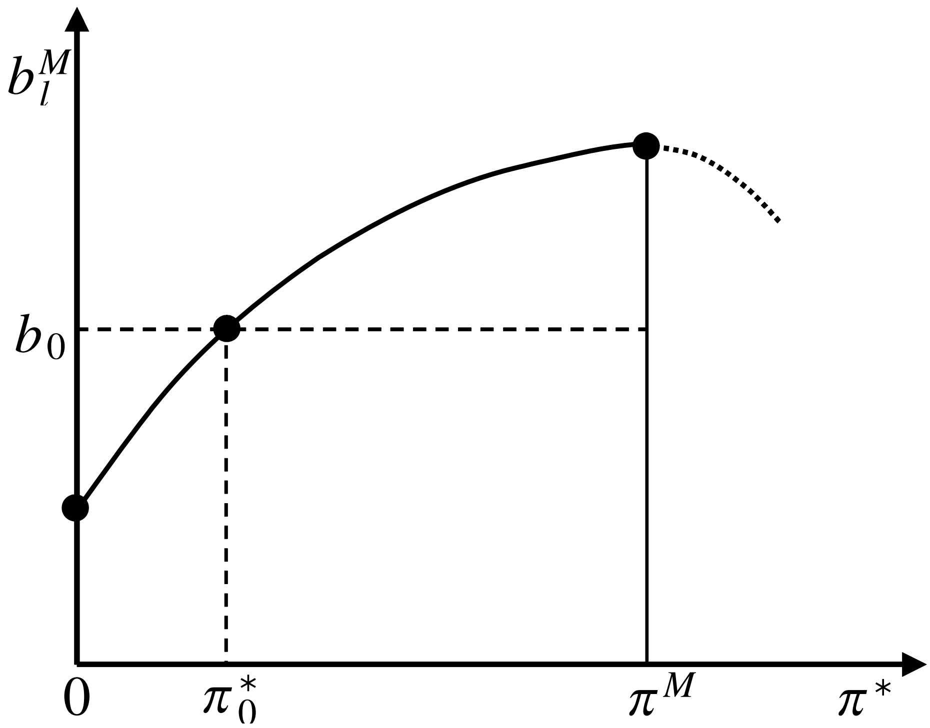

Figure 3: Maximum Debt and Inflation Target