Contestability and collateral in credit markets with adverse selection

Cesaroni, Giovanni

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CONTESTABILITY AND COLLATERAL IN CREDIT MARKETS WITH ADVERSE SELECTION

Abstract

The work discusses a basic proposition in the theory of competition in markets with adverse selection (Bester, 1985). By working out the sequence of market transactions, we show that the effectiveness of collateral in avoiding equilibrium rationing depends on an assumption of uncontestability of the loan market. If contestability is restored to its proper place, the separation of borrower by means of sufficient collateral does not impede the emergence of credit rationing, which results from a coordination failure among risk-neutral banks. As a consequence, even in a risk-neutral environment with suitable endowments, the use of collateral in credit contracts could not be a socially efficient screening-device. Our conclusion on rationing does not stand in contrast with the general result of Gale (1996).

1. Introduction

Since the seminal work of Bester (1985), it has been generally taken for granted that in a risk-neutral environment, when sufficient wealth is made available from a borrower's endowment, the competitive equilibrium of the credit market achieves perfect sorting, thus solving the adverse selection problem posed by Stiglitz-Weiss (1981). In other words, the separation of borrowers, that is accomplished by means of 'unlimited' collateral, impedes equilibrium rationing. This conclusion has become a real cornerstone in the literature on credit markets with asymmetric information, as it has been shared by several diverse contributions (e.g., Bester 1987, Besanko-Thakor 1987, Hellwig 1987, Stiglitz-Weiss 1992, Schmidt-Mohr 1997), and it has also been recalled by authors dealing with the more general problem of competition in markets with adverse selection (Gale 1996). Our paper discusses this proposition by rendering explicit the whole sequence of market transactions, and considering the symmetric incentive-compatible equilibrium of the resulting market game. We point out that Bester's claim relies on an *ad hoc* assumption, which removes contestability of the loan market in presence of a rationing equilibrium. In fact, an artificial restriction of the strategies of the incumbent banks is being imposed, while this restriction does not equally hold for the no-rationing equilibrium. Conversely, if the game is correctly specified, in the sense that the same extensive form and the same unrestricted strategy set is being considered for both kind of possible equilibria (rationing and no-rationing), then Bester's conclusion is not valid. The result extends easily to the case in which a recourse opening of the credit market is being added. Then, a competitive equilibrium with separation of borrowers can in fact show rationing of their demand for credit, even when they have sufficient collateral. As a direct consequence, one has that collateralization may turn out to be a socially-inefficient instrument to screen a borrower's riskiness. The paper is organized as follows: in section 2, we illustrate the basic model; in section 3, we introduce the definition of equilibrium and the extensive form of the game; section 4 deals with the central issue of the presence of rationing in an equilibrium; finally, section 5 discusses the results of our analysis and their relationship with the above mentioned strands of literature.

2. The model

Given his purpose, Bester[1] obviously employs a model that reproduces the

[1] In the following, when we mention Bester without any further indication we mean

same basic structure of Stiglitz-Weiss analysis. All agents are risk-neutral and there are two types of entrepreneurs or firms. Each entrepreneur has a fixed-scale investment opportunity, while the random returns of investments of the two classes of firms are ordered according to the mean preserving spread criterion (mps)[2]. Banks cannot directly observe the type of borrower they deal with, so adverse selection may arise. The supply of deposits to the banking system is a continuous and strictly increasing function of the rate of interest paid on deposits. In a credit contract, banks may charge collateral as a guarantee for loan repayments when the borrower defaults. Pledging collateral is costly to borrowers, who face a generic and constant unit cost. Moreover, they have a collateral endowment which is more than sufficient to fully guarantee the loan value. Symbols are as follows:

I is the fixed scale of investment, $\widetilde{R}_i$ is their random return, taking values in $[0, \bar{R}]$, where $i = a, b$ stands for the type of firm we are considering; $F_i(R) > 0$, $\forall R > 0$, is the distribution function: assuming that type b entrepreneurs are more risky than type a, in the sense of a mean-preserving spread, we have $E\left\{\widetilde{R}_a\right\} = E\left\{\widetilde{R}_b\right\}$, $\int_0^y \left[F_b(R) - F_a(R)\right] dR \geqq 0$, $\forall y \in \left[0, \bar{R}\right]$; W is the initial endowment of monetary wealth each entrepreneur can dispose of ($W < I$), $B = I - W$ is the amount of funds needed to start any investment project; $L^S(\pi)$ is the supply function of deposits; finally C and r denote, respectively, the collateral and the nominal rate of interest charged on a loan, k is the unit cost of collateral and π is the rate of interest banks pay on deposits. There are N_i entrepreneurs of type i. Note that the non-monetary wealth entrepreneurs

the 1985 article.

[2] Bester (1985, p. 851). On the definition of mean preserving spreads see Rothschild-Stiglitz (1970).

can offer as collateral is separated from their monetary holdings.

The debt contract the banks use to lend funds to firms declares the borrower to be insolvent anytime the sum of total return on investments and collateral is not sufficient for loan repayment: that is when $R_i + C < B(1+r)$. In this circumstance the bank seizes the whole disposable value $R_i + C$, otherwise it gets the contractual repayment $B(1+r)$. Bester considers only contracts for which the collateral does not exceed the face value of the loan $C \leq B(1+r)$ (otherwise default would be trivially excluded from the model). The bank's expected rate of return on a loan contract $\gamma \equiv (r, C)$ to borrower i is

$$\rho_i(\gamma) = E\left\{\min\left[B(1+r), \tilde{R}_i + C\right] - B\right\}/B \tag{1}$$

From the same contract, entrepreneur i expects to gain a total profit which is given as

$$\Pi_i(\gamma) = E\left\{\max\left[\tilde{R}_i - B(1+r) - kC, -(1+k)C\right]\right\} \tag{2}$$

For $C < B(1+r)$, the mps ordering implies that

$$\rho_a(\gamma) > \rho_b(\gamma) \tag{3}$$

and

$$\Pi_b(\gamma) > \Pi_a(\gamma) \tag{4}$$

For a given contract γ, a bank obtains a higher rate of return on a loan to a less-risky borrower, while the utility of a riskier borrower is higher than that of a safer one.

Totally differentiating expressions (1) and (2) with respect to r and C, and simplifying, we get the following marginal rates of substitution

$$\mu_i(\gamma) = -\frac{F_i(B(1+r) - C)}{B[1 - F_i(B(1+r) - C)]} \tag{5}$$

and

$$\sigma_i(\gamma) = -\frac{F_i(B(1+r)-C)+k}{B[1-F_i(B(1+r)-C)]} \tag{6}$$

These ratios clearly indicate that each kind of borrower has an indifference curve which is steeper in absolute value than its isoprofit curve, because $\sigma_i(\gamma) < \mu_i(\gamma)$. Moreover, it can easily be shown that both the isoreturn curves are strictly convex, so that functions $\Pi_i(\gamma)$ are quasiconvex. The property is not grasped by Bester, who believes these curves not to be concave everywhere[3]. However, in this model, a sufficient condition to obtain a separating equilibrium is represented by $F_b(R) > F_a(R)$, $\forall R \in [0, B(1+r)-C]$, an assumption which amounts to establishing a single-crossing property for the isoreturn curves. In fact, for any given contract, we immediately get that the indifference curve of riskier investors, those of type b, becomes steeper than that of less risky investors, $\sigma_b(\gamma) < \sigma_a(\gamma)$. The same goes for the isoprofit curves. Thus, the separate indifference (or isoprofit) curves will have one intersection point at most. As far as their shape is concerned, in what follows it will be more convenient to adopt Bester's representation. Finally, two important features of his model have to be emphasized:

Assumption 1: "banks act as perfect competitors, that is, each bank takes the rate of interest π on deposits and the set of credit offers by competing banks as given and as independent of its own actions";

[3] Convexity of the isoreturn curves holds when we consider both a continuous and a discrete random variable. On the contrary, Bester believes these curves to have mainly a concave behaviour, which in turn implies that the expected utility is quasiconcave. See Bester (1985), note 8, p. 851, and figures 1 and 2 -respectively at p. 852 and following. Standard renditions of this model employ concave isoreturn curves; cf. Goodhart (1989) and Freixas-Rochet (1998).

Assumption 2: an entrepreneur who is rationed at his preferred contract may successively apply for the other contract at the same bank[4].

As we shall see later on, assumption 2 is crucial for obtaining Bester's conclusion on the absence of rationing.

3. Definition of equilibrium and the extensive form of the market game

We are now going to get a preliminary insight into the basic definition of equilibrium and into the sequence of transactions, that are featured by the model we have just presented.

Definition 1

A *credit market equilibrium* is a situation in which "borrowers choose among contracts to maximize expected profits: (i) each contract γ_α^* and γ_β^* yields zero profit to the bank; (ii) any additional credit offer γ will make no profits; and (iii) there is no excess supply of funds"[5].

This equilibrium is shortly denoted as a tuple $\{(\gamma_\alpha^*, \gamma_\beta^*),\ (\lambda_\alpha^*, \lambda_\beta^*),\ \pi^*\}$, where λ_j, $0 < \lambda_j \leq 1$ $j = \alpha, \beta$, is the fraction of firms that receive credit under the terms of γ_j, that is to say each firm's identical probability of getting credit[6];

[4] Bester (1985), p. 852 ("In the following, it will be assumed...."). Please note that assumption 2 would have no sense if borrowers were allowed to apply to a different bank for the remaining contract. Because they are not known, in this eventuality type a entrepreneurs would rather apply to a new bank for their preferred contract. See fig. 1 below.

[5] Bester (1985), p. 852.

[6] Bester's credit rationing is that of type II (see Keeton 1979), because some

moreover, by requirement (i), $\rho_i(\gamma_j^*) = \pi^*$. Credit rationing occurs if some entrepreneur i faces a positive probability of being rejected at contract γ_j^*, which he prefers, and at the same time $\Pi_i(\gamma_j^*) > W(1+\pi^*)$. Notice that the sign of strict inequality comes from the need to satisfy entrepreneurs' participation constraint to the credit market:

$$\lambda_j^* \Pi_i(\gamma_j^*) \geq W(1+\pi^*) \tag{7}$$

where the second term is the alternative return the entrepreneur can get from his monetary wealth; this condition is also known as the individual rationality constraint (IR). In an equilibrium with separation of different borrowers, their incentive-compatibility constraints (IC) must also hold:

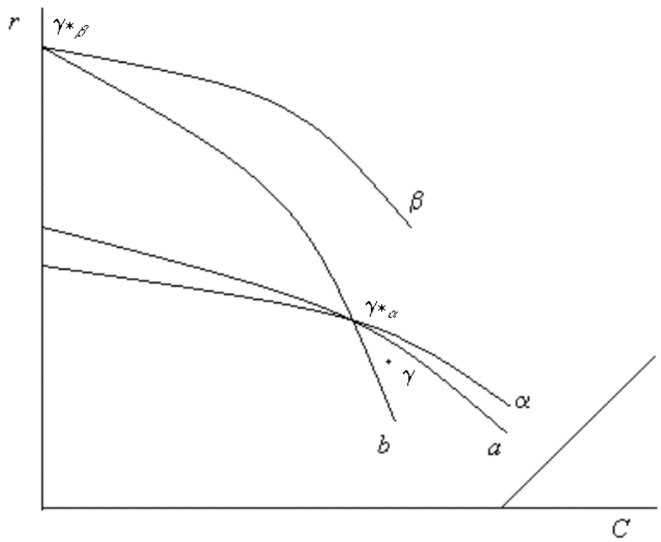
$$\lambda_\alpha^* \Pi_a(\gamma_\alpha^*) \geq \lambda_\beta^* \Pi_a(\gamma_\beta^*) \tag{8}$$

$$\lambda_\beta^* \Pi_b(\gamma_\beta^*) \geq \lambda_\alpha^* \Pi_b(\gamma_\alpha^*) \tag{9}$$

This equilibrium is depicted in fig.1, where a and b represent the borrowers' indifference curves while α and β denote a bank's isoprofit curve of the corresponding type. As long as the proportion between high-risk and low-risk borrowers is so high as to exclude the existence of a dominating pooling contract [7], the competitive equilibrium - a Nash equilibrium - exists, and is given by the separating equilibrium shown in fig. 1, $(\gamma_\alpha^*, \gamma_\beta^*)$. Bester's contention is that in this framework there can be no rationing of any type of borrowers at $(\gamma_\alpha^*, \gamma_\beta^*)$, that is to say $\lambda_\alpha^* = \lambda_\beta^* = 1$. To address this issue, it will be useful

randomly-chosen borrowers obtain no credit at all, while the remaining ones get the required loan-size.

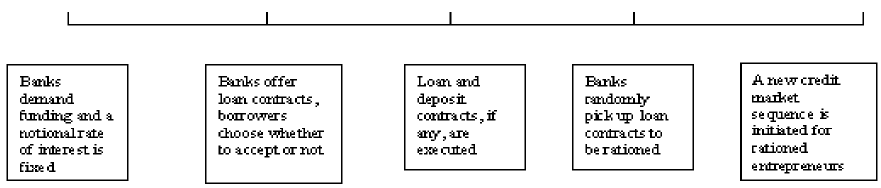
[7] The pooling isoprofit must not lie in the region that is delimitated by isoprofit α, indifference curve a, r-axis and contract γ_α^*. Bester assumes this condition to be satisfied, see Bester (1985), note 12, p. 853.



describing more precisely the sequence of market transactions which is posited by Bester.

With regard to the sequence of transactions, the timing of the model can be illustrated as follows (fig.2):

In the time-line above, we have uninformed agents (banks) moving first: they devise debt contracts to be offered to the informed ones (borrowers) (see Hellwig 1987). But price terms of this contract, i.e. γ_j, depend on the rate of interest on deposits, so an equilibrium on this market must be computed before banks can actually offer loans. If so, banks who have decided to enter the loans market must go first on the deposits market, where they demand a quantity of loanable funds corresponding to the credit probabilities they are going to offer and to the expected number of borrowers. The matching of these demands with the supply of funds from savers, $L^S(\pi)$, determines a notional equilibrium rate of interest on deposits, π^f. This, in principle, need not be the actual equilibrium rate as long as trading is not definitively closed (compare however assumption 3 below). On the other hand, when offers are made by



banks, a generic borrower can observe this rate and he accordingly chooses one of the two alternative options: investing his liquidity on the deposits market or applying for his preferred credit-offer at a single bank. Once credit applications by borrowers have been done, banks randomly choose the individual loans that are going to be financed.

We are now in the position to describe in some detail the functioning of markets along with the extensive form of the game which a representative bank plays with a generic borrower of a specified kind. In Bester's model a symmetric separating-equilibrium is considered, in which different banks and different entrepreneurs of a given risk-class behave the same way. The rate of interest on the market for deposits in a symmetric equilibrium in which banks offer credit with a positive probability, and in which firms demand loans, can be determined as

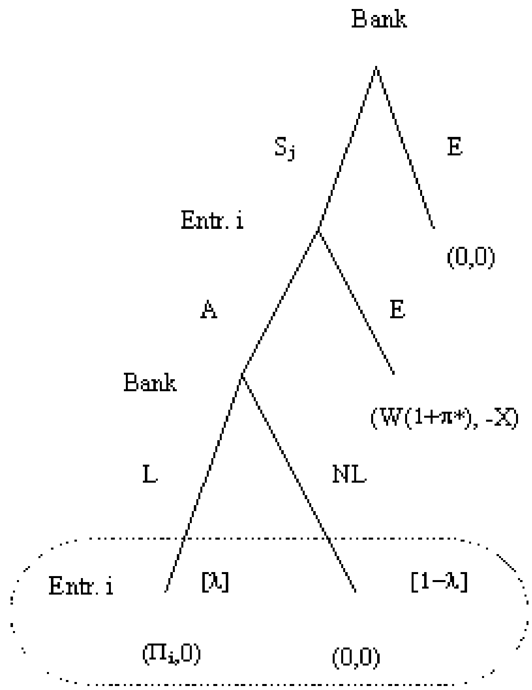
$$\pi^* : L^S(\pi) = B(\lambda^*_\alpha N_\alpha + \lambda^*_\beta N_\beta) \tag{10}$$

where, obviously, the right-hand term of the equation is the banks' total demand of funds.

There is a large number of banks, E, and the market for loans is contestable. A number of banks $H < E$ that is sufficient to serve the whole market for loans, will enter the credit market, therefore a borrower can be assured that λ_j is the actual probability of receiving a loan. The extensive form of the simple game played by a representative bank, which plans to enter the credit market, and by a generic borrower of type i may be depicted as in figure 3 below. Herein, $S_j \equiv (\gamma_j, \lambda_j)$ is the loan offer - which probability determines the equilibrium rate of interest on deposits π^*, $\rho_i(\gamma_j) = \pi^*$ and $\Pi_i \equiv \Pi_i(\gamma_j)$; while X is the absolute value of the loss that a bank incurs when entrepreneurs should refuse its contract offer: i.e. the value of the deposit contracts. Strategy E denotes exit from the credit market, A is the acceptance of the loan offer; L and NL, respectively, stand for according a loan or not, and such strategies are randomly chosen by the bank. As for trading on the market for deposits is concerned, it must be observed that the following assumption has to be made:

Assumption 3: Trading on this market is completed before borrowers accept credit offers, so the deposits market precedes the loans market, i.e. definitive fixing of the rate of interest and execution of the deposit contracts - by banks and savers - take place before the market for loans closes.

On the contrary, suppose that banks can sign deposit contracts after having observed the entrepreneurs' decision to accept or not the loan offer. Then, when borrowers found it favourable to refuse their preferred credit offer, be-



cause $\lambda_j \Pi_i(\gamma_j^*) < W(1+\pi^*)$, banks would not demand funding anymore on the deposits market and π^*would go to zero. If this occurs, payoffs from the equilibrium sequence $\{S_j, E\}$ would simply be $(W, 0)$, on this basis it cannot be excluded that $\lambda_j \Pi_i(\gamma_j^*) > W$. From this we have an obvious comment on the trading structure: if closure of trade on the deposits and on loans market was simultaneous then, in equilibrium, entrepreneurs could demand a loan even if their participation constraint (IR) to the credit market was violated. This explains the necessity of assumption 3.

Now that the basic framework of Bester's model has been described, let us consider a simple implication of his pivotal Assumption 2.

Lemma 1

Assumption 2 implies a second stage of the credit market, in which banks offer the new set of contracts to rationed borrowers and demand additional funding on the deposits market.

Proof. Suppose not. Then assumption 2 would entail a market in which banks demand funding just once and offer borrowers a lottery over loan contracts $(\gamma_\alpha^*, \gamma_\beta^*)$, where each lottery yields zero profit to a bank. The lottery is such that a type a firm will be offered contract γ_α^* and a credit-probability λ_α^*, while in the event that it should be rationed at this contract (an outcome which has probability $1 - \lambda_\alpha^*$) this firm will be offered contract γ_β^* and a credit-probability λ_β^*. Mutatis mutandum, the same goes for a type b firm. In this market environment, Bester's IR and IC constraints (7)-(9) and the zero-profit condition $\rho_i(\gamma_j^*) = \pi^*$ do not hold anymore, while the proof of his Theorem 1 is inapplicable.

Before we go further, another minor - but useful - observation can be advanced in relation to the differential information among banks, that can arise at stage two of the credit market: with a separating equilibrium, at the end of the first credit market an incumbent bank knows the risk features of its customers/borrowers, while a bank which is a potential entrant does not. This shows the feasibility of the pivotal assumption 2. Obviously, it is due to the fact that - in the first credit market - the equilibrium pair of contracts $(\gamma_\alpha^*, \gamma_\beta^*)$ is incentive compatible and self-selection of borrowers is induced. So, Lemma 1 tells us that, in the event of rationing, assumption 2 yields a second stage of the credit market, a stage in which incumbent banks offer a specific contract

- namely the one that was designed for the other type - to borrowers they have just rationed. Thus, with regard to the extensive form of the game, the presence of rationing in equilibrium adds a second game tree to the initial one; this new tree has the same general structure of the first (see fig. 3 above) and differs only in the agents payoffs. Accordingly, we can point out

Remark 1

The extensive form of the whole game is not indepedent from the type of equilibrium that has to be determined.

4. Equilibrium with rationing or not?

We are now going to discuss the central issue: what credit probabilities $(\lambda_\alpha^*, \lambda_\beta^*)$ must be assigned to the incentive-compatible equilibrium pair of contracts $(\gamma_\alpha^*, \gamma_\beta^*)$? Bester's theorem 1 argues that these probabilities equal one.

Proposition 1 (*Bester's result*)

Let $\{(\gamma_\alpha^*, \gamma_\beta^*),\ (\lambda_\alpha^*, \lambda_\beta^*),\ \pi^*\}$ be a credit market equilibrium at stage one, and let both contracts γ_α^* and γ_β^* be demanded by entrepreneurs. Moreover these contracts are incentive compatible. Then in a credit market equilibrium there is no rationing at γ_α^* or at γ_β^*, i.e. $\lambda_\alpha^* = \lambda_\beta^* = 1$.

Proof. See Bester (1985), pp. 853-854

Bester's proof is by contradiction. He denies that credit probabilities equal one in the incentive-compatible equilibrium, and then shows that, in presence of rationing of some fraction of firms, there exists a deviating credit offer which warrants a positive profit to a generic entrant bank. This is a contradiction to requirement (i) of the definition of an equilibrium[8], thus the proposition is proven.

Comment. Anyway, the deviating offer could be allowed only by the way of assumption 2. In fact, this assumption puts, in the second stage of the credit market, a completely-arbitrary restriction on the strategy set of the incumbent banks. In the first credit market, these banks can freely determine their credit offers, while in the second market assumption 2 forces their strategy set - for a rationed borrower of a given type - to be made up of the remaining contract offered at stage one: $(\lambda_\alpha^*, \gamma_\alpha^*)$ for a type b firm, $(\lambda_\beta^*, \gamma_\beta^*)$ for a type a firm. On the contrary, entrant banks - Bester's "competing banks" - have access to an unrestricted set of strategies. In this setting, incumbent banks do not actually offer the above contracts to rationed borrowers[9], while their reservation utility reduces to $W(1+\pi^*)$. If so, a deviating profitable offer by an entrant bank can be easily made to exist.

These observations clearly indicate that we could be in presence of a formal fallacy - due to assumption 2 - which should invalidate Bester's result. To check this, the logical problem has to be properly set up, that is:

a) the strategy set of incumbent banks must not be restricted when there is rationing- as it is actually contemplated for the no-rationing equilibrium;

b) the extensive form of the game tree must be given independently from the type of equilibrium we are solving for.

[8] See definition 1 given above, p. 6.

[9] An incumbent bank would earn a negative profit on type b borrowers. On the contrary, it could earn a positive profit on type a borrowers, but these won't demand this contract. See Bester (1985), p. 853.

These two logical requirements imply the removal of assumption 2. Therefore, we can limit ourselves to ascertain what happens in a credit market equilibrium which game tree is entirely described by figure 3 above, that is stage one of the credit market. Herein, we derive our main result

Proposition 2

Let $\{(\gamma_\alpha^*, \gamma_\beta^*),\ (\lambda_\alpha^*, \lambda_\beta^*),\ \pi^*\}$ be a credit market equilibrium at stage one, and let both contracts γ_α^* and γ_β^* be demanded by entrepreneurs. Moreover these contracts are incentive compatible and assumption 2 does not hold. Then in a credit market equilibrium there can be rationing at $(\gamma_\alpha^*, \gamma_\beta^*)$, i.e. $\lambda_\alpha^* = \lambda_\beta^* < 1$.

Proof. See appendix.

Comment. Essentially, we prove the impossibility to construct a profitable deviating offer in presence of rationing. It can be even remarked that an identical result is obtained if we are ready to abandon requirement b), thus allowing for a second stage of the credit market [10]. Nevertheless, note that this *recourse opening* would have no economic or behavioural justification in this static credit model: banks have already screened and rationed their borrowers, and have no motivation for offering rationed borrowers a new contract. We conclude that Bester's result on the absence of rationing is a circular reasoning based on the arbitrary restriction of the strategy set which is imposed, in the occurrence of rationing, on the sole incumbent banks.

[10] In this case it suffices to apply our proof to the subgame consisting of the second stage of the credit market. The equilibrium is a subgame-perfect Nash equilibrium with rationing both at stage one and stage two of the credit market.

5. Discussion

In this model, we have argued that an equilibrium with separation of borrowers is compatible with rationing. If banks fix a credit probability less than one, then equilibrium rationing[11] arises as a coordination failure among risk-neutral agents. When the market for loans is contestable[12] there is no way for banks to design a profitable deviant contract-pair that satisfies individual rationality of entrepreneurs and enhances their probability of credit.

The occurrence of rationing in a separating equilibrium questions the efficiency of collateral as a screening instrument of borrowers' riskiness. Hitherto, when compared to a pooling equilibrium, the costs associated with the introduction of collateral have been justified by the social gains ensuing from the elimination of rationing in a separating equilibrium, viz. a strict increase in the number of social efficient investment projects undertaken, i.e. projects for which $E\left\{\widetilde{R}\right\} - I(1+\pi^*) > 0$. Conversely, as we have argued, credit rationing can resist the introduction of collateral, then the number of these projects is not necessarily higher than that of a pooling equilibrium, while less-risky borrowers have to pay the cost for it, out of their collateralizable wealth. In the separating equilibrium we examined, social surplus V is given as:

$$V = N^* \left[E\left\{\widetilde{R}\right\} - I(1+\pi^*)\right] - N_\alpha \cdot kC_\alpha$$

where N^* is the number of entrepreneurs who receives a loan, while the second addend represents the cost of pledging collateral, borne by type a entrepreneurs. Clearly, if the number of financed projects is not sufficiently increasing,

[11] For a definition of equilibrium rationing, see Jaffe-Stiglitz (1990), pp.847-849.
[12] For a definition a contestable market see Baumol-Panzar-Willig (1982).

to compensate for collateral costs, social surplus will diminish and the use of collateral in credit contracts will not result in a socially-efficient outcome[13].

Finally, we can discuss the relationship of our analysis with the literature. To our knowledge, the flaws of Bester (1985) analysis have never been detected. For this reason, perhaps, it has been generally taken for granted that introducing collateral in an analytical framework à la Stiglitz-Weiss (1981) would have definitely excluded rationing from its credit market equilibrium[14]. We note that our conclusion offers a substantial, and totally unexpected, extension of the validity of the first Stiglitz-Weiss explanation for rationing, which relied on pooling and on a different concept of equilibrium. Basically, to be emphasized is that rationing in a separating allocation, with risk-neutral entrepreneurs and a solution that is not bound by borrower's endowment of collateral, can now be achieved. Contrary to Stiglitz-Weiss (1992), to explain equilibrium credit-rationing, we show that there is no need to assume the joint presence of adverse selection, moral hazard and the entrepreneurs' risk-aversion. But, most of all, the characteristic that confers generality to our result is the slackness of the collateral constraint. In fact, when indivisible projects are available, a binding constraint has constituted the key assumption to reach a credit-rationing equilibrium in presence of both collateral and separation of borrowers. This binding condition has been equally shared by models with diverse specifi-

[13] Anyway, consider that an increase in N^* will be partly offset by the following increase in the equilibrium rate of interest on deposits, and in the size of collateral costs as well.

[14] Stiglitz-Weiss (1992) themselves implicitly adhered to Bester's view, by considerably modifying their model in order to defend the possibility of rationing as an equilibrium phenomenon.

cations (see Besanko-Thakor 1987, Bester 1987, Stiglitz-Weiss 1992[15]). Not surprisingly, as we have examined Bester's model, our competitive-equilibrium concept differs from that of Stiglitz-Weiss (1981), where banks maximized the depositors' rate of return as they were monopolistic competitors on the market for loanable funds. Nevertheless, it is identical to the Stiglitz-Weiss (1992) concept. As for this point, observe that: a) the Stiglitz-Weiss (1981) definition of equilibrium is a more conducive assumption for equilibrium rationing (see Chan-Thakor 1987); b) considering banks that are monopolistic competitors for deposits would easily bring about equilibrium-rationing in Bester's framework[16]. Therefore, the use of a different concept of equilibrium adds to the significance of our main conclusion.

Remarkably, our proposition about the coexistence of screening and equilibrium rationing does not stand in contrast with the results of Gale (1996). This general contribution follows a walrasian approach, in a risk-neutral context where each contract makes up a single market and agents take the probability of being able to exchange any particular contract as given. Price terms are exogenous, i.e. defined only implicitly by contracts which are indeed identified with probability distributions over a set of outcomes, so that markets are to be balanced by adjusting the probability of trade. Herein, Gale proves that equilibrium rationing will not be observed unless agents are indifferent to trade

[15] Besanko-Thakor (1987) and Bester (1987), diverging partly from Stiglitz-Weiss (1981), employ a perfectly elastic supply of deposits along with risk-neutrality. On their part, Stiglitz-Weiss (1992) start from a not perfectly elastic supply of deposits and risk averse entrepreneurs.

[16] Here, the increase in the interest rate on deposits, which ensues to the deviant-bank offer, invalidates the proof of proposition 1.

(i.e. borrowers get their reservation utility), the result depending on an assumption which is "a generalization of the familiar single-crossing property". Nevertheless, we point out that rationing in our case does not contradict Gale's theorem, in that it can be easily ascertained that a single-crossing property holds but the first part of Gale's assumption is clearly violated[17]. In fact, in a Rothschild-Stiglitz kind of analysis, like that we followed, we have no finiteness of the space of contracts. This could indeed be a reason for that violation and for the fact that, if put in this different context, Gale's assumption may reveal to be quite stronger than the familiar single-crossing property[18].

Appendix

Proof of Proposition 2

The only route to prove the possibility of a profitable deviating offer is that of ascertaining - for any arbitrary credit probability $\lambda^* < 1$ - the existence of price terms γ_j such that $\Pi_i(\gamma_j) > \lambda^*\Pi_i(\gamma_j^*)$ and $\rho_i(\gamma_j) > 0$, $j = \alpha, \beta$ and $i = a, b$; hence, $\gamma_j \geq \gamma_j^*$. That is to say, the deviating offer we are searching for has credit probability $\lambda_j = 1$ and price terms γ_j, which are - for borrowers - slightly worse than γ_j^* in order to allow for a positive bank-profit. Following Bester, we can think of γ_j as being different from γ_j^* only because of a small increase in the rate of interest $r_j > r_j^*$, while collateral stays unchanged, i.e. $\gamma_j = (r_j, C_j^*)$. In principle, this problem may appear easy to solve. In fact,

[17] Part (i) of Assumption 1, see Gale (1996), par. 7, pp.221-223. Compare fig. 1 above.

[18] See Gale (1996), p. 212 on the finite support of the allocation functions. In his environment, Gale claims the first part of his assumption 1 to be "quite mild"; ibidem, p. 222.

expected-utility $\Pi_i(.)$ is a continuous function which is monotone with respect to r, because its partial derivative $\Pi_i^r(.)$ is always negative. Then function $\Pi_i(.)$ can be inverted and a threshold value $\overline{r}_j > r_j^*$, such that $\Pi_i(\gamma_j) = \lambda^* \Pi_i(\gamma_j^*)$, can be determined. Afterwards, we can choose r_j as the highest value that satisfies $r < \overline{r}_j$, so that $\Pi_i(\gamma_j) > \lambda^* \Pi_i(\gamma_j^*)$ with $r_j > r_j^*$ would generally follow. Note, however, that λ^* can be arbitrarily near to 1 and, as a consequence, r_j and r_j^* can be practically indistinguishable ($r_j \simeq r_j^*$). In practice, it is difficult to affirm that the sought after deviating offer will exist for any $\lambda^* < 1$. This is confirmed by a more rigorous analysis.

In an incentive-compatible equilibrium $i = j$. By means of Lagrange's formula on finite increments, we can express the positive increment in borrowers' expected utility, which would follow from an increase in the probability of receiving a loan, as

$$\Pi_j(\gamma_j^*)(1 - \lambda^*) = -\Pi_j^r({}'\gamma_j)(\overline{r}_j - r_j^*) \tag{11}$$

where ${}'\gamma_j = (r_j', C_j^*)$ and $r_j^* < r_j' < \overline{r}_j$, $\Pi_j^r(.) = -B\int_{B(1+r)-C}^{\overline{R}} f_j(R)dR = -B\left[1 - F_j(B(1+r) - C)\right] < 0$ $j = a, b$, will denote the partial derivative of the type-j utility function with respect to r. Note that $\overline{r}_j$ is the level of the rate of interest which absorbs all of the 'surplus' ensuing from the increase in the credit probability; $\overline{r}_j$ is a decreasing function of λ^*.

The increment in utility given by (11) will be partly offset by the increase in the rate of interest, which is required to yield a positive profit to a deviating bank. Employing again Lagrange's formula, the decrement in utility can be given as

$$\Pi_j^r({}''\gamma_j^*)(\underline{r}_j - r_j^*) \tag{12}$$

where ${}''\gamma_j = (r_j'', C_j^*)$ and $r_j^* < r_j'' < \underline{r}_j$; where $\underline{r}_j$ is the rate of interest fixed in the deviating offer. Clearly $\underline{r}_j \leq \overline{r}_j$.

Then, a necessary condition for the existence of the sought deviating credit-offer is

$$-\Pi_j^r({}'\gamma_j)(\overline{r}_j - r_j^*) + \Pi_j^r({}''\gamma_j^*)(\underline{r}_j - r_j^*) > 0 \tag{13}$$

which reduces to

$$\frac{(\overline{r}_j - r_j^*)}{(\underline{r}_j - r_j^*)} > \frac{\Pi_j^r({}''\gamma_j^*)}{\Pi_j^r({}'\gamma_j)} \tag{14}$$

Obviously, the condition cannot be satisfied whenever $\underline{r}_j = \overline{r}_j$. However, the required inequality need not to be satisfied even in the case of interest, that is when $\underline{r}_j < \overline{r}_j$. In fact, the second order derivative of the expected utility with respect to the rate of interest has a uniform sign: $\Pi_j^{rr} > 0$ This implies that $r_j'' < r_j'$, but then both members of inequality (14) are greater than one and we cannot tell whether this inequality is satisfied or not. Moreover, for values of λ^* approaching 1 it can be demonstrated that the inequality is certainly violated.

As for this, consider that $\lambda^* = 1$ implies $\underline{r}_j = \overline{r}_j = r_j^*$, while from $\lambda^* < 1$ it follows $\overline{r}_j > r_j^*$ and $\underline{r}_j > r_j^*$. Moreover, when $\lambda^* \simeq 1$ the left and right-hand members of the inequality are approximately equal to one, because $\underline{r}_j, \overline{r}_j, r_j'$ and r_j'' can be taken to be as practically coincident. Then, the effects of a decrease in λ^* - in the neighbourhood of 1 - can be reckoned by means of the first-order derivative with respect to $\underline{r}_j$, valued at $\underline{r}_j = \overline{r}_j$. The effects on the left-hand and on the right-hand member of (14) are, respectively:

$$-\frac{1}{(\underline{r}_j - r_j^*)} \tag{15}$$

$$\frac{\Pi_j^{rr}({}''\gamma_j^*)}{\Pi_j^r({}'\gamma_j)} \tag{16}$$

Both derivatives are negative, but observe that the modulus of the former is strictly greater than the latter, because $\underline{r}_j \simeq r_j^*$. This proves that (14) is violated.

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