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Can International Factor Mobility Lessen Wage Inequality in a Dual Economy?

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Abstract: We introduce international labor mobility in a three-sector general equilibrium model with rural-urban migration. We demonstrate that under some reasonable conditions an inflow of foreign skilled labor (capital) can reduce skilled-unskilled wage inequality.

Keywords: Skilled-unskilled wage inequality; rural-urban migration; Unemployment; International factor movements

JEL classification: F16, F22, O18

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1. Introduction

Liberalized trade and investment policies, according to the celebrated Stolper-Samuelson theorem, were expected to lower the skilled-unskilled wage inequality in the developing countries following increases in the prices of the export commodities as these are generally exporters of commodities that are intensive in the use of unskilled labour. But empirical studies\(^1\) strongly suggest that the wage inequality has increased in many Latin American and South Asian countries including India. The scanty theoretical literature explaining the deteriorating wage inequality in the Southern countries includes works of Feenstra and Hanson (1996), Marjit, Beladi and Chakrabarti (2004), Marjit and Kar (2005), Chaudhuri and Yabuuchi (2006), and Yabuuchi and Chaudhuri (2006). They have shown how trade liberalization, inflows of foreign capital and international mobility of labour both skilled and unskilled might produce unfavorable effects on the wage inequality in the South given the specific structural characteristics of the less developed countries, such as features of labour markets, structures of production, nature of capital mobility among others.

However, the theoretical literature on trade and development so far has adopted the full-employment framework and hence ignored the problem of unemployment, especially that of unskilled labour which is a salient feature of the developing countries. These economies are plagued by significant degree of skilled-unskilled wage inequality and high levels of unemployment of unskilled labour, especially in the urban areas. Large-scale international migration of workers from a developing country, irrespective of whether skilled or unskilled, may produce significant effects on overall employment and wages as well as on skilled-unskilled wage gap in that country.

In the circumstances, it is extremely important to analyze the consequences of international factor movements on the skilled-unskilled wage inequality in a dual economy setup.\(^2\) In a two-sector mobile capital Harris-Todaro (HT, hereafter) model, Corden and

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\(^2\) It may be mentioned that Yabuuchi and Chaudhuri (2006) have studied the consequences of international mobility of both skilled and unskilled labour on the wage inequality in terms of a three-sector general equilibrium model with distortion in the market for unskilled labour and have found
Findlay (1974)) there are only two factors of production: labour and capital. But, the
distinction between two types of labour with respect to their productive skills has gained extreme significance during the liberalized regime and some of the developing countries including India have become exporters of high-skill commodities e.g. computer software. Unfortunately, in the theoretical literature on trade and development no major attempts have been made to extend the HT model to include three mobile factors. A notable exception is, however, Yabuuchi (2006), who has employed a three-sector HT model to examine the effects of international labor mobility and other policies on the urban unemployment problem of unskilled labour and welfare in a small open economy. But to study the consequences of international factor movements on the skilled-unskilled wage inequality is equally important. This is exactly what the present note purports to do. The three-sector HT framework, as developed by Yabuuchi (2006), is employed for the purpose of analysis because it includes three intersectorally mobile factors of production: unskilled labor, skilled labor and capital.

The rest of the paper is organized as follows. Section 2 describes the basic model and its assumptions. In sections 3 and 4, we derive and interpret the consequences of the inflows of unskilled labor and skilled labor (as well as capital) on the skilled-unskilled wage inequality, respectively. Section 5 considers the relative efficacy of different policies from the perspective of an overpopulated and capital scarce developing economy. Some concluding remarks are provided in the final section.

2. Model and assumptions

Let us consider a small open dual economy with two sectors: rural and urban. The rural sector produces a primary exportable commodity, $X_1$, while the urban sector produces a manufacturing good, $X_2$. The production of each good requires unskilled labor ($L_j$), skilled labor ($H_j$), and capital ($K_j$). Thus, the production functions are as follows:
\[ X_j = F^j(L_j, H_j, K_j), \quad j = 1, 2 \]  
(1)

\( F^j \) is increasing in each factor, strictly quasi-concave, and homogeneous of degree one.

Under perfect competition, we have

\[ p_j = a_{ij} w_j + a_{ij} s + a_{kJ} r, \quad j = 1, 2 \]  
(2)

where \( a_{ij} \) is the amount of the \( i \)th factor used in the \( j \)th industry in order to produce one unit of output, \( w_j \) is the wage rate of unskilled labor in the \( j \)th sector, \( s \) is the wage rate of skilled labor, \( r \) is the rental of capital, and \( p_j \) is the price of the \( j \)th good \((j = 1, 2)\). We assume that all goods are tradable, and hence their prices are given internationally.

The urban sector faces an imperfect unskilled labour market in the form of a unionized labour market where unskilled workers receive a contractual wage, \( w_2 \), while their counterparts in the rural sector receive a low and competitive wage, \( w_1 \). \( L_2 \) and \( L_u \) represent the employed and unemployed labor in the urban region, respectively. The ratio of urban unemployment to urban employment, denoted \( \lambda \), is given by \( \lambda = L_u / L_2 (= L_u / a_{12}X_2) \).

Therefore, in the labor market equilibrium, the wage rate in sector 1 \( (w_1) \) equals the expected wage income in sector 2, which is equal to the manufacturing wage rate \( (w_2) \) multiplied by the probability of finding a job in the urban manufacturing sector, \( (L_2/(L_2 + L_u)) \)

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3 Following the theory of Harris and Todaro (1970), we assume that the manufacturing wage rate of unskilled labor is higher than the rural wage rate and that it is constant. The firms in the urban manufacturing sector have well-organized trade unions. One of the most important roles of the labour unions is to bargain with the respective employers in respect of the betterment of the working conditions. Through offer of negotiation, threat of strike, actual strike etc. they exert pressure on the employers (firms) in order to secure higher wages, reduced hours of work, share in profits and other benefits. Bhalotra (2002) has noted that in India before the initiation of economic reforms organized workers in large firms were been able to reap wages higher than the supply price of labour due to the job security and minimum wage legislations. Hence, a high and constant wage in the urban unskilled labour market should be the consequence of strong and successful collective bargaining between the labour unions and the firms.
i.e. \( (a_{L2}X_2 + (a_{L2}X_2 + L_u)) \). Therefore, the rural-urban labour allocation mechanism of unskilled labour is expressed as:

\[
w_1 = w_2 L_2 / (L_2 + L_u) \text{ or } w_1(1 + \lambda) = w_2, \tag{3}
\]

We assume full-employment of skilled labour. This is partly justified by the tendency for skilled labor, such as computer engineers or accountants, to be employed in either of the sectors under similar conditions. However, the diversity in characteristic features of the sectors can be shown by the difference in the factor intensities of the two sectors. In the HT model, it is supposed that sector 2 is manufacturing and sector 1 is agriculture. We also follow this tradition. Thus, it is natural to assume that sector 2 is more (probably highly) skilled labor intensive as well as capital intensive than sector 1 with respect to unskilled labor. In mathematical terms, these imply that \( \Lambda_{L2} = \lambda_{H2}\lambda_{L1} - (1 + \lambda)\lambda_{H1}\lambda_{L2} > 0 \) and \( \Lambda_{L2} = \lambda_{K2}\lambda_{L1} - (1 + \lambda)\lambda_{K1}\lambda_{L2} > 0 \), where \( \lambda_{ij} \) is the allocative share of factor \( i \) in the \( j \)th sector (e.g., \( \lambda_{L1} = a_{L1}X_1 / L \)). However, we cannot assume the sign of \( \Lambda_{iik} = \lambda_{H2}\lambda_{K1} - \lambda_{H1}\lambda_{K2} \) a priori because it depends on the technological, institutional and trade-related features of the economy.\(^4\)

Exogenously given endowments impose the resource constraints,

\[
a_{L1}X_1 + a_{L2}X_2 + \lambda a_{L2}X_2 = L + L^*, \tag{4}
a_{H1}X_1 + a_{H2}X_2 = H + H^*, \tag{5}
a_{K1}X_1 + a_{K2}X_2 = K + K^* \tag{6}
\]

where \( L, H, \) and \( K \) are the domestic endowments of unskilled labor, skilled labor, and capital, respectively, while \( L^*, H^*, \) and \( K^* \) are the magnitudes of foreign unskilled labor, skilled labor, and capital, respectively. This completes the specification of our model with the fixed endowment of factors and internationally determined prices.

Now using (4), equation (3) may be rewritten as follows.

\(^4\) It may be noted that the agricultural sector in a developing economy use both capital and skilled labour in lesser quantities than manufacturing sector. Despite this, the agricultural can use anyone of those two factors more intensively with respect to the other than the other sector. This has explained in more details in section 6.
The working of the model is as follows. There are six endogenous variables, \( w_1, s, r, X_1, X_2 \) and \( L_u \) in the system and the same number of equations, namely equations (2), (3'), and (4) – (6). The system does not possess the decomposition property. Solving equations (2), (3'), (5) and (6) simultaneously the equilibrium values of \( w_1, s, r, X_1 \) and \( X_2 \) are obtained. \( L_u \) is then found from (4). We should note that all the three factor prices, apart from commodity prices, depend on factor endowments.

There are three groups of unskilled workers in this system earning different wages. Unskilled workers employed in the rural and the urban sectors receive a competitive wage, \( w_1 \), and the unionized wage, \( w_2 \), respectively, while the unemployed urban workers earn nothing.

The average wage for unskilled labour is given by

\[
(w_2 / w_1) a_{L2} X_2 + a_{L1} X_1 = L
\]

(3')

The skilled–unskilled wage gap improves (worsens) in absolute terms if the gap between \( s \) and \( w_1 \) falls (rises). On the other hand, the wage inequality improves (deteriorates) both in absolute and relative terms if \( (s - w_1) < (>) 0 \).

3. Unskilled labor and wage inequality

Let us now analyze the consequences of international mobility of different factors of production on the skilled-unskilled wage inequality. Although skilled labour, unskilled labour

\[ \text{Footnote 5: The average wage of the workers (unskilled workers in this case) in an HT economy is equal to the rural sector wage. This is known as the ‘envelope property’.} \]
and capital move from one country to another simultaneously, to establish ideas we consider the effects of each of these changes one at a time.

Solving (A1) for $\hat{w}_i$ and $\hat{s}$, we obtain

$$\begin{aligned}
\hat{w}_i &= (I * \lambda L * / \Delta) [\Delta_{HK} \Theta_{HK}] + (h * \lambda H * / \Delta) [\Delta_{KL} \Theta_{HK}] \\
&+ (k * \lambda K * / \Delta) [\Delta_{HL} \Theta_{HK}]
\end{aligned}$$

(8)

and,

$$\begin{aligned}
\hat{s} &= (I * \lambda L * \Theta_{L1} \Theta_{K2} / \Delta) \Delta_{KH} + (h * \lambda H * \Theta_{L1} \Theta_{K2} / \Delta) \Delta_{KL} \\
&+ (k * \lambda K * \Theta_{L1} \Theta_{K2} / \Delta) \Delta_{HL}
\end{aligned}$$

(9)

where $\Delta$ is the value of the determinant of the coefficient matrix of the system, $\Theta_{HK} = \Theta_{K1} \Theta_{H2} - \Theta_{H1} \Theta_{K2}$, $I* = L* / L$, $H* = H* / H$, $K* = K* / K$; and, $\theta_{ij}$ is the distributive share of factor $i$ in the $j$th sector (e.g., $\theta_{H2} = sa_{H2} / p_2$). Note that $\text{sign } \Theta_{HK} = \text{sign } \Lambda_{HK}$ due to the formulation of the model, and $\Delta < 0$ if the system is stable.

Subtraction of (8) from (9) yields

$$\begin{aligned}
(\hat{s} - \hat{w}_i) &= (I * \hat{L} * \lambda / \Delta) \Delta_{KH} [\Theta_{L1} \Theta_{K2} - \Theta_{HK}] \\
&+ (h * \hat{H} * \lambda / \Delta) \Delta_{LH} [\Theta_{L1} \Theta_{K2} - \Theta_{HK}] \\
&+ (k * \hat{K} * \lambda / \Delta) \Delta_{LK} [\Theta_{L1} \Theta_{K2} - \Theta_{HK}].
\end{aligned}$$

(10)

Thus, we can see that

$$\begin{aligned}
((\hat{s} - \hat{w}_i) / \hat{L}*) &= (I * \lambda / \Delta) \Delta_{KH} [\Theta_{L1} \Theta_{K2} - \Theta_{HK}] < 0
\end{aligned}$$

(11)

if $\Theta_{HK} > 0$ and $\Theta_{L1} \Theta_{K2} > \Theta_{HK}$, since $\Delta < 0$ and $\text{sign } \Lambda_{HK} = \text{sign } \Theta_{HK}$.

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6 See Appendix A. Interested readers may check this and other mathematical expressions or can obtain the detailed derivations from the authors on request.

7 See Appendix B.
On the other hand, it is also straightforward that

$$\left( (\hat{\delta} - \hat{w}_1) / \hat{L}^* \right) = (I^* / \Delta) \Lambda_{KL} [\theta_{11} \theta_{22} - \Theta_{HK}] > 0,$$

if either (i) $\Theta_{HK} < 0$ or (ii) $\Theta_{HK} > 0$ and $\theta_{11} \theta_{22} < \Theta_{HK}$.

Hence, we can now establish the following proposition:

**Proposition 1:** The inflow of foreign unskilled labor decreases the skilled-unskilled wage inequality if $\Theta_{HK} > 0$ and $\theta_{11} \theta_{22} > \Theta_{HK}$. Conversely, the inflow increases the wage inequality if either (i) $\Theta_{HK} < 0$ or (ii) $\Theta_{HK} > 0$ and $\theta_{11} \theta_{22} < \Theta_{HK}$.

As factor prices depend on factor endowments, an inflow of foreign unskilled labor decreases the rural wage rate and leads to an expansion of the agricultural sector following a Rybczynski effect. The expansion of the sector also attracts both skilled-labor and capital from the manufacturing sector. The other two factor prices will also change depending on the factor intensity conditions between skilled labor and capital (i.e., $\Lambda_{HK}$ or $\Theta_{HK}$).

We obtain from equations (1) and (2) that under internationally given commodity prices and $\hat{w}_1 < 0$,

$$\theta_{H1} \hat{\delta} + \theta_{K1} \hat{r} = -\theta_{11} \hat{w}_1 > 0$$

and

$$\theta_{H2} \hat{\delta} + \theta_{K2} \hat{r} = 0.$$  

If the agricultural sector is skilled-labor intensive (with respect to capital) (i.e., $\Theta_{HK} < 0$), the decrease in $w_1$ raises the skilled wage rate ($s$), and lowers the rental of capital according to the Stolper-Samuelson effects. On the contrary, if the agricultural sector is capital-intensive (i.e. $\Theta_{HK} > 0$), the decrease in $w_1$ reduces the skilled wage rate. The paradox may occur in this case, and an inflow of unskilled labor eventually decreases the skilled-unskilled wage inequality if the proportionate decrease in the skilled wage rate is greater than that in the unskilled wage rate. The proposition provides the sufficient condition for the paradox. The condition signifies that the agricultural sector is capital-intensive (i.e. $\Theta_{HK} > 0$), but the difference in intersectoral capital intensities is not very large (i.e. $\theta_{11} \theta_{22} > \Theta_{HK}$).
The above interesting result crucially hinges on whether the agricultural sector is capital-intensive or skilled labor-intensive relative to the manufacturing sector i.e. on the nature of the production technologies employed in the two sectors of the economy in question. However, it may not be quite unrealistic to suppose that in the developing countries the agricultural sector is capital-intensive rather than skilled labor-intensive compared with the manufacturing sector. It is not difficult to find developing economies where the agricultural sector extensively uses modern agricultural machines, but it has not introduced Hi-Tech computational technologies as yet.

4. Skilled labor, capital inflow and wage inequality

Similar analysis can be conducted with foreign skilled labor and foreign capital. From (10) we have the following results on wage inequality of changes in the endowments of capital and skilled labor, respectively.

\[
\frac{(\hat{\delta} - \hat{\vartheta}_1)}{\hat{H}_*} = \left( h^* \frac{\delta}{\Delta} \right) \Lambda_{Li} \left[ \theta_{11} \theta_{22} - \Theta_{IHK} \right].
\]

and

\[
\frac{(\hat{\delta} - \hat{\vartheta}_1)}{\hat{K}_*} = \left( k^* \frac{\delta}{\Delta} \right) \Lambda_{K} \left[ \theta_{11} \theta_{22} - \Theta_{IHK} \right],
\]

(15) and (16) show that the inflow of skilled labor has qualitatively the same effect on the wage inequality as that of capital. This leads to the following proposition.

**Proposition 2:** The inflow of foreign skilled labor (or capital) decreases the skilled-unskilled wage inequality if \( \theta_{11} \theta_{22} > \Theta_{IHK} \). Conversely, the inflow increases the skilled-unskilled wage inequality if \( \theta_{11} \theta_{22} < \Theta_{IHK} \).

8 Many of the farmers in the agricultural sector of a developing economy like India have now introduced the HYV (High-yielding variety) technology that is intensive in the use of material inputs like fertilizers, pesticides, tractors, harvesters etc. Consequently, the capital-intensity of cultivation with the modern technology is significantly higher than that with the traditional cultivation. See Chaudhuri (2006) in this context.
An inflow of skilled labor exerts downward pressure on the skilled wage rate \((s)\) and also causes the other factor prices to change. The effect on \(w_i\) can be understood from (13) by applying the Stolper-Samuelson theorem and considering that \(\hat{s} < 0\). We find that the decrease in \(s\) raises (lowers) \(w_i\) if the agricultural sector (sector 1) is intensive in the use of skilled labour (capital) i.e. \(\Theta_{hk} < 0 \) (\(\Theta_{hk} > 0\)). So, the skilled-unskilled wage inequality unambiguously improves following an immigration of skilled labour if the agricultural sector is skilled labour-intensive with respect to capital.

The wage inequality may improve even if sector 1 is capital-intensive (i.e. \(\Theta_{hk} > 0\)) provided that difference between the capital intensities of the two sectors (with respect to skilled labour) is not sufficiently high. Let us explain this case in details. An inflow of skilled labor leads to an expansion of the manufacturing sector (sector 2) and a contraction of the agricultural sector (sector 1) following a Rybczynski effect as sector 2 is now skilled labour-intensive with respect to capital. Capital is drained out of sector 1 to sector 2. The extent of contraction in sector 1 depends on the magnitude of capital that moves out which would in turn crucially hinge on the difference in capital intensities of the two sectors. The lower the difference, the lesser would be the amount of capital transfer from sector 1 to sector 2 and hence the smaller would be the contraction in output and employment of unskilled labour in sector 1. This, in turn, leads to and a smaller decrease in the unskilled wage, \(w_i\). Our analysis suggests that if the difference in the intersectoral capital intensities\(^9\) is sufficiently small i.e. \(\theta_{l1} \theta_{k2} > \Theta_{hk}\), the rate of decrease in \(w_i\) would be less than in \(s\). Hence the skilled-unskilled wage inequality improves in this case following an immigration of skilled labour. However, if the capital-intensity difference is sufficiently large i.e. \(\theta_{l1} \theta_{k2} < \Theta_{hk}\) the wage inequality deteriorates as the unskilled wage falls more than the skilled wage. The latter case

\(^9\) It may be noted in this context that:
\[
\Theta_{hk} = \theta_{k1} \theta_{k2} ((\theta_{h2} / \theta_{k2}) - (\theta_{l1} / \theta_{k1})) = \theta_{l1} \theta_{h2} ((\theta_{h2} / \theta_{k2}) - (\theta_{l1} / \theta_{k1})).
\] So if sector 2 is highly skilled labour-intensive with respect to capital, sector 1 is highly capital-intensive with respect to skilled labour.
is highly interesting as it is contrary to the common wisdom. Equation (14) provides the necessary and sufficient conditions for the change in the wage inequality.

It is easy to derive from (15) the proposition on wage inequality of an inflow of foreign capital. An inflow of skilled labor has qualitatively the same effects on wages as those of capital. The result can, therefore, be interpreted similarly.

5. Comparison of different policies

Considering the economy in question is a developing country, it may be a conventional policy to send some of the unskilled workers to other countries to work. Thus, in this case, the proposition must be read as:

Proposition 1': An emigration of unskilled labor increases the skilled-unskilled wage inequality if $\theta_{l1}\theta_{K2} > \Theta_{HK} > 0$. Conversely, the wage inequality improves if either (i) $\Theta_{HK} < 0$ or (ii) $0 < \theta_{l1}\theta_{K2} < \Theta_{HK}$.

This implies that the emigration policy is successful in bringing down the relative wage inequality if either (i) $\Theta_{HK} < 0$; or, (ii) $0 < \theta_{l1}\theta_{K2} < \Theta_{HK}$ holds. In other words, the result is valid either if the agricultural sector is skilled-labor intensive (i.e., $\Theta_{HK} < 0$), or if the agricultural sector is highly capital-intensive with respect to skilled labour (i.e. $0 < \theta_{l1}\theta_{K2} < \Theta_{HK}$).

In the context of the developing economies which are usually plagued by high degree of relative wage inequality and serious unemployment problem of unskilled labour, the possible counterractive policy is to induce foreign skilled labor and/or capital into their economies so as to make enough employment opportunities for unskilled workers. The analysis of the previous section shows that the introduction of skilled labor or capital into the country decreases the wage inequality if $\theta_{l1}\theta_{K2} > \Theta_{HK}$. But, this policy does not deliver the goods if this condition does not hold. So, it is important to compare the efficacy of an inflow of foreign capital and/or skilled labour with that of the conventional emigration policy under different factor-intensity conditions.

If the agricultural sector is skilled-labor intensive (i.e., $\Theta_{HK} < 0$), all the three policies can successfully improve the wage inequality because the required condition
that $\theta_{11} \theta_{22} > 0 > \Theta_{HK}$ is automatically satisfied. Thus, any one of those three policies or any policy mix is effective for the purpose. On the contrary, if the manufacturing sector is skilled-labor intensive (i.e., $\Theta_{HK} > 0$) the policymakers have to select the appropriate policy taking into consideration the magnitude of difference between the intersectoral capital intensities. If this difference is sufficiently high (in the sense that $0 < \theta_{11} \theta_{22} < \Theta_{HK}$), the emigration policy should be employed in order to improve the wage inequality. On the contrary, if the manufacturing sector is skilled-labor intensive but the difference of capital intensities is small enough (in the sense that $0 < \Theta_{HK} < \theta_{11} \theta_{22}$), the policy to induce foreign skilled labor and/or capital into the country is effective.

### 6. Concluding remarks

Increasing skilled-unskilled wage inequality and unemployment of unskilled labour are two major exacerbating problems of the developing world in the liberalized economic regime. The economists and policymakers are searching for appropriate policies for tackling these problems. The conventional emigration policy of unskilled labour cannot produce the desired results in all situations. It is, therefore, extremely important to know when this policy works and what policy should be undertaken when it does not work. These are the two issues which have been addressed in this paper using a two-sector three-mobile-factor Harris-Todaro model.

We have found that all the three policies are effective in improving the wage inequality when the agricultural sector is skilled-labor intensive. But, if the primary sector is capital-intensive the conventional emigration policy of unskilled labour may not succeed in turning the relative wages to move in favour of unskilled labour. The conventional policy remains effective so long as the difference in intersectoral capital intensities is sufficiently high. An inflow of foreign capital and/or skilled labour should be given the top priority when the capital-intensity gap is sufficiently low.

However, it should be pointed out that the agricultural sector in a developing economy use both capital and skilled labour in lesser quantities vis-à-vis the manufacturing sector. Despite this, the agricultural sector can use anyone of those two factors more intensively with respect to the other than the secondary sector. Therefore, the first and the foremost task for the policymakers should be to determine empirically the relative factor intensities of the sectors. Once these are obtained, the appropriate policy, as suggested in the previous section, should be followed so as to improve the skilled-unskilled wage inequality.
Appendices

[A] Comparative statics

Differentiating equations (1) – (6) and writing in a matrix notation we obtain

\[
\begin{bmatrix}
0 & \theta_{H2} & \theta_{K2} & 0 & 0 & 0 \\
\theta_{L1} & \theta_{H1} & \theta_{K1} & 0 & 0 & 0 \\
\lambda_{L1}S_{L1}^1 & A & B (1+\lambda)\lambda_{L2} & \lambda_{L1} & \lambda\lambda_{L2} & \hat{r} \\
\lambda_{H1}S_{H1}^1 & C & D & \lambda_{H2} & \lambda_{H1} & 0 \\
\lambda_{K1}S_{K1}^1 & E & F & \lambda_{K2} & \lambda_{K1} & 0 \\
(1+\lambda) & 0 & 0 & 0 & 0 & \lambda
\end{bmatrix}
\begin{bmatrix}
\hat{w}_1 \\
\hat{\theta}_1 \\
\hat{p}_1 \\
\hat{\lambda}_2 \\
\hat{X}_1 \\
\hat{\lambda}
\end{bmatrix}
= \begin{bmatrix}
\hat{\theta}_2 \\
\hat{p}_2 \\
\hat{L} \\
\hat{H} \\
\hat{K} \\
0
\end{bmatrix}
\]

(A1)

where \( \lambda^* \equiv L^*/L, \ h^* \equiv H^*/H, \ k^* \equiv K^*/K, \ \theta_{ij} \) is the distributive share of factor \( i \) in the \( j \)th sector (e.g., \( \theta_{H2} = sa_{H2}/p_2 \)), \( \lambda_{ij} \) is the allocative share of factor \( i \) in the \( j \)th sector (e.g., \( \lambda_{L1} = a_{L1}X_1/L \)), \( S_{ij}^k \) is the partial elasticity of substitution between factors \( i \) and \( j \) in the \( k \)th sector (e.g., \( S_{LH}^2 \equiv (s/a_{L2})(\partial a_{L2}/\partial s) \)),

\[
A \equiv (1+\lambda)\lambda_{L2}S_{LH}^2 + \lambda_{L1}S_{L1}^2 > 0, \quad B \equiv (1+\lambda)\lambda_{L2}S_{LH}^2 + \lambda_{L1}S_{L1}^2 > 0,
\]

\[
C \equiv \lambda_{H2}S_{H1}^1 + \lambda_{H1}S_{H1}^1 < 0, \quad D \equiv \lambda_{H2}S_{H1}^1 + \lambda_{H1}S_{H1}^1 > 0,
\]

\[
E \equiv \lambda_{K2}S_{K1}^1 + \lambda_{K1}S_{K1}^1 > 0, \quad F \equiv \lambda_{K2}S_{K1}^1 + \lambda_{K1}S_{K1}^1 < 0.
\]

[B] Stability

Under the present setup, the dynamic adjustment process for the supply side of the model is specified as follows:

\[
\hat{X}_1 = d_1(p_1 - a_{L1}w_1 - a_{H1}s - a_{K1}r) \quad \text{(A2)}
\]

\[
\hat{X}_2 = d_2(p_2 - a_{L2}w_2 - a_{H2}s - a_{K2}r) \quad \text{(A3)}
\]

\[
\hat{w}_1 = d_3[a_{L1}X_1 + a_{L2}X_2 + \lambda a_{L2}X_2 - (L + L^*)] \quad \text{(A4)}
\]

\[
\hat{s} = d_4[a_{H1}X_1 + a_{H2}X_2 - (H + H^*)] \quad \text{(A5)}
\]

\[
\hat{r} = d_5[a_{K1}X_1 + a_{K2}X_2 - (K + K^*)] \quad \text{(A6)}
\]

\[
\hat{\lambda} = d_6[w_2 - (1+\lambda)w_1] \quad \text{(A7)}
\]

where “·” denotes differentiation with respect to time, and \( d_j \) is the positive coefficient measuring the speed of adjustment. A Marshallian adjustment process is assumed for
quantities when the demand price (i.e., the exogenously given price of goods) differs from the supply price (i.e., the average cost of producing a given commodity) in the goods markets. In the factor markets, we assume a Walrasian adjustment mechanism with the fixed endowment assumption, implying that returns will have to adjust.

The Jacobian matrix of the system of simultaneous equations (A2)-(A7) is

\[
J = \frac{d\lambda_{L1} + \lambda_{L2}}{p_1 p_2 LHK} \begin{bmatrix}
0 & 0 & 0 & \theta_{H2} & \theta_{K2} & 0 \\
0 & 0 & \theta_{L1} & \theta_{L1} & \theta_{K1} & 0 \\
\lambda_{L1} & (1 + \lambda)\lambda_{L2} & \lambda_{L1}\lambda_{L1} & A & B & \lambda\lambda_{L2} \\
\lambda_{H1} & \lambda_{H2} & \lambda_{H1}\lambda_{H1} & C & D & 0 \\
\lambda_{K1} & \lambda_{K2} & \lambda_{K1}\lambda_{KL} & E & F & 0 \\
0 & 0 & (1 + \lambda) & 0 & 0 & \lambda
\end{bmatrix}.
\]  

(A8)

Therefore, we can show that

\[
|J| = -(d_1 d_2 d_3 d_4 d_5 d_6 w_{srX_1 X_2} / p_1 p_2 p_3 LK V)\Delta.
\]  

(A9)

According to the Routh-Hurwitz theorem, a necessary condition for the local stability of the system is that \(\text{sign}|J| = (-1)^k\), where \(k\) is the number of rows (and hence, columns) of the system of the simultaneous equations. Hence, we assume that our equilibrium is stable, which implies that \(|J| > 0\); therefore, \(\Delta < 0\) from (A9).
References:


