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Strategic Asset Allocation and Intertemporal Demands: with Commodity Futures as an Asset Class

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[Abstract]

This paper analyzes the role of commodities in the process of strategic asset allocation, with an attempt of computing the weight of commodities relative to traditional assets in a multi-period portfolio choice problem and understanding the economic interpretations to its importance. We find U.S. investors have a significantly stable intertemporal hedging demand for commodities in the long horizons, even when they have access to foreign equity markets, for example, foreign stock market. Our results provide support to institutional investors attempting to include commodities into their strategic asset allocation decision.

1. Introduction

The idea of commodities as an investable asset class has been around since the 1970s. For example, Geer's (1978) study states that commodity future is a unique and conservative asset which is as liquidity, but less risky than common stocks and can be used to hedge inflation risk. Bodie and Rosansky's (1980) found that portfolios of commodity futures have similar risk-return characteristics to Standard and Poor's 500 stock indexes. However, strategic asset allocation in the past century still consists primarily of allocations to the three traditional asset classes: stocks, bonds and cash, while commodities receive little attention from investors. As the equity and fixed income market keep deteriorating, the returns of portfolios consisting of traditional asset classes are far lower than those during 1990s. To improve the risk-return characteristics of a strategic asset portfolio, institutional investors are expanding the investable universe beyond the three traditional asset classes and commodities have gained much prominence during the past few years.

The purpose of this paper is to study the role of commodities in strategic asset allocation and investors' portfolio choices. According to conventional wisdom, commodity futures returns have been especially effective in providing diversification of both stock and bond portfolios. Based on this, some observers view the commodity market as an attractive asset class to diversify traditional portfolios of stocks and bonds. For these reasons, it will be useful to obtain quantitative estimates of the percentage of portfolio allocation across commodities, stocks, and bonds. In this paper we have three principal objectives. First, we measure and analyze, in some detail, the demands for commodities as well as the traditional asset classes in asset allocation. Second, we try to gain some insights into the reasons for commodity's importance in portfolio choice. Third, we also compare the utility benefits *with* and *without* including commodities as an asset class.

Estimating the demands for asset classes in a multi-period portfolio choice problem is complicated by the fact that exact analytical solutions are generally not available. Finding a closed-form solution is, therefore, essential for discerning the demands for various asset classes. One representative stream of financial economists use discrete-state approximations to approximate the solutions to multi-period portfolio choice problems in the Merton model; for example, Balduzzi and Lynch (1999), Barberis (2000), Brennan et al. (1997, 1999), Cocco et al. (1998), Lynch (2001), and Lynch and Tan (2010). The other stream uses analytical approach to solve for multi-period portfolio choice problems by assuming long-lived investors have various form of utility functions (e.g., Campbell and Viceira, 1999, 2001, 2002; Schroder and Skiadas, 1999). The most recent stream by Campbell, Chan and Viceira(2003,; henceforth, CCV) combines the analytical method of Campbell and Viceira (1999, 2001, 2002) with a simple numerical method, vector autoregression (VAR). This approach has two significant advantages, compared to previous ones: (i) it can accommodate multi-period portfolio choice problems with large number of asset classes and (ii) it can decompose intertemporal hedging demands into components associated with individual asset classes. CCV uses this approach to analyze optimal dynamic asset allocation across U.S. bills, stocks, and bonds and find significant intertemporal hedging demands for U.S. stocks. Rapach and Wohar (2009) extend the analysis to the G7 countries as well as the U.S. by allowing domestic investors to access foreign equity markets. The analysis in this paper takes the advantage of this approach and tries to detect the importance of commodity asset class in the strategy asset allocation decision.

Both the implementation and empirical results are atypical in several aspects. Our first novel result is that, in addition to the mean total demand and hedging demand for stocks, the mean total and hedging demands for commodities are also large in magnitude for an U.S. investor. The demands are stable in magnitude, even when an investor can access to international stocks. The second singular result is that the mean intertemporal hedging demand for commodities are always significant according to the 90% confidence intervals, though its magnitude is generally smaller than those for domestic stocks. Another novel result is that there are large and significant mean myopic demands for foreign stocks, whenever an U.S. investor has access to international stock market. This is possibly the result of the better risk-return characteristics of the international stock market compared with U.S.

To the best of our knowledge, no paper to date has attempted to study the dynamic strategic asset allocation across the non-traditional commodity and the traditional stocks and bonds. A few papers have tried to show that commodities could be an attractive asset class to diversify traditional portfolios of stocks and bonds. One effort along this line is that of Gorton and Rouwenhorst (2006) who created an equally-weighted paper portfolio of commodity futures, in which they studied the simple properties of commodity futures as an asset class, rather than to investigate strategic asset allocation and portfolio choice directly, however.

The rest of this paper is organized as follows: Section 2 describes our empirical methodology, the CCV framework; Section 3 presents our empirical results; Section 4 tries to explain the sources of the importance of commodities; Section 5 presents the utility benefits by including commodities as an asset class; and Section 6 presents our conclusions.

2. Empirical Methodology

The investor in our multi-period portfolio choice problem can allocate after-consumption wealth among one benchmark asset and n additional risky asset classes. The expanded investment set includes bills, bonds, stocks, as well as commodities. To be consistent, we summarize the empirical approach using the same symbols as CCV; detailed discussion of the methodology is in Section 2 and 3 of CCV. By defining the real return on a benchmark asset as $R_{1.t+1}$, the investor's real portfolio return $R_{p,t+1}$ then

$$R_{p,t+1} = R_{1.t+1} + \sum_{i=2}^{n} \alpha_{i,t}(R_{i,t+1} - R_{1.t+1}),$$

where n is the number of risky asset classes available for investment; $\alpha_{i,t}$ is the portfolio weight on the ith risky asset class; In this paper, the benchmark asset is a 3-month treasury bill. The vector of log excess returns for the n risky assets x_{t+1} can thus be defined as

$$x_{t+1} = \begin{bmatrix} r_{2,t+1} - r_{1,t+1} \\ r_{3,t+1} - r_{1,t+1} \\ r_{4,t+1} - r_{1,t+1} \\ \vdots \\ r_{n,t+1} - r_{1,t+1} \end{bmatrix}$$

where $r_{i,t+1} = \log(R_{i,t+1})$ for $i = 1, 2, \ldots, n$. In addition to the n risky asset returns, the system includes $k = 3$ instrumental variables, for instance, nominal Treasury bill yield, log dividend yield and yield spread. These instrumental variables are put in the vector s_{t+1}. Thus, the who system of variables can be stacked into an $m \times 1$ vector z_{t+1} and

$$z_{t+1} = \begin{bmatrix} r_{t+1} \\ x_{t+1} \\ s_{t+1} \end{bmatrix},$$

where $r_{1,t+1}$ is the log return for the benchmark asset; x_{t+1} contains the log excess returns for the risky asset classes; s_{t+1} contains instrumental variables. CCV assumes that z_{t+1} can be captured by a $m \times 1$ first-order vector autoregressive system:

$$z_{t+1} = \Phi_0 + \Phi_1 z_t + v_{t+1},$$

where v_{t+1} is the unexpected shocks to the state variables and is assumed to be homoskedastic and independently distributed with a variance-covariance matrix $\sum_v$:

$$\sum_v = \begin{bmatrix} \sigma_1^2 & \sigma'_{1x} & \sigma'_{1s} \\ \sigma_{1x} & \sum_{xx} & \sum_{sx} \\ \sigma_{1s} & \sum_{xs} & \sum_{ss} \end{bmatrix},$$

CCV assumes that the investor has a recursive Epstein-Zin utility function, which can be written as

$$U(C_t, E_t(U_{t+1})) = [(1-\delta)C_t^{(1-\gamma)/\theta} + \delta(E_t(U_{t+1}^{1-\gamma}))^{1/\theta}]^{\theta/(1-\gamma)},$$

where C_t is the investor's consumption at time t. $\delta \epsilon (0,1)$ is the time discount factor. γ is the coefficient of constant relative risk aversion.

$\theta = (1-\gamma)/(1-\psi^{-1})$ and $\psi > 0$ is the elasticity of intertemporal substitution.

$E_t(.)$ is the expectation operator.

At time t, the investor makes optimal consumption and portfolio decisions by maximizing the Epstein-Zin utility function, subject to the intertemporal budget constraint,

$$W_{t+1} = (W_t - C_t)R_{p,t+1},$$

where is W_t wealth at time t .

CCV assumes that the optimal portfolio and consumption rules have the following form

$$\alpha_t = A_0 + A_1 z_t,$$

$$c_t - w_t = b_0 + B_1^{'} z_t + z_t^{'} B_2 z_t,$$

where A_0, A_1, $b_0, B_1^{'}, B_2$ are scalar coefficient matrices to be solved. Following Merton (1969), CCV solves the portfolio rule and partitioned the total demand for the assets into myopic and intertemporal hedging demand:

$$A_0 = (1/\gamma)\sum_{xx}^{-1}[H_x\Phi_0 + 0.5\sigma_x^2 = (1-\gamma)\sigma_{1x}] + [1-(1/\gamma)]\sum_{xx}^{-1} - \Lambda_0 /(1-\psi)],$$

$$A_1 = (1/\gamma)\Sigma_{xx}^{-1} H_x \Phi_1 + [1-(1/\gamma)]\Sigma_{xx}^{-1} - \Lambda_1 /(1-\psi)],$$

where H_x is a selection matrix that selects x_t from z_t. Λ_0 and Λ_1 are coefficient matrices. The first component on the right-hand-side of equations (10) and (11) are represents the myopic demand for assets; the second component on the right-hand-side of the two equations represents the intertemporal hedging demand for assets.

Then, by solving for the optimal consumption-wealth ratio, the value function - the maximized utility function can be expressed as

$$V_t = \exp\{-\frac{\psi}{1-\psi}\log(1-\delta) + \frac{b_0}{1-\psi} + \frac{B_1^{'}}{1-\psi} z_t + z_t^{'} \frac{B_2}{1-\psi} z_t\}$$

We can derive the unconditional mean of the value function $E(V_t)$, which is later used to calculate the utility of long-term investors under combinations of various asset classes.

3. Empirical Results

3.1. Commodity Futures as a Proxy of Commodities

Unlike financial assets such as stocks and bond, there are no well-accepted methods to measure the direct exposure to commodities. Traditional wisdom on commodities mainly uses three ways to approximate the exposure to commodities: (i) direct physical investment; (ii) a weighted index of commodity-related stocks; and (iii) commodity futures (Idzorek, 2006). However, a direct physical investment in commodities is not a good measurement, since it is hard to keep in the long time horizon. The weighted index of commodity-related stocks represents more of the traditional equity, instead of commodity itself, as it has high positive correlations with other equities. Commodity future contracts, though not perfect, provides better exposure to commodities through its direct connections with spot and expected future spot prices and its relationship with unexpected inflation shocks. In this paper, we use the Reuters/Jefferies Commodity Research Bureau (CRB) index, which a portfolio of commodity futures contracts. This index was originally created by the Commodity Research Bureau in 1957.

3.2. Data Description

The calibration results are based on monthly data for the U.S. market. The real return on Treasury bills is defined as the log return on a 3-month Treasury bill minus the log difference of the consumer price index. The log excess stock return is the log return on the S&P 500 stock index minus the log return on the 3-month Treasury bills. The log excess bond return is the log return on the 10-year government bonds minus the log return on the 3-month Treasury bills. The nominal yield on Treasury bills is the log yield on a 3-month Treasury bill and the term spread is the difference between the yields on a 10-year government bond and 3-month Treasury bill. Log excess commodity return is defined as the log difference of the Reuters/Jefferies Commodity Research Bureau (CRB) index.

Table 1 reports the mean, variance, and skewness for the bill, bond, stock and commodity returns and three instruments. The entries for mean and standard deviations are expressed in percentage. The Sharpe ratios for the bond, stock and commodity returns are reported in the last column. As expected, Treasury bill has low return as well as low volatility. The mean excess returns for stocks, bonds and commodities are 4.62%, 1.17% and 1.98%, respectively, and the standard deviations of them are 14.74%, 5.87% and 10.39%, respectively. Both the mean returns and volatility for stock and commodity are higher than for bond. The Sharpe ratio for stock, bond, and commodity are 0.31, 0.19 and 0.20. That is, Stock has the highest Sharpe ratio. Although commodity has higher volatility than bonds, its Sharpe ratio is almost the same as for bonds.

[Insert Table 1 Here]

3.3. VAR Estimation

Table 2 reports the estimation results for the VAR system. The top section of the table reports coefficients estimates and the R^2 statistics (with the p-value in the parentheses) for each equation in the VAR system. The bottom section of the table reports the cross-correlation matrix of the innovations.

The first row of the table corresponds to the real bill return equation. The lagged real bill return and commodity return have significant positive and negative coefficients, respectively. The second row corresponds to the equation for the excess stock return. None of variables are significant. This confirms that predicting stock returns is difficult. The third row is the equation for the excess bond return. The coefficients for the lagged bond return, excess stock return, commodity return, and yield spread are all significant. The fourth row reports the results for the equation of commodity. All the coefficients are insignificant, which implies that there are few correlations between commodities and other risky assets. This possibly implies commodity could be an important component of portfolio choice.

The bottom section reports the covariance structure of the innovations in the VAR system. Unexpected log excess stock returns has very low correlation with commodity returns, but are highly negatively correlated with shocks to the log dividend yield, which is consistent with previous empirical evidence (Campbell, 1991; Stambaugh, 1999). Unexpected log excess bond returns are negatively correlated with shocks to nominal bill rate, log dividend yield and commodity return, but positively correlated with the log excess stock return.

Altogether, the correlations between commodity and stock and bond returns suggest that commodity could play an important role in the process of strategic asset allocation. We will further explore their implications for optimal multiple-period portfolio choice.

[Insert Table 2 Here]

3.4. Demands for Domestic Assets

Table 3 reports the mean total, myopic, and intertemporal hedging demands (in percentage) for domestic bills, stocks, bonds, and commodities for a U.S. investor. The intertemporal elasticity of substitution is $\psi = 1$. The entries in each column are mean asset demands when the coefficient of relative risk aversion γ equal to 4, 7, and 10, respectively. Both the total mean demands and the mean myopic demands across the four assets sum to 100, while the mean hedging demands sum to 0. By comparing numbers within each column, we can study how the portfolio is allocated across the four risky assets and how much is the mean total demand, myopic demand, and hedging demand for each of the asset classes. By comparing numbers within each row, we can examine the incremental effects of relative risk aversion γ on asset allocation. In addition, the numbers in brackets under each entry are the 90% confidence intervals for the mean asset demands.

From the table, one can see that the total and myopic demand allocation is holding a long position on stocks, bonds and commodities, while shorting bills. The mean total demand for stocks is about 1.7 and 1.5 times those for bonds and commodities. The significant mean total and myopic demand for stocks is consistent with the theory that there is higher demand for the asset with the largest Sharpe ratio. In addition, there are positive mean hedging demands for stocks and commodities. The mean hedging demand for commodities is significant in the 90% confidence interval, though the mean hedging demand for stocks is larger in magnitude. Both the mean hedging demands for bonds and bills are negative, which is consistent with the findings in Campbell, Chan, and Viceira (2003) and Rapach and Wohar (2009). By comparing each row, as we would expect, the mean total, myopic and hedging demands for all the risky assets decrease as relative risk aversion γ increase.

The large mean demands for stocks can be explained by the negative correlation between innovations to log excess stock returns and the log dividend yield. As the stock returns have large positive Sharpe ratio, investors usually take long positions in stocks. An increase in expected stock returns represents an improvement in the investment opportunity set, while a decrease in expected stocks returns represents a worsening of the investment set. In addition, the VAR estimation results suggest that lagged dividend yield has a positive effect on the expected stock returns. Given the negative correlation between innovations to excess stock returns and dividend yield, one should expect that a negative shock to excess stock return next period are accompanied by a positive shock to the dividend yield next period. In turn, a positive shock to the log dividend yield next period can lead to higher expected stock returns in the future. Thus, investors can use stocks to hedge against the negative shocks to futures returns.

[Insert Table 3 Here]

Figure 1 plots the estimated hedging demands for domestic stocks, bonds, and commodities in order to show more intuitive picture of the intertemporal hedging demands for each of the three asset class. Overall, the hedging demand for commodity appears to be the most stable compared with those for stocks and bonds. And the hedging demand for commodity and stock are well above the hedging demand for bonds over most of the sample period.

[Insert Figure 1 Here]

A recent theoretical work by Bhamra and Uppal (2006) suggests that the elasticity of intertemporal substitution ψ can affect the magnitude, but not the sign, of the intertemporal hedging demand for the risky asset. Here we compute the mean demands for stock, bond, and commodity by setting values of intertemporal substitution ψ equal to 0.3, 1, and 1.5 and the results are presented in Table 4. The mean total and hedging demands for stocks increase largely as the intertemporal substitution ψ increases and gradually becomes positively significant. That is, investors are becoming more willing to make a trade-off between contemporary and future consumption by taking more long positions on stocks. Both the mean total and hedging demand for commodities do not change very much, but are always positively significant as the value of ψ increases. The stable mean total and hedging demands for commodities over various ψ values provide support for the argument that commodity is an attractive asset class for multi-period portfolio choice. In addition, our results are also consistent with Bhamra and Uppal's theoretical results that ψ only affects the magnitude, but not the sign, of the mean hedging demands for risky assets.

[Insert Table 4 Here]

3.5. What Explains the Demands for Commodities?

What explains the striking and significant mean total and intertemporal hedging demand for commodities? One explanation might be based on modern portfolio theory which states the interaction of asset classes with each other provides diversification. The commodity future return is negatively correlated with bond return by -0.13 and a very low correlation with the most risky asset, stock return. These findings suggest commodities have the ability to help diversify stock and bond portfolios. Furthermore, the low correlation between innovations to stock and commodity returns is 0.05, which almost approaches zero. Consequently, the large positive intertemporal hedging demand for stocks does not reduce the demand for commodities, which may explain why the positive demand for commodities is significant during the long horizons. The portfolios constructed based on the estimation results provide further evidence to support the explanation. In figure 3, the portfolio with commodity futures is more efficient, has a higher ratio of return to risk, than the portfolio without commodity futures.

[Insert Figure 3 Here]

The second tentative explanation is from the aspect of the return distributions. It is a well established fact that traditional asset returns, for example, stock returns, are negatively skewed, and the distribution of commodity returns is positively skewed. The positive skewness of commodity return together with its lower volatility relative to stock return, imply that commodity has lower downward risk compared to equities like stocks. In this study, the skewness for monthly average excess stock and commodity returns are -0.62 and 0.49, while the volatilities for them are 14.74% and 10.39%, respectively. If the tail events can happen simultaneously for the two asset classes, commodity, as an independent asset class, can provide large diversification benefits to the portfolio allocation.

Another explanation comes from the correlation between commodity returns and inflation. This is because the ultimate function of portfolios is for consumption. Thus, investors should consider the real purchasing power of their returns; that is, the asset classes' ability of hedging against inflation Traditional asset classes such as stocks and bonds are negatively correlated with inflation and are not good asset classes for hedging against inflation. However, commodities represented by commodity futures, may be a better hedge against inflation because (i) they have a positive correlation with inflation in the long run; (ii) commodity prices are directly linked to unexpected inflation shocks, which is an important component of inflation. These together may explain why commodities are a better asset class of hedging against inflation risk than stocks and bonds.

In summary, the three tentative explanations together with the significant hedging demand for commodities found in this paper suggest that commodity can be an attractive asset class to diversify traditional portfolio of stocks and bonds.

4. Controlling for International Stock Markets

To check for the robustness and stableness of the estimated hedging demand for commodities, we expand the analysis by allowing the investors to access international equity markets, in addition to domestic stocks, bonds, commodities, and bills. We use the MSCI World Equity Index ex U.S. as a proxy for foreign stock market, which makes the calibration not too complicated. In this case, investors can make a strategic asset allocation across domestic bills, bonds, stocks, commodities, as well as foreign stocks. We estimate the expanded VAR system and use the CCV approach to approach the mean total, myopic, and hedging demands for each of the asset classes. To reserve space, we will not report the VAR estimation results.

Table 5 reports the mean total, myopic, and intertemporal hedging demands (in percentage) for domestic bills, stocks, bonds and commodities as well as foreign stocks for a U.S. investor. The results are computed by setting the intertemporal elasticity of substitution $\psi = 1$ and the coefficient of relative risk aversion γ equal to 4, 7, and 10, respectively. The demands for various asset classes decrease, as the relative risk aversion γ increases. The first striking finding is U.S. investors continue to have relative large mean total and intertemporal hedging demands for domestic stocks when they can invest in foreign equity markets, although these demands are no more significant according to the 90% confidence intervals. However, the mean myopic demand for domestic stock is essentially low compared to when they can only allocate across domestic asset classes. The large magnitude in mean total and hedging demands for domestic stocks is consistent with the well-established theoretical and empirical finance literature that U.S. investors have home bias (e.g., Cooper and Kaplanis, 1994; Coval and Moskowitz, 1999; Norman and Xu, 2003; Barron and Ni, 2008). The second striking finding is the significant mean total and myopic demand for foreign stocks, although the mean total demand is lower relative to the mean total demand for domestic stocks. This interesting phenomenon can be intuitively explained based on some summary statistic characteristics of excess domestic and foreign stock returns. The standard deviations of domestic and foreign stock returns are almost equal, but the log excess foreign stock returns are almost two times that for domestic stock. Thus, the foreign stock returns have higher Sharpe ratio than domestic stock returns. All else equal, investors should have higher myopic demands for assets with higher Sharpe ratio, which explains why the myopic demand for domestic stocks is lower for U.S. investors.

The most striking results in Table 5 is the mean intertemporal hedging demands for commodities. The mean intertemporal hedging as well as the mean total and myopic demand for commodities are fairly stable compared to when investors only have access to domestic asset classes. Furthermore, the intertemporal demand for commodities is still significant to a certain degree, even after investors can access foreign asset classes. This implies the intertemporal hedging demands for commodities is stable and confirms commodities should be a conservative component in strategic asset allocation. To give a direct explanation, we plot the intertemporal hedging and demands for domestic bonds, stocks, and commodities, as well as foreign stocks in Figure 2. Overall, the hedging demand for commodity appears to be the most stable as compared to those for both domestic and foreign stocks and bonds. Also, the hedging demand for commodity and stock are well above the hedging demand for bonds and foreign stocks over most of the sample period.

[Insert Table 5 Here]

[Insert Figure 2 Here]

We also compute the mean demands for stock, bond, and commodity by setting values of intertemporal substitution ψ equal to 0.3, 1, and 1.5. As shown in Table 6, investors become willing to make more trade-off between contemporary and future consumption, i.e., the hedging demands for stock increase substantially and gradually become positively significant, as the intertemporal substitution ψ increases from 0.3 to 1.3. Similarly, when investors can only access domestic asset classes, the mean hedging demand for commodities changes very little and are always positively significant, as the value of ψ increases. It confirms again that commodity is effective in portfolio diversification and is an attractive asset class which should be seriously concerned in the strategic asset allocation process. In summary, investors in U.S. have significant intertemporal hedging demands for commodities, in addition to domestic stocks. This intertemporal hedging demand remains significant in amount, even when investors have opportunities to invest in other asset classes, for example, foreign stocks.

In summary, the results above indicate that commodity is one of the important determinants in the process of strategic asset allocation and multi-period portfolio choice. There exists a significant and relatively stable intertemporal hedging demand for commodities, as well as for domestic stocks for U.S. investors.

[Insert Table 6 Here]

5. The Utility Benefits from Including Commodities

In this section, we differentiate out the importance of the commodity asset class by comparing the utility of an investor who has access to commodities with the utility of an investor who does not. Table 7 reports the mean value function when values of γ are set to 4, 7, 10, and 20. Panel A compares the mean value function when two investors can only allocate across domestic asset classes. Panel B compares the mean value function when two investors can allocate across domestics as well as international asset classes. The value function is normalized so that a doubling from one portfolio to another implies that an investor would require twice as much as wealth to obtain the same utility with the worse portfolio than with the better one.

A comparison of portfolio 1 of panel A, in which commodities are not included, with portfolio 2 shows that commodities generate large welfare gains for all investors. Both aggressive and conservative investors gain by allocating some weights to commodities, which can help hedge against the long positions in domestic stocks and inflation risk of real interest rates. One can draw the same conclusion by comparing portfolio 3 and 4 in panel B. In addition, a comparison of portfolio 1 with portfolio 3 and portfolio 2 with portfolio 4 suggest that the addition of foreign stocks to an investor's portfolio also creates large gains for all investors.

[Insert Table 7 Here]

6. Conclusions

Using the Reuters/Jefferies Commodity Research Bureau (CRB) index as a proxy for commodity, this paper has documented relatively strong and stable intertemporal hedging demands of U.S. investors for commodity. The result is robust when other traditional assets, for example, foreign stocks, are included in the portfolio choice. We also provide evidence that the intertemporal hedging demand for commodity are relatively stable and permanent in magnitude by setting the intertemporal substitution ψ and the relative risk aversion γ to various values. In addition, the results referring to stock market are consistent with previous findings that there are large mean total and myopic demands for domestic stock.

A more difficult question is *why* U.S. investors have a strong intertemporal hedging demand for the commodity asset class. In this paper, we have tried to make progress on this question by providing some tentative interpretations based on modern portfolio theory, return characteristics, as well as the ability of hedging against inflation. The results presented in this study show, perhaps interestingly, the significant intertemporal hedging demand for commodity, because commodity return has low and negative correlations with traditional asset classes, while having a lower downward risk by having higher positive skewness. In addition, the surprisingly significant intertemporal hedging demand for commodity seems to come through its increased ability to hedge against the unexpected future inflation, compared to traditional assets.

Institution investors have been increasingly interested in commodities during the past few years. There are currently many intense debates on the role of commodities in strategic asset allocation. This study provides some new empirical evidence for advocating commodities as an asset class in portfolio choice. Despite this, future efforts are needed to build an well-accepted theoretical model on commodity pricing and to analyze the sources of commodity returns and their role in asset allocation.

References

Barron, J. M., and Ni, J., 2008, Endogenous Asymmetric Information and International Equity Home Bias: The Effects of Portfolio Size and Information Costs, *Journal of International Money and Finance*, 27, 4, 617-35.

Bodie, Zvi and Victor Rosansky., 1980, "Diversification Returns And Asset Contributions", *Financial Analysts Journal*, (May/June): 26-32.

Balduzzi, P., Lynch, A., 1999, Transaction costs and predictability: some utility cost calculations, *Journal of Financial Economics*, 52, 47--78.

Barberis, N.C., 2000, Investing for the long run when returns are predictable, *Journal of Finance*, 55, 225--264.

Bhamra, H.S., Uppal, R., 2006, The role of risk aversion and intertemporal substitution in dynamic consumption-portfolio choice with recursive utility, *Journal of Economic Dynamics and Control,* 30 (6), 967--991.

Brennan, M.J., Schwartz, E.S., Lagnado, R., 1997, Strategic asset allocation, *Journal of Economic Dynamics and Control*, 21, 1377--1403.

Campbell, J.Y., 1991. A variance decomposition for stock returns. Economic Journal 101, 157--179.

Campbell, J.Y., Viceira, L.M., 1999, Consumption and portfolio decisions when expected returns are time varying, *Quarterly Journal of Economics*, 114, 433--495.

Campbell, J.Y., Viceira, L.M., 2000, Consumption and portfolio decisions when expected returns are time varying: erratum, *Unpublished paper, Harvard University*, available on the authors' websites.

Campbell, J.Y., Viceira, L.M., 2001, Who should buy long-term bonds? *American Economic Review*, 91, 99--127.

Campbell, J.Y., Viceira, L.M., 2002, Strategic Asset Allocation: Portfolio Choice for Long-Term Investors, *Oxford University Press, Oxford*.

Campbell, J.Y., Chan, Y.L., Viceira, L.M., 2003, A multivariate model of strategic asset allocation, *Journal of Financial Economics*, 67 (1), 41--81.

Cooper, L., and Kaplanis, E., 1994, Home Bias in Equity Portfolios, Inflation Hedging, and International Capital Market Equilibrium, *Review of Financial Studies*, 7, 1, 45-60.

Coval, J.D., and Moskowitz, T.J., 1999, Home Bias at Home: Local Equity Preference in Domestic Portfolios, *Journal of Finance*, 54, 6, 2045-73.

Gorton, Gary, and Geert Rouwenhorst. 2006, Facts and Fantasies about Commodity Futures. *Financial Analysts Journal*, 62, 47-68.

Greer, Robert J., 1978, Conservative Commodities a Key Inflation Hedge, *Journal of Portfolio Management*, 4(4), 26-29.

Idzorek, M.T., 2008, Strategic Asset Allocation and Commodities, *Ibboston Associates Reports.*

Merton, R.C., 1969, Lifetime portfolio selection under uncertainty: the continuous time case, *Review of Economics and Statistics*, 51, 3, 247--257.

Norman, S., and Xu. X., 2003, Understanding the Equity Home Bias: Evidence from Survey Data, *Review of Economics and Statistics*, 85, 2, 307-12.

Rapach, E.D., and Wohar, E.M., 2009, "International Asset Allocation with Regime Shifts," *Journal of International Money and Finance*, 28, 1137--1387.

Lynch, A.W., 2001, Portfolio choice and equity characteristics: characterizing the hedging demands induced by return predictability, *Journal of Financial Economics*, 62, 67--130.

Lynch, A.W., Tan, S., 2010, Multiple Risky Assets, Transaction Costs and Return Predictability: Allocation Rules and Implications for U.S. Investors, *Journal of Financial and Quantitative Analysis*, Forthcoming.

Stambaugh, R.F., 1999. Predictive regressions, *Journal of Financial Economics*, 54, 375--421.

Table 1. Summary Statistics

variables	mean	standard deviation	Skewness	Sharpe ratio	ρ
U.S., 1956:10-2004:05					
rtb_t	1.391	0.989	-0.185		
xr_t	4.624	14.739	-0.615	0.314	
xb_t	1.170	5.873	0.067	0.199	
xc_t	1.973	10.394	0.493	0.190	
y_t	-0.018	1.072	-0.175		
div_t	1.125	0.385	-0.020		
spr_t	1.424	1.206	-0.870		
xfr_t	7.390	16.921	-0.399	0.437	

Notes: rtb_t =real Treasury bill return, xr_t =excess domestic stock return, xb_t =excess bond return, xc_t =excess commodity return, y_t =nominal Treasury bill yield, div_t =log dividend yield, spr_t = yield spread. xfr_t =excess foreign stock

Table 2. VAR Estimation Results

Dependent Variables	rtb_{t-1} (t)	xr_{t-1} (t)	xb_{t-1} (t)	xc_{t-1} (t)	y_{t-1} (t)	div_{t-1} (t)	spr_{t-1} (t)	R^2 (p)
VAR estimation results								
rtb_t	0.379	0.003	0.005	-0.017	0.00007	0.0001	0.00009	0.196
	(7.974)	(1.213)	(0.703)	(-3.342)	(-0.456)	(0.356)	(0.856)	(0.000)
xr_t	0.900	-0.001	0.212	-0.085	-0.004	0.007	0.002	0.043
	(1.458)	(-0.022)	(1.700)	(-1.272)	(-1.693)	(1.276)	(0.816)	(0.001)
xb_t	0.656	-0.063	0.152	-0.070	0.001	0.0002	0.002	0.083
	(2.283)	(-3.303)	(2.414)	(-2.153)	(0.974)	(0.106)	(2.825)	(0.000)
xc_t	-0.738	-0.035	-0.039	0.004	0.0007	-0.002	0.002	0.013
	(-1.468)	(-1.108)	(-0.371)	(0.068)	(0.378)	(-0.626)	(1.203)	(0.001)
y_t	-9.629	1.481	-5.991	3.857	0.873	-0.016	0.038	0.802
	(-1.020)	(2.587)	(-2.490)	(3.726)	(27.028)	(-0.253)	(1.998)	(0.000)
div_t	-1.038	0.017	-0.231	0.081	0.005	0.994	-0.0008	0.987
	(-1.634)	(0.292)	(-1.827)	(1.208)	(2.017)	(175.941)	(-0.408)	(0.000)
spr_t	-2.520	-0.228	2.623	-2.658	-0.006	0.0004	0.938	0.889
	(-0.329)	(-0.479)	(1.414)	(-3.420)	(-0.220)	(0.007)	(51.876)	(0.000)

Cross-correlation of residuals

	rtb	xr	xb	xc	y	div	spr
rtb	1.000	0.049	-0.035	-0.007	0.112	-0.074	-0.141
xr		1.000	0.138	0.050	-0.038	-0.968	-0.089
xb			1.000	-0.131	-0.652	-0.133	0.024
xc				1.000	0.067	-0.042	0.039
y					1.000	0.027	-0.735
div						1.000	0.100
spr							1.000

Notes: rtb_t = real Treasury bill return, xr_t = excess domestic stock return, xb_t = excess bond return, xc_t = excess commodity return, y_t = nominal Treasury bill yield, div_t = log dividend yield, spr_t = yield spread. xfr_t = excess foreign stock

Table 3. Mean Demands for Domestic Asset Classes

CRRA		$\gamma = 4$	$\gamma = 7$	$\gamma = 10$
Stocks	Total demand	**120.889**	**84.224**	**65.284**
		[29.43,196.24]	[22.74,153.33]	[10.36,121.10]
	Myopic demand	**60.339**	**34.333**	**23.930**
		[12.26, 72.41]	[6.30, 40.63]	[4.32, 28.33]
	Hedging demand	60.549	49.891	41.354
		[11.64,133.75]	[6.15, 112.37]	[8.54, 101.86]
Bonds	Total demand	70.157	36.890	24.582
		[-70.33,269.44]	[-39.77,154.31]	[-29.11,107.85]
	Myopic demand	98.847	56.787	39.964
		[-76.33,251.84]	[-43.98,143.71]	[-30.56,100.74]
	Hedging demand	-28.690	-19.897	-15.381
		[-48.56, 29.84]	[-32.59, 20.69]	[-28.17, 13.00]
Commodities	Total demand	**80.678**	**49.080**	**35.827**
		[15.41, 159.16]	[9.63, 93.58]	[7.54, 67.27]
	Myopic demand	61.604	35.260	24.723
		[9.04, 140.30]	[5.20, 80.32]	[3.62, 56.33]
	Hedging demand	**19.074**	**13.820**	**11.104**
		[-0.19, 32.81]	[0.44, 22.77]	[1.60, 18.36]
Bills	Total demand	-171.723	-70.194	-25.693
		[-404.42,38.79]	[-226.65,42.92]	[-146.17,53.11]
	Myopic demand	-120.790	-26.380	11.383
		[-305.91,77.90]	[-132.03,86.99]	[-70.74, 82.40]
	Hedging demand	-50.933	-43.813	-37.077
		[-161.94,-1.32]	[-140.05,-9.98]	[-108.88, 0.41]

Notes: This table reports mean monthly total, myopic, and hedging asset demands in percentage for stocks, 10-year government bonds, commodities and 3-month Treasury bills (cash) for an investor with a unitary elasticity of intertemporal substitution ($\psi = 1$); time discount factor equals $0.92^{1/12}$; and coefficient of relative risk aversion (γ) equal to 4, 7, or 10. Numbers in brackets are Bootstrapped 90% confidence interval. A bold entry indicates significance according to the 90% confidence interval.

Table 4. Mean Demands for Domestic Asset Classes Assuming Different Values for the Elasticity of Intertemporal Substitution (ψ)

CRRA		ψ = 0.3	ψ = 1.0	ψ = 1.3
Stocks	Total demand	63.833	**84.224**	**108.646**
		[16.28,152.46]	[22.74,153.33]	[12.57,148.42]
	Myopic demand	**34.333**	**34.333**	**34.333**
		[9.16, 42.94]	[6.30, 40.63]	[7.57, 40.84]
	Hedging demand	29.501	49.891	**74.313**
		[8.18, 122.59]	[6.15, 112.37]	[3.94, 115.96]
Bonds	Total demand	32.525	36.890	42.651
		[-26.86,160.95]	[-39.77,154.31]	[-50.72,138.94]
	Myopic demand	56.788	56.787	56.787
		[-28.19,147.36]	[-43.98,143.71]	[-28.73, 148.45]
	Hedging demand	-24.262	-19.897	-14.136
		[-32.29, 20.83]	[-32.59, 20.69]	[-33.56, 20.99]
Commodities	Total demand	**49.256**	**49.080**	**48.703**
		[13.04, 98.45]	[9.63, 93.58]	[0.72, 92.10]
	Myopic demand	35.260	35.260	35.260
		[6.11, 82.90]	[5.20, 80.32]	[2.52, 81.54]
	Hedging demand	**13.995**	**13.820**	**13.443**
		[2.17, 22.86]	[0.44, 22.77]	[0.46, 20.91]
Bills	Total demand	-45.614	-70.194	-100.000
		[-224.27,41.22]	[-226.65,42.92]	[-224.86, 45.94]
	Myopic demand	-26.380	-26.380	-26.380
		[-135.99, 65.45]	[-132.03, 86.99]	[-128.46, 85.65]
	Hedging demand	-19.233	-43.813	**-73.620**
		[-131.28, 7.15]	[-140.05,-9.98]	[-133.74, 5.62]

Notes: This table reports mean monthly total, myopic, and hedging asset demands in percentage for stocks, 10-year government bonds, commodities and 3-month Treasury bills (cash) for an investor with a unitary elasticity of intertemporal of relative risk aversion (γ) equal to 4. Numbers in brackets are Bootstrapped 90% confidence interval. A bold entry indicates significance according to the 90% confidence interval.

Table 5. Mean Demands for Domestic and International Asset Classes

CRRA		$\gamma = 4$	$\gamma = 7$	$\gamma = 10$
Domestic Stocks	Total demand	77.01	58.37	46.80
		[-52.30,165.25]	[-33.47,134.21]	[-22.24,115.66]
	Myopic demand	8.60	4.69	3.12
		[-45.57, 57.00]	[-26.15, 33.60]	[-21.50, 19.67]
	Hedging demand	68.41	53.69	43.68
		[-17.88,160.44]	[-13.13,134.95]	[-6.44, 118.56]
Domestic Bonds	Total demand	127.00	69.27	46.96
		[-17.01,363.00]	[-14.35,205.01]	[-12.26,142.90]
	Myopic demand	141.02	81.08	57.10
		[-17.88,337.15]	[-9.87, 193.28]	[-6.67, 135.73]
	Hedging demand	-14.02	-11.81	-10.14
		[-52.75, 65.19]	[-37.36, 42.45]	[-29.67, 29.65]
Commodities	Total demand	90.24	55.75	**40.99**
		[34.54, 185.67]	[24.52, 113.92]	[16.41, 79.62]
	Myopic demand	63.95	36.62	25.69
		[22.05, 162.01]	[12.79, 92.76]	[8.19, 64.09]
	Hedging demand	**26.29**	**19.13**	**15.29**
		[4.49, 48.52]	[4.82, 34.31]	[3.97, 25.94]
Domestic Bills	Total demand	-262.15	-122.13	-61.86
		[-560.42,-25.35]	[-350.23,-5.78]	[-202.00,52.93]
	Myopic demand	-179.07	-59.80	-12.09
		[-409.47,-17.48]	[-191.68,32.76]	[-104.56,52.86]
	Hedging demand	-83.07	-62.33	-49.77
		[-204.14, 35.68]	[-158.30,-19.19]	[-131.86,11.81]
Foreign Stocks	Total demand	**67.90**	**38.74**	**27.12**
		[-30.03, 115.64]	[-16.55, 67.67]	[-11.67, 47.47]
	Myopic demand	**65.50**	**37.41**	**26.18**
		[-28.58, 122.47]	[-16.24, 69.93]	[-11.31, 48.93]
	Hedging demand	2.39	1.33	0.94
		[-9.56, 18.13]	[-6.72, 12.32]	[-4.20, 10.15]

Notes: This table reports mean monthly total, myopic, and hedging asset demands in percentage for stocks, 10-year government bonds, commodities, 3-month Treasury bills (cash) and foreign stocks for an investor with a unitary elasticity of intertemporal substitution ($\psi = 1$); time discount factor equals $0.92^{1/12}$; and coefficient of relative risk aversion (γ) equal to 4, 7, or 10. Numbers in brackets are Bootstrapped 90% confidence interval. A bold entry indicates significance according to the 90% confidence interval.

Table 6. Mean Demands for Domestic and International Asset Classes Assuming Different Values for Elasticity of Intertemporal Substitution (ψ)

CRRA		ψ = 0.3	ψ = 1.0	ψ = 1.3
Domestic Stocks	Total demand	28.03	58.37	**121.19**
		[-29.82,136.66]	[-33.47,134.21]	[-26.25,140.56]
	Myopic demand	4.69	4.69	4.69
		[-25.92, 27.65]	[-26.15, 33.60]	[-28.15, 33.83]
	Hedging demand	23.35	53.69	**116.51**
		[-8.10, 135.21]	[-13.13,134.95]	[-20.32,125.89]
Domestic Bonds	Total demand	63.90	69.27	82.51
		[-11.80,202.45]	[-14.35,205.01]	[-14.98,196.10]
	Myopic demand	81.08	81.08	81.08
		[-14.93,192.35]	[-9.87, 193.28]	[0.86, 189.29]
	Hedging demand	-17.18	-11.81	1.44
		[-39.66. 39.37]	[-37.36, 42.45]	[-37.59, 37.94]
Commodities	Total demand	54.86	55.75	56.00
		[16.98, 112.65]	[24.52, 113.92]	[19.77, 112.32]
	Myopic demand	36.62	36.62	36.62
		[5.92, 91.74]	[12.79, 92.76]	[8.78, 91.76]
	Hedging demand	**18.24**	**19.13**	**19.38**
		[1.81, 30.46]	[4.82, 34.31]	[4.89, 32.74]
Domestic Bills	Total demand	-86.69	-122.13	-196.40
		[-305.45,30.41]	[-350.23,-5.78]	[-293.89,24.86]
	Myopic demand	-59.90	-59.80	-59.80
		[-193.18,41.07]	[-191.68,32.76]	[-167.34,45.80]
	Hedging demand	-26.89	-62.33	-136.61
		[-175.51,10.47]	[-158.30,-19.19]	[-163.24,20.06]
Foreign Stocks	Total demand	**39.90**	**38.74**	**36.70**
		[-16.95, 69.14]	[-16.55, 67.67]	[-20.97, 73.35]
	Myopic demand	**37.41**	**37.41**	**37.41**
		[-21.66, 65.02]	[-16.24, 69.93]	[-21.79, 74.26]
	Hedging demand	2.49	1.33	-0.71
		[-6.76, 12.32]	[-6.72, 12.32]	[-7.20, 12.88]

Notes: This table reports mean monthly total, myopic, and hedging asset demands in percentage for stocks, 10-year government bonds, commodities, 3-month Treasury bills (cash) and foreign stocks for an investor with elasticity of intertemporal substitution ($\psi = 0.3, 1, 1.3$); time discount factor equals $0.92^{1/12}$; and coefficient of relative risk aversion (γ) equal to 4. Numbers in brakcets are Bootstrapped 90% confidence interval. A bold entry indicates significance according to the 90% confidence interval.

Table 7. Mean Value Function

γ	$E(V_t)$	
Panel A.		
	Portfolio 1 Bills, bonds, and domestic stocks	Portfolio 2 Bills, bonds, domestic stocks, and commodities
4	0.035	0.070
7	0.013	0.020
10	0.009	0.012
20	0.005	0.006
Panel B.		
	Portfolio 3 Bills, bonds, domestic stocks, and international stocks	Portfolio 4 Bills, bonds, domestic stocks, international stocks, and commodities
4	0.222	0.490
7	0.036	0.057
10	0.017	0.024
20	0.007	0.008

Notes: This table reports the mean value function for investors with a unitary elasticity of intertemporal substitution ($\psi = 1$); time discount factor equals $0.92^{1/12}$ and coefficient of relative risk aversion (γ) equal to 4, 7, 10 or 20. Portfolio 1 is a benchmark portfolio (traditional portfolio), which allocates across domestic asset classes without including commodities. Portfolio 2 allocates across domestic classes, including commodities. Portfolio 3 allocates across both domestic and international asset classes without including commodities. Portfolio 4 allocates across both domestic and international asset classes including commodities.

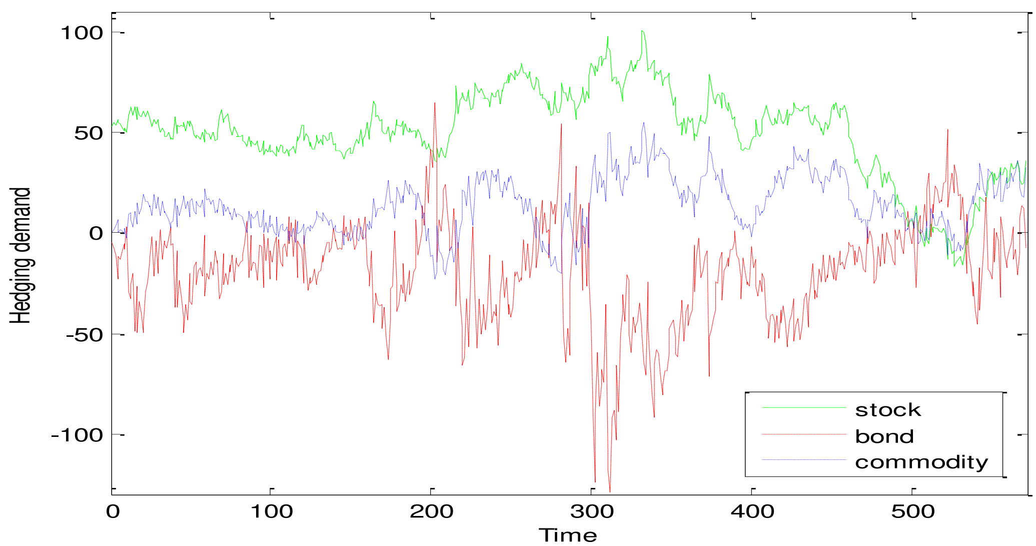

Fig 1. Historical intertemporal hedging demands for domestic stocks, bonds and commodities for U.S. investors.

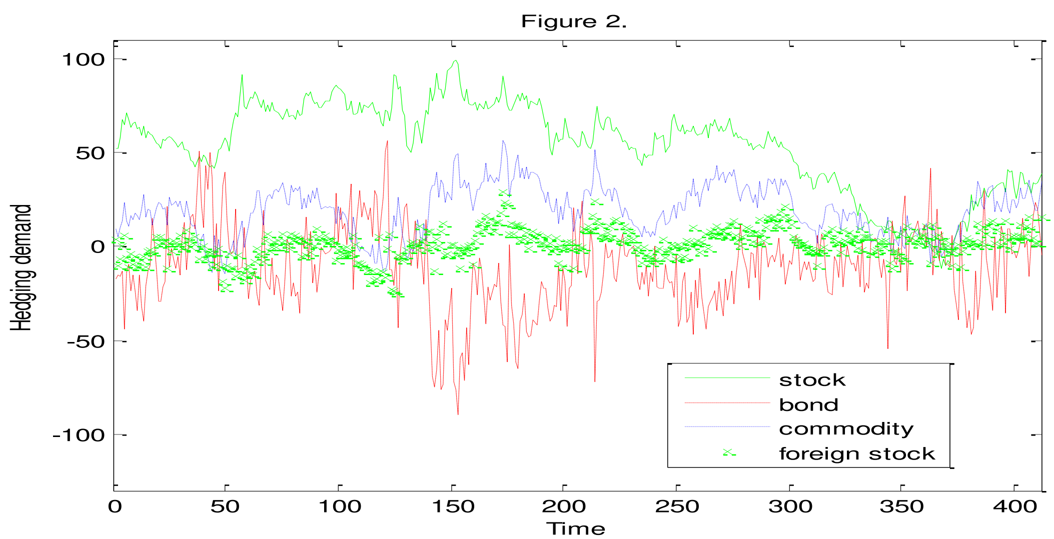

Fig 2. Historical intertemporal hedging demands for domestic stocks, bonds and commodities, as well as foreign stocks for U.S. investors.

Efficient Frontiers

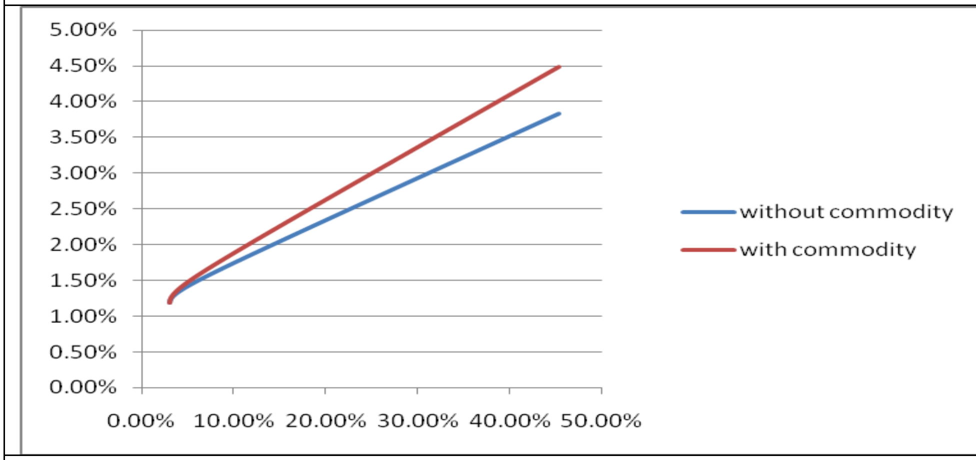

Fig 3. Efficient frontier with and without commodity as an asset class