Beta estimates for leveraged ETF

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Abstract

Leveraged ETF are mandated to provide a multiple of the return on an index for intraday time periods. I present statistical estimates of beta for two leveraged ETF and one index at sampling rates from one to twenty five minute sampling. I find that beta is close to the leverage factor for sampling rates between ten and twenty five minutes, which suggests the assets are being well priced.

Keywords Leverage ETF, CAPM, Linear Dependence

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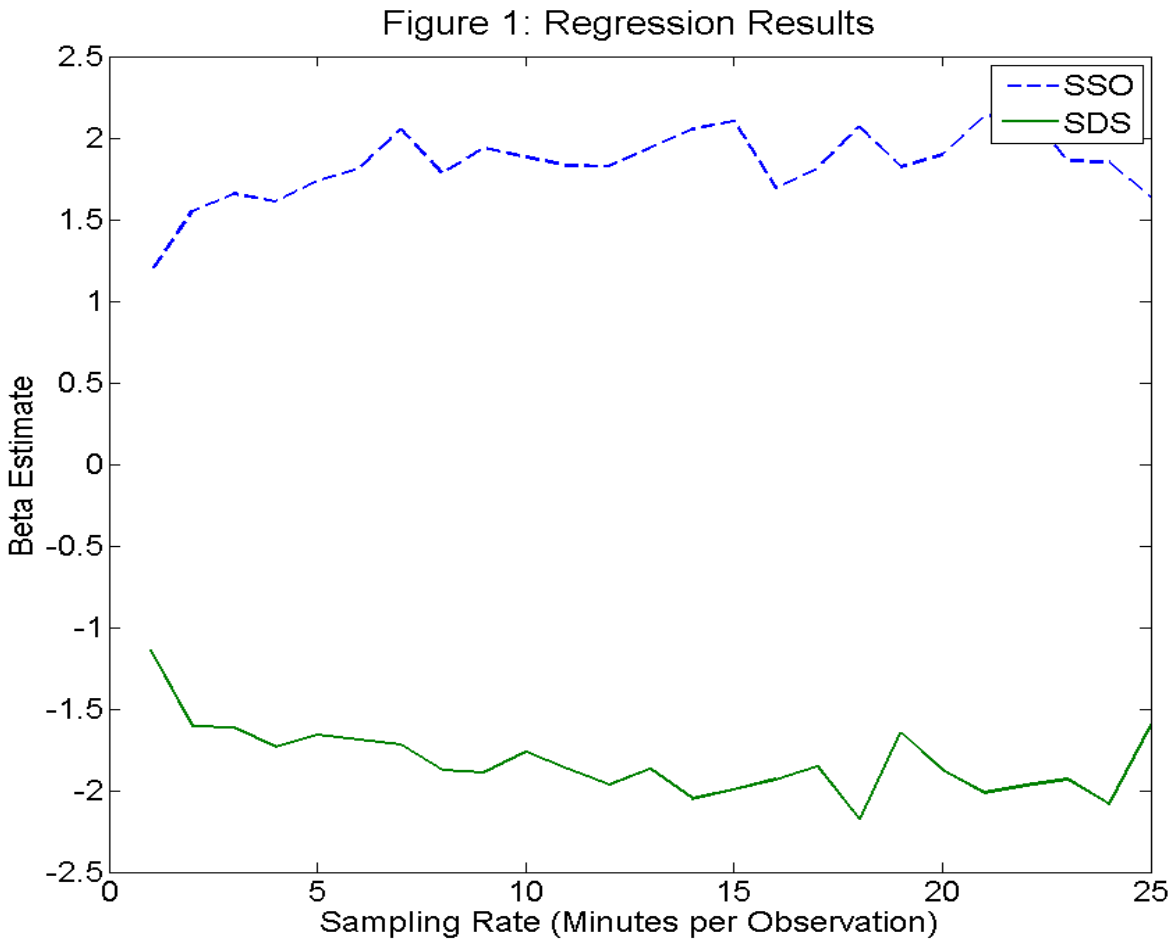
1 Introduction

With the new, popular, Leveraged Exchange Traded Funds (LETF) we have great mathematical issues. See Jarrow (2010) for more. These LETF are mandated to provide a multiple of the return on an index at intraday time scales; when the index and LETF trade, there is linear dependence between asset returns. This is special.

The Capital Asset Pricing Model (CAPM) is enduring and useful in the case of linear dependence in asset returns. This paper shows that CAPM beta is close to the leverage factor for two LETF and one index: SPY, SSO, and SDS on November 16, 2010. The leverage factor for SSO is 2, and SDS is -2. For sampling rates 10 minutes and slower I report that the beta confidence intervals contain the leverage factor for both LETF.

2 Result

In Figure 1, I present estimates of the beta coefficients for SSO and SDS against SPY. For each sampling rate in (1, 2, 3, ..., 25) I estimate beta for SSO and SDS; if sampling rate equals 25, then I use price observations that are 25 minutes apart. The estimates are presented in a graph. In the Appendix is a table of two logical variables that show when the confidence interval of beta estimates includes the leverage factor for each asset; this happens more often as sampling rate slows down.



The linear dependence of LETF and index returns holds on intraday time scales (within one day). The graph, Figure 1, shows that beta estimates for SSO and SDS are close to the leverage factor for all sampling rates considered. In line with Epps (1979), the fastest sampling rate has the weakest beta. It is reassuring to find that beta tends towards the leverage factor with real data, taken from a regular trading day.

Since the beta estimates are close to the leverage factor, I claim that both assets are well priced.

3 Conclusion

The appearance of linear dependence between asset returns is exciting. For the assets involved, there is potential for arbitrage. The beta coefficient of LETF is determined by issuers and monitored by traders. This paper shows that SPY, SSO and SDS are well priced assets. More generally, this method can show assets that are being poorly priced and guide trading of them.

For future work, I intend to explore the records of the National Best Bid and Offer (NBBO) for the same LETF in search of arbitrage. Increased attention should be paid to arbitrage, given the mandated linear dependence between asset returns. Also, the data I used is from a data provider and in the future I would like to make custom price series from transaction data.

Appendix

Sampling Rate	SSO, Contains 2	SDS, Contains -2
1	0	0
2	0	0
3	0	0
4	0	0
5	0	0
6	0	0
7	1	0
8	1	1
9	1	1
10	1	0
11	1	1
12	1	1
13	1	1
14	1	1
15	1	1
16	0	1
17	0	1
18	1	1
19	1	0
20	1	1
21	1	1
22	1	1
23	1	1
24	1	1
25	0	0

Table: Shows 1 if confidence interval for Beta estimate includes the asset leverage factor, shows 0 if not.

References

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