Coopetitive game solutions for the eurozone economy

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COOPETITIVE GAME SOLUTIONS
FOR THE EUROZONE ECONOMY
Introduction°.

The severe financial and economic crisis which affected the Greek economy and its effects on the whole Eurozone economy have become the main topic in the agenda of the EU Governments, notably those of the Euro area, and of the international institutions.

Governments of the Eurozone and IMF recently agreed to provide Greece with enough financing to cover its refinancing needs for three years, while the Greek government commits to an additional tough austerity program.

At the same time Germany is the country of the Eurozone which has a large trade surplus with Greece and other Euro partners, hence strong trade imbalances occur within the Eurozone economy, which inevitably influence its growth path.

This paper aims at offering, through a game theory model of coopetition, possible solutions in a cooperative perspective for the divergent interests which drive the economic policies in Germany and Greece, but that affect all the economies of the Eurozone.

Germany is the country who has profited most from the Euro for the past 11 years, that is, since the start of the European Monetary Union, according to Adam Posen (2010) of the Peterson Institute of International Economics. As the benefits enjoyed by the German economy have been possible only in a cooperative economic system, our main aim in this paper is to analyze win-win arrangements among Eurozone countries that imply improvements in Germany’s domestic demand.

This work, however, does not analyze the causes of the financial crisis in Greece and its relevant political and institutional effects on the European Monetary Union and on the European Union. It focuses, instead, on the real aspects of the Eurozone economy. In particular it concentrates on stability and growth, which should become the central goals of the Eurozone and drive the economic policy of Germany, Greece and the other Euro countries.

Yet, we share the view that if the Eurozone economy likes to survive in the medium-long term, it will be necessary that its institutions must provide a crisis resolution system, better fiscal policy co-ordination (which should take in perspective towards a fiscal union) and policies to reduce intra-eurozone imbalances. Furthermore, the Eurozone should have a single European bond.

The paper is organized as follows: the first section examines the Greek crisis, its implications for the Eurozone economies. This part which focuses on Germany, the major economy of the Euro area and the biggest exporter, suggests a possible way out to reduce the intra-eurozone imbalances through coopetitive solutions within a growth path. The second section explains the notion of coopetition through a short review of the literature upon it. In the third section David Carfì provides a game theory analytical framework of coopetition with two original models of coopetitive games applied to the Eurozone context, showing their solutions. Conclusions end up the paper.

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° The Introduction and section 1. of this paper are written by D. Schilirò, section 2. is written by M. Magaudda, section 3. by D. Carfì, Conclusions have been drawn by the three authors.
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1. The Eurozone and the Greek crisis: a coopetitive solution?

The European Monetary Union (EMU) and the Eurozone economy started in 1999 with the introduction of the Euro. Over this decade the Euro area became a major economic region of relative stability with a high per capita income, strong trade balance and a low inflation rate. The global crisis of 2008-2009, probably the worst for the world economy since the 1930s, hit the European economies severely. But above all the recent deep financial crisis of Greece, which was almost causing the default of its sovereign debt, determining financial instability throughout the European markets and the devaluation of the Euro, has revealed the contradictions that have been characterized the Eurozone economy and the Euro since their start.

The institutional setting of the European Monetary Union is featured by the European Central Bank which is independent from political pressure and which has an explicit mandate to preserve price stability. Moreover, the setting is characterized by the Stability and Growth Pact that guards against the emergence of public deficits and debt. Therefore the budget policy in each country of the Eurozone should be under control, even if in practice this is not so, because there isn’t a true and effective mechanism of enforcement in the Pact. Actually, the political and economic governance of the EMU has been criticized because of the fragmentation of economic responsibilities across member states, which determines the fragmentation of the Single Market, the lack of a central fiscal authority and the unclear role of EU institutions. Consequently the Eurozone economy has a king (the Euro), but not a country, as Thomas Mayer wrote.

The Eurozone economy is also featured by the presence of two countries which have a major and increasing political and economic role: Germany and France. Germany is especially considered Europe’s soundest economy. First of all, it accounts for about one-third of the Eurozone economy. Secondly, it is the world’s second-biggest exporter, but its wide commercial surplus is originated mainly by the exports in the Eurozone, that accounts for about two thirds. Furthermore, since 2000 its export share has gradually increased vis-à-vis industrial countries. Thirdly, its government has not allowed itself the extraordinary budget deficits that are threatening economies elsewhere on the continent, like Ireland, Italy, Greece, Portugal and Spain. Despite this positive records, the contribution of domestic demand to real GDP from 1999 onwards in Germany has been weak. It is clear, from such a context, that the Germany’s growth path has been export driven. We do not discuss in this paper the factors explaining Germany’s increase in export share, but we observe that its international competitiveness has been improving, with the unit labor cost which has been kept fairly constant, since wages essentially kept pace with productivity. Therefore the prices of the German products have been relatively cheap, favouring the export of German goods towards the Euro countries and towards the markets around the world, especially those of the emerging economies (China, India, Brasil, Russia).

Greece, on the contrary, is a country with a total population of 11 million and it represents 2,6% of the Eurozone’s GDP. This country adopted the euro in 2001, then interest rates fell to near German levels, the lowest in the Euro area, fueling consumer spending and house prices.

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1 The European Commission has recently proposed tougher rules to enforce fiscal discipline in the Eurozone and to set up a permanent crisis management mechanism to prevent sovereign debt disasters. The initiatives would represent the most significant advance in Eurozone economic governance since the Euro’s launch in 1999. Among the most important proposals is the idea that national governments should assess each other’s annual budgets in greater detail and much earlier than is now the case. The Stability and Growth Pact would be tightened so that more emphasis is placed on the need for governments to cut public debt.

2 See Pisani-Ferry, Posen (2009), p.179.

3 For an analysis of these factors see Danninger and Joutz (2007).
Since joining the Eurozone economy, Greece has lost competitiveness and because of that Greek’s unit labor cost rose 34 percent from 2000 to 2009. Thus, Greece relied on state spending to drive growth. With the outbreak of the crisis, debt in Greece has surged as in the other countries, but in 2009 Greece recorded a deficit/GDP ratio of 13.6%, higher of many of the countries belonging to the Eurozone. This has created deep concerns about its fiscal sustainability. Table 1 describes the deficit and the debt as percentage of GDP for the 16 countries of the Eurozone.

Table 1.

Deficit and the debt as percentage of GDP for the 16 countries of the Eurozone in 2009.

<table>
<thead>
<tr>
<th></th>
<th>Deficit/ GDP (percentage values)</th>
<th>Debt/GDP (percentage values)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>3.4</td>
<td>66.5</td>
</tr>
<tr>
<td>Belgium</td>
<td>6.0</td>
<td>96.7</td>
</tr>
<tr>
<td>Cyprus</td>
<td>6.1</td>
<td>56.2</td>
</tr>
<tr>
<td>Finland</td>
<td>2.2</td>
<td>44.0</td>
</tr>
<tr>
<td>France</td>
<td>7.5</td>
<td>77.6</td>
</tr>
<tr>
<td>Germany</td>
<td>3.3</td>
<td>73.2</td>
</tr>
<tr>
<td>Greece</td>
<td>13.6</td>
<td>115.1</td>
</tr>
<tr>
<td>Ireland</td>
<td>14.3</td>
<td>64.0</td>
</tr>
<tr>
<td>Italy</td>
<td>5.3</td>
<td>115.8</td>
</tr>
<tr>
<td>Luxemburg</td>
<td>0.7</td>
<td>14.5</td>
</tr>
<tr>
<td>Malta</td>
<td>3.8</td>
<td>69.1</td>
</tr>
<tr>
<td>Netherlands</td>
<td>5.3</td>
<td>60.9</td>
</tr>
<tr>
<td>Portugal</td>
<td>9.4</td>
<td>76.8</td>
</tr>
<tr>
<td>Slovakia</td>
<td>6.8</td>
<td>35.7</td>
</tr>
<tr>
<td>Slovenia</td>
<td>5.5</td>
<td>35.9</td>
</tr>
<tr>
<td>Spain</td>
<td>11.2</td>
<td>53.1</td>
</tr>
<tr>
<td>Euro Area</td>
<td>6.3</td>
<td>78.7</td>
</tr>
</tbody>
</table>

Source: Eurostat

The table shows that nearly all the countries of Eurozone accumulated large deficit/GDP ratio in 2009. The average value for the whole Eurozone was 6.3%. While the average growth rate of real GDP for the 16 member countries was, in the period 2001-2009, 1.08%; in particular it was 0.6% in 2008 and – 4.1% in 2009. This means that Eurozone’s GDP has been growing much less
than budget deficit and public debt, thus growth must become again the primary goal for Eurozone countries without relying on public expenditure.

Greece has also accumulated a huge debt of about 310 billion euros, thus its financial exposition prevents the Greek government to find capital in the financial markets; Greece, therefore, has become at risk of sovereign default. The countries of the Eurozone, after a period of uncertainty which raises the cost of the bailout, have decided to help Greece financially also with the support of IMF\(^4\). This financial contribution is likely to be given until 2012 and it will be very substantial\(^5\). But tough austerity conditions are requested in return for the emergency loans, which are to be paid with interest rates below the market rates. The Greek government is required to take courageous and specific actions that will lastingly and credibly consolidate the public budget. First, to recover from the budget disequilibrium, Greece is expected to improve the primary balance of 10 percent of GDP over the next three years\(^6\). Moreover, the package includes measures to reduce the size of Greece’s public sector, cuts in public sector salaries and pensions, a rise in value added tax and other tax increases. It also includes measures to enhance competition in many sectors which are still protected; thus the country is expected to reduce its budget deficit from 13.6 per cent of gross domestic product to below 3 per cent by 2014.

However, although a restrictive fiscal policy and budget austerity are necessary, they could be insufficient for Greece to overcome its crisis. The austerity measures would probably hit hard the Greek economy, since its growth is expected to be negative for this year and the next year, making also the financial recovery more problematic\(^7\). Furthermore, exports are much less than imports, so the trade balance shows a deficit around 10%. Therefore, the emphasis should go to the real economy and growth must become the major goal for Greece as for the whole Eurozone. This would also help the adjustment process in Greece and other vulnerable countries. To ensure growth, Germany, which has spent less than its income and run current-account surpluses (like other surplus countries of the Eurozone, i.e. Netherlands), should commit to stimulating domestic demand and relying less on exports to the Euro area. Germany, as Adam Posen from the Peterson Institute for International Economics underlined\(^8\), has benefited from being the anchor economy for the Eurozone over the last 11 years. In fact, it benefited most in terms of productivity growth, and has also enjoyed a wider and deep range of trade in the Euro currency than it had under the Deutsch Mark. For instance, in 2009, during a time of global contraction, Germany has been a beneficiary, being able to run a sustained trade surplus with its European neighbours. Germany exported, in particular, 6.7 billions euros worth of goods to Greece, but imported only 1.8 billion euros worth in return. It is true that not only Greece has been living beyond its real economic possibilities as well as hiding the true figures of its public deficit until recently, but it has also a harsh problem of competitiveness. Clearly a policy which aims at growth in Germany, in Greece and in the whole Eurozone is more important, specially if we take a medium-long term perspective and if we consider that the rate of unemployment in the Euro area has reached 10.1%\(^9\), the highest rate in almost 12 years. Another important aspect to highlight is that the European institutions, like the European Commission, the Ecofin and the ECB, together with the IMF have given perhaps too much emphasis at the financial aspects to solve the crisis. This

\(^{4}\) An agreement has been reached on May 2\(^{nd}\), between the Eurogroup, the IMF and the Greek Government.

\(^{5}\) The total sum given to Greece in three years should be of 110 billion euros.

\(^{6}\) This is an heavy task, but other economies like Lettonia and Hungary have succeeded in the recent past with the help and the assistance of the IMF and EU.

\(^{7}\) This view, of course, is not shared by the economists who believe that fiscal adjustments not always cause recessions (Giavazzi, Pagano, 1990; Von Hagen, Strauch, 2001; Alesina, Adagna, 2009).

\(^{8}\) Posen (2010), see also Abadi (2010).

\(^{9}\) Source: Eurostat. The figure refers to april 2010.
strategy, although necessary, could reveal itself not sufficient and short lasting. Despite the new huge rescue plan of 750 billion euros supported by the EU and IMF to avoid the contagion of the Greek crisis to the whole Eurozone economy, the recent turmoil in the financial markets and the consequent weakness of the Euro seem to confirm - apart from the excesses of market speculation - the poorly optimistic expectation of the financial markets on the future of the Greek economy and of the Euro\textsuperscript{10}. In this context we share the view that a partial debt restructuring by the Greek government might become a sensible and realistic solution\textsuperscript{11}.

We believe that a policy that aims at adjusting budget and trade imbalances and looks at improving the growth path of the real economy in the medium and long term in Greece, but also in all the other Euro economies, is the only possible one to assure a stable re-balancing of the Greek economy and the stability of the Eurozone. As we have already argued, German modest wage increases and weak domestic demand favoured the export of German goods towards the Euro countries. This is why Posen, as reported by \textit{Business Week} on March 31, 2010, said that Germany should boost domestic demand and increase wages to ease the lopsided euro-region trade flows that restrict growth in economies like Greece and Portugal. Therefore he suggests a “\textit{win-win solution}”\textsuperscript{12} for the Eurozone, which entails that Germany, which still represents the leading economy, should stimulate domestic demand, increase wages in its own country, so that to make its own people better off, and thereby ease some of the pressure on these Euro countries. In Posen’s proposal there is a clear suggestion to Germany to re-balance its trade surplus. Of course, we are aware that this is a mere hypothesis. After the Greek crisis, because of the turmoil in the financial markets, the German government, in fact, have decided to take austerity fiscal measures, which consists of a seven years plan of government budget consolidations of 70 billion euros (10 billion euros for each year), based mainly on structural spending cuts to welfare payments and reduction in the public sector\textsuperscript{13}. This plan, however, will also favour investment in education and research to improve Germany’s capacity to compete at a global level\textsuperscript{14}.

On the other hand, Greece must fulfil the conditions of the agreement signed with the Eurozone Governments and the IMF for their financial help. For this reason it must implement a fiscal policy of government budget consolidations, with current spending cuts and tax increases, thus reducing its public and private debt. These changes in current variables (taxes, incentives, provision of public services) would probably also change the expectations about future fiscal policy\textsuperscript{15}. In our opinion, to favour growth Greece must keep its wages and salaries under control and, at the same time, focus on investments and exports as the two main strategic variables to improve the structure of production and to shift the aggregate demand towards a higher growth path. However, aiming at exports for a country like Greece that has a low “Extra Euro Area” export share on GDP (about 4%) does not mean to rely on the external demand, for instance

\textsuperscript{10} Investors are looking for a credible plan that indicated public finances in Greece but also in whole Euro area could be kept at a sustainable level.

\textsuperscript{11} See Mussa (2010) on the alternatives of an immediate versus a later debt restructuring.

\textsuperscript{12} A \textit{win-win solution} is the outcome of a game which is designed in a way that all participants can profit from it in one way or the other. In conflict resolution a \textit{win-win strategy} is a process that aims to accommodate all disputants.

\textsuperscript{13} The German government’s plan is coherent with the view expressed in Von Hagen and Strauch (2001) about the quality and the success of fiscal consolidations and also with the so called “German view” based on changes in expectations about future fiscal policy (Giavazzi, Pagano,1990).

\textsuperscript{14} Actually, Germany has been pursuing a strategy of competitiveness based on investments in technology and R&D on the one hand, and on industrial relations which are featured by cooperative behaviors between labor and capital on the other since 2003. We believe that this cooperative attitude, which is an hallmark of German capitalism, can be also taken with respect to its Euro partners and the Greece in particular.

\textsuperscript{15} Regarding the indirect positive effect on aggregate demand see Hellwig, Neumann (1987) that merge the Keynesian view and the expectations view or “German view” on budget cutting. See also Giavazzi, Pagano (1990).
through the devaluation of the euro, rather to follow an appropriate medium term strategy. In this medium term strategy, Greece should focus on innovative investments, specially investments in knowledge\textsuperscript{16}, to change and improve its production structure and to increase its production capacity and its productivity, which is made possible by the structural change process. As a result of that its competitiveness will raise. An economic policy that focuses on investments and exports, instead of consumptions, will address Greece towards a sustainable growth and, consequently, its financial reputation and stability will get improved. This policy will also benefit the Euro and contribute to the stability of the whole Eurozone\textsuperscript{17}.

In this paper we provide a new set of tools based on the notion of coope}tition, that could be fruitful for the setting of the Eurozone policy issues. Thus we suggest a model of coopetitive games which intends to offer possible solutions to the partially divergent interests of Germany and Greece in a perspective of a cooperative attitude that should drive their policies.

2. Coopetition and Game Theory.

The concept of coope}tition has been devised following different theoretical approaches. Essentially the literature on coopetitive games has a microeconomic origin and has an important point of reference in the seminal paper of Brandenburger and Nalebuff (1995), who studied the strategic behaviour of firms applying some basic notion of game theory and elaborated their theoretical original concept of coopetition. They start their analysis considering firms as independent entities which are competing with other firms in a hostile market environment. In this competitive environment, each firm tends to choose a win-lose strategy following Porter’s strategy of competitive advantage\textsuperscript{18}. Yet, Brandenburger and Nalebuff believe that the essence of business success lies in making sure you are playing the right game (1995, p.57). Firms realize that in their business relations there are times when you want to create a win-win situation. Brandenburger and Nalebuff suggest the term coopetition (a situation in which the firm must cooperate and compete at the same time) to indicate a situation in which the firm thinks about both cooperative and competitive ways to change the game (1995, p.59).\textsuperscript{19} Under certain circumstances it pays for the firm to consider both cooperative and competitive strategies to be successful; in these cases you have coopetitive games. Hence, firms look for win-win solutions, as well as for win-lose ones. In this framework rules are very important, since they determine how the game is played by limiting the possible reactions to any action. Moreover, no game is an island, therefore games are linked across space and over time. Thus coopetition, according to Brandenburger and Nalebuff, is defined as the set of two strategies: the first is a non-cooperative game that tends to a win-lose solution, that brings to a Nash equilibrium solution. The second is a cooperative game that tends to a win-win solution, that brings to a Pareto-optimum solution\textsuperscript{20}.

Another approach to coopetition is that offered by Padula and Dagnino (2007), who, differantly from Brandenburger and Nalebuff, define coopetition as the intrusion of competitive

\textsuperscript{16} Schilirò (2008).
\textsuperscript{17} In this context ECB should facilitate the process of adjustment in Greece and in the whole Eurozone through an adequate monetary policy.
\textsuperscript{18} This competitive positioning approach can be related more generally to the strategic positioning paradigm, which is concerned with the industry structure and the pursuit of generic strategies to achieve competitive advantage.
\textsuperscript{19} Brandenburger and Nalebuff implicitly assume that the competitive environment is an oligopolistic market structure.
\textsuperscript{20} It is important to consider both strategies, since the win-lose strategies often “backfire”, that is, they will go against you and they become lose-win or lose-lose.
elements into a cooperative environment, because of the partially divergent interests among the partners. For Padula and Dagnino coopetition represents the synthesis between the competitive paradigm (Porter, 1985) and the cooperative paradigm (Gulati, Nohria, Zaheer, 2000), a sort of integrative framework between the two, that should provide a more realistic representation of the interdependence among the firms. According to them, in a situation of coopetition you get a trade-off between the cooperative aspects and the competitive ones. Thus coopetition is a complex construct and it is the result of the interplay between competition and cooperation. To represent a coopetitive game we have to start defining a situation of pure competition, where the structure of interests is totally divergent, as we have to consider a situation of pure cooperation, where the structure of interests is totally convergent, coopetition will be a solution that will be in between of these two ‘extreme’ points, so that it will determine a sort of “coopetitive matrix”. The game of reference must be the Pareto curve and its solutions, which are Pareto-optimum, are on this curve. However, Padula and Dagnino did not provide a game theory model of coopetition.

The term coopetition has also been used in other contexts (i.e. the analysis of industrial districts, Fortis, 2006)\footnote{It is generally assumed that in the districts there is a cooperative environment, but at the same time there is competition among the firms that determines an efficiently-grounded process of selection. Coopetition is interpreted as the best strategy to shift the industrial competition from the costs of production towards product innovation, speed of time to market and capacity of quickly changing the services provided.}. Sociologists also used the notion of coopetition to develop network analysis.

In this paper we suggest an original analytical framework of coopetitive games applied at a macroeconomic level to the Eurozone context worked out by David Carfì, with the aim to enrich the set of tools of macroeconomic policy in the Euro area. Our model of coopetition is closer to the approach that regards coopetition as a complex construct rooted in a cooperative environment.

3. An Analytical Framework of Coopetitive Games.

In this section we provide a general analytical framework of coopetition with two models of coopetitive games, applying them to the two Eurozone countries, Germany and Greece. The two coopetitive models will show the possible solutions feasible in a particular coopetitive context, defined by the set of strategy profiles chosen by the two countries through a convenient ex ante agreement. This suggested analytical framework enables us to widen the set of possible solutions in a coopetitive context and it allows “to share the pie fairly” in a win-win scenario. At the same time, it permits to examine the range of possible economic outcomes along a coopetitive dynamic path. Finally, it limits the space within which the coopetitive solutions can be determined.

3.1 The general analytical framework

The basic definition we propose of coopetitive game is the following one.

**Definition (of coopetitive game).** Let $E$, $F$ and $C$ be three nonempty sets. We define two person coopetitive gain game carried by the strategic triple $(E, F, C)$ any pair of the form $G = (f, >)$, where $f$ is a function from the Cartesian product $E \times F \times C$ into the real Euclidean plane and $>$ is
the usual order of the Cartesian plane, defined, for every couple of points \( p, q \), by \( p > q \) iff \( p_i > q_i \), for each index \( i \).

**Remark.** The difference among a two person normal-form gain game and a two person coopetitive game is simply the presence of the third strategy Cartesian-factor \( C \).

**Terminology and notation.** Let \( G = (f, >) \) be a two person coopetitive gain game carried by the strategic triple \((E, F, C)\). We will use the following terminologies:

- the function \( f \) is called the **payoff function of the game** \( G \);
- the first component \( f_1 \) of the payoff function \( f \) is called the **payoff function of the first player** and analogously the second component \( f_2 \) is called the payoff function of the second player;
- the set \( E \) is said the **strategy set of the first player**, the set \( F \) the **strategy set of the second player**;
- the set \( C \) the **cooperative strategy set of the two players**.
- the Cartesian product \( E \times F \times C \) is called the **coopetitive strategy space of the game** \( G \).

**Memento.** The first component \( f_1 \) of the payoff function \( f \) of a coopetitive game \( G \) is the function of the strategy space of the game \( G \) into the real line defined by \( f_1(x,y,z) = \text{pr}_1(f(x,y,z)) \), analogously we proceed for the second component \( f_2 \).

**Interpretation.** We have two players, each of them has a strategy set in which to choose his strategy; moreover, the two players can cooperatively choose a strategy \( z \) in a third set \( C \). The two players will choose their cooperative strategy \( z \) to maximize (in some sense) the gain function \( f \).

**Bargaining solutions of a coopetitive game.** The payoff function of a two person coopetitive game is (as in the case of normal-form game) a vector valued function with values belonging to the Cartesian plane \( \mathbb{R}^2 \); so that we should consider the maximal Pareto boundary of the payoff space \( \text{im}(f) \) as an appropriate zone for the bargaining solutions.

**The family of normal form games associated with a coopetitive game.** For any cooperative strategy \( z \) selected in the cooperative strategy space \( C \) there is a corresponding normal form game

\[
G_z = (f_z, >)
\]

upon the strategy pair \((E, F)\) and with payoff function the section

\[
f(, z): E \times F \to \mathbb{R}^2,
\]

of the payoff function \( f \) of the coopetitive game (the section is defined, as usual, on the competitive strategy space \( E \times F \) by \( f(, z)(x) = f(x, z) \), for every bi-strategy \( x \) in the bi-strategy space \( E \times F \)).

**General solution.** The two players should choose the cooperative strategy \( z \) in order that, for instance
- the Nash equilibria of $G_z$ are “better” than the Nash equilibria in each other game $G_{z'}$;
- the supremum of $G_z$ is greater than the supremum of any other game $G_{z'}$;
- the Pareto maximal boundary of $G_z$ is “higher” than that of any other game $G_{z'}$;
- the Nash bargaining solution is better in $G_z$ than that in $G_{z'}$;
- and so on, fixed a common kind of solution for any game $G_z$, say $S(z)$ the set of these kind of solutions, we can consider the problem to find the optimal solutions in set valued path $S$, defined on the cooperative strategy set $C$;

we note the fundamental circumstance that in general the above criteria are multi-criteria and so they generate multi-criteria optimization problems.

Let us formalize the concept of game-family associated with a coopetitive game.

**Definition (the family associated with a coopetitive game).** Let $G = (f, >)$ be a two person coopetitive gain game carried by the strategic triple $(E, F, C)$. We naturally can associate with the game a family of competitive games $G = (G_z)_{z \in C}$, which we will denote by the same symbol $G$ and which we call the family of normal-form games associated with the coopetitive game $G$.

**Applicative remark.** It is clear that with any family of normal form games $G = (G_z)_{z \in C}$ we can associate

- a family of payoff spaces $(\text{im}(f_z))_{z \in C}$,
- a family of Pareto maximal boundary $(\text{bd}^*G_z)_{z \in C}$;
- a family of suprema $(\sup G_z)_{z \in C}$;

and so on.

And we can interpret any of the above families as set-valued paths in the strategy space $E \times F$.

It is just the study of these induced families which becomes of great interest in the study of a coopetitive game $G$.

### 3.2 Two models of coopetitive games

In our analysis Germany is the first exporting country among the Eurozone countries that has also experienced a weak domestic demand due to a modest wage increases. Thus our hypothesis is to stimulate Germany’s domestic demand and to re-balance its trade surplus in favour of Greece.

On the other hand, Greece is the country that showed a high and rising public debt, which determined its sovereign debt at risk of default. Given that Greece must pursue a budget austerity program externally imposed by the Euro area Governments and by IMF in exchange of their
financial help, this country has anyway experienced a declining competitiveness of its products. Therefore our hypothesis is that Greece aims at growth by undertaking innovative investments and by increasing its exports towards Germany and also the other Euro countries\textsuperscript{22}.

The coopetitive models that we propose hereunder must be interpreted as normative models, in the sense that they will show the more appropriate solutions of a win-win strategy chosen within a cooperative perspective.

The main variables of the two models are:

- **strategies** \( x \) of Germany (*the consumptions of Germany*), which directly influence only Germany pay-off;
- **strategies** \( y \) of Greece (*the investments of Greece*) which increase only Greece pay-off function;
- a **shared strategy** \( z \) which is determined *ex ante* together by the two countries, Germany and Greece (\( z \) is a given amount of Greek exports imported by Germany).

Therefore, in the two models we assume that Germany and Greece define the set of coopetitive strategies.

**First coopetitive model.**

**Main Strategic assumptions.** We assume that a real number \( x \), in the unit interval \( U = [0,1] \), is the consumption of Germany and a real number \( y \), in the same unit interval \( U \), is the investment of Greece, moreover a real number \( z \), again in \( U \), is the amount of Greek exports which is imported by Germany.

We also consider as payoff function of Germany its domestic demand, that we represent in our model as the algebraic sum of the variables \( x \) and \( z \), and also of the exports of Germany as a reaction function with respect to its domestic consumption.

**Payoff function of Germany**

*We assume that the payoff function of Germany is the function \( g \) of the square \( U \times U \) into the real line, defined by*

\[
g(x, z) = x + 1/(x + 1) - z,
\]

*for every pair \((x,z)\) in the square \( U \times U \); where the reaction function \( E \), of \( U \) into the real line, defined by*

\[
E(x) = 1/(x+1),
\]

*for every consumption \( x \) of Germany in \( U \), is the export of Germany corresponding to the level \( x \)*

\textsuperscript{22} The potential benefit coming from a better trade balance can also contribute to ease the government budget constraint and improve its public debt.
of consumption; E is a decreasing function, randomly chosen, and within certain limits, this choice does not diminish the generality of the model.

**Payoff function of Greece**

We consider as payoff function of Greece the algebraic sum of the variables y and z.

We assume that the payoff function of Greece is the function \( e \) of the square \( U \times U \) into the real line, defined by

\[
e(y,z) = y + z + my + nz = (1+m) y + (1+n)z
\]

for every pair \((y,z)\) in the Cartesian square \( U \times U \); we note that the function \( e \) does not depend upon the strategy \( x \) chosen by Germany and that \( e \) is a linear function. The term \( my \) represents the quality and quantity effect of investments on the exports. In fact, the investments, specially innovative investments, contribute at improving the competitiveness of Greek goods, favouring the exports. The term \( nz \) is the cross-effect of the cooperative variable \( z \) that represents the additive level of investment required to support the production of \( z \). We assume \( m \) and \( n \) strictly positive.

**Payoff function of the game**

We so have build up a gain game with payoff function given by

\[
p(x,y,z) = (x + 1/(x+1) - z, (1+m) y + z) = (x + 1/(x+1), (1+m) y) + z (-1,1+n)
\]

with \( x,y,z \) in \([0,1]\).

**Study of the game** \( G = (p, >) \).

Note that, fixed a cooperative strategy \( z \) in \( U \), the game \( G(z) = (p(z), >) \) with payoff function \( p(z) \), defined on the square \( U \times U \) by

\[
p(z)(x,y) = p(x,y,z),
\]

is the translation of the game \( G(0) \) by the vector \( v(z) = z(-1,1+n) \), so that we can study the game \( G(0) \) and then we can translate the various informations of the game \( G(0) \) by the vector \( v(z) \).

So let us consider the game \( G(0) \). Let the strategic square \( S \) be with vertices \( A,B,C,D \), where \( A \) is the origin, \( B \) is the first canonical vector \( C \) the sum of the two canonical vectors and \( D \) be the second canonical vector.

**Pareto Boundary in the payoff space**

The transformation of the side \([A, B]\) is the trace of the curve

\[
c(x) = p(x,0,0) = (x + 1/(x+1), 0),
\]
that is the segment
\[ [A', B'] = [(1,0), (3/2,0)]. \]

The transformation of the segment \([A, D]\) is the trace of the curve
\[ c(y) = p(0,y,0) = (1, (1+m) y), \]
that is the segment
\[ [A', D'] = [(1,0), (1,1)]. \]

The transformation of the segment \([B, C]\) is the trace of the curve
\[ c(y) = p(1,y,0) = (1 + 1/2, (1+m) y), \]
that is the segment \([B', C'] = [(3/2,0), (3/2,1+m)].\]

So that the payoff space of the game \(G(0)\) is the rectangle with vertices \(A', B', C', D'\).

**The payoff space of the coopetitive game** \(G\), the image of the payoff function \(p\), is the union of the family of payoff spaces
\[ (\text{im } p(z))_z, \]
that is the convex envelope of the of the four points \(A', B', C', D'\) and of their translations by the vector \(v(1)\).

The Pareto maximal boundary of the payoff space \(f(S)\) is the segment \([P', Q']\), with \(P' = C'\) and \(Q' = C' + v(1)\).

It is important to note that the slope of the Pareto boundary is \(1 + n\). Thus the collective payoff \(g + e\) of the game is not constant on the Pareto boundary and, therefore, the game implies growth.

The Nash bargaining solution and the Kalai-Smorodinsky bargaining solution, with respect to the infimum of the Pareto boundary, coincide with the medium point of the segment \([P', Q']\).

**Transferable utility solution.** In this coopetitive context it is more convenient to adopt a transferable utility solution: indeed the point of maximum collective gain is the point

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23 The classic Kalai-Smorodinsky solution, that we applied in both models, coincides with the solution on the coopetitive Nash path; this result allows us to provide a construct of coopetition which is only “weakly”cooperative, in the sense that it not necessary to cooperate at every stage of the decision process.
Q' = (1, 3/2 + m + n).

Thus we propose a new kind of coopetitive solution, as it follows (in the case m=0):

First, we consider the coopetitive rectangle R having: a) two sides on the straight lines of equations Y=1 and Y = 2 + n; b) two vertices in (1/2,1) and (1/2,2 + n); c) the diagonal on the straight line S of equation Y + X = 2.5 + n.

Second, We consider the segment S’ of vertices (3/2,1) (supremum of the game G(0)) and the supremum of the rectangle R, that is the point (3/2+n,2+n).

Third, our best payoff coopetitive compromise K is the intersection of S and S’.

This K represents a win-win solution with respect to the initial supremum (3/2,1).

Second coopetitive model

Let us consider now that a fraction ax of Germany consumption comes from consumption of Greek goods, apart from the given amount of Greek exports that Germany has already determined through an ex ante agreement with Greece (z).

Payoff function of Greece

e(x,y,z) = by + z + ax

Payoff function of the game

p(x,y,z) = (x + 1/(x+1) - z, ax+by + cz) = (x + 1/(x+1), ax+ by) + z (-1,c)

with a, x,y,z in [0,1] and b,c>1.

Similarly to the previous coopetitive model, but through a more complex procedure, we deduce that the Pareto boundary of the coopetitive game G = (p, >) – in the payoff space - is the above segment [P’, Q’] translated by the vector (0,a).

The Nash bargaining solution and the Kalai-Smorodinsky bargaining solution, with respect to the infimum of the Pareto boundary, coincide with the medium point of the segment [P’, Q’] + (0,a), which is the optimum of the game G_{1/2}.

Conclusions.

This paper has aimed at providing, through a game theory model of coopetition, feasible solutions in a cooperative perspective to the problems that affect the Eurozone economy after the Greek crisis. In particular, it has focused on stability and growth as the primary goals, which should drive the Eurozone economy in consequence of the financial and economic crisis of the Greek economy with its effects throughout the Euro area.
This work has underlined two aspects which emerged from the crisis. First, the necessity of government budget consolidation of Greece; second, the opportunity to re-balance the trade surplus of Germany with respect to Greece and also with respect to the other Euro countries that have a deficit trade balance.

By means of two coopetitive models derived by an original general analytical framework of coopetition, we have showed in our paper the strategies that could bring to feasible solutions in a cooperative perspective between Germany and Greece, where these feasible solutions aim at “sharing the pie fairly”, by offering a win-win outcome for both countries, within a growth path represented by a non-zero sum game. In fact, our analytical results allow us to find a “fair” amount of Greek exports which Germany must import, in order to re-balance the trade surplus of Germany, as well as the investments necessary to improve the Greek economy, thus contributing to growth and to the stability of the Eurozone.

Finally, a remarkable analytical result of the paper consists in the determination of the win-win solution by a new selection method on the transferable utility Pareto boundary of the coopetitive game.

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