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Abstract

This paper develops a macroeconomic model of the interaction between consumer debt and firm debt over the business cycle. I incorporate interest rate spreads generated by firm and household loan default risk into a real business cycle model. I estimate the model on US aggregate data. This allows me to analyse the quantitative importance of possible feedback effects between the debt levels of firms and households, and the relative contributions of financial and supply shocks to economic fluctuations. While firm level credit market frictions significantly amplify the response of investment to shocks, they do not amplify output responses. In general equilibrium, higher external financing spreads for households contribute to lower external financing spreads for firms, contrary to traditional Keynesian predictions. Furthermore, total factor productivity shocks remain an important source of business cycles in my model. They are responsible for 71 - 74% of the variance of output and 56 - 69% of the variance of consumption in the model. Financial shocks are important in explaining interest rate spreads and leverage ratios, but they account for less than 11% of the fluctuations in output. My results suggest that other factors, beyond credit market frictions on their own, are necessary to justify an important role for financial shocks in aggregate output fluctuations.

JEL classification: E3, E4, G3.

Key words: Financial frictions, external finance premium, DSGE models, Bayesian estimation, business cycles.

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1 Introduction

Financing frictions are often suggested as a prime candidate for amplifying the effects of shocks hitting the economy. The basic idea, often called the financial accelerator, is that in the presence of credit constraints exogenous shocks can generate a positive feedback effect between the financial health of borrowing firms or households and output (Bernanke, Gertler and Gilchrist (1999) [9]). The standard approach to analysing credit constraints over the business cycle focuses either on households or on firms in isolation (see Christiano et al. (2010) [23] and Iacoviello and Neri (2009) [38] for prominent examples). This assumes that the strength and the effect of the two types of financing frictions are independent of each other. However, if both household and firm level financing frictions create financial accelerators which on their own amplify output fluctuations, then intuitively there could be a positive interaction between them. In that case, focusing on only one type of financing frictions at a time could significantly underestimate their overall effect on business cycles. To quote Bernanke et al (1999) [9]:

" By enforcing the standard consumption Euler equation (in the firm financial accelerator model), we are effectively assuming that financial market frictions do not impede household behavior... An interesting extension of this model would be to incorporate household borrowing and associated frictions. With some slight modification, the financial accelerator would then also apply to household spending, strengthening the overall effect."

This paper builds a dynamic stochastic general equilibrium (DSGE) model that integrates both firm and household debt, while improving in several dimensions on the most popular existing models such as Bernanke et al (1999) [9] and Iacoviello (2005) [37]. I incorporate external financing spreads faced by firms and households due to loan default risk, generating endogenous movements in the debt to collateral ratios (henceforth also leverage ratios) of borrowers. The modeling framework that I develop has a rich pattern of interactions between external financing conditions, consumption and production decisions. It provides a partial microfoundation for preference and risk premium shocks that some models without explicit financial frictions have found to be important (e.g Smets and Wouters (2007) [55]). At the same time it is tractable enough to be used more generally in business cycle and monetary policy models. [1]I use a mixture of Bayesian estimation and calibration to determine the model's parameters. Using these estimates, I examine the possibility of feedback effects between the strength of financing frictions affecting firms and households, and evaluate the relative importance of financial and supply shocks as alternative sources of business cycles.

[1]For example, Daracq-Paries et al (2010) [25] have extended the model in an earlier version of this paper to analyse bank capital regulation in an environment with banking frictions and nominal price rigidities.

A key premise underlying the possibility of positive interaction is that worse household financial conditions contribute to reductions in aggregate economic activity. Mian and Sufi (2009) [49] provide some empirical evidence on this point. They rank US counties by the growth rate in household leverage (measured by debt to income) in 2002-2006. They find that in 2006-2008 the unemployment rate increased by about 2.5% more in the top 10% leverage growth counties than in the bottom 10% leverage growth counties. This sort of evidence is suggestive of a positive relation between the leverage level of households and the sensitivity of aggregate economic activity to shocks, which can then affect the tightness of firms' borrowing constraints through a financial accelerator effect. However, it does not necessarily represent a causal link from worse financial conditions for households to worse financial conditions for firms. Mian and Sufi's correlations are also compatible with a situation in which a common factor caused both higher business and household leverage in certain counties in the early 2000's, subsequently making these counties more vulnerable to a recession. To gain a better understanding of the causal relation between firm and household sector borrowing requires a more structural general equilibrium approach.

The main economic agents in my model are financially constrained firms that can borrow by using revenue and capital as collateral, financially constrained households that use debt collateralised by housing, and financially unconstrained households that fund borrowers. I focus on real estate collateralised loans for households since this is by far the most common type of loan, accounting for 81.5% of household debt in the US Survey of Consumer Finances of 2001 (Campbell and Hercowitz (2006) [13]). I follow other macroeconomic models of financial frictions in using differences in the level of impatience of agents to generate equilibrium borrowing and lending (e.g. Iacoviello (2005) [37]). Both borrowing firms and households are affected by idiosyncratic shocks to their collateral values. While borrowers can insure themselves against the idiosyncratic risk, as in representative agent business cycle models, the lender cannot seize the proceeds of the insurance. As a result, borrowers default on their loans when the value of their collateral is below the repayment promised to the lender. In case of default, the lender can seize collateral at a cost. The combination of insurance and limited liability partially preserves the effects of risk averse/consumption-smoothing behaviour of agents despite the ex-ante heterogeneity among agents and the nonlinear default decision. [2] The possibility of default generates a trade-off for borrowers between a bigger loan relative to the collateral (the leverage ratio) and a lower interest rate on the loan. This is a realistic feature of loan contracts that is missing from models without equilibrium default. In light of the renewed interest in studying the effect of turbulence in financial markets on the macroeconomy I add financial shocks to the model, measured as exogenous changes in the efficiency of the collateral seizure technology. This can be seen as a reduced form modeling of changes in the level of asymmetric information or moral hazard related to financially distressed households or firms, or more generally financial intermediaries whose

[2] These insurance contracts are mathematically equivalent to the the large household assumption in Shi (1997) [54] and other labour market and monetary search models.

portfolios are heavily loaded with loans to distressed borrowers. [3]

As a benchmark, I focus on a model with flexible price and wage adjustment. I find that credit constraints on their own are not enough to overturn the important role of total factor productivity (henceforth TFP) shocks in generating business cycles. At the posterior mode parameter estimates these shocks account for 71% - 74% of the long run variance of output (depending on the definition of the trend). While financial constraints amplify the response of investment to TFP shocks, they do not significantly amplify the response of output except for the first period or two after a TFP shock. I find that in general equilibrium worse credit frictions for households tend to reduce credit frictions for firms, though other common factors can generate positive comovement between firm and household credit spreads. Finally, household credit frictions do not create bigger fluctuations in housing investment and prices relative to a frictionless model.

TFP shocks remain important in my model because financing frictions on their own do not generate sufficient fluctuations in output in the short run, when capital is approximately fixed. Absent movements in TFP, we need a strong procyclicality of labour demand and the capital utilisation rate to move output. In my framework, financing frictions affect hiring and capital utilisation decisions of firms. In partial equilibrium a higher cost of external financing reduces labour demand and the capital utilisation rate in a recession, and vice versa in a boom. In general equilibrium the more procyclical price of capital in the model with financing frictions reduces the procyclicality of capital utilisation, by making capital depreciation more costly in expansions and less costly in recessions. Combining these effects, we still need exogenous changes in TFP to generate realistic output movements. The reduced procyclicality of capital utilisation rates relative to a frictionless benchmark helps explain why financing frictions do not significantly increase the volatility of output fluctuations. [4]

That higher household financing frictions can reduce firm financing frictions may seem surprising, at least if one expects household financing frictions to act like a financial accelerator. A recession in my model worsens borrowing households' external financing conditions and reduces their consumption more severely than without financing frictions. A traditional Keynesian analysis would be that by reducing firms' revenue that are part of collateral, this should worsen financing conditions for firms. In my model prices are flexible, so that output is not aggregate demand determined for some firms in the short run as in sticky price models. Nevertheless, with external financing costs that in general equilibrium are in part a function of aggregate output and with adjustment costs hindering the reallocation of resources between production of consumption and investment goods, it may still be possible

[3] See Jermann and Quadrini (2008) [40] and Christiano et al. (2010) [23] for alternative ways of introducing financial or credit shocks.

[4] Another way of thinking about this is that while financing frictions affecting labour demand and capital utilisation can produce countercyclical "markups" that amplify output fluctuations, similar to imperfect competition models (e.g Jaimovich and Floetotto (2008) [39]), this countercylicality is relatively weak for my estimated parameter values.

to get a positive interaction between household and firm financing frictions even with flexible price adjustment. At the same time there are other effects going in the opposite direction. First, higher financing frictions for households reduce their demand for loans, decreasing interest rates faced by firms. This lowers firms' external financing frictions. Second, greater difficulties in getting external financing encourage borrowers to increase labour supply to compensate, similar to the effect of a decline in household wealth. This reduces wage rates and raises firms' output. Since output is partially used as collateral, this increases the value of firms' collateral and improves their financing conditions. The overall result for my estimated parameter values is that worse financing conditions for firms actually lead to a slight improvement in firms' financing conditions.

The paper proceeds as follows: section 2 describes the model and provides a theoretical analysis of the impact of financial frictions. I start with a partial equilibrium analysis, and then examine the general equilibrium links between firm and household credit frictions. Section 3 discusses the estimation of the model and its quantitative implications in terms of the response of the economy to shocks, the sources of fluctuations in the model and the model's prediction for macroeconomic time series statistics. Section 4 concludes.

1.1 Related Literature

Most existing models of household borrowing with aggregate fluctuations follow Kiyotaki and Moore (1997) [42] or Iacoviello (2005) [37] in using a pure quantity borrowing constraint and assuming it always binds. The assumption of an always binding quantity borrowing constraint may fail for large shocks, and it may severely distort the dynamics of borrowers and the rest of the economy in those circumstances. It also eliminates endogenous movements in borrowers' leverage level. The equilibrium default mechanism in my model (with interest rates rising smoothly as a function of borrowing) gets around this issue. Furthermore, the model proposed here can at least qualitatively match the countercyclical leverage ratio of firms and households found in the US by Adrian and Shin (2008) [1]. At the same time, the financial shocks in my model can potentially generate episodes of procyclical leverage, like the early 2000's in the US.

Bernanke et al. (henceforth BGG) (1999) [9], and Carlstrom and Fuerst (1997) [15] introduced equilibrium default of firms into DSGE models. To facilitate aggregation, they assumed risk neutral entrepreneurs. In contrast, I use a setup with both equilibrium default and risk averse agents. The consumption smoothing motive and risk aversion of owners may play an important role in the decisions of firms. Assuming risk neutrality eliminates these factors from the analysis. The risk aversion of borrowers in my model also makes it more applicable to households. Aoki et al (2004) [5] model equilibrium default on loans collateralised by housing in a DSGE model with aggregate shocks. They adapt the BGG framework to housing by positing the existence of a special class of risk neutral home owners that rent homes to households. The financing frictions in their model apply to these home owners as opposed to the risk averse households. In order to model an effect of housing wealth on household consumption they are forced to adopt an ad-hoc dividend payment rule between home owners and households as well as assuming rule of thumb consumers that simply consume all their wealth each period.

To my knowledge, this is the first business cycle model that takes into account default risk on both consumer and firm loans. This allows me to examine the impact of endogenous time varying interest rate spreads and leverage ratios. My model of firms' financial constraints allows me to consider a more standard formulation of entrepreneur balance sheets than the less conventional balance sheets used by BGG or Carlstrom and Fuerst to make their models tractable. In particular, firms in my model own their capital stock, as in more sophisticated heterogeneous agent models of financing constraints, and do not have to repurchase it or rent it each period as in BGG or Carlstrom and Fuerst (assumptions which are no longer without loss of generality in the presence of financing frictions). My model also allows the researcher to consider other nonlinearities in the balance sheet of financially constrained firms, such as imperfect competition or labour adjustment costs. This flexibility may be important in extensions to study the interaction of credit constraints and pricing or labour demand in more detail.

Iacoviello (2005) [37] and Gerali et al (2010) [32] also model financing frictions affecting both households and firms. Both of these papers rely on quantity borrowing constraints as in Kiyotaki and Moore [42] to model credit frictions and assume the borrowing constraints always bind. The analysis in this paper of an environment with default costs and actual lending spreads provides an alternative perspective. These papers do not explicitly examine the effect of modeling both types of financing frictions as opposed to just one type, choosing to focus on other issues. They assume a fixed stock of structures, eliminating any role for residential investment, or forcing a counterfactual negative correlation between business structures and residential investment. Finally, they analyse models with nominal rigidities which mix real effects of financing frictions with other channels. In contrast, this paper isolates the interaction between household and firm financing frictions in the flexible price and wage equilibrium of the economy. This equilibrium defines an output gap relative to the equilibrium with sticky prices or wages that is critical in monetary policy analysis. Therefore, the results should also be relevant to models with nominal rigidities. [5]

[5] At the same time, the importance of nominal price and wage rigidities for aggregate output dynamics is not without controversy (e.g Barro (1977) [7], Pissarides (2008) [51] and Williamson and Wright (2010) [58]). From this perspective, understanding the effects of financing constraints with flexible prices and wages may be even more important.

2 A Model of Firm and Household Leverage

There are two types of households distinguished by their discount factors. Patient households with a relatively high discount factor lend to other households and firms, as well as owning some of the firms in the economy. Impatient households with a lower discount factor borrow from the patient households to finance housing and consumption subject to financing frictions. Financially constrained entrepreneurs own the capital of the economy and produce final output. They borrow from patient households to finance consumption , investment and wages subject to credit constraints. These entrepreneurs are also more impatient than the lenders. [6] Capital and housing producers transform final output into new capital and housing subject to adjustment costs, and are owned by the patient households.

2.1 The Household Sector

2.1.1 Patient Households (savers)

There is a measure θ^s of patient households that have a relatively high discount factor, and access to complete financial markets without any financing constraints. Households derive utility from non durable consumption, leisure and housing. They provide loans through banks to firms and households. Following BGG(1999) [9] and Iacoviello(2005) [37] I assume that the deposits are risk free in aggregate. The representative saver picks non-negative sequences of consumption, working hours, housing and deposits at the bank

$$\{c_{s,t}, n_{s,t}, h_{s,t}, d_t\}_{t=0}^{\infty}$$

to maximise

$$E_0 \Sigma_{t=0}^{\infty} \beta^t u_{s,t}, \tag{1}$$

where

$$\begin{aligned} u_{s,t} &= \frac{(c_{s,t}^{\xi_c} h_{s,t}^{\xi_h} (1-n_{s,t})^{\xi_n})^{1-\sigma}}{1-\sigma}\ , \text{ for } \sigma \neq 1, \\ u_{s,t} &= \xi_c \ln c_{s,t} + \xi_h \ln h_{s,t} + (1-\xi_c-\xi_h)\ln(1-n_{s,t}), \text{ for } \sigma = 1, \\ \xi_n &= 1-\xi_c-\xi_h. \end{aligned} \tag{2}$$

[6] For other models using discount factor differences to generate equilibrium borrowing, see Iacoviello(2005) [37] and Krusell and Smith(1998)[45] for households, Carlstrom and Fuerst(1997,1998[15][16]) and Kiyotaki and Moore (1997) [42] for firms. There is some evidence from estimation of structural consumption models supporting this heterogeneity in preferences (Cagetti(2003)[12]). Higher impatience is also a reduced form proxy for higher expected income growth (see Browning and Tobacman's(2007)[10]) , or Carroll (2000)[17]). Under this interpretation, the impatient agents roughly correspond to young homeowners with an upward sloping expected wage profile that use borrowing to enjoy some of their higher future salaries in terms of consumption today. The patient households can be thought of as middle aged households with relatively low expected salary growth and higher savings. Survey of Consumer Finances data on the lifecycle pattern of wage growth and financial asset holdings are consistent with this interpretations (Hintermaier and Koeniger (2009) [36]). The entrepreneur‘s relative impatience can also be interpreted as reflecting a higher expected future profit growth relative to more mature but financially unconstrained firms, or a higher death rate of entrepreneurial firms relative to households.

subject to a sequence of constraints

$$c_{s,t} + q_t[h_t^s - (1-\delta_h)h_{s,t-1}] + d_t = R_t d_{t-1} + w_t n_{s,t} + \Pi_{s,t}, \tag{3}$$

where Π_t^s are profits from housing and capital producers.

2.1.2 Impatient Households (borrowers)

There is a measure $\theta^{bo} \equiv 1 - \theta^s$ of impatient households. They have the same intra-period preferences over housing,consumption and leisure as patient households, but they have a lower discount factor than lenders(patient households):

$$\beta^{bo} < \beta.$$

The lower discount factor means that impatient households will be borrowers in a neighborhood of the steady state. Without any frictions their borrowing would be unbounded in the steady state. Financing frictions make borrowing $l_{bo,t}$ bounded. The value of borrowers' housing stock is subject to idiosyncratic shocks $\varepsilon_{bo,t}$ that are i.i.d across borrowers and across time. $\varepsilon_{bo,t}$ has a CDF

$$F(\varepsilon_{bo,t}), \text{ with } F'(\varepsilon_{bo,t}) = f(\varepsilon_{bo,t}), \text{ and } E(\varepsilon_{bo}) = 1. \tag{4}$$

As in Bernanke, Gertler and Gilchrist(1999) [9], I assume $\varepsilon_{bo,t}$ follows a log normal distribution, so that

$$\frac{d\left(\frac{\bar{\varepsilon} f(\bar{\varepsilon})}{1-F(\bar{\varepsilon})}\right)}{d\bar{\varepsilon}_e} > 0$$

over the range that is relevant for the optimal debt contract. It is useful to define resources before debt repayment and new loans

$$\begin{aligned} A_{bo,t} &= \varepsilon_{bo,t} q_t (1-\delta_h) h_{bo,t-1} + n_{bo,t} w_t, \\ \bar{A}_{bo,t} &= q_t (1-\delta_h) h_{bo,t-1} + n_{bo,t} w_t. \end{aligned} \tag{5}$$

Lending in this economy is only possible through 1-period debt contracts that require a constant repayment $R_t^l l_{bo,t-1}$ independent of $\varepsilon_{bo,t}$ for the borrower to avoid costly loan monitoring or enforcement, where R_t^l is the loan rate. The borrower can default and refuse to repay the debt. Savers cannot force borrowers to repay. Instead lending must be intermediated by banks that have a loan enforcement technology allowing them to seize collateral

$$\varepsilon_{bo,t} \tilde{A}_{bo,t} = \varepsilon_{bo,t} q_t (1-\delta_h) h_{bo,t-1} \tag{6}$$

at a proportional cost $\mu_{bo,t} \varepsilon_{bo,t} \tilde{A}_{bo,t}$ when the borrower defaults. $\mu_{bo,t} \in (0,1)$ determines the deadweight cost of default. To introduce financial shocks, I assume this parameter is stochastic with

$$\begin{aligned} \ln\left(\frac{\mu_{bo,t+1}}{1-\mu_{bo,t+1}}\right) - \ln\left(\frac{\mu_{bo}}{1-\mu_{bo}}\right) &= \rho_\mu \left[\ln\left(\frac{\mu_{bo,t}}{1-\mu_{bo,t}}\right) - \ln\left(\frac{\mu_{bo}}{1-\mu_{bo}}\right)\right] + \varepsilon_{\mu,t}, \\ \varepsilon_{\mu,t} &\sim N(0, \sigma_\mu). \end{aligned} \tag{7}$$

The timing of the innovation $\varepsilon_{\mu,t}$ is such that $\mu_{bo,t+1}$ is known by both parties when signing the loan contract to be repaid at $t+1$. If the borrower defaults, the bank can seize the collateral $\varepsilon_{bo,t}\tilde{A}_{bo,t}$. Suppose first that the borrower does not have access to any insurance against the idiosyncratic $\varepsilon_{bo,t}$ shock. When

$$\varepsilon_{bo,t} < \bar{\varepsilon}_{bo,t}$$

the borrower prefers to default and lose

$$\varepsilon_{bo,t}\tilde{A}_{bo,t} < R_t^l l_{bo,t-1} \equiv \bar{\varepsilon}_{bo,t}\tilde{A}_{bo,t} \tag{8}$$

when the bank enforces the contract. On the other hand when

$$\varepsilon_{bo,t} \geq \bar{\varepsilon}_{bo,t}$$

the borrower prefers to pay $R_t^l l_{bo,t-1}$ rather than lose

$$\varepsilon_{bo,t}\tilde{A}_{bo,t} \geq R_t^l l_{bo,t-1}.$$

To be able to use a representative agent framework while maintaining the intuition of the default rule above, I make two assumptions. First, borrowers' labour supply is predetermined with respect to the idiosyncratic shock. Second, borrowers have access to insurance contracts providing them with payments conditional on the realisation of $\varepsilon_{bo,t}$,

$$p_{bo}(\bar{A}_{bo,t} - A_{bo,t} + \min[\varepsilon_{bo,t}, \bar{\varepsilon}_{bo,t}]\tilde{A}_{bo,t} - [(1-F(\bar{\varepsilon}_{bo,t}))\bar{\varepsilon}_{bo,t} + \int_0^{\bar{\varepsilon}_{bo,t}} \varepsilon dF]\tilde{A}_{bot})$$

with

$$0 \leq p_{bo} \leq 1.$$

The insurer can fully diversify $\varepsilon_{bo,t}$ across many households. Averaging across all households it makes zero profits. I restrict contracts to complete insurance with $p_{bo} = 1$, guaranteeing the borrower the expected ex-ante value of his house net of loan repayments. Risk averse borrowers willingly buy this contract, which completely diversifies the risk related to $\varepsilon_{bo,t}$. The insurance payments cannot be seized by the bank. The borrower cannot commit to always repay the loan even though from an ex-ante perspective it is optimal to do so. Therefore, as in the case of uninsured risk, the borrower defaults for low values of the collateral. The borrower repays the lender

$$\min[\varepsilon_{bo,t}, \bar{\varepsilon}_{bo,t}]\tilde{A}_{bo,t}.$$

After taking into account the insurance, this leaves the borrower with total resources before new loans of

$$\bar{A}_{bo,t} - RP(\bar{\varepsilon}_{bo,t})\tilde{A}_{bo,t}, \text{ where} \tag{9}$$

$$RP(\bar{\varepsilon}_{bot}) = (1-F(\bar{\varepsilon}_{bo,t}))\bar{\varepsilon}_{bo,t} + \int_0^{\bar{\varepsilon}_{bo,t}} \varepsilon_{bo} dF.$$

The repayment function $RP(\bar{\varepsilon}_{bot})$ gives the proportion of the collateral's expected value lost to the lender. [7] While this level of insurance is clearly exaggerated, it provides a significant gain in tractability. The $p_{bo} = 1$ case of full insurance can also be used as a point around which to approximate more realistic partial insurance or the uninsurable risk case with $p_{bo} = 0$ through perturbation methods. [8] For these reasons, the full insurance assumption is a useful benchmark for starting the analysis. Even with full insurance, we can still have a nondegenerate distribution of default rates and loan positions for arbitrary initial resource distributions among borrowers. To further simplify the model, I assume a symmetric initial distribution of housing among borrowers. This leads to a symmetric distribution of assets and income across borrowers, and allows us to discuss their choices using a representative borrower (henceforth also called the borrower).

Define the rate of return required on loans made at $t-1$ as $\bar{R}_t$ (to be distinguished from the loan rate R_t^l). Since $\varepsilon_{bo,t}$ is idiosyncratic the bank can diversify it by lending to a large number of borrowers. Therefore it only requires that the loan is profitable in expectation. Loan rates depend on the aggregate state of the economy as in Bernanke et al. (1999) [9]. In order to participate in the loan, the bank requires that

$$[1 - F(\bar{\varepsilon}_{bo,t})]\,\bar{\varepsilon}_{bo,t}\tilde{A}_{bo,t} + (1-\mu_{bo,t})\int_0^{\bar{\varepsilon}_{bo,t}} \tilde{A}_{bo,t}\varepsilon_{bo}dF \geq \bar{R}_t l_{bo,t-1}. \tag{10}$$

The bank participation constraint above will act as a borrowing constraint in this model. Competition among banks makes the participation constraint bind. Defining

$$G(\bar{\varepsilon}_{_{bo,t}}) = RP(\bar{\varepsilon}_{bot}) - \mu_{bo,t}\int_0^{\bar{\varepsilon}_{bo,t}} \varepsilon_{bo}dF, \tag{11}$$

we can rewrite the participation constraint as

$$G(\bar{\varepsilon}_{bo,t})\tilde{A}_{bo,t} = \bar{R}_t l_{bo,t-1}. \tag{12}$$

Since deposits are required to be risk-free, $\bar{R}_t = R_t$, the risk free rate. $G(\bar{\varepsilon}_{bo,t})$ is the loan to collateral ratio, a measure of leverage. [9] $G(\bar{\varepsilon}_{bo,t})$ is increasing in the default threshold $\bar{\varepsilon}_{bot}$ at

[7] This default rule only holds without moving costs for housing. In a model with realistic non-convex moving costs, the default rule would be adjusted so that the household only considers defaulting when it decides to move(e.g Garriga et al (2009) [30]). My derivation of the default rule also assumes that deposits are not seizable by the bank. This is without loss of generality: Suppose borrowers' deposits $d_{bo,t-1}$(or another safe asset such as money) can be seized by lenders to repay debt at a cost $\mu_d R_t d_{bo,t-1}$ with $\mu_d \geq 0$. If $\mu_d > 0$, a borrower would never want to hold deposits and loans simultaneously. In the special case of $\mu_d = 0$ and $\bar{R}_t = R_t$(deposits are a perfect collateral), $l_{bo,t} - d_{bo,t}$ is indeterminate, and we can set $d_{bot} = 0$ without loss of generality.

[8] See Algan et al (2009)[2] for an example of using a model without idiosyncratic shocks or with insurance against idiosyncratic shocks as a point around which to approximate a model without insurance.

[9] Another frequently used definition of leverage is the ratio of collateral to equity in the loan. This measure is an increasing function of the loan to collateral ratio.

an optimum. Therefore we can solve for a function $\bar{\varepsilon}_{bo}(\frac{R_t l_{bo,t-1}}{\tilde{A}_{bo,t}})$, with the default threshold $\bar{\varepsilon}_{bo,t}$ increasing in leverage. For a given idiosyncratic risk distribution the default rate is increasing in the default threshold, so that a higher debt to collateral ratio also increases the probability of default. [10]

Given the household's choices for consumption, housing, loan size and labour supply, the lender confronts the borrower with a schedule of loan rates R_t^l that respect the bank participation constraints. With perfect competition among banks for customers, and using the equation linking R_t^l and $\bar{\varepsilon}_{bo,t}$, we can represent the problem of borrowers as if they choose default thresholds as a function of the aggregate states directly, subject to the bank's participation constraints. The representative borrower picks non-negative sequences of consumption,housing, loans, deposits, labour supply and default threshold functions

$$\{c_{bo,t}, h_{bo,t}, l_{bo,t}, d_{bo,t}, n_{bo,t}, \bar{\varepsilon}_{bo,t}\}_{t=0}^{\infty}$$

to maximise

$$E_0 \Sigma_{t=0}^{\infty} \beta^{bo^t} u_{bo,t} \tag{13}$$

where

$$\begin{aligned} u_{bo,t} &= \frac{(c_{bo,t}^{\xi_c} h_{bo,t}^{\xi_h} (1-n_{bo,t})^{\xi_n})^{1-\sigma}}{1-\sigma} \text{ for } \sigma \neq 1 \\ u_{bo,t} &= \xi_c \ln c_{bo,t} + \xi_h \ln h_{bo,t} + \xi_n \ln(1-n_{bo,t}) \text{ for } \sigma = 1, \end{aligned} \tag{14}$$

subject to a sequence of budget constraints

$$c_{bo,t} + q_t h_{bo,t} + d_{bo,t} + RP(\bar{\varepsilon}_{bo,t}) q_t (1-\delta_h) h_{bo,t-1} = q_t(1-\delta_h) h_{bo,t-1} + n_{bo,t} w_t + l_{bo.t} + d_{bo,t-1} R_t \tag{15}$$

, and participation constraints of the bank

$$G(\bar{\varepsilon}_{bo,t}) \tilde{A}_{bo,t} = G(\bar{\varepsilon}_{bo,t}) q_t (1-\delta_h) h_{bo,t-1} = \bar{R}_t l_{bo,t-1}. \tag{16}$$

In a neighbourhood of the steady state impatient households will set $d_{bo,t} = 0$ and $l_{bo,t-1} > 0$ for all t.[11]

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[10] The caveat is that if a new unexpected shock at time t significantly lowers the value of $\tilde{A}_{bo,t}$ it may be impossible to find a default threshold that allows the bank to break even on the loan with the risk free rate. This should not be a major concern except for very low aggregate shock values. We can completely eliminate this possibility in the version of the model in which loan rates are predetermined with respect to the aggregate state, detailed in appendix C. See the appendix in BGG(1999)[9] for a discussion of the same issue in their model.

[11] It is impossible to simultaneously have $l_{bo,t} > 0$ $d_{bo,t} > 0$. What cannot be excluded completely is that impatient households may wish to become savers for large enough positive shocks. This would be particularly troublesome in a model with a fixed borrowing limit as in Carroll (2001).[18]Here, we have a procyclical borrowing limit, and an impatient household would want to increase its borrowing in response to a higher limit.

2.2 Production

2.2.1 Financially Constrained Entrepreneurs

There is a measure 1 continuum of risk averse entrepreneurs that use capital and labour to produce final output. Just like borrowing households, I assume that their discount factor is below that of savers to guarantee that they borrow in equilibrium in a neighborhood of the steady state. Entrepreneurs' capital and output are subject to common multiplicative idiosyncratic shocks ε_{e_t}. These shocks are independent and identically distributed across time and across entrepreneurs with $E(\bar{\varepsilon}_{e,t}) = 1$, a lognormal PDF $f(\bar{\varepsilon}_{et})$ and CDF $F(\bar{\varepsilon}_{e,t})$. Production is also subject to an aggregate TFP shock $z_{e,t}$.

$$z_{e,t} = G_{z,e}^t \tilde{z}_{e,t}, \text{ where} \tag{17}$$

$$\ln \tilde{z}_{e,t+1} = \rho \ln \tilde{z}_{e,t} + \varepsilon_{z,t+1}, \; \varepsilon_{z,t+1} \sim N(0, \sigma_z) \tag{18}$$

Define expected output conditional on the aggregate shock

$$\begin{aligned} y_{e,t} &= z_{e,t}((u_{e,t}k_{e,t})^{\alpha} n_{e,t}^{1-\alpha})^{\theta}, 0 < \theta < 1 \text{ and} \\ \bar{A}_{e,t} &= (1-\delta_{e,t})q_t^k k_{e,t} + y_{e,t}. \end{aligned} \tag{19}$$

[12] The entrepreneurs' financial contract is similar to that of borrowers. Entrepreneurs are restricted to using one-period debt contracts in which the loan rates can be made contingent on aggregate shocks z_t but not on the idiosyncratic shock $\varepsilon_{e,t}$.They have access to insurance contracts that completely diversify the idiosyncratic risk after loan contracts are settled, but cannot commit to sharing the proceeds of this insurance with banks. Banks can seize collateral $\varepsilon_{e,t}\tilde{A}_{e,t}$ when the entrepreneur refuses to pay at a cost of $\mu_e \varepsilon_{e,t}\tilde{A}_{e,t}$. As in other models of financial frictions such as Jerman and Quadrini (2008) [40] and Carlstrom and Fuerst (1998) [16], a fraction a of the wage bill must be paid before production occurs, requiring an intratemporal loan from the bank. [13] The wages in advance of production requirement implies that the entrepreneur must choose the amount of labour before knowing the value of $\varepsilon_{e,t}$. The new intratemporal loan modifies the default rule: the entrepreneur defaults if and only if

$$\begin{aligned} \varepsilon_{e,t}\tilde{A}_{e,t} &< R_{e,t}^l l_{e,t-1} + R_t^w a w_t n_{e,t} = \bar{\varepsilon}_{e,t}\tilde{A}_{e,t}, \text{ where} \\ \tilde{A}_{e,t} &= (1-\delta_{e,t})q_t^k k_{e,t} + (1-s_y)y_{e,t}, \text{ and} \\ \delta_{e,t} &= \delta_e u_{et}^{\phi_e}, \; \phi_e > 1. \end{aligned} \tag{20}$$

[12] $\theta < 1$ is necessary to get a solution to the non stochastic balanced growth path when trying to match data on average firm default rates, leverage ratios and credit spreads. Trying to match these targets with $\theta = 1$ would lead to an overidentified system of equations. The assumption that θ is below 1, but close to 1 is consistent with empirical evidence (see Atkeson and Kehoe (2007) [6]) and can be interpreted as a reflection of limited span of control for entrepreneurs.

[13] This sort of working capital financing friction has been found to play a potentially important role in helping to explain the effect of credit frictions in response to news and credit shocks (see Inaba and Kobayashi (2007) [44] and Jermann and Quadrini (2008) [40]). It can help account for Chari et al's (2007) [20] finding of an important role for the labour wedge in their Business Cycle Accounting framework.

Here, I have adopted Burnside and Eichenbaum's (1996) [11] capital utilisation rate specification , in which more intensive utilisation of capital increases its depreciation rate. I assume that the capital utilisation rate $u_{e,t}$ is predetermined with respect to the idiosyncratic shock to facilitate aggregation. $s_y \geq 0$ reflects differences in the ability to collateralise capital and revenue, due to the possibility that wages must be paid before creditors in default or because proceeds from sales are easier to hide from creditors than structures or equipment. [14] As for borrowers, I define

$$RP(\bar{\varepsilon}_{e,t}) = (1 - F_e(\bar{\varepsilon}_{e,t}))\bar{\varepsilon}_{e,t} + \int_0^{\bar{\varepsilon}_{e,t}} \varepsilon dF_e \text{ and} \tag{21}$$

$$G(\bar{\varepsilon}_{e,t}) = RP(\bar{\varepsilon}_{e,t}) - \mu_{e,t} \int_0^{\bar{\varepsilon}_{e,t}} \varepsilon dF_e. \tag{22}$$

$\mu_{e,t}$ is subject to the same financial shock as borrowers' $\mu_{bo,t}$:

$$\ln\left(\frac{\mu_{e,t+1}}{1-\mu_{e,t+1}}\right) - \ln\left(\frac{\mu_e}{1-\mu_e}\right) = \rho_\mu \left[\ln\left(\frac{\mu_{e,t}}{1-\mu_{e,t}}\right) - \ln\left(\frac{\mu_e}{1-\mu_e}\right)\right] + \varepsilon_{\mu,t}. \tag{23}$$

The assumption of a common credit shock is a simplification meant to capture in reduced form the idea that changes in conditions in financial intermediaries' funding markets tend to have common effects on consumer and business loan spreads.

Given our financial contracting environment, the representative entrepreneur picks nonnegative sequences of consumption, capital, utilisation rates, labour demand, default thresholds, loans and deposits

$$\{c_{e,t}, k_{e,t+1}, u_{e,t}, n_{e,t}, \bar{\varepsilon}_{e,t}, l_{e,t}, d_{e,t}\}_{t=0}^{\infty}$$

to maximise

$$E_0 \Sigma_{t=0}^{\infty} \beta^{e^t} \ln c_{e,t} \tag{24}$$

subject to a sequence of constraints

$$c_{e,t} + (1-a)w_t n_{e,t} + q_t^k k_{t+1}^e + d_{e,t} = \bar{A}_{e,t} - RP(\bar{\varepsilon}_{e,t})\tilde{A}_{e,t} + l_{e,t} + R_t d_{e,t-1} \tag{25}$$

and the bank's break-even constraints

$$G(\bar{\varepsilon}_{e,t})\tilde{A}_{e,t} = R_t l_{e,t-1} + a w_t n_{e,t} \tag{26}$$

In a neighbourhood of a balanced growth path $d_{e,t} = 0$ and $l_{e,t} > 0$ for all t.

[14] Our setup can also be interpreted as having each entrepreneur own shares in a continuum of firms that are his only source of income and for which the allocation of production factors must be determined before knowing the idiosyncratic firm specific shock. From this perspective our entrepreneurs can be interpreted more generally as including rich shareholders that own most of the capital stock of the economy. While stock market participation rates in the US have increased in the 1990's, stock ownership is highly concentrated. The top 1% of the wealth distribution still own 50% of the stock market (Zawadowski (2010) [59]). According to Guvenen (2009) [35], even in periods of high participation rates the top quintile of the distribution owns 98% of the stock market value.

2.2.2 Financially Unconstrained Firms:

For some purposes it will be useful to compare the model where firms are financially constrained with a model where firms are financially unconstrained. In this case, I replace the entrepreneurs with a representative firm owned by savers that picks non-negative sequences of capital, utilisation rates and labour

$$\{k_{u,t+1}, u_{u,t}, n_{u,t}\}_{t=0}^{\infty}$$

to maximise its value

$$E_0\Sigma_{t=0}^{\infty}\beta^t\lambda_t^s[z_{u,t}(u_{u,t}k_{u,t})^{\alpha}n_{u,t}^{1-\alpha} - w_t n_{u,t} - q_t^k(k_{u,t+1} - (1-\delta_{u,t})k_{u,t})], \tag{27}$$

where $0 < \alpha < 1$, and $\delta_{u,t} = \delta_e u_{u,t}^{\phi_u}$.

2.2.3 Capital and Housing Production

I use a standard investment adjustment cost model as in Christiano and Fisher (2003) [22] for both residential and non-residential investment. This formulation of the adjustment cost has better empirical properties than the more traditional capital adjustment costs (Topel and Rosen (1988) [57], Christiano and Fisher (2003) [22]). It can be seen as a reduced form for time to build and other frictions affecting the supply of new capital and housing. A representative firm owned by the savers produces new housing. The firm purchases I_t^h units of the consumption good and turns it into

$$I_t^h = h_t - (1-\delta_h)h_{t-1} \tag{28}$$

units of housing while paying an adjustment cost of

$$AC_{h,t} = \frac{\gamma^h}{2}(\frac{I_t^h}{I_{t-1}^h} - G_y)^2 I_t^h. \tag{29}$$

$\frac{1}{\gamma_h}$ is the elasticity of housing investment to a temporary increase in house prices (See Christiano and Fisher (2003) [22]). With these assumptions, the housing producer's problem reduces to picking sequences of $\{I_t^h\}$ to maximise

$$E_0\Sigma_{t=0}^{\infty}\beta^t\lambda_t^s[(q_t - 1)I_t^h - AC_{h,t}] \tag{30}$$

Along the balanced growth path, $AC_{h,t} = 0$, $q = 1$ and the housing producer makes no profits. The representative capital producer faces the same problem as housing producers, except that I allow for investment shocks. The capital producer purchases investment goods I_t at a cost of $P_{I,t}I_t$ from zero profit competitive investment goods producers. It incurs an adjustment cost

$$AC_{k,t} = \frac{\gamma}{2}(\frac{I_t}{I_{t-1}} - G_y)^2 I_t. \tag{31}$$

The capital producer picks sequences of $\{I_t\}_{t=0}^{\infty}$ to maximise

$$E_0\Sigma_{t=0}^{\infty}\beta^t\lambda_t^s[(q_{k,t}-P_{I,t})I_t-AC_{k,t}] \tag{32}$$

Without aggregate fluctuations, $P_{I,t}=q_{k,t}=1$ and $AC_{k,t}=0$. With aggregate fluctuations, $P_{I,t}$ is subject to shocks:

$$\ln P_{I,t+1}=\rho_I\ln P_{i,t}+\varepsilon_{I,t+1},\ \sigma_I\sim N(0,\sigma_I). \tag{33}$$

2.3 Competitive Equilibrium

Combining the budget constraints of the model's agents gives us the resource constraint of the economy:

$$Y_t=C_t+P_{I,t}I_t+I_t^h+AC_t+FF_t \tag{34}$$

, where C_t, I_t, I_t^h are aggregate consumption net of financial services, investment in capital and investment in housing, AC_t are non financial adjustment costs of capital and housing and FF_t are the dead-weight costs of default on debt contracts, and Y_t is aggregate final output.

In addition labour and credit market clearing require

$$\theta_s n_{s,t}+\theta_{bo}n_{bo,t} = n_{e,t}, \tag{35}$$

$$\theta_s d_t = \theta_{bo}l_{bo,t}+l_{e,t} \tag{36}$$

Other market clearing conditions, can be similarly derived by aggregating over the decisions of households and entrepreneurs. A competitive equilibrium consists of a set of prices and risk-free interest rates $\{q_t, q_t^k, R_{t+1}, w_t\}$ for all possible states and for all $t\geq 0$ such that all markets clear when households and firms solve their maximisation problems while taking these prices and risk-free interest rates as given.

2.4 Understanding the Effect of Financing Constraints in the Model

Before proceeding to estimate and quantify the effects of the model's financing frictions, I will develop more theoretical insight about how these frictions affect the dynamics. I start with the borrowers and entrepreneurs in partial equilibrium. Finally I bring them together and discuss some general equilibrium considerations.

2.4.1 Borrowers

The first order conditions for the borrower's problem are:

$$c_{bo,t} \quad : \quad \lambda_{bo,t} = \xi_c \frac{(c_{bo,t}^{\xi_c} h_{bo,t}^{\xi_h} (1 - n_{bo,t})^{\xi_n})^{1-\sigma}}{c_{bo,t}}, \tag{37}$$

$$\bar{\varepsilon}_{bo,t} \quad : \quad \psi_{bo,t} = \lambda_{bo,t} \left[\frac{RP'(\bar{\varepsilon}_{bo,t})}{G'(\bar{\varepsilon}_{bo,t})} \right] \equiv \lambda_{bo,t} efp_{bo,t}, \tag{38}$$

$$l_{bot} \quad : \quad \lambda_{bo,t} = \beta^{bo} R_{t+1} E_t \lambda_{bo,t+1} efp_{bo,t+1} \tag{39}$$

$$h_{bo,t} \quad : \quad q_t \lambda_{bo,t} = \frac{\xi_h}{\xi_c} \frac{c_{bo,t}}{h_{bo,t}} \lambda_{bo,t} + \beta^{bo} (1 - \delta_h) E_t \lambda_{bo,t+1} q_{t+1} A_{hh}(\bar{\varepsilon}_{bo,t+1}). \tag{40}$$

$$n_{bot} \quad : \quad \frac{\xi_n}{1 - n_{bo,t}} = \xi_c \frac{w_t}{c_{bo,t}}, \tag{41}$$

where

$$A_{hh}(\bar{\varepsilon}_{bo,t}) = 1 - RP(\bar{\varepsilon}_{bo,t}) + efp_{bo,t} G(\bar{\varepsilon}_{bo,t}).$$

[15] The first order condition for borrowing has the same form as the saver's first order condition for deposits except that the effective gross interest rate is now $R_{t+1} efp(\bar{\varepsilon}_{bo,t+1})_{bo,t+1}$.

$$efp_{bo,t} = \frac{RP'(\bar{\varepsilon}_{bo,t})}{G'(\bar{\varepsilon}_{bo,t})} > 1 \tag{42}$$

is an external finance premium faced by borrowers on loans relative to the risk free interest rate R_t. We can also think about it as a microfounded intertemporal wedge in the spirit of Chari et al's (2007) [20] Business Cycle Accounting framework. First, hold the loan enforcement cost parameter $\mu_{bo,t}$ fixed.

$$\frac{d}{d\bar{\varepsilon}_{bo}} \left(\frac{\bar{\varepsilon} f(\bar{\varepsilon}_{bo})}{1 - F(\bar{\varepsilon}_{bo})} \right) > 0$$

at an optimum implies that $efp_{bo,t}$ is increasing in $\bar{\varepsilon}_{bo,t}$, and therefore in the default rate $F(\bar{\varepsilon}_{bo,t})$ as well. [16]That the external finance premium is increasing in the default rate is quite intuitive. This relation implies that if we reproduce the countercyclical default rates found in the data, the model will generate a countercyclical external finance premium. The link between the two will also allow us to interchangeably interpret any result obtained for the effect of an increase in the default rate in terms of an increase in the external finance premium (for a fixed $\mu_{bo,t}$).

[15] Here and throughout the paper, optimality also requires the transversality conditions to hold. For debt, this condition is guaranteed to hold for bounded fluctuations around the balanced growth path (BGP) because of the finite BGP debt level.

[16] Omitting time subscripts, $efp'(\bar{\varepsilon}_{bo}) = \frac{RP''(\bar{\varepsilon}_{bo})G'(\bar{\varepsilon}_{bo}) - RP'(\bar{\varepsilon}_{bo})G''(\bar{\varepsilon}_{bo})}{G'(\bar{\varepsilon}_{bo})^2} > 0$ iff $f(\bar{\varepsilon}_{bo})^2 \bar{\varepsilon}_{bo} + [1 - F(\bar{\varepsilon}_{bo})](\bar{\varepsilon}_{bo} f'(\bar{\varepsilon}_{bo}) + f(\bar{\varepsilon}_{bo})) > 0$. This is equivalent to $\left(\frac{\bar{\varepsilon}_{bo} f(\bar{\varepsilon}_{bo})}{1 - F(\bar{\varepsilon}_{bo})} \right)' > 0$.

An increase in the expected future external finance premium reduces current consumption relative to future consumption (holding constant labour supply and housing in the case of nonseparable preferences), just like an increase in the risk free interest rate. From the bank's participation constraint we know that the default rate is increasing in leverage ($\bar{\varepsilon}'_{bo}(\frac{R_t l_{bo,t-1}}{\tilde{A}_{bo,t}}) > 0$). On impact, with $l_{bo,t-1}$ and $h_{bo,t-1}$ predetermined, any shock that increases house prices reduces the loan to collateral ratio. This decreases the default rate, and lowers the external finance premium. The effect of the shock in the next periods depends on how borrowers adjust loan demand and housing in response to the shock. With risk neutrality, the increase in loan demand in response to a positive shock would ultimately lead to a higher loan to collateral ratio and a higher default rate. But with a diminishing marginal utility of consumption, this does not have to be the case.

The impatient household behaves like the consumption smoothing patient household with a bias towards debt financed consumption instead of saving. For a fixed level of financial frictions and desired housing, a consumption smoothing borrower reacts to an increase in wealth by increasing savings. This increase in saving can be accomplished through a combination of reduced borrowing and increased accumulation of collateral in the form of housing. At the same time an increase in the value of collateral encourages higher borrowing. If the first effect dominates, then the leverage ratio $\frac{R_t l_{bo,t-1}}{\tilde{A}_{bo,t}}$ is countercyclical and therefore the external finance premium is also countercyclical beyond the initial impact of a shock. The analysis is similar for a credit shock that directly changes the monitoring cost parameter $\mu_{bo,t+1}$, except that now a reduction in the level of financial frictions is compatible with a higher leverage. A decline in $\mu_{bo,t+1}$ directly reduces the external finance premium $efp_{bo,t+1}$ and raises the leverage ratio. As long as consumption smoothing moderates the agent's desire to increase borrowing relative to his collateral, a lower $\mu_{bo,t+1}$ will reduce the strength of financing frictions. The overall effect on leverage is ambiguous.

To highlight the differences and similarities between the model with financial constraints and a standard permanent income consumption model, I derive a perfect-foresight consumption function for the special case of log-utility with exogeneous housing and labour supply. This will also allow me to introduce the credit (or external financing) spread as an alternative measure of financing frictions. By combining the bank loan participation constraint with the budget constraint, and iterating forward on the resulting expression for $l_{bo,t-1}$, we obtain

$$\begin{aligned} c_{bo,t} &= \frac{R_t + s_{bo,t}}{1+\sum_{j=1}^{\infty}\beta^{bo^j}\Pi_{k=1}^{j}\frac{R_{t+k}efp_{bo,t+k}}{R_{t+k}+s_{bo,t+k}}}\sum_{j=0}^{\infty}\frac{n_{bo,t+j}w_{t+j}-q_{t+j}I^h_{bo,t+j}}{\Pi_{k=0}^{j}(R_{t+k}+s_{bo,t+k})} \qquad (43) \\ &\quad -\frac{(R_t+s_{bo,t})l_{bo,t-1}}{1+\sum_{j=1}^{\infty}\beta^{bo^j}\Pi_{k=1}^{j}\frac{R_{t+k}efp_{bo,t+k}}{R_{t+k}+s_{bo,t+k}}}, \text{ where} \\ s_{bo,t+k} &= \frac{\mu_{bo,t+k}\tilde{A}_{bo,t+k}\int_0^{\bar{\varepsilon}_{bo,t+k}}\varepsilon dF_{bo}}{l_{bo,t+k-1}}. \qquad (44) \end{aligned}$$

$s_{bo,t+k}$ is a credit spread compensating the lender for the cost of default. Financial frictions affect consumption by modifying the effective interest rates facing the household through factors that depend on default thresholds $\bar{\varepsilon}_{bo,t+k}$ (or equivalently on default rates for fixed idiosyncratic risk distributions and enforcement cost parameter μ_{bo}). They also affect consumption decisions through the loan balance from the previous period $l_{bo,t-1}$. The consumption function without financial frictions can be obtained by setting $efp_{bo,t+k} = 1$ and $s_{bo,t+k} = 0$ for all periods. The credit spread $s_{bo,t+k}$ is increasing in $\bar{\varepsilon}_{bo,t+k}$ as long as it is lower than $(efp_{bo,t+k} - 1)R$.[17] This condition holds in a neighbourhood of the steady state. Increasing external finance premia $efp_{bo,t+k}$ reduce the present value of the household's wealth net of housing investments, which leads to a reduction in consumption. The effect of changes in future credit spreads is more ambiguous. On one hand higher $s_{bo,t+k}$'s reduce the present value of future income net of housing investment. On the other hand they reduce the effect of higher $efp_{bo,t+k}$ on the present value of that future income. Finally increases in $l_{bo,t-1}$ or $s_{bo,t}$ reduce consumption by raising the household's debt burden. While these effects are conditional on a fixed level of housing investment and labour supply, the estimated dynamics below confirm that there is a negative link between default risk, debt repayments and borrower consumption.

Financial frictions also distort the household's choice between housing and non durable consumption. In a neighbourhood of the steady state $A_{hh}(\bar{\varepsilon}_{bo,t+1})$ is greater than one in the first order condition for $h_{bo,t}$. This makes the marginal value of housing investment more sensitive to the future expected value of housing than in the model without financing frictions. The effect of a change in the expected future external finance premium is less clear-cut. For a given expected house price appreciation an increase in the expected future default rate (and hence in the external finance premium) increases the value of housing as collateral. [18] Fixing consumption across periods, this makes the borrower's housing investment increasing in the expected external finance premium. At the same time, there is an indirect effect of financing frictions on housing investment through the the interaction of these frictions with the marginal utility of consumption $\lambda_{bo,t}$. From the first order condition for loans, a reduction in the future external finance premium reduces the effective discount rate applied to housing investment ($\frac{\lambda^{bo}_{t+1}}{\lambda^{bo}_t}$ increases). This encourages the household to increase its housing stock when the expected external finance premium declines. In the special case when agents have perfect foresight, the relationship between the expected external finance premium and

[17] Using the bank participation constraint, $\frac{R+s(\bar{\varepsilon}_{bo})}{R} = \frac{RP(\bar{\varepsilon}_{bo})}{G(\bar{\varepsilon}_{bo})}$. $s_{bo}(\bar{\varepsilon}_{bo})$ is increasing in $\bar{\varepsilon}_{bo}$ iff

$$\frac{d}{d\bar{\varepsilon}_{bo}}\left(\frac{R+s(\bar{\varepsilon}_{bo})}{R}\right) = \frac{RP'(\bar{\varepsilon}_{bo})G(\bar{\varepsilon}_{bo}) - RP(\bar{\varepsilon}_{bo})G'(\bar{\varepsilon}_{bo})}{G(\bar{\varepsilon}_{bo})^2} > 0$$

iff $\frac{R+s(\bar{\varepsilon}_{bo})}{R} = \frac{RP(\bar{\varepsilon}_{bo})}{G(\bar{\varepsilon}_{bo})} < \frac{RP'(\bar{\varepsilon}_{bo})}{G'(\bar{\varepsilon}_{bo})} = efp(\bar{\varepsilon}_{bo})$, that is iff $s(\bar{\varepsilon}_{bo}) < R(efp(\bar{\varepsilon}_{bo}) - 1)$.

[18] $\frac{d}{d\bar{\varepsilon}_{bo}}(\frac{\bar{\varepsilon}f(\bar{\varepsilon}_{bo})}{1-F(\bar{\varepsilon}_{bo})}) > 0$ implies that $A_{hh}(\bar{\varepsilon}_{bo,t+1})$ is increasing in $\bar{\varepsilon}_{bo,t+1}$: $\frac{d(A_{hh}(\bar{\varepsilon}_{bo,t}))}{d\bar{\varepsilon}_{bo,t}} = [-RP'(\bar{\varepsilon}_{bo,t}) + G'(\bar{\varepsilon}_{bo,t})efp(\bar{\varepsilon}_{bo,t}) + G(\bar{\varepsilon}_{bo,t})efp'(\bar{\varepsilon}_{bo,t})] = G(\bar{\varepsilon}_{bo,t})efp'(\bar{\varepsilon}_{bo,t}) > 0$.

the housing to non durable consumption ratio $\frac{h_{bo,t}}{c_{bo,t}}$ can be easily determined . Combining the loan and housing first order conditions we obtain:

Proposition 1 *With perfect foresight,* $\frac{\partial(h_{bo,t}/c_{bo,t})}{\partial\bar{\varepsilon}_{bo,t+1}} < 0$.
Proof. *See appendix A.* ■

This implies that the borrower's housing investment is decreasing in the external finance premium if consumption is also decreasing in the external finance premium. Due to certainty equivalence, the proposition also holds in a linear approximation of the dynamics. In general case of aggregate uncertainty and nonlinear approximation, we cannot determine analytically which effect dominates. But the perfect foresight result should be a good guide for small levels of aggregate uncertainty.

Finally financial frictions distort borrowers' labour supply through their effect on non-durable consumption. For example, if the cost of external financing declines in a boom borrowers increase their nondurable consumption by more than savers, and their labour supply will increase by less than that of savers'. Intuitively, better external financing conditions act like an increase in borrower wealth, reducing labour supply.

2.4.2 Entrepreneurs

As for the borrowers, we can use the relation between the Lagrange multiplier on the bank participation constraint and the marginal utility of consumption to obtain the following first order conditions for the entrepreneurs:

$$c_{e,t} : \frac{1}{c_{e,t}} = \lambda_{e,t} \tag{45}$$

$$\bar{\varepsilon}_{e,t} : \psi_{e,t} = \lambda_{e,t}\frac{RP'(\bar{\varepsilon}_{e,t})}{G'(\bar{\varepsilon}_{e,t})} \equiv \lambda_{e,t}efp_{et}. \tag{46}$$

$$l_{e,t} : \lambda_{e,t} = \beta^e R_{t+1}E_t\lambda_{e,t+1}efp_{e,t+1}. \tag{47}$$

$$k_{e,t+1} : \lambda_{e,t}q_t^k = \beta^e E_t\lambda_{e,t+1}[A_{kk}(\bar{\varepsilon}_{e,t+1})q_{t+1}^k(1-\delta_{e,t+1}) + A_{ky}(\bar{\varepsilon}_{e,t+1})\alpha\theta\frac{y_{e,t+1}}{k_{e,t+1}}]. \tag{48}$$

$$n_{e,t} : A_{ky}(\bar{\varepsilon}_{e,t})(1-\alpha)\theta\frac{y_{e,t}}{n_{e,t}} = (1-a+aefp_{e,t})w_t \tag{49}$$

$$u_{e,t} : A_{ky}(\bar{\varepsilon}_{e,t})\alpha\theta\frac{y_{e,t}}{k_{e,t}} = \phi_e A_{kk}(\bar{\varepsilon}_{e,t})\delta_{e,t}q_t^k, \tag{50}$$

where

$$\begin{aligned} A_{kk}(\bar{\varepsilon}_{e,t}) &= 1 + efp_{e,t}G(\bar{\varepsilon}_{e,t}) - RP(\bar{\varepsilon}_{e,t}) \text{ and} \\ A_{ky}(\bar{\varepsilon}_{e,t}) &= 1 + (1-s_y)[efp_{e,t}G(\bar{\varepsilon}_{e,t}) - RP(\bar{\varepsilon}_{e,t})]. \end{aligned} \tag{51}$$

The analysis of these equations parallels in many respects the previous analysis for borrowing households. The key modification of entrepreneur behaviour relative to a standard financially unconstrained firm comes through the evolution of the external finance premium on the bank loan, $efp_{e,t+1}$ which is increasing in the default threshold $\bar{\varepsilon}_{e,t+1}$. As for households, we can define a credit spread

$$s_{e,t} = \frac{\mu_{e,t}\tilde{A}_{e,t}\int_0^{\bar{\varepsilon}_{e,t}} \varepsilon dF_e}{l_{e,t-1} + aw_t n_{e,t}}. \tag{52}$$

This spread is increasing in $\bar{\varepsilon}_{e,t}$ in a neighbourhood of the steady state. The default threshold is increasing in the loan to collateral ratio. The first order condition for loans implies that an increase in the expected future external finance premium reduces current consumption relative to future consumption for the entrepreneur. Consider a boom that raises the entrepreneur's collateral. In response to an increase in the value of collateral, on one hand the entrepreneur wants to expand his loan. This tends to raise $efp_{e,t+1}$. On the other hand the entrepreneur will smooth changes in his consumption by moderating the increase in the loan and by raising investment in the collateral asset, capital. This tends to reduce $efp_{e,t+1}$.The effect of changes in $\mu_{e,t+1}$ is also similar to the effect of changes in households' $\mu_{bo,t+1}$. The impact of financing frictions on investment and labour demand can be approximated by the following results for small levels of uncertainty :

Proposition 2 *a) With perfect foresight and fixed capital utilisation,* $\frac{\partial k_{e,t+1}}{\partial \bar{\varepsilon}_{e,t+1}} < 0$ *for fixed employment. With fixed capital utilisation and variable employment* $\frac{\partial n_{e,t}}{d\bar{\varepsilon}_{e,t}} < 0$ *is a sufficient condition for* $\frac{\partial k_{e,t+1}}{\partial \bar{\varepsilon}_{e,t+1}} < 0$.

b) With fixed capital utilisation, $a \geq 1 - s_y$ *implies that* $\frac{\partial n_{e,t}}{\partial \bar{\varepsilon}_{e,t}} < 0$.

c) $s_y > 0$ *implies* $\frac{\partial u_{e,t}}{\partial \bar{\varepsilon}_{e,t}} < 0$.

Proof. See appendix A. ■

Start with the first part of the proposition on investment. Consider an increase in the future external finance premium. This raises the entrepreneur's effective discount rate for investment $R_{t+1}efp_{e,t+1}$. Because of diminishing marginal productivity of capital, the higher discount rate reduces the entrepreneur's desired capital investment. At the same time, an increase in the future external finance premium also raises the marginal value of capital as collateral. [19] This tends to raise desired investment. The proposition states that the first effect dominates and investment is decreasing in the future external finance premium.

The labour demand decision of entrepreneurs is directly distorted by financing frictions as long as $s_y < 1$ or $a > 0$. In the presence of working capital requirements on wages ($a > 0$), a higher external finance premium acts as a tax on hiring and lowers labour demand. At the same time, if revenue can be used as collateral ($s_y < 1$), then an increase in the external

[19] $A_{kk}(\bar{\varepsilon}_{e,t+1})$ is increasing in $\bar{\varepsilon}_{e,t+1}$, and $\bar{\varepsilon}_{e,t+1}$ is increasing in $efp_{e,t+1}$.

finance premium raises the value of labour as an input into the collateral that can be offered by the entrepreneur. This tends to stimulate labour demand in response to a higher external finance premium. The proposition says that labour demand must decrease in the level of financing frictions if the wages in advance requirement is strong enough or if revenue is relatively hard to collateralize.

The capital utilisation rate is decreasing in the external finance premium for $s_y > 0$. With capital easier to collateralise than revenue ($s_y > 0$), an increase in the external finance premium encourages the entrepreneur to reduce the depreciation rate of valuable collateral. This can be achieved by decreasing the utilisation rate. In addition, the financing frictions will affect the entrepreneurs' utilisation rate decision indirectly. A decline in the external finance premium stimulates labour demand This increases the marginal product of capital which in turn increases the desired capital utilisation rate. In the other direction, if a lower external finance premium stimulates entrepreneur demand for capital, then in general equilibrium this leads to a higher price of capital. The higher price of capital reduces capital utilisation by making it more costly to depreciate existing capital.

2.4.3 Some General Equilibrium Considerations

Here, I focus on some specific issues arising from the joint modeling of firm and household borrowing constraints. One question highlighted in the introduction is whether the incorporation of more pervasive financing constraints affecting both firms and households could increase the importance of these frictions in amplifying the response of macroeconomic quantities to shocks. The possibility for interactions can be seen from the bank break-even constraint for the loan to entrepreneurs:

$$\begin{aligned} G(\bar{\varepsilon}_{e,t}) &= \frac{R_t l_{e,t-1} + a w_t n_{e,t}}{(1-\delta_{e,t}) q_t^k k_{e,t} + (1-s_y) y_{e,t}}, \\ G'(\bar{\varepsilon}_{e,t}) &> 0. \end{aligned} \tag{53}$$

This equation shows that in general equilibrium changes in the financing conditions of borrowers affect firms' leverage and external financing conditions through their effect on the risk-free interest rate, wages and demand for firms' goods.

Consider a negative shock that increases financial frictions for households (for example by depressing house prices that serve as collateral for loans). Traditional Keynesian analysis would suggest a positive interaction between the strength of financing frictions affecting households and firms. Higher credit spreads for households reduce their consumption which may depress the sales of some firms and raise their credit spreads. Even though my model has flexible prices and wages, a reduction in sales of consumption goods to borrowing households could depress output and indirectly increase firms' external financing spreads if investment adjustment costs limit reallocation of resources between the production of consumption and investment goods. [20]

In the other direction there are several channels for negative interaction between household and firm credit frictions. An increase in the external financing spread reduces households' demand for loans for any given risk free interest rate. By the loan market equilibrium condition,

$$\theta_s d(R_{t+1}, *) = \theta_{bo} l_{bo}(s_{bo,t+1}, \mu_{t+1}, R_{t+1}, *) + l_e(s_{e,t+1}, \mu_{t+1}, R_{t+1}, *), \tag{54}$$

this puts downward pressure on the risk-free rate interest rate. From the bank loan participation constraint above, this reduces financing spreads for entrepreneurs. Now consider the effect of this negative shock on labour supply and wages. Higher spreads reduce borrowers' current consumption and increase their desired labour supply. Absent opposing movements in the labour supply of savers, aggregate labour supply will increase due to the increase in household external financing spreads, leading to a lower wage rate for firms. Movements in the risk free interest rate in the model with household borrowing frictions will tend to dampen the decline in savers' consumption and increase the fall in their labour supply in a recession. Therefore, the overall effect of household financing frictions on labour supply may seem ambiguous. However, by aggregating over the labour supply of the two types of households we find that

$$\begin{aligned} 1 - N_t^s &= \theta_s(1 - n_{s,t}) + \theta_{bo}(1 - n_{bo,t}) = \theta_s \frac{\xi_n c_{s,t}}{\xi_c w_t} + \theta_{bo} \frac{\xi_n c_{bo,t}}{\xi_c w_t} = \frac{\xi_n}{\xi_c w_t} C_t^{hholds}, \qquad (55) \\ \text{where } C_t^{hholds} &= \theta_s c_{s,t} + \theta_{bo} c_{bo,t}. \end{aligned}$$

The aggregate labour supply equation above implies that as long as increasing household financing frictions generate a bigger decline in aggregate household consumption C_t^{hholds}, they will raise labour supply and reduce the wage rate faced by firms. Going back to the break-even constraint on entrepreneur borrowing, this reduction in the wage improves entrepreneur financing conditions by reducing borrowing for the wage bill and stimulating entrepreneur output. For our parameter values, higher financing spreads in a recession tend to decrease aggregate household consumption and increase labour supply. This generates another channel for negative interaction between credit conditions for households and firms.

[20] Note that entrepreneurs in our model produce a general good $y_{e,t}$. This implicitly means that an entrepreneur facing lower demand from borrowing households in the model can easily switch to producing other goods for other customers. In reality, many firms may be specialised in consumption goods, and their financing spreads may be more directly linked to sales to borrowing households. This channel is missing from the current model, but it may be worthwhile to explore in future work.

3 Estimation and Quantitative Results

I solve the model using a loglinear approximation around the deterministic balanced growth path (henceforth the BGP). I use a mixture of calibration and Bayesian estimation to obtain parameter values. [21] Since the loglinearised model is in state space form, we can evaluate its likelihood $p(Y|\theta_p)$ using the Kalman filter. The posterior density of the parameters $p(\theta_p|Y)$ is then proportional to the product of the likelihood function and the prior density of the parameters $p(Y|\theta_p)\pi(\theta_p)$. I follow the standard practice of first finding the mode of the posterior density using a robust optimisation algorithm. Then, I use the resulting mode and covariance matrix estimates to run a random walk Markov Chain Monte Carlo (MCMC) simulation. The output of the MCMC simulation approximates the posterior distribution of the parameters, allowing me to form probability intervals around the parameters and functions of the parameters such as variance decompositions. [22]

I estimate the model using quarterly US data from 1955:Q1 to 2004:Q4. This sample avoids the Korean war and immediate transition phase from World war 2, as well the most recent financial crisis that may be hard to model with the loglinear approximation used in estimation. There are 3 shocks in the model: the TFP shock, the investment shock and the credit shock. Therefore, I can use 3 time series while avoiding stochastic singularity. I use real consumption on nondurables and services, real private non residential investment and real residential investment. I define the GDP measure in the data corresponding to the model's economy to be the sum of these variables. This is in line with my omission of government and consumer durables from the model. All of these series are obtained by dividing nominal data by the GDP implicit price deflator and the civilian non institutional population over the age of 16 (see the appendix for more information on the data). I take

[21] Bayesian estimation has become the method of choice for inference on DSGE models (see An and Schorheide (2007) [3] for a survey). This partly reflects the attractiveness of quantifying the uncertainty in a policy oriented model in terms of posterior probabilities as opposed to classical confidence intervals. It also reflects the difficulty of direct application of maximum likelihood methods to highly misspecified models. Regardless of model misspecification, under mild regularity conditions and as long as the model is identified, the Bayes estimator converges asymptotically to the maximum likelihood estimator, and the effect of the priors vanishes. In finite samples, the priors will of course still affect the estimation. This can be seen as a disadvantage, but at the same time it allows the researcher to introduce information from other sources that may be hard to directly incorporate in maximum likelihood estimation.

[22] After detrending all variables by the BGP growth rate of output, I use Klein's (2000) [43] generalised Schur decomposition method to solve for the loglinear approximation. I have also explored the effects of using second order perturbation methods as in Schmitt Grohe and Uribe (2004) [34]. While there is some evidence of nonlinearity, it does not appear strong enough to justify the significantly larger computational burden of replacing the Kalman filter by a nonlinear particle filter in estimation. I use the Covariance Matrix Adaptation Evolutionary Strategy algorithm (CMAES) to maximise the posterior $p(\theta_p|Y)$. As documented by Andreasen (2008) [4] for DSGE models, this algorithm has excellent global optimisation properties while being significantly faster than other common alternatives such as simulated annealing. Given the global nature of the algorithm, I perform 10 optimisation runs and pick the best, using the prior distribution to pick sufficiently dispersed starting points.

out a separate linear trend and a sample average from the natural logarithm of each variable to match the corresponding variables in the model's state-space representation. I calibrate the financial contract parameters, the preference parameters and discount factors to match long run averages and micro evidence. These parameters are often hard to identify with only aggregate macroeconomic quantity data, and the calibration allows me to incorporate information from micro data and studies to identify them. I estimate the two investment adjustment cost parameters and the exogenous shock process parameters. [23] I run three chains of 500,000 draws each, starting from the posterior mode and the posterior mode +/- 2 standard deviations. I drop the first 40% of the draws for each chain as a burn-in, and mix all the remaining draws for inference on the posterior. This gives me a final sample of 900,000 draws. [24]

3.1 Calibrated Parameters

I calibrate the parameters of the model controlling balanced growth path ratios to long run averages of the US economy at a quarterly frequency (table 1).The curvature coefficient of the utility function σ is set to 2, which is in line with the empirical upper bounds on relative risk aversion established in Chetty (2006) [21] and the estimates in Basu and Kimball (2002) [8]. The results are qualitatively quite similar in the common case of log utility ($\sigma = 1$). The patient households' discount factor β is set to match an average annual real risk-free rate of 4%. This is a common choice in the macroeconomic literature. While it is higher than the average rate of return on US 3 months T-bill rate, it matches the rate of return on longer term US T-bonds which as argued by McGrattan and Prescott (2003) [48] is free of liquidity premia and transaction costs considerations that are not modelled here. The housing and consumption share parameters in the utility function ξ_h and ξ_c are set to deliver an annual housing stock to output ratio of around 1.3 as in Davis and Heathcote (2005) [26] and an hours of work share of 0.32. The measure of GDP in the data most close to my model is the sum of consumption expenditures on nondurables and services, private residential investment

[23] The financial contract parameters should be well identified in the Bayesian estimation, if I included financial time series in my estimation data. To do this I would have to add more shock processes , enrich the financial shock processes with news on future shocks as in Christiano et al (2010) [23], or I would have to add measurement errors. While adding more variables to the estimation is certainly a worthwhile and relatively straightforward extension, I have opted to start with a minimal shock structure, relying on calibration (a rough method of moments) to incorporate additional information such as average leverage ratios or default rates.

[24] Using a scaling factor for the covariance matrix of $\frac{2.38^2}{\dim(\theta_p)}$ (see Rosenthal (2010) [52] for a justification of this choice), leads to an acceptance rate for the draws of around 27%. This is close to the recommended acceptance rate. My computation of the posterior assumes that all draws lead to a unique rational expectations solution. This was the case for 99.95% of the draws. Convergence to the posterior was assessed by examining the recursive means of the draws in each chain and by comparing the variance of the draws in the mixed overall chain to the average variance within the chains as in Gelman and Shirley (2010) [31]. In all cases, the results suggested convergence of the chains.

and private non residential investment. Based on this measure and my estimation sample period I find an average quarterly growth rate of GDP of .51%, leading to $G_y = 1.0051$. I set the returns to scale parameter $\theta = 0.95$, as in Atkeson and Kehoe (2007) [6]. The capital share α is set to target a BGP firm investment to output ratio of 0.153, the average of the ratio of private non residential investment to model GDP over my sample of 1955-2004.

For the depreciation rates on housing and business capital I rely on the BEA data for 1948-2001 in Davis and Heathcote (2005) [26]. Therefore, I set $\delta_h = 0.016/4$ and $\delta_e = 0.056/4$. Combining this with the first order condition for the capital utilisation rate in the BGP fixes ϕ_e, the curvature of the capital depreciation function.

We now come to the parameters related to the financial frictions. I set the share of impatient households at 40%, based on the proportion of US households with negative net financial assets reported in Scoccianti (2009) [53] and Diaz and Luengo-Prado (2010) [27]. This number is a bit higher than that used in models with rule of thumb households that consume all their income or in models using the Iacoviello framework, but my definition of being credit constrained is weaker than the quantity rationing imposed in those models.

I set the share of wages that must be paid in advance to 50% ($a = 0.5$), halfway between the typical assumption of models with this channel that all wages must be paid in advance and the other extreme that all wages can be paid out of realised revenue. For the proportion of revenue that can be collateralised I start with $s_y = 0.5$ as a benchmark. This is the midpoint between 2 extremes that are common in the literature: one in which only capital can be seized as in Kiyotaki and Moore (1997)[42], and one in which all revenue can be seized as in Carlstrom and Fuerst (1998) [16].

A priori it is not clear whether collateral liquidation or foreclosure costs are higher on consumer or firm debt: one can envision more efficient mechanisms for resolving firm default proceedings, but on other hand firms are more complex entities with higher possibilities for fraud. For now, I set $\mu_{bo} = \mu_e = \mu$. I calibrate the discount factors of borrowers and entrepreneurs, the loss proportion of a bank in default μ and the volatilities of idiosyncratic shocks σ_e and σ_{bo} to match firm and household leverage ratios, the credit spread on firm borrowing, and firm and household default rates. Covas and Den Haan (2010) [24] report an average debt to assets ratio for nonfinancial Compustat corporations of 0.587 over 1971-2004. I use this as my firm leverage target. For the borrowing households' leverage target, I use the average loan to value ratio on single family conventional mortgages over 1973-2006 of 0.76 (Iacoviello and Neri 2009) [38]. I target an annual spread on firm borrowing of around 1.27%. This is close to the estimated value of the credit spread for the US in De Graeve (2008) [33] using the BGG model. I reach this number by taking the average spread between the prime loan rate and the 3 month commercial paper rate over 1971-1996 as in Carlstrom and Fuerst (1997) [15]and adjusting it by a factor of 68% to better reflect the component due to default risk. This is in the middle of the range of the proportion of corporate bond spreads accounted for by default risk as computed in Longstaff et al (2005) [47]. For firms, I target the 3% average annual default rate on US bonds over 1971-2005 (Fuentes-Albero

(2009) [29]). For households, I interpret default in the model to be similar to foreclosure on a mortgage in the data. I use the average annual foreclosure rate of 1.4% in 1990-2004 from Garriga and Shlagenhauf (2009) [30] to calibrate the BGP default rate households. To match these targets, I set

$$\mu = 0.45, \beta^{bo} = 0.949\,,\ \beta^{e} = 0.95, \sigma_e = 0.209,\ \sigma_{bo} = 0.099.$$

These parameter values generate an annual spread on household loans of around 0.6%. In comparison, the actual spread between the typical 30 year fixed rate mortgage and a 30 year government bond over 1977-2008 was 1.5% (Sommer et al 2009) [56]. Matching a higher spread for households with the lower default rate than that of firms would require us to assume an implausibly low discount factor β^{bo} or to calibrate a separate monitoring parameter μ_{bo} significantly above 0.5, which is again implausible. This suggests that there are other unmodeled factors accounting for most of the interest rate spread on consumer mortgages.

3.2 Prior Distribution for Estimated Parameters

Table 2 describes the prior and posterior distributions. For most parameters I use priors consistent with earlier studies (e.g Justiniano et al (2009) [41]). The capital investment adjustment cost parameter γ follows a Gamma distribution with a mean of 4 and a standard deviation of 1. For the shock persistence parameters ρ_z, ρ_μ, ρ_I I use a Beta distribution with a mean of 0.75 and a standard deviation of 0.15. I use a uniform distribution on $[0, 0.05]$ for the standard deviation of the innovations to TFP and to the price of investment goods, σ_z and σ_I. For the housing investment adjustment cost parameter γ_h there is less prior information I use a Gamma distribution with a mean of 1 (in line with the estimates in Topel and Rosen (1988) [57]), and a standard deviation of 0.5. Finally, I use a uniform distribution on $[0, 0.5]$ for the credit shock standard deviation σ_μ. This reflects the high level of uncertainty on this parameter, given the small number of previous papers allowing for this shock. [25]

3.3 Posterior Distribution of the Estimated Parameters

I will focus on the posterior mode estimates, since the posterior means and medians are very similar to the modes. In general, the data appear to be informative about the estimated parameters. While for some parameter the posterior mode is close to the prior mean, the posterior probability intervals are significantly tighter than the prior intervals for all parameters . The estimated investment adjustment cost $\hat{\gamma}$ of around 4 is lower than that in other estimated models with financial frictions, where it ranges from around 6 (De Graeve (2008)

[25] To my knowledge the only other papers estimating this sort of credit shock to μ_t are Levin and Natalucci (2004) using US firm level data and BGG's model of financing frictions, and Fuentes-Albero's estimation of the BGG model (2009) [29].

[33]) to around 30 (Christiano et al (2010) [23]) . [26]This probably reflects the fact that I only use aggregate quantities as data, while De Graeve also includes interest rates and Christiano et al add the stock market to their observable variables. The difference in estimates is a symptom of the tension with investment or capital adjustment costs between matching asset price volatility and matching the volatility of investment. The estimated housing investment adjustment costs $\hat{\gamma}_h = 0.178$ are significantly below the prior mean. This is necessary to allow the model to match the very high volatility (around 14% per quarter) of residential investment. As in most DSGE models, we need relatively persistent shock processes to match the data, with persistence parameters ranging from $\hat{\rho}_I = 0.86$ for the investment shock to $\hat{\rho}_\mu = 0.97$ for the credit shock. The volatility of the TFP shock innovation $\hat{\sigma}_z = 0.51\%$ is significantly below the volatility of $0.7 - 1\%$ typically used in calibrated real business cycle models. Finally the credit shock is extremely volatile with $\hat{\sigma}_\mu = 23.2\%$. In the first order approximation,

$$\frac{\mu_{t+1} - \mu}{\mu} \simeq \rho_\mu \frac{\mu_t - \mu}{\mu} + (1 - \mu)\varepsilon_{\mu,t}. \tag{56}$$

A 1 standard deviation shock to the μ_t process increases μ_t by approximately 13.61%. Starting at the balanced growth path, this would raise the proportion of collateral the bank loses in default from $\mu = 0.45$ to $\mu_t = 0.511$. While this change still seems like a large number, note that it applies only to the approximately $0.26\% - 0.41\%$ of borrowers' assets that are seized in default procedures every quarter. Therefore, the overall direct impact on the leverage ratio of borrowers is not that large. [27]

3.4 The Effect of Aggregate Shocks

Here I use impulse response function analysis and variance decomposition to examine the effect of shocks hitting the economy. To isolate the effect of financial frictions, I also plot the responses of the economy when shutting down the financial frictions. I compare the model with all financing frictions (blue circles), only firm financing frictions (green stars), only household financing frictions (red diamonds) and the model without any financing frictions (light green/turquoise crosses) at the posterior mode parameter values. There are several conclusions we can draw from this exercise. First, TFP shocks are still central to generating business cycles. Second, financial shocks have difficulty in generating big recession on their own. Third, in general equilibrium worse credit frictions for households tend to reduce credit frictions for firms, though other common factors lead to comovement between firm

[26]Liu et al. (2010) [46] estimate $\hat{\gamma}$ of only 0.19 in a model with collateral constrained entrepreneurs, but they assume a fixed stock of structures (infinite adjustment costs), and define investment as the sum of equipment investment and durable consumption.

[27]To the degree that μ_t in the model is a general measure of the efficiency of the financial system in dealing with distressed loans, beyond pure bankruptcy costs, it is difficult to judge the economic plausibility of this volatility based on purely microeconomic evidence. Fuentes - Albero (2009) [29] incorporates shocks to μ_t in the BGG framework. She finds a volatility of these shocks that is even higher.

and household credit spreads. Fourth, household credit frictions have difficulty in generating bigger fluctuations in housing investment and prices.

3.4.1 Total Factor Productivity and Investment Shocks

Consider a 1 standard deviation decline in TFP. This leads to a decline in output, consumption, firm investment and residential investment (figure 1). Firm financing frictions amplify the response of business investment and consumption. Comparing the model with only firm financing frictions to the frictionless model, the absolute value of the percentage deviation from the BGP of investment is 34% - 85% higher, and the absolute value of the percentage deviation of consumption is 30% - 80% higher during the first 10 periods after the shock. Firm financing frictions dampen the response of housing investment in the initial periods after a shock. For the first 5 periods the absolute value of the percentage deviation from the BGP of residential investment is around 50% lower in the model with firm financing constraints. The overall result is that except on impact financial frictions reduce the response of output to a TFP shock, though there is significant initial output amplification if we look at total value added omitting the deadweight costs of financing frictions (figure 2). [28]

The entrepreneur's external finance premium increases strongly on impact because the loan is predetermined and the decline in productivity reduces the value of the entrepreneurs' collateral by decreasing revenue and the value of the entrepreneur's capital stock (figure 2). In subsequent periods consumption smoothing prevents entrepreneurs from reducing their borrowing 1 to 1 with their collateral. This increases the entrepreneur's leverage, external financing spread (figure 4), and external financing premium (figure 2). The spread returns gradually to its long run value as the entrepreneur reduces his borrowing and the price of capital climbs back to its steady state value. The increase in the external finance premium and the entrepreneur's desire to smooth consumption reduce investment. The higher financing frictions also forces the entrepreneur to significantly decrease consumption. [29] The strong effect of changes in financing conditions on investment has an indirect general equilibrium impact on the entrepreneur's capital utilisation decision. The bigger drop in investment in the economy with firm financing constraints leads to a bigger decline in the price of capital. This reduces the cost of using capital more intensively. As a result capital utilisation is less procyclical with firm financing constraints (figure 2), which helps explain why these constraints don't amplify the response of output to the shock.

Borrowing households also suffer from increasing external financing spreads (figures 3 and 4). The decline in the demand for housing lowers house prices and reduces the value of their collateral. To smooth their consumption, these households reduce their borrowing by less than the decline in their collateral. This leads to higher leverage and financing

[28] These costs increase in a TFP shock driven recession as default rates go up.

[29] The strong reaction of entrepreneur consumption is in line with micro evidence that consumption of the wealthiest households in the US is much more procyclical than that of other households (Parker and Vissing Jorgensen (2009) [50]).

frictions, though here the effect is less persistent than for entrepreneurs. The worsening financial condition of borrowers reduces their consumption and housing investment significantly more than those of financially unconstrained households (who actually increase their housing investment). It also increases borrowers' labour supply.

The overall effect of adding household financing frictions seems to be a modest reduction in the impact of firm financing frictions on aggregate investment and output. Moving from the economy with only firm financing constraints to the economy with all financing constraints reduces the magnitude of the decline in investment (measured by the percentage deviation from the BGP) by around 11%, and the magnitude of the decline in output by around 5% - 8% during the first 10 periods after a shock. The increase in borrowers' labour supply and the reduction in their demand for loans improve firms' financing conditions indirectly by lowering wages and reducing the risk free interest rate. As a result, the entrepreneurs' external finance premium percentage deviation from the BGP increases by around 6 - 11% less during the first 10 periods in the economy with household credit constraints. The effect on aggregate quantities is modest because savers' consumption declines less, and savers' labour supply declines more in the economy with both types of credit constraints relative to the economy with only firm credit constraints.

Household financing frictions in the model do not amplify movement in housing investment or house prices (figure 1). On the other hand, firm financing frictions do have a significant impact on the behaviour of residential investment. In their presence, residential investment responds significantly less to a negative TFP shock on impact, but its decline is more persistent. The weaker decline in residential investment in the economy with firm financing frictions is explained by the smaller increase in the risk free interest rate in comparison to the economy with financially unconstrained producers. This relative decrease in the interest rate (the return on saving through market loans or capital) generates an incentive to increase saving in the form of housing (non-market capital) relative to the financially unconstrained production economy. The relative decline in interest rate can be traced back to the no arbitrage relation between the entrepreneur's return on capital and the risk free interest. Consider for simplicity the special case of perfect foresight and $s_y = 0$. The entrepreneur's first order condition for capital is

$$\begin{aligned} R_{t+1}R^{k_e}(\bar{\varepsilon}^e_{t+1}) &= \frac{(1-\delta_{e,t+1})q^k_{t+1} + \alpha\theta\frac{y_{e,t+1}}{k_{e,t+1}}}{q^k_t}, \\ \text{where } R^{k_e\prime}(\bar{\varepsilon}^e_{t+1}) &> 0. \end{aligned} \tag{57}$$

The recession increases the default probability of entrepreneurs (a higher $\bar{\varepsilon}_{e,t+1}$) and raises the effective discount rate for investment $(R^{k_e}(\bar{\varepsilon}_{e,t+!}))$, which tends to reduce R_{t+1}. [30] As for household financing frictions, it is true that on impact borrowers dramatically reduce

[30] The same effect holds when $s_y > 0$, except now the capital gain component $\frac{(1-\delta_e)q^k_{t+1}}{q^k_t}$ and the dividend component $\alpha\theta\frac{y_{e,t+1}}{q^k_t k_{e,t+1}}$ of the return on capital investment no longer have exactly the same effective discount

their housing investment. But savers increase their housing investment significantly, so that in aggregate housing investment is barely affected by the presence of borrowers (in fact it's slightly less responsive to TFP shocks). Aggregate housing investment adjustment costs and the decline in borrower and aggregate housing investment induce an initial decline in house prices, which is reversed over time as the economy converges back to the BGP. Unlike for borrowers, savers' housing investment decisions are purely a function of interest rates and house prices. The upward sloping path of house prices together with the smaller increase in the risk free interest rate in the economy with household borrowers encourages savers to initially expand their housing stock in a TFP shock recession. [31]

Figure 5 displays the response of the model economy to an investment shock. The impact of investment shocks on output and investment are dampened by firm credit constraints, as in other models of financial frictions (e.g De Graeve (2008) [33]). An increase in the relative price of investment in the model pushes up the price of capital and reduces investment as in a frictionless model. But the higher price of capital raises the value of entrepreneur collateral and reduces the financing frictions that they face (figure 6). As a result consumption actually increases and investment declines by less. Our disaggregation of the capital stock into housing and business capital also reveals that the investment shock produces counterfactual negative comovement between business investment and both consumption and residential investment. The addition of financing frictions reduces the negative comovement between residential and non residential investment significantly, but does not eliminate it.

3.4.2 Credit Shocks

A 1 standard deviation increase in the cost of monitoring distressed loans μ_{t+1} generates a significant hump shaped decline in investment peaking at -1.5% (figure 7). Aggregate consumption declines initially by around 0.4% and then recovers, though it is still below the BGP for several more periods. Residential investment actually increases by up to 4%, leading to an increase in house prices. The strong increase in housing investment means that the decline in output due to the credit shock is quite small (at most around -0.1% if measured from the production function, or -0.3% if measured using total value added that

rate. The interest rate in both economies is countercyclical for productivity shocks because of the presence of investment adjustment costs. These adjustment costs induce procyclical movements in the price of capital that by the no arbitrage condition between capital and financial loans increase the risk free interest rate in a recession and decrease it in a boom.

[31] The size of swings in agents' housing investment may seem surprising at first. However, note that agents care about the stock of housing, which given its very low depreciation rate is barely affected even by large swings in agents' housing investment. For example if housing investment of savers drops to 0 in a period, this leads to a decline in their housing stock of only 0.4%. Given this insensitivity of the housing stock to housing investment, agents are willing to allow significant adjustment in housing investment in response to small changes in the path of interest rates and house prices. Also note that since the economy is stationary around the BGP with a constant housing stock (after detrending by the BGP growth factor), any increase in the housing stock above the BGP level must be matched by a reduction in the housing stock of an agent in future periods.

excludes the deadweight cost of financing frictions). Certainly, we are far from generating the deep recession typically associated with a credit crunch.

To understand the aggregate effects of a credit shock I start with the behaviour of entrepreneurs (figure 8). For entrepreneurs, a rise in default monitoring costs increases credit frictions. Both the credit spread and the external finance increase and borrowing declines significantly. The entrepreneur responds by reducing investment and consumption. In general equilibrium the decline in investment depresses the price of capital and lowers the value of collateral. This leads to a further rise in the external financing spread. It also tends to increase leverage, while the direct effect of higher monitoring costs is to lower leverage (figure 10). The tension between these two effects means that leverage rises in the first periods of the shock before declining.

Now consider the savers (figure 9). On impact, the fall in the risk-free interest rate encourages savers to substitute from financial assets towards housing as an alternative form of saving. The decline in interest rates also stimulates savers' consumption and reduces their labour supply. As the interest rate converges back to its BGP value, savers' consumption and housing investment decline.

Next, add the borrowers to the picture (figure 9). If we examine a model where some households are credit constrained but the production sector is financially unconstrained, the credit shock has virtually no impact on aggregates. While monitoring costs increase, the decline in borrowing is big enough to make the increase in the borrower's external finance premium tiny. Now drop the borrowing households into an economy with financially constrained entrepreneurs. In response to higher monitoring costs the borrower's external finance premium and credit spread actually decline initially. How is this possible when the same increase in μ_{t+1} has increased the external financing costs of entrepreneurs? The answer lies in the different type of collateral used by these two types of borrowers. Households borrow against housing. The increase in μ_{t+1} generates an increase in housing investment and house prices. This improves the credit constrained households' collateral position and reduces their financing costs. As a result they increase their consumption and reduce their labour supply. The profile of the increase in house prices still induces them to reduce housing investment despite the lower financing costs.

So far we have explained why housing investment increases by appealing to interest rate movements. Why does the interest rate decrease sharply on impact? The lower demand for loans from entrepreneurs reduces the interest rate on deposits, but this was also the case for a negative TFP shock even though the interest rate actually increased. There is another factor explaining the bigger decline in interest rates for a credit shock. Interest movements play an important role in reconciling changes in the demand for the components of GDP and the supply of output. Because the credit shock does not affect TFP, it puts a bigger burden on shifts in labour demand and the capital utilisation rate to generate a decline in output. The need to borrow for wage payments generates a negative link between the external finance premium and labour demand that reduces production. At the same time the fall in the price of capital reduces the cost in depreciated capital of a higher capital utilisation rate. This encourages an increase in the capital utilisation rate that raises production (figure 8). The direct effect of rising financing spreads on labour demand is not enough to counter the change in capital utilisation, so that the overall response of output (and value added) is weak. The resource constraint of the economy implies that the weak response of output and the strong decline in firm investment must lead to compensating increases in either consumption or residential investment. In equilibrium this requires a decline in the risk free interest rate, which given the higher sensitivity of residential investment to interest rates leads to a housing boom in the middle of the recession. We can eliminate the comovement problem between residential and nonresidential investment in response to a credit shock by raising housing adjustment costs. For example, when increasing γ_h by a factor of 10,000 residential investment barely increases in the first periods after a shock, and it declines a bit in latter periods. But the response of output to a higher μ_{t+1} is still economically insignificant with a maximum decline of 0.15%.

3.4.3 Variance Decomposition

I examine the contribution of each shock to the long run variance of macroeconomic quantities and financial variables at the posterior mode of the parameters. I follow the advice of Canova (1998) [14] in examining the results for several common definitions of the trend. Table 3 reports the variance decomposition of the linearly detrended variables in levels (the closest to the definition of fluctuations in the original state-space representation of the model), growth rates of variables and HP filtered variables (a common definition of business cycle frequencies). The TFP shock accounts for 71% - 73.9% of the variance of output and 56.1 - 69.4% of the variance of consumption. However it only accounts for 9.3% - 18.6% of the variance of non-residential investment and 7.2% - 11.3% of the variance of residential investment, implying that the other shocks are also important in accounting for aggregate fluctuations. Investment shocks explain 66.6% - 77.4% of the variance of non-residential investment and 35.5% - 49% of the variance of residential investment. Finally, financial shocks explain most of the fluctuations in credit spreads, external finance premia and leverage ratios, but only 4.2% - 10.9% of the variance of output, 10% - 29.6% of the variance of consumption and 13.3% - 15.7% of investment fluctuations. They do explain 39.7% - 53.2% of the variance of residential investment, though the prediction of a strong increase in housing investment in response to an increase in the loan default cost μ_t that helps generate this result seems counterfactual.

3.5 Comparing Data and Model Statistics

Here I compare time series moments based on model simulations to US data moments. In particular I examine the model's ability to match the standard deviations of consumption, residential and non residential investment and output, as well as some correlations among these variables over the estimation sample period. I analyse the data in log-levels, in growth rates and with HP filtering. I simulate time series of the same length as the data (after dropping some initial observations) for different draws from the posterior. The model does a reasonably good job in matching the volatilities of the levels of output, consumption and investment. All the model based 90% confidence intervals contain the data for these variables. In contrast, the level of housing investment is too volatile relative to the data. The model does very well in matching the correlations between the levels of consumption, business investment and consumption, but it does less well in matching some of the correlations between growth rates or HP filtered variables. One major discrepancy is that the model predicts a counterfactual negative correlation between residential and non residential investment, regardless of the filtering used. The negative comovement of the two types of investment is a common problem in DSGE models without adjustment costs (see Davis and Heathcote (2005) [26] for a discussion and possible solutions). Moderate investment adjustment costs were sufficient to generate positive comovement for TFP shocks, but this was not the case for investment and financial shocks. As for growth rates, the model captures quite well the volatilities of the residential and non residential investment growth rates, but it predicts a volatility of consumption growth which is significantly above what we find in the data. For HP filtered variables, the model predicts the magnitude of investment fluctuations quite well, but it overpredicts the volatility of consumption and underpredicts output volatility.

4 Conclusion

I have developed and estimated a dynamic stochastic general equilibrium model of the interaction between firm and household debt levels, in an environment with credit default risk and endogenous leverage ratios for both business and consumer loans. I embedded these financial frictions in a real business cycle model with residential and non residential investment. In this context, I found that total factor productivity shocks are still the main driver of business cycles in the model, at least for output and consumption. While the financial shocks as modeled here are important in explaining fluctuations in credit spreads and leverage ratios, they do not produce plausible business cycles on their own. In particular they generate a counterfactual strong negative comovement between residential and nonresidential investment. This is due to the lack of sufficient procyclicality in labour demand and capital utilisation rates in response to financial shocks, that limits the ability of these shocks to generate comovement in all output components. While the joint consideration of household and firm financing frictions allows us to address more empirical facts and provides some interesting insights, overall allowing for more pervasive financing frictions did not lead to a bigger amplification of the response of output to shocks. In fact, adding household financing frictions to an environment with firm financing frictions dampens the effect of the firm level financial accelerator in the model. Higher external financing frictions for households encourage them to reduce their demand for loans and increase their labour supply. In general equilibrium, these responses of households reduce interest rates and wages, decreasing external financing frictions faced by firms.

An interesting extension of the analysis in this paper would investigate the robustness of the results to a more realistic setup with uninsured idiosyncratic risk. In such an environment households that are saving are themselves indirectly affected by financing frictions through the possibility that they may want to borrow in the future. This leads to precautionary saving and generates a wedge relative to the complete markets consumption Euler equation that is similar in some respects to my external financing premium. [32] In particular it is also countercyclical, with precautionary saving increasing in a recession. The interaction in debt markets between household and firm loan demand is also more complex in an economy where the proportion of borrowing households and firms can change across the business cycle. In the current model all firms are net borrowers in each period, so that an increase in loan demand by households in a boom makes things worse for firms by raising their borrowing costs. In a model with heterogeneous firms, in each period some firms will be net savers. For these firms, an increase in the risk-free interest rate due to higher loan demand by borrowers makes it easier to accumulate assets that may relax their future borrowing constraints. Solving a model where both firms and households are affected by uninsurable idiosyncratic shocks presents significant difficulties using the standard simulation based Krusell and Smith (1998) [45] algorithm for heterogenous agent models with aggregate uncertainty. However, this objective may be easier using more recent techniques that reduce the use of simulation to solve the model (Algan et al. (2009) [2]).

5 Appendix A, proofs of propositions

For convenience, throughout these proofs we will also use $H(\bar{\varepsilon}) \equiv -RP(\bar{\varepsilon})$.

Proposition1: With perfect foresight, $\frac{\partial(h_{bo,t}/c_{bo,t})}{\partial\bar{\varepsilon}_{bo,t+1}} < 0$.

Proof: With perfect foresight we can combine the loan and the housing Euler equations to obtain

$$q_t = \frac{\xi_h}{\xi_c}\frac{c_{bo,t}}{h_{bo,t}} + (1-\delta_h)q_{t+1}\frac{A_{hh}(\bar{\varepsilon}_{bo,t+1})}{R_{t+1}efp(\bar{\varepsilon}_{bo,t+1})}.$$

[32] See Challe and Ragot (2010) [19] for a derivation of this wedge in a simplified uninsured idiosyncratic risk model.

$$\frac{d\left(\frac{A_{hh}(\bar{\varepsilon}_{bo,t+1})}{efp(\bar{\varepsilon}_{bo,t+1})}\right)}{d\bar{\varepsilon}_{bo,t+1}} = \frac{-efp'(\bar{e}_{bo,t+1})}{efp^2_{bo,t+1}} + [\frac{H'(\bar{\varepsilon}_{bo,t+1})}{efp_{bo,t+1}} + G'(\varepsilon_{bo,t+1}) - H(\bar{\varepsilon}_{bo,t+1})\frac{efp'(\bar{\varepsilon}_{bo,t+1})}{efp^2_{bo,t+1}}]$$
$$= -\frac{efp'(\bar{\varepsilon}_{bo,t+1})}{efp^2_{bo,t+1}}[1 + H(\bar{\varepsilon}_{bo,t+1})] < 0,$$

where the last inequality follows from $efp'(\bar{\varepsilon}_{bo,t+1}) > 0$ and $H(\bar{\varepsilon}_{bo,t+1}) > -1$. Therefore, an increase in $\bar{\varepsilon}_{bo,t+1}$ must lead to a rise in $\frac{c_{bo,t}}{h_{bo,t}}$ for given house prices.

Proposition 2:a) With perfect foresight and fixed capital utilisation, $\frac{\partial k_{e,t+1}}{\partial \bar{\varepsilon}_{e,t+1}} < 0$ for fixed employment. With variable employment $\frac{\partial n_{e,t}}{d\bar{\varepsilon}_{e,t}} < 0$ is a sufficient condition for $\frac{\partial k_{e,t+1}}{\partial \bar{\varepsilon}_{e,t+1}} < 0$.

b) With fixed capital utilisation, $a \geq 1 - s_y$ implies that $\frac{\partial n_{e,t}}{\partial \bar{\varepsilon}_{e,t}} < 0$.

c) $\frac{\partial u_{e,t}}{\partial \bar{\varepsilon}_{e,t}} < 0$.

Proof:a) The proof is similar to that of proposition 1. Combining the loan and capital Euler equations, we have

$$q_t^k = \frac{1}{R_{t+1}efp_{e,t+1}}[A_{kk}(\bar{\varepsilon}_{e,t+1})q^k_{t+1}(1-\delta_e) + A_{ky}(\bar{\varepsilon}_{e,t+1})\alpha\theta\frac{y_{e,t+1}}{k_{e,t+1}}].$$

Using the fact that $A_{ky}(\bar{\varepsilon}_{e,t+1})/efp(\bar{\varepsilon}_{e,t+1})$ and $A_{kk}(\bar{\varepsilon}_{e,t+1})/efp(\bar{\varepsilon}_{e,t+1})$ are both decreasing in $\bar{\varepsilon}_{e,t+1}$ and the diminishing marginal productivity of capital we find $\frac{\partial k_{e,t+1}}{\partial \bar{\varepsilon}_{e,t+1}} < 0$ for fixed $n_{e,t+1}$. Now allow for variable labour supply with $\frac{\partial n_{e,t}}{\partial \bar{\varepsilon}_{e,t}} < 0$. Rewrite the first order condition for $n_{e,t}$ as

$$\frac{1 + (1-s_y)[H(\bar{\varepsilon}_{e,t}) + efp_{e,t}G(\bar{\varepsilon}_{e,t})]}{(1 - a + aefp_{e,t})}(1-\alpha)\theta\frac{y_{e,t}}{n_{e,t}} = w_t = B_{n_e}(\bar{\varepsilon}_{e,t})(1-\alpha)\theta\frac{y_{e,t}}{n_{e,t}} = w_{t.}$$

By diminishing marginal productivity of labour, $\frac{\partial n_{e,t}}{\partial \bar{\varepsilon}_{e,t}} < 0$ if and only if $\frac{dB_{n_e}(\bar{\varepsilon}_{e,t})}{d\bar{\varepsilon}_{e,t}} < 0$. Solving for n_{et} as a function of capital and substituting the result into the Euler equation for capital,

$$q_t^k = \frac{1}{R_{t+1}efp_{e,t+1}}[A_{kk}(\bar{\varepsilon}_{e,t+1})q^k_{t+1}(1-\delta_e) + A_{ky}(\bar{\varepsilon}_{e,t+1})\left(\frac{(1-\alpha)\theta z_{e,t+1}}{w_{t+1}}B_{n,e}(\bar{\varepsilon}_{e,t+1})\right)^{\frac{(1-\alpha)\theta}{1-(1-\alpha)\theta}} k_{e,t+1}^{\frac{\alpha\theta}{1-(1-\alpha)\theta}-1}].$$

$\frac{dB_{n_e}(\bar{\varepsilon}_{e,t})}{d\bar{\varepsilon}_{e,t}} < 0$ implies that as before the right hand side of this equation is decreasing in $\bar{\varepsilon}_{e,t+1}$. Since $k_{e,t+1}^{\frac{\alpha\theta}{1-(1-\alpha)\theta}-1}$ is decreasing in $k_{e,t+1}$, a higher $\bar{\varepsilon}_{e,t+1}$ must lead to a lower $k_{e,t+1}$.

Proof of 2,b): From the previous proof, we know that $\frac{\partial n_{e,t}}{\partial \bar{\varepsilon}_{e,t}} < 0$ if and only if $\frac{dB_{n_e}(\bar{\varepsilon}_{e,t})}{d\bar{\varepsilon}_{e,t}} < 0$, where

$$\frac{1 + (1-s_y)[H(\bar{\varepsilon}_{e,t}) + efp_{e,t}G(\bar{\varepsilon}_{e,t})]}{(1 - a + aefp_{e,t})} = B_{n_e}(\bar{\varepsilon}_{e,t}).$$

.

$$\begin{aligned}
\frac{dB_{n_e}(\bar{\varepsilon}_{e,t})}{d\bar{\varepsilon}_{e,t}} &= \frac{efp'(\bar{\varepsilon}_{e,t})}{(1-a+aefp_{e,t})^2}\{(1-s_y)[(1-a+aefp_{e,t})G(\bar{\varepsilon}_{e,t})-a(H(\bar{\varepsilon}_{e,t})+efp_{e,t}G(\bar{\varepsilon}_{e,t}))]-a\} \\
&= \frac{efp'(\bar{\varepsilon}_{e,t})}{(1-a+aefp_{e,t})^2}\{(1-s_y)[G(\bar{\varepsilon}_{e,t})(1-a)-a(H(\bar{\varepsilon}_{e,t})]-a\} \\
&= \frac{efp'(\bar{\varepsilon}_{e,t})}{(1-a+aefp_{e,t})^2}\{(1-s_y)[G(\bar{\varepsilon}_{e,t})+a\mu_{e,t}\int_0^{\bar{\varepsilon}_{e,t}}\varepsilon dF]-a\}
\end{aligned}$$

where the last equality uses $H(\bar{\varepsilon}_{e,t})+G(\bar{\varepsilon}_{e,t}) = -\mu_{e,t}\int_0^{\bar{\varepsilon}_{e,t}}\varepsilon dF$. $\frac{efp'(\bar{\varepsilon}_{e,t})}{(1-a+aefp_{e,t})^2} > 0$, since $efp'(\bar{\varepsilon}_{e,t}) > 0$. Therefore, $\frac{dB_{n_e}(\bar{\varepsilon}_{e,t})}{d\bar{\varepsilon}_{e,t}} < 0$ if and only if

$$(1-s_y)[G(\bar{\varepsilon}_{e,t})+a\mu_{e,t}\int_0^{\bar{\varepsilon}_{e,t}}\varepsilon dF]-a < 0$$

, that is

$$G(\bar{\varepsilon}_{e,t})+a\mu_{e,t}\int_0^{\bar{\varepsilon}_{e,t}}\varepsilon dF < \frac{a}{1-s_y}.$$

Since

$$G(\bar{\varepsilon}_{e,t})+a\mu_{e,t}\int_0^{\bar{\varepsilon}_{e,t}}\varepsilon dF < 1$$

, $a \geq 1-s_y$ is a sufficient condition for

$$(1-s_y)[G(\bar{\varepsilon}_{e,t})+a\mu_{e,t}\int_0^{\bar{\varepsilon}_{e,t}}\varepsilon dF]-a < 0.$$

c) Rewrite the first order condition for capital utilisation:

$$u_{e,t} : \frac{A_{ky}(\bar{\varepsilon}_{e,t})}{A_{kk}(\bar{\varepsilon}_{e,t})}\alpha\theta z_e k_{e,t}^{a\theta-1} n_{e,t}^{\theta(1-\alpha)} = \phi_e\delta_e u_{e,t}^{\phi-\alpha\theta} q_t^k.$$

We will show that $\frac{A_{ky}(\bar{\varepsilon}_{e,t})}{A_{kk}(\bar{\varepsilon}_{e,t})}$ is decreasing in $\bar{\varepsilon}_{e,t}$. Since $\phi > \alpha\theta$, this implies that $\frac{\partial u_{e,t}}{\partial\bar{\varepsilon}_{e,t}} < 0$. Ignoring time subscripts,

$$\begin{aligned}
\frac{A_{ky}(\bar{\varepsilon}_e)}{A_{kk}(\bar{\varepsilon}_e)} &= 1+\frac{A_{ky}(\bar{\varepsilon}_e)-A_{kk}(\bar{\varepsilon}_e)}{A_{kk}(\bar{\varepsilon}_e)} \\
&= 1-\frac{s_y(efp_e(\bar{\varepsilon}_e)G(\bar{\varepsilon}_e)-RP(\bar{\varepsilon}_e))}{1+efp_e(\bar{\varepsilon}_e)G(\bar{\varepsilon}_e)-RP(\bar{\varepsilon}_e)} = 1-\frac{s_y}{1+\frac{1}{efp_e(\bar{\varepsilon}_e)G(\bar{\varepsilon}_e)-RP(\bar{\varepsilon}_e)}}.
\end{aligned}$$

$efp_e(\bar{\varepsilon}_e)G(\bar{\varepsilon}_e)-RP(\bar{\varepsilon}_e)$ is increasing in $\bar{\varepsilon}_e$, so $\frac{A_{ky}(\bar{\varepsilon}_e)}{A_{kk}(\bar{\varepsilon}_e)}$ is decreasing in $\bar{\varepsilon}_e$.

6 Appendix B, the balanced growth path:

I approximate the model around a balanced growth path (BGP) where $k_{e,t}, I_{e,t}y_{e,t}, h_{s,t}$, $h_{bo,t}, c_{s,t}, c_{bo,t}, d_t, l_{e_t}, l_{bo,t}$ and w_t all grow at the same growth rate $G_y - 1$. hours of work, capacity utilisation rates, default thresholds and the real interest rate on deposits are all constant in the BGP. This implies that the growth rates of total factor productivity in the BGP are $G_{z_e} = G_y^{1-\alpha\theta}$ and $G_{z_u} = G_y^{1-\alpha}$. Let $\tilde{x}_t = \frac{x_t}{G_x^t}$, where G_x is the gross growth rate of x in the BGP. $\hat{x}_t = \frac{x_t - x}{x}$ where x is the BGP value of $\tilde{x}_t$. I normalize $z_u = z_e = 1$. To find the BGP, first detrend all first order conditions and constraints.

1) We start by computing the default thresholds and idiosyncratic shock volatilities to match BGP default rates.

For $i = e, bo$ $\bar{\varepsilon}_i$ solves

$1 = \beta^i G_{\lambda_i} efp_i(\bar{\varepsilon}_i)R.$

Note that this solution ignores the constraint that entrepreneur loans $l_e \geq 0$ which may in theory be violated due to the wages in advance constraint affecting part of the wage bill. I solve the model conjencturing that this constraint does not bind. Afterwards, I can compute l_e and check that this is indeed the case. For my calibrations, this was never a problem with $\frac{Rl_e}{G_e\tilde{A}} > 0$ by a large margin.

There are two solutions to each of the above equations. The monotone hazard rate assumption implies that the optimal solution is the lower $\bar{\varepsilon}_i$ solution. After picking the lower solution interval, a simple bisection finds each $\bar{\varepsilon}_i$.

We can then solve for various other expressions that are functions of $\bar{\varepsilon}_i$ such as $H_i, G_i, efp_{i,}$.

2) We need to compute the market clearing wage. This can be done by a bisection, in which we solve for the model quantities conditional on a given wage, check if the labour market equilibrium has converged and iterate if required. Since entrepreneurial financing frictions tend to lower the demand for labour relative to that of financially unconstrained firms, the wage in a model with financially unconstrained firms is a reliable upper bound for the solution of the model where all firms are financially constrained.

2.1) Conditional on the guess for the wage rate we compute entrepreneur variables:

We already have $\bar{\varepsilon}_e$ from 1). We use the labour demand first order condition to express n_e as a function of k_e:

$n_e = \left[\frac{(1-\alpha)\theta z_e(u_e k_e)^{\alpha\theta}}{A_{ne}w}\right]^{\frac{1}{1-\theta(1-\alpha)}}$, where $A_{ne} = \frac{1-a+aefp_e}{1+(1-s_y)(H_e+G_e efp_e)}$.

This allows us to use the euler equation for entrepreneur capital to solve for k_e.

$k_e = \frac{1}{G_y}\left[\frac{\frac{G_y}{\beta^e}-(1-\delta_e)A_{kk}}{\alpha\theta A_{yk}(G_{z_e}z_e)^{\frac{1}{1-\theta(1-\alpha)}}(\frac{(1-\alpha)\theta u_e}{G_y w A_{ne}})^{\frac{\theta(1-\alpha)}{1-\theta((1-\alpha)}}}\right]^{\frac{1-\theta(1-\alpha)}{\theta-1}}$, where $A_{yk} = 1 + (1 - s_y)(H_e + efp_e G_e)$ and $A_{kk} = 1 + (1 - s_k)(H_e + efp_e G_e)$.

It can be shown that the expressions above are always positive.

Next, we can find ϕ_e from the first order condition for u_e :

$\phi_e = \frac{A_u}{\delta_e}\left(\frac{\frac{G_y}{\beta^e}-(1-\delta_e)A_{kk}}{A_{yk}}\right)$, where $A_u = \frac{A_{yk}}{A_{kk}}$.

We normalise $u_e = 1$, allowing us to identify δ_e with the average depreciation rate in the sample.

Given the variables above it is easy to find y_e, $\tilde{A}_e, \bar{A}_e$, $l_e = (\tilde{A}_e G_e - awn_e)/R$ and $c_e = \bar{A}_e + H_e\tilde{A}_e - (1-a)wn_e + l_e G_y - k_e G_y$.

2.2) Now we compute the borrower variables.

We already have the default threshold $\bar{\varepsilon}_{bo}$ and functions of this threshold from 1).

Using the first order conditions for housing and labour supply we can solve for

$h_{bo} = A_{hbo}c_{bo}$ where $A_{hbo} = \frac{\xi_h}{\xi c G_y(1-\beta^{bo}G_\lambda(1-\delta_h)[1+(1-s_h)(H_{bo}+efp_{bo}G_{bo})])}$, and

$n_{bo} = 1 - A_{nbo}\frac{c_{bo}}{w}$ where $A_{nbo} = \frac{\xi_n}{\xi_c[1+(1-s_w)(H_{bo}+efp_{bo}G_{bo})]}$.

We can then also solve for $\bar{A}_{bo} = (1-\delta_h)h_{bo} + wn_{bo}$, $\tilde{A}_{bo}$ and $l_{bo} = G_{bo}\tilde{A}_{bo}/R$ as functions of c_{bo}.

To compute c_{bo} we plug the expressions above into the budget constraint and solve for c_{bo} :

$c_{bo} = \frac{X_{nbo}}{1+A_{hbo}X_{hbo}+A_{nbo}X_{nbo}}w$ where $X_{nbo} = 1 + (1-s_w)(\tilde{H}_{bo} + \frac{G_y}{R}G_{bo})$ and $X_{hbo} = G_y - (1 - \delta_h)[1 + (1-s_h)(H_{bo} + G_{bo}\frac{G_y}{R})]$

Again, it can be shown that the expressions for c_{bo} and h_{bo} are always positive. $n_{bo} > 0$ for any calibration in which $n_s > 0$ since $n_{bo} > n_s$.

Given c_{bo}, we can now go back and solve for the other borrower variables using the expressions derived earlier.

2.3) Next we compute the variables for the savers.

For the savers, the solution is similar to that for the borrowers. We use the first order conditions for housing and labour supply to express $h_s = A_{hs}c_s$, where $A_{hs} = \frac{\xi_h}{\xi_c G_y[1-\beta G_\lambda(1-\delta_h)]}$ and $n_s = 1 - \frac{\xi_n}{\xi_c}\frac{c_s}{w}$. We also determine deposits from the market clearing condition $d = \frac{\theta_e l_e + \theta_{bo}l_{bo}}{\theta_s}$.

We can then substitute the expressions above, using the results from all the previous steps, into the savers' budget constraint to solve for $c_s = \frac{(R-G_y)d+w}{1+(G_y-(1-\delta_h))A_{hs}+\frac{\xi_n}{\xi_c}}$.

3) We now check if the excess demand $|n_e - \theta_s n_s - \theta_{bo}n_{bo}| < \varepsilon^n$, and update the wage guess if necessary.

This concludes the solution of the BGP.

Alternatively, we can solve for the equilibrium wage analytically if we know the targeted proportion of hours worked in the BGP. Suppose we have a target in mind for n_e. The first order condition for capital can be solved for the equilibrium $\frac{y_e}{k_e}$ ratio as a function of $\bar{\varepsilon}_e$. $\frac{y_e}{k_e} = z_e\left(\frac{k_e}{n_e}\right)^{\alpha\theta-1} n_e^{\theta-1}$ allows us to solve for $\frac{k_e}{n_e}$. The labour demand function can be written as $\theta(1-\alpha)\frac{y_e}{n_e} = \theta(1-\alpha)\left(\frac{k_e}{n_e}\right)^{\alpha\theta} n_e^{\theta-1} = A_{n_e}(\bar{\varepsilon}_e)w$. Since we know $\frac{k_e}{n_e}$, n_e and $\bar{\varepsilon}_e$ we can solve for w analytically. To make the labour market clear we adjust the labour supply through the ratio $\frac{\xi_n}{\xi_c}$ for a given ξ_h until $|\theta_s n_s + \theta_{bo}n_{bo} - n_e| < \varepsilon_N$ for a small ε_N. A good upper bound for the labour market clearing $\frac{\xi_n}{\xi_c}$ can be found by solving the economy where only borrowers work, while savers just earn income from deposits (this bound can be computed analytically for $G_y = 1$, and it is usually also an upper bound for $G_y > 1$ but close to 1).

7 Appendix C, the model with a predetermined loan rate with respect to aggregate shocks

For concreteness, I focus on the borrowing household. Similar derivations apply to the entrepreneur.

Suppose that now R_t^l must be determined before knowing the aggregate state of the economy at t. Since $l_{bo,t-1}$ is predetermined at t, This makes the repayment without default $R_t^l l_{bo,t-1} = \bar{\varepsilon}_{bo,t}\tilde{A}_{bo,t}$ predetermined.

As before, the bank can still diversify its exposure to the borrowers' idiosyncratic shocks. We can still write the bank's expected repayments as

$$G(\bar{\varepsilon}_{bo,t})\tilde{A}_{bo,t} = \bar{R}_{bo,t} l_{bo,t-1}$$

for some $\bar{R}_{bo,t} \geq 0$, where

$$G(\bar{\varepsilon}_{_{bo,t}}) = [1 - F(\bar{\varepsilon}_{bo,t})]\bar{\varepsilon}_{bo,t} + (1 - \mu_{bo,t}) \int_0^{\bar{\varepsilon}_{bo,t}} \varepsilon_{bo} dF.$$

$\bar{\varepsilon}_{bo,t}$ must satisfy two conditions:

$$\bar{\varepsilon}_{bo,t} = \frac{R_t^l l_{bo,t-1}}{\tilde{A}_{bo,t}}$$

and

$$G(\bar{\varepsilon}_{bo,t}) = \frac{\bar{R}_{bo,t} l_{bo,t-1}}{\tilde{A}_{bo,t}}.$$

Before we assumed that $\bar{R}_{bo,t} = R_t$ and allowed $R_t^l l_{bo,t-1}$ to adjust as a function of aggregate conditions to make the first condition trivially hold. But now $R_t^l l_{bo,t-1}$ is independent of the aggregate state. As a result, if $\bar{R}_{bo,t}$ is independent of the aggregate state we have a system of 2 equations in one unknown $\bar{\varepsilon}_{bo,t}$ that (generically) has no solution. Therefore, $\bar{R}_{bo,t}$ must adjust as a function of the aggregate state. The contract can no longer be reduced to picking a schedule of default thresholds $\bar{\varepsilon}_{bo,t}$. Instead, the representative borrower now picks

$$\left\{c_{bo,t}, h_{bo,t}, l_{bo,t}, d_{bo,t}, R_{bo,t+1}^l, n_{bo,t}, \bar{\varepsilon}_{bo,t}\right\}_{t=0}^{\infty}$$

to maximise

$$E_0 \Sigma_{t=0}^{\infty} \beta^{bo^t} u_{bo,t}$$

subject to a sequence of budget constraints

$$c_{bo,t} + q_t h_{bo,t} + d_{bo,t} = q_t(1-\delta_h)h_{bo,t-1} + n_{bo,t}w_t - RP(\bar{\varepsilon}_{bo,t})(1-s_h)q_t(1-\delta_h)h_{bo,t-1} + l_{bo.t} + d_{bo,t-1}R_t$$

,the participation constraints of the bank $G(\bar{\varepsilon}_{bo,t})\ (1-s_h)q_t(1-\delta_h)h_{bo,t-1}=\bar{R}_{bo,t}l_{bo,t-1}$ and $\bar{\varepsilon}_{bo,t}=\frac{R^l_{bo,t}l_{bo,t-1}}{\tilde{A}_{bo,t}}$.

The expected rates of return conditional on the aggregate state $\bar{R}_{bo,t}$ must now satisfy $\lambda_{s,t}=\beta E_t\lambda_{s,t+1}\bar{R}_{bo,t+1}$.

This can be shown using either a decentralisation where savers are assumed to directly make risky loans to borrowers, or an equivalent (by the Modigliani-Miller theorem) but more realistic decentralisation where savers provide banks that they own with risk free deposits and the banks lend out those funds at the risky rate $\bar{R}_{bo,t+1}$, ,repay the borrowers' deposits and distribute all profits(negative if they suffer a loss) to the savers.

Note that the deterministic balanced growth path solution, around which we approximate the dynamics, is the same as for the model where R^l_t is conditional on the aggregate state.

For the entrepreneur, we get the same results if $a=0$, otherwise we need to think more carefully about the joint modeling of the intratemporal working capital loan and the intertemporal loan. The most direct generalisation of the previous setup is to assume that the predetermined loan rate $R^l_{e,t}$ is the same on both types of loans. That is $\bar{\varepsilon}_{e,t}\tilde{A}_{e,t}=R^l_{e,t}(l_{e,t-1}+aw_tn_{e,t})$. With this assumption, the analysis for the entrepreneur financial contract is similar to that of the of the borrower's contract. Otherwise, we will need to model these two types of loans separately.

8 Data Appendix

I use US aggregate data from the first quarter of 1955 to the 4th quarter of 2004. All time series are from the Federal Reserve Economic Data at http://research.stlouisfed.org/fred2/ :

1) C^{obs}_t : sum of personal consumption expenditures on services and nondurables ($PCESV+PCND$) divided by the GDP implicit price deflator and civilian non-institutional population over the age of 16

2) $I^{h,obs}_t$: Private residential fixed investment ($PRFI$) divided divided by the GDP implicit price deflator and civilian non-institutional population over the age of 16.

3) I^{obs}_t : Private non residential fixed investment ($PNFI$) divided divided by the GDP implicit price deflator and civilian non-institutional population over the age of 16.

The model's GDP is defined as the sum of the three variables above. To match the observable GDP in estimation I use the model's value-added (before financial frictions costs) and add the financial frictions related to households, which correspond to NIPA's inclusion of financial services in its services expenditure measure (see NIPA handbook, chapter 5 [28]).

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Table 1
Calibrated parameters

Parameter	Description	Value,Target
β	Patient households' discount factor	$\beta = 0.997, R - 1 = 0.01$
β^e	Entrepreneurs' discount factor	$\beta^e = 0.95$, leverage$= 0.587$
β^{bo}	Borrower discount factor	$\beta^{bo} = 0.949$, leverage$= 0.76$
σ	relative risk aversion coefficient	2
$G_y - 1$	trend growth rate of GDP	0.51%
α	capital share	$\alpha = 0.384$, $\frac{I}{y} = 0.153$
δ_e, ϕ_e	capital depreciation rate	$\delta_e = 1.4\%$, $\phi_e = 3.21$
δ_h	housing depreciation rate	0.4%
θ	returns to scale	0.95
ξ_c, ξ_n, ξ_h	$u(.)$ weights of consumption, leisure, housing	housing/GDP≈ 1.3, $N^s = 0.32$
θ_{bo}	proportion of borrowers	0.4
μ	monitoring cost, entrepreneurs	0.45, 1.27%/4 credit spread
σ_{bo}	idiosyncratic shock std. dev, borrowers	$\sigma_{bo} = 0.0993$, 1.4%/4 default rate
σ_e	idiosyncratic shock std. dev, entrepreneurs	$\sigma_e = 0.2086$, 3%/4 default rate
s_y	output proportion seizable by lender	0.5
a	wages in advance proportion	0.5

Table 2
Estimated parameters

	Prior			Posterior					
	Distr.	Mean/LB	St. Dev/UB	mode	mean	St. Dev	5%	50%	95%
γ	Gamma	4	1	3.999	4.086	0.5182	3.3	4.049	4.989
γ_h	Gamma	1	0.5	0.178	0.183	0.021	0.151	0.181	0.219
ρ_z	Beta	0.75	0.15	0.934	0.933	0.0113	0.914	0.934	0.951
ρ_μ	Beta	0.75	0.15	0.968	0.966	0.0088	0.95	0.966	0.979
ρ_I	Beta	0.75	0.15	0.858	0.859	0.0139	0.836	0.859	0.881
σ_z	Uniform	0	0.05	0.0051	0.0052	0.0003	0.0047	0.0051	0.0056
σ_μ	Uniform	0	0.5	0.232	0.241	0.023	0.206	0.239	0.281
σ_I	Uniform	0	0.05	0.0252	0.0254	0.0014	0.023	0.0253	0.028

For uniform priors, the table provides the lower bound (LB) instead of the mean and the upper bound (UB) instead of the standard deviation. The last columns provide percentiles of the posterior distribution (90% probability intervals and median).

Table 3,

Variance decomposition for level of variables

Var	$\varepsilon_{z,t}$		$\varepsilon_{\mu,t}$		$\varepsilon_{I,t}$	
y_t	0.71	[0.693, 0.708, 0.721]	0.042	[0.04, 0.042, 0.053]	0.248	[0.239, 0.25, 0.254]
c_t	0.577	[0.574, 0.575, 0.58]	0.10	[0.097, 0.102, 0.108]	0.323	[0.311, 0.323, 0.327]
I_t	0.186	[0.181, 0.186, 0.191]	0.148	[0.114, 0.145, 0.191]	0.666	[0.628, 0.669, 0.695]
I_t^h	0.113	[0.101, 0.114, 0.126]	0.397	[0.356, 0.396, 0.438]	0.49	[0.436, 0.49, 0.542]
$efp_{bo,t}$	0.089	[0.057, 0.088, 0.128]	0.611	[0.571, 0.616, 0.661]	0.3	[0.282, 0.296, 0.301]
$efp_{e,t}$	0.136	[0.104, 0.134, 0.163]	0.662	[0.637, 0.665, 0.686]	0.202	[0.199, 0.201, 0.21]
$s_{bo,t}$	0.083	[0.053, 0.083, 0.121]	0.638	[0.594, 0.642, 0.689]	0.279	[0.258, 0.275, 0.285]
$s_{e,t}$	0.162	[0.128, 0.161, 0.193]	0.632	[0.597, 0.635, 0.672]	0.206	[0.2, 0.204, 0.21]
$G_{bo,t}$	0.008	[0.004, 0.008, 0.015]	0.966	[0.952, 0.966, 0.978]	0.026	[0.018, 0.026, 0.034,]
$G_{e,t}$	0.073	[0.041, 0.075, 0.119]	0.863	[0.781, 0.86, 0.92]	0.064	[0.038, 0.065, 0.1]

Variance decomposition in growth rates (first difference of log-levels)

Var	$\varepsilon_{z,t}$		$\varepsilon_{\mu,t}$		$\varepsilon_{I,t}$	
y_t	0.739	[0.687, 0.738, 0.779]	0.109	[0.084, 0.111, 0.146]	0.152	[0.137, 0.151, 0.167]
c_t	0.694	[0.646, 0.693, 0.731]	0.234	[0.195, 0.236, 0.285]	0.072	[0.069, 0.071, 0.074]
I_t	0.093	[0.081, 0.094, 0.107]	0.133	[0.118, 0.132, 0.148]	0.774	[0.771, 0.774, 0.776]
I_t^h	0.113	[0.089, 0.115, 0.145]	0.532	[0.478, 0.533, 0.581]	0.355	[0.33, 0.352, 0.377]
$efp_{bo,t}$	0.08	[0.055, 0.08, 0.111]	0.626	[0.597, 0.632, 0.661]	0.294	[0.284, 0.289, .291]
$efp_{e,t}$	0.20	[0.157, 0.197, 0.232]	0.552	[0.537, 0.556, 0.573]	0.248	[0.231, 0.247, 0.27]
$s_{bo,t}$	0.086	[0.059, 0.085, 0.119]	0.605	[0.572, 0.61, 0.644]	0.309	[0.298, 0.304, 0.308]
$s_{e,t}$	0.229	[0.186, 0.227, 0.263]	0.522	[0.499, 0.526, 0.554]	0.249	[0.238, 0.247, 0.26]
$G_{bo,t}$	0.116	[0.08, 0.116, 0.157]	0.487	[0.457, 0.493, 0.531]	0.397	[0.385, 0.39, 0.391]
$G_{e,t}$	0.293	[0.247, 0.293, 0.336]	0.516	[0.464, 0.519, 0.575]	0.191	[0.178, 0.188, 0.2]

Variance decomposition for HP detrended variables (smoothnes parameter $\lambda = 1600$)

Var	$\varepsilon_{z,t}$		$\varepsilon_{\mu,t}$		$\varepsilon_{I,t}$	
y_t	0.734	0.721,0.764, 0.799]	0.109	[0.083, 0.102, 0.129]	0.157	[0.118, 0.133, 0.149]
c_t	0.561	0.521, 0.557, 0.585]	0.296	[0.223, 0.265, 0.313]	0.143	[0.166, 0.178, 0.192]
I_t	0.097	0.121, 0.137, 0.152]	0.157	[0.174, 0.205, 0.238]	0.746	[0.64, 0.657, 0.673]
I_t^h	0.072	0.082, 0.097, 0.111]	0.453	[0.465, 0.528, 0.583]	0.475	[0.335, 0.375, 0.424]
$G_{bo,t}$	0.049	0.034, 0.048, 0.063]	0.782	[0.775, 0.782, 0.789]	0.169	[0.162, 0.171, 0.177]
$G_{e,t}$	0.241	0.202,0.24, 0.276]	0.594	[0.497, 0.548, 0.597]	0.165	[0.201, 0.212, 0.227]

For each shock the first column provides the contribution to the variance of each variable at the posterior mode. The second column provides 5%,50% and 95% percentiles of the posterior distribution of the contribution to the variance, approximated using 1000 draws from the posterior distribution of θ_p. I take logs for output, consumption, residential and non residential investment and the leverage ratios $G_{bo,t}, G_{e,t}$. x% percentile variance contributions need not add up to 1.

Table 4
Model simulation based statistics versus the data in (log) levels

var	σ_x				corr(Y,X)			
	data	5%	50%	95%	data	5%	50%	95%
y	3.1%	1.79%	2.56%	3.84%	1	1	1	1
c	3%	1.56%	2.16%	3.19%	0.798	0.658	0.822	0.914
I	10.4%	8.08%	11.28%	15.75%	0.473	0.255	0.534	0.73
I^h	14.6%	14.76%	20.15%	27.39%	0.592	-0.129	0.237	0.543

var	corr(I,X)			
	data	5%	50%	95%
y	0.473	0.255	0.534	0.723
c	-0.071	-0.276	0.052	0.372
I	1	1	1	1
I^h	0.069	-0.744	-0.51	-0.163

Data is linearly detrended variables in logs. Model simulations are detrended by common BGP trend and in logs. I simulate 1000 draws from the posterior distribution of the estimated parameters. For each posterior draw, I simulate 400 samples of length 300, dropping the first 100 observations as burn-in. After the US sample statistics, I report medians and 90% confidence intervals of model simulations. The first columns examine standard deviations, the next columns examine contemporaneous correlations with output and investment.

Table 5
Model simulation based statistics versus the data for growth rates

var	σ_x				corr(Δy,Δx)			
	data	5%	50%	95%	data	5%	50%	95%
Δy	0.76%	0.81%	0.93%	1.07%	1	1	1	1
Δc	0.51%	0.84%	0.98%	1.16%	0.783	0.91	0.931	0.947
ΔI	2.36%	2.31%	2.66%	3.08%	0.719	0.445	0.533	0.61
ΔI^h	4.57%	4.41%	5.08%	5.89%	0.63	-0.227	-0.085	0.058

var	corr(ΔI,Δx)			
	data	5%	50%	95%
Δy	0.719	0.445	0.533	0.61
Δc	0.313	0.172	0.281	0.385
ΔI	1	1	1	1
ΔI^h	0.1848	-0.654	-0.559	-0.449

Growth rates are computed as the first difference of the log-levels. I simulate 1000 draws from the posterior distribution of the estimated parameters. For each posterior draw, I simulate 400 samples of length 300, dropping the first 100 observations as burn-in. After the US sample statistics, I report medians and 90% confidence intervals of model simulations. The first columns examine standard deviations, the next columns examine contemporaneous correlations with output and investment.

Table 6

Model simulation based statistics versus the data, HP filter (smoothness parameter $\lambda = 1600$)

var	σ_x				corr(Y,X)			
	data	5%	50%	95%	data	5%	50%	95%
y	1.57%	0.98%	1.18%	1.42%	1	1	1	1
c	0.83%	0.97%	1.19%	1.46%	0.878	0.773	0.856	0.91
I	4.64%	4.07%	5.19%	6.58%	0.795	0.292	0.504	0.669
I^h	9.81%	8.025%	10.24%	12.99%	0.705	-0.346	-0.096	0.166

var	corr(I,X)			
	data	5%	50%	95%
y	0.795	0.292	0.504	0.669
c	0.617	-0.12	0.123	0.354
I	1	1	1	1
I^h	0.227	-0.808	-0.677	-0.479

I take logs and HP filter (with smoothness parameter $\lambda = 1600$) both the data and the model simulations. I simulate 1000 draws from the posterior distribution of the estimated parameters. For each posterior draw, I simulate 400 samples of length 300, dropping the first 100 observations as burn-in. After the US sample statistics, I report medians and 90% confidence intervals of model simulations. The first columns examine standard deviations, the next columns examine contemporaneous correlations with output and investment.

Figure 1: 1 standard deviation TFP shock. Blue circles: model with all credit frictions on. Green stars: model with only firm credit frictions. Red diamonds: model with only household credit frictions. Green crosses: no credit frictions

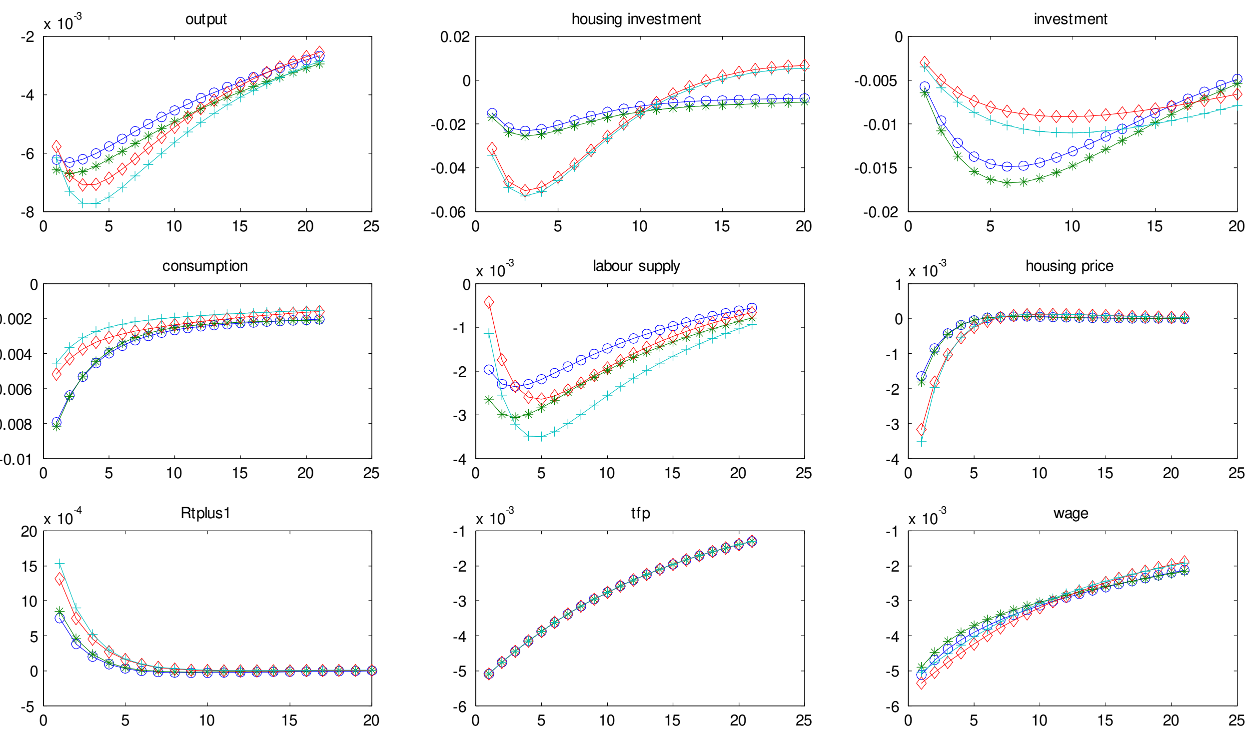

Figure 2: 1 standard deviation TFP shock. Blue circles: model with all credit frictions on. Green stars: model with only firm credit frictions. Red diamonds: model with only household credit frictions. Green crosses: no credit frictions

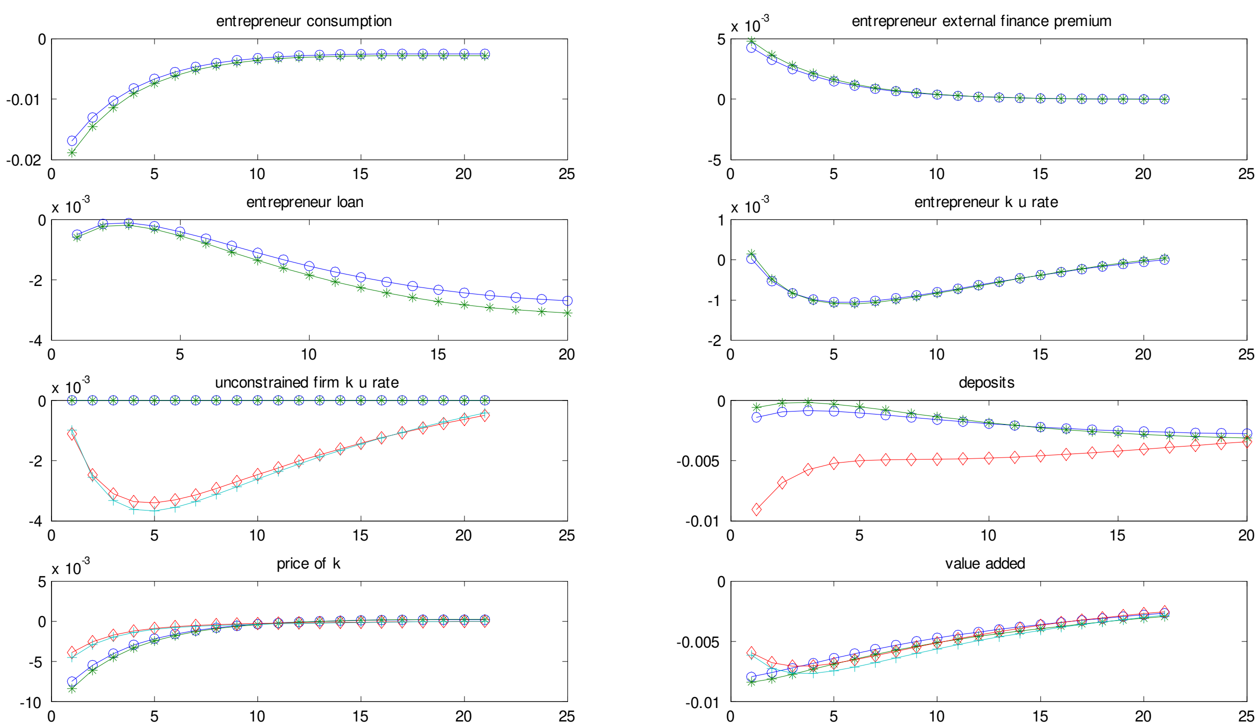

Figure 3: 1 standard deviation TFP shock. Blue circles: model with all credit frictions on. Green stars: model with only firm credit frictions. Red diamonds: model with only household credit frictions. Green crosses: no credit frictions

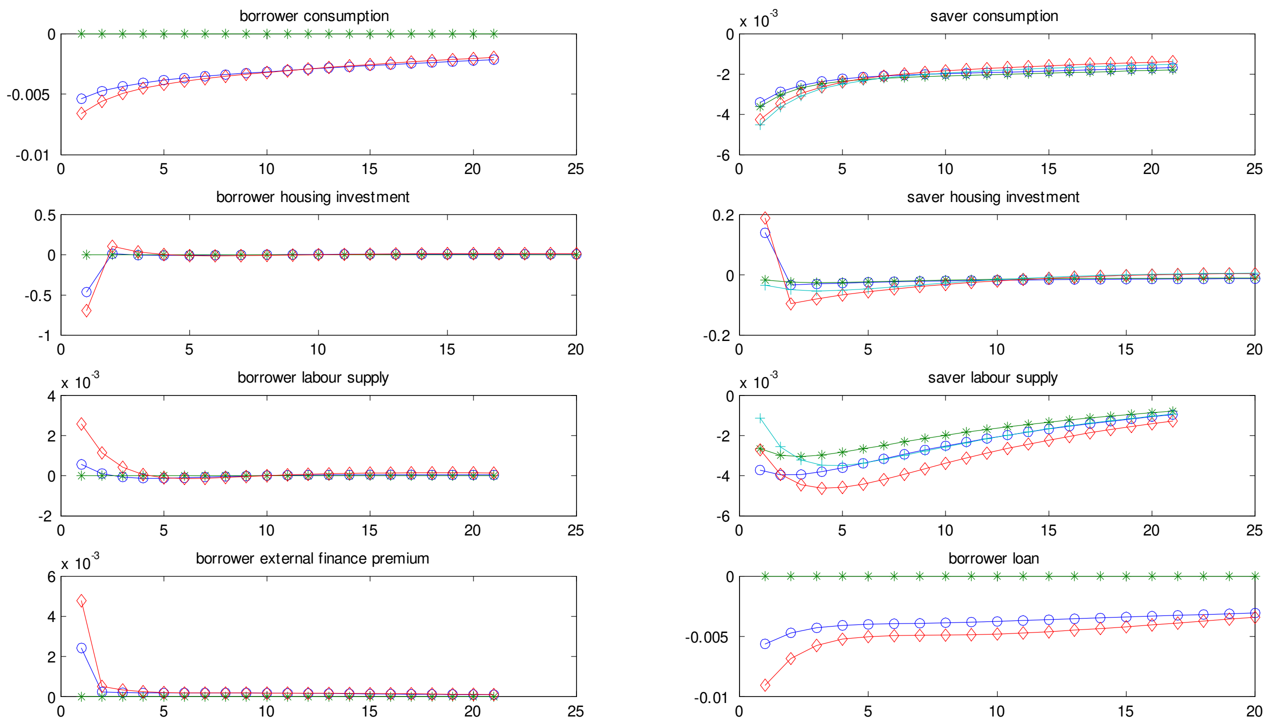

Figure 4: 1 standard deviation TFP shock. Blue circles: model with all credit frictions on. Green stars: model with only firm credit frictions. Red diamonds: model with only household credit frictions. Green crosses: no credit frictions

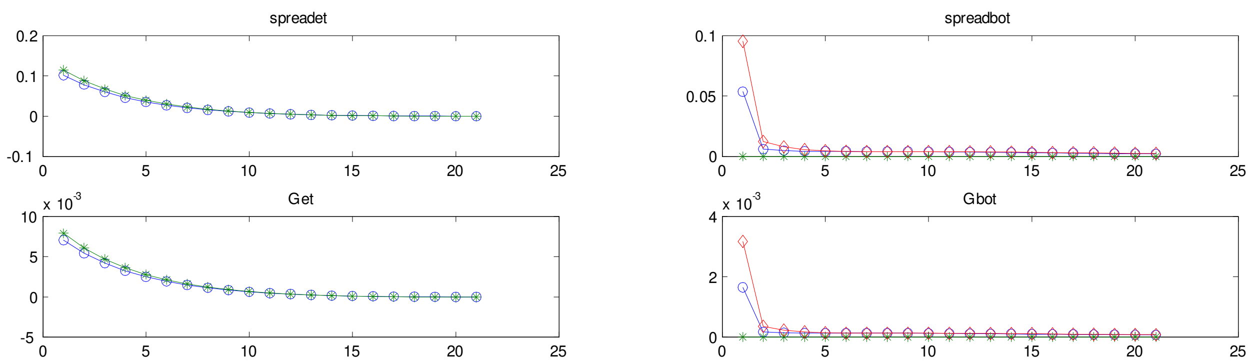

Figure 5: 1 standard deviation investment shock. Blue circles: model with all credit frictions on. Green stars: model with only firm credit frictions. Red diamonds: model with only household credit frictions. Green crosses: no credit frictions

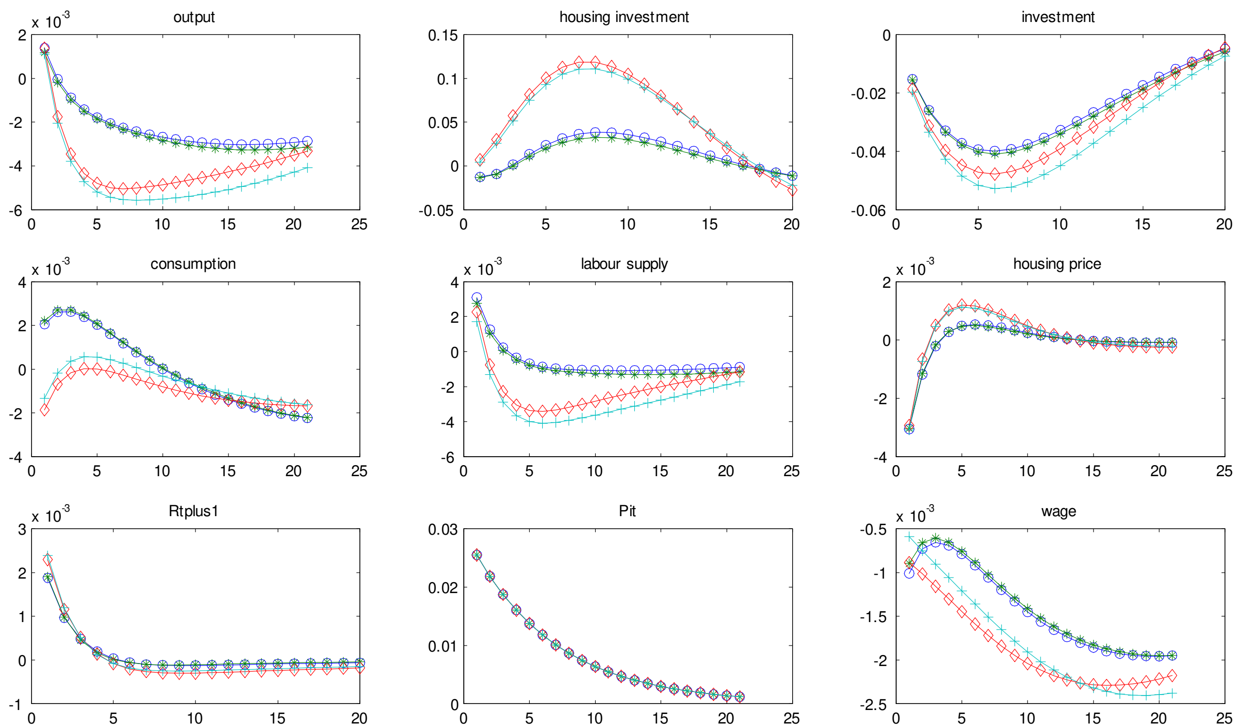

Figure 6: 1 standard deviation investment shock. Blue circles: model with all credit frictions on. Green stars: model with only firm credit frictions. Red diamonds: model with only household credit frictions. Green crosses: no credit frictions

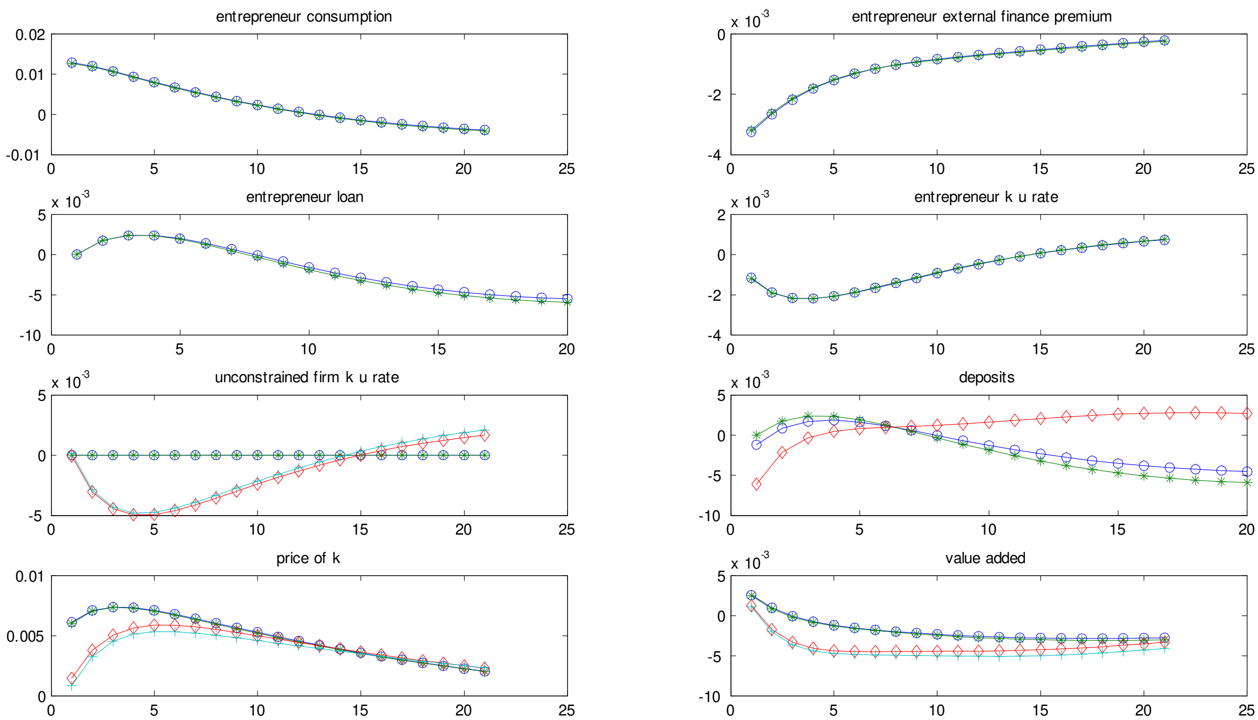

Figure 7: 1 standard deviation credit shock. Blue circles: model with all credit frictions on. Green stars: model with only firm credit frictions. Red diamonds: model with only household credit frictions. Green crosses: no credit frictions

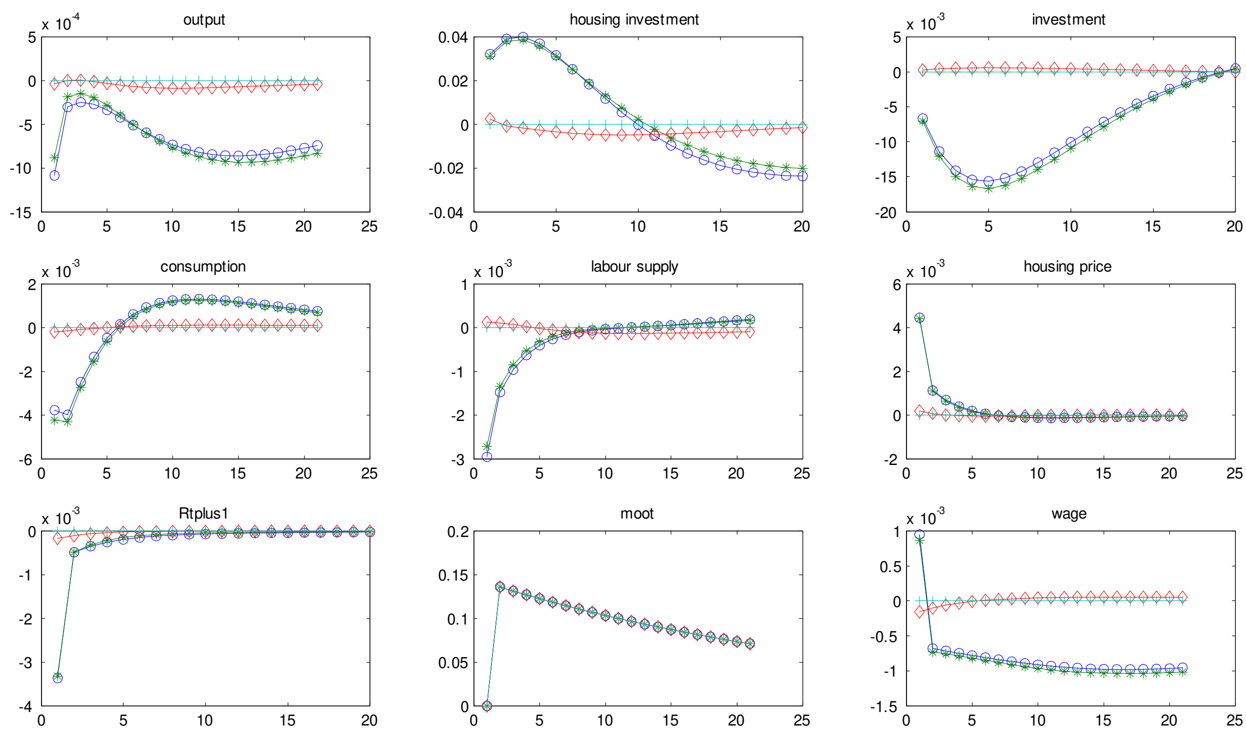

Figure 8: 1 standard deviation credit shock. Blue circles: model with all credit frictions on. Green stars: model with only firm credit frictions. Red diamonds: model with only household credit frictions. Green crosses: no credit frictions

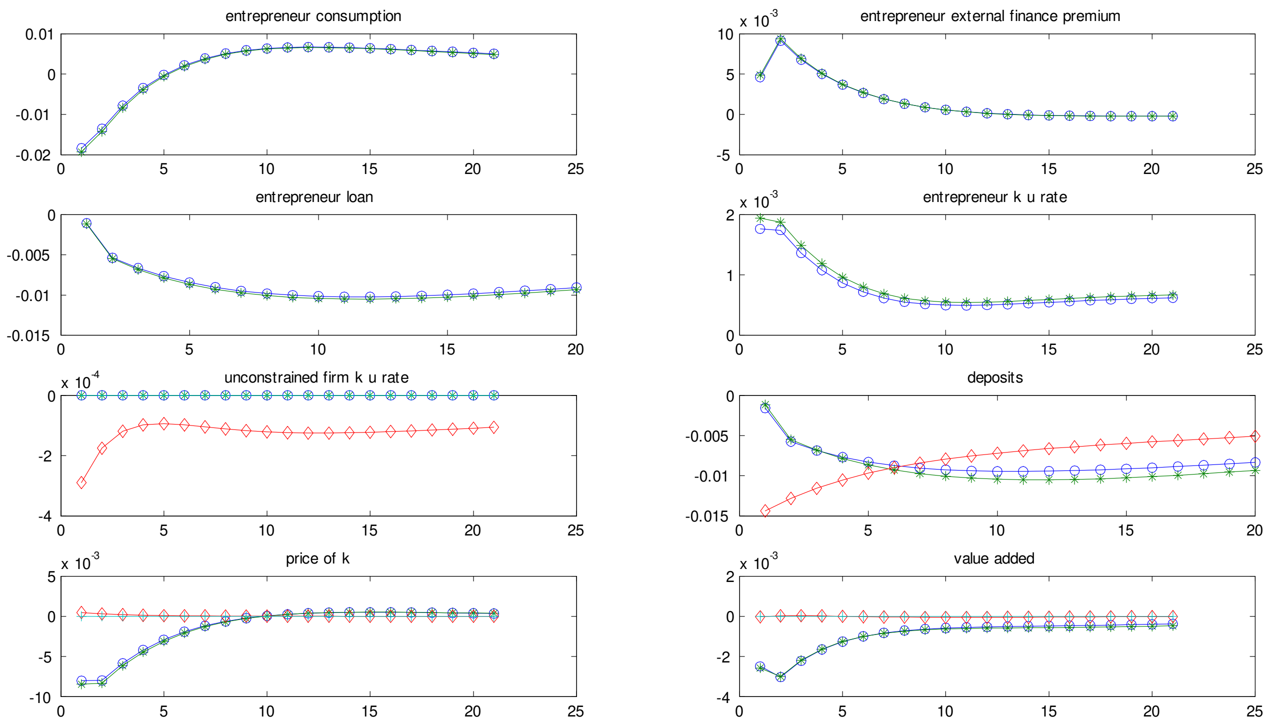

Figure 9: 1 standard deviation credit shock. Blue circles: model with all credit frictions on. Green stars: model with only firm credit frictions. Red diamonds: model with only household credit frictions. Green crosses: no credit frictions

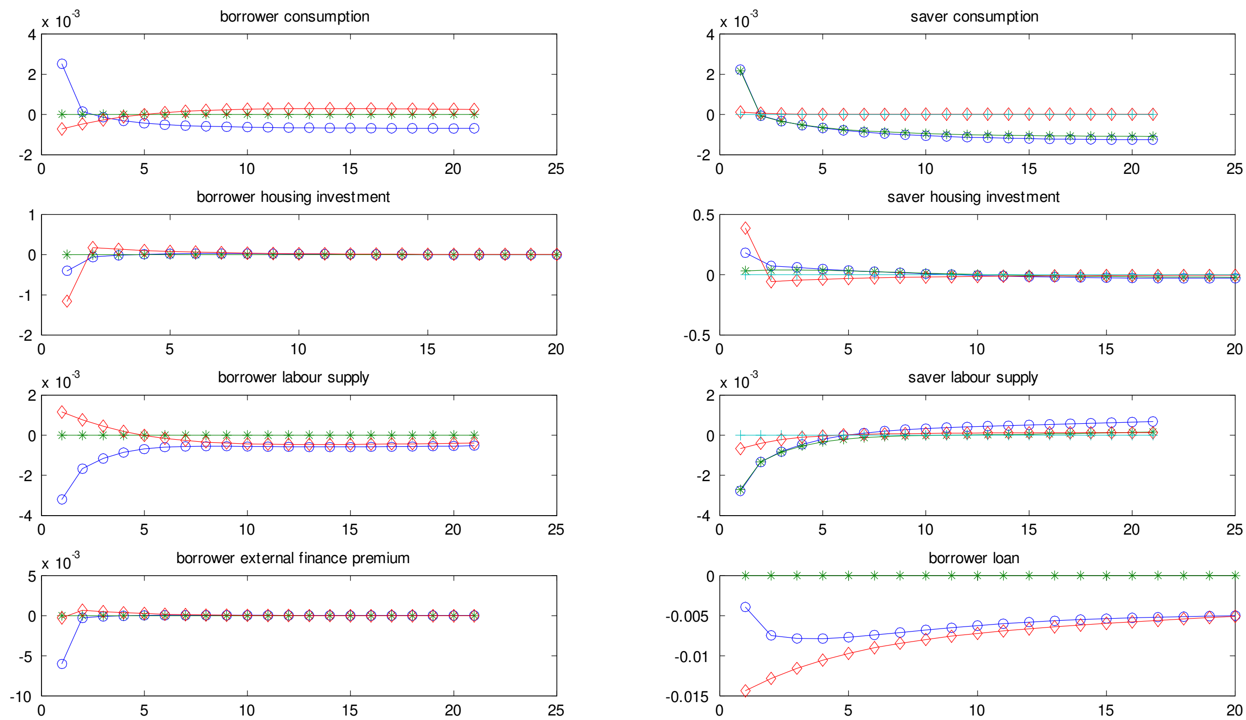

Figure 10: 1 standard deviation credit shock. Blue circles: model with all credit frictions on. Green stars: model with only firm credit frictions. Red diamonds: model with only household credit frictions. Green crosses: no credit frictions