A trend deduction model of fluctuating oil prices

Xu, Haiyan and Zhang, ZhongXiang

East-West Center

19 August 2010

The previous version: August 19, 2010
This version: November 17, 2010

A Trend Deduction Model of Fluctuating Oil Prices

Haiyan Xu[a,c] **ZhongXiang Zhang**[b*]
[a] Institute of International Studies, Fudan University, Shanghai 200129, China
[b] Research Program, East-West Center, 1601 East-West Road, Honolulu, HI 96848-1601, USA
[c] Center for Energy Economics and Strategy Studies, Fudan University, Shanghai, China

Abstract

Crude oil prices have been fluctuating over time and by a large range. It is the disorganization of oil price series that makes it difficult to deduce the changing trends of oil prices in the middle- and long-terms and predict their price levels in the short-term. Following a price-state classification and state transition analysis of changing oil prices from January 2004 to August 2009, this paper first verifies that the observed crude oil price series during the soaring period follow a Markov Chain. Next, the paper deduces the changing trends of oil prices by the limit probability of a Markov Chain. We then undertake a probability distribution analysis and find that the oil price series have a log-normality distribution. On this basis, we integrate the two models to deduce the changing trends of oil prices from the short-term to the middle- and long-terms, thus making our deduction academically sound. Our results match the actual changing trends of oil prices, and show the possibility of re-emerging soaring oil prices.

Keywords: Oil price; Log-normality distribution; Limit probability of a Markov Chain; Trend deduction model; OPEC

JEL Classification: Q41; Q47; C12; C49; F01; O13.

* Corresponding author: ZhongXiang Zhang, Senior Fellow, Research Program, East-West Center, 1601 East-West Road, Honolulu, HI 96848-1601, USA. Tel.: +1-808-944 7265; fax: +1-808-944 7298. E-mail address: ZhangZ@EastWestCenter.org.

I. Introduction

Since 2004, crude oil prices had tended to fluctuate at high level and by a large range. After four-year price soaring, oil prices had been extraordinarily soaring from August 2008 for a half year and then fell straightly to the starting level in early 2004. This was followed by a new round of climbing oil prices to a high level. It is the disorganization of oil price series that makes it difficult to deduce the changing trends of oil prices in the middle- and long-terms and predict their price levels in the short-term.

There have been few studies on crude oil prices based on the application of a Markov Chain. Kosobud and Stokes (1978) have applied a Markov probability model to verify the pattern of "best market share rules", and have concluded that after the Organization of the Petroleum Exporting Countries (OPEC) has taken shape, the probability of conflicts among suppliers has reduced whereas such a probability among consumers has increased. Holmes and Wang (2003) apply a Markov switching model in studying the influence of soaring oil prices on the growth of British GDP, and reach the conclusion that during the increasing period of business cycle the soaring oil prices and growth of GDP are asymmetric to various extent. Wei et al. (2006) have classified the time series of oil prices into three states as increasing by a large range, increasing by a small range and decreasing by a large range states. They identify the duration of each state and conclude that the Markov Chain model is superior to an auto-regression model. Song (2005) has conducted the prediction of oil prices by one-state transition matrix without testing the existence of a Markov Chain and calculating the convergence value of transition state matrix. As a result, the outcome is far from the reality, thus concluding that the Markov method could not predict the evolution of the event perfectly. Vo (2009) discusses a stochastically fluctuating regime of oil market by a Markov transition model to catch the factors which influence oil market, and points out that the fluctuation of oil prices is consistent. All the literatures cited above have not touched on the deduction of trends and prediction of oil prices directly by a Markov Chain. Moreover, none of them recognizes the potential role of the limit probability of a Markov Chain in deducing the trends of oil prices.

In this paper, following a price-state classification and state transition analysis of changing oil prices from January 2004 to August 2009, we first verify that the observed crude oil price series during the soaring period follow a Markov Chain. Next, an attempt is made to deduce the changing trends of oil prices by the limit probability of a Markov Chain. We then undertake a probability distribution analysis and find that the oil price series have a log-normality distribution. On this basis, we integrate the two models to deduce the changing trends of oil prices from the short-term to the middle- and long-terms, thus making our deduction academically sound. Our results match the actual changing trends of oil prices that immediately followed the sample period, and show the possibility of re-emerging soaring oil prices.

2. The Oil Price Series and Oil Price Transition States

Appendix 1 provides the monthly average prices of OPEC basket of crude oils[1] from January 2001 to April 2010. During the 76 months, although the oil price series feature chaotic characteristic, stage-transition states of oil prices can be clearly distinguished. They can be classified as six states: low state, middle-low state, middle state, middle-high state, high state and super-high state. These states constitute the following full space for stochastic events of crude oil prices:

$$(0, 40)\cup[40, 60)\cup[60, 80)\cup[80, 100)\cup[100, 120)\cup[120, 140)$$

Figure 1 shows a moving process of these six transition states with its main distinguishing features including the occurrence of oil prices extraordinarily soaring or steeply falling.

---

[1] The OPEC collects price data on a “basket” of crude oils, and uses average prices for these oil streams to develop an OPEC reference price to monitor world oil market conditions. From January 1, 1987 to June 15, 2005, OPEC calculated an arithmetic average of seven crude oil streams, including: Algeria’s Saharan Blend, Indonesia Minas, Nigeria Bonny Light, Saudi Arabia Arab Light, Dubai Fateh, Venezuela Tia Juana and Mexico Isthmus (a non-OPEC oil) to estimate the OPEC basket price. At its 136th meeting to review oil markets on June 15, 2005, OPEC decided to change both the composition of the basket and the way that it is calculated. Effective June 16, 2005, OPEC’s reference basket now consists of eleven crude streams representing the main export crudes of all member countries, weighted according to production and exports to the main markets. The crude oil streams in the basket are: Saharan Blend (Algeria), Minas (Indonesia), Iran Heavy (Islamic Republic of Iran), Basra Light (Iraq), Kuwait Export (Kuwait), Es Sider (Libya), Bonny Light (Nigeria), Qatar Marine (Qatar), Arab Light (Saudi Arabia), Murban (UAE) and BCF 17 (Venezuela) (OPEC, 2010).

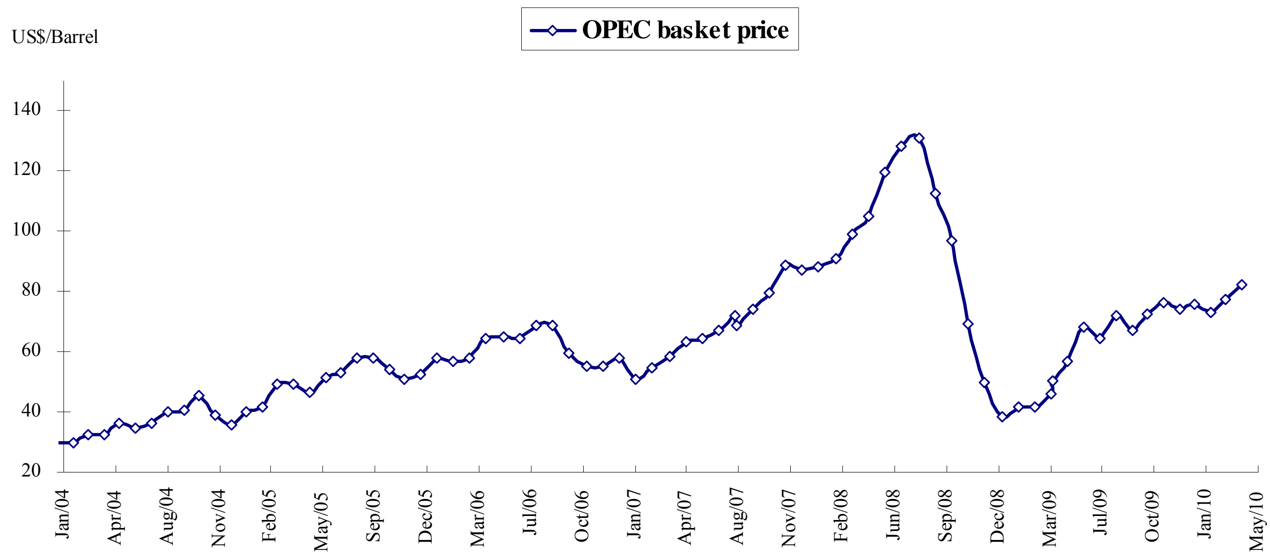

Figure 1 Monthly-Average Price of OPEC Basket of Crude Oils from January 2004 to April 2010

Source: Drawn based on data from a compilation based on OPEC (2010).

The weighted average of oil prices is US$ 60 per barrel. So we treat the [60, 80) interval as a middle state of fluctuating oil prices, the [40, 60) interval as a middle-low state, and the [80, 100) interval as a middle-high state. The three states can be broadly termed as the middle state. By contrast, we treat the (0, 40) interval as a low state of oil prices, the [100, 120) interval as a high state, and the [120, 140) interval as a super-high state. Suppose that E represents oil price state (event). Let E_l represent the (0, 40) interval of low-state oil price, E_{ml} the [40, 60) interval, E_m the [60, 80) interval, E_{mh} the [80, 100) interval, E_h the [100, 120) interval, and E_{eh} the [120, 140) interval. The oil price transition process from January 2004 to April 2010 can be then induced as follows:

$$E_l \Longrightarrow E_l \Longrightarrow E_l \Longrightarrow E_l \Longrightarrow E_l \Longrightarrow E_l \Longrightarrow E_l \Longrightarrow E_{ml} \Longrightarrow E_{ml} \Longrightarrow E_{ml} \Longrightarrow E_l \Longrightarrow E_l \Longrightarrow E_{ml} \Longrightarrow E_{ml} \Longrightarrow$$
$$E_{ml} \Longrightarrow E_{ml} \Longrightarrow E_{ml} \Longrightarrow E_{ml} \Longrightarrow E_{ml} \Longrightarrow E_{ml} \Longrightarrow E_{ml} \Longrightarrow E_{ml} \Longrightarrow E_{ml} \Longrightarrow E_{ml} \Longrightarrow E_{ml} \Longrightarrow E_{ml}$$
$$\Longrightarrow E_{ml} \Longrightarrow E_m \Longrightarrow E_m \Longrightarrow E_m \Longrightarrow E_m \Longrightarrow E_m \Longrightarrow E_{ml} \Longrightarrow E_{ml} \Longrightarrow E_{ml} \Longrightarrow E_{ml} \Longrightarrow E_{ml} \Longrightarrow E_{ml}$$
$$\Longrightarrow E_{ml} \Longrightarrow E_m \Longrightarrow E_m \Longrightarrow E_m \Longrightarrow E_m \Longrightarrow E_m \Longrightarrow E_m \Longrightarrow E_m \Longrightarrow E_{mh} \Longrightarrow E_{mh} \Longrightarrow E_{mh} \Longrightarrow E_{mh} \Longrightarrow$$
$$E_{mh} \Longrightarrow E_h \Longrightarrow E_h \Longrightarrow E_{eh} \Longrightarrow E_{eh} \Longrightarrow E_h \Longrightarrow E_{mh} \Longrightarrow E_m \Longrightarrow E_{ml} \Longrightarrow E_l \Longrightarrow E_{ml} \Longrightarrow E_{ml} \Longrightarrow E_{ml}$$
$$\Longrightarrow E_{ml} \Longrightarrow E_{ml} \Longrightarrow E_m \Longrightarrow E_m \Longrightarrow E_m \Longrightarrow E_m \Longrightarrow E_m \Longrightarrow E_m \Longrightarrow E_m \Longrightarrow E_m \Longrightarrow E_m \Longrightarrow E_m \Longrightarrow E_{mh}$$

There are 76 states and 75 state transitions which constitute an oil price transition process. It looks like a chain linking one state with another. So we call it a state transition chain. In the next section, we will examine its properties.

3. Oil Price State Transition Chain as a Markov Chain

Table 1 shows the state-transition-frequency matrix of oil-price six state transition chain. This transition chain has a χ^2 distribution if it follows a Markov Chain. To test this, we use the following formula (Jing, 1985; Xu, 2001):

Table 1 State Transition Frequency of Oil Prices from January 2004 to April 2010

	E_l(0, 40)	E_{ml}[40, 60)	E_m[60, 80)	E_{mh} [80, 100)	E_h[100, 120)	E_{eh}[120, 140)	$n_{i.}$
E_l(0，40)	7	3	0	0	0	0	*10*
E_{ml} [40，60)	2	26	3	0	0	0	*31*
E_m[60，80)	0	2	19	2	0	0	*23*
E_{mh} [80，100)	0	0	1	4	1	0	*6*
E_h[100，120)	0	0	0	1	1	1	*3*
E_{eh}[120，140)	0	0	0	0	1	1	*2*
$n_{.j}$	*9*	*31*	*23*	*7*	*3*	*2*	**n=75**

$$\chi^2 = \sum_{i=1}^{m}\sum_{j=1}^{m}\frac{(n_{ij} - n_{i.}n_{.j}/n)^2}{(n_{i.}n_{.j}/n)} \tag{1}$$

where $n_{i}.\ (i = 1,2, \ldots\ m)$, and $n_{.j}\ (j = 1, 2, \ldots\ 6)$ are the frequency of state *i* and state *j*, respectively. This has a χ^2 distribution with $(m-1)^2$ degrees of freedom, where *m* refers to the number of states. The test results are reported in Table 2.

Table 2 χ^2 Testing Results of Crude Oil-Price State Transition Chain

$\frac{(n_{ij} - n_{i.}n_{.j}/n)^2}{(n_{i.}n_{.j}/n)}$	j=1 (E_l)	j=2 (E_{ml})	j=3 (E_m)	j=4 (E_{mh})	j=5 (E_h)	j=6 (E_{eh})
i=1 (E_l)	28.0333	0.3108	3.0667	0.9333	0.4000	0.2667
i=2 (E_{ml})	0.7953	13.5709	4.4534	2.8933	1.2400	0.8267
i=3 (E_m)	2.7600	5.9274	20.2348	0.0100	0.9200	0.6133
i=4 (E_{mh})	0.7200	2.4800	0.3835	21.1314	2.4067	0.1600
i=5 (E_h)	0.3600	1.2400	0.9200	1.8514	6.4533	10.5800
i=6 (E_{eh})	0.2400	0.8267	0.6133	0.1867	10.5800	16.8033
$\sum_{i=1}^{m}$	32.9086	24.3557	29.6717	27.0062	22.0000	29.2500
$\sum_{i=1}^{m}\sum_{j=1}^{m}$						**165.1922**

With m=6, so the degrees of freedom $(m-1)^2 = (6-1)^2 = 25$. Using a 5% significance level (that is, $\alpha = 0.05$), and referring to the χ^2 tables with 25 degrees of freedom, we find that $\chi^2_{(25,\ \alpha=0,05)} = 37.7$. The observed value of the sample statistics χ^2 is 165.1922, much higher than $\chi^2_{(25,\ \alpha=0,05)} = 37.7$. Thus, we reject the null hypothesis that states are independent. As a result, it confirms that a state transition chain of OPEC basket of crude oil prices from January 2004 to April 2010 follows a Markov Chain.

4. Taking the Limit Probability of a Markov Chain to Induce the Changing Trends of Oil Prices in the Middle- and Long-Terms

Fisz (1980) and Wang (1979) discuss the Ergodic Theorem of a Markov Chain. Its connotation is that in a Markov Chain when the number of the transition steps is large enough, the transition probability from any particular state E_i will eventually get stabilized at its limit value P_j. Thus P_j is called a limit probability. At that point, the row vectors of the state transition matrix are all equal, indicating that the state transition process has been at the steady state.

Let $Z_{(1)}$ represent the first-stage transition matrix and $Z_{(n)}$ represent the n-th stage transition matrix. Their link is given as follows:

$$Z_{(n)} = Z_{(1)}^{n} \tag{2}$$

As this power continues, it will tend to its limit. This essentially provides a method to approach limit (Fisz, 1980; Lu, 1987; Arimoto, 1985).

Having each element n_{ij} of respective row from Table 1 divided by the sum of its row (n_i), then we obtain the first-stage state transition matrix:

$$Z_{(1)} = \begin{array}{c} \\ E_l \\ E_{ml} \\ E_m \\ E_{mh} \\ E_h \\ E_{eh} \end{array} \begin{array}{c} \begin{array}{cccccc} E_l & E_{ml} & E_m & E_{mh} & E_h & E_{eh} \end{array} \\ \begin{bmatrix} 0.7000 & 0.3000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0645 & 0.8387 & 0.0968 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0870 & 0.8261 & 0.0870 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1667 & 0.6667 & 0.1667 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.3333 & 0.3333 & 0.3333 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.5000 & 0.5000 \end{bmatrix} \end{array}$$

Similarly, we can derive the second-stage state transition matrix, …, and the convergence state transition matrix as follows:

$$Z_{(1)}^{2} = \begin{array}{c} \\ E_l \\ E_{ml} \\ E_m \\ E_{mh} \\ E_h \\ E_{eh} \end{array} \begin{array}{c} \begin{array}{cccccc} E_l & E_{ml} & E_m & E_{mh} & E_h & E_{eh} \end{array} \\ \begin{bmatrix} 0.5094 & 0.4616 & 0.0290 & 0.0000 & 0.0000 & 0.0000 \\ 0.0993 & 0.7312 & 0.1611 & 0.0084 & 0.0000 & 0.0000 \\ 0.0056 & 0.1448 & 0.7053 & 0.1298 & 0.0145 & 0.0000 \\ 0.0000 & 0.0145 & 0.2488 & 0.5145 & 0.1667 & 0.0556 \\ 0.0000 & 0.0000 & 0.0556 & 0.3333 & 0.3333 & 0.2778 \\ 0.0000 & 0.0000 & 0.0000 & 0.1667 & 0.4167 & 0.4167 \end{bmatrix} \end{array}$$

……….

$$Z_{(1)}^{256} = \begin{array}{c} \\ E_l \\ E_{ml} \\ E_m \\ E_{mh} \\ E_h \\ E_{eh} \end{array} \begin{array}{c} \begin{array}{cccccc} E_l & E_{ml} & E_m & E_{mh} & E_h & E_{eh} \end{array} \\ \begin{bmatrix} 0.0634 & 0.2948 & 0.3281 & 0.1712 & 0.0856 & 0.0571 \\ 0.0634 & 0.2948 & 0.3281 & 0.1712 & 0.0856 & 0.0571 \\ 0.0634 & 0.2948 & 0.3281 & 0.1712 & 0.0856 & 0.0571 \\ 0.0634 & 0.2948 & 0.3281 & 0.1712 & 0.0856 & 0.0571 \\ 0.0634 & 0.2948 & 0.3281 & 0.1712 & 0.0856 & 0.0571 \\ 0.0634 & 0.2948 & 0.3281 & 0.1712 & 0.0856 & 0.0571 \end{bmatrix} \end{array}$$

The row-vector, which has been converged to the same value, is the Markov Chain's limit probability of oil price series. It implies that the oil price series have become stabilized after a continuous state transition process. At the moment, P_j refers to the probability of each state E_{ij} in the whole process. In other words, it means the share and proportion of each state E_{ij} in the ultimate state of the series. This convergence process is the changing trends of crude oil prices, and the row-vector as the limit probability is the ultimate state of oil price series. It is expressed as follows:

	$E_l(0,\ 40)$	$E_{ml}[40,\ 60)$	$E_m[60,\ 80)$	$E_{mh}[80,\ 100)$	$E_h[100,\ 120)$	$E_{eh}[120,\ 140)$
Limit Probability Value of Markov Chain	[0.0634	0.2948	0.3281	0.1712	0.0856	0.0571]

This limit probability vector indicates the ultimate probability of six states in the crude oil price series or the ultimate proportion of them in the crude oil price series. The probability of low level state $E_l(0,\ 40)$ is 0.0634, meaning that the proportion in the series is 6.34%; the probability of middle-low level state $E_{ml}[40,\ 60)$ is 0.2948, the proportion is 29.48%; the probability of middle level state $E_m[60,\ 80)$ is 0.3281, the proportion is 32.81%; the probability of middle-high level state $E_{mh}[80,\ 100)$ is 0.1712, the proportion is 17.12%; the probability of high level state $E_h[100,\ 120)$ is 0.0856, the proportion is 8.56%; the probability of super-high level state $E_{eh}[120,\ 140)$ is 0.0571, the proportion is 5.71%, respectively.

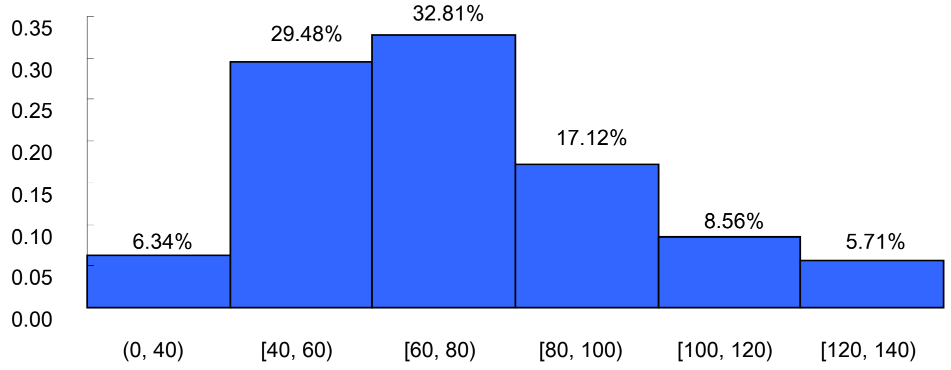

Figure 2 The Limit Probability of OPEC Monthly-Average Crude Oil Prices

Figure 2 illustrates the ultimate states of oil prices, which are revealed by a Markov Chain. It can be seen that the limit probabilities of a Markov Chain constitute a full-probability interval, in which middle states (including middle-low state, middle state and middle-high state) dominate, accounting for 79.41%. By contrast, the low state accounts for 6.34%, and high state and super-high state together account for 14.27%, respectively.

5. The Probability Distribution of the Changing Trends of Oil Prices in the Short-Term

The limit probability of oil-price state transition chain as a Markov Chain is the ultimate state of oil price series. It approximates the changing trends of oil price in the medium- and long-terms, but not in the short-term. Generally speaking, an actual distribution of oil price series reflects the short-term changing trends of oil prices. We have replaced an actual distribution by a probability simulation of actual oil price distribution. Thus it has a more generalized implication and is more academically sound.

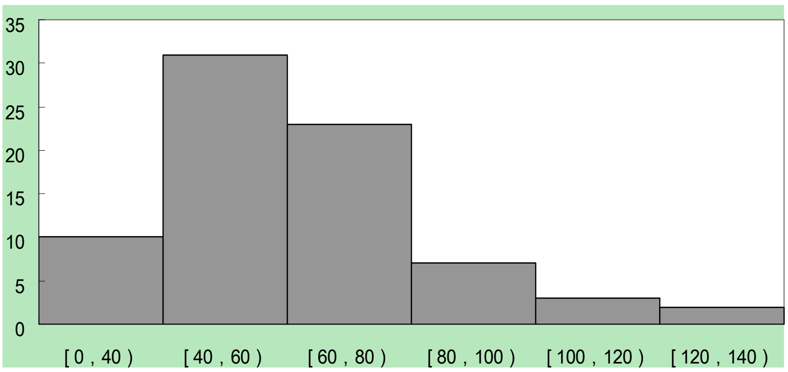

Figure 3 A Distribution of Crude Oil Prices from January 2004 to April 2010

As shown in Figure 3, the distribution of oil prices inclines toward the left of the whole interval. The hypothesis test of this distribution confirms that oil price series conform to a log-normality distribution[2]. The function of a log-normality distribution is as follows:

$$f(x)=\begin{cases}\dfrac{1}{x\sigma\sqrt{2\pi}}\exp\left[-\dfrac{(\ln x-a)^2}{2\sigma^2}\right], & x>0,\\ 0, & x\le 0\end{cases} \qquad (3)$$

[2] The confirmation of probability distribution is undertaken in Excel.

where a and σ as the mathematical characteristics of a log-normality distribution are the mathematical expectation value and average variance, respectively. They refers to the average value and average variance of $\ln x$ when the statistics get logarithmic, and are estimated to be that

$\bar{x}_{\ln x} = 4.0570$, $s^2_{\ln x} = 0.1297$, $s_{\ln x} = 0.3601$.

Because mathematical statistics reveal trends from a batch of statistics, hence the frequency of one sample in each interval cannot be too small. Statistically speaking, it is considered appropriate to take the frequency of each interval $\mu \geq 5$. Given that the frequency of the (0, 20) interval is zero, therefore it needs to be combined with the [20, 40) interval. Meanwhile, the frequencies of the [100, 120), [120, 140) and [140, 160) intervals are three, two and zero, respectively, so that they should be merged together as well.

Taking the end-point value of an interval as upper and lower limits while considering probability distribution function as an integrand function, we can then calculate the probability value of each interval as follows:

$$p_i = \int_{\text{the lower limit of interval}}^{\text{the upper limit of interval}} \frac{1}{x\sigma\sqrt{2\pi}} \exp\left[-\frac{(\ln x - a)^2}{2\sigma^2}\right] dx \qquad (4)$$

For the χ^2 test, we use

$$\chi^2 = \sum_{i=1}^{m} \frac{(f_i - np_i)^2}{np_i} \qquad (5)$$

Table 3 χ^2 Testing Values of a Log-normality Distribution

Price interval	[0, 40)	[40, 60)	[60, 80)	[80, 100)	>100	The sum of testing values	The boundary values of χ^2 distribution at different significant levels (the degrees of freedom *m*=2)			
							$\alpha = 0.3$	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.01$
Log-normality distribution	0.2164	0.0698	0.1942	0.4691	0.0011	**0.9506**	2.401	4.61	5.99	9.21

The test results are given in Table 3. We can see that $\chi^2_{\log-normality} = 0.9506 < \chi^2_{(2,0.05)} = 5.99$ and $< \chi^2_{(2,\,0.1)} = 4.61$, even that $\chi^2_{\log-normality} = 0.9506 < \chi^2_{(2,0.3)} = 2.401$. Therefore, the log-normality distribution fits into the actual distribution of oil prices very well.

In the following sections, all the discussions are based on a log-normality distribution. Substituting $\overline{x}_{\ln x} = 4.0570$, $s^2_{\ln x} = 0.1297$ and $s_{\ln x} = 0.3601$ in equation (4) yields the following log-normality distribution:

$$\begin{aligned} f(x) &= \frac{1}{0.3601\sqrt{2\pi}x}\exp\left[-\frac{(\ln x - 4.0570)^2}{2\times 0.1297}\right] \\ &= \frac{1}{0.9026x}\exp\left[-\frac{(\ln x - 4.0570)^2}{0.2594}\right] \end{aligned} \qquad (6)$$

Substituting the end-point value of each interval,

$$p_i = \int_{x_i}^{x_i+20} \frac{1}{0.9026x}\exp\left[-\frac{(\ln x - 4.0570)^2}{0.2594}\right]dx, \quad x_i = 0,\ 20, \ldots\ ,120\ ,\ldots \qquad (7)$$

we have the respective probability value of each interval, as given in Table 4. Then plotting the statistics from Table 4, we yield a fitting map for a probability distribution function of oil price series as shown in Figure 4.

Table 4 Probability Values of Observed Oil Prices

Observed interval	(0, 20)	[20, 40)	[40, 60)	[60, 80)	[80, 100)	[100, 120)	[120, 140)	[140, 160)	>200
Probability value of the interval	0.00160	0.15168	0.38797	0.27536	0.11941	0.04272	0.01424	0.00467	0.00028
Observed prices	20	40	60	80	100	120	140	160	200
Accumulated probability value	0.00160	0.15329	0.54125	0.81661	0.93602	0.97875	0.99299	0.99765	0.99972

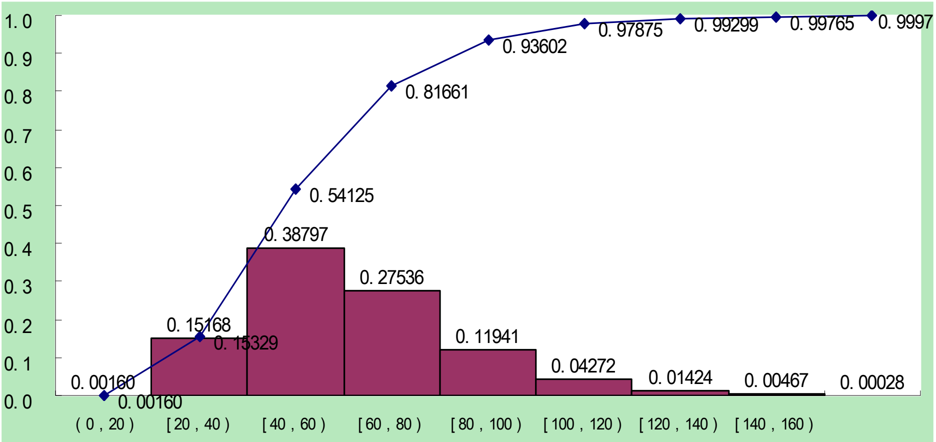

Figure 4 Probability Distribution of Monthly-Average Prices of OPEC Basket of Crude Oils from January 2004 to April 2010

As shown in Table 4, the probability of oil prices falling in the interval of US$ 20-40 per barrel is 0.15168, 0.38797 in the interval of US$ 40-60 per barrel, 0.27536 in the interval of US$ 60-80 per barrel, 0.11941 in the interval of US$ 80-100 per barrel respectively, whereas the probability of oil prices higher than US$ 100 per barrel is 0.06398. By contrast, the probability of oil prices below 20 US$/barrel is merely 0.16%. This probability distribution of oil prices puts the probability of recent oil prices in the range of US$ 20 to US$ 120 per barrel at 0.97715. It can be labeled as an inevitable event. The probability of oil prices in the interval of US$ 20-100 per barrel is 0.93442. It is a fairly high probability event. By contrast, the probability of oil prices over 140 US$/barrel is 0.00701. It is a low probability event. The probability of oil prices over 200 US$/barrel is 0.00028. It is almost an impossible event. However, it should be pointed out that this probability distribution fittings are based on the recent statistics so that they only reveal recent changes in oil prices. Thus, this deduction is only meaningful for the recent changing trends of oil prices. In the next section, we will discuss deducing changing trends of oil prices in the middle- and long-terms.

6. A Deduction Model of Integrating the Limit Probability of a Markov Chain with a Probability Distribution

6.1 Deducing the Changing Trends of Oil Prices from the Short-term to the Middle- and Long-Terms

Sections 4 and 5 discuss the Markov Chain model and the probability distribution function model, separately. By integrating the two models, we can infer the changing trends of oil prices from the short-term to the middle- and long-terms.

Table 5 A Comparison between Log-normality Distribution Fitting and Limit Probability of a Markov Chain of Oil Price Series

Price interval	(0，40]	(40，60]	(60，80]	(80，100]	(100，120]	(120，140]
Probability value of a log-normality distribution	0.1533	0.3880	0.2754	0.1194	0.0427	0.02125*
Limit probability value of Markov chain	0.0634	0.2948	0.3281	0.1712	0.0856	0.0571
The difference between the two probabilities	-0.0898	-0.0932	0.0527	0.0518	0.0429	0.03585
The percentage of the above difference (%)	-58.59	-24.02	19.14	43.38	100.47	168.71

*0.02125 is the probability of the crude oil prices higher than US$ 120 per barrel.

As shown in Table 5, the probability of oil prices being 40 US$/barrel or less is 0.1533 in the short-term, while such a probability is 0.0634 in the middle- and long-terms, 58.59% less than that in the short-term. The probability of oil prices being in the (40, 60] interval is reduced from 0.3880 in the short-term to 0.2948 in the middle- and long-terms by 24.02%. By contrast, the probability of oil prices falling in the (60, 80] interval or above has all gone up, with an increase ranging from 19.14% for the (60, 80] interval to168.71 % for the (120, 140] interval. To put it simply in Table 6, taking 60 US$/barrel as a dividing line, we can see that the probability of oil prices below 60 US$/barrel is decreasing from the short-term to the middle- and long-terms, while the corresponding probability of being over 60 US$/barrel is increasing.

It can be seen from Table 6, in the future period of time, oil prices of 60 US$/barrel or less will be reduced by 18.31%, while the oil prices being higher than 60 US$/barrel will increase by 18.31%. On the other hand, the expectation value of a log-normality distribution of recent changing trends of oil prices is 62.7 US$/barrel, while the expectation value of a Markov Chain reflecting the middle-long term changing trends of oil prices is 71.8 US$/barrel. There is the difference of 9.1 US$/barrel. This means that the monthly-average oil prices will have an absorbing capacity and changing range of about 9 US$/barrel as oil prices tend to go up in the future.

Table 6 A Two-State Comparison between a Log-normality Distribution Fitting and the Limit Probability of Markov Chain of Oil Prices Series

	(0, 60]	(60, 140]
Log-normality probability	0.5413	0.45875
Limit probability of a Markov chain	0.3582	0.6418
The difference between the two probabilities	-0.1831	0.1831

6.2 Will a Period of Soaring Oil Prices Reemerge?

From 2004-2009, oil prices had experienced a trend-circle "fluctuating at low level—fluctuating at high level—soaring extraordinarily—falling swiftly—rising slowly". Does this circle reoccur or will oil prices soar extraordinarily again? In what follows, we will address this issue by comparing the probability distribution and the limit probability of a Markov Chain of changing oil prices.

Suppose that the length of a deduction period is the same as 76 months of the sample statistic. Multiplying the probability of a log-normality distribution and the limit probability of a Markov Chain in Table 5 by the sample observations of 76, we have the theoretical frequency of a log-normality distribution and the limit frequency of a Markov chain of oil prices, as shown in Table 7.

Table 7 A Comparison between the Theoretical Frequency of a Log-normality Distribution and the Limit Frequency of a Markov Chain

	(0,40]	(40, 60]	(60, 80]	(80, 100]	(100, 120]	(120, 140]
Theoretical frequency of log-normality distribution	11.65	29.49	20.93	9.07	3.25	1.62*
Limit frequency of a Markov Chain	4.82	22.40	24.93	13.01	6.50	4.34
The difference between the two frequencies	-6.83	-7.09	4.00	3.94	3.25	2.72
The percentage of the above difference (%)	-58.63	-24.04	19.13	43.44	100.00	167.90
The sum of positive and negative numbers in the above rows	-14		14			

*1.62 is the theoretical frequency of a log-normality distribution of crude oil prices higher than US$ 120 per barrel.

It can be observed from Table 7 that the number of months of oil prices falling in the (0, 60] interval is decreasing. Specifically, the number of monthly oil prices in the (0, 40] interval has been reduced from 11.65 to 4.82 by 6.83, and the number in the (40, 60] interval has dropped from 29.49 to 22.40 by 7.09. They together drop by 13.92≈14. By contrast, the number of oil prices in the (60, 140] interval is increasing. The number of monthly oil prices in the (60, 80] interval, in the (80, 100] interval, in the (100, 120] interval and in the (120, 140] interval has increased by 4.00, 3.94, 3.25 and 2.72, respectively. The sum of the total increased number is 13.91≈14, which equals to the total decreased number. But the increasing range of each interval is different. The higher the interval is, the faster it increases. It is worth noting that in the next 76 months the frequency of oil prices falling in the (0, 60] interval will decrease approximately by 14, that is, 14 months. By contrast, the number of months in which oil prices are in the (60, 140] interval will increase by 14. Of the total, 4 out of 14 have moved into the middle-level state of oil prices, while all the rest will jump into the (80, 140] interval. This clearly shows that the possibility of shifting to the super-high state is even higher than that to the middle-level state. This kind of interval-state shifting can be considered as a jump. It indicates the reoccurrence of oil price soaring. At some point in the future, oil price will soar again after maintaining at the level in the (60, 80] interval. Moreover, the duration of future price soaring would last longer than the previous one. The soaring range would be broadened as well. However, it should be pointed out that from the general distribution of oil prices, the probability of oil prices below 120 US$/barrel is 0.9788, indicating that this price level or less would prevail in the world oil market. Meanwhile, the probability of oil prices below 100 US$/barrel is 0.9360, and the probability of oil prices below 80 US$/barrel is 0.8166, all of which would have played a role in stabilizing oil prices.

According to our deduction, oil prices will not soar extraordinarily. In 1998, when oil price exceeded 100 US$/barrel, Goldman Sachs had estimated that oil prices could go up to reach 200 US$/barrel in 2009 (Associated Press, 2009). History has shown that this was the exaggeration of reality. From our analysis of a probability distribution, the probability of oil prices above 200 US$/barrel is less than 0.0003. It is considered an impossible event.

6.3 Verification on our Trend Deduction of Oil Prices by Actual Oil Prices

Using sample statistics up to April 2010, we have deduced the changing trends of oil prices by the limit probability of a Markov Chain. The validity and practicability of this deduction can be verified by the actual changing trends of oil prices from May 2010 to October 2010, the months that immediately follow our sample period, as shown in Table 8.

Table 8 The Spot Monthly-Average Oil Prices of OPEC Basket of Crude Oils from May 2010 to October 2010 (US$/Barrel)

May 2010	June 2010	July 2010	August 2010	September 2010	October 2010
74.48	72.95	72.51	74.30	79.86	80.55

Source: Compiled based on OPEC (2010).

All these monthly prices fall in the [40, 100) interval. This confirms our aforementioned judgment that oil prices will fall into this most possible interval by a probability of 0.7828.

Moreover, all these 6 monthly prices are in the middle-level state and middle- and high-level state of the interval, namely, the E_m[60, 80) and E_{mh}[80, 100) intervals, of which 5 monthly oil prices are in the E_m[60, 80) interval. This shows that deducing trends of fluctuating oil prices by the limit probability of a Markov Chain is reliable, thus reinforcing practical value of this model.

7. Conclusions

Crude oil prices have been fluctuating over time and by a large range. It is the disorganization of oil price series that makes it difficult to deduce the changing trends of oil prices in the middle- and long-terms and predict their price levels in the short-term. Our paper has established a trend deduction model of fluctuating oil prices. This model integrates the probability distribution of oil price series with the limit probability distribution of a Markov Chain. The probability distribution of oil price series reveals the short-term changing trends of oil prices, while the limit probability of oil price series as a Markov Chain reflects the middle-long term changing trends of oil prices. The difference between them indicates specific changes in a variety of oil price states from the short-term to the middle- and long-terms.

Based on the integrated model, we have deduced the changing trends of oil prices in the next 76 months. Our results match the actual changing trends of oil prices in the 6 months that immediately followed our sample period of up to April 2010, and show that oil prices will continue to increase. The probability of oil prices below 40 US$/barrel has been reduced by 58.6% while that of oil prices over 80 US$/barrel has increased markedly. However, the probability of middle-level oil prices ranging from 40 to 100 US$/barrel approximates 80%. This price level would prevail in the world oil market. Although it is still possible that future oil prices will soar extraordinarily, these are the occasional events. They cannot shake the overall changing trends of oil prices.

References

Arimoto, S. (1985), *The Contemporary Information Theory*, Posts & Telecom Press, Beijing, pp. 33-38.

Associated Press (2009), Analyst: $200 Oil Could Happen This Year, May 6, Available at: http://www.msnbc.msn.com/id/24482312/ns/business-autos.

Fisz, M. (1980), *Probability Theory and Mathematical Statistics,* 3rd Edition, Krieger Publishing, Huntington, New York.

Holmes, M.J. and P. Wang (2003), Oil Price Shocks and the Asymmetric Adjustment of UK Output: a Markov-switching Approach, *International Review of Applied Economics*, Vol. 17, No. 2, pp.181–192.

Jing, Y., Wang, S., and Q. Yuan (1986), *The Application of Markov Process in Geology*, Geological Publishing House, Beijing, pp. 48-50.

Kosobud, R.F. and H. H. Stokes (1978), Oil Market Share Dynamics: A Markov Chain Analysis of Consumer and Producer Adjustments, *Empirical Economics*, Vol. 3, No. 4, pp. 253-275.

Lu, D. (1987), *Stochastic Process and Application*, Tsinghua Press, Beijing, pp. 38-44.

Organization of the Petroleum Exporting Countries (OPEC, 2010), OPEC Basket Price, Geneva, Available at: http://www.opec.org/opec_web/en/data_graphs/40.htm.

Song, D. (2005), The Applications of Markov Law in Predicting the Price of Petroleum, *Value Engineering*, No.12, pp.105-106.

Vo, M.T. (2009), Regime-switching Stochastic Volatility: Evidence from the Crude Oil Market, *Energy Economics*, Vol. 31, No. 5, pp.779-788.

Wang, Z. (1979), *Basics of Probability Theory and its Application*, China Science Press, Beijing, pp.181-188.

Wei, W., Chen, Z. and J. Wang (2006), A Research on the Three States Markov-Switching Mode - An Application in the Analysis of World Oil Price Fluctuation, *The Study of Finance and Economics*, No.6, pp.120-131

Xu, H. (2001), The Application of Markov Chain in the Research of Import Commodities Market in Kazakstan, in M.M. Shyngysbaeva (ed.), *Kazakstan Economic Reform: Experience, Theory and Practice*, Almaty, Kazakhstan, pp. 129-139.

Appendix 1 Spot Monthly Prices of OPEC Basket of Crude Oils from January 2004 to April 2010 (US$/Barrel)

	2004	2005	2006	2007	2008	2009	2010
January	29.820	40.240	58.173	50.79	88.50	41.54	76.01
February	29.560	41.480	56.634	54.56	90.81	41.41	72.99
March	32.230	49.070	57.864	58.59	99.03	45.78	77.21
April	32.350	49.462	64.386	63.55	105.16	50.20	82.33
May	36.270	46.620	65.178	64.48	119.40	56.98	
June	34.610	51.512	64.568	66.89	128.34	68.36	
July	36.290	53.217	68.975	71.89	131.22	64.59	
August	40.270	57.837	68.81	68.71	112.41	71.73	
September	40.360	57.990	59.34	74.18	96.85	67.17	
October	45.370	54.394	54.97	79.36	69.16	72.67	
November	38.960	51.100	55.42	88.99	49.76	76.29	
December	35.700	52.482	57.95	87.19	38.60	74.01	

Source: Compiled based on OPEC (2010).