A First Stochastic General Framework to Model the Project Finance Cash Flows under Monopolistic Situations

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Abstract

The main aim of this work is to model the cash flows and cost dynamics for a Project Finance. Large scale capital-intensive projects usually require substantial investments up front and only generate revenues to cover their costs in the long term.

The abandonment flexibility affects each project independently. This is the only one that we consider in this study and it is quite different from the idea to abandon due to a common (specific) catastrophic event. This option is exercised under those situations of expected costs to completion higher than the expected cash flow, that is, during the investment period in the development phase. Including this flexibility in project finance is the same as valuing a project with an implicit American put option.

1. Modeling the Cash Flows Dynamics in a Project Finance

The asset for the company consists in a Project Finance (PF). The flexibility inherent in this project is captured by real option methodology. The life for the project is in average twenty years. Once the project is completed, the company sometimes enjoys of a monopolistic situation until the expiration of the contract[3]. Since this moment, the cash flow decreases until reach zero profits. Another possibility would be that the firm runs out the PF without completion.

We can identify three types of uncertainties in the project finance cash flows: the first one is about the investment cost required for the completion of the development project stage. The reason is a learning process that occurs while there is investment. We capture this one with a cost dynamic described in Pindyck (1993) and Schwartz (2004). The second uncertainty concerns about the possible free cash flows (FCF) that the company will obtain once have been finished the completion phase. For capturing this one, we will proposal three types of processes as Piñeiro and León (2004) recommended. The third uncertainty is described by catastrophic events that could lead the failure and abandon of the project. This one will be described by a Poisson process.

In all stages the possibility of abandon can be exercise if the completion expected cost is higher than the expected cash flow.

2. Continuous time model

The development stage for project finance is long and it has an associated costs that can vary depend on the development phase will take place. If the project finance overcomes every phase, it will ready to generate profits. In our model we implemented a different expected cost to completion in each phase for our empirical evaluation.

2.1 Cost Dynamic

We follow, in spirit, the modeling of cost uncertainty in irreversible investment projects described in Schwartz (2003) and León y Piñeiro (2004). The dynamics of the conditional expected remaining costs to completion are given by:

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[3] E.g. Infrastructure Projects.

$$dK_s(t) = -I_s dt + \sigma_s \sqrt{I_s K_s(t)} dw_s(t), \tau^*_{s-1} < t < \tau_s^* \tag{1}$$

where $K_s(t)$ is the expected real cost to complete the ongoing phase before starting the next phase. The interpretation for equation (1) is straightforward. As the firm continues investing in the R&D, the expected remaining cost to completion decreases. However, the firm also learns more about its ability to complete the project on time and on budget.

Prior to the beginning of Phase *i*, the firm expects that the total cost to complete the Phase *i* research to be $K_i(0)$. Negative shocks to the project development at this stage delays the Phase *i* completion and increase the total development cost for the phase, while positive shocks shorten development time and reduces the development cost.

The drift component in equation (1), which is the rate of investment I_j, is a control variable: the larger is the investment rate; the lower is the expected cost to completion. This means that the investment implies a "learning process" and the expected cost decreases only when there is investment. The uncertainty $dw_s(t)$ is called by Pindyck (1993) *"technical uncertainty"* and only can be solved by investing. Because the variance is linear in investment; there will be only two possible solution values for the control: invest zero or the maximum possible rate.

Remark that the stochastic process for the cost dynamic is a reasonable representation of uncertainty about expected cost in the project finance investment initial stage according to our methodology.

2.2 Free Cash Flow Dynamic

We implemented a cash flow model that consists in three parts: from the development project stage completion date until the threshold peak[4]; from the peak to the maturity of the project; and from the maturity to the time that the firm obtains zero profits.

2.2.1 Stage 1

For the first stage, we use a Brownian motion model given by:

[4] According to some papers, it is achieved in the 4 year.

$$dC_1(t) = \alpha^* C_1(t)dt + \phi C_1(t)dz_c^*(t) \tag{2}$$

where alpha is the risk-adjusted drift:

$$\alpha^* = \alpha - \eta \tag{3}$$

and η is the risk premium. ϕ is the volatility parameter and $dz_c^*(t)$ is the increment of a wiener process under the risk neutral measure.

2.2.2 Stage 2

For the second stage we propose three alternative cash flow models, that is, a Brownian motion, an Orstein-Uhlenbeck and a random walk. The key for introducing them is to avoid a possible overvaluation of the project (Bollen 1999, León y Piñeiro 2004).

2.2.3 Stage 3

In the third stage, we assume that the behavior of the cash flow will be decreasing since the monopolistic advantage shows time decay profile:

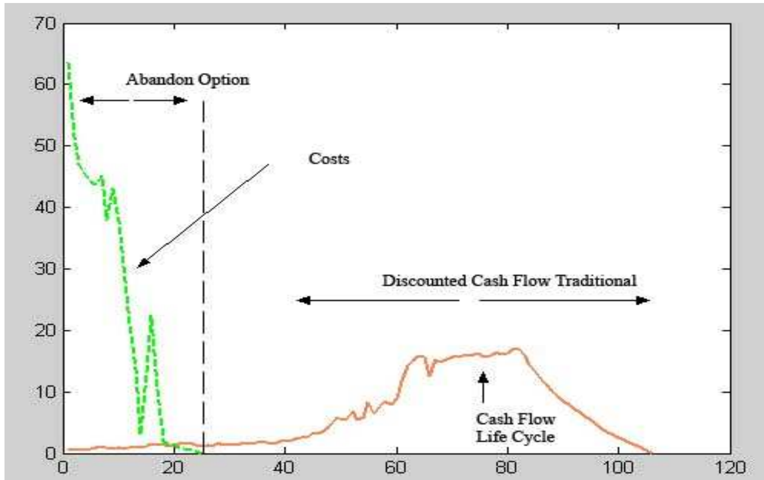
$$dC_2 = -\delta(t)C_1(T)dt, T \le t \le T^* \tag{4}$$

where T is the project finance maturity, $C_1(T)$ is the cash flow starting value for stage two and T^* is the time in that the profits are zero and $\delta(t)$ is the delta function with values decreasing from T to T^* (it captures the decreasing effect in profits):

$$\delta(t) = \begin{cases} \delta_1; t \in [T, t_1) \\ \ldots \\ \delta_m; t \in [t_{m-1}, T^*] \end{cases} \tag{5}$$

We also let correlation between $dw_i(t)$ and $dz_c^*(t)$ given by:

$$dw_i(t)dz_c^*(t) = \rho_{ci}dt \tag{6}$$



70
60
50
40
30
20
10
0
0
20
40
60
80
100
120
Abandon Option
Costs
Discounted Cash Flow Traditional
Cash Flow
Life Cycle

References

[1] Caperaa and Genest: A nonparametric estimation procedure for bivariate extreme value copulas. Biometrika, Sept. 1997, vol. 84.

[2] Chan Chiou: Multivariate Continuous Time Models through copulas (2004).

[3] Duffie, D. and K. Singleton (1999): “Simulating Correlated Defaults”, Working Paper, Graduate School of Business, Stanford University.

[4] Duffie, D. and K. Singleton (2000): “Simulating Correlated Defaults. A revision”, Working Paper, Graduate School of Business, Stanford University.

[5] Devroye, L.: Non uniform random variables generation.Mc Graw Hill. Chapter 10, pages 548-558.

[6] Embechts, McNeil and Straumann (1999): Correlation and dependence in risk management. Cambridge University Press.

[7] Embrechts, Frey and McNeil (2001): Quantitative Risk Management. Princeton University Press.

[8] Frees, E. W. and E. A. Valdez: Understanding relationship using copulas. North American Actuarial Journal. 2(1):1-25 (1998).

[9] Genest, Christian: Statistical inference procedure for Archimedean copula models. Journal of American Statistical Association. Vol. 88, September 1993.

[10] Kole, Eric: Testing copulas to model financial dependence. Erasmus University, Rotterdam.

[11] León and Piñero (2004): Valuation of a biotech company: A real option approach. Cemfi Working Paper 0420.

[12]Marshall and Olkin (1988): Families of Multivariate Distributions. Journal of the American Statistical Association.

[12] Nelsen (1999): An introduction to copulas. Lectures Notes in Statistics. Risk Conference. New York.

[13] Nolan, J. Stable Distributions: Models for heavy tailed Data. American University Working Paper (1994).

[14] Roncalli. Copulas in Finance: A reading guide and some applications. Groupe de Recherche Opérationnelle. Crédit Lyonnais. France.

[15] Shang Chan Chiou: Multivariate Continuous Time models through copulas. Berkeley Working Paper.

[16] Schwartz (2004): “Patents and R&D as Real Options” *Economic Notes* 33, pp. 23-54.

[17] Schombücher (2004): Modelling Dynamic portfolio credit risk. Department of Mathematics. Imperial College, London.

[18] Schombücher (2004): Simple and not so simple loan loss distribution. Bonn University working paper.

[18] Schömbucher: Correlated defaults. Conference on credit risk at Salomon center, NY, May 2004.

[19] Whelan, Ian: Quantitative Finance. Sampling from Archimedean Copulas (2003).

[220] Yu, Fan: Default correlations in reduced form models. California University working paper.