Bundling revisited: substitute products and inter-firm discounts

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Abstract

This paper extends the standard model of bundling to allow products to be substitutes and for products to be supplied by separate sellers. Whether integrated or separate, firms have an incentive to introduce bundling discounts when demand for the bundle is elastic relative to demand for stand-alone products. Separate firms often have a unilateral incentive to offer inter-firm bundle discounts, although this depends on the detailed form of substitutability. Bundle discounts mitigate the innate substitutability of products, which can relax competition between firms and induce an integrated firm to lower all of its prices when it follows a bundling strategy.

1 Introduction

Bundling—the practice whereby consumers are offered a discount if they buy several distinct products—is used widely by firms, and is the focus of a rich economic literature. However, most of the existing literature discusses the phenomenon under relatively restrictive assumptions, namely:

- a consumer's valuation for a bundle of several products is the sum of her valuations for consuming the items in isolation, and
- bundle discounts are only offered for products sold by the same firm.

The two assumptions are related to some extent, in that when valuations are additive it is less often the case that a firm would wish to reduce its price if its customer also buys a

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product from another seller. This paper analyzes the incentive to engage in bundling when one or both of these assumptions is relaxed.

There are many situations in which modelling products as substitutes is relevant. For instance, when visiting a city a tourist may gain some extra utility from visiting art gallery A if she has already visited art gallery B, but the incremental utility is likely to be smaller than if she only visits A. Joint purchase discounts on products offered by separate sellers are perhaps rarer, though several examples can be found, including:

- Museums in a city may collaborate to offer a "museum pass", so that a tourist can visit all participating museums at a discount on the individual entry fees.

- Online music stores retail music by many different companies to final consumers, often using bundling discounts. Separately-owned television channels may be retailed separately as well as being offered as a bundle to viewers. Separately owned academic journals are marketed individually and as part of a collection to libraries.

- Pharmaceuticals are sometimes used in isolation and sometimes as part of a "cocktail" with one or more drugs supplied by other firms. Drugs companies can set different prices depending on whether the drug is used on a stand-alone basis or in a cocktail. (One way to do this is for a firm to use a different name for the same chemical in two different uses, and to obtain regulatory approval for one name to be used in the cocktail and the other name to be used for stand-alone treatment.)

- Separately-owned products are often marketed together, with discounts for joint purchase. Thus, supermarkets and gasoline stations may cooperate to offer a discount when both services are consumed. Airlines and car rental firms may link up for marketing purposes, and sometimes credit cards offer discounts proportional to spend towards designated flights or hotels. Currently, *Amazon.co.uk* offers its customers discounts (in the form of vouchers enclosed when its books are delivered) for seemingly unrelated products (such as wine) offered by independent sellers.

- Bundling of various kinds is prevalent in markets for transport services. Sometimes customers can obtain inter-firm bundling discounts, as is the case with alliances between airlines or when neighboring ski-lifts agree to offer a combined ticket.

This paper focusses on the case where products are substitutes, in the sense that a consumer's value for a bundle is lower than the sum of her stand-alone valuations. When linear prices are used an integrated monopolist tends to raise its price when its products become more substitutable, while separate sellers tend to lower their price when products are more substitutable. The impact of a bundle discount is to mitigate or reverse the extent of product substitutability, since the reduced price for joint consumption acts to reduce the innate disutility of joint consumption. As such, when an integrated firm engages in bundling, it may reduce *all* its prices. When separate sellers engage in inter-firm bundling, this may induce them to raise their regular prices. Indeed, when separate sellers coordinate on a bundle discount in advance of price competition, this could act as an instrument of collusion.

In more detail, the plan of the paper is as follows. In section 3, we present a fairly general analysis of the incentive to introduce bundling discounts. Both situations where products are supplied by an integrated monopolist and where separate products are supplied by separate monopolists are covered. In broad terms, there is a motive to offer a bundle discount when consumer demand for the bundle is relatively elastic compared to demand for stand-alone items. We show that when separate sellers choose the bundle discount without coordination, they will choose too small a discount.

In section 4, we consider further the case where an integrated firm supplies both products, and specialise the framework to the case where the two products are symmetric. When products are partial substitutes, an integrated firm has an incentive to bundle whenever the proportion of those consumers who buy a product at price p and who go on to buy the other product at the same price decreases with p. In examples, it is often the case that when an integrated firm engages in bundling all its prices fall relative to the situation with linear pricing.

In section 5 we focus on the situation where products are supplied by separate sellers. When valuations are additive, a firm has a unilateral incentive to offer a bundle discount when valuations for products are negatively correlated. When products are substitutes, whether a firm has a unilateral incentive to introduce a discount depends on the precise way that preferences are modelled. When there is a constant disutility of joint consumption, separate sellers typically wish to offer a joint-purchase discount: the fact that a customer has purchased the rival product implies that her incremental valuation for the firm's own item has fallen, and this usually implies that the firm would like to reduce its price to this customer. Alternatively, if a proportion of buyers can only consume a single item (for instance, a tourist in a city might only have time to visit a single museum) while other consumers have additive preferences, separate sellers would like, if feasible, to charge a *premium* when a customer also buys the rival product. In this context, the fact that a consumer wants to buy both products implies that she has additive valuations, and there is no competition between sellers for these consumers.

Finally, in section 6 we investigate partial coordination between separate sellers. (The earlier parts of the paper considered the two polar cases where separate sellers did not coordinate their tariffs at all and—in the integrated firm analysis—where the two suppliers fully coordinated their tariffs.) Specifically, we suppose that firms first agree on an inter-firm discount (which they fund equally), and subsequently they choose their regular prices without coordination. When valuations are additive, we show that such a scheme will usually raise each firm's profit, and, at least when valuations are independent, its operation will also boost total welfare. However, when sellers offer substitute products, the negotiated bundle discount acts to reduce the effective substitutability between products, inducing firms to raise their prices. Thus, the scheme can induce collusion and harm consumers.

This paper is not the first to investigate these and related issues. A number of papers have investigated whether or not "code sharing"—i.e., coordinated pricing by separately-owned airlines for multi-flight itineraries—is an efficient practice. Multi-flight itineraries are products made up of complementary components, and so the inefficiency of uncoordinated pricing by separate airlines is due to double marginalization. An early theoretical contribution to this literature is Brueckner (2001), who provides a model in which two airlines need to cooperate to prevent double-marginalisation on some city-pair routes, but compete on other routes. In his model, if the two firms can coordinate their prices on all routes, the benefits of price reductions on the non-competitive routes tend to outweigh the harm done by allowing collusion on the competitive routes.

The incentive for an integrated seller to offer a discount for the purchase of multiple items is discussed by Stigler (1963), Adams and Yellen (1976), Long (1984) and McAfee, McMillan, and Whinston (1989), among many others. The latter two papers showed that it is optimal to introduce a bundle discount when the distribution of valuations is statistically independent and valuations are additive, suggesting that a degree of joint pricing is optimal even for entirely unrelated products. Except for Long, these papers assume that valuations are additive.[1] Long's note proposed an intuitive "economic" analysis of the incentive to bundle, which is adopted to a large extent in the current paper and is discussed in detail in section 2.

The early literature on bundling also included papers by Schmalensee (1982) and Lewbel (1985), who studied the incentive for a single-product monopolist unilaterally to offer a bundling discount if its customers also purchased a competitively-supplied product. Since the two products can be independent or substitutes in their analysis, their argument is distinct from the idea that tying a monopoly product with a competitively-supplied *complementary* product can be used as a metering device. Consider Schmalensee's argument in more detail. There are two items for sale to a population of consumers, and item A is available at marginal cost due to competitive pressure, while item B is supplied by a monopolist. Valuations are additive, but are not independent in the statistical sense, and that fact that a consumer is willing to buy item A is informative to the monopolist. If there is negative correlation in the values for the two items, the fact that a consumer buys item A is "bad news" for the monopolist, who then has an incentive to set a lower price to its customers who also buy A. Lewbel performs a similar exercise but allows the two items to be partial substitutes. In this case, the fact that a consumer buys item A is also bad news for the monopolist, and provides an additional reason for it to offer a discount for joint consumption.

Bundling arrangements between separate firms are analyzed by Gans and King (2006), who investigate a model with two kinds of products (gasoline and food, say), and each kind of product is supplied by two differentiated firms. When all four products are supplied by separate firms and firms set their prices independently, there is no interaction between the two kinds of product. However, two firms (one offering each of the two kinds of product) can enter into an alliance and agree to offer consumers a discount if they buy both products from the alliance. (In the model, the joint pricing mechanism is similar to that used in section 6 below: the firms decide on their bundle discount, which they agree to fund equally, and then they set their prices non-cooperatively.) Gans and King observe that

[1] Venkatesh and Kamakura (2003) analyze an integrated firm's incentive to engage in bundling when products are either complements or substitutes. The analysis is carried out using a specific uniform example, and a consumer's valuation for the bundle is some constant proportion (greater or less than one, depending on whether complements are substitutes are present) of the sum of her stand-alone valuations. The focus of their analysis is mostly on whether pure bundling is superior to linear pricing.

when a bundle discount is offered for joint purchase of otherwise independent products, those products are then converted into "complements". In their model, in which consumer tastes are uniformly distributed, a pair of firms does have an incentive to enter into such an alliance, but when both pairs of firms do this, their equilibrium profits are unchanged from the situation when all four firms set independent prices, although welfare and consumer surplus fall. Indeed, the equilibrium discounts are so large that all consumers buy both products from one alliance or the other, so there is "pure bundling".[2]

There is a substantial literature on "meet the competition" offers by firms, whereby a retailer offers to refund the difference (or more than the difference) if a customer documents a lower price for the same item at a different store. (See Salop (1986) for early discussion of this practice.) In effect, such a policy conditions a firm's price on rival *prices*, while in the current paper we suppose that a firm can condition its price only on whether a consumer buys from another firm. Because price-matching guarantees can blunt incentives to undercut rivals, this apparently pro-consumer policy can act as an instrument of collusion, just as agreements to offer bundle discounts can do in the current paper.

This paper investigates when a seller wishes unilaterally to makes its price contingent on whether a customer also purchases from another seller. The only other paper I know of which analyzes the same form of price discrimination is Calzolari and Denicolo (2009). They propose a model where each consumer has linear demands for the two products, and where a firm offers a nonlinear tariff which is a function of a consumer's demand for its own product *and* the consumer's demand for the other firm's product. They find that the use of these kinds of tariffs can in equilibrium harm consumers compared to the situation in which firms base their tariff only on their own supply. Their model differs in two ways from the one presented in this paper. First, in their model consumers have variable demands, rather than unit demands, for the two products. Thus, they must consider general nonlinear tariffs, while the firms in my model merely need to choose a pair of prices and this makes the analysis far more tractable. Second, in my model consumers differ in richer way, and a consumer might like product 1 but not product 2, and can vary in the degree of substitutability between products. In Calzolari and Denicolo (2009), consumers differ by

[2]Brito and Vasconcelos (2010) modify the model of Gans and King so that rival suppliers of the same products are vertically rather than horizontally differentiated. They find that when two pairs of firms form an alliance all prices rise relative to the situation when all four products are marketed independently. This result resembles the analysis in section 6 below, where an agreed bundle discount acts to induce collusion in the market.

only a scalar parameter (the demand intercept for both products), and so all consumers view the two products when consumed alone as perfect substitutes.

Finally, Lucarelli, Nicholson, and Song (2010) discuss the case of pharmaceutical cocktails. Although the focus of their analysis is on situations in which firms set the same price for a drug, regardless of whether it is used in isolation or as part of a cocktail, they also consider situations where firms can set two different prices for the two kinds of uses. They document how a firm selling treatments for HIV/AIDS in the early 2000s set very different prices for similar chemicals depending on whether the drug was part of a cocktail or not. They estimate a demand system for colorectal cancer drugs, where there are at least 12 major drug treatments, 6 of which were cocktails which combine drugs from different firms. Although in this market firms do not price drugs differently depending whether the drug is used in a cocktail (unlike the HIV/AIDS market), they estimate the impact when one firm engages in this form of price discrimination. They find that a firm will typically (but not always) reduce the price for stand-alone use and raise the price for bundled use.

2 An Economic Model of Bundling

In a clever note, Long (1984) presents what could be termed an "economic" model of bundling. Rather than focussing on a diagrammatic exposition concentrating on the details of joint distributions of two-dimensional consumer valuations, he uses standard tools from demand theory to derive conditions under which a bundling discount is optimal. Here, I recapitulate his analysis in its simplest, symmetric form. (Long also analyzes the situation where products are asymmetric.)

Suppose there are two symmetric products supplied by an integrated monopolist, labelled 1 and 2, each of which has constant marginal cost c. A consumer wishes to buy either zero or one unit of each product (and may wish to buy a unit of both products). Due to the assumed symmetry of demand and cost, suppose the firm sets the same price p for buying either product. Potentially, the firm offers a discount δ if the consumer buys both products, so that the total price for buying both products is then $2p - \delta$. Write the proportion of all potential consumers who buy just one item as X_1 and the proportion who buy both items as X_2. The firm's profit is therefore

$$\pi = (p - c)X_1 + (2p - \delta - 2c)X_2 \ ,$$

which can be re-written as

$$\pi = \delta N + (P - c)X \ , \tag{1}$$

where $N \equiv X_1 + X_2$ is the proportion of consumers who buy something from the firm, $X \equiv X_1 + 2X_2$ is the total number of units supplied, and $P = p - \delta$ is the incremental price of one product given the consumer buys the other product. Thus, (1) shows how the bundling tariff can be viewed as a two-part tariff comprising a fixed charge δ and marginal price P. Viewing the two demands N and X as functions of (δ, P), standard demand theory indicates that cross-price effects are symmetric, so that $N_P \equiv X_\delta$ (where subscripts denote partial derivatives).

The question whether it is optimal for the monopolist to introduce a bundling discount is therefore equivalent to whether it is optimal to have a positive fixed charge in the two-part tariff. Let P^* be the monopolist's most profitable price when no bundle discount is offered, i.e., P^* maximizes $(P - c)X(0, P)$. Starting from this situation with linear pricing, consider the impact on profit of introducing a small discount $\delta > 0$, keeping the marginal price fixed at P^*. From (1), the impact on profit is

$$\left.\frac{\partial \pi}{\partial \delta}\right|_{\delta=0} = N + (P^* - c)X_\delta = N + (P^* - c)N_P = N - \frac{X}{X_P}N_P \overset{sign}{=} -\frac{\partial}{\partial P}\frac{X}{N}$$

(where every term on the right-hand side of the above is evaluated at $\delta = 0$). Here, the third equality follows from the first-order condition for the optimality of P^*. Thus, introducing a bundle discount raises profits if average demand per consumer, X/N, falls with price when $\delta = 0$. More exactly, if the firm offers linear price p for either item (and no bundle discount), write $x_1(p)$ and $x_2(p)$ respectively for proportion of consumers who buy only one item and who buy two items. Since when $\delta = 0$ we have

$$\frac{X}{N} = \frac{x_1 + 2x_2}{x_1 + x_2} = 1 + \frac{x_2/x_1}{1 + x_2/x_1} \ ,$$

the condition requires that the ratio x_2/x_1 decreases with price, so that demand for a single item is less elastic than demand for the bundle. (This discussion presumes that there is some two-item demand, so that $x_2 > 0$.) We summarize this result as:

Result (Long, 1984): *Suppose an integrated monopolist supplies two symmetric products. The firm has an incentive to introduce a discount for buying the bundle whenever the elasticity of demand for buying a single item is lower than the elasticity of demand for* *buying both items, so that*

$$-\frac{x_1'(p)}{x_1(p)} < -\frac{x_2'(p)}{x_2(p)} \; . \tag{2}$$

In economic terms, the elasticity condition (2) is intuitive. If the firm initially charges the same price for buying a single item as for buying a second item, and if demand for the latter is more elastic than demand for the former, then the firm would like to reduce its price for buying a second item.

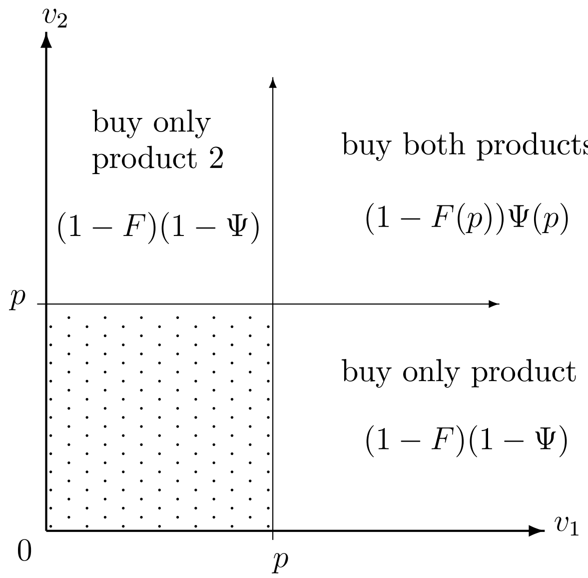

Figure 1: Pattern of demand with additive valuations

Consider the knife-edge case where a consumer's value for the bundle is simply the sum of her individual stand-alone values. That is, the stand-alone value for product $i = 1, 2$ is v_i and her value for the bundle is $v_1 + v_2$. With additive values, if the firm offers the linear price p for buying either item the consumer's buying decision is simple: she should buy product i whenever $v_i \geq p$, as shown on Figure 1. Suppose that the marginal c.d.f. for either value v_i is $F(v_i)$. A useful way to capture the extent of correlation in values is the function

$$\Psi(p) \equiv \Pr\{v_2 \geq p \mid v_1 \geq p\} \; . \tag{3}$$

Then, as shown on the figure, we have

$$x_1(p) = 2(1 - F(p))(1 - \Psi(p)) \; ; \; x_2(p) = (1 - F(p))\Psi(p) \; .$$

It follows that (2) holds whenever

$$\Psi(p) \text{ is strictly decreasing in } p \ . \tag{4}$$

Clearly, condition (4) holds if v_1 and v_2 are independently distributed, but it also applies much more widely. Indeed, the beauty of Long's approach is that condition (2) applies just as well to situations in which valuations are not additive, as we discuss in more detail in the following analysis.

3 Bundling Revisited

Consider a market with two products, labeled 1 and 2, where there is a constant marginal cost of supplying product i equal to c_i. Depending on the context, we will consider situations where a monopolist supplies both products, as well as situations where the two products are supplied by separate firms. Each consumer wishes to buy either zero or one unit of each product. A consumer is willing to pay v_i for product $i = 1, 2$ on its own, and to pay v_{12} for the bundle of both products. Thus a consumer's preferences are entirely described by the vector (v_1, v_2, v_{12}), and this vector is distributed across the population of consumers according to some known distribution. Unlike most of the bundling literature, we allow for non-additive preferences so that $v_{12} \neq v_1 + v_2$. We say that a consumer views the two products as (partial) substitutes whenever $v_{12} \leq v_1 + v_2$. Whenever there is free disposal (so that a consumer can discard one item without incurring any cost), we require that $v_{12} \geq \max\{v_1, v_2\}$ for all consumers.

Consumers face three relevant prices: p_1 is the price for consuming product 1 on its own; p_2 is the price for product 2 on its own, and $p_1 + p_2 - \delta$ is the price for consuming the bundle of both products. Thus, δ is the discount for buying both products (which is zero if there is a linear price for each product, or negative if consumers are charged a premium for joint consumption). A consumer will choose the option which leaves her with the highest net surplus, i.e., she will buy both items whenever

$$v_{12} - [p_1 + p_2 - \delta] \geq \max\{v_1 - p_1, v_2 - p_2, 0\} \ ,$$

she will buy product $i = 1, 2$ on its own whenever

$$v_i - p_i \geq \max\{v_{12} - [p_1 + p_2 - \delta], v_j - p_j, 0\} \ ,$$

and otherwise she will buy nothing.

As functions of the three tariff parameters (p_1, p_2, δ), denote by Q_1 the proportion of potential consumers why buy only product 1, Q_2 the proportion of consumers who buy only product 2, and Q_{12} is the proportion of consumers who buy both products. It will also be useful to define demand functions when no discount is offered, so let $q_i(p_1, p_2) \equiv Q_i(p_1, p_2, 0)$ and $q_{12}(p_1, p_2) \equiv Q_{12}(p_1, p_2, 0)$ be the corresponding demand functions when $\delta = 0$. Products are gross substitutes if total demand for product i, $q_i + q_{12}$, is increasing with the other product's price p_j. Products are gross complements if $q_i + q_{12}$ decreases with p_j. In this paper we focus on the case with substitutes, although parallel analysis often applies when products are complements. As one would expect, if all consumers view products as partial substitutes, products are then gross substitutes:

Lemma 1: *Suppose that $v_{12} \leq v_1 + v_2$ for all consumers. Then demand for product i, $q_i + q_{12}$, weakly increases with p_j.*

Proof: Suppose that $\delta = 0$ so that linear prices are used. A type (v_1, v_2, v_{12}) consumer buys product 1 if and only if

$$\max\{v_{12} - p_1 - p_2, v_1 - p_1\} \geq \max\{v_2 - p_2, 0\} \ . \tag{5}$$

The left-hand side is the consumer's maximum surplus if she buys product 1 (either as part of a bundle or on its own), while the right-hand side is the consumer's maximum surplus if she does not buy product 1. We claim that difference between the two sides in (5), that is

$$\max\{v_{12} - p_1 - p_2, v_1 - p_1\} - \max\{v_2 - p_2, 0\} \ , \tag{6}$$

is weakly increasing in p_2. (This then implies that the set of consumer types who buy product 1 is increasing, in the set-theoretic sense, in p_2, and so in particular the measure of such consumers is increasing in p_2.) The only way in which expression (6) could strictly decrease with p_2 is if

$$v_{12} - p_1 - p_2 > v_1 - p_1 \text{ and } v_2 - p_2 < 0 \ .$$

However, since products are substitutes we have $v_{12} \leq v_1 + v_2$, which implies that the above pair of inequalities are contradictory. This establishes the result. ■

Importantly, when a bundle discount is offered, this result can be reversed. That is to say, if products are partial substitutes then when a bundle discount is offered, the demand for a product can *decrease* with the stand-alone price of the other product. For instance, if there is a fixed disutility from joint consumption in the sense that $v_{12} \equiv v_1 + v_2 - z$, then if $\delta > z$ the bundle discount outweighs the disutility z and the net result is that the products act like complements, not substitutes, in terms of cross-price elasticities. The observation that a bundle discount can mitigate or overturn the innate substitutability of products will play a major role in the following analysis.

Regardless of whether the underlying products are complements or substitutes, the three discrete purchasing *options* (buy product 1 only, buy product 2 only, or buy both products) are necessarily substitutes, in the sense that cross-price effects are non-negative:

$$\frac{\partial Q_i}{\partial \delta} \leq 0 \ ; \frac{\partial Q_j}{\partial p_i} + \frac{\partial Q_j}{\partial \delta} \geq 0 \ ; \frac{\partial Q_{12}}{\partial p_i} + \frac{\partial Q_{12}}{\partial \delta} \geq 0 \ . \tag{7}$$

(Concerning the second and third inequalities here, note that if price p_i and discount δ rise by the same amount, the price for the bundle is unchanged but the stand-alone price for item i rises.) We also necessarily have symmetry of cross-price effects:

$$\frac{\partial Q_2}{\partial p_1} + \frac{\partial Q_2}{\partial \delta} \equiv \frac{\partial Q_1}{\partial p_2} + \frac{\partial Q_1}{\partial \delta} \ ; \ \frac{\partial Q_{12}}{\partial p_i} + \frac{\partial Q_{12}}{\partial \delta} \equiv -\frac{\partial Q_i}{\partial \delta} \ . \tag{8}$$

Note that the right-hand expression in (8) implies that

$$\left. \frac{\partial (Q_i + Q_{12})}{\partial \delta} \right|_{\delta=0} = -\frac{\partial q_{12}}{\partial p_i} \ , \tag{9}$$

so that the impact of a small bundle discount on the total demand for product i is equal to the impact of a corresponding price cut on the demand for the bundle. To avoid tedious caveats involving corning solutions, suppose that over the relevant range of linear prices there is some two-item demand, so that $q_{12} > 0$.

An integrated monopolist: Suppose an integrated monopolist supplies both products. The firm's profit with bundling tariff (p_1, p_2, δ) is

$$\pi = (p_1 - c_1)(Q_1 + Q_{12}) + (p_2 - c_2)(Q_2 + Q_{12}) - \delta Q_{12} \ . \tag{10}$$

Given that the three purchase options are substitutes, the most profitable bundling tariff will involve above-cost pricing for each option, so that

$$p_i \geq c_i \ ; \ p_i + p_2 - \delta \geq c_1 + c_2 \ .$$

Consider the firm's incentive to offer a bundling discount. Starting from any pair of linear prices (p_1, p_2), by differentiating (10) we see that the impact on profit of introducing a small discount $\delta > 0$ is

$$\left.\frac{\partial \pi}{\partial \delta}\right|_{\delta=0} = \left.\left\{(p_1 - c_1)\frac{\partial}{\partial \delta}(Q_1 + Q_{12}) + (p_2 - c_2)\frac{\partial}{\partial \delta}(Q_2 + Q_{12}) - Q_{12}\right\}\right|_{\delta=0}$$
$$= -(p_1 - c_1)\frac{\partial q_{12}}{\partial p_1} - (p_2 - c_2)\frac{\partial q_{12}}{\partial p_2} - q_{12} \ , \tag{11}$$

where the second equality follows from (9). Let (p_1^*, p_2^*) be the most profitable linear prices. Therefore,

$$(p_1^*, p_2^*) \text{ maximizes } (p_1 - c_1)(q_1 + q_{12}) + (p_1 - c_2)(q_2 + q_{12}) \ ,$$

which has first-order condition for p_i^* given by

$$q_i + q_{12} + (p_1^* - c_1)\frac{\partial}{\partial p_i}(q_1 + q_{12}) + (p_2^* - c_2)\frac{\partial}{\partial p_i}(q_2 + q_{12}) = 0 \ . \tag{12}$$

If the products are gross substitutes, both price-cost margins are positive, and in particular $(p_2^* - c_2)\frac{\partial}{\partial p_1}(q_2 + q_{12}) \geq 0$ and $(p_1^* - c_1)\frac{\partial}{\partial p_2}(q_1 + q_{12}) \geq 0$. The first-order condition (12) therefore implies that

$$p_i^* - c_i \geq \frac{q_i + q_{12}}{-\partial(q_i + q_{12})/\partial p_i} \quad for\ i = 1, 2 \ . \tag{13}$$

Substituting this pair of inequalities into (11) shows that offering a bundle discount is profitable whenever condition (14) holds, as summarized in this result:

Proposition 1: *Suppose that products are gross substitutes and that*

$$\frac{q_1 + q_{12}}{q_{12}} \frac{\partial q_{12}/\partial p_1}{\partial(q_1 + q_{12})/\partial p_1} + \frac{q_2 + q_{12}}{q_{12}} \frac{\partial q_{12}/\partial p_2}{\partial(q_2 + q_{12})/\partial p_2} > 1 \ . \tag{14}$$

Then the integrated monopolist has an incentive to offer a discount when its customers buy both products.

Condition (14) is satisfied when demand for the bundle is not "too much" less elastic than the overall demand for each product. A simple sufficient condition for (14) to hold is that each term on the left-hand side is greater than a half, so that a price rise which causes demand for a particular product to fall by 10% causes demand for the bundle to fall by more than 5%.

Two separate sellers: Next, suppose that each product is supplied by a separate seller. When firms offer linear prices—i.e., prices which do not depend on whether the consumer also purchases the other product—firm i chooses its price p_i^*, given its rival's price, to maximize $(p_i - c_i)(q_i + q_{12})$. It some circumstances, a firm can condition its price on whether a consumer also buys the rival product. For instance, a museum could ask a visitor to show her entry ticket to the other museum to claim a discount. The next result describes when a firm has a unilateral incentive to offer a discount when a customer also buys the other firm's product.

Proposition 2: *Suppose that demand for the bundle is more elastic than demand for firm i's stand-alone product, i.e., that*

$$-\frac{1}{q_{12}}\frac{\partial q_{12}}{\partial p_i} > -\frac{1}{q_i}\frac{\partial q_i}{\partial p_i} \ . \tag{15}$$

Starting from the situation where both firms set the equilibrium linear prices p_1^* and p_2^*, firm i has an incentive to offer a discount on its product to those consumers who buy product j.

Proof: Firm i's equilibrium linear price p_i^*maximizes $(p_i - c_i)(q_i + q_{12})$, so that

$$0 = q_i\left[1-(p_i^*-c_i)\frac{-\partial q_i/\partial p_i}{q_i}\right] + q_{12}\left[1-(p_i^*-c_i)\frac{-\partial q_{12}/\partial p_i}{q_{12}}\right] \ . \tag{16}$$

Suppose now that firm i offers a discount $\delta_i > 0$ from its price p_i^* to those consumers who purchase product j as well. (Those consumers who only buy product i continue to pay p_i^*.) Then firm i's profit is

$$\pi_i = (p_i^* - \delta_i - c_i)(Q_i + Q_{12}) - \delta_i Q_{12} \ , \tag{17}$$

and the impact of a small joint purchase discount is governed by the sign of $\left.\frac{d\pi_i}{d\delta_i}\right|_{\delta=0}$, which from (9) is equal to

$$-q_{12} - (p_i^* - c_i)\frac{\partial q_{12}}{\partial p_i} \ . \tag{18}$$

When (15) holds, the second term $[\cdot]$ in (16) must be strictly negative, i.e., expression (18) is strictly positive. Therefore, offering a small discount for joint purchase will raise the firm's profit. ■

Thus, discounts for joint purchase can arise even when products are supplied by separate firms and when a firm chooses, and funds, the discount unilaterally. The reason for this is straightforward: since demand for the bundle is more elastic than demand for its stand-alone product, a firm wants to offer a lower price to those consumers who also buy the other product.

If condition (15) holds for firm i, then demand for the bundle is more elastic than total demand for that firm's product, and so

$$\frac{q_i + q_{12}}{q_{12}} \frac{\partial q_{12}/\partial p_i}{\partial (q_i + q_{12})/\partial p_i} > 1 \ .$$

Therefore, condition (14) applies (at least for the same pair of linear prices), and so whenever at least one separate seller has an incentive to bundle, we expect that an integrated firm does also (but not necessarily *vice versa*).

In asymmetric cases, it is possible that condition (15) holds for one firm but not for the other. Thus, one firm has an incentive to offer a joint purchase discount when its customers buy the other product, while the other firm does not. On the other hand, it is possible that both firms wish to offer a lower price to its customers when they buy the rival product. If firm i offers the price $p_i - \delta_i$ when the consumer buys the rival's product, a consumer who buys both products pays the price $p_1 + p_2 - \delta_1 - \delta_2$. The issue arises as to how the "double" joint purchase discount $\delta = \delta_1 + \delta_2$ is implemented. For instance, in many cases a consumer must buy the two items in *order*, and both firms cannot simultaneously require proof of purchase from the other seller when they offer their discount. However, the inter-firm bundling discount could easily be implemented via some kind of joint marketing body or electronic sales platform.[3] Beyond this modest coordination, there is no need for firms to coordinate their actual tariffs.

A major difference between inter-firm bundling discounts and the discount offered by an integrated supplier is that with separate sellers the bundle discount is chosen non-cooperatively. A bundle is, by definition, made up of two "complementary" components, namely, firm 1's product and firm 2's product, and the total price for the bundle, $p_1 + p_2 - \delta_1 - \delta_2$, is the sum of each firm's component price $p_i - \delta_i$. Thus, as usual with separate

[3] For instance, a website could display the total prices for the various options, and firms receive directly the revenue from their stand-alone products as well as their share of the revenue of the bundle. Similarly, TV channels sold via a broadcasting platform could choose the prices for viewing a channel conditional on which other channels are purchased, and then viewers choose the appropriate bundle of channels and pay the stipulated price to each channel.

supply of complementary components, we expect that double marginalization will result and the overall discount will be too *small.*

In more detail, with separate sellers firm i's profit is (17), and so the first-order condition for firm i's joint purchase discount δ_i is

$$0 = (p_i - c_i)\frac{\partial(Q_i + Q_{12})}{\partial \delta_i} - \delta_i \frac{\partial Q_{12}}{\partial \delta} - Q_{12} = -(p_i - c_i)\frac{\partial Q_{12}}{\partial p_i} - \delta_i \frac{\partial Q_{12}}{\partial \delta} - Q_{12}$$

(where the second equality follows from (8)). Adding the corresponding expression for the other firm shows that the total bundle discount $\delta = \delta_1 + \delta_2$ satisfies

$$2Q_{12} = -(p_1 - c_1)\frac{\partial Q_{12}}{\partial p_1} - (p_2 - c_2)\frac{\partial Q_{12}}{\partial p_2} - \delta \frac{\partial Q_{12}}{\partial \delta} \ .$$

This can be written more succinctly as

$$-\frac{1}{Q_{12}} \times \frac{d}{dt} Q_{12}(p_1 + t(p_1 - c_1), p_2 + t(p_2 - c_2), \delta + t\delta)\Big|_{t=0} = 2 \ .$$

In words, this expression states that when the price-cost margins in the bundling tariff are each "amplified" by 1%, say, the resulting decrease in demand for the bundle is 2%.

By contrast, with integrated supply the firm's profit is (10), and so the first-order condition for the bundle discount δ is

$$\begin{aligned} 0 &= (p_1 - c_1)\frac{\partial(Q_1 + Q_{12})}{\partial \delta} + (p_2 - c_2)\frac{\partial(Q_2 + Q_{12})}{\partial \delta} - \delta \frac{\partial Q_{12}}{\partial \delta} - Q_{12} \\ &= -(p_1 - c_1)\frac{\partial Q_{12}}{\partial p_1} - (p_2 - c_2)\frac{\partial Q_{12}}{\partial p_2} - \delta_i \frac{\partial Q_{12}}{\partial \delta} - Q_{12} \ . \end{aligned}$$

Therefore,

$$-\frac{1}{Q_{12}} \times \frac{d}{dt} Q_{12}(p_1 + t(p_1 - c_1), p_2 + t(p_2 - c_2), \delta + t\delta)\Big|_{t=0} = 1$$

and a 1% amplification of the tariff causes demand for the bundle to fall by only 1%. (Of course, the stand-alone prices p_i are different with integrated and separate supply.) The fact that with separate sellers the equilibrium elasticity of demand (broadly interpreted) is so great represents the so-called Cournot effect seen when complementary items are priced non-cooperatively. It implies that the equilibrium bundle discount chosen by separate sellers is too small, and a joint increase in each δ_i would boost each firm's profit (and total welfare).[4]

[4] If each firm's discount δ_i is increased by $\varepsilon > 0$, the change in firm i's profit has the sign of

$$2(p_i - c_i)\frac{\partial(Q_i + Q_{12})}{\partial \delta_i} - 2\delta_i \frac{\partial Q_{12}}{\partial \delta} - Q_{12} = Q_{12} > 0 \ ,$$

where the equality follows from the first-order condition for firm i's choice of discount.

Without specifying consumer tastes in more detail, it is hard to derive further results. In the next sections—which cover respectively the cases of integrated and separate supply—we specialise the framework in various ways to obtain further insight.

4 Integrated Supply

For maximum transparency of the analysis, suppose now that the two products are symmetric, so that $c_1 = c_2 = c$ and the same density of consumers have taste vector (v_1, v_2, v_{12}) as have the permuted taste vector (v_2, v_1, v_{12}). As in section 2, let $x_1(p)$ denote the proportion of consumers who buy a single item when the price for either item is p and let $x_2(p)$ denote the proportion of consumers who buy both items when the linear price is p. Then section 2 shows that an integrated monopolist wishes to introduce a bundle discount whenever (2) holds. In the special case of additive utility, the condition requires that (4) be satisfied.

Suppose that the products are substitutes, so that $v_{12} \leq v_1 + v_2$ for all consumers. For a type (v_1, v_2, v_{12}) consumer, define

$$v_{[1]} = \max\{v_1, v_2\}$$

to be her maximum utility if she buys only one item, and

$$v_{[2]} = v_{12} - \max\{v_1, v_2\}$$

to be her incremental utility from buying two items rather than one. The assumption that products are substitutes implies that

$$v_{[2]} = v_{12} - \max\{v_1, v_2\} \leq v_1 + v_2 - \max\{v_1, v_2\} = \min\{v_1, v_2\} \leq v_{[1]}$$

so that the support of $(v_{[1]}, v_{[2]})$ lies under the 45^0 line, as shown on Figure 2. Note that, by construction, we have $v_{12} = v_{[1]} + v_{[2]}$, so that valuations are additive after this change of variables.

With a linear price p for either item, a type $(v_{[1]}, v_{[2]})$ consumer will buy both items whenever $v_{[2]} \geq p$, and will buy only one item whenever $v_{[1]} \geq p$ and $v_{[2]} < p$, as depicted on the figure. As in expression (3), define

$$\Phi(p) \equiv \Pr\{v_{[2]} \geq p \mid v_{[1]} \geq p\} \ . \tag{19}$$

If we write $G(P) \equiv \Pr\{v_{[1]} \le P\}$ for the marginal c.d.f. for $v_{[1]}$, by examining Figure 2, we see that[5]

$$x_1(p) = (1 - G(p))(1 - \Phi(p)) \ ; \ x_2(p) = (1 - G(p))\Phi(p) \ . \tag{20}$$

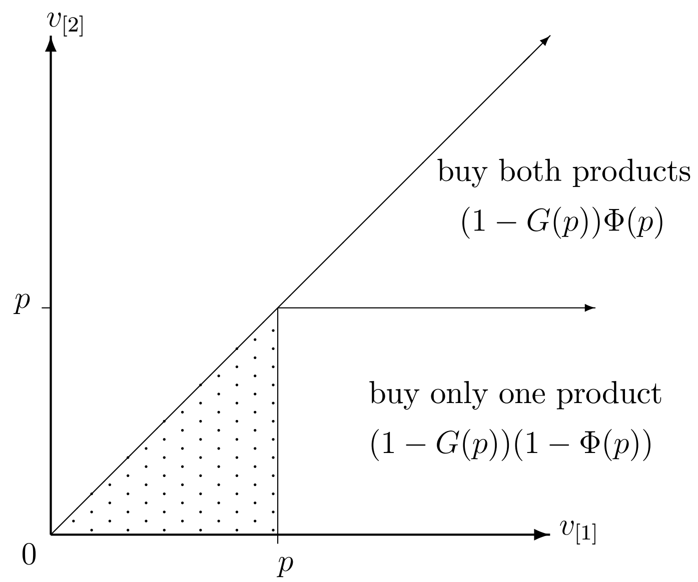

Figure 2: Pattern of demand with substitutes

It follows immediately that when Φ is decreasing condition (2) holds, and we can deduce the following generalization of Long's original condition (4) to the case where products are partial substitutes:

Proposition 3: *Suppose products are substitutes and Φ in (19) is strictly decreasing. Then an integrated monopolist has an incentive to offer a bundle discount.*

When products are partial substitutes rather than independent, this often makes the integrated firm's demand less elastic. From (20) we see that the firm's total demand with linear price p is $(1 - G(p))(1 + \Phi(p))$. Consider for instance the two polar cases where valuations are additive and where each consumer wants only a single item. In the former case, $v_{[2]} = \min\{v_1, v_2\}$, and Figure 1 shows that

$$\Phi(p) = \Psi(p)/(2 - \Psi(p)) \ , \tag{21}$$

[5]In terms of the marginal c.d.f. $F(\cdot)$ and correlation function $\Psi(\cdot)$ in (3), we have

$$1 - G(p) = (1 - F(p))(2 - \Psi(p)) \ .$$

and the firm's most profitable price maximizes $(p-c)(1-G(p))(1+\Psi(p)/(2-\Psi(p)))$. In the latter case, $v_{[2]}=0$ and $\Phi(p)=0$, so the firm chooses p to maximize $(p-c)(1-G(p))$. A revealed preference argument shows that the latter price is higher than the former whenever Ψ is decreasing. More generally, as illustrated in Figure 4 below, we expect that more pronounced substitutability between products will usually induce the integrated firm to set a higher linear price. (Interestingly, when products are *complements*, this also tends to induce the integrated firm to choose a higher linear price compared to when values are additive. We discuss this in more detail when the particular case of a constant disutility of joint consumption is analyzed.)

To illustrate, consider an example where the stand-alone values (v_1, v_2) are uniformly distributed on the unit square $[0,1]^2$ and given (v_1, v_2) the bundle value v_{12} is uniformly distributed on the interval $[\max\{v_1, v_2\}, v_1+v_2]$. (With free disposal we require that v_{12} be at least $\max\{v_1, v_2\}$, and we require that $v_{12} \le v_1 + v_2$ if products are substitutes.) Then the density for $v_{[2]}$ given $v_{[1]}$, where $v_{[2]} \le v_{[1]}$, is

$$\frac{1}{v_{[1]}}\log\frac{v_{[1]}}{v_{[2]}} \ .$$

Therefore, if $p < v_{[1]}$ then

$$\Pr\{v_{[2]} \ge p \mid v_{[1]}\} = 1 - \frac{p}{v_{[1]}}\left(\log v_{[1]} - \log p + 1\right) \ .$$

Therefore

$$\Phi(p) = 1 + \frac{2p\log p}{1-p^2} \ , \tag{22}$$

which is indeed a decreasing function, and so the integrated firm will wish to offer a bundle discount.

If $c = 0$, the integrated monopolist's most profitable linear price maximizes $p(1-G(p))(1+\Phi(p))$, where $G(p) = p^2$. It follows that the optimal linear price is approximately $p \approx 0.540$, which yields industry profit of 0.406. Note that about 70% of potential consumers buy something given this price, although only 4% of consumers buy both items. A more laborious calculation shows that the firm's optimal bundling tariff is

$$p \approx 0.527 \ ; \ \delta \approx 0.149 \tag{23}$$

which yields slightly higher industry profit 0.415. Notice that, compared to the corresponding example with additive values, the bundling discount is far less pronounced.[6]

[6]When $c_1 = c_1 = 0$, (v_1, v_2) is uniformly distributed on $[0,1]^2$ and $v_{12} \equiv v_1 + v_2$, then one can check that $p = \frac{2}{3}$ and $\delta = \frac{\sqrt{2}}{3} \approx 0.47$.

In this example, bundling acts to reduce *all* prices paid by consumers, unlike the additive case where the stand-alone price typically rises when bundling is used. Some intuition for this goes as follows. As discussed earlier in this section, a more pronounced substitutability between products usually leads the firm to raise its linear price (as it does in this example). Proposition 3 demonstrates that an integrated firm very often wishes to introduce a bundle discount. But a bundle discount acts to mitigate, or even overturn, the impact of substitution, since the reduced price for the second product reduces or reverses the reduced incremental utility due to substitution. Thus, the use of a bundle discount acts endogenously to weaken product substitutability, and this in turn can lead the firm to reduce its (stand-alone) price.

In the following we consider two special forms of product substitutability. In the first there is a constant disutility of joint consumption, while the second supposes that a fraction of consumers are constrained to buy only one item. (We will revisit these special cases when we consider supply by separate sellers.) As before, let $F(.)$ denote the marginal c.d.f. for either stand-alone valuation v_i, and in the following special cases we assume that F has an increasing hazard rate, so that

$$\frac{f(v)}{1-F(v)} \text{ strictly increases with } v \ . \tag{24}$$

Constant disutility of joint consumption: Consider the situation in which for all consumers we have

$$v_{12} = v_1 + v_2 - z \tag{25}$$

for some constant $z > 0$. Here, to ensure free disposal we need to assume that the minimum possible realization of v_i is greater than z. Then with a linear price p_i for buying product i, the pattern of demand is as shown on Figure 3.[7] The next result provides a sufficient condition for bundling to be profitable in this setting:

Proposition 4: *Suppose that bundle valuations are given by (25). Then an integrated monopolist has an incentive to offer a bundle discount whenever condition (4) holds.*

[7] Note that the pattern of demand with linear pricing and a disutility of joint consumption $z > 0$ is the same as that corresponding to additive valuations and a tariff *premium* for buying both items. (The latter is illustrated in Figure 8 in Long (1984).) Thus, just as a bundle discount can convert independent products into complements, a bundle premium converts these products into substitutes.

Proof: From Figure 3 we see that with linear price p for either product we have

$$x_2(p) = (1 - F(p+z))\Psi(p+z) \ ; \ x_1(p) = (1 - F(p))(2 - \Psi(p)) - x_2(p) \ ,$$

and so (19) is given by

$$\Phi(p) = \frac{x_2(p)}{x_1(p) + x_2(p)} = \frac{(1 - F(p+z))\Psi(p+z)}{(1 - F(p))(2 - \Psi(p))} \ .$$

Differentiating shows that Φ is strictly decreasing if and only if

$$\frac{\Psi'(p)}{2 - \Psi(p)} + \frac{\Psi'(p+z)}{\Psi(p+z)} < \frac{f(p+z)}{1 - F(p+z)} - \frac{f(p)}{1 - F(p)} \ .$$

Since F is assumed to have an increasing hazard rate, the right-hand side of the above is positive, while if condition (4) holds then the left-hand side is negative. Therefore, Φ is strictly decreasing and Proposition 3 implies the result. ■

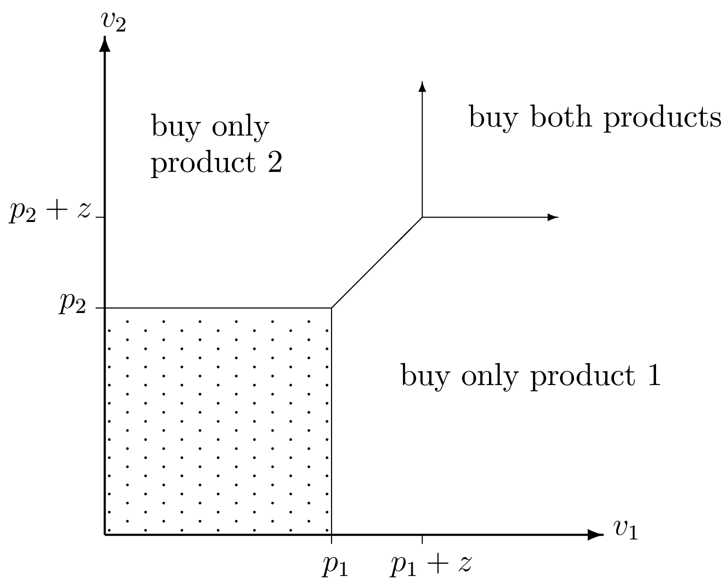

Figure 3: Pattern of demand with constant disutility of joint purchase

To illustrate, suppose that (v_1, v_2) is uniformly distributed on the unit square $[1, 2]^2$, that $z = \frac{1}{2}$ and that $c = 1$.[8] Then an integrated monopolist which uses linear prices will choose price

$$p = 1 + \frac{1}{\sqrt{3}} \approx 1.58 \ ,$$

[8]This example gives rise to a linear demand system when linear prices are used, and when prices are such that there is some two-item demand and some consumers who buy nothing, Figure 3 shows that the total demand for product i is equal to $k - p_i + \frac{1}{2}p_j$ for a constant k.

and this generates profit $\frac{2}{3\sqrt{3}} \approx 0.385$. Since $p + z > 2$, at this price there is no two-item demand at all (see Figure 3). By contrast, the most profitable bundling tariff is

$$p = 1 + \frac{1}{\sqrt{3}} \approx 1.58 \; ; \; \delta = \frac{1}{\sqrt{3}} - \frac{1}{6} \approx 0.41 \; , \tag{26}$$

which generates profit of about 0.403, and one in nine consumers buy both items. In particular, and similarly to the example in (23) above, the use of bundling means that the firm weakly lowers all its prices, thus boosting both consumer surplus and total welfare.[9] However, in contrast to that earlier example, the size of the bundle discount here is not much lower than the corresponding situation without substitutes (i.e., when $z = 0$, in which case the bundle discount is $\delta = \frac{\sqrt{2}}{3} \approx 0.47$).

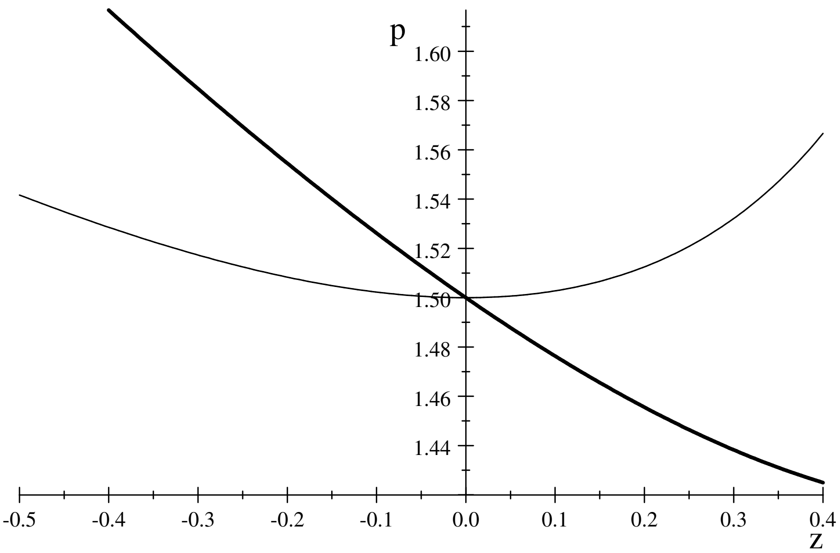

Figure 4: The equilibrium linear price given z (bold line corresponds to seperate sellers, feint line corresponds to the integrated firm)

This example shows again how the integrated firm sets a higher linear price when products are substitutes compared to when values are additive. (If $z = 0$ in this example, then the most profitable linear price is $p = 1.5$.) It turns out that price also rises in the corresponding setting when products are partial complements, i.e., when $z < 0$ in (25).[10] (When $z < 0$, then the pattern of demand is similar to that shown in Figure 6 below.) The feint line in Figure 4 shows the integrated firm's optimal linear price in the example

[9]Note that in this example, since there is no two-item demand with linear pricing, the firm has no *local* incentive to introduce a bundle discount, although it does have a global incentive to do so.

[10]The observation that the linear price is lowest when values are additive was also made in a slightly different framework by Venkatesh and Kamakura (2003). (See their Figure 2.)

where (v_1, v_2) is uniformly distributed on $[1,2]^2$ and $c = 1$ for a range of positive and negative values for z, and it can be seen that price is lowest when preferences are additive. (The bold line shows the equilibrium price with separate sellers, which is monotonically decreasing in the degree of substitution, z.) This observation helps to explain why when values are additive, the integrated firm's stand-alone price typically rises when bundling is used. When values are additive, the firm wishes to offer a bundle discount unless values are strongly positively correlated. When it chooses discount $\delta > 0$, this converts the products into complements (i.e., $z = -\delta$ in this particular setting), and this in turn induces the firm to raise its stand-alone price.

Time-constrained consumers: A natural reason why products might be substitutes is that some buyers are only able to consume a restricted set of products, e.g., due to time constraints.[11] For instance, a tourist may have the time only to visit a single museum in a city. To that end, suppose that an exogenous fraction λ of consumers have valuation v_i for stand-alone product $i = 1, 2$ and valuation $v_{12} = v_1 + v_2$ for the bundle, while the remaining consumers can only buy a single item (and have valuation v_i if they buy item i). For simplicity, suppose that the distribution for (v_1, v_2) is the same for the two groups of consumers. Let the marginal c.d.f. for each v_i be $F(v)$, and let $\Psi(\cdot)$ be as defined in (3). (See Figure 5 for an illustration.)

The central feature of this scenario is that the time-constrained consumers have zero incremental value for the second item (so for them $v_{[2]} = 0$). It is then straightforward to show that

$$\Phi(p) = \lambda \frac{\Psi(p)}{2 - \Psi(p)} \ ,$$

so that Φ is decreasing if and only if Ψ is decreasing. Proposition 3 therefore has the corollary:

Proposition 5: *When some consumers are time-constrained, an integrated firm has an incentive to offer a bundle discount whenever (4) holds, i.e., under the same conditions as when all consumers have additive preferences.*

[11] In the context of competitive *intra*-firm bundling, Thanassoulis (2007) also analyzes the situation where an exogenous fraction of consumers wish to buy a single product.

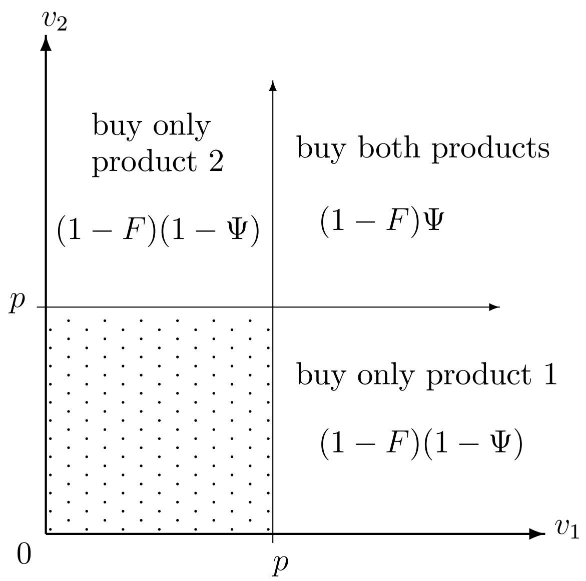

Figure 5A: Additive valuations

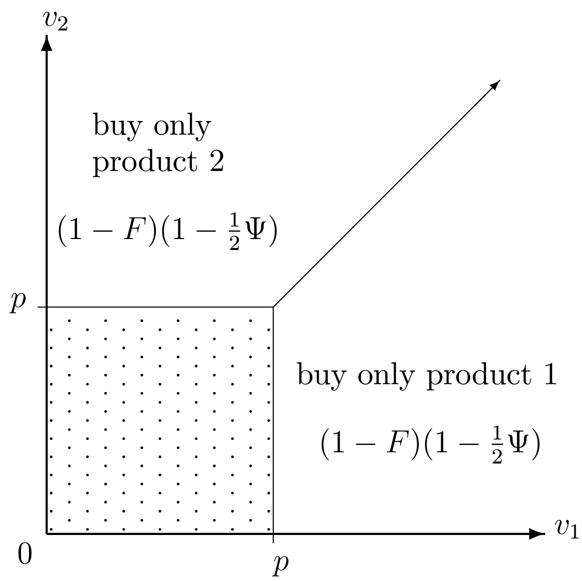

Figure 5B: Time-constrained consumers

Summary: In this section we focussed on the case when an integrated firm supplies products which are partial substitutes. We derived a general condition (Proposition 3) which governs when the firm wishes to offer a bundle discount, and a number of special cases were solved. We saw no evidence that product substitutability made the firm less inclined to offer a bundle discount. (In the case of time constrained consumers, the condition governing when bundling is used was exactly the same as when values were additive, and when there was a fixed disutility of joint consumption, bundling was profitable in more cases relative to the additive case.) No systematic evidence was seen as to whether the bundle discount was significantly smaller than the corresponding case with additive utility. When products are substitutes, we saw that the use of bundling often led the firm to reduce all prices compared to when linear pricing was used. Intuitively, the bundle discount reduces the importances of substitutability, which induces the firm to lower its stand-alone price. By contrast, when values are additive, the use of a bundle discount converts independent products into complements, and this often induces the firm to raise its stand-alone price.

5 Separate Sellers

In this section we turn to the situation where the two products are supplied by separate sellers. We first consider the situation where the sellers do not compete, in the sense that valuations are additive. We then consider the two special cases where consumers have a constant disutility of joint consumption and where some consumers are time-constrained.

Additive valuations: Consider first the special case where valuations are additive, so that $v_{12} = v_1 + v_2$ for all consumers. With separate sellers, there is no particular benefit in assuming that the products are symmetric. Let $F_i(v_i)$ and $f_i(v_i)$ be respectively the marginal c.d.f. and the marginal density for v_i, and define

$$H_i(p_i \mid v_j) = \Pr\{v_i \leq p_i \mid v_j\}$$

to be the conditional c.d.f. for value v_i when the other value is v_j. The next result provides a sufficient condition for a firm to offer a discount to those of its customers who buy the other product:

Proposition 6: *Suppose that valuations are additive. Starting from the situation where firms set equilibrium linear prices, firm i has an incentive to offer a discount to those consumers who buy the other product whenever $H_j(p_j \mid v_i)$ strictly increases with v_i (where p_j is firm j's equilibrium linear price).*

Proof: From Figure 1, we see that

$$q_i(p_i, p_j) = \int_{p_i}^{\infty} H_j(p_j \mid v_i) f_i(v_i) dv_i \ ; \ q_{12}(p_i, p_j) = \int_{p_i}^{\infty} (1 - H_j(p_j \mid v_i)) f_i(v_i) dv_i \qquad (27)$$

and

$$-\frac{\partial q_i}{\partial p_i} = H_j(p_j \mid p_i) f_i(p_i) \ ; \ -\frac{\partial q_{12}}{\partial p_i} = (1 - H_j(p_j \mid p_i)) f_i(p_i) \ .$$

Since H_j is assumed to be strictly increasing in v_i, it follows from (27) that

$$q_i(p_i, p_j) > H_j(p_j \mid p_i)(1 - F_i(p_i)) \ ; \ q_{12}(p_i, p_j) < (1 - H_j(p_j \mid p_i))(1 - F_i(p_i))$$

and so

$$-\frac{1}{q_i} \frac{\partial q_i}{\partial p_i} < \frac{f_i(p_i)}{1 - F_i(p_i)} < -\frac{1}{q_{12}} \frac{\partial q_{12}}{\partial p_i}$$

and Proposition 2 implies the result. ■

Thus, whenever the valuations are negatively correlated, in the strong sense that $H_j(p_j \mid v_i)$ strictly decreases with v_i, a firm has an incentive to offer a discounted price for joint purchase. It is intuitive that negative correlation is associated with the incentive to engage in inter-firm bundling when valuations are additive (see Schmalensee, 1982, for earlier discussion of this point). If firm i knows that a potential consumer has purchased firm j's product, i.e., the consumer has a relatively high value for item j, then negative correlation implies that this is bad news for the consumer's likely value for i's product, and this will usually induce the firm to lower its price to this consumer. By contrast, if there is no correlation in the values for the two items (so that H_j does not depend on v_i), the observation that a consumer has purchased item j gives no reason for firm i to adjust its price.

In the remainder of this section, the two special cases analyzed in section 7 are considered in the context of separate supply. In contrast to the case with integrated supply, here the two examples have very different implications for a firm's unilateral incentive to offer a joint purchase discount.

Constant disutility of joint consumption: Here, the pattern of consumer demand is as illustrated in Figure 3. Note first that a more pronounced substitutability between products, in the sense that z increases, tends to cause a firm's demand to become more elastic, since the competitive frontier in Figure 3 (the upward-sloping margin between consumers who buy only product 1 and consumers who buy only product 2) lengthens. Thus, we expect that competing firms will then set lower linear prices. This is illustrated as the bold line on Figure 4 above, in the example where (v_1, v_2) is uniform on $[1, 2]^2$ and $c = 1$. This implies that with separate sellers the typical impact of more pronounced substitutability is the opposite to that when an integrated firms supplies both products (although in the example in the region $z < 0$ where products are complements, the price with separate sellers and with integrated monopoly both increase with the degree of complementarity.)

For simplicity, we focus on the situation where v_1 and v_2 are identically and independently distributed. (From Proposition 6, we already know that negative correlation will tend to give an incentive to offer a unilateral bundle discount.) The next result shows that a firm typically does have a unilateral incentive to offer a bundle discount.

Proposition 7: *Suppose that v_1 and v_2 are identically and independently distributed with c.d.f. satisfying (24) and that the bundle valuations satisfy (25). When the two products are supplied by separate sellers, each seller has an incentive to offer a discount to those* *consumers who buy the rival product.*

Proof: If F and f are respectively the c.d.f. and density for each valuation v_i, by examining Figure 3 we see that

$$-\frac{\partial q_{12}}{\partial p_1} = f(p+z)(1-F(p+z))$$

and

$$-\frac{\partial q_1}{\partial p_1} = f(p)F(p) + \int_p^{p+z} (f(v))^2 dv$$

(where these derivatives are evaluated at symmetric prices $p_1 = p_2 = p$). At the symmetric price p we have

$$q_{12} = (1-F(p+z))^2 \ ; \ q_1 = \tfrac{1}{2}\left(1-(F(p))^2-(1-F(p+z))^2\right) \ .$$

We need to show that inequality (15) holds so that Proposition 2 can be applied.

Since F has an increasing hazard rate in (24), we have

$$\begin{aligned}\int_p^{p+z} (f(v))^2 dv &= \int_p^{p+z} \frac{f(v)}{1-F(v)} f(v)(1-F(v))dv \\ &\leq \frac{f(p+z)}{1-F(p+z)} \int_p^{p+z} f(v)(1-F(v))dv \\ &= \frac{1}{2}\frac{f(p+z)}{1-F(p+z)}\left((1-F(p))^2-(1-F(p+z))^2\right) \ .\end{aligned}$$

Therefore, a sufficient condition for (15) to hold is that

$$\frac{f(p+z)}{1-F(p+z)} > \frac{2f(p)F(p) + \frac{f(p+z)}{1-F(p+z)}\left((1-F(p))^2-(1-F(p+z))^2\right)}{1-(F(p))^2-(1-F(p+z))^2}$$

which can be rearranged to give

$$\frac{f(p+z)}{1-F(p+z)} > \frac{f(p)}{1-F(p)} \ .$$

Since F has a strictly increasing hazard rate, the claim is established. ■

It is economically intuitive that products being substitutes of the form (25) will give an incentive to a firm to offer a discount when its customers have purchased the rival product. If the potential customer has already purchased the other product, this is bad news for the firm as the customer's incremental value for its product has been shifted downwards by z, and typically this will give an incentive to offer the customer a lower price. (See Lewbel, 1985, for earlier discussion of this point.)

Consider the same example as presented in section 4 (that is, (v_1, v_2) uniform on $[1, 2]^2$, $z = \frac{1}{2}$ and $c = 1$) applied to the case with separate sellers. The equilibrium with linear pricing has price $p = 17/12 \approx 1.417$ and industry profit is about 0.347. Consumer surplus is around 0.274. Less than 1% of consumers buy both items with this linear price. Numerical calculations show that the equilibrium inter-firm bundling tariff is

$$p_1 = p_2 = 1.454 \; ; \; \delta_1 = \delta_2 = 0.102 \; .$$

Thus, the discount $\delta = \delta_1 + \delta_2$ when a consumer buys the second product is about 15% of the stand-alone price. This bundle discount is approximately half the discount with integrated supply (see expression (26) above), reflecting the discussion in section 3 that separate firms will unilaterally choose too small a discount. Industry profit is now 0.376, and around 6% of consumers buy both items. However, relative to linear pricing, consumer surplus falls to 0.245. In particular, the use of inter-firm discounts may harm consumers, despite their apparent pro-consumer effect. Intuitively, when firms offer a bundle discount, this reduces the effective degree of substitution between products, which in turn relaxes competition between firms. In particular, and in contrast to the case of an integrated firm, when bundling is used the regular price increases relative to the situation with linear pricing.

Time-constrained consumers: Finally, consider the situation with time-constrained consumers when separate sellers supply the products:

Proposition 8: *Suppose that v_1 and v_2 are identically and independently distributed with c.d.f. satisfying (24) and that some consumers are time-constrained. When the two products are supplied by separate sellers, a seller has no incentive to offer a discount to those consumers who buy the rival product. (They would, if feasible, like to charge their customers a higher price when a customer buys the rival product.)*

Proof: By examining Figure 5, we see that

$$-\frac{\partial q_{12}}{\partial p_1} = \lambda f(1 - F) \; ; \; q_{12} = \lambda(1 - F)^2$$

and

$$-\frac{\partial q_1}{\partial p_1} = fF + (1-\lambda)\int_p^\infty (f(v))^2 dv \ ; \ q_1 = \lambda F(1-F) + \tfrac{1}{2}(1-\lambda)(1-F^2)$$

(where these expressions are evaluated at symmetric prices $p_1 = p_2 = p$ and the dependence of f and F on p is suppressed). We need to show that inequality (15) is reversed.

Since F has an increasing hazard rate, we have

$$\begin{aligned}
\int_p^\infty (f(v))^2 dv &= \int_p^\infty \frac{f(v)}{1-F(v)} f(v)(1-F(v))dv \\
&> \frac{f}{1-F}\int_p^\infty f(v)(1-F(v))dv \\
&= \tfrac{1}{2}\frac{f}{1-F}(1-F)^2 \\
&= \tfrac{1}{2} f(1-F) \ .
\end{aligned}$$

Thus (15) is reversed whenever

$$\frac{f}{1-F} < \frac{2fF + (1-\lambda)f(1-F)}{2\lambda F(1-F) + (1-\lambda)(1-F^2)}$$

which some rearranging shows to be always the case provided $\lambda < 1$. ■

This implies that, starting from the situation in which firms set their equilibrium linear price, if feasible a firm would wish to charge a *higher* price to its customers who also buy the rival's product. In this framework, the observation that a consumer wishes to buy both items implies she is in the "non-competitive" group of consumers, and a firm would like to exploit its monopoly position over those consumers if possible.

Summary: In this section we considered a firm's incentive to offer a discount when a customer also buys the rival product. Two broad forces may provide such an incentive. First, if a consumer's value for one product is negatively correlated with the other, the information that consumer has purchased the rival product (i.e., its value for the rival product is relative high) is bad news for a firm, and typically induces it to lower its price to that customer. Second, if purchasing the rival product causes a consumer's incremental value for the firm's product to fall, due to substitution, then the firm may wish to reduce its price to these customers (Proposition 7). However, Proposition 8 shows that an alternative, but natural, form of substitution makes a firm wish to set a higher price when its customers buy the rival product. Thus, the precise form in which products are substitutes is important for a firm's incentive to offer inter-firm bundling discounts.

It is plausible that framework studied here, where customers are final consumers, could sometimes be extended to situations where rival manufacturers supply products to a retailer (who then ultimately supplies one or both products to final consumers). If the manufacturers supply products which are partial substitutes, this analysis suggests that one manufacturer could have an incentive to charge a lower price if the retailer also chooses to supply the rival product. This is the opposite pricing pattern to the "loyalty pricing" which often worries antitrust authorities. On the other hand, if the situation is more like the time-constrained consumer case, then a supplier has an incentive to charge the retailer less if the retailer does *not* stock the rival product, which is the more conventional prediction. (In the retailing context, it might be that some retailers can only stock one of the two products, for instance because of shelf or refrigeration constraints, in which case Proposition 8 might be more relevant.)

6 Partial Coordination Between Sellers

The analysis to this point has considered the two extreme cases where (a) there is no tariff coordination between separate sellers, and (b) where there is complete tariff coordination between sellers. (The integrated-firm analysis in section 4 describes the outcome when two sellers coordinate their pricing to maximize industry profit.). The problem with complete coordination is that any competition between rivals is eliminated. As discussed in section 3, though, the problem with a policy of permitting no coordination between sellers is that the resulting bundle discount may be inefficiently small (or non-existent). It would be desirable, is feasible, to obtain the efficiency gains which may accrue to bundling without permitting the firms to collude over their regular prices.[12] One way this might be achieved is if firms first negotiate an inter-firm bundle discount and then compete in the usual way by choosing their stand-alone prices independently.

To consider this situation in more detail, suppose that two symmetric firms supply two products. The firms interact in two stages in a similar manner to the procedure in the four-firm analysis of Gans and King (2006) and Brito and Vasconcelos (2010). First, the

[12] A similar dilemma is encountered when considering policy towards code-sharing by airlines. Ideally, one would like to allow airlines to coordinate their pricing when they jointly offer multi-flight itineraries so as to avoid double marginalization, but not when they compete along similar routes.

two firms agree on a bundle discount, δ say, which they agree to fund equally. That is to say, if firm $i = 1, 2$ chooses stand-alone price p_i, the consumer pays this price if she buys only that firm's product (and the firm receives that revenue), but if she buys both products she pays $p_1 + p_2 - \delta$ and firm i receives revenue $p_i - \frac{1}{2}\delta$. After δ is chosen, firms choose their stand-alone prices unilaterally. Far-sighted firms will choose δ after taking into account how this discount will affect their competitive interaction in the second stage. Since separate firms tend to set lower prices when products are more substitutable (see the bold line in Figure 4 for an illustration of this), and since a bundle discount mitigates or overturns a consumer's view of the products as substitutes, it will usually be the case that an agreed bundle discount δ will induce firms to set higher stand-alone prices. To the extent this is so, a joint-pricing scheme of this form could act as an instrument of collusion.

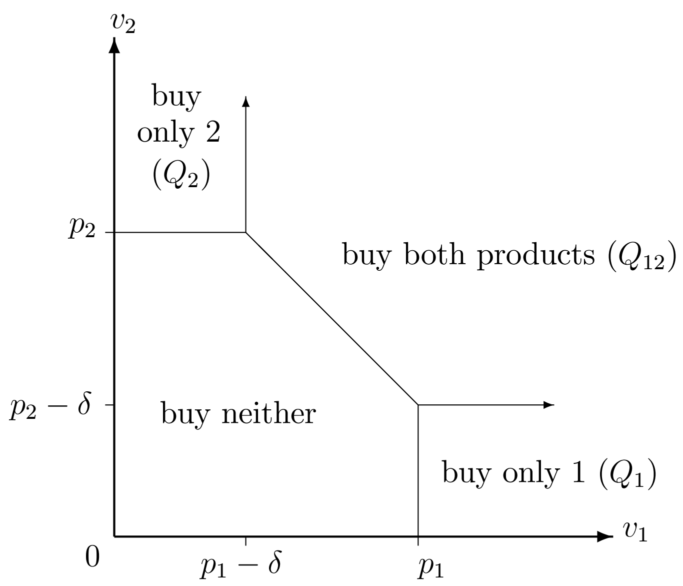

Figure 6: Pattern of demand with additive values and bundling discount $\delta > 0$

Consider first the case in which valuations are additive. Then for an agreed inter-firm discount δ, the pattern of demand for the two firms is as illustrated in Figure 6. The following result shows that this joint pricing scheme leads to higher industry profit, and describes when the scheme also increases total welfare:

Proposition 9: *Suppose that products are symmetric and valuations are additive. The marginal c.d.f. for either value v_i satisfies (24). For given $\delta > 0$ consider the joint pricing scheme in which if firm $i = 1, 2$ sets the stand-alone price p_i then the price for buying* *both products is $p_1 + p_2 - \delta$ and firm i receives revenue $p_i - \frac{1}{2}\delta$ when a bundle is sold. If condition (4) holds, for sufficiently small $\delta > 0$ this inter-firm bundling scheme increases each firm's profit, relative to the situation where the products are marketed independently. In addition, if the function $H(p, v) \equiv \Pr\{v_2 \le p \mid v_1 = v\}$ weakly increases with v, the scheme increases total welfare for small δ. If $H(p, v)$ weakly decreases with v, the scheme induces the firms to increase their stand-alone prices when δ is small.*

Proof: See the appendix.

The reason that a small agreed inter-firm discount will boost profits is intuitive. A small $\delta > 0$ will have some effect on the firms' choice of stand-alone price, but this has no first-order impact on a firm's profit. (A small change in the firm's own price does not significantly affect its profit, since the original price was at the optimal level. And with additive valuations a small change in the other firm's price does not significantly affect the firm's profit when the bundle discount small, as can be seen from Figure 6.) The first-order impact on industry profit is that, for a *fixed* stand-alone price p, the introduction of a bundle discount boosts profit whenever expression (4) is satisfied. The impact on total welfare is more complex, as the impact of the discount on equilibrium prices needs to be considered. A bundle discount tends to induce firms to raise their stand-alone prices. (The result shows this is always true when values are independently distributed or positively correlated in the sense that $H(p, v)$ decreases with v.) A bundle discount converts independent products into complements, and this typically induces separate firms to set higher prices. However, when values are independently distributed or negatively correlated (in the sense that $H(p, v)$ increases with v), the impact of the price rise is not large enough to outweigh the efficiency benefits of the bundle discount, and total welfare then rises when the scheme is used.

To illustrate, consider the example where (v_1, v_2) is uniformly distributed on the unit square $[0, 1]^2$ and $c = 0$. Using Figure 6, one can show that each firm's equilibrium stand-alone price as a function of the agreed discount is

$$p(\delta) = \frac{3\delta + 2\delta^2 + 2}{3\delta + 4} ,$$

which is indeed increasing in δ as required. For small δ, we have shown that this scheme benefits the firms and efficiency.[13] However, in this example the scheme harms aggregate consumer surplus.[14]

While the operation of this joint pricing scheme appears to be relatively benign when values are additive, this can easily be reversed when firms offer substitutable products. Consumers benefit, and total welfare rises, when firms are forced to set low prices due to products being substitutes. However, an agreed inter-firm discount can reduce the effective substitutability of products, and thus relax competition between suppliers. While this effect can be demonstrated more generally, for maximum clarity consider the following simple example:

Example: There are two museums in a city, and the marginal cost of a museum visit is zero. All tourists have identical tastes, and the two museums are homogenous in the sense that if a tourist visits just one museum, she does not mind which one it is. A tourist values visiting any single museum at V_1 and gains incremental utility $V_2 < V_1$ from visiting the second museum. Because of the declining marginal value of visits, the two museums compete to some extent. If each museum sets an independent entry charge, one can check that the equilibrium entry charge is the incremental value of a second visit, V_2. The result is that tourists visit both museums, and obtain strictly positive surplus $V_1 - V_2$. Suppose the two museums are free to choose their own entry charge but agree in advance to offer a discount δ on the sum of stand-alone prices if a tourist visits both museums, and they fund this discount equally. (That is to say, if museum i chooses the entry fee p_i, the charge for visiting both museums is $p_1 + p_2 - \delta$ and museum i receives revenue $p_i - \frac{1}{2}\delta$ when a tourist visits both museums.) Since with a bundle discount δ a tourist's incremental utility from a second visit is now $V_2 + \delta$, the equilibrium stand-alone price with discount $\delta \le V_1 - V_2$ is $p = V_2 + \delta$, with the result that tourists visit both museums and pay the joint price $2V_2 + \delta$. In particular, by choosing $\delta = V_1 - V_2$ firms can induce the fully collusive outcome. Thus, the apparently pro-consumer policy of offering a discount for joint purchase can act as a device to sustain collusion.

This example suggests that inter-firm discounting schemes operated by rivals should be

[13] One can check that the most profitable choice of δ for the firms is $\delta \approx 0.38$.

[14] When $\delta = 0$, we have $p' = \frac{3}{8}$. Therefore, when δ is small that half of the consumer population who only buy one item experience a price rise of $\frac{3}{8}\delta$, while that quarter of consumers who buy both items experience a price fall of $\frac{1}{4}\delta$. Thus, the net impact on consumers is a loss of $\frac{1}{8}\delta$.

viewed with some suspicion by antitrust authorities.

7 Conclusions

This paper has extended the standard model of bundling to allow products to be partially substitutable and for products to be supplied by separate sellers. We found that bundling often continued to be a profitable strategy for an integrated firm, and that separate sellers often wished unilaterally to offer joint-purchase discounts when their customers buy the rival products. Because bundle discounts act to mitigate the innate substitutability of the products, when separate sellers agree on a bundle discount this can enable them to collude.

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APPENDIX

Proof of Proposition 9: Firm i's profit under the proposed joint-pricing scheme is

$$(p_i - c)(Q_i + Q_{12}) - \tfrac{1}{2}\delta Q_{12}\ . \tag{28}$$

The impact of introducing a small $\delta > 0$ on firm i's equilibrium profit is therefore governed by the sign of

$$\frac{d}{d\delta}\left\{(p_i - c_i)(Q_i + Q_{12}) - \tfrac{1}{2}\delta Q_{12}\right\}\Big|_{\delta=0}$$

$$= \frac{dp_i}{d\delta}\frac{\partial}{\partial p_i}[(p_i - c_i)(Q_i + Q_{12})]\Big|_{\delta=0,p_i=p_i^*} + \frac{dp_j}{d\delta}\frac{\partial}{\partial p_j}[(p_i - c_i)(Q_i + Q_{12})]\Big|_{\delta=0,p_i=p_i^*} \tag{29}$$

$$- \tfrac{1}{2}Q_{12}\big|_{\delta=0} + (p_i^* - c_i)\frac{\partial}{\partial\delta}(Q_i + Q_{12})\Big|_{\delta=0}$$

$$= -\tfrac{1}{2}q_{12} - (p^* - c)\frac{\partial q_{12}}{\partial p_i} \tag{30}$$

(where this final expression is evaluated at optimal linear price p^*). Here, the terms in line (29) vanish, the first because p^* is the optimal price for firm i when firms choose linear prices (i.e., p_i maximizes $(p_i - c_i)(q_i + q_{12})$), and the second because changing the other firm's price has no impact on a firm's demand when there is no bundling discount (i.e., $q_i + q_{12}$ does not depend on p_j when values are additive). The final expression follows from (9). Following by-now familiar arguments, the term (30) is strictly positive whenever (4) holds.

Consider next the impact of the joint pricing scheme on total welfare. To calculate this we need to understand how the introduction of δ affects equilibrium prices p_i. Firm i's profit is given by (28) and so the first-order condition for p_i given δ (and p_j) is

$$Q_i + Q_{12} + (p-c)\frac{\partial(Q_i + Q_{12})}{\partial p_i} - \frac{1}{2}\delta\frac{\partial Q_{12}}{\partial p_i} = 0 \ . \tag{31}$$

This expression then determines the symmetric stand-alone price $p(\delta)$ as a function of the discount δ. Totally differentiating (31) with respect to δ yields

$$0 = \frac{\partial(Q_i + Q_{12})}{\partial \delta} + 2p'\frac{\partial(Q_i + Q_{12})}{\partial p_i} + p'\frac{\partial(Q_i + Q_{12})}{\partial p_j}$$

$$+(p-c)\left[\frac{\partial^2(Q_i + Q_{12})}{\partial p_i \partial \delta} + p'\frac{\partial^2(Q_i + Q_{12})}{\partial p_i^2} + p'\frac{\partial^2(Q_i + Q_{12})}{\partial p_i \partial p_j}\right] - \frac{1}{2}\frac{\partial Q_{12}}{\partial p_i} \ ,$$

where $p' = \frac{d}{d\delta}p(\delta)$. When $\delta = 0$ this simplifies to

$$0 = -\frac{3}{2}\frac{\partial q_{12}}{\partial p_i} - 2fp' + (p-c)\left[-\frac{\partial^2 q_{12}}{\partial p_i^2} - p'f'\right] \ . \tag{32}$$

Note that

$$-\frac{\partial q_{12}}{\partial p_1} = f(p_1)(1 - H(p_2 \mid p_1))$$

and so

$$\begin{aligned} -\left.\frac{\partial^2 q_{12}}{\partial p_1^2}\right|_{p_1=p_2=p} &= f'(p)(1 - H(p \mid p)) - f(p)\frac{\partial}{\partial p_1}H(p_2 \mid p_1) \\ &\leq f'(p)(1 - H(p \mid p)) \\ &= -\frac{f'(p)}{f(p)}\frac{\partial q_{12}}{\partial p_1} \ , \end{aligned} \tag{33}$$

where the inequality follows when $H(p \mid v)$ weakly increases in v. Thus, expression (32) implies

$$\begin{aligned}[2f+(p-c)f']p' &= -\frac{3}{2}\frac{\partial q_{12}}{\partial p_i} - (p-c)\frac{\partial^2 q_{12}}{\partial p_i^2} \\ &\le -\frac{\partial q_{12}}{\partial p_i}\left[\frac{3}{2} + \frac{f'}{f}(p-c)\right] \\ &\le -\frac{\partial q_{12}}{\partial p_i}\left[2 + \frac{f'}{f}(p-c)\right] \\ &= -\frac{1}{f}\frac{\partial q_{12}}{\partial p_i}\left[2f + f'(p-c)\right] \ .\end{aligned} \tag{34}$$

Here, the first inequality follows from (33), and the second follows from the fact that $\frac{\partial q_{12}}{\partial p_i}$ is negative. Since the term $[2f + f'(p-c)]$ is strictly positive due to the second-order condition for p to be the equilibrium price when $\delta = 0$ (the second-order condition is sure to be satisfied given (24)), we deduce that

$$fp' \le -\frac{\partial q_{12}}{\partial p_i} \ . \tag{35}$$

By inspecting Figure 6, one can see that the impact of a small discount δ on total welfare is equal to

$$W' = 2f(p)(p-c)\left\{(1-H(p \mid p))(1-p') - H(p \mid p)p'\right\} \ .$$

(Here, the first term represents the welfare gain when more single-item consumers buy two items, as the incremental cost of the second item falls to $p(\delta) - \delta$, while the second term represents the welfare loss when some single-item consumers decide to buy nothing due to the price rising to $p(\delta)$.) This welfare change has the sign of

$$f\left\{1 - H - p'\right\} = -\frac{\partial q_{12}}{\partial p_i} - fp' \ge 0 \ ,$$

where the inequality follows from (35). Thus, when $H(p \mid v)$ weakly increases with v, the joint pricing scheme will increase total welfare when δ is small.

Finally, we show how the stand-alone price changes when a bundle discount is agreed. If $H(p \mid v)$ decreases with v, then inequality (33) is reversed. It follows that inequality (34) is also reversed. Assumption (24) implies that at the equilibrium linear price p, we have

$$f + (p-c)f' > 0 \ ,$$

which in turn implies that both $[2f + (p-c)f']$ and $[\frac{3}{2} + \frac{f'}{f}(p-c)]$ are strictly positive. Since (34) is reversed, we deduce that $p' > 0$ and the agree discount raises the equilibrium stand-alone prices. ■