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Abstract

We make a case for price-increasing competition on "competitive bottleneck" two-sided markets. Unlike previous literature on price-increasing competition and two-sided markets, we abstract from product/platform differentiation, structural differences, scale effects, search costs, and capacity constraints, which would per se favor the one or the other market structure. We argue that demand interrelation as given on many competitive bottleneck two-sided markets might be sufficient to cause either no observable price effect of competition or price-increasing competition under certain conditions. We derive these conditions and illustrate the economic intuition. Under price equality, virtually everything except for the number of platform operators is identical in monopoly and duopoly. Nevertheless, total demand on both market sides in the duopoly market exceeds total demand in the monopoly market. Furthermore, even though there is no observable price effect, there is still a competitive effect that becomes manifest in total duopoly equilibrium profits being strictly smaller than monopoly profits. The relationship of total welfare is ambiguous in subsidization cases, while it is strictly greater in duopoly, if no subsidization takes place.

Keywords: two-sided markets, platform competition, price-concentration relationship, welfare analysis, price-increasing competition

JEL Classification: D42, D43, K20, L12, L13, L51

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1. Introduction

When teaching students the basic insights of microeconomics, most economists claim that competition decreases prices and increases welfare as compared to a monopoly market. This standard view of the relationship of competition and prices is further deepened when teaching the standard Bertrand-model of duopolistic competition, i.e. when claiming that price competition among two firms is sufficient to create perfect competition. But is it really that clear, easy, and straightforward? A number of theoretical contributions present models that predict the opposite effect. This literature basically argues that product differentiation or search costs may lead to price-increasing competition. We argue that the "two-sidedness" of markets might also be an explanation for this phenomenon without relying on product differentiation or the like.

The economic literature on two-sided markets studies markets in which intermediary services, called "platforms", facilitate interaction between distinct and distinguishable groups of agents, called "market sides". The distinct feature of these platform industries is that each market side's utility from joining a platform is affected by the platform's diffusion on the other market side. In other words, each agent's decision to join or not to join a platform exerts an externality on agents on the other market side (Rochet and Tirole (2003)). We assume a "competitive bottleneck" two-sided market (Armstrong (2006)) and argue that under specific conditions there is a *demand-enhancing effect of competition* on the single-homing market side, which drives prices upwards. On the other hand, platform competition has the opposite effect on prices. We show that it is possible that the latter effect does not fully compensate the former effect, which either causes no observable price effect or price-increasing competition. To focus solely on the effects of the two-sidedness, we neither allow for platform differentiation, nor do we impose structural differences in terms of costs, scale effects or capacity constraints that might per se favor the one or the other market structure.

The question of price changes with regard to market structure becomes relevant when policy makers consider subsidies in order to attract new entrants to given monopoly markets. Reversely, antitrust authorities need to assess the impact of mergers on prices. A positive correlation of price and market concentration is also the fundamental assumption of the empirical price-concentration literature that aims at measuring the price effect of market concentration.

The paper is constructed as follows: After briefly reviewing the relevant literature, we develop a monopoly model and derive the monopolist's optimal pricing policy. In Section 4, we suggest a model of duopolistic competition that is founded on the same assumptions as the monopoly model, thus being fully comparable. In Section 5, we compare the equilibrium outcome in duopoly and the monopolist's optimum. In Section 6 we illustrate our main propositions using numerical examples. Finally, we conclude and highlight some implications of our findings in Section 7.

2. Literature Review

The term "two-sided" or "multi-sided market" describes a situation in which two or more distinct and distinguishable groups of agents interact via an intermediary. This intermediary, usually called "platform", charges all groups per transaction and/or for platform access, thereby determining total transaction costs (Rochet and Tirole (2006)). Agents may or may not obtain intrinsic utility from joining a platform. More importantly, demand of one market side affects the utility the other market side obtains from joining a platform. The platform therefore needs "to get both sides on board" (Rochet and Tirole (2003)). Generally, it is hard to draw conclusions from general formulations of two-sided market models, because a number of contradicting assumptions seem plausible, depending on the context.

In Rochet and Tirole (2003)'s credit card example, the number of retailers connected to a specific credit card network, positively affects the utility of consumers, who wish to pay using this credit card and vice versa. In media economics two-sidedness is present in the relation of the media provider, media consumers, and advertisers. However, unlike in the credit card example, most theoretical models assume that media consumers wish to consume the media content only, and are "coerced" to consume advertising as well (see e.g. Anderson and Coate (2005), Gabszewicz et al. (2004), Gal-Or and Dukes (2003), Peitz and Valletti (2008))[1]. Empirical results support this "ad-aversion" assumption for television viewers (see e.g. Danaher (1995), Wilbur (2007)), but find "ad-liking" for print media (Kaiser and Song (2009), Rysman (2004)).

Credit cards and media products also serve as an example to illustrate a second crucial aspect: Excludability of agents who are not willing to pay. While a credit card service can refuse to accommodate specific consumers and retailers, and therefore charge both market sides for its services, media consumers cannot be excluded from free-to-air broadcasting services, so they cannot be charged for consuming it (see the seminal paper by Anderson and Coate (2005)). Third, credit cards are goods that allow for joint consumption or "multi-homing" as it is called in the two-sided market literature. That is, consumers are able to own more than one credit card and retailers can accept more than one card, while in media applications

[1] Models that consider "ad-liking" are e.g. Gabszewicz et al. (2006), Häckner and Nyberg (2008), Kind and Stähler (2010), Rasch (2007).

consumers are often assumed to single-home, e.g. a moviegoer has to decide for exactly one cinema on a Saturday night, but a firm can place advertisements in more than one movie theater.

These selected aspects illustrate, why literature on two-sided markets tailors models around specific examples. For instance, Armstrong (2006) gives a media economic example in his seminal study of pricing in a "competitive bottleneck" scenario. A "competitive bottleneck" is a two-sided market in which one market side is restricted to single-homing, while the other market side is able to multi-home. Armstrong (2006) derives optimal pricing rules for monopoly and duopoly markets in terms of elasticities and finds that there is no difference in advertising prices between monopolies and oligopolies. He argues that competition only emerges on the market for media consumers: Media providers compete for consumers, since advertisers' demand depends on the number of consumers that are exposed to the advertisement, and consumers -by assumption- need to single-home. However, media providers are still monopolists when providing their consumers' attention to the advertiser, because a specific consumer can only be reached by advertising with the platform this consumer chose. If the advertisers' willingness to pay is sufficiently high, the platform might reduce its prices for consumers even below marginal costs, i.e. it "subsidizes" consumers out of advertising revenues.

Counter-intuitive price effects with regard to competition on a competitive bottleneck duopoly have already been reported by Chandra and Collard-Wexlar (2009). They compare pricing under duopolistic competition and joint management of two platforms. Similar to our paper, they find that under specific conditions a merger might lead to a price decrease instead of a price increase. However, their argument and setting is completely different than ours. In their model, platforms can exert negative externalities on each other's profit by increasing their prices. Under joint management this externality would be internalized, which might result in lower monopoly prices. The basic economic effect in their model is analog to the well-known effects occurring when merging two firms with complementary products on a traditional one-sided market.

Price-increasing competition on one-sided markets has also been reported as the result of product differentiation (see Chen and Riordan (2008), Melzer and Morgan (2009), and Bertoletti et al. (2008) for recent contributions and the references therein) and in the presence of search costs (Janssen and Moraga-González (2004), Schulz and Stahl (1996)). In contrast, we explicitly assume a homogeneous good and perfect information of all agents, and focus solely on the two-sidedness of the market to explain price-increasing competition.

3. The Monopoly Model

Consider a two-sided market for a consumption good that is offered in combination with advertising. To foster intuition and readability, we will label customers on the one market side "consumers" and customers on the other market side "advertisers". In this section, we assume that the market is served by a monopolistic platform operator. Similar to Anderson and Gabszewicz (2006), we assume that consumers are homogeneous, except for their preference for the good (say e.g. "movie theater experience"). Let the individual net utility function be additive-separable and given by

(1) $$U_c(q_c) = \theta \cdot q_c - \alpha \cdot n_a \cdot q_c - p_c \cdot q_c, \quad q_c \in \{0,1\},$$

where p_c is the price for one unit of the consumption good, while θ is the taste parameter or marginal willingness to pay, q_c is the quantity, and α is a parameter for the influence of advertising quantity n_a on the individual's utility. Heterogeneous preferences for the good are reflected by θ, which is assumed to be uniformly distributed on the interval $[0, \overline{\theta}]$, where $\overline{\theta} >$ 0 determines the market size for the considered goods market. We limit q_c to the values 0 and 1, so that the individual's decision problem is reduced to whether or not to consume a single unit of the good (e.g. whether to go to the movies or not). Obviously, an individual demands the good, if its net utility of doing so is greater than the net utility of refraining from consumption, that is if

$$U_c(1) \geq U_c(0) = 0 \Leftrightarrow \theta \geq \alpha \cdot n_a + p_c$$

holds. Therefore, we obtain

$$(2)\ n_c(p_c, n_a) = \int_{\alpha \cdot n_a + p_c}^{\overline{\theta}} 1 \cdot d\theta = \overline{\theta} - \alpha \cdot n_a - p_c$$

as the aggregated consumer demand function on the market.

Similar to Armstrong (2006), firms are assumed to generate constant net profits from advertising. For simplicity, we assume advertisements to be standardized, so that firms only decide whether or not to place an advertisement. Therefore a single firm's advertising demand q_a is either 0 or 1. The firm's net profit is given by

$$(3)\ \Pi_a(q_a) = \mu \cdot q_a - p_a \cdot n_c \cdot q_a, \quad q_a \in \{0,1\},$$

where n_c is the number of consumers, p_a is the per-contact advertising price, so that the firm has to pay $p_a \cdot n_c$ to place an advertisement. μ is the parameter that describes the gross benefit of advertising. Firms are assumed to be heterogeneous with respect to μ, and μ is assumed to be uniformly distributed on $[0, \overline{\mu} \cdot n_c]$. The expression $\overline{\mu} \cdot n_c$, with $\overline{\mu} > 0$ determines the size of the advertising market. Note that the net profit to be gained from advertising and therefore the size of the advertising market depends on consumer demand. The economic intuition is straightforward: The higher the demand for the good, the more consumers will be exposed to the advertisement, the more profitable advertising becomes to a firm. Firms are willing to advertise, if their net profit from doing so is positive, that is if

$$\Pi_a(1) \geq \Pi_a(0) = 0 \Leftrightarrow \mu \geq p_a \cdot n_c .$$

Hence, total advertising demand is given by

$$(4)\ n_a(p_a, n_c) = \int_{p_a \cdot n_c}^{\overline{\mu} \cdot n_c} 1 \cdot d\mu = \overline{\mu} \cdot n_c - p_a \cdot n_c .$$

This specific functional form assures that $n_a(p_a,0) = 0$, which implies that there is no demand for advertising if consumer demand is equal to zero.

Solving equations (2) and (4) for p_c and p_a yields the inverse demand functions. We assume fixed and variable costs to be zero and capacity constraints to be non-binding. Thus, the monopolist's optimization problem is

$$\max_{n_c, n_a} \Pi_M = n_c \cdot \left(\overline{\theta} - \alpha \cdot n_a - n_c\right) + n_a \cdot n_c \cdot \left(\overline{\mu} - \frac{n_a}{n_c}\right),$$

yielding the first order conditions

$$n_c = \frac{1}{2} \cdot \left(\overline{\theta} - n_a \cdot \eta\right), \text{ where } \eta = \alpha - \overline{\mu}$$

$$n_a = -\frac{1}{2} \cdot n_c \cdot \eta$$

Therefore, the optimal monopoly solution is

$$(5)\ n_c^M = \frac{2 \cdot \overline{\theta}}{4 - \eta^2} \qquad (6)\ n_a^M = \frac{\eta \cdot \overline{\theta}}{\eta^2 - 4}$$

$$(7)\ p_c^M = \frac{(\overline{\mu} \cdot \eta + 2) \cdot \overline{\theta}}{4 - \eta^2} \qquad (8)\ p_a^M = \frac{\alpha + \overline{\mu}}{2},$$

and yields a monopoly profit of

$$(9)\ \Pi^M = \frac{\overline{\theta}^2}{4 - \eta^2} .$$

An economically plausible solution requires that quantities and profit are nonnegative, i.e. $n_c^M, n_a^M, \Pi^M \geq 0$. Note that optimal pricing on two-sided markets might involve prices below marginal cost ("subsidization") for one market side. To determine economically plausible parameter sets, we consider three cases:[2]

If consumers are ad-averse ($\alpha > 0$), $(\overline{\mu}, \alpha) \in \{(\overline{\mu} > 0, \max(0, \overline{\mu} - 2) < \alpha \leq \overline{\mu})\}$ yields plausible solutions. For ad-neutral consumers ($\alpha = 0$), $0 < \overline{\mu} < 2$ is required and in case consumers are ad-likers ($\alpha < 0$), the model yields plausible results for $(\overline{\mu}, \alpha) \in \{(0 < \overline{\mu} < 2, \overline{\mu} - 2 < \alpha < 0)\}$.

4. A Model of Duopolistic Competition

In this section, we develop a model of duopolistic competition in order to identify competitive effects on the market that has been presented in the former section. Since our paper focuses on the comparison of monopoly and duopoly markets, all assumptions of Section 2 remain, except that we now assume the market to be served by two identical platforms, denoted $i = 1,2$.

The consumption good offered by both platforms is assumed to be perfectly homogenous (e.g. two multiplexes offering the same menu of movies in direct proximity). Just like the monopolist, the duopolists are assumed to produce without variable and fixed costs. Consumers are assumed to be the same utility-maximizing individuals they were in the previous section. Additionally, we assume that consumers are required to single-home, that is, if they join, they will have to decide for one and only one platform to join (e.g. a moviegoer can only be in one cinema at the same time)[3]. Obviously, consumers will prefer the platform that offers most net utility. If consumers' net utility is equal on both platforms, aggregate demand is assumed to be equally shared among the two operators. Thus, using equation (1) for $q_c = 1$ and equation (2), the consumer demand function platform operator i faces is

$$(10)\quad n_c^i\left(p_c^i, n_a^i\right) = \begin{cases} 0 \ \mathit{for}\ \theta - \alpha \cdot n_a^i - p_c^i < \theta - \alpha \cdot n_a^j - p_c^j \\ \dfrac{\overline{\theta} - \alpha \cdot n_a^i - p_c^i}{2} \ \mathit{for}\ \theta - \alpha \cdot n_a^i - p_c^i = \theta - \alpha \cdot n_a^j - p_c^j \\ \overline{\theta} - \alpha \cdot n_a^i - p_c^i \ \mathit{for}\ \theta - \alpha \cdot n_a^i - p_c^i > \theta - \alpha \cdot n_a^j - p_c^j \end{cases} \qquad i, j = 1,2\,,\ i \neq j$$

Unlike consumers, advertisers are allowed to multi-home, which implies that they can place a single unit of advertising on one platform only or on both platforms simultaneously. Therefore, the advertisers' decision problem only depends on the advertising price and the

[2] Remember that we restricted $\overline{\theta} > 0$. Since throughout (5) - (9) $\overline{\theta}$ only appears -if at all- as a factor in the numerator, and therefore only has a scaling function, we will ignore $\overline{\theta}$ in the parameter sets to simplify notation. We will apply this simplification throughout the paper.

[3] Note that going to the movies at another time of the day or at another day would be product differentiation, which we intentionally abstract from, to focus solely on the effect of "two-sidedness".

consumer demand of the corresponding platform. In other words, platform i's advertising demand does not directly depend on platform j's behavior.[4] Using equation (4), advertising demand of platform i is therefore given by

$$(11)\ n_a^i\left(p_a^i, n_c^i\right) = \overline{\mu} \cdot n_c^i - p_a^i \cdot n_c^i = n_c^i \cdot \left(\overline{\mu} - p_a^i\right),$$

which is analog to Section 2.

We assume that both platforms compete in a Bertrand-type pricing game, simultaneously choosing prices p_c^i and p_a^i. Since the platforms are perfectly identical, we focus on symmetric equilibria.[5] Generally, a symmetric solution for $i,j = 1,2,\ i \neq j$ is characterized by

$$p_c^i = p_c^j = p_c^s \text{ and } p_a^i = p_a^j = p_a^s,$$

which implies

$$(12)\ n_a^i = n_a^j = n_a^s = \left(\overline{\mu} - p_a^s\right) \cdot \frac{n_c^s}{2},$$

where n_a^s is the advertising demand faced by one platform operator, while n_c^s is the aggregate consumer demand in the market, equally shared among the operators, given any symmetric solution p_c^s and p_a^s. In order to calculate n_c^s, we have to take into account that single-homing consumers are interested in the amount of advertising on *each platform*, which is n_a^s. The *total number* of ads $n_a^i + n_a^j = 2 \cdot n_a^s$ is not relevant for consumers' decision making. Thus, using equations (2) and (12), aggregate consumer demand is given by

$$(13)\ n_c^s = \overline{\theta} - \alpha \cdot n_a^s - p_c^s = \overline{\theta} - \alpha \cdot \left(\overline{\mu} - p_a^s\right) \cdot \frac{n_c^s}{2} - p_c^s \Leftrightarrow n_c^s = \frac{\overline{\theta} - p_c^s}{1 + \frac{1}{2} \cdot \alpha \cdot \left(\overline{\mu} - p_a^s\right)},$$

so that n_a^s can be expressed as

$$(14)\ n_a^s = \left(\overline{\mu} - p_a^s\right) \cdot \frac{\overline{\theta} - p_c^s}{2 + \alpha \cdot \left(\overline{\mu} - p_a^s\right)}.$$

Therefore, for any given p_c^s and p_a^s, firm i's profit in the symmetry case is

$$(15)\ \Pi_i^s\left(p_c^s, p_a^s\right) = \frac{\overline{\theta} - p_c^s}{2 + \alpha \cdot \left(\overline{\mu} - p_a^s\right)} \cdot \left(p_c^s + p_a^s\left(\overline{\mu} - p_a^s\right) \cdot \frac{\overline{\theta} - p_c^s}{2 + \alpha \cdot \left(\overline{\mu} - p_a^s\right)}\right).$$

[4] It is, of course, indirectly dependent of j's behavior, because n_a^i depends on n_c^i, and by (10), n_c^i depends on n_c^j.
[5] See Nilssen and Sørgard (2001) for a model with heterogeneous platforms and asymmetric equilibria.

Suppose that the candidate equilibrium (p_c^*, p_a^*) is characterized by

$$p_c^{i*} = p_c^{j*} = p_c^* \qquad p_a^{i*} = p_a^{j*} = p_a^* .$$

In this case, platform operator i's deviation strategies can be defined as

$$p_c^{i/low}, p_a^{i/low} \;\; for\; \theta - \alpha \cdot n_a^{i/low} - p_c^{i/low} < \theta - \alpha \cdot n_a^{j*}\left(p_c^{j*}, p_a^{j*}\right) - p_c^{j*}$$
$$p_c^{i/equal}, p_a^{i/equal} \;\; for\; \theta - \alpha \cdot n_a^{i/equal} - p_c^{i/equal} = \theta - \alpha \cdot n_a^{j*}\left(p_c^{j*}, p_a^{j*}\right) - p_c^{j*},$$
$$p_c^{i/high}, p_a^{i/high} \;\; for\; \theta - \alpha \cdot n_a^{i/high} - p_c^{i/high} > \theta - \alpha \cdot n_a^{j*}\left(p_c^{j*}, p_a^{j*}\right) - p_c^{j*}$$

where $(p_c^{i/low}, p_a^{i/low})$ is a strategy that implies lower consumer utility, $(p_c^{i/equal}, p_a^{i/equal})$ is a strategy that implies the same consumer utility, and $(p_c^{i/high}, p_a^{i/high})$ is a strategy that implies higher consumer utility than strategy (p_c^*, p_a^*). Note that platform operator i might deviate by changing one or both prices, and that the operator might alter both prices in the same direction or in opposing directions. Therefore the indices *low*, *equal*, and *high* do not imply prices in the deviation strategy being higher, equal or lower than the equilibrium candidate prices.

The well-known condition for a Nash equilibrium is that operator i cannot deviate profitably, which means that (p_c^*, p_a^*) is an equilibrium, if and only if

$$\Pi_i^s\left(\left(p_c^{i*}, p_a^{i*}\right), \left(p_c^{j*}, p_a^{j*}\right)\right) \geq \Pi_i\left(\left(p_c^{i/low}, p_a^{i/low}\right), \left(p_c^{j*}, p_a^{j*}\right)\right)$$
$$\Pi_i^s\left(\left(p_c^{i*}, p_a^{i*}\right), \left(p_c^{j*}, p_a^{j*}\right)\right) \geq \Pi_i\left(\left(p_c^{i/equal}, p_a^{i/equal}\right), \left(p_c^{j*}, p_a^{j*}\right)\right),$$
$$\Pi_i^s\left(\left(p_c^{i*}, p_a^{i*}\right), \left(p_c^{j*}, p_a^{j*}\right)\right) \geq \Pi_i\left(\left(p_c^{i/high}, p_a^{i/high}\right), \left(p_c^{j*}, p_a^{j*}\right)\right)$$

Thus, in order to find equilibria, we will have to analyze these cases separately. We will do this, using the following propositions:

Proposition 1: If platform operator i deviates by choosing any $(p_c^{i/low}, p_a^{i/low})$, the resulting profit is always $\Pi_i^{low}(.) = 0$.

Proof: Equation (10) implies that $n_c^i(.) = 0$, which means that demand for platform i is taking the value zero as all consumers will decide to use the rival platform j. In addition, using equation (11) we obtain $n_a^i(.) = 0$, because advertisers are not willing to place an ad on platform i, when there are no consumers. Thus, for any $(p_c^{i/low}, p_a^{i/low})$ i's profit is zero.

(q.e.d.)

As economic intuition suggests, it is not profitable for a platform operator to deviate by offering less consumer utility than the rival platform. Since any equilibrium with $\Pi_i(.) < 0$ is not economically plausible, there will never be an incentive for operator i to charge

($p_c^{i/low}$, $p_a^{i/low}$). Therefore, this strategy can be neglected for further analysis of candidate equilibria.

Proposition 2: Suppose platform operator *i* is maximizing her profit, while offering the same utility as operator *j* by charging any *($p_c^{i/equal}$, $p_a^{i/equal}$)*. Then, it is always profit-maximizing to charge the monopoly advertising price.

Proof: Since both platforms offer equal consumer utility, we know that

$$(16)\ \theta - \alpha \cdot n_a^{i/equal} - p_c^{i/equal} = \theta - \alpha \cdot n_a^{j*} - p_c^{j*} \Leftrightarrow p_c^{i/equal} = \alpha \cdot n_a^{j*} - \alpha \cdot n_a^{i/equal} + p_c^{j*}$$

must hold, which also implies that *i*'s consumer demand, denoted by $n_c^{i/equal}$, is fixed. Solving (11) for the advertising price gives the inverse advertising demand function as

$$p_a^{i/equal} = \overline{\mu} - \frac{n_a^{i/equal}}{n_c^{i/equal}}.$$

Therefore, operator *i*'s (constrained) maximization problem is

$$\max \Pi_i\left(d_a^{i/equal}\right) = \left(\alpha \cdot n_a^{j*} - \alpha \cdot n_a^{i/equal} + p_c^{j*}\right) \cdot n_c^{i/equal} + \left(\overline{\mu} - \frac{n_a^{i/equal}}{n_c^{i/equal}}\right) \cdot n_c^{i/equal} \cdot n_a^{i/equal},$$

yielding

$$(17)\ n_a^{i/equal*} = \frac{-n_c^{i/equal} \cdot \eta}{2} \Leftrightarrow p_a^{i/equal*} = \frac{\overline{\mu} + \alpha}{2},$$

from which we see that the profit-maximizing advertising price is equal to the optimal monopoly advertising price (8).

(q.e.d.)

Proposition 2 implies a very important result: In any symmetric situation, operator *i*'s deviation profit implied by (17) is at least as great as the profit in the symmetric situation (proof: see Appendix 1). Therefore, in any symmetric situation there is an incentive to charge the monopoly advertising price. Since the platforms are identical, we can expect that $n_a^{j*} = n_a^{i/equal*}$, so that (16) simplifies to $p_c^{i/equal} = p_c^{j*}$. Thus, we can tentatively conclude that a symmetric equilibrium requires

$$(18)\ p_a^{i*} = p_a^{j*} = \frac{\overline{\mu} + \alpha}{2}$$

for any $p_c^{i*} = p_c^{j*}$. However, at this stage of our analysis the equilibrium level of the consumer price remains unspecified.

Proposition 3: When offering consumers more net utility than her rival firm, thus charging ($p_c^{i/high}$, $p_a^{i/high}$), platform operator i's profit-maximizing strategy is either (i) the monopoly solution or (ii) $n_c^{i/high} = \overline{\theta} - p_c^{j*}$.

Proof: In order to attract all consumers, the constraint

$$(19)\ \theta - \alpha \cdot n_a^{i/high} - p_c^{i/high} > \theta - \alpha \cdot n_a^{j*} - p_c^{j*} \quad \Leftrightarrow \quad -\alpha \cdot n_a^{i/high} - p_c^{i/high} > -\alpha \cdot n_a^{j*} - p_c^{j*}$$

must be satisfied. From equation (10) we know that consumer demand for operator i is

$$(20)\ n_c^{i/high} = \overline{\theta} - \alpha \cdot n_a^{i/high} - p_c^{i/high} \quad \Leftrightarrow \quad p_c^{i/high} = \overline{\theta} - \alpha \cdot n_a^{i/high} - n_c^{i/high}.$$

As long as (19) is satisfied, rival platform j's consumer demand is equal to zero, which implies that j's advertising demand is also zero. Therefore, we assume that consumers anticipate that $n_a^{j*} = 0$, so that (19) simplifies to

$$(21)\ -\alpha \cdot n_a^{i/high} - \left(\overline{\theta} - \alpha \cdot n_a^{i/high} - n_c^{i/high}\right) > -p_c^{j*} \quad \Leftrightarrow \quad n_c^{i/high} > \overline{\theta} - p_c^{j*}.$$

The corresponding (Kuhn-Tucker-) optimization problem for operator i can be expressed as

$$\max \Pi_i\left(n_c^{i/high}, n_a^{i/high}\right) = n_c^{i/high} \cdot \left(\overline{\theta} - \alpha \cdot n_a^{i/high} - n_c^{i/high}\right) + \left(\overline{\mu} - \frac{n_a^{i/high}}{n_c^{i/high}}\right) \cdot n_c^{i/high} \cdot n_a^{i/high}$$
$$+ \lambda\left(n_c^{i/high} - \overline{\theta} + p_c^{j*}\right),$$

which yields the monopoly solution of Section 2 if (21) is not binding, i.e. $\lambda = 0$. In case (21) is binding ($\lambda > 0$)[6], operator i will choose the slightest possible $n_c^{i/high}$ without violating the constraint. The resulting solution is approximately

$$(22)\ n_c^{i/high*} = \overline{\theta} - p_c^{j*}$$

(q.e.d.).

Since case (i) of Proposition 3 does not contain equilibria, we now further investigate case (ii). This case holds as long as $n_c^{i/high*} \leq n_c^M$ which corresponds to

$$p_c \leq \hat{p}_c = \frac{\overline{\theta} \cdot \left(\eta^2 - 2\right)}{\left(\eta^2 - 4\right)},$$

where $\hat{p}_c$ solves $n_c^{i/high*} = n_c^M$. Given (22), $p_a^{i/high*}$ still matches the monopoly solution and $n_a^{i/high*}$ becomes

$$n_a^{i/high*} = \frac{\left(p_c^{j*} - \overline{\theta}\right) \cdot \eta}{2}.$$

Using $n_a^{i/high*}$ as well as equations (20) and (22), the resulting consumer price is described by

[6] It can be shown that there is no equilibrium in the non-binding case as deviation to the monopoly solution would always be profitable, and the monopoly solution is not an equilibrium solution.

$$p_c^{i/high*} = \frac{\alpha \cdot \left(\overline{\theta} - p_c^{j*}\right) \cdot \eta}{2} + p_c^{j*}.$$

Therefore, operator i's deviation profit can be expressed as

$$(23)\ \Pi_i^{high}\left(p_c^{j*}\right) = \frac{1}{4} \cdot \left(\overline{\theta} - p_c^{j*}\right) \cdot \left[\eta^2 \cdot \left(\overline{\theta} - p_c^{j*}\right) + 4 \cdot p_c^{j*}\right],$$

which is defined for all $p_c^{j*} \leq \hat{p}_c$.

Proposition 4: For ad-averse consumers $(\alpha > 0)$, symmetric equilibria are characterized by $p_a^* = p_a^M$ and $\underline{p}_c^* \leq p_c^* \leq \overline{p}_c^*$, where

$$\overline{p}_c^* = \frac{\eta \cdot \left[\alpha \cdot \left(20 + \eta^2 \cdot (\alpha \cdot \eta - 8)\right) - 12 \cdot \overline{\mu}\right] \cdot \overline{\theta}}{\zeta}, \text{ and } \underline{p}_c^* = \frac{\left(\overline{\mu}^2 - \alpha^2\right)}{\eta^2 - 8}, \text{ with}$$

$$\zeta = \eta \cdot \left[\alpha \cdot \left(44 + \eta\left(\alpha \cdot \left(\eta^2 - 12\right) + 8 \cdot \overline{\mu}\right)\right) - 12 \cdot \overline{\mu}\right] - 32.$$

For ad-neutral consumers $(\alpha = 0)$ and ad-liking consumers $(\alpha < 0)$, there is no equilibrium solution.

Proof: Symmetric equilibria are characterized by the results of Propositions 2 and 3. As stated before, Proposition 2 implies that both platform operators charge the monopoly advertising price in equilibrium, given any consumer price. Equilibrium consumer prices can be obtained by the results of Proposition 3, because there is no incentive for deviation, if the deviation profit, given by (23), does not exceed the platform's profit in the symmetry case. Therefore, in equilibrium

$$\Pi_i^{high}\left(p_c^*\right) \leq \Pi_i^s\left(p_c^*, p_a^*\right)$$

must hold.

Given (18) and using equations (15) and (23), it is easy to verify that $\Pi_i^{high}\left(p_c^*\right)$ and $\Pi_i^s\left(p_c^*, p_a^*\right)$ intersect exactly once at $\overline{p}_c^*$, where

$$(24)\ \overline{p}_c^* = \frac{\eta \cdot \left[\alpha \cdot \left(20 + \eta^2 \cdot (\alpha \cdot \eta - 8)\right) - 12 \cdot \overline{\mu}\right] \cdot \overline{\theta}}{\zeta}.$$

Assuming ad-aversion $(\alpha > 0)$, and given the corresponding parameter restrictions of the monopoly model we see that

$$\left.\frac{\partial \Pi_i^{high}}{\partial p_c}\right|_{\overline{p}_c^*} > \left.\frac{\partial \Pi_i^s}{\partial p_c}\right|_{\overline{p}_c^*} \text{ and } \frac{\partial \Pi_i^{high}}{\partial p_c} > 0,$$

which implies that $\overline{p}_c^*$ is the upper bound of the continuum of equilibrium consumer prices. The lower bound is given by the root of $\Pi_i^s\left(p_c^*, p_a^*\right)$, that is, where duopolists realize zero profits, i.e. at

$$\underline{p}_c^* = \frac{\left(\overline{\mu}^2 - \alpha^2\right)}{\eta^2 - 8}.$$

Assuming ad-liking $(\alpha < 0)$ or ad-neutrality $(\alpha = 0)$, and the corresponding restrictions of the monopoly model, there are no equilibria, because $\Pi_i^{high}\left(p_c^*\right) > \Pi_i^s\left(p_c^*, p_a^*\right) \forall p_c \in \left\lfloor \underline{p}_c^*, \hat{p}_c \right\rfloor$.

(q.e.d.)

Proposition 4 states that the duopoly model only yields equilibrium solutions for the case of ad-averse consumers, while there are no equilibria in the cases of ad-neutrality and ad-liking. This outcome corresponds to economic intuition. If consumers are ad-averse, the deviating platform gains from two specific economic effects: By offering more net-utility, the firm is able to exploit the whole consumer demand, instead of equally sharing the market. In addition, a higher number of consumers on this platform shifts the advertising demand function, resulting in a higher level of advertising. Thus, the deviating platform is able to generate a higher profit. However, ad-averse consumers are facing a disutility from advertising, which implies that the higher advertising level in turn shifts the aggregate consumer demand function on the market downwards. This effect makes deviation less profitable.

In the case of ad-neutrality, a deviating firm still gains from serving the whole consumer demand and the resulting enhancing effect on advertising demand, while there is no ad disutility for consumers and, consequently, no negative effect on consumer demand. Therefore, the profit from deviation is, ceteris paribus, higher than in the case of ad-aversion, making deviation more likely to occur. This also holds for ad-liking consumers, where deviation is even more profitable, because higher advertising levels increase consumers' willingness to pay. Obviously, the deviation profit for $\alpha < 0$ and $\alpha = 0$ is higher than in the case of ad-aversion $(\alpha > 0)$, which intuitively underpins our finding that there are no equilibria in these cases.

5. Analysis of the Model

In the previous sections we developed a competitive bottleneck two-sided market model and determined the monopolist's optimum as well as the duopoly equilibria. In this section we are going to study these outcomes more deeply. We will specifically focus on the comparison of monopoly and duopoly in terms of prices, quantities, and welfare. Furthermore, we will focus rather on those cases that are counterintuitive or contrary to common economic knowledge from traditional one-sided markets, i.e. cases in which equilibrium duopoly prices equal or even exceed optimal monopoly prices.

Since there are no equilibria in the cases of ad-neutrality and ad-liking, we strictly focus further analysis on the case of ad-averse consumers, where a continuum of equilibrium consumer prices exists. We assume that both platform operators select the equilibrium solution that generates the highest consumer price.[7] Thus, the resulting equilibrium strategy is $\left(p_a^* = p_a^M, p_c^* = \overline{p}_c^*\right)$. The corresponding equilibrium is therefore characterized by (17) and (24), yielding

$$(25)\ n_a^* = n_a^{i*} = n_a^{j*} = \frac{4\cdot\overline{\theta}\cdot\eta\cdot(2-\alpha\cdot\eta)}{\zeta}$$

$$(26)\ n_c^* = n_c^{i*} = n_c^{j*} = \frac{8\cdot\overline{\theta}\cdot(\alpha\cdot\eta-2)}{\zeta}$$

$$(27)\ \Pi^* = \Pi^{i*} = \Pi^{j*} = \frac{8\cdot\overline{\theta}^2\cdot\eta\cdot\left(\alpha\cdot\left(\eta^2-6\right)+2\cdot\overline{\mu}\right)(\alpha\cdot\eta-4)(\alpha\cdot\eta-2)}{\zeta^2},$$

where an economically plausible symmetric equilibrium solution obviously requires

$$\Pi_i^s\left(p_c^*, p_a^*\right), n_c^s\left(p_c^*, p_a^*\right), n_a^s\left(p_c^*, p_a^*\right) \geq 0,$$

which in case of ad-aversion ($\alpha > 0$), is given for parameter sets satisfying

$$(\overline{\mu},\alpha)\in\{(\overline{\mu}>0, \tau\leq\alpha\leq\overline{\mu})\},\ \text{where}$$

$$\tau = R_2 \text{ of } x^3 + 2\cdot\overline{\mu}\cdot\left(1-x^2\right)+\left(\overline{\mu}^2-6\right)\cdot x.$$ [8,9]

Prices and Quantities

To see if there exist parameter tuples ($\overline{\mu},\alpha$) given which duopoly prices equalize or exceed monopoly prices, we first remember that by Proposition 2 the duopoly advertising price in equilibrium equals the monopoly advertising price. The intuition of this result follows the standard argument of Armstrong (2006), who argues that on a competitive bottleneck two-

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[7] The qualitative results and conclusions of the following sections do not depend on the selected equilibrium as it can be shown that all results also hold for the equilibrium strategy $\left(p_a^* = p_a^M, p_c^* = \underline{p}_c^*\right)$, which implies that both operators charge the lowest possible consumer price (see Appendix 3).

[8] R_l, $l = 1,\ldots,n$, denotes the l-th real-valued polynomial root in ascending order of the corresponding polynomial of degree n.

[9] Since $\overline{\theta}$ already turned out to be a nonnegative scaling factor only, we will suppress it in the notation, that is, we will give tuples ($\overline{\mu},\alpha$) only.

sided market each platform is a monopolist towards the multi-homing market side, because a specific agent on the single-homing side can only be reached by joining the platform this agent chose. Our analysis therefore needs to focus on consumer prices only. Using (7) and (24), the parameter sets we are interested in solve

$$(28)\ \bar{p}_c^* = \frac{\eta \cdot \left[\alpha \cdot \left(20 + \eta^2 \cdot (\alpha \cdot \eta - 8)\right) - 12 \cdot \bar{\mu}\right] \cdot \bar{\theta}}{\zeta} \geq \frac{(\bar{\mu} \cdot \eta + 2) \cdot \bar{\theta}}{4 - \eta^2} = p_c^M .$$

These parameter sets can be expressed as

$$(29)\ (\bar{\mu}, \alpha) \in \left\{ \left(\max(\beta, \delta) \leq \bar{\mu} \leq \frac{\alpha^2 - 1 + \sqrt{4 \cdot \alpha^2 + 1}}{\alpha}, \sqrt{2} \leq \alpha \right) \right\}, \text{ where}$$

$\beta = R_1$ of $x^4\alpha^2 + 4x^3\alpha(1-\alpha^2) + 2x^2(3\alpha^4 - 8\alpha^2 - 2) + 4x(-\alpha^5 + 5\alpha^3 - 2\alpha) + \alpha^6 - 8\alpha^4 + 12\alpha^2 + 32$, and

$\delta = R_3$ of $x^4\alpha^2 + 4x^3\alpha(1-\alpha^2) + 2x^2(3\alpha^4 - 8\alpha^2 - 2) + 4x(-\alpha^5 + 5\alpha^3 - 2\alpha) + \alpha^6 - 8\alpha^4 + 12\alpha^2 + 32$.

Cases with consumer prices being equal in monopoly and duopoly are those cases satisfying (28) with equality. These cases are given by the lower bounds of $\bar{\mu}$ in (29) that is if $(\bar{\mu}, \alpha) \in \left\{ \left(\max(\beta, \delta), \sqrt{2} \leq \alpha \right) \right\}$. For $\sqrt{2} \leq \alpha < \gamma \approx 3.955$, $\max(\beta, \delta) = \beta$, and the result is a subsidization solution[10]. For $\alpha > \gamma$, $\max(\beta, \delta) = \delta$, and there is no subsidization. In all these cases, there is no *observable* price effect of competition. To gain deeper insight into the economics of this phenomenon, we evaluate demands and profits at any of these ($\bar{\mu}, \alpha$) and see that

$$n_c^* > n_c^M,\ n_a^* < n_a^M < 2 \cdot n_a^*, \text{ and } 2 \cdot \Pi^* < \Pi^M .$$

Remember that n_a^* is advertising quantity per duopolist, hence $2 \cdot n_a^*$ is total advertising on the duopoly market. $(\bar{\mu}, \alpha) \in \left\{ \left(\max(\beta, \delta), \sqrt{2} \leq \alpha \right) \right\}$ describes a situation, in which the mere fact that the market is served by two identical firms instead of one, causes an increase in total consumer demand. While in the traditional one-sided world, increases in total consumer demand stem from the fact that duopolistic competition yields lower prices, the situation described here is more complex, because there is no *observable* price effect of competition. The reason for the enhancement in total consumer demand is the two-sidedness of the market or more precisely, the effect of a decreasing amount of advertising per platform on consumer utility. To illustrate the economics of this case, we do the following gedankenexperiment: Starting from some monopolistic optimum $n_c^M > 0$, $n_a^M > 0$, we imagine that -all else equal-

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[10] The special case $(\bar{\mu}, \alpha) = \left(2\sqrt{2}, \sqrt{2}\right)$ with $\max(\beta, \delta) = \delta = 2\sqrt{2}$, which is discussed in Appendix 3, is also a subsidization solution.

the monopolist is replaced by two identical, but independent platforms. In this case, total consumer demand n_c^M will be equally divided among the two platforms. As a consequence, advertising with one platform only reaches half of the consumers, which will reduce advertising demand per platform to some $n_a^n < n_a^M$. Since we assumed $\alpha > 0$, the decrease in advertising exposure increases the willingness to pay of each consumer by an amount equal to $\alpha\left(n_a^M - n_a^n\right)$. Since the willingness to pay of each consumer increases, the demand function shifts to the outside. If both platforms colluded, they would exploit this additional willingness to pay by setting a higher price (see Appendix 2). Since there is no *observable* price effect in either direction, there must be a countervailing effect - the *unobservable* price effect of platform competition, which in this case just compensates the price increasing effect of increased willingness to pay. Therefore, prices remain equal, but total consumer demand increases to some quantity $n_c^n > n_c^M$.

Price-increasing competition occurs, if (28) is satisfied with strict inequality. The corresponding parameter sets are subsets of (29), i.e.

$$(\bar{\mu},\alpha)\in\left\{\left(\max(\beta,\delta)<\bar{\mu}\leq\frac{\alpha^2-1+\sqrt{4\cdot\alpha^2+1}}{\alpha},\sqrt{2}\leq\alpha\right)\right\}$$

As is the case of price equality, there are subsidization solutions and solutions with positive prices. However, it is now also possible that the monopolist charges negative consumer prices, while the duopolists do not. In the case of subsidization, less negative prices in duopoly can be interpreted as a lower subsidization of consumers as compared to monopoly. In case of price-increasing competition, the effect of platform competition does not fully compensate the effect of demand-enhancement. Therefore, the relationships of quantities are less straightforward and partly ambiguous without further refinement of the parameter sets. Before doing so, let us first note some results that hold for the whole case of price-increasing competition: Evaluated at any of the corresponding ($\bar{\mu}$, α), we see that

$$2\cdot\Pi^* < \Pi^M \text{ and } n_a^* < n_a^M,$$

which is so far consistent with the results obtained for price equality. There is a competitive effect on profits, because despite of the price increase, industry wide profits are lower in duopoly than in monopoly. Furthermore, advertising demand per platform is lower in duopoly than in monopoly. However, because of the negative effect of consumer price on consumer demand, the demand-enhancing effect of lower advertising that shifts the consumer demand function to the outside is partially compensated by the price increase, which corresponds to a move along the (shifted) demand function, ($n_c^* \gtreqless n_c^M$), and total advertising quantity does not necessarily exceed the monopoly level ($2 \cdot n_a^* \gtreqless n_a^M$).

Parameter sets contained in

$$(\bar{\mu}, \alpha) \in \left\{ \left(\max(\beta, \omega) < \bar{\mu} \le \frac{\alpha^2 - 1 + \sqrt{4 \cdot \alpha^2 + 1}}{\alpha}, \sqrt{2} < \alpha \right) \right\}, \text{ where}$$

$$\omega = \mathrm{R}_1 \text{ of } e^2\alpha^3 + (8\alpha - 3\alpha^3)x^2 + (12 - 16\alpha^2 + 3\alpha^5)x - \alpha^5 + 8\alpha^3 - 20\alpha,$$

yield negative consumer prices. The relationships of n_c^* and n_c^M and n_a^M and $2 \cdot n_a^*$ are ambiguous in this case. The same holds for the mixed cases, in which the monopolist will subsidize while the duopolists will not, which are described by

$$(\bar{\mu}, \alpha) \in \left\{ \left(\frac{\alpha + \sqrt{8 + \alpha^2}}{2} < \bar{\mu} \le \delta, \alpha > \kappa \approx 2.265 \right) \right\}.$$

Only in case of positive prices, the relationships of quantities are unambiguous and equal to the results in case of price equality, i.e. $n_c^* > n_c^M$, $n_a^* < n_a^M < 2 \cdot n_a^*$, and $2 \cdot \Pi^* < \Pi^M$. Positive prices result, if

$$(\bar{\mu}, \alpha) \in \left\{ \left(\max(\beta, \delta) < \bar{\mu} < \frac{\alpha + \sqrt{8 + \alpha^2}}{2}, \kappa < \alpha \right) \right\}.$$

Table 1 summarizes the findings.

Case		$n_c^* \gtreqless n_c^M$	$n_a^* \gtreqless n_a^M$	$2 \cdot n_a^* \gtreqless n_a^M$	$2 \cdot \Pi^* \gtreqless \Pi^M$
$p_c^* = p_c^M$	Subsidization	$>$	$<$	$>$	$<$
	Positive Prices	$>$	$<$	$>$	$<$
$p_c^* > p_c^M$	Subsidization	$\gtreqless$	$<$	$\gtreqless$	$<$
	Mixed	$\gtreqless$	$<$	$\gtreqless$	$<$
	Positive Prices	$>$	$<$	$>$	$<$

Table 1: Relationship of quantities and profits, if consumer prices are equal or if the duopoly consumer price exceeds the monopoly consumer price.

Welfare

To complete our analysis, we will study the welfare effects imposed by our model. Since we assumed zero costs of production, monopoly profit and the sum of both providers' profits is equal to producer surplus. Traditional "consumer surplus" here is the sum of the surpluses created on both market sides. In the monopoly case, the market side we labeled "consumers" realizes a benefit of

$$\int_0^{n_c^M} p_c^M\left(n_c, n_a^M\right)dn_c - n_c^M \cdot p_c^M = \frac{2 \cdot \overline{\theta}}{\left(\eta^2 - 4\right)^2},$$

where n_c^M is given by (5), n_a^M is given by (6), p_c^M is given by (7), and $p_c^M(\cdot)$ is the inverse of (2). In the duopoly case the consumer side realizes a surplus of

$$\int_0^{2n_c^i} p_c^d\left(n_c, n_a^*\right)dn_c - 2 \cdot n_c^* \cdot p_c^* = \frac{128 \cdot (2 - \alpha \cdot \eta)^2 \cdot \overline{\theta}^2}{\zeta^2},$$

where n_c^* is given by (26), n_a^* is given by (25), p_c^* is given by (24), and $p_c^d(\cdot)$ is the inverse of (13). Advertisers obtain a surplus of

$$\int_0^{n_a^M} p_a^M\left(n_a, n_c^M\right)dn_a - n_a^M \cdot p_a^M = \frac{\eta^2 \cdot \overline{\theta}}{4 \cdot \left(4 - \eta^2\right)},$$

where n_c^M is given by (5), n_a^M is given by (6), p_a^M is given by (8), and $p_a^M(\cdot)$ is the inverse of (4) in the monopoly case, and

$$2 \cdot \left(\int_0^{n_a^*} p_a^D\left(n_a, n_c^*\right)dn_a - n_a^* \cdot p_a^* \right) = \frac{2 \cdot \eta^2 \cdot (\alpha \cdot \eta - 2) \cdot \overline{\theta}}{\zeta}$$

in the duopoly case, where n_c^* is given by (26), n_a^* is given by (25), $p_a^* = p_a^M$ is given by (8), and $p_a^D(\cdot)$ is the inverse of (12).

Welfare is given by the sum of producer, consumer, and advertiser surplus. Table 2 summarizes the relation of total welfare in monopoly optimum and duopoly equilibrium as well as the relations of the individual welfare components.

Case		Consumer Surplus	Advertiser Surplus	Producer Surplus	Total Welfare
$p_c^* = p_c^M$	Subsidization	$\gtreqless$	$\gtreqless$	$<$	$\gtreqless$
	Positive Prices	$>$	$>$	$<$	$>$
$p_c^* > p_c^M$	Subsidization	$\gtreqless$	$\gtreqless$	$<$	$\gtreqless$
	Mixed	$\gtreqless$	$\gtreqless$	$<$	$\gtreqless$
	Positive Prices	$>$	$>$	$<$	$>$

Left hand side = duopoly equilibrium; right hand side = monopoly optimum

Table 2: Comparison of welfare effects for ad-averse consumers.

From Table 2 we see that clear-cut welfare predictions can only be made, if no subsidization takes place. In this case, total welfare can unambiguously be improved by adding a second homogeneous platform to the industry or by permitting platform merger, even if consumer prices in duopoly are higher than in monopoly. The decrease of total producer surplus due to duopolistic competition is overcompensated by the increased consumer welfare stemming from lower advertising levels per platform, and the increased advertiser surplus, because in total more consumers can be reached.

6. Numerical Examples

In this section we illustrate the economics of our model using numerical examples and graphical representations. We start by presenting a case, which yields results that fit economic intuition from oligopolistic competition (Example I). Example II shows a situation in which the upper bound of the equilibria is equal to the monopoly price, representing the case of price equality from the previous section. Example III represents the case of price-increasing competition.

Example I: Intuitive price effect of competition

$\left(\overline{\mu},\overline{\theta},\alpha\right)=\left(1{,}3{,}0.7\right)$			
p_c^M	1.304	$p_a^* = p_a^M$	0.85
$\left[\underline{p}_c^*,\overline{p}_c^*\right]$	[-0.193, 0.035]	n_a^M	0.230
$\hat{p}_c$	1.476	$\left[\underline{n}_a^*,\overline{n}_a^*\right]$	[0.228, 0.211]
		$\left[2\underline{n}_a^*,2\overline{n}_a^*\right]$	[0.455, 0.423]
n_c^M	1.535	Π^M	2.302
$\left[\frac{1}{2}\underline{n}_c^*,\frac{1}{2}\overline{n}_c^*\right]$	[1.517, 1.408]	$\left[\underline{\Pi}^*,\overline{\Pi}^*\right]$	[0, 0.303]
$\left[\underline{n}_c^*,\overline{n}_c^*\right]$	[3.034, 2.817]	$\left[2\underline{\Pi}^*,2\overline{\Pi}^*\right]$	[0, 0.606]

Table 3: Numerical evaluation of the models with parameters of Example I.

Example I illustrates a situation with ad-averse consumers. It is a “normal” situation in terms of economic intuition from one-sided markets that is equilibrium consumer prices are below the monopoly price. Figure 1 depicts profit functions in terms of the consumer price p_c, with advertising price p_a being fixed at the monopoly price, which is 0.85 (see Table 3). The grey dotted curve represents monopoly profit, being maximized at p_c^M, given in Table 3 as 1.304.

The grey dashed curve represents each platform's profit in a symmetric situation, i.e. $\Pi_i^s\left(p_c, p_a^*\right)$. The black dashed curve represents deviation profit of firm i, given firm j remains at the symmetric situation. The function is truncated, when (19) starts binding, which is at $\hat{p}_c = 1.476$. The black curve represents Nash equilibrium profits, i.e. deviation profits being lower than profits in the symmetric situation, with the upper and lower bound corresponding to $\overline{p}_c^* = -0.035$ and $\underline{p}_c^* = -0.193$ in Table 3. Note that in equilibrium prices may exceed marginal cost, which, since we assumed marginal costs to be zero, means that in equilibrium prices can be positive. The reason is that consumers anticipate that advertising on the non-deviating platform will fall to zero, which ceteris paribus increases consumer utility from joining the non-deviating platform. The deviating platform anticipates this, and hence, does not deviate in marginal steps (see (19) in Proposition 3). If the minimum step size required for a deviating strategy is sufficiently high, positive prices may be equilibrium solutions.

At the lower bound of the equilibrium consumer price interval, duopolists gain zero profits. From Figure 1, it becomes clear that zero profits do not correspond to zero consumer prices, but to subsidization of consumers. In a zero-profit equilibrium, all revenues from the monopoly-like position on the multi-homing market side are used to subsidize the single-homing side.

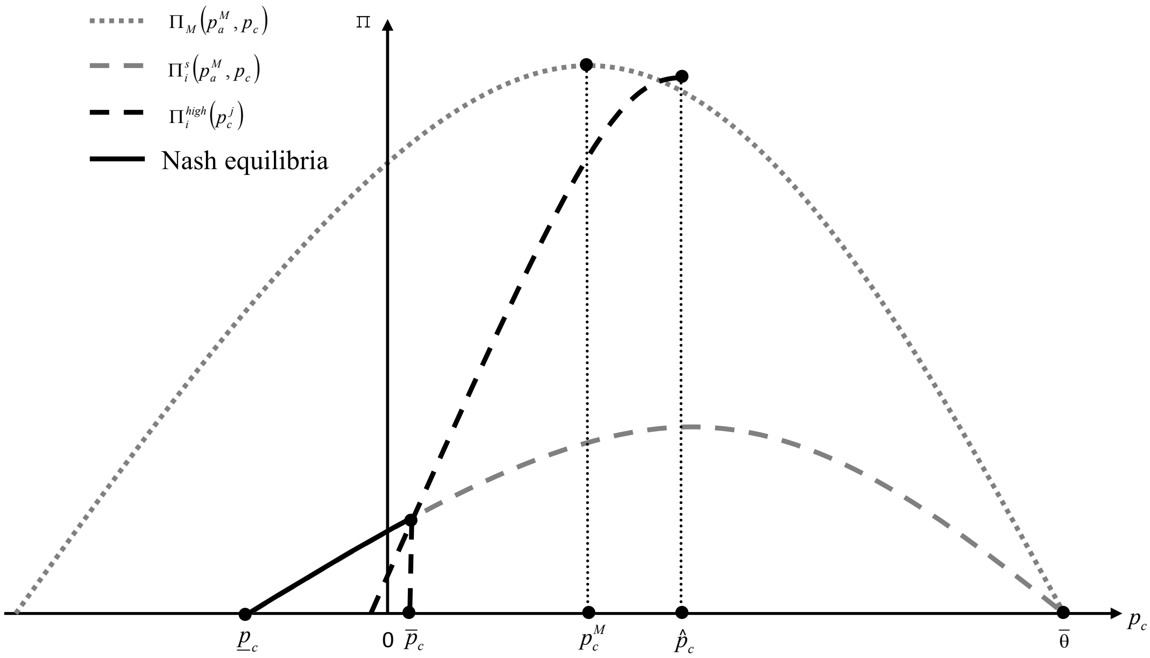

Figure 1: Profit functions of Example I with fixed $p_a = p_a^* = p_a^M$.

Example II: Duopoly price equals monopoly price

$(\overline{\mu}, \overline{\theta}, \alpha) = (4.058, 10, 3.65)$			
p_c^M	0.899	$p_a^* = p_a^M$	3.854
$\left[\underline{p}_c^*, \overline{p}_c^*\right]$	[-4.014, 0.899]	n_a^M	1.064
$\hat{p}_c$	4.783	$\left[\underline{n}_a^*, \overline{n}_a^*\right]$ $\left[2\underline{n}_a^*, 2\overline{n}_a^*\right]$	[1.042, 0.676] [2.083, 1.353]
n_c^M	5.217	Π^M	26.085
$\left[\frac{1}{2}\underline{n}_c^*, \frac{1}{2}\overline{n}_c^*\right]$	[5.106, 3.316]	$\left[\underline{\Pi}^*, \overline{\Pi}^*\right]$	[0, 11.625]
$\left[\underline{n}_c^*, \overline{n}_c^*\right]$	[10.213, 6.632]	$\left[2\underline{\Pi}^*, 2\overline{\Pi}^*\right]$	[0, 23.251]

Table 4: Numerical evaluation of the models with parameters of Example II.

Example II also assumes ad-averse consumers. However, in this case, the monopoly price equals $\overline{p}_c^* = 0.899$. This situation, which is visualized in Figure 2, represents a case in which there is no observable price effect of competition, assuming $\overline{p}_c^*$ is the resulting Nash equilibrium (as we did previously in the formal analysis of Section 5).

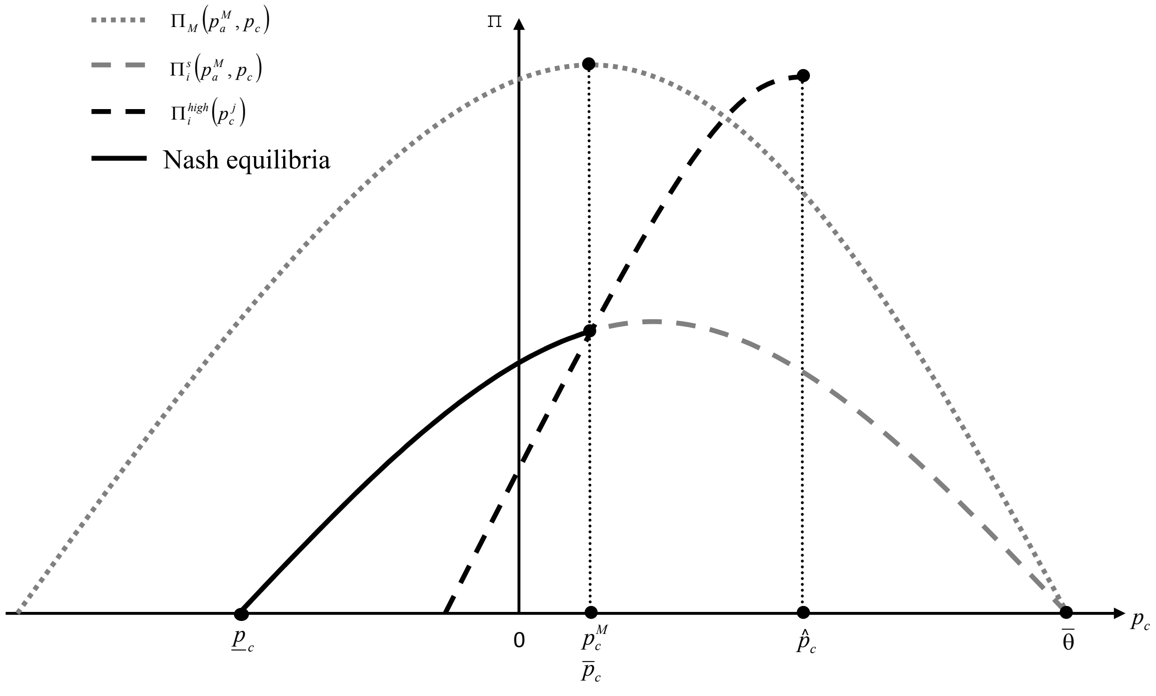

Figure 2: Profit functions of Example II with fixed $\mathrm{p_a = p_a^* = p_a^M}$.

From Table 4 we see that the total advertising quantity at $\left(p_a = p_a^* = p_a^M, p_c = \overline{p}_c^* = p_c^M\right) = (3.854, 0.899)$ in duopoly exceeds the monopoly level $\left(2\overline{n}_a^* = 2.083 > n_a^M = 1.064\right)$. Analogously, total consumer demand in duopoly exceeds the monopoly level $\left(\overline{n}_c^* = 6.632 > n_c^M = 5.217\right)$. The economic intuition is that consumers single-home, and are therefore exposed to the advertising quantity of one platform only, which is lower in duopoly than in monopoly $\left(\overline{n}_a^* = 0.676 < n_a^M = 1.064\right)$. Hence, the disutility due to the negative externality is lower in duopoly, which yields the demand-enhancing effect of lower market concentration.

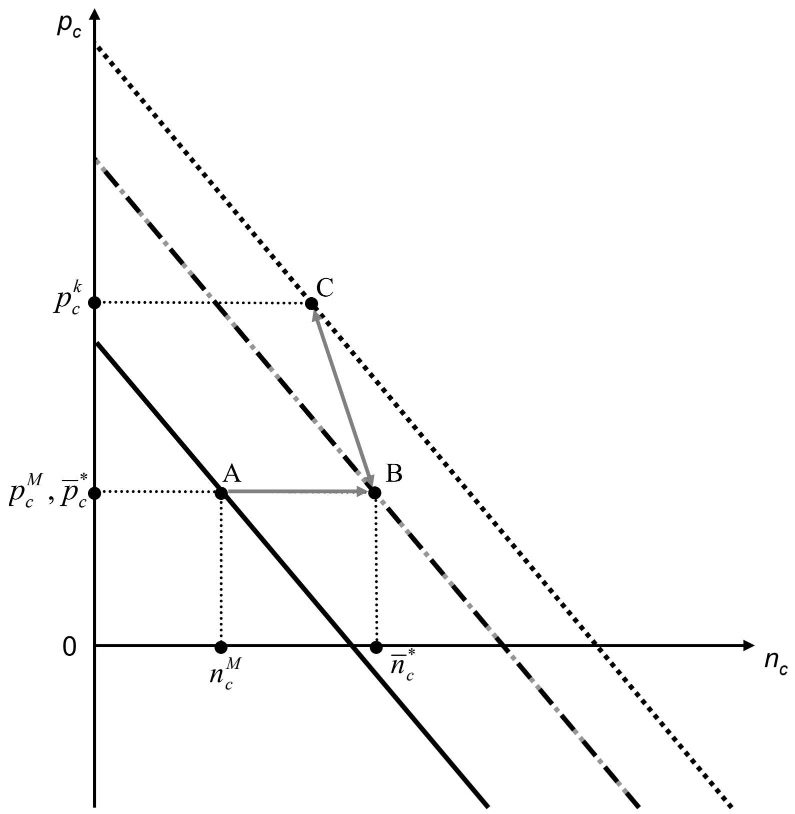

Figure 3: Demand-enhancing effect and competitive effect in case of price equality.

Figure 3 shows the shift in consumer demand caused by the reduction of advertising per platform when switching from monopoly to duopoly. The black solid line represents consumer demand under the monopoly advertising quantity as a function of the consumer price. The monopoly optimum is represented by point A. Switching to a duopoly reduces advertising quantities per platform, which shifts the consumer demand function to the outside (grey dotted line; Point B). Platform operators would like to exploit the additional willingness to pay, and indeed joint management or joint profit-maximizing collusion would yield a price above the monopoly price (see Appendix 2). This solution is represented by Point C in Figure 3. Note that every move on a given demand curve implies a shift of the curve, because advertising quantities change. Therefore, Point C is on a different curve than Point B. Since the platform operators do not collude, competition drives prices downwards to the black dashed line. In Example II, these two effects exactly compensate, i.e. the observable price in duopoly is the same as in monopoly. In Figure 3 this is represented by moving back to Point B.

Example III: Duopoly price above monopoly price

$(\overline{\mu}, \overline{\theta}, \alpha) = (4.1, 10, 3.65)$			
p_c^M	0.408	$p_a^* = p_a^M$	3.875
$\left[\underline{p}_c^*, \overline{p}_c^*\right]$	[-4.473, 0.836]	n_a^M	1.185
$\hat{p}_c$	4.783	$\left[\underline{n}_a^*, \overline{n}_a^*\right]$ $\left[2\underline{n}_a^*, 2\overline{n}_a^*\right]$	[1.154, 0.731] [2.308, 1.462]
n_c^M	5.267	Π^M	26.333
$\left[\frac{1}{2}\underline{n}_c^*, \frac{1}{2}\overline{n}_c^*\right]$	[5.130, 3.248]	$\left[\underline{\Pi}^*, \overline{\Pi}^*\right]$	[0, 11.915]
$\left[\underline{n}_c^*, \overline{n}_c^*\right]$	[10.26, 6.50]	$\left[2\underline{\Pi}^*, 2\overline{\Pi}^*\right]$	[0, 23.830]

Table 5: Numerical evaluation of the models with parameters of Example III.

Parameters in Example III are only slightly varied as compared to Example II. As can be seen from Table 5 and Figure 4, the upper bound of equilibrium consumer prices now exceeds the monopoly price $\left(\overline{p}_c^* = 0.836 > p_c^M = 0.408\right)$, hence, we have a case of price-increasing competition. The negative effect of platform competition does not offset the positive effect of increased willingness to pay, so that some equilibrium prices exceed the monopoly price. Compared to Figure 3, we see that in Figure 5 the competitive effect does not fully compensate the price increasing effect of the shift in the demand curve. Analog to Figure 3, A represents the monopoly solution. Switching to a duopoly reduces advertising quantities, which shifts the demand curve to the outside. The distance AB represents the demand-enhancing effect of lower market concentration. Due to the increased willingness to pay, colluding operators would choose Point C. Different from Figure 3, platform competition only partially compensates this effect, and pushes the consumer price down to Point D.

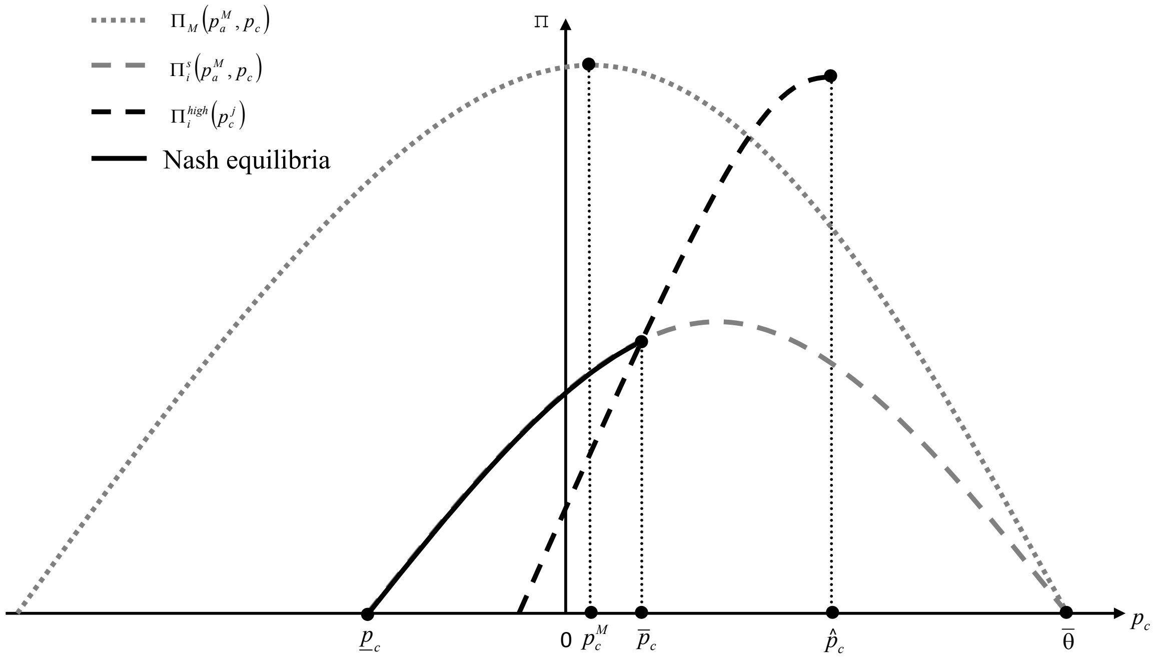

Figure 4: Profit functions of Example III with fixed $p_a = p_a^* = p_a^M$.

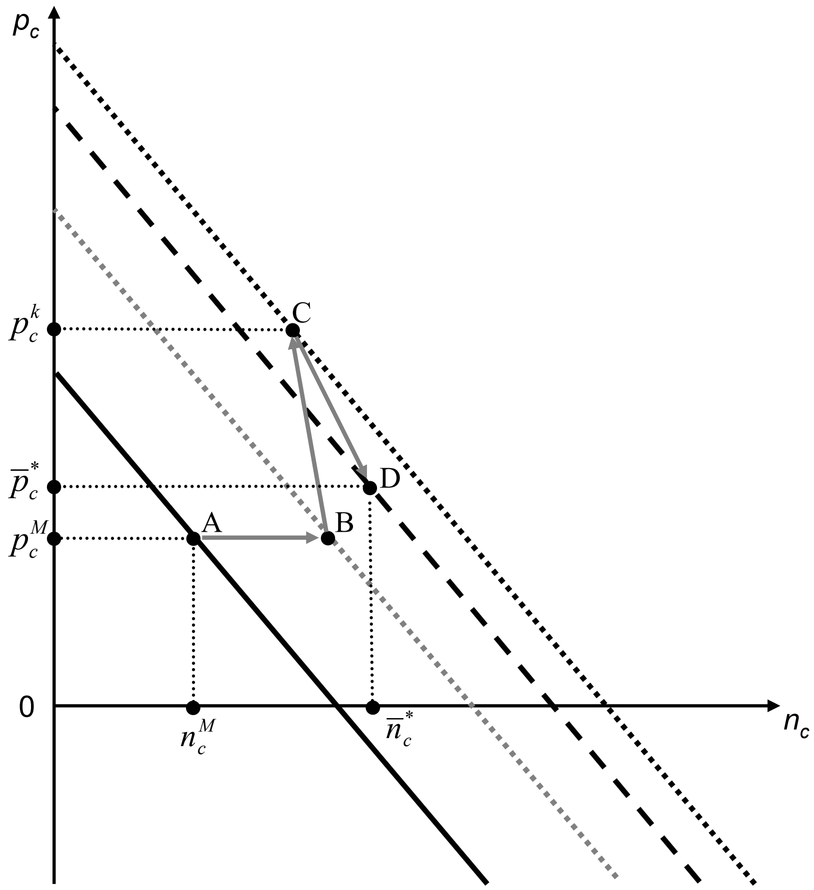

Figure 5: Demand-enhancing effect and competitive effect in case of price-increasing competition.

7. Conclusion and Implications

The results of the above analysis have implications on multiple fields of economic research. Keep in mind that we are studying a competitive bottleneck two-sided private goods market with perfect information, and that the analysis of our model is focused on those parameter sets that are economically plausible *and* yield price effects contrary to economic intuition from one-sided markets, namely duopoly equilibrium prices being at least as high as monopoly prices. Furthermore it turned out that an equilibrium solution requires ad-averse consumers that is a negative impact of demand from the multi-homing market side to the single-homing one.

As summarized in Table 1, there are cases in which total consumer demand in the duopoly equilibrium exceeds consumer demand in the monopoly optimum. On first sight, this result does not seem too surprising as it is a well-known relationship and a reason to foster competition. On second thought, the reason for higher equilibrium consumer demand in textbook oligopoly models is that oligopolistic competition yields lower consumer prices than under monopoly, and therefore demand increases. In our analysis, we explicitly study cases in which prices are equal in monopoly and duopoly. Hence, the two-sidedness of the market holds a demand-enhancing effect, caused by the mere fact that total consumer demand is now equally split between two platforms instead of being served by one platform only.

The two-sidedness of the market also contributes an alternative explanation for a missing price effect of competition. On one-sided markets, there are broadly speaking two explanations for missing price effects: Either the monopolist does not or cannot make use of her monopoly power for some reason or the oligopolists collude implicitly or explicitly. On two-sided markets, there might just be no price effect of competition. As Table 1 suggests, there is a competitive effect that causes total profits in the duopoly equilibrium to be strictly lower than in the monopoly case, and total advertising demand to be lower in monopoly than in duopoly. We neither restrict the monopolist's optimization problem artificially nor do we hinder competition between the duopolists. Still and regardless of competition taking place, there is no observable price effect or competition even increases prices given any of the parameter sets described by (29).

A price effect of competition, however, is the underlying assumption of empirical price-concentration studies. These studies presume that prices increase with the concentration of the market, and try to estimate the magnitude of this effect. Our results suggest that this relationship might be negative, given certain exogenous conditions. Therefore, empirical analyses yielding a negative price-concentration effect do not necessarily suffer from methodological or technological mistakes. Furthermore, if conditions are such that there is no observable price effect of competition, then there is obviously no price-concentration effect that could be measured. This implies that the absence of significant empirical results cannot be interpreted as lack of competition. In this light, it is also not sensible to study the sum of the prices, which the two-sided market literature usually calls the "price level", as compared to "price balance", which describes the allocation of the price level between the two market sides.

Regarding the welfare effects of competition, we obtain ambiguous results and need to distinguish our conclusions as in Table 2. In case of positive prices, i.e. in case of prices above marginal cost, total welfare is always higher in duopoly than in monopoly, even though consumer prices might be lower in monopoly. In case of subsidization, this is not necessarily true. Therefore, policy makers as well as regulators aiming at welfare maximization will have to obtain in-dept knowledge of the environment (in the terminology of our model: "the parameter set") they are facing before being able to act optimally. A brief glance at the prevailing price level or price balance will not suffice to make a sensible judgment. Unlike on common one-sided markets, fostering competition will not necessarily increase welfare. Similarly, merger control becomes more difficult. Under conditions of positive prices, mergers generally have a negative impact on total welfare. Under conditions of subsidized consumer prices, we cannot draw general conclusions. If, for some exogenous reason, a merger has to take place anyway, it will virtually always imply that one platform closes down (proof: see Appendix 2). This is in line with the regulator's objective of welfare maximization, because welfare increases, if the operator of the two merged platforms closes down one of them. It even holds for distributive objectives, i.e. consumer surplus, advertiser surplus, and producer surplus all increase, if the operator closes down one platform in case of a merger (see also Appendix 2).

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Appendix 1:

Proof: Proposition 2 implies that operator *i*'s deviation profit is at least as great as the profit in the corresponding symmetric situation:

Respecting that $p_c^{i/equal}$ is restricted by (16), the deviation profit departing from any symmetric situation (p_a^s, p_c^s) is given by

$$\Pi_i^{equal} = (\alpha \cdot n_a^s - \alpha \cdot n_a^{i/equal} + p_c^s) \cdot n_c^{i/equal} + \frac{\overline{\mu} + \alpha}{2} \cdot n_a^{i/equal} \cdot n_c^{i/equal} .$$

Deviation from the symmetric situation will take place, iff the profit from doing so is not lower than the profit in the symmetric situation, which is

$$\Pi_i^s\left(p_c^s, p_a^s\right) = n_c^s \cdot \left(p_c^s + p_a^s \cdot n_a^s\right).$$

Proposition 2 assumes a deviation strategy which yields the same consumer utility as the corresponding symmetrical situation. Therefore $n_c^{i/equal} = n_c^s$ is fixed.

Using (17), evaluating the inverse demand function obtained from (11) at $(n_a^s,\ n_c^s)$, and respecting that economic plausibility implies $n_c^s \geq 0$, it is easy to see that $\Pi_i^{equal} \geq \Pi_i^s$ becomes after some algebraic manipulation

$$n_c^{i/equal} \frac{\left(-\eta\right)^2}{4} \geq -n_a^s \cdot \eta - \frac{\left(n_a^s\right)^2}{n_c^{i/equal}}$$

Note that all expressions in this inequality are fixed, except for d_a^s on the right hand side. The expression on the right hand side has its maximum at

$$n_a^s = \frac{-\eta \cdot n_c^{i/equal}}{2} .$$

Evaluating the inequality at this maximum yields

$$n_c^{i/equal} \frac{\left(-\eta\right)^2}{4} \geq n_c^{i/equal} \frac{\left(-\eta\right)^2}{4} ,$$

which is always true. *(q.e.d.)*

Appendix 2:

Explicit collusion or merger on the duopoly market:

Assume, both platform operators are able and willing to cooperate in order to maximize joint profits. Given (10) and (11), the operators have two options: Either they equally divide consumer demand between their platforms or they close down one platform and create a monopoly. In the first case -for reasons to be seen soon, we label it "hypothetical collusion case"- the optimization problem is

$$\max_{n_c, n_a^i} \Pi_k = \sum_{i=1}^{2} n_a^i \cdot \frac{n_c}{2} \cdot p_a^i\left(n_a^i, \frac{n_c}{2}\right) + \frac{n_c}{2} \cdot p_c^i\left(n_a^i, \frac{n_c}{2}\right)$$

which yields a maximum profit of

$$\Pi_k = \frac{2 \cdot \overline{\theta}^2}{8 - \eta^2},$$

optimal quantities

$$n_c^k = \frac{4 \cdot \overline{\theta}}{\eta^2 - 8} \text{ and } n_a^{1,k} = n_a^{2,k} = n_a^k = \frac{\eta \cdot \overline{\theta}}{\eta^2 - 8},$$

and optimal prices

$$p_c^{1,k} = p_c^{2,k} = p_c^{i,k} = \frac{(\eta \cdot \overline{\mu} + 4) \cdot \overline{\theta}}{8 - \eta^2} \text{ and } p_a^{1,k} = p_a^{2,k} = p_a^{i,k} = p_a^M.$$

Given the nonnegativity constraints on $\overline{\mu}$ and $\overline{\theta}$, and the parameter restrictions implied by the nonnegativity of Π_k and Π_M, the maximum hypothetical collusion profit never exceeds the optimal monopoly profit (9), and consumer prices never exceed monopoly consumer prices. Furthermore, there is only one corner solution, in which both profits and consumer prices become equal. Therefore, explicit collusion or merger always implies that the operators close down one platform to play the monopoly solution, except, if $(\overline{\mu}, \alpha) = (\alpha, \alpha > 0)$, in which case the operators are indifferent between keeping both platforms open and closing down one.

Assume that for some exogenous reason it is not possible to close down one platform. In case of a merger, this might be due to obligations of a regulating authority. To study the welfare effects in this case, we compute hypothetical consumer surplus as

$$\int_0^{n_c^k} p_c^k\left(n_c, n_a^k\right) dn_c - n_c^k \cdot p_c^k = \frac{8 \cdot \overline{\theta}^2}{\left(\eta^2 - 8\right)^2}, \text{ where}$$

$$p_c^1 = p_c^2 = p_c^k = \overline{\theta} - \alpha \cdot n_a - n_c.$$

Hypothetical advertiser surplus is

$$2 \cdot \left(\int_0^{n_a^k} p_a^k\left(n_a, n_c^k\right) dn_a - n_a^k \cdot p_a^k \right) = \frac{\eta^2 \cdot \overline{\theta}}{2 \cdot \left(8 - \eta^2\right)}, \text{ where}$$

$$p_a^1 = p_a^2 = p_a^k = \overline{\mu} - \frac{2 \cdot n_a}{n_c}.$$

Comparing consumer, advertiser, and producer surplus of the hypothetical collusion case and the monopoly optimum, we find that each of these welfare components is at least as great in the monopoly case as it is in the hypothetical collusion case. Therefore total welfare in the hypothetical collusion case also never exceeds total welfare in the monopoly case.

Comparing welfare outcomes of hypothetical collusion and duopoly equilibrium case, we need to distinguish the cases known from Section 4 and obtain the results presented in Table A1.

Case		Consumer Surplus	Advertiser Surplus	Producer Surplus	Total Welfare
$p_c^* = p_c^M$	Subsidization	$\gtreqless$	$\gtreqless$	$\leq$	$\gtreqless$
	Positive Prices	$>$	$>$	$<$	$>$
$p_c^* > p_c^M$	Subsidization	$\gtreqless$	$\gtreqless$	$\leq$	$\gtreqless$
	Mixed	$\gtreqless$	$\gtreqless$	$\leq$	$\gtreqless$
	Positive Prices	$>$	$>$	$<$	$>$
Left hand side = duopoly equilibrium; right hand side = duopoly collusion					

Table A1: Welfare in the case of explicit collusion

Appendix 3:

Quantities at the lower bound of the Nash-equilibria

Proposition 4 describes upper and lower bounds for equilibrium prices. Our exposition focused on the upper bound that yields positive equilibrium profits. In this appendix, we show that our qualitative results can also be obtained using the lowest equilibrium price that is when using the equilibrium

$$\left(p_a^* = p_a^M = \frac{\overline{\mu} + \alpha}{2}, p_c^* = \underline{p}_c^* = \frac{\left(\overline{\mu}^2 - \alpha^2\right)}{\eta^2 - 8}\right).$$

In this case, each platform realizes an advertising quantity of

$\underline{n}_a^* = \underline{n}_a^{i*} = \underline{n}_a^{j*} = n_a\left(\underline{p}_c^*, p_a^*\right) = \frac{2 \cdot \overline{\theta} \cdot \eta}{\eta^2 - 8}$, and attracts $\underline{n}_c^* = \underline{n}_c^{i*} = \underline{n}_c^{j*} = n_c\left(\underline{p}_c^*, p_a^*\right) = \frac{8 \cdot \overline{\theta}}{8 - \eta^2}$ consumers.

Respecting that $\underline{\Pi}_i^s, \underline{n}_c^s, \underline{n}_a^s, n_c^M, n_a^M, \Pi^M \overset{!}{\geq} 0$, $\underline{p}_c^* = p_c^M$ results for

$$\left(\overline{\mu}, \alpha\right) \in \left\{\left(2\sqrt{2}, \sqrt{2}\right), \left(\sigma, \alpha > \sqrt{2}\right)\right\}, \text{ where}$$

$$\sigma = R_1 \text{ of } e \cdot x^3 + \left(6 - 3 \cdot e^2\right) \cdot x^2 + e \cdot \left(3 \cdot e^2 - 12\right) \cdot x - e^4 + 6 \cdot e^2 - 16 .$$

Comparing with (29), we see that $\alpha = \sqrt{2}$ is the lower bound of α, and that in this case $\max\left(\beta, \delta\right) = \delta = 2\sqrt{2}$, so that $\underline{p}_c^* = \overline{p}_c^*$, i.e. the equilibrium is unique for $\left(\overline{\mu}, \alpha\right) = \left(2\sqrt{2}, \sqrt{2}\right)$.

Comparison of monopoly and duopoly quantities reveals that $\underline{n}_c^* > n_c^M$, $\underline{n}_a^* < n_a^M < 2 \cdot \underline{n}_a^*$, and $2 \cdot \underline{\Pi}^* = 0 < \Pi^M$, which is consistent with our findings for the equilibrium $\left(p_a^*, \overline{p}_c^*\right)$.

Price-increasing competition $\left(\underline{p}_c^* > p_c^M\right)$ results for

$$\left(\overline{\mu}, \alpha\right) \in \left\{\left(\sigma < \overline{\mu} \leq \frac{\alpha^2 - 1 + \sqrt{4 \cdot \alpha^2 + 1}}{\alpha}, \alpha > \sqrt{2}\right)\right\}.$$

As with $\left(p_a^*, \overline{p}_c^*\right)$, $2 \cdot \underline{\Pi}^* = 0 < \Pi^M$ and $\underline{n}_c^* \leq n_c^M$ holds, while the relationship of $\underline{n}_c^*$ and n_c^M and n_a^M and $2 \cdot \underline{n}_a^*$ is ambiguous.