Industrialization and the role of government

Takeuchi, Nobuyuki

Kobe University

November 2010

Industrialization and the Role of Government

Nobuyuki Takeuchi[*]

Abstract

We construct a two-sector endogenous growth model to examine the role of government in industrialization. Three main features of this model are (a) household preference is non-homothetic; (b) government's sector-specific spending is introduced as a production factor and (c) technological progress occurs only in the manufacturing sector through learning-by-doing. By using the model with these features, we derive the optimal policy for government resource allocation, optimal tax rate and share of government spending for each sector, to maximize the household's utility. In addition, we examine the dynamics of the model.

The model reveals that (a) increments in both agricultural productivity and manufacturing productivity cause labour to move from the agricultural sector to the manufacturing sector; (b) depending on the relative elasticity of production with respect to government's spending between the two sectors, the optimal tax rate will shrink or expand with the passage of time and will stay at a level of balanced growth path in the long run and (c) as the industrialization progresses, the optimal share of government spending for the agricultural sector will decline.

Key words: industrialization, productive government spending,
learning-by-doing, economic growth
JEL: O11, O41

[*] Graduate School of International Cooperation Studies, Kobe University, Nada, Kobe 657-8501, Japan.
Email: ntakeuchi@people.kobe-u.ac.jp

1. Introduction

Industrialization plays a key role in economic development and growth in many countries. In pursuing industrialization, governments in many developing countries have emphasized expansion of the manufacturing sector, often at the expense of the agricultural sector. However, in some countries, such as India, this policy has resulted in stagnant economic growth or in expansion of poverty in rural areas, where most people engage in agriculture.

Several studies concerning the relation between industrialization and economic growth have appeared since the early 1950s. Lewis (1954), Fei and Ranis (1964) and Jorgenson (1961, 1967) are some early attempts on this subject. The model in these studies, known as the dual economy model, mainly argued that the continuous progress in agricultural productivity, not manufacturing productivity, is necessary to avoid food shortages and to achieve sustained growth.[1] Thus, these studies emphasized that to improve agricultural productivity governments should prioritize their policies, such as land improvement or irrigation policies. Recently, Matsuyama (1992) and Eswaran and Kotwal (1993) shed new light on the relation between industrialization and each sector's productivity, by using general equilibrium models that are more sophisticated than earlier dual economy models. Matsuyama (1992) showed that in a closed economy improvements in agricultural productivity, not in manufacturing productivity, lead to industrialization and economic development, yielding results similar to those of the earlier dual economy models. However, he showed that, in a small open economy improvements in manufacturing productivity promote technological progress in manufacturing sector and cause economic development.[2]

Although these studies explained which sector—agriculture or manufacturing—plays a key role in industrialization and economic development, they did not treat the government sector explicitly and did not mention the concrete role of government. How should government provide infrastructure to promote industries? To the best of the author's knowledge, Ortiz (2004) and Chang, Chen and Hsu (2006) are the only attempts to address such questions. Ortiz (2004) derived optimal tax rate for industrialization by using a model that incorporated the productive government spending argued in Barro (1990) into the model of Matsuyama (1992). In his model, government spending takes the form of infrastructure development and is nonrival among sectors in the economy. It leaves no room to consider the case that government divides the productive government spending between any two sectors depending on their characteristics. However, actually some of the productive government spending, such as that on land reforming and industrial water service, are sector-specific, and we should regard such spending as sector-specific production factors. Chang, Chen and Hsu (2006) extended Matsuyama (1992) by introducing a government that supplies the manufacturing sector with infrastructure to promote learning-by-doing in that sector. While their model explicitly showed the role of government in manufacturing, they did not mention government's role in the agricultural sector.

On the basis of earlier studies, including those mentioned above, this study aims to examine the relationship among the role of government, industrialization and economic development. In particular, it considers government's role in agriculture and manufacturing simultaneously and examines how government should adjust its policy weight between the two sectors along the path of economic growth. To achieve so, we incorporate sector-specific productive government spending into Matsuyama's (1992) model and construct an endogenous growth model with the two sectors:

agriculture and manufacturing. The model has three notable features. First, both sectors employ two factors of production: labour and sector-specific government spending. Therefore, government's budgetary decisions for each sector are an important component of the model. Second, following Matsuyama (1992), the growth engine for the economy is technological progress in manufacturing arising from learning-by-doing. On the other hand, there is no technological progress in agricultural sector. Finally, household preferences are non-homothetic, and income elasticity of demand for agricultural goods is less than 1, implying that Engel's law holds.[3] Non-homothetic preferences are regarded as a cause of industrialization in many studies on economic development. Under non-homothetic preference, rising per capita income results in a decline in budgetary share of the commodity with lower income elasticity of demand, such as agricultural goods, which in turn leads to industrialization. Echevarria (1997, 2000), Kongsamut et al. (2001) and Gollin et al. (2002) are representative studies investigating the implications of non-homothetic preference for industrialization.

By using our model, we derive an optimal policy for allocation of government resources among sectors, an optimal tax rate and the dynamics of economic variables, such as GDP and the manufacturing sector's share of the total labour endowment. Our analysis is restricted to a closed economy, while Matsuyama (1992) examined both closed and open economy cases. Because of our model's complexity, the case of the open economy must be analysed separately from the present study.

The paper is organized as follows: The next section, Section 2, specifies notations and assumptions and sets up the model. The features, existence and uniqueness of the static equilibrium are analysed in Section 3, together with some comparative statics analyses. In Section 4, we examine how government decides its policy. Section 5 deals with dynamics of the model. The last section summarizes the main results and concludes.

2. The Model

2-A. The Production and the Factor Market

The economy consists of two production sectors: agriculture (A) and manufacturing (M). Agricultural goods are produced by competitive firms using two factor inputs: labour (L_A) and productive government spending (g_A). Here, g_A (and g_M to appear later) is non-rival among firms within the sector and is provided without charges. These features are the same as those defined by Barro (1990) and Barro and Sala-i-Martin (1992, 1995), which show that productive government spending affects the long-run growth rate. But, g_i ($i = A, M$) differs from Barro's assumption in being a sector-specific factor, which we believe is more realistic than Barro's assumption. Consider the typical examples of agricultural irrigation and industrial water service, both of which are public services but differ in function. Therefore, it is reasonable to assume that public services are sector-specific production factors.

Technology is subject to diminishing returns with respect to each factor. We specify the agricultural production function as follows:

$$Y_A = AL_A^a g_A^\alpha, 0 < a < 1, 0 < \alpha < 1 \qquad (1)$$

where A implies total factor productivity in agriculture. Manufacturing goods are produced using efficient labour (mL_M) and productive government spending (g_M). Technology exhibits diminishing returns to each factor, the same as in the agriculture sector:

$$Y_M = M(mL_M)^b g_M^\beta, 0 < b < 1, 0 < \beta < 1 \qquad (2)$$

where M, m and L_M denote total factor productivity in manufacturing, labour productivity and labour input in the manufacturing sector, respectively. α and β in equations (1) and (2) implies that the elasticities of output with respect to government spending on agriculture and manufacturing, respectively, and these are important in deciding patterns of transitional dynamics in our model.

There is no factor accumulation. Instead, we assume that labour productivity in manufacturing improves alongside the process through which manufacturing production proceeds and we assume that its increment depends on production per labour, not on total production.[4] This formalization is along the lines of the external learning-by-doing process à la Romer (1986). Thus, the increment in *m* is expressed by

$$\dot{m} = \frac{\delta Y_M}{L_M} = \delta M m^b L_M^{b-1} g_M^{\beta}, \quad 0 < \delta < 1 \qquad (3)$$

where δ denotes the degree of learning-by-doing.

Government levies sales tax on each sector as a constant rate of revenue τ , and labour is assumed to be fully mobile between sectors. These assumptions, along with those of perfect competition and profit maximization in each sector, imply that the wage rate (w) in terms of agricultural goods is equal to

$$w = (1-\tau) a A L_A^{a-1} g_A^{\alpha} = (1-\tau) p b M m^b L_M^{b-1} g_M^{\beta} \qquad (4)$$

where p denotes the relative price of manufacturing goods.

2-B. Behaviour of Representative Household

We assume that all households are identical and have the following utility function

$$U = \theta \ln(c_A - \gamma) + \ln c_M, \qquad \gamma, \theta \text{ given} \qquad (5)$$

where c_i is consumption of commodity i $(i = A, M)$. This specification is the same as that in Matsuyama (1992) and is known as the Stone-Geary type utility function. It has two specific properties: First, γ implies the minimum quantity of consumption of agricultural goods required for survival. Second, income elasticity of demand for agricultural goods is less than 1. This second property, known as Engel's law, causes industrialization through changes in household demand and is important in our model. From equation (5) and households' utility maximization, consumer demand for the two goods (agricultural and manufacturing goods) satisfies

$$p\theta c_M = c_A - \gamma . \qquad (6)$$

We also assume that the number of households is equal to the total labour endowment: L.

2-C. The Government Sector

Government collects sales taxes from firms in each sector and uses the revenue for productive government spending. In our model, government is assumed to obey the balanced-budget rule. The budget constraint for the government and the amount of spending for each sector are as follows:

$$g = (1/p)\cdot \tau \cdot (Y_A + pY_M), \qquad (7)$$

$$g_A = s_A g \qquad (8)$$

and $$g_M = (1 - s_A)g \qquad (9)$$

where s_A and $(1 - s_A)$ denote the share of expenditures from total tax revenue for the agricultural sector and the manufacturing sector, respectively, and $0 < s_A < 1$.[5] g is total government expenditure in terms of manufacturing goods. Here, s_A can be regarded as the policy weight between sectors: a large s_A implies that the government emphasizes agricultural development, whereas a small s_A implies that the government encourages manufacturing development.

We also assume that government purchases products from the manufacturing sector and uses them in providing governmental services.

2-D. Market Equilibrium

To conclude the model, we specify equilibrium conditions in markets for production factors and commodities. The factor market equilibrium condition is

$$L_A + L_M = L. \quad (10)$$

As for commodity market equilibrium, we note that demand for agricultural goods derives only from household consumption. On the other hand, demand for manufacturing goods is the sum of household consumption and government expenditure. Therefore, the market equilibrium conditions for agriculture and manufacturing are, respectively,

$$c_A L = Y_A \qquad (11)$$

and $$c_M L + g = Y_M . \qquad (12)$$

This completes the description of the model. We have 12 equations (1)–(12), with 12 endogenous variables $Y_A, Y_M, L_A, L_M, g_A, g_M, g, p, m, c_A, c_M$ and U; and six parameters $\theta . \gamma, \tau, s_A, L$ and m_0.

3. Static Equilibrium

We define static equilibrium as a set of $Y_A, Y_M, L_A, L_M, g_A, g_M, g, p, c_A, c_M$ and U, which satisfies the 11 equations (1),(2) and (4)–(12) for given m, τ and s_A. We define sustainable equilibrium as the situation in which agricultural output exceeds the minimum total household consumption, γL. First, we show the existence and uniqueness of the sustainable static equilibrium in this section. Combining equations (1), (2), (4), (6)–(12) and rearranging for *g* yields

$$g^{\alpha} = \frac{\gamma L (L - L_M)^{1-a}}{A(s_A)^{\alpha}\left\{(1+\tau\theta)(L-L_M) - (1-\tau)\theta\left(\frac{a}{b}\right)L_M\right\}}. \qquad (13)$$

Second, substituting equations (1), (2), (4), and (8)–(10) into (7) and rearranging for g yields

$$g^{1-\beta} = \tau M m^{b} L_M^{b-1} (1-s_A)^{\beta} \left\{ \left(\frac{b}{a} \right) (L - L_M) + L_M \right\}. \qquad (14)$$

Equations (13) and (14) comprise a system of two equations with two unknown variables, g and L_M. If we solve equations (13) and (14) and substitute the resulting two variables into other equations comprising the equilibrium, we obtain the static equilibrium. From the solution to equations (13) and (14), we obtain the following proposition:

***Proposition 1*: ** A unique sustainable static equilibrium exists if and only if the following inequality holds:

$$\frac{\tau^{\frac{b}{1-\beta}} (1-\tau)^{\frac{a}{\alpha}}}{\{b\tau + (1-\tau)a\}^{\frac{b}{1-\beta}+\frac{a}{\alpha}}} \cdot s_A (1-s_A)^{\frac{\beta}{1-\beta}} \cdot A^{\frac{1}{\alpha}} \cdot M^{\frac{1}{1-\beta}} \cdot m^{\frac{b}{1-\beta}} \cdot L^{\frac{b}{1-\beta}-\frac{1-a}{\alpha}} \cdot a^{\frac{a}{\alpha}} \cdot b^{\frac{b}{1-\beta}} \cdot \gamma^{-\frac{1}{\alpha}} > 1. \quad (15)$$

Proof. See Appendix A.

From Proposition 1, we see that increments in either parameter regarding productivities (A, M and m) will extend the range of the government's decision to be viable, because such improvements enable the economy to satisfy the above condition more easily. However, it is difficult to add further intuitive economic implications to the above condition.

Figure 1. Uniqueness of a sustainable static equilibrium

The static equilibrium is graphically shown in Figure 1. The downward sloping curve and the upward solid curve depict equation (14) and equation (13), respectively. The intersection of these two solid lines represents the static equilibrium. If the equilibrium is sustainable, which means condition (15) is satisfied, the intersection always appears in the area above the dashed line; otherwise, it will be a point below the dashed line. (See Appendix A.)

To know the effects of a change in parameters, we totally differentiate equations (13) and (14) and rearrange the terms, resulting in the following equation:

$$\begin{pmatrix} a_{1L} & a_{1g} \\ a_{2L} & a_{2g} \end{pmatrix}\begin{pmatrix} dL_M \\ dg \end{pmatrix} = \begin{pmatrix} b_{1A}dA \\ b_{2M}dM + b_{2m}dm \end{pmatrix} \qquad (16)$$

where

$$a_{1L} = -A(s_A g)^{\alpha}(L-L_M)^{a-2}\left\{(1+\tau\theta)(L-L_M)a + (1-\tau)\theta\frac{a}{b}(L-aL_M)\right\} < 0, \quad (17a)$$

$$a_{2L} = \frac{b}{a}\tau\, Mm^b(1-s_A)^{\beta}L_M{}^{b-2}\{(a-b)L_M - (1-b)L\} < 0, \quad (17b)$$

$$a_{1g} = \alpha A s_A^{\alpha} g^{\alpha-1}(L-L_M)^{a-1}\left\{(1+\tau\theta)(L-L_M) - (1-\tau)\theta\frac{a}{b}L_M\right\} > 0, \quad (17c)$$

$$a_{2g} = -(1-\beta)g^{-\beta} < 0, \quad (17d)$$

$$b_{1A} = -(s_A g)^{\alpha}(L-L_M)^{a-1}\left\{(1+\tau\theta)(L-L_M) - (1-\tau)\theta\frac{a}{b}L_M\right\} < 0, \quad (17e)$$

$$b_{2M} = -\tau\, m^b(1-s_A)^{\beta}L_M{}^{b-1}\left\{\frac{b}{a}(L-L_M) + L_M\right\} < 0, \quad (17f)$$

$$b_{2m} = -b\tau\, Mm^{b-1}(1-s_A)^{\beta}L_M{}^{b-1}\left\{\frac{b}{a}(L-L_M) + L_M\right\} < 0. \quad (17j)$$

The determinant of the coefficient matrix is positive:

$$|D| = a_{1L}a_{2g} - a_{1g}a_{2L} > 0.$$

Solving the above matrix system, we obtain the comparative statics result as follows:

$$\frac{\partial L_M}{\partial A} = \frac{1}{|D|}b_{1A}a_{2g} > 0, \frac{\partial g}{\partial A} = -\frac{1}{|D|}b_{1A}a_{2L} < 0, \qquad (18a)$$

$$\frac{\partial L_M}{\partial M} = -\frac{1}{|D|} a_{1g} b_{2M} > 0 , \frac{\partial g}{\partial M} = \frac{1}{|D|} a_{1L} b_{2M} > 0 , \tag{18b}$$

$$\frac{\partial L_M}{\partial m} = -\frac{1}{|D|} a_{1g} b_{2m} > 0 , \frac{\partial g}{\partial m} = \frac{1}{|D|} a_{1L} b_{2m} > 0 . \tag{18c}$$

From these results, we can establish the following proposition:

***Proposition 2*:** An increase in any technology parameter (A , M or m) causes labour to move from agriculture to manufacturing. However, the effect on L_M will be minor when L_M is sufficiently close to $\overline{L}_M \equiv \frac{(1+\tau\theta)L}{(1+\tau\theta)+(1-\tau)\theta(a/b)}$. Moreover, the effect of an increment in agricultural productivity (A) differs from an increment in manufacturing productivity (M or m) in the sense that improvement of the former reduces total government spending (g) while improvement of the latter increases total government spending.

Proof. See equations (18a), (18b) and (18c). In addition, when $L_M = \frac{(1+\tau\theta)L}{(1+\tau\theta)+(1-\tau)\theta(a/b)} \equiv \overline{L}_M$, a_{1g} and b_{1A} become 0. Therefore, $\frac{\partial L_M}{\partial A}$, $\frac{\partial L_M}{\partial M}$ and $\frac{\partial L_M}{\partial m}$ become 0.

This result differs from results in Matsuyama (1992) and Ortiz (2004), which showed that the increase in manufacturing productivity does not affect allocation of labour between sectors. In their model, an increase in M causes a decrease in the relative price of manufacturing goods at the same rate as the increase in the marginal productivity of labour in manufacturing. Therefore, the value of marginal productivity in the manufacturing sector remains unchanged, and M does not affect the labour allocation between sectors. However, in our model, an increase in M causes labour movement and does not affect labour allocation in the long run.[6] That is because an increase in M leads to an increase in production in the manufacturing sector. This effect also causes the government revenue and government spending in each sector to increase. Thus, the value of marginal productivity of labour in both sectors increases, but the effect is stronger in manufacturing.

4. The Optimal Tax Rate and the Share of Government Spending

So far, we have assumed that the tax rate (τ) and the share of government spending (s_A) are fixed. In this section, we examine how the government decides these policy parameters. We assume that government chooses τ and s_A at each moment of time so as to maximize representative household utility. Accordingly, a problem for the government is to maximize equation (8) under constraints (1), (2), (4) and (6)–(12), where the control variables are τ and s_A. Since we have proven the existence and the uniqueness of the static equilibrium in Section 3, we now solve, by using all constraints of this problem, for c_i ($i = A, M$) as a function of τ and s_A:

$$c_A = c_A(\tau, s_A) \text{ and } c_M = c_M(\tau, s_A).$$

Hence, the optimality conditions for the government are

$$\frac{\partial u}{\partial \tau} = \frac{\theta}{c_A - \gamma}\frac{\partial c_A}{\partial \tau} + \frac{1}{c_M}\frac{\partial c_M}{\partial \tau} = 0 \qquad (19a)$$

and $$\frac{\partial u}{\partial s_A} = \frac{\theta}{c_A - \gamma}\frac{\partial c_A}{\partial s_A} + \frac{1}{c_M}\frac{\partial c_M}{\partial s_A} = 0. \qquad (19b)$$

To derive $\partial c_i/\partial \tau$ and $\partial c_i/\partial s_A$ ($i = A, M$), we require the comparative statics of the constraints. To begin, substituting equation (4) into equation (6) and rearranging the terms, using equations (8)–(10), yield

$$bMm^b L_M^{b-1}(1-s_A)^\beta g^\beta (c_A - \gamma) - aA(L - L_M)^{a-1} s_A^\alpha g^\alpha \theta C c_M = 0. \qquad (20)$$

Then, combining equations (1), (2), (4) and (7)–(11) yields

$$\tau M m^b L_M^{b-1}\left(1-s_A\right)^\beta g^\beta \left\{\left(b/a\right)\left(L-L_M\right)+L_M\right\}-g=0\,.^7 \quad (21)$$

Further, combining (1), (8), (10) and (11) results

$$c_A L = A\left(L-L_M\right)^a \left(s_A g\right)^\alpha . \qquad (22)$$

Finally, combining (2), (9) and (12) yields

$$c_M L = M\left(mL_M\right)^b \left\{\left(1-s_A\right)g\right\}^\beta - g\,. \qquad (23)$$

Up to this point, equations (20)–(23) comprise a system that has four endogenous variables, c_A , c_M , g and L_M and two policy parameters, τ and s_A . Hence, by totally differentiating equations (20)–(23), we calculate $\partial c_A/\partial\tau$, $\partial c_M/\partial\tau$, $\partial c_A/\partial s_A$ and $\partial c_M/\partial s_A$. Substituting these results into the optimality conditions (19) and rearranging the terms, we obtain

$$\beta c_M L + (\beta-1)g + \frac{\alpha c_A L}{p} = 0 \qquad (24)$$

$$s_A = -\frac{\alpha c_A L}{g}\frac{\left[(\beta-1)g\Lambda - \tau\gamma L\Omega\{\beta c_M L + (\beta-1)g\}\right]}{\left[(\alpha c_A L + \beta p c_M L)\Lambda - \tau\gamma L\Omega\cdot\alpha c_A L\right]}. \quad (25)$$

where $\Lambda = (1+\theta)z - p\theta c_M L\Omega - \tau\theta c_A L\Omega > 0$, $\Omega = \dfrac{(a-b)L_M - (1-b)L}{L_M\left(L-L_M\right)} < 0$ and

$z = aA\left(L-L_M\right)^{a-1}\left(s_A g\right)^\alpha = pbMm^b L_M^{b-1}\left\{\left(1-s_A\right)g\right\}^\beta$. Details of derivations of equations (24) and (25) are presented in Appendix B. Combining equations (7), (11), (12) and (24) and rearranging for τ , we obtain the property of optimal tax rate:

$$\tau = \frac{\alpha Y_A + \beta p Y_M}{Y_A + pY_M}. \qquad (26)$$

Rearranging equation (25) by using equations (7), (11), (24) and (26) yields the property of optimal share of government spending:

$$s_A = \frac{\alpha Y_A}{\alpha Y_A + \beta\, pY_M}. \qquad (27)$$

These results lead to the following proposition:

Proposition 3**:** If government chooses its policy at each moment of time so as to maximize the representative household's utility, the tax rate and the share of government spending, respectively, satisfy the following two equations:

$$\tau = \frac{\alpha Y_A + \beta\, pY_M}{Y_A + pY_M} \text{ and } s_A = \frac{\alpha Y_A}{\alpha Y_A + \beta\, pY_M}.$$

The interpretation of equations (26) and (27) is as follows. From equations (7) and (26), total government spending is $pg = \alpha Y_A + \beta\, pY_M$. Substituting this into (27) yields $p = \alpha Y_A / s_A g = \alpha Y_A / g_A$. Further, the marginal product of government spending in the agricultural sector is equal to $\alpha Y_A / g_A$. Combining these two relations, we see that, when government sets up its policy according to equations (26) and (27), the marginal product of government spending in the agricultural sector is equal to the price of manufacturing goods. Similarly, it is shown that the marginal value product of government spending in the manufacturing sector is equal to the price of manufacturing goods. Recalling that government procures the products from the manufacturing sector, equations (26) and (27) guarantee the productive efficiency of government spending. This result is an extension of Barro (1990), which showed the tax rate for maximizing social welfare is equal to the elasticity of production with respect to government spending. In fact, if we assume that production technologies of both sectors are the same, i.e. $\alpha = \beta$, then the optimal tax rate becomes $\alpha\ (= \beta)$. Hence, our model can replicate Barro's result.[8] However, we should note that equations (26) and (27) do not express the quantities of τ and s_A explicitly, because the right-hand sides of these equations are also the function of τ and s_A. In brief, these equations imply a system of τ and s_A. Therefore, we must solve the system to obtain optimal tax levels and share explicitly. In Section 5, we derive the explicit solutions of optimal policy and the dynamics of the model.

5. Dynamics of the Model and the Balanced Growth Path (BGP)

So far, we have analysed the static equilibrium of the model and derived the optimal policy pairing. We now examine the dynamics of the model under the optimal policy. The optimal policy pairing is given by (26) and (27). We add these two equations to the model and turn policy parameters (τ and s_A) into endogenous variables. For simplicity, we assume that the production functions are Cobb-Douglas functions: $a = 1-\alpha$ and $b = 1-\beta$. The model is characterized by 14 equations, 14 endogenous variables $Y_A, Y_M, L_A, L_M, g_A, g_M, g, p, m, c_A, c_M, U, \tau$ and s_A; and four parameters $\theta . \gamma, L$ and m_0.

By using equations (7) and (26), we obtain

$$g = \frac{1}{p}\left(\alpha Y_A + \beta\, pY_M\right). \qquad (28)$$

Combining equations (8), (9), (27) and (28) yields

$$g_A = \frac{\alpha Y_A}{p} \text{ and } g_M = \beta Y_M . \qquad (29)$$

Substituting equation (29) into equations (1) and (2) and rearranging for Y_A and Y_M, using equation (10), yields

$$Y_A = A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} \left(L - L_M\right) p^{\frac{\alpha}{\alpha-1}} \qquad (30)$$

and $$Y_M = M^{\frac{1}{1-\beta}} \beta^{\frac{\beta}{1-\beta}} mL_M . \qquad (31)$$

Further, substituting equations (30) and (31) for equation (4) and rearranging for p gives us

$$p=\left[\frac{(1-\alpha)A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}}{(1-\beta)M^{\frac{1}{1-\beta}}\beta^{\frac{\beta}{1-\beta}}}\right]^{1-\alpha} m^{\alpha-1}. \tag{32}$$

Combining equations (6), (11), (12) and (28) yields

$$(1-\beta)p\theta Y_M=(1+\theta\alpha)Y_A-\gamma L. \tag{33}$$

Further, by substituting equations (30), (31) and (32) into equation (33) and rearranging for L_M , we obtain

$$L_M=\frac{(1+\theta\alpha)\phi m^{\alpha}L-\gamma L}{(1+\theta)\phi m^{\alpha}}=\frac{(1+\theta\alpha)\phi-\gamma/m^{\alpha}}{(1+\theta)\phi}L \tag{34}$$

where $\phi=A\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}M^{\frac{\alpha}{1-\beta}}\beta^{\frac{\alpha\beta}{1-\beta}}(1-\beta)^{\alpha}$ .We assume that hereafter the right-hand side of equation (34) is positive.[9] By using equations (32) and (34), we can derive all the endogenous variables and GDP in terms of agricultural goods as a function of m :

$$Y_A=L\frac{(1-\alpha)\theta\phi+\gamma/m^{\alpha}}{(1+\theta)}m^{\alpha}, \tag{35}$$

$$Y_M=M^{\frac{1}{1-\beta}}\beta^{\frac{\beta}{1-\beta}}L\frac{(1+\theta\alpha)\phi-\gamma/m^{\alpha}}{(1+\theta)\phi}m, \tag{36}$$

$$\tau=\frac{(1-\alpha)(\alpha\theta+\beta)\phi+(\alpha-\beta)(\gamma/m^{\alpha})}{(1-\alpha)(1+\theta+\alpha\theta-\beta\theta)\phi+(\alpha-\beta)(\gamma/m^{\alpha})}, \tag{37}$$

$$s_A=\frac{\alpha(1-\beta)\{(1-\alpha)\theta\phi+(\gamma/m^{\alpha})\}}{(1-\alpha)(\alpha\theta+\beta)\phi+(\alpha-\beta)(\gamma/m^{\alpha})}, \tag{38}$$

and $$GDP=\frac{(1-\alpha)(1+\theta+\alpha\theta-\beta\theta)\phi+(\alpha-\beta)(\gamma/m^{\alpha})}{(1-\beta)(1+\theta)}m^{\alpha}L. \tag{39}$$

Finally, combining equations (3) and (31), we obtain an autonomous differential equation of m :

$$\dot{m}=\delta M^{\frac{1}{1-\beta}}\beta^{\frac{\beta}{1-\beta}}m. \tag{40}$$

Thus, we obtain the following lemma regarding the growth rate of m .

***Lemma 1*:** If the government chooses its policy at each moment of time so as to maximize the representative household's utility, m grows at a positive, constant rate.

Here, we define the balanced growth path (BGP) as a situation in which all endogenous variables change at constant rates, which are not necessarily identical, and can be zero. The following proposition characterizes the BGP of the economy.

***Proposition 4*:** On the balanced growth path, Y_M, g, g_A, g_M and m grow at the common rate n, where n is $\delta M^{\frac{1}{1-\beta}} \beta^{\frac{\beta}{1-\beta}}$. Y_A and GDP in terms of agricultural goods grow at the same common rate αn. On the contrary, p declines at rate $(1-\alpha)n$.

Proof. From Lemma 1, the growth rate of m is constant. As m continues to grow, it becomes so large that we can regard γ / m^{α} as 0 in the long run. In this situation, L_M becomes constant because equation (34) does not contain the time variant variables. From equations (37) and (38), τ and s_A are also constant on the BGP for the same reason. We denote the growth rate of m by n, where n is $\delta M^{\frac{1}{1-\beta}} \beta^{\frac{\beta}{1-\beta}}$ from equation (40). By taking the logarithm and then differentiating equations (28), (29), (32), (35), (36) and (39) with respect to t, we obtain

$$\frac{\dot{Y}_M}{Y_M} = \frac{\dot{g}}{g} = \frac{\dot{g}_A}{g_A} = \frac{\dot{g}_M}{g_M} = n, \ \frac{\dot{Y}_A}{Y_A} = \frac{GD\dot{P}}{GDP} = \alpha n \text{ and } \frac{\dot{p}}{p} = -(1-\alpha)n.$$

From Proposition 4 and equation (40), an increment in manufacturing productivity (M) or the degree of learning-by-doing (δ) causes an increase in the long-run growth rate, αn, while an increment in A or L does not affect the growth rate in the long run. These results differ markedly from those of Matsuyama (1992), who finds that an increment of agricultural productivity brings higher growth rate. This is because we assume that an increment of m is dependent on production per labour in manufacturing, not the scale of production. In our model, an improvement in A causes labour to move to manufacturing sector, as shown in Section 3, and increases manufacturing production. However, since both increments arise at the same rate and do not change production per labour in manufacturing, the improvement in A has no effect on m and the growth rate in the long run. As for the effects of L, our model does not engender the 'scale effect', which means that the size of the economy, which is given by factors such as population, affects the long-run growth rate for the same reason as for A.

Further, we investigate the transitional dynamics of the GDP growth rate, the manufacturing sector's share of total labour endowment ($l_M \equiv L_M / L$), the tax rate and the share of government spending. For this purpose, we solve the differential equation (40) and use n, to obtain $m = m_0 e^{nt}$ where m_0 is the initial state of m. Substituting this equation into equations (34), (37) and (38) yields

$$l_M \equiv \frac{L_M}{L} = \frac{(1+\theta\alpha)\phi - \gamma / m_0^\alpha e^{\alpha nt}}{(1+\theta)\phi}, \tag{41}$$

$$\tau = \frac{(1-\alpha)(\alpha\theta+\beta)\phi + (\alpha-\beta)(\gamma / m_0^\alpha e^{\alpha nt})}{(1-\alpha)(1+\theta+\alpha\theta-\beta\theta)\phi + (\alpha-\beta)(\gamma / m_0^\alpha e^{\alpha nt})} \tag{42}$$

$$\text{and } s_A = \frac{\alpha(1-\beta)\{(1-\alpha)\theta\phi + (\gamma / m_0^\alpha e^{\alpha nt})\}}{(1-\alpha)(\alpha\theta+\beta)\phi + (\alpha-\beta)(\gamma / m_0^\alpha e^{\alpha nt})} \tag{43}$$

Substituting $m = m_0 e^{nt}$ into equation (39) and differentiating with respect to t, we obtain the growth rate of GDP as follows:

$$g_{GDP} \equiv \frac{\dot{GDP}}{GDP} = \frac{\alpha n(1-\alpha)(1+\theta+\alpha\theta-\beta\theta)\phi}{(1-\alpha)(1+\theta+\alpha\theta-\beta\theta)\phi + (\alpha-\beta)(\gamma / m_0^\alpha e^{\alpha nt})}. \tag{44}$$

Equations (41)–(44) lead to the following proposition about the transitional dynamics:

Proposition 5**:** Properties of the transitional dynamics of our model are as follows:

(a) While the manufacturing sector's share of total labour endowment gradually increases on the transitional path, the share of government expenditure for the agriculture sector gradually decreases. In the long run, each reaches its own BGP level.

(b) If $\alpha < \beta$ (or $\alpha > \beta$), the optimal tax rate is initially low (or high). It gradually increases (or decreases) and, in the long run, reaches its own BGP levels.

(c) If $\alpha < \beta$ (or $\alpha > \beta$), the growth rate of GDP in terms of the agricultural goods is initially high (or low). It gradually decreases (or increases) and, in the long run, reaches its own BGP levels.

Proof. (a) Differentiating equations (41) and (43) with respect to t yields

$$\dot{l}_M = \frac{\alpha n \gamma / m_0^{\alpha} e^{\alpha n t}}{(1+\theta)\phi} > 0$$

and $$\dot{s}_A = -\frac{\alpha^2 \beta n (1-\alpha)(1-\beta)(1+\theta)\left(\gamma / m_0^{\alpha} e^{\alpha n t}\right)}{\left\{(1-\alpha)(\alpha\theta+\beta)\phi + (\alpha-\beta)\left(\gamma / m_0^{\alpha} e^{\alpha n t}\right)\right\}^2} < 0 .$$

(b) Differentiating equation (42) with respect to t yields

$$\dot{\tau} = -\frac{\alpha n (\alpha-\beta)(1+\theta)(1-\alpha)(1-\beta)\left(\gamma / m_0^{\alpha} e^{\alpha n t}\right)}{\left\{(1-\alpha)(1+\theta+\alpha\theta-\beta\theta)\phi + (\alpha-\beta)\left(\gamma / m_0^{\alpha} e^{\alpha n t}\right)\right\}^2} .$$

Since all terms in the equation above, except $(\alpha-\beta)$ in the numerator are positive, $\dot{\tau}$ is positive if and only if β is larger than α.

(c) Differentiating equation (44) with respect to t yields

$$\dot{g}_{GDP} = \frac{\alpha^2 n^2 \phi (1-\alpha)(1+\theta+\alpha\theta-\beta\theta)(\alpha-\beta)\left(\gamma / m_0^{\alpha} e^{\alpha n t}\right)}{\left\{(1-\alpha)(1+\theta+\alpha\theta-\beta\theta)\phi + (\alpha-\beta)\left(\gamma / m_0^{\alpha} e^{\alpha n t}\right)\right\}^2} .$$

Along the same lines with (b), we can show that $\dot{g}_{GDP}$ is negative if and only if β is larger than α.

Figure 2. Transitional dynamics under optimal policy.

Figure 2 depicts the possible patterns of growth. Proposition 5 provides some interesting considerations. First, the manufacturing sector's share in total labour endowment (l_M) increases in any case, and the share of government expenditure for the agricultural sector (s_A) declines in any case. The dynamics of s_A implies that the government shifts its policy emphasis from agriculture to manufacturing along the transitional path. Besides, the dynamics of l_M indicate the movement of labour from agriculture to manufacturing. This phenomenon suggests Petty-Clark's law. Second, regarding the optimal tax rate (τ), there are two possible patterns that depend on the relative size of the elasticity of the product with respect to government spending in each sector, α and β.[9] If $\alpha > \beta$, which implies that the agricultural sector uses government spending more intensively than the manufacturing sector, the optimal tax rate declines in its transitional dynamics. In contrast, if $\alpha < \beta$, the optimal tax rate continues to increase until it reaches its own BGP level. Third, the path of the GDP growth rate also depends on the relative magnitude of the technological parameter between sectors. In the case of $\alpha > \beta$, the growth rate of GDP gradually increases. In contrast, if $\alpha < \beta$, the economy's growth rate gradually decreases, consistent with neoclassical growth theory.

In our model, the minimum quantity of the consumption of agricultural goods (γ) plays an important role in the transitional dynamics.[10] If γ is zero, and therefore the income elasticity of agricultural demand is one, then the economy has no transitional dynamics in our model and is always on BGP because there is no time-variant term in the equations (42)–(45). In contrast, if γ is less than zero, and hence the agricultural goods are regarded as luxurious goods, we observe a result opposite to Proposition 5.

6. Concluding Remarks

To examine the government's role in industrialization, we constructed an endogenous growth model with sector-specific government spending and considered the optimal policy and dynamics of the economy. The main findings are as follows. (1) Increments in both agricultural productivity and manufacturing productivity cause the movement of labour from agriculture to manufacturing, implying industrialization. However, such labour movements cease to exist on a balanced growth path. (2) Depending on the relative elasticity of output with respect to government spending between the two sectors, the optimal tax rate shrinks or expands over time. In the long run, however, it stays at the level of balanced-growth-path. (3) As industrialization progresses, the optimal share of government spending on the agricultural sector declines. In other words, the policy weight shifts from agriculture to manufacturing.

Our model has some limitations. First, it omits capital accumulation. This omission probably causes overestimation of government's role because capital is a key factor in determining productive capacity. By considering capital accumulation, we may examine the relation between the roles of government and private capital. Second, we assume that the government is short sighted in that it chooses the policy at each moment of time so as to maximize instantaneous utility of households and does not consider the learning-by-doing effect in manufacturing. Third, our model treats government spending purely as spending on public goods. Therefore, we do not take into account the congestion and inefficiencies, such as corruption, which are frequently associated with the public sector in many countries. Despite these limitations, our attempt offers greater possibilities for considering the role of government than earlier related studies.

Appendix A

This appendix presents the proof for Proposition 1. First, we use superscript 'I' to denote the value of g and L_M satisfying equation (13). Consider the relation between g and L_M by taking the latter on the horizontal axis. Then, the slope of equation (13) is calculated as

$$\frac{dg^{\mathrm{I}}}{dL_M} = \frac{1}{\alpha g^{\alpha-1}} \frac{\gamma L(L-L_M)^{-a}}{A(s_A)^{\alpha}\{\bullet\}^2}\left[(1+\tau\theta)(L-L_M)a + (1-\tau)(L-aL_M)\theta\frac{a}{b}\right] > 0 \quad \text{(A1)}$$

where $\{\bullet\} = \left\{(1+\tau\theta)(L-L_M) - (1-\tau)\theta\frac{a}{b}L_M\right\}$. As $L_M \to 0$, g^{I} approaches the value $\underline{g}^{\mathrm{I}}$, where

$$\underline{g}^{\mathrm{I}} \equiv \lim_{L_M \to 0} g^{\mathrm{I}} = \left\{\frac{\gamma L^{1-a}}{A(s_A)^{\alpha}(1+\tau\theta)}\right\}^{1/\alpha} > 0. \quad \text{(A2)}$$

Furthermore, as L_M approaches L, $\{\bullet\}$ becomes negative, and we cannot define g^{I} appropriately by using equation (13). To avoid this situation, we solve the inequality $\{\bullet\} > 0$ to obtain the upper limit of L_M. It is

$$\overline{L_M} = \frac{(1+\tau\theta)L}{(1+\tau\theta) + (1-\tau)\theta(a/b)}.$$

As $L_M \to \overline{L_M}$, g^{I} approaches the value $\overline{g}^{\mathrm{I}}$, where

$$\overline{g}^{\mathrm{I}} \equiv \lim_{L_M \to \overline{L_M}} g^{\mathrm{I}} = +\infty. \quad \text{(A3)}$$

Further, we consider the properties of equation (14) as we considered those of g^{I} and use superscript 'II' to denote g and L_M determined by equation (14). The slope of equation (14) is calculated as

$$\frac{dg^{\mathrm{II}}}{dL_M} = \frac{g^{\beta}}{(1-\beta)}\tau M m^b (1-s_A)^{\beta} L_M^{b-2}\frac{b}{a}\left[(a-b)L_M - (1-b)L\right] < 0. \quad \text{(A4)}$$

As $L_M \to 0$, g^{II} approaches $\underline{g}^{\mathrm{II}}$, where

$$\underline{g}^{\mathrm{II}} \equiv \lim_{L_M \to 0} g^{\mathrm{II}} = +\infty . \qquad \text{(A5)}$$

As $L_M \to L$, g^{II} approaches the value $\overline{g}^{\mathrm{II}}$, where

$$\overline{g}^{\mathrm{II}} \equiv \lim_{L_M \to L} g^{\mathrm{II}} = \left[\tau M m^b L^b (1 - s_A)^\beta\right]^{\frac{1}{1-\beta}} > 0 . \qquad \text{(A6)}$$

Since $\underline{g}^{\mathrm{I}} < \underline{g}^{\mathrm{II}}, \overline{g}^{\mathrm{I}} > \overline{g}^{\mathrm{II}}$ and g^{I} and g^{II} are monotonically decreasing and increasing in L_M, respectively, there exists a unique and positive pair of solutions to equations (13) and (14) within the range of L_M between 0 and $L_{\overline{M}}$.

For a sustainable equilibrium, which means all endogenous variables are positive, we consider the following condition:

$$Y_A = A(L - L_M)^a (s_A g)^\alpha > \gamma L \qquad \text{(A7)}$$

This condition implies that production of agricultural goods exceeds the total subsistence levels of all households. If the equilibrium does not satisfy this condition, this economy cannot display positive consumption of manufacturing goods from equations (6) and (11). On the $L_M - g$ plane, the condition indicates that the intersection of equations (13) and (14) must lie in the area above the boundary indicated by equation (A7). By using superscript 'B' to denote g and L_M being determined by equation (A7) with the inequality replaced by equality, the properties of the boundary are characterized by

$$\frac{dg^B}{dL_M} = \frac{1}{\alpha g^{\alpha-1}} \frac{a\gamma L}{A s_A^\alpha (L - L_M)^{a+1}} > 0 , \qquad \text{(A8)}$$

$$\underline{g}^B \equiv \lim_{L_M \to 0} g^B = \left[\frac{\gamma L^{1-\alpha}}{A s_A^\alpha}\right]^{\frac{1}{\alpha}} > \underline{g}^{\mathrm{I}} , \qquad \text{(A9)}$$

and $$\overline{g}^B \equiv \lim_{L_M \to L} g^B = +\infty > 0 \qquad \text{(A10)}$$

This boundary is depicted by the dashed line in Figure 1. From the properties of the boundary and equation (13), these two curves must intersect only once in $0 < L_M < \overline{L_M}$. To derive the value of L_M at this intersection, we subtract the value of g satisfying equation (A7), with the inequality replaced by equality, from the value of g satisfying equation (13), which yields

$$g^{\mathrm{I}\alpha} - g^{B\alpha} = \frac{\gamma L}{A(s_A)^{\alpha}(L-L_M)^{a}\{\bullet\}}\left[\left\{\tau\theta + (1-\tau)\theta\frac{a}{b}\right\}L_M - \tau\theta L\right].$$

Then, we obtain the following relationship:

$$g^{\mathrm{I}} \geq g^{B} \leftrightarrow L_M \geq \frac{b\tau}{b\tau + (1-\tau)a}L = L_M^T \qquad \text{(A11)}$$

From equation (A11), if the intersection between equations (13) and (14) exists in the range greater than L_M^T, the equilibrium must be in the area above the boundary indicated by equation (A7), and thus the condition (A7) must be satisfied. The condition for this situation to appear is $g^{\mathrm{II}} \geq g^{\mathrm{I}} = g^{B}$ when L_M is equal to L_M^T. (See Figure 1) Hence, we take the ratio of the values of g satisfying equations (14) and (A7), with inequality replaced by equality, which must be greater than 1:

$$\left.\frac{g^{\mathrm{II}}}{g^{B}}\right|_{L_M = L_M^T} = \frac{\tau^{\frac{b}{1-\beta}}(1-\tau)^{\frac{a}{\alpha}}}{\{b\tau + (1-\tau)a\}^{\frac{b}{1-\beta}+\frac{a}{\alpha}}} \cdot s_A(1-s_A)^{\frac{\beta}{1-\beta}} \cdot A^{\frac{1}{\alpha}} \cdot M^{\frac{1}{1-\beta}} \cdot m^{\frac{b}{1-\beta}} \cdot L^{\frac{b}{1-\beta}-\frac{1-a}{\alpha}} \cdot a^{\frac{a}{\alpha}} \cdot b^{\frac{b}{1-\beta}} \cdot \gamma^{-\frac{1}{\alpha}} > 1$$

Thus, we have Proposition 1.

Appendix B

This appendix presents the derivation of equations (24) and (25) in Section 3. Totally differentiating equations (20)–(23) and using the equilibrium conditions of the model, we obtain

$$\begin{pmatrix} \frac{z}{p} & -\theta z & z\theta c_M\Omega & \frac{z\theta c_M}{g}(\beta-\alpha) \\ 0 & 0 & \frac{\tau c_A L}{p}\Omega & \beta-1 \\ -L & 0 & -z & \frac{\alpha c_A L}{g} \\ 0 & -L & \frac{z}{p} & \frac{\beta c_M L}{g}+(\beta-1) \end{pmatrix} \begin{pmatrix} dc_A \\ dc_M \\ dL_M \\ dg \end{pmatrix} = \begin{pmatrix} 0 & z\theta c_M\left(\frac{\beta}{1-s_A}+\frac{\alpha}{s_A}\right) \\ -\frac{g}{\tau} & \frac{\beta}{1-s_A}g \\ 0 & -\frac{\alpha c_A L}{s_A} \\ 0 & \frac{\beta}{1-s_A}(c_M L+g) \end{pmatrix} \begin{pmatrix} d\tau \\ ds_A \end{pmatrix}$$

where $z = aA(L-L_M)^{a-1}(s_A g)^{\alpha} = pbMm^b L_M^{b-1}\{(1-s_A)g\}^{\beta}$ and $\Omega = \frac{(a-b)L_M-(1-b)L}{L_M(L-L_M)} < 0$.

If we let the determinant of the coefficient matrix be $|A|$,

$$|A| = \frac{1}{p^2 g}\left[pz^2 gL(1-\beta)(1+\theta)+p^2 zc_M gL^2\theta\Omega(\beta-1)+\tau zc_A L^2\Omega\{p\theta g(\beta-1)-\alpha\gamma L\}\right] > 0.$$

Then we have

$$\frac{\partial c_A}{\partial \tau} = \frac{1}{|A|}\begin{vmatrix} 0 & -\theta z & z\theta c_M\Omega & \frac{z\theta c_M}{g}(\beta-\alpha) \\ -\frac{g}{\tau} & 0 & \frac{\tau c_A L}{p}\Omega & \beta-1 \\ 0 & 0 & -z & \frac{\alpha c_A L}{g} \\ 0 & -L & \frac{z}{p} & \frac{\beta c_M L}{g}+(\beta-1) \end{vmatrix}, \tag{B1}$$

$$\frac{\partial c_M}{\partial \tau} = \frac{1}{|A|}\begin{vmatrix} \frac{z}{p} & 0 & z\theta c_M\Omega & \frac{z\theta c_M}{g}(\beta-\alpha) \\ 0 & -\frac{g}{\tau} & \frac{\tau c_A L}{p}\Omega & \beta-1 \\ -L & 0 & -z & \frac{\alpha c_A L}{g} \\ 0 & 0 & \frac{z}{p} & \frac{\beta c_M L}{g}+(\beta-1) \end{vmatrix}, \tag{B2}$$

$$\frac{\partial c_A}{\partial s_A} = \frac{1}{|A|}\begin{vmatrix} z\theta c_M\left(\frac{\beta}{1-s_A}+\frac{\alpha}{s_A}\right) & -\theta z & z\theta c_M\Omega & \frac{z\theta c_M}{g}(\beta-\alpha) \\ \frac{\beta}{1-s_A}g & 0 & \frac{\tau c_A L}{p}\Omega & \beta-1 \\ -\frac{\alpha c_A L}{s_A} & 0 & -z & \frac{\alpha c_A L}{g} \\ \frac{\beta}{1-s_A}(c_M L+g) & -L & \frac{z}{p} & \frac{\beta c_M L}{g}+(\beta-1) \end{vmatrix} \tag{B3}$$

and $$\frac{\partial c_M}{\partial s_A}=\frac{1}{|A|}\begin{vmatrix} \frac{z}{p} & z\theta c_M\left(\frac{\beta}{1-s_A}+\frac{\alpha}{s_A}\right) & z\theta c_M\Omega & \frac{z\theta c_M}{g}(\beta-\alpha) \\ 0 & \frac{\beta}{1-s_A}g & \frac{\tau c_A L}{p}\Omega & \beta-1 \\ -L & -\frac{\alpha c_A L}{s_A} & -z & \frac{\alpha c_A L}{g} \\ 0 & \frac{\beta}{1-s_A}(c_M L+g) & \frac{z}{p} & \frac{\beta c_M L}{g}+(\beta-1) \end{vmatrix}. \quad \text{(B4)}$$

First, let us derive equation (24). Substituting equations (B1) and (B2) into (19a) yields

$$\frac{\partial u}{\partial \tau}=\frac{\theta}{c_A-\gamma}\frac{1}{|A|}\begin{vmatrix} 0 & -\theta z & z\theta c_M\Omega & \frac{z\theta c_M}{g}(\beta-\alpha) \\ -\frac{g}{\tau} & 0 & \frac{\tau c_A L}{p}\Omega & \beta-1 \\ 0 & 0 & -z & \frac{\alpha c_A L}{g} \\ 0 & -L & \frac{z}{p} & \frac{\beta c_M L}{g}+(\beta-1) \end{vmatrix}+\frac{1}{c_M}\frac{1}{|A|}\begin{vmatrix} \frac{z}{p} & 0 & z\theta c_M\Omega & \frac{z\theta c_M}{g}(\beta-\alpha) \\ 0 & -\frac{g}{\tau} & \frac{\tau c_A L}{p}\Omega & \beta-1 \\ -L & 0 & -z & \frac{\alpha c_A L}{g} \\ 0 & 0 & \frac{z}{p} & \frac{\beta c_M L}{g}+(\beta-1) \end{vmatrix}$$

$$=\frac{1}{c_M}\frac{1}{|A|}\left[\frac{1}{p}\begin{vmatrix} 0 & -\theta z & z\theta c_M\Omega & \frac{z\theta c_M}{g}(\beta-\alpha) \\ -\frac{g}{\tau} & 0 & \frac{\tau c_A L}{p}\Omega & \beta-1 \\ 0 & 0 & -z & \frac{\alpha c_A L}{g} \\ 0 & -L & \frac{z}{p} & \frac{\beta c_M L}{g}+(\beta-1) \end{vmatrix}+\begin{vmatrix} \frac{z}{p} & 0 & z\theta c_M\Omega & \frac{z\theta c_M}{g}(\beta-\alpha) \\ 0 & -\frac{g}{\tau} & \frac{\tau c_A L}{p}\Omega & \beta-1 \\ -L & 0 & -z & \frac{\alpha c_A L}{g} \\ 0 & 0 & \frac{z}{p} & \frac{\beta c_M L}{g}+(\beta-1) \end{vmatrix}\right]$$

$$=\frac{1}{c_M}\frac{1}{|A|}\begin{vmatrix} 0 & -(1+\theta)\frac{z}{p} & z\theta c_M\Omega & \frac{z\theta c_M}{g}(\beta-\alpha) \\ -\frac{g}{\tau} & 0 & \frac{\tau c_A L}{p}\Omega & \beta-1 \\ 0 & L & -z & \frac{\alpha c_A L}{g} \\ 0 & -\frac{L}{p} & \frac{z}{p} & \frac{\beta c_M L}{g}+(\beta-1) \end{vmatrix}$$

$$=\frac{1}{c_M}\frac{1}{|A|}\frac{z}{\tau p}\{(1+\theta)z-p\theta c_M L\Omega\}\left\{\beta c_M L+(\beta-1)g+\frac{\alpha c_A L}{p}\right\}=0. \quad \text{(B5)}$$

where we use equation (6). Since the first bracket of equation (B5) is always positive, $\partial u/\partial\tau$ becomes 0 if and only if the last bracket of equation (B5) is 0:

$$\beta c_M L + (\beta - 1)g + \frac{\alpha c_A L}{p} = 0 .$$

Thus we obtain equation (24).

Further, we consider equation (25). Similar to the derivation for equation (24), substituting equations (B3) and (B4) into equation (19b), we obtain

$$\frac{\partial u}{\partial s_A} = \frac{1}{c_M}\frac{1}{|A|}\begin{vmatrix} \frac{z\theta c_M\{\beta s_A + \alpha(1-s_A)\}}{s_A(1-s_A)} & -(1+\theta)\frac{z}{p} & z\theta c_M\Omega & \frac{z\theta c_M}{g}(\beta-\alpha) \\ \frac{\beta g s_A}{s_A(1-s_A)} & 0 & \frac{\tau c_A L}{p}\Omega & \beta - 1 \\ -\frac{\alpha c_A L(1-s_A)}{s_A(1-s_A)} & L & -z & \frac{\alpha c_A L}{g} \\ \frac{\beta(c_M L + g)s_A}{s_A(1-s_A)} & -\frac{L}{p} & \frac{z}{p} & \frac{\beta c_M L}{g} + (\beta-1) \end{vmatrix}$$

$$= \frac{1}{c_M}\frac{1}{|A|}\frac{z}{p^2 g s_A(1-s_A)}\left[s_A|s_1| - |s_2|\right]$$

where

$$|s_1| = \begin{vmatrix} \theta c_M(\beta-\alpha) & -(1+\theta) & p\theta c_M\Omega & \theta c_M(\beta-\alpha) \\ \beta g & 0 & \tau c_A L\Omega & (\beta-1)g \\ \alpha c_A L & pL & -pz & \alpha c_A L \\ \beta c_M L + \beta g & -L & z & \beta c_M L + g(\beta-1) \end{vmatrix}$$

and $$|s_2| = \begin{vmatrix} -\alpha\theta c_M & -(1+\theta) & p\theta c_M\Omega & \theta c_M(\beta-\alpha) \\ 0 & 0 & \tau c_A L\Omega & (\beta-1)g \\ \alpha c_A L & pL & -pz & \alpha c_A L \\ 0 & -L & z & \beta c_M L + g(\beta-1) \end{vmatrix}.$$

Therefore, $\partial u/\partial s_A$ is 0 if and only if $s_A = |s_2|/|s_1|$. Calculating $|s_2|/|s_1|$ yields

$$s_A = -\frac{\alpha c_A L}{g}\frac{[(\beta-1)g\Lambda - \tau\gamma L\Omega\{\beta c_M L + (\beta-1)g\}]}{[(\alpha c_A L + \beta p c_M L)\Lambda - \tau\gamma L\Omega\cdot\alpha c_A L]}.$$

where $\Lambda = (1+\theta)z - p\theta c_M L\Omega - \tau\theta c_A L\Omega > 0$. Thus, we obtain equation (25).

Notes

1. The comprehensive survey of the dual economy model is presented by Kanbur and McIntosh (1988) and Temple (2005).
2. While Matsuyama (1992) is essentially based on the Ricard-Viner-Jones model, Wong and Yip (1999) incorporate capital accumulation in the manufacturing sector into Matsuyama (1992) and obtain a result similar to Matsuyama in a small open economy.
3. Sato and Niho (1971) and Niho (1974) were the first attempts explicitly to incorporate Engel's law into the dual economy model.
4. Matsuyama (1992), formalizes the degree of learning-by-doing as a function of total output. However, if we apply the same formalization to our model, the transitional dynamics of the model becomes extremely complicated.
5. Under our specialization of production technology, productive government spending is necessary to produce the commodity in both sectors. Thus, s_A cannot take the value of 0 or 1.
6. As we show in Section 5, labour productivity (m) is always growing. Therefore, whatever the initial L_M is, L_M grows gradually and, in due time, approaches $\overline{L_M}$ because of equation (18c).
7. Equation (21) is the same as equation (14).
8. Ortiz (2004) also obtained the same result as Barro (1990) by using productive government spending as a flow, as in this study. On the other hand, Futagami, Shibata and Morita (1993) and Turnovsky (1997, 2000) showed that Barro's result is not optimal if government spending is treated as a stock.
9. If the initial L_M is positive, then L_M is always positive throughout the time path because m grows at a positive, constant rate, which is proven later.
10. In the endogenous growth model with productive government spending treated as a flow, there is no model in which the transitional dynamics arise, as far as we know.

References

Barro, R. J. 1990. Government Spending in a Simple Model of Endogenous Growth. *Journal of Political Economy* 98, no. 5: S103-S125.

Barro, R. J. and X. Sala-i-Martin. 1992. Public Finance in Models of Economic Growth. *Review of Economic Studies* 59, no. 4: 645-661.

Barro, R. J. and X. Sala-i-Martin. 1995. *Economic Growth*. N.Y.: McGraw-Hill.

Chang, J. J., B. L. Chen and M. Hsu. 2006. Agricultural Productivity and Economic Growth: Role of Tax Revenues and Infrastructures. *Southern Economic Journal* 72, no. 4: 891-914.

Echevarria, C. 1997. Changes in Sectoral Composition Associated with Economic Growth. *International Economic Review* 38, no. 2: 431-452.

Echevarria, C. 2000. Non-homothetic preferences and growth. *Journal of International Trade & Economic Development* 9, no. 2: 151-171.

Eswaran, M. and A. Kotwal. 1993. A Theory of Real Wage Growth in LDCs. *Journal of Development Economics* 42, no. 2: 243-269.

Fei, J. C. H. and G. Ranis. 1964. *Development of the Labor Surplus Economy: Theory and Policy*. Homewood, IL: Irwin.

Futagami, K., Y. Morita and A. Shibata. 1993. Dynamic Analysis of an Endogenous Growth Model with Public Capital. *Scandinavian Journal of Economics* 95, no. 4: 607-625.

Gollin, D., S. Parente and R. Rogerson. 2002. The Role of Agriculture in Development. *American Economic Review* 92, no. 2:160-164.

Jorgenson, D. W. 1961. The Development of Dual Economy. *Economic Journal* 71, no. 282: 309-334.

Jorgenson, D. W. 1967. Surplus Agricultural Labor and the Development of a Dual Economy. *Oxford Economic Papers* 19, no. 3: 288-312.

Kanbur, S. M. R. and J. McIntosh. 1988. Dual Economy Models: Retrospect and Prospect. *Bulletin of Economic Research* 40, no. 2: 83-113.

Kongsamut, P., S. Rebelo and D. Xie. 2001. Beyond Balanced Growth. *Review of Economic Studies* 68, no. 4: 869-882.

Lewis, W. A. 1954 Economic Development with Unlimited Supplies of Labour. *Manchester School of Economic and Social Studies* 22: 139-191.

Matsuyama, K. 1992. Agricultural Productivity, Comparative Advantage, and Economic Growth. *Journal of Economic Theory* 58, no. 2: 317-334.

Niho, Y. 1974. Population Growth, Agricultural Capital, and the Development of a Dual Economy. *American Economic Review* 64, no. 6: 1077-1085.

Ortiz, C. H. 2004. An economic growth model showing government spending with reference to Colombia and learning-by-doing. *Colombian Economic Journal* 2, no. 1: 157-188.

Romer, P. M. 1986. Increasing Returns and Long-Run Growth. *Journal of Political Economy* 94, no. 5: 1002-1037.

Sato, R and Y. Niho. 1971. Population Growth and the Development of a Dual Economy. *Oxford Economic Papers* 23, no. 3: 418-436.

Temple, J. R. W. 2005. Dual Economy Models: A Primer for Growth Economists. *Manchester School* 73, no. 4: 435-478.

Turnovsky, S. J. 1997. Fiscal Policy in a Growing Economy with Public Capital. *Macroeconomic Dynamics* 1, no. 03: 615-639.
Turnovsky, S. J. 2000. *Methods of Macroeconomic Dynamics*. 2nd ed.: MIT Press.
Wong, Kar-yiu and C. K. Yip. 1999. Industrialization, Economic Growth, and International Trade. *Review of International Economics* 7, no. 3: 522-540.

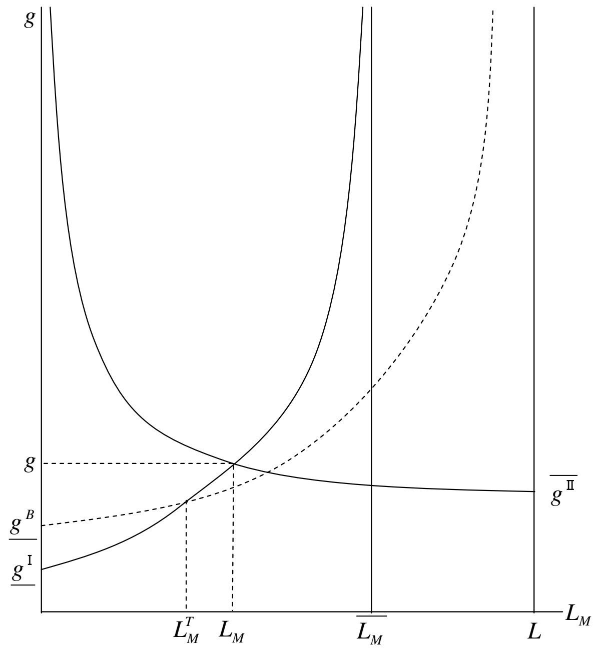

Figure 1. Uniqueness of a sustainable static equilibrium

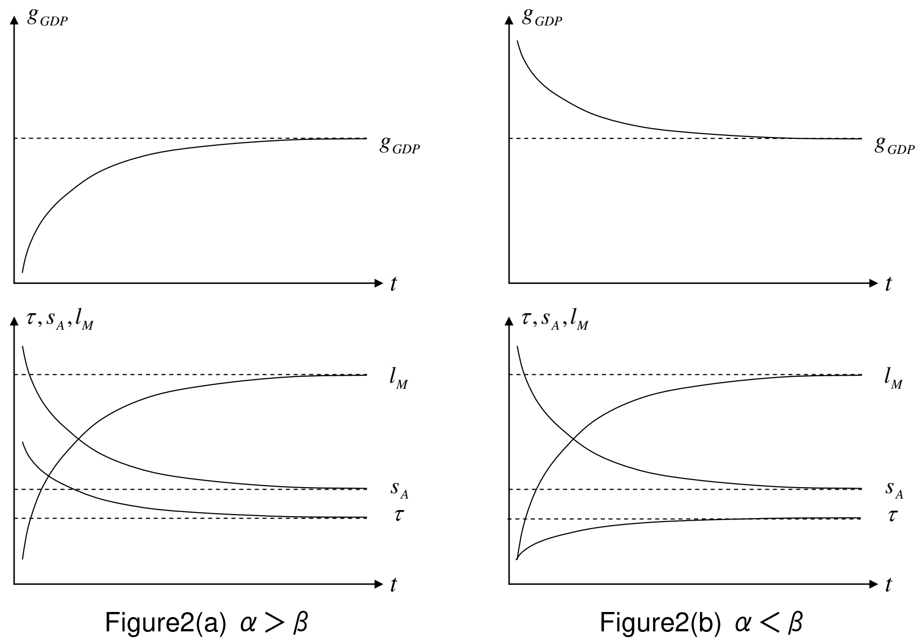

Figure2(a) $\alpha > \beta$ Figure2(b) $\alpha < \beta$

Figure 2. Transitional dynamics under optimal government policy.