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Abstract

We model how the choices by students to "rush" a fraternity, and the choices by a fraternity of whom to admit, interact with the signals that firms receive about student productivities to determine labor market outcomes. Both the fraternity and students care about future wages and fraternity socializing values. We first show that if the signals firms receive about students are either perfectly informative or perfectly noisy, then fraternity membership has no impact on labor market outcomes. For intermediate signaling technologies, however, three types of equilibria can exist: pessimistic beliefs by firms about the abilities of fraternity members can support an equilibrium in which no one pledges; optimistic beliefs can lead to higher wages for fraternity members than non-members, so that in equilibrium everyone whom the fraternity would like to admit actually pledges; and an equilibrium in which most fraternity members have intermediate abilities—less able students apply, hoping to be mixed in with better students, but are rejected unless they have high fraternity socializing values, while most very able students do not apply to avoid being tainted in labor market outcomes due to being mixed in with less able fraternity members. We provide sufficient conditions for this latter "hump-shaped" equilibrium to exist, take the model to the data and show that this equilibrium can reconcile the ability distribution of fraternity members at the University of Illinois. Finally, we estimate the welfare impact of the fraternity on different students.

Keywords: Signaling, fraternities, statistical discrimination
JEL: J31, D82, H4

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1 Introduction

To many, the word “fraternities” brings to mind images of beer, parties and fun. Yet, fraternity membership also enters prominently the job seeking process of many students: resumes often devote scarce space to highlighting a student’s society memberships in addition to the standard information about education, work experience, awards, etc. This suggests that fraternity membership helps employers evaluate a person’s productivity. On first impression it is not clear why fraternity or sorority membership should matter for labor market outcomes. In particular, while fraternities make significant time demands — members must spend considerable time picking up trash on highways, raising money for charitable causes, and so on — these activities appear largely unrelated to skill development for future careers. Nonetheless, fraternities draw many applicants who eagerly spend money and devote time to these activities, and employers seem to weigh membership information positively.

We develop a theory of fraternity membership and filtering by firms that makes sense of these observations. Students are distinguished by a fraternity socializing value and their productivity as a worker. Fraternities value both the future wages generated by members and their socializing values. Firms combine information in noisy signals about student productivities with fraternity membership status to set wages. To emphasize the key economic forces, we suppose that fraternity socializing values are not directly valued by firms and also that they are uncorrelated with worker productivities. We further assume away all standard club features for fraternities as in Buchanan [1965], so that there are no consumption spillovers due to the presence of other students. So, too, we assume away any networking services that a fraternity might provide. As a result, the fraternity membership statuses of other students only affect job market outcomes for a given student via the equilibrium beliefs that firms form about the distribution of abilities of fraternity members and non-members.

We first identify sufficient conditions for fraternity membership not to matter for job market outcomes. In particular, we show that if the signal that firms receive about a student’s productivity is either perfectly informative or perfectly noisy, then equilibrium wages do not depend on a student’s fraternity membership. As a result, whether a student rushes a fraternity depends only on his fraternity socializing value. If signals are perfectly informative, fraternities trade off between productivity and socializing value in admission, but a student's wage will equal his known productivity, rendering membership irrelevant for labor market outcomes. If, instead, signals are perfectly noisy, a fraternity would like to commit to excluding low ability students with high socializing value, and to accepting high ability students with low socializing value. However, with perfectly noisy productivity signals, firms have no source other than fraternity membership for evaluating a student's ability, so that fraternities weigh only socializing values in admission. As a result, fraternity membership conveys no information to firms about ability, so that wages do not hinge on fraternity membership.

In sum, we show that for fraternity membership to affect job market outcomes, firms must receive signals about a student productivities that are noisy, but not perfectly so. Then, because more productive students tend to earn higher wages, fraternities trade off between productivity and fraternity socializing values when deciding which pledge applicants to accept. In particular, fraternities accept students with low socializing values who are sufficiently able. Students may face a different type of trade off—more able students may incur a labor market cost from joining a fraternity, as their fraternity membership may lump them in with intermediate quality students, making it harder for the able students to distinguish themselves in the eyes of firms. In such a situation, sufficiently more able students may be reluctant to pledge fraternities.

We then turn to a three-signal setting in which we can explicitly solve for the multiple equilibria that emerge in the fraternity game. We identify three types of equilibria: (a) an "empty fraternity" equilibrium in which no student applies to the fraternity, supported by beliefs of firms that any student who joins the fraternity is especially lacking in ability; (b) a "hump-shaped" equilibrium in which most fraternity members have intermediate abilities—less able students apply, but are rejected unless they have high fraternity socializing values, while very able students who do not have very high fraternity socializing values do not apply; and (c) an equilibrium in which employer beliefs about the abilities of fraternity members are more optimistic—so that fraternity membership would increase the expected wage of each student type. In these latter two equilibria, relatively low ability students expect higher wages if they gain fraternity membership than if they do not; while in the second equilibrium, but not the third, higher ability students may anticipate lower wages if they join. That is, fraternity membership may taint labor market outcomes for high ability students, but not low.

We return to a more general signal framework, in which we only assume that the conditional distribution of ability signals that firms receive satisfies the monotone likelihood ratio property—more able students are more likely to generate higher signals. We then provide gross sufficient conditions for non-trivial equilibria to have the hump-shaped feature. In particular, these sufficient conditions imply that the wage premium due to fraternity membership declines with ability—the lowest ability fraternity members always receive a particularly large wage premium, as their membership ensures that they are separated away from all lower ability types, and are mixed in with relatively higher proportions of more able types, while high ability types gain less (or lose) from being mixed in with less able fraternity members. The hump-shaped equilibrium then emerges, due to the filtering by fraternities of low ability students, and, when membership costs are of an appropriate magnitude relative to socializing values, the reluctance of high ability students to join.

Finally, we investigate whether equilibria of our three-signal model are consistent with actual practice. To do this, we obtain data on cumulative GPAs of seniors at the University of Illinois for fraternity members and non-members. Using GPA as a noisy indicator of ability, we find that the equilibrium in which most fraternity members have intermediate abilities—where high ability students are tainted by membership in fraternities—can generate a distribution over probability of membership conditional on ability that closely mirrors the distribution over the probability of fraternity membership conditional on GPA found in the data. We back out plausible estimates of primitives—the time costs of fraternity participation, and the tradeoffs of both students and fraternity between socializing values and future wages. We use these estimates to derive how the presence of the fraternity affects the welfare of different student types.

We next review the literature and then provide a brief overview of fraternities. In section 2 we develop our model and analysis. Section 3 considers a three signal setting. Section 4 returns to a general setting and provides sufficient conditions for all non-trivial equilibria to exhibit the hump-shaped pattern that we find in the data. Section 5 provides our empirical analysis of fraternities. Section 6 concludes. Some proofs are in an appendix.

1.1 Related Literature

Our model and analysis can be integrated into the endogenous statistical discrimination literature. The closest papers are Moro and Norman [2004] and Austen-Smith and Fryer [2005]. Moro and Norman [2004] consider a setting in which individuals choose whether to make a costly investment in education, when firms receive noisy signals of that investment.[1] They show how a productivity irrelevant aspect such as racial identity can affect investment choices if firms believe that one population is more likely to invest.[2] In particular, one can support multiple equilibria, one where beliefs do not depend on race, and one where they do.[3,4] So, too, in our economy, the essence is how the beliefs of firms about the abilities of workers who generate different signals are affected by the equilibrium fraternity membership status. One difference between our economy and this literature is that fraternity membership in our economy is not productivity enhancing, and uncertainty does not relate to whether or not an investment was made. More importantly, race is exogenous, while fraternity membership is endogenous; in particular, fraternity membership is *not* solely determined by the student, but rather is the outcome of an admission game played by the fraternity and students.

Austen-Smith and Fryer [2005] consider a setting in which a "peer group" generates additional utility from leisure for its members, and this drives members to shift time allocation toward less education, which leads to lower wages. Those who are rejected by the peer group study more, "acting white".

[1]See also Coate and Loury [1993], Fang and Norman [2006] and Norman [2003] for related models.

[2]Fang [2001] considers a variant in which firms interpret participation in an irrelevant activity as a signal that the agent has a low investment cost, and assign agents who signal to the job where such investment is productive. By way of contrast, ours is not a signaling model (indeed, sufficiently higher ability students are *reluctant* to join), and the information content of fraternity membership is determined by the equilibrium interaction between the fraternity and different student types.

[3]We could augment our economy to have firm beliefs depend on race as well, with the result that race will enter both the decision of whether to apply to a fraternity, and the decision by the fraternity of whether to accept an applicant.

[4]Mailath, Samuelson, and Shaked [2000] develop a related search model in which firms choose which populations to search, and each population makes investments in skills based on beliefs about firm search intensities. Again, asymmetric search intensities can give rise to asymmetric investment choices in two otherwise identical populations.

1.2 Fraternities and Sororities

To ease presentation, we drop gender differences and refer to both fraternities and sororities as "fraternities". The first club-like fraternity with a centralized organization, was Kappa Alpha Society, founded in 1825. For a history and current status of fraternities, see Anson and Marchesani [1991].

Fraternities require pledge applicants to submit extensive information about themselves: their school GPA, recommendations, interests and useful skills. Fraternities devote far more time to evaluating applicants than do potential employers. In particular, almost all fraternity applicants are interviewed, and applicants take part in an extensive series of activities during the evaluation process. For example, Sigma Chi requires a potential member to spend one year working for the fraternity before the pledge. This suggests that fraternities are well-situated to evaluate a pledge applicant's ability, so that fraternity membership can provide firms with valuable information.

Fraternities rely on membership fees and donations to fund activities. A substantial share of a fraternity's income comes from alumni donations. Because high income alumni donate more, fraternities care about the future job market outcomes of members. An indication of the value that fraternities place on productive members is that GPA-based stipends to fraternity members are widespread. Fraternities frequently reject pledge applicants. Conversely, many highly-productive students choose not to apply to fraternities. Finally, students almost never join more than one fraternity. This reflects both secrecy issues (secret handshakes, for example, allow one member to verify the membership status of others), and because fraternity activities are quite time-consuming.

2 The Fraternity Game

There is a population of measure 1 of students. A student is fully described by his future employment productivity θ and his fraternity socializing value, μ. Students have separable preferences over income and fraternity membership: a non-member's payoff corresponds to his expected net lifetime income, M, and a fraternity member with socializing value μ derives utility

$$M + n\mu,$$

where $n > 0$. M equals the student's expected lifetime future wage minus the monetary value c of the time costs of fraternity service activities. Note that to sever all links with the club-good literature, we assume away any externalities from the socializing values of other fraternity members. So, too, to ensure that there is no direct link between productivity and membership, we assume that θ and μ are uncorrelated in the population. That is, the density over θ and μ is given by

$$h(\theta, \mu) = h_\theta(\theta) h_\mu(\mu),$$

where the bounded supports of θ and μ are given by$[\underline{\theta}, \bar{\theta}]$ and $[\underline{\mu}, \bar{\mu}]$, respectively, and $\underline{\theta}$ and $\bar{\mu}$ are both positive. The associated cdf is H, and the measure m, used in some proofs, is based on H. We emphasize that while we believe that socializing skills and productivity may be correlated in practice, we assume such correlation away in order to highlight the impact of application decisions by students and filtering by fraternities on the equilibrium distribution of abilities in the fraternity.[5]

There is a single representative fraternity that chooses which "rush" applicants to admit. The fraternity cares about both the future market wages that its members will obtain, and the socializing values of its members. For simplicity, we assume that the fraternity has separable linear preferences over wages and socializing values, so that the fraternity's payoff from members (θ, μ) in the set C of fraternity members is given by

$$\int_{(\theta,\mu)\in C} [W_1 E_{\widetilde{\theta}}(w_C(\widetilde{\theta}|\theta)) + W_2\mu] h(\theta, \mu) d\theta d\mu,$$

where $W_1 > 0$ and $W_2 \geq 0$. We assume that the fraternity is limited by space constraints to admitting at most a measure Γ of students: in practice, a fraternity house has a limited number of bedrooms. This means that the fraternity will tradeoff between μ and θ in admission—trading off future higher contributions from more able and hence wealthier alumni against their social contribution.

[5]Obviously, if social skills and productivity are positively correlated in the population, and the fraternity values social skills, then this exogenous correlation will lead to fraternity members receiving higher wages than non-members; we wanted to avoid building this result trivially into our model. We assume away any network services that a fraternity might provide for the same reason.

Our analysis is qualitatively unaffected by alternative preferences of the fraternity that continue to induce the fraternity to tradeoff between socializing value and ability in admissions. In particular, qualitatively identical outcomes obtain if the fraternity did not face a space constraint, but instead cared about the average socializing value of its members (say due to externalities), in addition to the future wages that members earn. So, too, outcomes are qualitatively unaffected if the fraternity, rather than facing a space constraint, incurred costs that were a convex function of the measure of members (say due to cramming more students into each room), or if the fraternity cared about the market value of the time contributions of its members.

After graduation, students are employed by firms. We assume that several risk-neutral firms make simultaneous wage offers to students. The firms do not observe an individual student's productivity θ or fraternity socializing value μ. However, firms do observe whether a student is a member of a fraternity. Firms also observe a common signal $\tilde{\theta}$ about the student's productivity θ, where $\tilde{\theta}$ is distributed according to $F_{\tilde{\theta}}(\cdot|\theta)$. We assume that more able students are more likely to generate higher signals: $F_{\tilde{\theta}}(\tilde{\theta}|\theta)$ is strictly decreasing in θ for all $(\theta, \tilde{\theta})$ with $F_{\tilde{\theta}}(\tilde{\theta}|\theta) \in (0, 1)$. Competition drives firms to offer each individual a wage equal to his expected productivity given his fraternity membership status and ability signal, $\tilde{\theta}$.

There are four stages to our "fraternity rush" game. At stage one, each student type (θ, μ) decides whether to apply for fraternity membership. We let $a(\theta, \mu)$ be an indicator function taking on the value 1 if student type (θ, μ) applies, and taking on the value 0 if the student type does not apply. We sometimes use the set $A = \{(\theta, \mu)|a(\theta, \mu) = 1\}$. At stage 2, the fraternity chooses which applicants to accept. We let $b_A(\theta, \mu)$ be an indicator function taking on the value 1 if, given the set of applicants A, the fraternity would admit a student type (θ, μ) who applied, and taking on the value 0 otherwise. We use $B_A = \{(\theta, \mu)|b_A(\theta, \mu) = 1\}$ to represent the set of admitted student types. Then, the set of fraternity member types is $C_A = \{(\theta, \mu)|a(\theta, \mu)b_A(\theta, \mu) = 1\}$, and the set of nonmembers is $\bar{C}_A = \{(\theta, \mu)|a(\theta, \mu)b_A(\theta, \mu) = 0\}$. At stage 3, firms see whether an individual is a fraternity member, and they see a noisy signal of his ability, but do not observe their types—firms must form beliefs about which student types actually join the fraternity. Let $\rho_F(\theta, \mu)$ denote firm beliefs about fraternity membership for each type (θ, μ), where $\rho_F(\theta, \mu) = 1$ if firms believe type (θ, μ) is a member of the fraternity, and $\rho_F(\theta,\mu) = 0$ if not. Finally, $w_C(\tilde{\theta})$ denotes the wage of a fraternity member who emits the signal $\tilde{\theta}$, and $w_{\overline{C}}(\tilde{\theta})$ denotes the wage of a non-member who generates signal $\tilde{\theta}$. At stage 4, a worker with productivity θ produces output with value θ.

An **equilibrium** is a collection of functions, $\{a(\theta,\mu), b_A(\theta,\mu), w_C(\tilde{\theta}), w_{\overline{C}}(\tilde{\theta})\}$ and firm beliefs $\rho_F(\theta,\mu)$ such that

i) Students optimize: $a(\theta,\mu) = 1$ if $E[w_C(\tilde{\theta})|\theta] + n\mu - c \geq E[w_{\overline{C}}(\tilde{\theta})|\theta]$; 0 otherwise. We let A^* be the associated set of fraternity applicants.

ii) For every A the fraternity optimizes: B_A solves

Problem 1

$$\max_{B_A} \int_{(\theta,\mu)\in A\cap B_A} [W_1 E[w_C(\tilde{\theta})|\theta] + W_2\mu] h(\theta,\mu) d\theta d\mu \tag{1}$$

$$\textit{subject to} \quad m(A\cap B_A) \leq \Gamma.$$

iii) wages are competitive given beliefs by firms $\rho_F(\theta,\mu)$:

$$w_C(\tilde{\theta}) = \frac{\int_C \theta h(\theta,\ \mu)\rho_F(\theta,\ \mu) f_{\tilde{\theta}}(\tilde{\theta}|\theta) d\theta d\mu}{\int_C h(\theta,\ \mu)\, \rho_F(\theta,\ \mu)\, f_{\tilde{\theta}}(\tilde{\theta}|\theta) d\theta d\mu}; \quad w_{\overline{C}}(\tilde{\theta}) = \frac{\int_{\overline{C}} \theta h(\theta,\ \mu)\,(1-\rho_F(\theta,\ \mu)) f_{\tilde{\theta}}(\tilde{\theta}|\theta) d\theta d\mu}{\int_{\overline{C}} h(\theta,\ \mu)\,(1-\rho_F(\theta,\ \mu))\, f_{\tilde{\theta}}(\tilde{\theta}|\theta) d\theta d\mu}.$$

iv) Firm beliefs are consistent with choices of student types and fraternity: For a.e. (θ,μ), $\rho_F(\theta,\mu) = a(\theta,\mu) b_{A^*}(\theta,\mu)$.

Off-equilibrium path characterizations are not intrinsically interesting, and to ease presentation, we only characterize equilibrium path outcomes. Moreover, measure zero perturbations of the fraternity's acceptance set B_{A^*} are uninteresting—any measure zero perturbation to B_{A^*} is also part of an equilibrium, so we focus on a best response of the fraternity that is a **good set**, i.e., a set B_{A^*} that is equal to the closure of its own interior. Finally, to simplify notation, we omit the A^* index on the equilibrium acceptance set.

We begin by providing conditions under which fraternity membership has no effect on labor market outcomes.

Proposition 1 *Suppose that firms either receive perfect signals about students, i.e.* $\tilde{\theta} = \theta$, *a.e.,* θ, *or firms receive perfectly uninformative signals,* $F_{\tilde{\theta}}(\cdot|\theta) = F_{\tilde{\theta}}(\cdot|\theta')$, *for all* θ, θ'. *Then, in equilibrium, a student's wage does not depend on whether he is in the fraternity or not,* *i.e.,* $w_C(\tilde{\theta}) = w_{\overline{C}}(\tilde{\theta})$*, for all* $\tilde{\theta}$*. Hence, a student type* (θ, μ) *applies for membership in the fraternity if and only if* $n\mu - c \geq 0$*.*

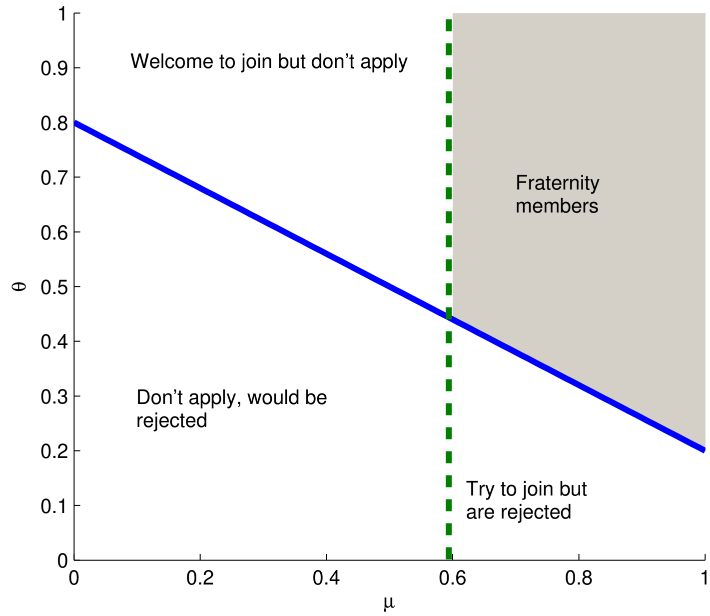

Figure 1: Perfect signaling equilibrium.

Suppose signaling is perfect. Then $w_C(\tilde{\theta}) = w_{\bar{C}}(\tilde{\theta}) = \tilde{\theta} = \theta$ (a.e.). Optimization by students then implies that a student type (θ, μ) applies if and only if $n\mu - c \geq 0$, independently of θ. In contrast to students, the fraternity selectively admits higher θ applicants who will earn higher wages. In particular, the fraternity trades off between μ and θ in admission; letting $\mu_B(\theta)$ denote the boundary of the admission set, indifference implies that the boundary has slope $\frac{d\mu_B(\theta)}{d\theta} = \frac{-W_1}{W_2}$. Figure 1 illustrates the equilibrium. The solid line is the fraternity's equilibrium cutoff rule—all types to the right of the line who apply are accepted, while all those to the left are rejected. The vertical dashed line represents the accept-or-reject line of students—student types to the right apply in equilibrium, i.e., are in the set A. Hence, the equilibrium set C of fraternity members consists of those types to the right of both the dashed and solid lines, and the measure of the set C is at most Γ.

If, instead, signals are completely uninformative, then all individuals receive the same wage, $w_C(\tilde{\theta}) = w_{\bar{C}}(\tilde{\theta}) = E[\theta]$. A fraternity would like to commit to excluding low θ students with high socializing values μ, and to accepting high θ students with low socializing values. However, since firms have no source other than fraternity membership for evaluating a student's ability, all fraternity members must receive the same wage. But then, given *any* beliefs that firms hold about the abilities of fraternity members, the fraternity's optimal admission policy only depends on μ, admitting a type (θ, μ) if and only if μ exceeds some critical cutoff. Hence, fraternity membership conveys no information to firms about θ. As a result, in equilibrium, both fraternity and non-fraternity members receive wage $E[\theta]$. Since wages do not depend on membership, it follows that only students with $n\mu \geq c$ apply. Figure 2 illustrates this uninformative signal case.

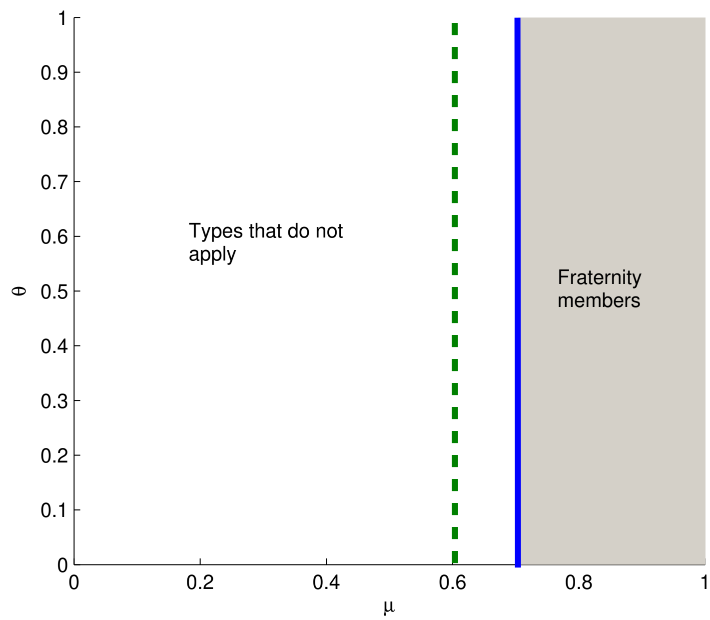

Figure 2: Contentless signaling equilibrium.

Although we do not explore it further, the case of completely uninformative signals about abilities highlights the gains that fraternities may achieve from an ability to commit to their admission policies. In particular, the fraternity would like to commit to excluding low ability students who have moderately high socializing values, and to accepting high ability students with lower socializing values. In practice, imperfect commitment devises that fraternities use include having university officials report the average GPA of members, and having the Greek council forbid fraternity participation to students with GPAs below some standard. This commitment induces a fraternity to weigh ability in admission, raising wages of members, and thereby raising the fraternity's payoff.

The central implication of Proposition 1 is that for fraternity membership to affect job market outcomes, firms must receive signals about student productivities that are noisy, but not perfectly so. Then, because more productive students tend to earn higher wages, fraternities value both productivity and fraternity socializing value, and will tradeoff between the two in admission. We now examine the choice problems of students and the fraternity in more detail.

2.1 Student's Problem

Students compare the expected payoffs from being a fraternity member and not, taking into account both the consequences for expected wages, and his fraternity socializing value. Optimization implies that a student type (θ, μ) applies for fraternity membership if and only if

$$E_{\tilde{\theta}}\left[w_C(\tilde{\theta})|\theta\right] + n\mu - c \geq E_{\tilde{\theta}}\left[w_{\overline{C}}(\tilde{\theta})|\theta\right]. \tag{2}$$

That is, a student applies to the fraternity either to obtain higher expected wages, or because his fraternity socializing value μ is sufficiently high.[6] The following result follows straightforwardly.

Proposition 2 *If $a(\theta, \mu) = 1$, then $a(\theta, \mu') = 1$, for all $\mu' > \mu$.*

Proof. The expected wages of individuals (θ, μ) and (θ, μ') are the same, but a type (θ, μ') student gains strictly more utility from joining the membership. ■

Corollary 1 *The equilibrium supply of fraternity applicants is summarized by a continuous function $\mu_A(\theta)$ such that a type (θ, μ) student applies if and only if $\mu \geq \mu_A(\theta)$.*

Proof. The result follows because expected wages of fraternity and non-fraternity members are continuous in θ. Therefore, student payoffs and hence choices are continuous in expected wages. ■

[6]If (2) holds, then student type (θ, μ) applies even if he expects to be rejected by the fraternity. If students face positive costs of applying, then to reconcile the observation that some students apply, but are not admitted, one must integrate additional uncertainty/noise, so that a student does not always know whether he or she will be admitted. Such uncertainty complicates presentation and analysis, while providing limited benefits. Accordingly, we abstract away.

2.2 Fraternity's Problem

In any equilibrium, the fraternity offers membership to the set of students B that solves Problem 1. The sets A and B implicitly define the set of fraternity members $C = A \cap B$ and the set of nonmembers $\overline{C}$. Since the fraternity's payoff is increasing in the socializing values of its members, we have

Proposition 3 *For almost all (μ, θ) in $A \bigcap B$, almost all types (θ, μ') with $\mu' > \mu$ also belong to B.*

Proof. See the appendix. ∎

We next show that we can extend this characterization to establish that the fraternity also wants to admit students who are more able as long as expected wages are increasing in θ; and expected wages are increasing in θ if $w_C(\tilde{\theta})$ is increasing in $\tilde{\theta}$. As a preliminary step, we present an implication of the MLRP property on signals (see Milgrom [1981]) for equilibrium wages.

Lemma 1 *Assume that $f(\theta|\tilde{\theta}) > 0$ on $[\underline{\theta}, \bar{\theta}]$, and that $f(\theta|x)$ satisfies the MLRP property. Fix a set $D \subset \Theta \times \tilde{\Theta}$ with $P(\tilde{\theta} = k|\tilde{\theta} \in D) < 1$ for all signals k. Let $Q(\tilde{\theta}) = E(\theta|\tilde{\theta}, D)$ be a firm's estimate of ability given signal $\tilde{\theta}$ and set D. Then from the perspective of a student with ability θ, his expected wage $E_{\tilde{\theta}}[Q(\tilde{\theta})|\theta, D]$, increases with his ability θ.*

Proof. See the appendix. ∎

Proposition 4 *Suppose that the signals that firms receive about student abilities have the MLRP property. Then for almost all (θ, μ) if $b(\theta, \mu) = 1$, we have $b(\theta', \mu) = 1$ for almost all $\theta' > \theta$.*

Proof. By Lemma 1, the expected wage $E_{\tilde{\theta}}[w_C(\tilde{\theta})|\theta]$ is an increasing function of θ. The logical construction of Proposition 3 then applies. ∎

Propositions 3 and 4 pin down the attributes of the set B of student types that the fraternity would admit. For example, if every student whom the fraternity would want to admit applies, then B is defined by a negatively-sloped curve in (θ, μ) space, $\mu_B(\theta)$: The fraternity

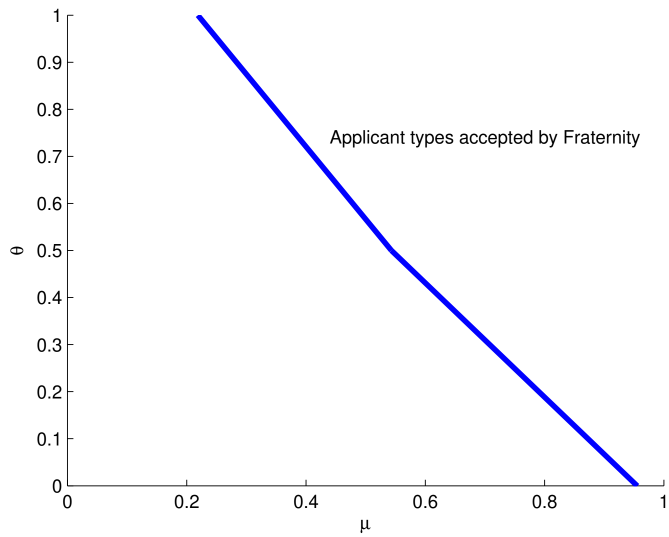

Figure 3: Unconstrained admissions set B.

admits almost every student type above (Proposition 4) and to the right (Proposition 3) of this curve (see Figure 3), i.e., $B = \{(\theta, \mu) | \mu \geq \mu_B(\theta)\}$, and $C = A \bigcap B$. Both $\mu_A(\theta)$ and $\mu_B(\theta)$ are continuous in θ, reflecting the continuity of expected wages in θ.

More generally, for almost all θ where the fraternity's admission decision is not constrained by student application, i.e., for almost all θ with $\mu_B(\theta) > \mu_A(\theta)$, the fraternity trades off linearly between expected wage and fraternity socializing value in admission. That is, for θ_1, θ_2 with $\mu_B(\theta_j) > \mu_A(\theta_j)$, $j = 1, 2$, we have

$$W_1 E(w_C(\tilde{\theta})|\theta_1) + W_2 \mu(\theta_1) = W_1 E(w_C(\tilde{\theta})|\theta_2) + W_2 \mu(\theta_2).$$

That is, marginal contributions of these marginal types, $(\theta_1, \mu_B(\theta_1))$ and $(\theta_2, \mu_B(\theta_2))$ are equal.[7]

2.3 Existence of equilibrium

We first characterize when the "empty fraternity" is an equilibrium. In this "Groucho Marx" equilibrium, the fraternity would accept anyone who applies, but no one applies because firms

[7] This result extends if we relax the structure on the fraternity's preferences, so that preferences over aggregate wages and socializing values are non-linear, $W(m(Ew_C(\tilde{\theta})), m(\mu|C))$. Then, the appropriate marginal derivatives, W_1, W_2, evaluated at the aggregates, describe the indifference relationship for the fraternity.

believe that anyone who joins the fraternity has low ability $\underline{\theta}$ and hence would be given wage $w_C(\tilde{\theta}) = \underline{\theta}$. If no one joins the fraternity, then someone who generates signal $\tilde{\theta}$ receives wage $w_{\bar{C}}(\tilde{\theta}) = E[\theta|\tilde{\theta}]$. Let $\underline{w} = E_{\tilde{\theta}}[w_{\bar{C}}(\tilde{\theta})|\underline{\theta}]$ be the wage that a student with lowest ability $\underline{\theta}$ expects if he does not join the fraternity in this scenario.

Proposition 5 *Suppose that the signaling technology has a full support property, $f(\theta|x) > 0, \forall x$. Then an equilibrium exists with $A = C = \emptyset$ if and only if $n\bar{\mu} - c \leq \underline{w} - \underline{\theta}$.*

Proof. See the appendix. ■

If $n\bar{\mu} - c \leq \underline{w} - \underline{\theta}$, then pessimistic firm beliefs can support the empty fraternity equilibrium. However, if the inequality does not hold, then sufficiently inept students with high socializing values would prefer to join the fraternity because they also expect to receive low enough wages outside the fraternity that the maximum wage cost from joining the fraternity is more than offset by their high socializing values.

We next prove that an equilibrium always exists to this fraternity game, establishing a fixed point to a mapping from conjectured optimal student application and fraternity admission choices by firms to the best responses to those conjectures by students and fraternities. To do so, we exploit Propositions 2 and 4 and consider continuous student and fraternity choice functions $\mu_A(\cdot)$ and $\mu_B(\cdot)$, where a student type (θ, μ) is a member of the fraternity if and only if $\mu \geq \max\{\mu_A(\theta), \mu_B(\theta)\}$. Existence of equilibrium then follows from standard fixed point theorems.

Proposition 6 *An equilibrium exists to the fraternity game.*

Proof. See the appendix. ■

3 Three Signal Economy

To gain explicit insight into the equilibria of this fraternity game, we next consider an economy in which student productivities and fraternity socializing values are uniformly distributed on the unit square, i.e., (θ, μ) are uniformly distributed on $[0;1] \times [0;1]$, and students generate one of three possible signals, $\tilde{\theta} \in \{H, M, L\}$. In particular, we suppose that more able students with $\theta > 0.5$ generate either medium or high signals, where the probability of a high signal is linearly increasing in ability; and that less able students with $\theta < 0.5$ generate either low or medium signals:

$$\begin{aligned} Prob\,(H|\theta) &= 2\theta - 1,\ \theta > \frac{1}{2} \quad \text{and 0 otherwise.} \\ Prob\,(L|\theta) &= 1 - 2\theta,\ \theta < \frac{1}{2} \quad \text{and 0 otherwise.} \\ Prob\,(M|\theta) &= 1 - Prob\,(L|\theta) - Prob\,(H|\theta)\,. \end{aligned}$$

This signal technology obviously satisfies the MLRP property. Its central feature is that a student with $\theta < 0.5$ hopes to get lucky and receive a medium signal, and thereby be indistinguishable from a student with $\theta > 0.5$ who unluckily receives a medium signal.

Let $w_C(\tilde{\theta})$ be the wage that a fraternity member who generates signal $\tilde{\theta}$ receives and $w_{\bar{C}}(\tilde{\theta})$ be the wage that a non-member who generates signal $\tilde{\theta}$. The expected wage of a student with ability θ who joins the fraternity is

$$E\left(w_C\left(\tilde{\theta}\right)|\theta\right) = w_C\,(H)\,Prob\,(H|\theta) + w_C\,(M)\,Prob\,(M|\theta) + w_C\,(L)\,Prob\,(L|\theta)\,.$$

An analogous expression describes wages of students who are not fraternity members.

The piecewise linear structure of the signaling technology implies that the expected wage functions are piecewise linear in θ with a single kink at $\theta = \frac{1}{2}$. It follows that the boundary describing the set of students that the fraternity would admit, where not limited by students' application decisions, is also linear with a kink at $\theta = \frac{1}{2}$. Since the difference in wages of fraternity members and non-members is linear with a kink at $\theta = \frac{1}{2}$, the boundary of the set of applicants to the fraternity, $\mu_A\,(\theta)$, is also linear with a kink at $\theta = \frac{1}{2}$. Therefore, the set of fraternity members, $\{(\mu, \theta)|\mu \geq \max\{\mu_A(\theta), \mu_B(\theta)\}\}$, is described by a continuous piecewise-linear function from $[0, 1]$ to $[0, 1]$ that has one or two kinks, where one kink is at $\theta = 0.5$, and the other (if it exists) is at the intersection of the fraternity and student cutoff rules.

One equilibrium is obviously the "empty fraternity", but there are also more interesting equilibria. In particular, given Γ, we search for (i) an equilibrium in which the boundary $\mu_A(\theta)$ of A is everywhere to the left of the boundary $\mu_B(\theta)$ of B, i.e., where every student that the fraternity would want is admitted and (ii) an equilibrium in which $\mu_A(\theta)$ and $\mu_B(\theta)$ intersect, so the piecewise-linear function describing the frontier of the set of fraternity members has two kinks. This latter equilibrium is described by a system with thirteen unknowns (the slope and intercepts of the three lines plus the intersection point of the student and fraternity frontier, plus six wages) and thirteen equations (6 equations from the firm's problem — wages equal expected skill given signal realization and membership status, 4 equations from the fraternity and 3 equations from the students). We solve this system numerically for the associated equilibrium outcome, when it exists.

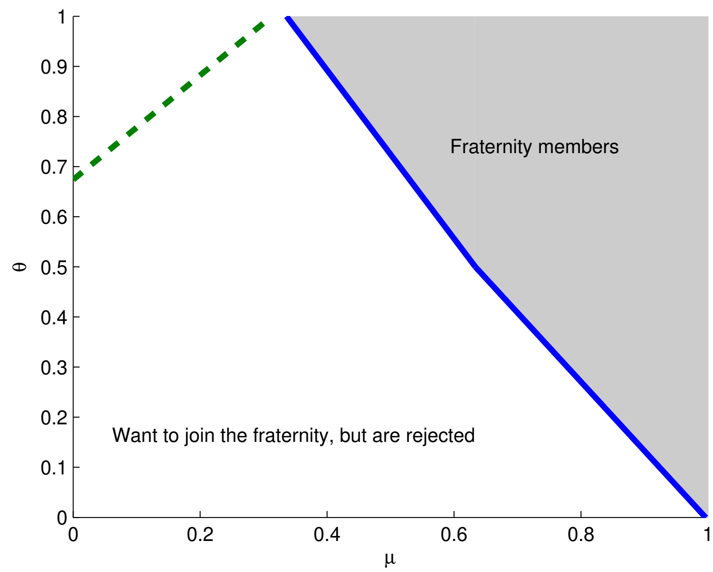

Figure 4: Application-unconstrained equilibrium.

In our base parameterization, student utilities are $M + n\mu$, with $n = 0.18$, student time costs of participating in the fraternity are $c = 0.09$, the fraternity trades off between wages and socializing value according to $\frac{W_1}{W_2} = 1.1$, and the fraternity's capacity is $\Gamma = 0.35$. Figure 4 illustrates the unique "application-unconstrained" equilibrium. In this equilibrium firms have optimistic beliefs about the productivities of fraternity members, so that given any signal emitted by a student, his wage is higher if he is a member of a fraternity than if he is a non-member. As a result, in this equilibrium every student whom the fraternity would like to admit chooses to apply—and, indeed, because $w_C(H) = 0.8480 > w_{\overline{C}}(H) = 0.8143$, only very productive people with especially low socializing values choose not to apply (and while less productive students apply, most are rejected).

However, for exactly the same parameterization, there is also an "application constrained" equilibrium in which some students whom the fraternity would like to admit do not apply.

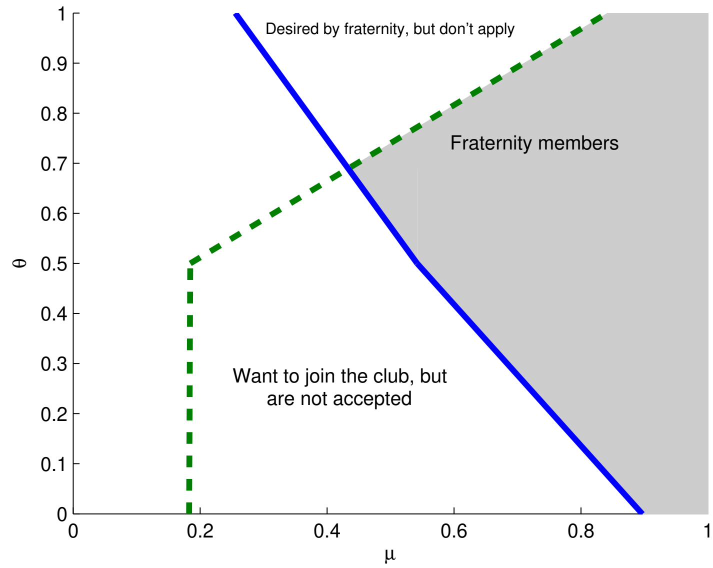

Figure 5: Application constrained fraternity game equilibrium.

Figure 5 depicts this equilibrium: the solid line denotes the fraternity's cutoff rule, and dashed line denotes locus of students who are indifferent between joining the fraternity and not. In this equilibrium, firms hold more pessimistic beliefs about the abilities of fraternity members, so that higher ability students are more reluctant to join the fraternity. Intermediate quality students remain eager to join, and the fraternity's composition is radically shifted to reflect this population. Comparing the fraternity's cutoff line in Figure 4 with that in Figure 5 reveals that the fraternity is less "picky" when its choice set is constrained by the reluctance of able students to apply. Because able types $\theta = 1$ expect lower wages inside the fraternity than out, $w_C(H) = 0.7940 < w_{\bar{C}}(H) = 0.8555$, the fraternity attracts only a small fraction of able students, and the bulk of its members have intermediate abilities.

Figure 6 presents the expected wage that a student with ability θ would receive as a fraternity member and non-member for these two equilibria. Notice the crossing of wages in the application-constrained equilibrium. This reflects that while all lower ability student receive higher wages as fraternity members, higher θ students in the application-constrained equilibrium accept a direct loss in wage by joining the fraternity, for which they are compensated by high socializing values.

Note that in the application-unconstrained equilibrium, were we to increase Γ slightly,

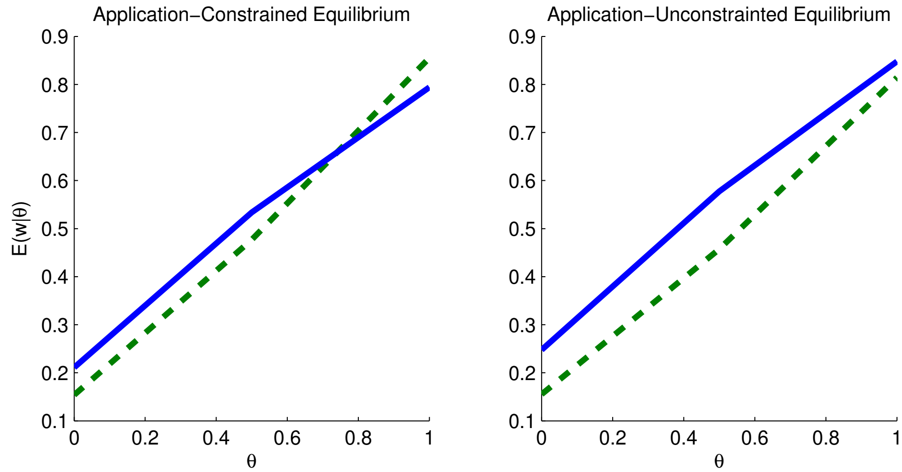

Figure 6: Equilibrium Wages. The solid line is the expected wage of a fraternity member (as a function of θ), and the dashed line is the expected wage of a nonmember.

then all existing members of the fraternity would still apply, and the fraternity's payoff would be increased. This observation implies that were we to replace the fraternity's capacity constraint with a strictly convex cost function of admitting more members, the fraternity would admit more members when beliefs of firms about member abilities are optimistic, thereby encouraging able students to apply.

Figure 7 reveals how the fraternity's capacity affects equilibrium outcomes. Interestingly, raising capacity can *raise* the wages of able fraternity members. Essentially, when Γ is increased, the mix of students that the fraternity admits shifts slightly toward more able students with lower socializing values, i.e., toward students with higher θs and lower values of μ. But this raises the expected wages of able students who join the fraternity. But then, able students are more willing to join—there is a significant increase in the measure of able students who apply to the fraternity. Notice also that as Γ increases, the slope of the boundary characterizing the application decision of less able students with $\theta \in [0, \frac{1}{2}]$ changes. This result reflects a change in the relative slope of wage functions: when, among students with $\theta \in [0, \frac{1}{2}]$, most of those with relatively high productivities are in the fraternity, then receiving the signal M and being outside the fraternity has a smaller premium than being in the fraternity and getting signal M (relative to receiving signal L in both cases).

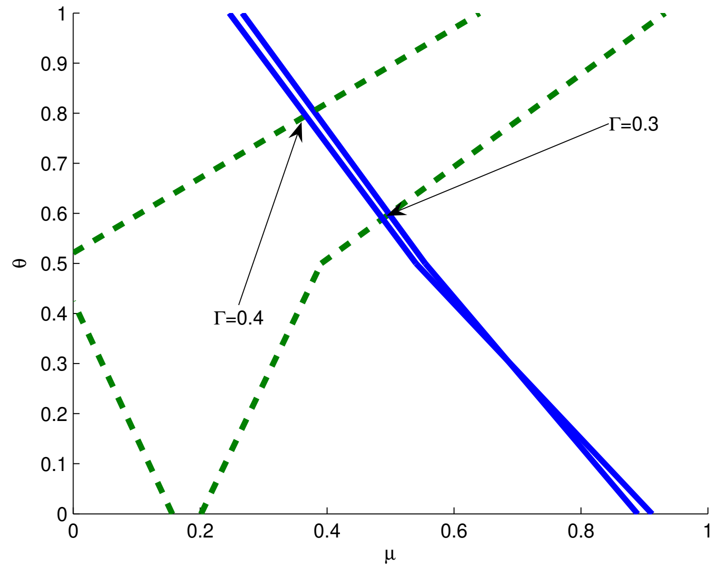

Figure 7: Application-constrained equilibrium for different capacities; arrows denote the intersection of fraternity's and students' choices in corresponding equilibrium.

4 Hump-Shaped Equilibria

Our three-signal setting shows that one possible equilibrium fraternity composition (see e.g., Figure 5) is where membership is "hump-shaped" in student ability. Empirically (see Figure 8), we will see that conditioning on student ability, among sufficiently able students, the percentage who are members is a declining function of ability. This makes it important to understand when and how the hump-shaped equilibrium emerges in a more general setting, and, in particular, to ensure that it is not the three-signal setting that underlies the hump-shaped outcome. Accordingly, we now identify sufficient conditions under which the hump-shape is a characteristic of the fraternity equilibrium.

We say that an equilibrium set of fraternity members is *hump-shaped* if:

1. $A \cap B \neq B$ and $A \cap B \neq \emptyset$.

2. The students' acceptance threshold $\mu_A(\theta)$ is increasing in θ for $(\theta, \mu_A(\theta)) \in B$.

The first condition says that there are students who would be accepted by the fraternity, $(\theta, \mu) \in B$, but choose not to apply. When the fraternity only values the future wages of its members, the first condition always holds as long as the costs c of joining are neither too small (possibly negative) that even the students with the lowest socializing values want to join, nor so large that even the students with the highest socializing values do not want to join. When the fraternity also values the socializing values of its members by enough that it does not accept the most able student with the least socializing value, then the necessary lower bound on c is higher.

We begin by showing that if the wage premium from membership in the fraternity falls with higher signals (recall, for example, Figure 6), then more able students are more reluctant to join the fraternity.

Lemma 2 *If $w_C(\tilde{\theta}) - w_{\bar{C}}(\tilde{\theta})$ is a decreasing function of $\tilde{\theta}$ then the students' acceptance threshold $\mu_A(\theta)$ is an increasing function of θ.*

Proof. See the Appendix. ■

We next establish conditions under which the hump-shaped equilibrium emerges when the fraternity only cares about the wages of its members, and not their socializing values. Then, it follows that the fraternity admits every students whose ability exceeds some cutoff, θ_0, and further that as capacity Γ falls, θ_0 rises. We make two weak assumptions. The first is technical in nature, and the second says that the cost of membership is such that students with the highest socializing value want to join, but those with the lowest socializing value do not.

Assumption 1 *Either the support for signals $\tilde{\theta}$ is finite, or the support of $f_{\tilde{\theta}}(\tilde{\theta}|\bar{\theta})$ is non-trivial.*

Assumption 2 *Suppose that the cost c of joining the fraternity satisfies*

$$n\underline{\mu} + \bar{\theta} - E[\theta] < c < n\bar{\mu} + \bar{\theta} - E[\theta].$$

Proposition 7 *Suppose that Assumptions 1 and 2 hold and the fraternity does not care about the socializing values of its members. Then when the fraternity is small enough, the equilibrium is hump-shaped. That is, there exists a $\underline{\Gamma} > 0$, such that for all $\Gamma < \underline{\Gamma}$, any non-trivial equilibrium is hump-shaped.*

Proof. See the appendix. ■

The role of $\bar{\Gamma}$ is to provide a gross sufficient condition for a declining wage premium for fraternity membership—it ensures that the fraternity is sufficiently picky. The lowest ability fraternity members always receive a large wage premium, as their membership ensures that they are separated away from all lower ability types, and are mixed in with relatively higher proportions of more able types. The smaller is Γ, the higher is the wage premium for a given low ability type, due to the increased filtering out of lower ability students by the fraternity. High ability types gain less, because they only benefit from separation from low ability non-members, whom they may be unlikely to be confused with (as they are unlikely to generate the signals sent by low ability students), and fraternity membership lumps them in with intermediate ability students. When costs of membership are of an appropriate magnitude, they cause higher ability students to become increasingly reluctant to join, giving rise to the hump-shaped equilibrium. That is, the fraternity's filtering eliminates higher proportions of lower ability students, while on the high ability end, increasing proportions of higher and higher ability students choose not to join to avoid the increasing wage "penalty" for membership.

The logic of the proof of Proposition 7 extends immediately to settings where the fraternity cares about socializing values, so that $\mu_B(\theta)$ is a decreasing function of θ, but not so much that $\mu_B(\bar{\theta}) < \underline{\mu}$. As the weight W_2 that the fraternity places on socializing values increases, the analysis follows directly, albeit inelegantly, if we replace Assumption 2 with an assumption that we write implicitly in terms of equilibrium values:

Assumption 3 *Suppose that the cost c of joining the fraternity satisfies*

$$n\mu_B(\bar{\theta}) + \bar{\theta} - E[\theta] < c < n\bar{\mu} + \bar{\theta} - E[\theta].$$

In essence, if the fraternity places a sufficient weight on socializing values, it will not admit high ability students who have low socializing values, so it may already be filtering out the set of high ability students who are reluctant to join. As a result, a higher cost of membership may be required to support a hump-shaped equilibrium.

Proposition 8 *Suppose that the fraternity places a positive weight on both socializing values and wages ($W_1, W_2 > 0$). Then when Assumptions 1 and 3 hold and the fraternity is small* *enough, the equilibrium is hump-shaped. That is, there exists a* $\underline{\Gamma} > 0$*, such that for all* $\Gamma < \underline{\Gamma}$*, any non-trivial equilibrium is hump-shaped.*

Proof. The argument follows directly along the lines of the proof of Proposition 7, with the added structure on c guaranteeing that $\bar{\mu} > \mu_A(\bar{\theta}) > \mu_B(\bar{\theta})$. ■

Summing up, the hump-shaped nature of the equilibrium is generated not by any particular specification of the signal structure that must be imposed to solve explicitly for equilibrium. Rather, the hump-shaped property derives only from the monotone likelihood property of the distribution of signals that firms receive about students combined with the conflicting interests of students and the fraternity that necessarily emerges whenever the fraternity is sufficiently selective and membership is costly.

- The monotone likelihood ratio property means that when the fraternity filters out low ability students, it is lower ability student types who gain more from fraternity membership, as most people who would generate low signals are rejected by the fraternity.

- The filtering out of low ability students by the fraternity, combined with the fraternity's trade off between future earnings and socializing values, initially leads to student participation in the fraternity being an increasing function of ability (for low θ types).

- The MLRP signal structure implies that higher and higher ability students gain less and less, or are even hurt in terms of wages by fraternity membership due to mixing in with less able students; and a cost-benefit calculation eventually causes higher ability students to become increasingly reluctant to join. This implies that when Γ is small, student participation in the fraternity eventually declines in ability.

5 Empirical Analysis

To see whether our model can reconcile the actual application and selection process of fraternities, we obtained data on the cumulative GPAs of the 8634 seniors at the University of Illinois in the fall semester of 2007 (excluding international students on temporary visas), and a random sampling of 701 seniors who were fraternity or sorority members. GPAs only

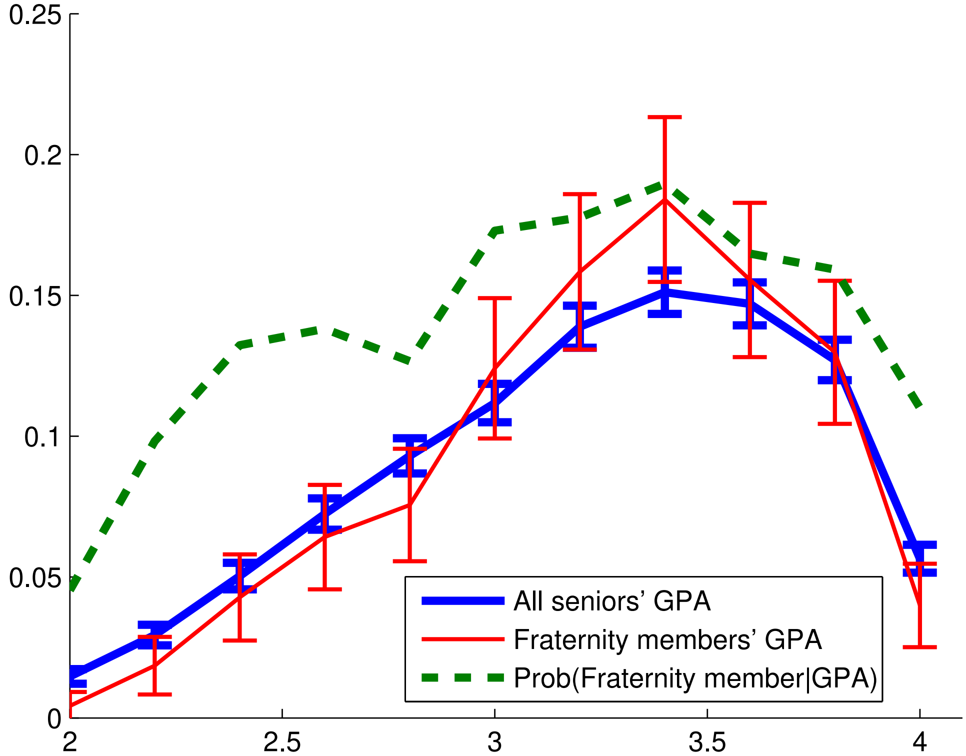

Figure 8: Conditional Distributions of GPAs

NOTES: The dashed line is the conditional probability that a student is a fraternity member given his or her GPA (rounded to the nearest 0.2). The thin line graphs the distribution of GPAs for 701 seniors who are fraternity members (fall 2007), and the thick line graphs the probability distribution for all 8634 seniors at the University of Illinois (fall 2007). The bars indicate 2 standard deviation confidence intervals.

reflect courses taken at the University of Illinois (i.e., omitting transferred courses), but the senior classification is based on all hours accumulated prior to the end of the fall, 2007 term.

Figure 8 presents the conditional probability that a student is a member of a fraternity given his or her GPA. Figure 8 reveals that the conditional probability that a student with a low GPA of 2.0 is a fraternity member is less than 0.05, but that this probability more than triples for intermediate GPAs between 3 and 3.4, before falling by more than a third for students with high GPAs. Interpreting a student's GPA as a noisy indicator of his or her ability, this inverted U-shaped pattern is precisely what emerges in the equilibrium to our fraternity game where able students are reluctant to join fraternities to avoid being tainted in labor market outcomes, while intermediate and less able students are eager to join, and the fraternity screens out most of the less able students, i.e., those who do not have high socializing values.

This finding suggests that it is plausible to estimate our three-signal model formally, and to extract the implications of our structural estimates for student welfare. We illustrate our model's potential by exploiting our limited data on grades and fraternity membership as far as possible. To be consistent with the premises of our three-signal model, we assume that θ corresponds to the quantile of the GPA distribution so that θ is distributed uniformly; and that fraternity-socializing values, μ, are independently and uniformly distributed. Fixing ability, our model indicates that fraternities admit higher μ types, allowing us to estimate $1 - \mu(\theta)$. In particular, letting Φ be the event that a randomly-selected person is in the fraternity, the fraction of students with ability θ who are in the fraternity is

$$1 - \max(\mu_A(\theta), \mu_B(\theta)) = P(\Phi|\theta) = \Pr(\Phi)\frac{f_\theta(\theta|\Phi)}{f_\theta(\theta)},$$

where $\mu_A(\theta)$ and $\mu_B(\theta)$ are the cutoff rules of students and fraternity, $f_\theta(\cdot)$ is the density of θ, and $\Pr(\Phi)$ is the probability that a randomly-selected senior is a member of the fraternity. Our estimate of $\Pr(\Phi)$ is $\frac{1345}{8634}$, the number of senior fraternity members divided by the number of seniors. To estimate $f_\theta(\cdot)$ we use the sample of all senior students with GPAs of at least 2,[8] and we use the sample of fraternity members' GPAs exceeding 2 to estimate $f_\theta(\cdot|\Phi)$.[9] The densities are estimated using a kernel estimator, and we use them to smooth the conditional probability of being in the fraternity conditional on GPA (the dashed line in Figure 8). We then take 20 equally spaced θs between 0.05 and 0.95 as our pseudosample, and evaluate $\hat{\mu}(\theta) = \max(\mu_A(\theta), \mu_B(\theta))$ from our smoothed conditional probability estimator at these points. They are represented by dots in Figure 9.

The boundary in our model describing the set of fraternity members is piecewise linear with two kinks, one at $\theta = 0.5$, and another that we estimate. We first do this non-structurally, simply finding the slopes and intercepts of the three line segments, and the second kink position by minimizing the SSE, without imposing the equilibrium consistency requirements (i.e., consistency of firm's beliefs over the distribution of fraternity member

[8]We drop the few students with GPAs below 2, as they are subject to screening by the University (and, indeed, only students with GPAs of at least 2 can graduate).

[9]We caution that in this exercise, we are treating GPA as ability, rather than as a noisy signal of ability; however, since we aggregate individual observations and only use the aggregate distribution in our estimation, it follows that estimates are qualitatively unaffected if the distribution of noisy signals closely corresponds to the true distribution of ability.

types with the set of fraternity members implied by the estimates). To do this, we use a two-step estimation procedure to estimate the kink. We first take a possible kink value as given, and find the slopes of the cut-off rules that minimize the SSE; our estimate of the kink location minimizes the SSE overall.

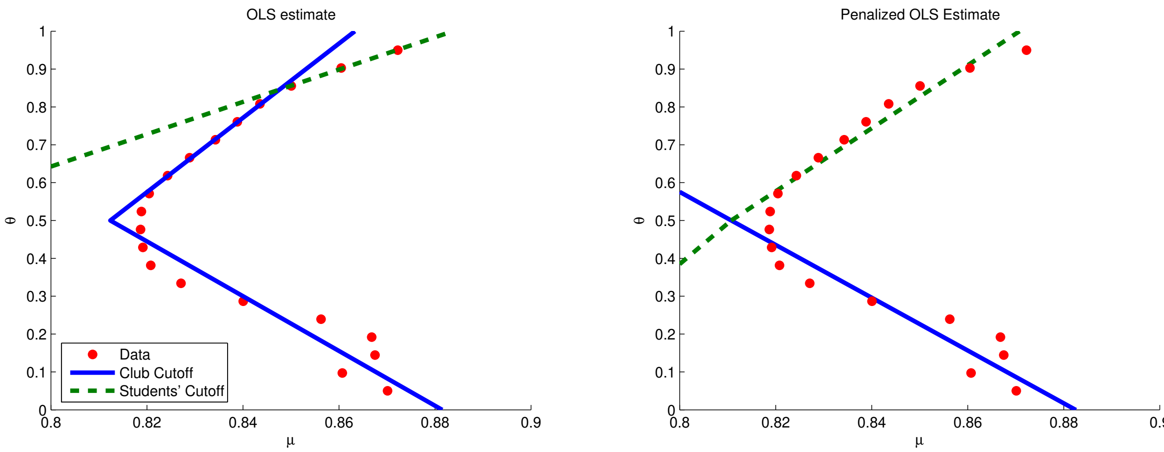

Figure 9: Estimated Unconstrained Model and Structural Model Fits

Next, we contrast this non-structural estimate with that obtained when we penalize cut-off rules that are inconsistent with equilibrium. Direct structural estimation is complicated by the extreme nonlinearity of the equilibrium requirement. This leads us to adopt a lasso-type estimation approach, in which we minimize the residual sum of squares plus a quadratic measure of the distance from the equilibrium. In equilibrium, $\frac{W_1}{W_2} = \frac{b_1}{2(w_C(M)-w_C(L))} = \frac{b_2}{2(w_C(H)-w_C(M))}$, where b_1 is the slope of the club's cutoff rule below $\theta = 0.5$, and b_2 is the slope for $\theta > 0.5$. We use the penalty function

$$10\left[b_1(w_C(H) - w_C(M)) - b_2(w_C(M) - w_C(L))\right]^2.$$

The first panel of Figure 9 presents the estimated cut-off rules from unconstrained estimation approach. This fit is far from an equilibrium; most obviously, the cut-off rule for the club is not a monotonically decreasing function of θ. The panel on the right presents the estimated cut-off rules from the penalized estimation approach. The value of the penalty is less than 10^{-10}, indicating that the estimated model is very close to an equilibrium model. An F-test[10] indicates that the differences between the structural and non-structural models are not statistically significant.

[10]The F-value is 2.93, (1,13) degrees of freedom, p-value of 0.11; we are omitting considerations of the

The penalized estimation approach implies estimates of the primitives for students: the cost c is 0.23, or about 46% of the unconditional expected wage, and the fraternity socializing parameter is $n = 0.28$. We bootstrap the estimator to obtain 95% confidence intervals. While the confidence intervals for n and c are wide ($[0.11, 0.52]$ and $[0.08, 0.44]$, respectively), the fraction $\frac{c}{n} = 0.82$ has a tight 95% confidence interval of $[0.73, 0.84]$; and it is this ratio that determines whether a student gains a net utility benefit from joining the fraternity in a full information setting where firms know a student's ability. Figure 9 shows that most fraternity members are above this threshold, i.e., most fraternity members have socializing values of $\mu > \frac{c}{n} \approx 0.82$. Thus, relative to a full information setting, the wage-setting mechanism impedes the efficiency of club participation: there are too many fraternity members with intermediate abilities and too few low and high ability students with high socializing values. Our estimate of the club's relative weighting on member wages versus socializing values, $\frac{W_1}{W_2}$ of 0.22 has a wide confidence interval of $[0.06, 0.33]$, but the fraternity's capacity is precisely estimated (95% confidence interval of [15.44%, 15.76%]).

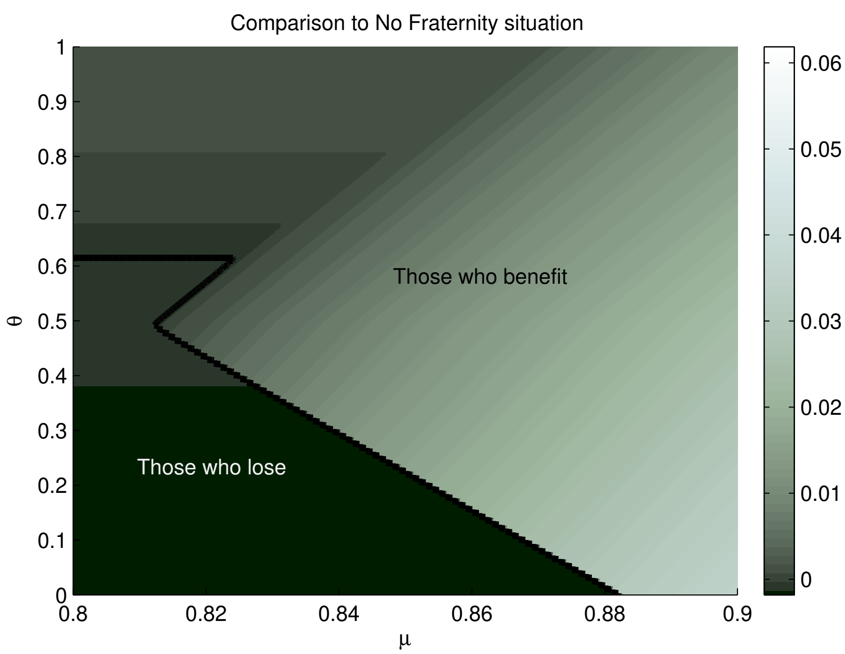

Figure 10: Impact of Fraternity on Student Welfare

Our estimates allow us to explore how the presence of the fraternity affects the welfare of

non-normality of errors and the nonlinearity of both the model and restrictions. The asymptotic distribution of this test is $\chi^2(1)$, with a p-value of 0.08.

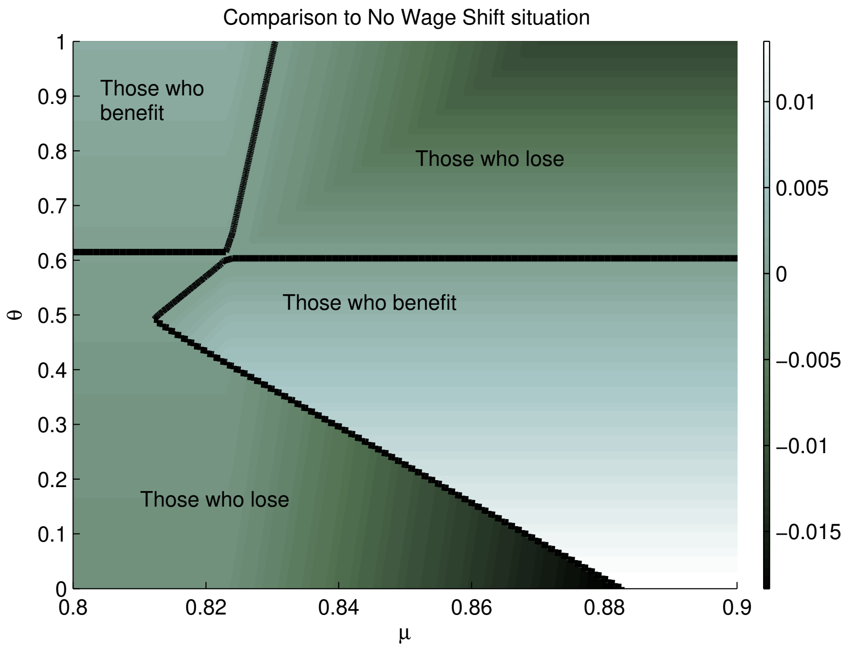

Figure 11: Impact of Fraternity's Wage Redistribution on Student Welfare

different student types. Figure 10 presents welfare gains and losses of different student types relative to a setting in which there is no fraternity [or equivalently relative to the "empty" fraternity equilibrium]. The solid line divides the population of student types into those who benefit and those who are hurt, and the darker is the shade in the figure, the more the fraternity's presence hurts/benefits less a student. The figure reveals that all fraternity members actually are made better off by the presence of the fraternity, gaining from the socializing values of fraternity membership. In addition, able types, $\theta > 0.61$ who are not members gain because they receive higher market wages—firms believe that most highly productive types do not join the fraternity.

Figure 11 contrasts student welfare in the equilibrium with that which obtains when firms ignore the information in fraternity membership, or equivalently where the fraternity does not have better information about θ than firms, and hence only weighs socializing values in admission. Two groups of student types benefit when the fraternity weighs both expected wage/ability and socializing values in admission: (i) low ability types with high enough socializing values that they are admitted benefit from wage gain associated with being mixed in with more able types; and (ii) high ability types with lower socializing values who do not join benefit from the higher wages due to the partial separation from mediocre ability types generated by the fraternity admission process. Low ability types with moderately high socializing values are hurt the most, as they both would gain socializing values were the fraternity not to weigh ability, and they are punished by lower wages due to their exclusion. The other group hurt consists of high ability/high socializing value students who join in both environments, but are tarred by association with lesser types when the fraternity weighs ability in admission, and therefore receive lower wages.

One should recognize caveats with our empirical analysis. In particular, one would like better measures of ability. For example, an alternative explanation of the empirical relationship between GPA and fraternity membership that we document is that not only does ability influence GPAs, but so does fraternity membership—fraternity cheat sheets may help low ability students, but a fraternity party environment may make it difficult for high ability students to study. Ideally, one would obtain measures of ability such as high school grade or ACT scores that are not affected by fraternity membership. We also note that to show that the ability distribution in the fraternity that we obtain theoretically is not driven by direct factors, we assume that firms do not value fraternity socializing values and that socializing values are uncorrelated with ability. Still, our empirical finding that high GPA students are reluctant to join fraternities indicates that, in practice, firms cannot value those skills by too much, and that the correlation between ability and socializing value cannot be too high.

6 Conclusion

On first impression, it is not clear why fraternity or sorority membership should matter for labor market outcomes—fraternity activities seem to have little to do with skill development for future careers. Nonetheless, resumes regularly highlight fraternity membership, suggesting that membership augments the other signals that employers use to evaluate a person's productivity. Our paper provides insights into when fraternity membership matters for labor market outcomes. We first show that if firms can either evaluate student productivities perfectly, or are completely incapable of screening job applicants, then fraternity membership has no impact on labor market outcomes. Otherwise, fraternity membership matters. In particular, we identify two equilibria in which fraternity membership is valued by some students for labor market outcomes. In one equilibrium, optimistic beliefs by firms about the abilities of fraternity members lead to higher wages for fraternity members than non-members. As a result, everyone whom the fraternity would like to admit chooses to pledge. We also identify an equilibrium in which able students are harmed in the labor market by fraternity membership, but less able students benefit. In this equilibrium, most fraternity members have intermediate abilities: less able students apply, hoping to be mixed in with better students, but are rejected unless they have high fraternity socializing values, while very able students who lack high socializing values do not apply to avoid being tainted in labor market outcomes due to being mixed in with less able fraternity members. We find that this latter equilibrium can reconcile the qualitative features of the ability distributions of fraternity members and non-members at the University of Illinois.

While we pose our analysis in the context of fraternities, the central economic story extends with some variations to filtering by other organizations. For example, ROTC (reserve officer training corps) may value both intellectual ability and leadership skills that firms value, but also physical fitness that does not contribute productively in many occupations. As a result, even were ROTC not to directly build skills of its officers, our model indicates that firms may rationally weigh ROTC membership positively in their evaluations of job-seekers.

7 Appendix

Proof of Proposition 3: First observe that in light of Proposition 2, if (θ, μ) applies to the fraternity in equilibrium, then so does (θ, μ'). Suppose the proposition were false. Then for $\varepsilon > 0$, sufficiently small, the set

$$\Theta_\varepsilon = \{\theta | \exists z_\theta : \int_{-\infty}^{z_\theta} a(\theta, x) b(\theta, x) h_\mu(x) dx > \varepsilon, \int_{z_\theta}^{+\infty} (1 - b(\theta, x)) \, h_\mu(x) dx > \varepsilon\}$$

has positive measure, i.e., there exists $\delta > 0$ such that $\int_{\Theta_\varepsilon} h_\theta(x) dx > \delta$. For every θ in Θ_ε pick a set of $K = \{(\theta, \mu) | \mu < z_\theta, a(\mu, \theta) b(\mu, \theta) = 1\}$ and $L = \{(\theta, \mu) | \mu > z_\theta, b(\mu, \theta) = 0\}$ such that $\int_K a(\theta, \mu) b(\theta, \mu) h_\mu(x) dx = \int_L a(\theta, \mu)(1 - b(\theta, \mu)) h_\mu(x) dx = \varepsilon$. But then the fraternity decision rule

$$\hat{b}(\theta, \mu) = b(\theta, \mu)(1 - I((\theta, \mu) \in K)) + I((\theta, \mu) \in L)$$

strictly raises the fraternity's payoff as $E(\mu|K) < E(\mu|L)$ and the expected wages generated by members and fraternity size are unchanged. Therefore, b could not have been an equilibrium strategy for the fraternity. $\square$

Proof of Lemma 1: Consider two signals, $x > y \in \tilde{\Theta}$, and two productivities, $\theta_2 > \theta_1 \in \Theta$, such that (x, θ_1), (x, θ_2) and $(y, \theta_2) \in D$. By the MLRP property,

$$\frac{f(\theta_2|x)}{f(\theta_1|x)} > \frac{f(\theta_2|y)}{f(\theta_1|y)}.$$

Notice that for every $(j, k) \in D$, $f(j|k, D) = \frac{f(j|k) I((j,k) \in D)}{\int_\Theta I((\theta, \tilde{\theta}) \in D) dF(\theta|\tilde{\theta})} = f(j|k) \frac{I((j,k) \in D)}{P(D)}$. Rewrite the MLRP condition:

$$\frac{f(\theta_2|x, D)}{f(\theta_1|x, D)} = \frac{\frac{f(\theta_2|x) I((\theta_2, x) \in D)}{P(D)}}{\frac{f(\theta_1|x) I((\theta_1, x) \in D)}{P(D)}} = \frac{\frac{f(\theta_2|x)1}{P(D)}}{\frac{f(\theta_1|x)1}{P(D)}} > \frac{\frac{f(\theta_2|y)1}{P(D)}}{\frac{f(\theta_1|y)1}{P(D)}} \geq \frac{\frac{f(\theta_2|y) I((\theta_2, y) \in D)}{P(D)}}{\frac{f(\theta_1|y) I((\theta_1, y) \in D)}{P(D)}} = \frac{f(\theta_2|y, D)}{f(\theta_1|y, D)}.$$

Therefore, if the MLRP condition holds for the entire support, it holds for a subset D of that support. This condition ensures that $E(\theta|\tilde{\theta}, D)$ is an increasing function of $\tilde{\theta}$. By $F(x|\theta_2, D) \succeq F(x|\theta_1, D)$,

$$\begin{aligned} E_{\tilde{\theta}}[E(\theta|\tilde{\theta}, D)|\theta_2, D] &= \int_{\tilde{\Theta}} E(\theta|\tilde{\theta}, D) dF(\tilde{\theta}|\theta_2, D) \\ &> \int_{\tilde{\Theta}} E(\theta|\tilde{\theta}, D) dF(\tilde{\theta}|\theta_1, D) = E_{\tilde{\theta}}[E(\theta|\tilde{\theta}, D)|\theta_1, D]. \quad \square \end{aligned}$$

Proof of Proposition 5: We first show that if $n\bar{\mu} - c \geq \underline{w} - \underline{\theta}$, then the empty fraternity cannot be an equilibrium. If no one joins the fraternity, then equilibrium demands that $\underline{\theta}$ expect wage $\underline{w}$ if he does not join; and the expected wages of students with ability greater than $\underline{\theta}$ who do not join exceed $\underline{w}$. With the full support assumption, following any signal realization $\tilde{\theta}$, firms can hold equilibrium beliefs that the anyone who joins the fraternity and generated that signal has ability $\underline{\theta}$. These beliefs minimize the wage of any student who joins the fraternity. Given these beliefs, since expected wages are continuous in θ, if $n\bar{\mu} - c + \underline{\theta} > \underline{w}$, then all students in a sufficiently small neighborhood of $(\underline{\theta}, \bar{\mu})$ would apply for fraternity membership, and since their measure is less than Γ, the fraternity would accept them. Hence, the empty fraternity cannot be an equilibrium.

Conversely, if $n\bar{\mu} - c \leq \underline{w} - \underline{\theta}$, then given the pessimistic beliefs by firms, $w_C(\tilde{\theta}) = \underline{\theta}$ so that $(\underline{\theta}, \bar{\mu})$ at least weakly prefers not to apply to the fraternity; and all other types strictly prefer not to apply. Hence, no one applying to the fraternity is an equilibrium. $\square$

Proof of Proposition 6: To prove existence, it suffices to characterize student and fraternity choices via the continuous functions $\mu_A(\cdot)$ and $\mu_B(\cdot)$ (see Propositions 2 and 4), proving the existence of an equilibrium in which a student type (θ, μ) is a member of the fraternity if and only if $\mu \geq max\{\mu_A(\theta), \mu_B(\theta)\}$. In particular, given $w_C(\cdot)$ and $w_{\bar{C}}(\cdot)$, $\mu_A(\theta)$ solves equation (2) at equality, for $\mu_A(\theta) \in (\underline{\mu}, \bar{\mu})$. Since $\mu_A(\cdot)$ is uniquely defined, it follows that $\mu_B(\cdot)$ is uniquely defined. We have established that $\mu_j : [\underline{\theta}, \bar{\theta}] \to [\underline{\mu}, \bar{\mu}]$, $j = A, B$, is continuous. The space of such functions, endowed with the weak* topology, is compact. So, too, we can focus on beliefs by firms about which student types are fraternity members that are summarized by continuous functions $\hat{\mu}_A(\cdot)$ and $\hat{\mu}_B(\cdot)$ about which student types apply and which ones are accepted by the fraternity, where (θ, μ) is a conjectured fraternity member if and only if $\mu \geq max\{\hat{\mu}_A(\theta), \hat{\mu}_B(\theta)\}$. These beliefs, $\hat{\mu}_A(\cdot), \hat{\mu}_B(\cdot)$ then determine competitive wage functions,

$$(\hat{w}_C(\tilde{\theta}),\ \hat{w}_{\overline{C}}(\tilde{\theta})) = (E[\theta|\tilde{\theta}, \mu \geq max\{\hat{\mu}_A(\theta), \hat{\mu}_B(\theta)\}\}],\ E[\theta|\tilde{\theta}, \mu < max\{\hat{\mu}_A(\theta), \hat{\mu}_B(\theta)\}]),$$

and these wage functions, in turn, imply optimal student and fraternity best response choices, $\mu_A(\cdot), \mu_B(\cdot)$. Hence, we have a mapping from $(\hat{\mu}_A(\theta), \hat{\mu}_B(\theta))$ to $(\mu_A(\theta), \mu_B(\theta))$. Equilibrium is given by a fixed point to this mapping from [conjectured by firms] optimal student and fraternity choices to the best response optimal student and fraternity choices; and we have just established that this mapping satisfies the conditions of Kakutani's fixed point theorem. $\square$

Proof of Lemma 2: $\mu_A(\theta)$ solves

$$E(w_C(\tilde{\theta})|\theta) + n\mu_A(\theta) - c = E(w_{\bar{C}}(\tilde{\theta})|\theta) \;\Leftrightarrow\; E(w_C(\tilde{\theta})|\theta) - E(w_{\bar{C}}(\tilde{\theta})|\theta) = c - n\mu_A(\theta).$$

Then $\mu_A(\theta)$ is increasing in θ if and only if

$$\frac{\partial}{\partial\theta}E(w_C(\tilde{\theta}) - w_{\bar{C}}(\tilde{\theta})|\theta) = \frac{\partial}{\partial\theta}\int_{-\infty}^{\infty} w_C(\tilde{\theta}) - w_{\bar{C}}(\tilde{\theta}) f_{\tilde{\theta}}(\tilde{\theta}|\theta)d\tilde{\theta} < 0.$$

Consider $g(\tilde{\theta}|\theta) \equiv \frac{\partial}{\partial\theta}f_{\tilde{\theta}}(\tilde{\theta}|\theta)$. As $g(\tilde{\theta}|\theta)$ is the change in the distribution of signal $\tilde{\theta}$ due to a change in θ, the integral of g over the support of $\tilde{\theta}$ is zero. The MLRP assumption implies that $g(\tilde{\theta}|\theta)$ is increasing in $\tilde{\theta}$:

$$\text{MLRP:}\frac{f_{\tilde{\theta}}(t_2|\theta+\Delta_\theta)}{f_{\tilde{\theta}}(t_2|\theta)} > \frac{f_{\tilde{\theta}}(t_1|\theta+\Delta_\theta)}{f_{\tilde{\theta}}(t_1|\theta_1)}\ \forall t_2 > t_1,\ \Delta_\theta > 0 \text{ implies}$$

$$\frac{1}{f_{\tilde{\theta}}(t_2|\theta)}\frac{\Delta f_{\tilde{\theta}}(t_2|\theta)}{\Delta_\theta} > \frac{1}{f_{\tilde{\theta}}(t_1|\theta_1)}\frac{\Delta f_{\tilde{\theta}}(t_1|\theta)}{\Delta_\theta}$$

Therefore,

$$\frac{\partial}{\partial\theta}\ln f_{\tilde{\theta}}(t_2|\theta) \geq \frac{\partial}{\partial\theta}\ln f_{\tilde{\theta}}(t_1|\theta)$$

implying that $g(\tilde{\theta}|\theta)$ is nondecreasing in $\tilde{\theta}$; and since g integrates to zero, there exists a K such that $g(\tilde{\theta}|\theta) \leq 0$ for $\tilde{\theta} \leq K$, and $g(\tilde{\theta}|\theta) \geq 0$ for $\tilde{\theta} > K$.

Remember, we want to establish when

$$\frac{\partial}{\partial\theta}\int_{-\infty}^{\infty}\left[w_C(\tilde{\theta}) - w_{\bar{C}}(\tilde{\theta})\right] f_{\tilde{\theta}}(\tilde{\theta}|\theta)d\tilde{\theta} = \int_{-\infty}^{\infty}\left[w_C(\tilde{\theta}) - w_{\bar{C}}(\tilde{\theta})\right]\frac{\partial f_{\tilde{\theta}}(\tilde{\theta}|\theta)}{\partial\theta}d\tilde{\theta} < 0$$

Subtracting $[w_C(K) - w_{\bar{C}}(K)]\int_{-\infty}^{\infty}\frac{\partial f_{\tilde{\theta}}(\tilde{\theta}|\theta)}{\partial\theta}d\tilde{\theta} = 0$ from the integral and breaking the multiplicands under the integral into two parts yields

$$\int_{-\infty}^{\infty}\left(\left(w_C(\tilde{\theta}) - w_{\bar{C}}(\tilde{\theta})\right) - (w_C(K) - w_{\bar{C}}(K))\right)\frac{\partial f_{\tilde{\theta}}(\tilde{\theta}|\theta)}{\partial\theta}d\tilde{\theta}$$

$$= \int_{-\infty}^{K}\left(\left(w_C(\tilde{\theta}) - w_{\bar{C}}(\tilde{\theta})\right) - (w_C(K) - w_{\bar{C}}(K))\right)g(\tilde{\theta}|\theta)d\tilde{\theta}$$

$$+ \int_{K}^{\infty}\left(\left(w_C(\tilde{\theta}) - w_{\bar{C}}(\tilde{\theta})\right) - (w_C(K) - w_{\bar{C}}(K))\right)g(\tilde{\theta}|\theta)d\tilde{\theta}.$$

From the premise that $w_C(x) - w_{\bar{C}}(x)$ is decreasing, the difference in the w terms in the first integral is positive, and negative in the second; by construction, the g term in the first integral is negative, and positive in the second integral. Therefore, the integral is the sum of two negative values.

Finally, inspection reveals that an analogous argument holds if there are a finite number of signals, $\tilde{\theta}$, interpreting the signal density $f_{\tilde{\theta}}(\tilde{\theta}|\theta)$ as the probability mass on signal $\tilde{\theta}$. $\square$

Proof of Proposition 7: Denote the set of students with $\theta < \theta_0$ as P, the set of students in fraternity as P_2, and the rest P_1; and let $m(\cdot)$ be the measure of students in the argument set. Then

$$w_C(\tilde{\theta}) = E(\theta|\tilde{\theta}, P_2) \qquad \text{and} \qquad w_{\bar{C}}(\tilde{\theta}) = E(\theta|\tilde{\theta}, P \cup P_1).$$

Rewrite $w_{\bar{C}}$ as

$$w_{\bar{C}}(\tilde{\theta}) = \frac{m(P)}{m(P)+m(P_1)} E(\theta|\tilde{\theta}, P) + \frac{m(P_1)}{m(P)+m(P_1)} E(\theta|\tilde{\theta}, P_1).$$

Take two signals, H and L, with $H > L$. Then the expected wage premium is decreasing if

$$\begin{aligned} & E(\theta|H, P_2) - \left[\frac{m(P)}{m(P)+m(P_1)} E(\theta|H, P) + \frac{m(P_1)}{m(P)+m(P_1)} E(\theta|H, P_1)\right] \\ < \ & E(\theta|L, P_2) - \left[\frac{m(P)}{m(P)+m(P_1)} E(\theta|L, P) + \frac{m(P_1)}{m(P)+m(P_1)} E(\theta|L, P_1)\right]. \end{aligned} \tag{3}$$

Observe that $\theta_0 \to \bar{\theta}$ implies:

$$\begin{aligned} E(\theta|H, P_2) \to \bar{\theta} \qquad \text{and} \qquad E(\theta|L, P_2) &\to \bar{\theta} \\ \frac{m(P)}{m(P)+m(P_1)} E(\theta|H, P) + \frac{m(P_1)}{m(P)+m(P_1)} E(\theta|H, P_1) &\to E(\theta|H) \\ \frac{m(P)}{m(P)+m(P_1)} E(\theta|L, P) + \frac{m(P_1)}{m(P)+m(P_1)} E(\theta|L, P_1) &\to E(\theta|L). \end{aligned}$$

Therefore, as $\theta_0 \to \bar{\theta}$, (3) approaches

$$\bar{\theta} - E(\theta|H) < \bar{\theta} - E(\theta|L),$$

which holds as by the MLRP assumption, $E(\theta|H) > E(\theta|L)$. As the distribution of θ has full support, and there are no atoms in the distribution, $E(\theta|H, \theta > \theta_0)$ and $E(\theta|H, \theta < \theta_0)$ are continuous in θ_0. Therefore, there exists a $\hat{\theta}_0(H, L) < \bar{\theta}$ such that for all $\theta_0 \geq \hat{\theta}_0(H, L)$, the expected wage premium of fraternity members is decreasing in $\tilde{\theta}$. This bound on θ_0 depends on the signals H and L; however, Assumption 1 ensures the existence of a uniform bound. In particular, if the support $\tilde{\theta}$ is finite, then the uniform bound is the maximum of the bounds for each signal pair; and if the support of $\tilde{\theta}$ is not finite, but the support of $f_{\tilde{\theta}}(\tilde{\theta}|\bar{\theta})$ is non-trivial, then the expected wage difference that $\bar{\theta}$ expects places strictly positive probability on signals bounded away from $\hat{\theta}$. Hence, there exists a small enough $\underline{\Gamma} > 0$ such that for $\Gamma < \underline{\Gamma}$, the equilibrium expected wage premium from fraternity membership is declining in ability. $\square$

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