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Chan, Rosanna

University of Michigan (Ann Arbor), World Bank

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Financial Constraints, Working Capital and the Dynamic Behavior of the Firm

Rosanna Chan*

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Abstract

Financial constraints are widespread in developing countries, where even short term credit is limited. Finance held by firms as working capital is a substantial proportion of sales revenue, yet the role of working capital is largely neglected by existing models of financial constraints. I present a dynamic model of the firm that incorporates working capital by introducing a delay between factor payments and the receipt of revenue. In contrast with previous models, the working capital model predicts that firms under binding constraints will substitute between labor and capital in response to demand shocks, causing investment to be countercyclical. For firms near the margin of being constrained, constraints bind when positive production opportunities arise. Output growth is therefore constrained in response to positive shocks but not to negative shocks. Simulations suggest that models without working capital may understate the predicted effects of financial constraints on production efficiency, firm profit and growth over time. I test the predictions using the recently completed Bangladesh Panel Survey for manufacturing firms. Consistent with the theory, I find evidence that constraints bind when output price increases, that investment by constrained firms is countercyclical, and that output response to positive shocks is dampened for firms that are sometimes constrained. The results also are important for policy. In order to maximize growth, efforts to relieve credit constraints should be focused on periods when demand shocks are high.

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1 Introduction

Financial constraints are a prevailing problem facing firms in developing countries where capital is scarce and financial institutions are underdeveloped.¹ The World Bank Investment Climate Surveys, covering more than 26,000 firms across 53 developing countries, find that the cost and access to finance² is considered by firms to be among the top 5 problems they face (Hallward-Driemeier and Smith (2005)). The functioning of financial markets and the availability of credit affect the ability of firms to grow. They also influence the firms' incentives to hire labor and invest, which in turn affect economic growth and poverty reduction.

An often ignored mechanism by which financial constraints can affect the firm is working capital. Working capital is needed to cover costs of operations before revenue is received. For example, the farmer needs to purchase seeds and fertilizer before his crop is harvested, the garment maker must buy fabric and pay workers before delivering the clothing and the stall owner must pay for produce before it can be sold. The need for working capital thus arise from the difference in the timing of when costs are incurred and when revenue is received. In some instances, financial arrangements can help overcome the timing problem, either through prepayment of accounts receivable (i.e. online shopping) or delayed payment of current liabilities (i.e. trade credit³), however the majority of production requires cash to purchase inputs before goods or services are delivered.⁴

Working capital accounts for a substantial proportion of firms' financial needs, particularly in developing countries. Working capital is therefore likely to be an important avenue by which financial constraints can affect firm behavior. Table 1 presents the amount of working capital relative to sales revenue held on average by a sample of firms in the US and in Bangladesh within similar manufacturing industries in 2002. Working capital is measured as the firms' net short term liquid assets: current assets (inventories, accounts receivable⁵, cash and short term credit) minus current liabilities (accounts payable and any short term debt). On average, US firms hold approximately 22 percent of sales revenue as working capital while Bangladeshi firms hold on average 35 percent. Firms in Bangladesh rely more on non-cash working capital (mainly inventories) compared to US firms, which is consistent with less available credit. In Bangladesh, working capital is considerably greater than investment. The average cost of investment spending relative to sales is less than 5%.

Recent business cycle models of emerging economies have relied on working capital as a propagation mechanism to transmit interest rate shocks to real outcomes (see Neumeier and Perri (2005); Oviedo (2004)). The responses to interest rate shocks are magnified in these models because the need for working capital imposes additional borrowing requirements. In these models, the firm is assumed to always borrow the entire cost of production. Internally generated revenue is not considered as a source of finance. My model incorporates the option of internal finance. Accounting for the role of internal revenue is critical for understanding working capital, as the delay in revenue is the very mechanism that creates the need for working capital. Allowing for internally generated

¹Constraints to external finance may arise due to a number of factors: credit market imperfections, scarcity of financial resources, volatile environments or the lack of contract enforcement mechanisms. (Stiglitz & Weiss(1981)).

²Cost to finance refers to the interest rate charged for loans. Access to finance refers to the need for collateral and the availability of loans.

³An interesting body of literature looks at the role of trade credit in financial development. See Fisman and Love (2004); Fisman (2001); Fisman and Love (2003); Burkart and Ellingsen (2004)

⁴In the 2003-2005 Bangladesh Panel Survey of Manufacturing Firms, the median percent of sales paid at delivery is 100 percent.

⁵Accounts receivable is money owed to the firm.

finance is also important considering that, empirically, the largest source of financing is from internal finance. This is particularly true for firms in developing countries. Amongst a sample of Bangladeshi manufacturing firms, approximately 75 percent of the financing of new investments and 60 percent of additional working capital come from internal funds (shown in Figure 1).⁶

Accounting for working capital and internal finance has real economic implications when financial constraints exist.⁷ First of all, working capital directly affects the firm's decision making. A factory owner with limited cash must ration financial resources between purchases of different factor inputs at suboptimal levels. This alters the decision from one where finance is only needed for one factor. Second, working capital affects the firm's response to shocks when constrained. For example, if a credit constrained factory owner faces an increase in price for her output today, the urgency to increase output immediately to take advantage of the short-term profit opportunity will lead her to delay investment in order to purchase more production inputs. Third, working capital propagates the effects of financial constraints intertemporally through the accumulation of revenue. If poor firms cannot afford the inputs to produce at an optimal level, then revenue falls, limiting the ability to purchase inputs in the next period as well. As a result, financially constrained firms grow much more slowly and have lower expected profits. Not accounting for working capital understates the effects of financial constraints on the growth of the firm over time.

This paper has three goals. First, I extend the existing theory of firm behavior with financial constraints to allow delays in the receipt of revenue. This generates the need for working capital. Although a large body of literature has looked at the effects of financial constraints on the firm, previous models start with the assumption that the firm requires financing for physical capital only⁸, restricting *a priori* the effects to one factor of production and foregoing the possible allocation of finance between factors under financial constraints. This may be a reasonable starting assumption for firms in developed countries where short term credit is readily available but it is inappropriate for firms in developing countries where credit is scarce. Empirical studies have shown that firms facing financial constraints reallocate finances for working capital to smooth investment (Fazzari and Petersen (1993)). It is natural to ask what the reallocation of finances implies for production when funds are diverted away from short term purchases. In the model I develop, firms must choose between allocating cash for investment or for immediate production needs. Thus firms facing financial constraints need to trade off future production with present production in response to changes in production opportunities. Such substitution effects have been neglected by existing models.

The model produces an analytically tractable solution that characterizes the optimal constrained and unconstrained behavior of the firm. The results show that properly accounting for working capital and internal finance changes the predictions for firm behavior, especially those concerning the firm's response to demand shocks. Under financial constraints, the reallocation of financial resources between factors in response to shocks causes investment to be countercyclical. When current demand is high, constrained firms forgo investment to allocate scarce resources toward current production.

⁶In the US, Carpenter and Petersen (2002) has also found that small firms are constrained by internal finance.

⁷Although the need for working capital is observed in reality, the optimal behavior of the firm does not differ from a model without working capital when external credit is freely available to facilitate the intertemporal substitution required to solve the production timing problem. The only difference is that the cost of borrowing would enter additionally to the cost of purchasing inputs.

⁸See Bond and Van Reenen (2007); Hubbard (1998); Love (2003); Lorenzoni and Walentin (2007); Tybout (1983); Whited (1992); Bigsten et al. (2005); Bloom et al. (2006); Bond et al. (2003); Bratkowski et al. (2000); Bond and Meghir (1994)

When demand is low, firms produce less and have lower costs, relaxing the liquidity constraint and enabling them to allocate more resources to investment.

The model also describes the conditions under which firms would move between being constrained and unconstrained. Whether a firm is constrained depends on both its assets and on demand shocks. Financial constraints bind when firms wish to increase output but cannot finance a larger input bill. Firms may be unconstrained at moderate demand levels but become constrained when a higher than average demand shock occurs. As a result, output response to positive shocks is limited. Output response is not limited in response to negative shocks.

The key theoretical predictions of the working capital model are important as they imply that financial constraints limit output of constrained firms just when good production opportunities arise and cause constrained firms to disinvest just when investment should increase.

The second goal of this paper is to examine how financial constraints affect firm output, efficiency and growth over time when working capital is taken into account. I solve the model numerically and subject the model to simulated stochastic shocks over time to illustrate the extent to which financial constraints cause scale and production inefficiencies. Holding initial conditions and parameters constant across the working capital model and the standard investment model, simulations show that constrained firms on average produce 38 percent of optimal output versus the 60 percent predicted by a standard model.⁹ Labor to capital ratios are higher than optimal under constraints and the costs of generating a dollar of revenue are higher for constrained firms than for unconstrained firms. These two factors create a loss in producer surplus; numerical results show that the constrained firm achieves on average only 8 percent of possible optimal profits. As firms must rely on internal finance to grow, the reduced profits substantially slow the growth of the firm over time. The time to reach maturity (in terms of being able to produce optimally) is estimated to be around 3 times longer than that predicted by standard investment models under the same financial constraints¹⁰.

One of the empirical challenges in the literature has been to identify financially constrained firms. Many studies unsatisfactorily use endogenous firm characteristics such as size, outward orientation, or dividend payment as proxies to categorize affected firms (see Hubbard (1998) for review, Fazzari and Petersen (1993); Ganesh-Kumar et al. (2001)). My results suggest that firms' dynamic behavior can reveal whether they are credit constrained. Results also speak to the ongoing debate in the literature about whether investment cashflow sensitivity indicate financially constrained firms (Kaplan and Zingales (1997, 2000); Fazzari et al. (1988, 2000); Fazzari and Petersen (1993))

Third, I take the model's predictions to the data by testing when constraints are likely to bind, and how investment and output of Bangladeshi manufacturing firms respond to demand shocks under financial constraints. The Bangladesh Survey Panel contains unique survey questions that enable me to estimate demand shocks at the firm level. I find strong empirical support for the model's predictions. Constraints bind when firms experience positive price shocks. This is consistent with the working capital model and in contradiction with the competing thesis that firms become more constrained during downturns.¹¹ There is evidence that investment of financially constrained firms

⁹The exact magnitude of the difference between models depends on parameter choice.

¹⁰Recall that a period references the time from production to receipt of revenue, i.e. turnover time. This would differ from industry to industry and may range from 30 days to a quarter or longer. For example, in construction the appropriate time frame of a period would be close to a year. For food manufacturing, a period may reference a month or a couple of weeks. Regardless of the time frame, numerical simulation shows, under standard parameterization, the working capital model predicts a longer time to maturity and slower long run growth than the standard investment model.

¹¹Accelerator model, such as Bernanke and Gertler (1989); Bernanke et al. (1996), borrowing is dependent on

is countercyclical. The output response to price shock is different for firms that are unconstrained and firms that are sometimes constrained.

The next section presents the working capital model of the firm and theoretical results. Section 3 illustrates the implications of working capital and financial constraints on long term growth by simulating the model over time. Section 4 outlines the estimation strategy and presents empirical results. Section 5 concludes.

2 The Model

The introduction presented two key observations about firms in developing countries: working capital is an important component of financial requirements and internal finance is the primary source of finance. The model of the firm developed in this section captures both these components by introducing the demand for working capital due to a delay in the receipt of revenue. It is a partial equilibrium model designed to isolate the dynamic responses of the firm to output price or productivity shocks.

A representative firm seeks to maximize the present value of profits over an infinite horizon. The maximization problem is the following:

$$\begin{aligned}
 & \max_{L_t, K_t} E_t \left[\sum_{t=0}^{\infty} \beta^t (\beta P_t F(K_t, L_t) - w L_t - p^k I_t) \right] \\
 (1) \quad & s.t. \quad w L_t + p^k I_t + b_t = P_{t-1} F(K_{t-1}, L_{t-1}) + (1+r)b_{t-1} \\
 (2) \quad & s.t. \quad K_{t+1} = (1-\delta)K_t + I_t \\
 & s.t. \quad b_t \geq bc \\
 & \quad \quad \lim_{t \rightarrow \infty} b_t = 0 \\
 & \quad \quad K_0 \quad \text{given} \\
 & \quad \quad b_0 \quad \text{given}
 \end{aligned}$$

The setup of the firm's maximization problem follows the standard dynamic model of the firm except for the delay in revenues. Production requires two factor inputs: capital and labor. Capital is a durable factor that brings a future stream of benefits. It evolves according to Equation (2). The depreciation rate, δ , is assumed to be less than one and time invariant. Labor is a short-term variable input that is perfectly elastically supplied. It can also represent raw materials, energy or other adjustable inputs.¹² At each time period, the firm chooses inputs to maximize the stream of expected profits subject to the budget constraint given by Equation (1), where bond holdings are denoted by b . The discount factor β is assumed to equal $\frac{1}{1+r}$. The price of investment, p^k , the wage, w , and the interest rate r are exogenous and time invariant.

Working capital is introduced via a one period delay in the receipt of revenue. The firm's profit, $\beta P_t F(K_t, L_t) - w L_t - p^k I_t$, discounts the value of revenue by one period due to the delay. The budget constraint, Equation (1), includes the revenue from last period's production $P_{t-1} F(K_{t-1}, L_{t-1})$ and thus takes into account of internally generated funds. The borrowing constraint bc is introduced

network that decreases during downturns.

¹²There could also be other quasi-fixed inputs that share the characteristics of capital in the model.

as an exogenous parameter that can be any negative number including zero.¹³ Capital has no adjustment costs and requires no time to install.¹⁴ Capital stock can be re-sold at the prevailing market price. Thus, physical capital is assumed to be a liquid asset.¹⁵ Firms can transfer financial resources across time through bonds or capital assets.

The only stochastic variable in the model is the output price given by P_t , where $P_t = \bar{P} + \varepsilon_t$ and $\varepsilon \sim (0, \sigma_\varepsilon)$ and is i.i.d. This variable may be interpreted alternatively as a technology shock or any exogenous shock that changes the value of output. The firm knows with certainty the price it will receive before input decisions are made. One can think of the firm as receiving orders for its product and signing contracts that set the price it will receive upon delivery of the order.¹⁶ However, the firm faces uncertainty over the price in future years. Table 2 shows the order in which production is undertaken and when revenue is received. Cash in hand is defined as the sum of revenue and bond holdings, $X_t = P_{t-1}F(K_{t-1}, L_{t-1}) + (1+r)b_{t-1}$.

2.1 Solution

To solve the infinite horizon maximization problem, I reformulate the problem as a Bellman equation. The budget constraint may be written in terms of cash in hand, X , that yields the transition equation of wealth over time: $X_{t+1} = (1+r)[X_t - w_t L_t - p^k I_t] + P_t F(K_t, L_t)$ The cash in hand describes all of the financial resources available to the firm. The state variables are capital stock, K_{t-1} , and cash in hand, X_t . Control variables labor and capital are denoted as L_t and K_t . The associated Bellman equation is:

$$(3) \quad \begin{aligned} V(X, K_{-1}) = \max_{L, K} \quad & \beta P(\varepsilon)F(K, L) - wL - p^K(K - (1 - \delta)K_{-1}) + \beta EV(X', K) \\ \text{s.t.} \quad & X' = P(\varepsilon)F(K, L) + (1+r) [X - wL - p^k(K - (1 - \delta)K_{-1})] \\ \text{s.t.} \quad & X - wL - p^k(K - (1 - \delta)K_{-1}) \geq bc \end{aligned}$$

Denoting the multiplier in the borrowing constraint as v^b , the first order conditions are the following:

$$(4) \quad (\beta P F_L(K, L) - w) \left(1 + E \left[\frac{\partial V(x', P')}{\partial x'} \right] \right) = w v^b$$

$$(5) \quad (\beta P F_K(K, L) - p^k + \beta p^k (1 - \delta)) \left(1 + E \left[\frac{\partial V(x', P')}{\partial x'} \right] \right) = p^k v^b$$

Equations (4) and (5) show how the firm weighs the future value of cash $\left(1 + E \left[\frac{\partial V(x', P')}{\partial x'} \right] \right)$ against the shadow value of loosening the current period's borrowing constraint, v^b , in its choice of factors. The solution

¹³Alternatively, limits to borrowing may be modeled through the cost of borrowing. However, I have chosen to depict borrowing constraints as a set amount because there is a tendency for firms in developing countries to be more constrained by access than by the cost of finance. Lacking in sufficient collateral, firms can rarely borrow as much as they wish at prevailing market interest rates. The borrowing constraint or the interest rate could also be modeled as endogenous to net worth or business cycles (such as Bernanke and Gertler (1989); Bernanke et al. (1996)). Although these alternative approaches would affect the likelihood of *when* constraints become binding, they do not take away from the key prediction of the firm's behavior *under* constraints as long as the working capital assumption holds (i.e. the receipt of revenues are delayed)

¹⁴I explore a working capital model with one period time to build capital adjustment in another paper. The solution becomes forward looking -the choice of capital depends on the present returns to labor as well as the expected returns to labor and capital. When constraints are not binding, the model follows the same first order conditions as that of the Jorgenson model of investment. When constraints are binding, the behavior of the firm varies according to the magnitude of the shocks: encompassing precautionary savings behavior of cash during large negative shocks as well as the capital labor substitution as demonstrated by the model presented in this paper.

¹⁵This departs from the strict accounting definition of working capital (physical capital is not considered a current asset).

¹⁶If prices are not known, firms make all decisions based on expectations which are invariant to temporary shocks. Firms will only react to changes in their internal revenue or permanent changes in expectations.

can be simplified as follows: If v^b equals zero, the firm is not constrained and input choices are governed by optimal conditions. If v^b is nonzero, the firm is credit constrained in which case its behavior will be governed by constrained optimal conditions. We can then write the first-order conditions case-by-case:

Unconstrained:

$$(6) \quad \beta PF_L(K, L) = w$$

$$(7) \quad \beta PF_K(K, L) = p^k \cdot \frac{(r+\delta)}{(1+r)}$$

L^* and K^* is the solution to the firm's maximization problem if and only if:

Equation (6) and (7) hold and $x - wL^* - p^k(K^* - (1 - \delta)K_{-1}) > bc$.

Otherwise, the solution is given by Equations (8) and (9) below:

Constrained:

$$(8) \quad \frac{\beta PF_L(K, L)}{w} = \frac{\beta PF_K(K, L)}{p^k} + \frac{(1-\delta)}{(1+r)}$$

$$(9) \quad X = wL + p^k(K - (1 - \delta)K_{-1}) + bc$$

Under non binding constraints, $X - wL^* + p^k(K^* - (1 - \delta)K_{-1}) \geq bc$, the shadow value v^b is equal to zero. The amount of cash in hand is irrelevant to the unconstrained optimal decision of the firm. Labor and capital are chosen such that the marginal product is equated to marginal cost as defined by Equations (6) and (7).

Under binding constraints, the firm cannot achieve optimal production and instead reach a constrained optimum. The firm needs to consider the expected benefits of cash the next period, $\left(1 + E \left[\frac{\partial V(x', P')}{\partial x'} \right] \right)$, along with the cost of binding constraints today, v^b , when making factor input choices. As both the future benefit of cash and the present shadow value of cash enter the two first order conditions (Equation (4) and (5)), the ratio of the two conditions yields:

$$\frac{(\beta PF_L(K, L) - w)}{w} = \frac{(\beta PF_K(K, L) - p^k + \beta p^k(1 - \delta))}{p^k}$$

This ratio simplifies to Equation (8) above.

Note that even though the firm is optimizing dynamically, the forward looking terms cancel out and current actions can be described independently of expectations, which yields an analytically tractable solution. Under constraints, the firm need only compare the present opportunity cost of funds, w and p^k , and the relative returns, $(\beta PF_L(K, L) - w)$ and $(\beta PF_K(K, L) - p^k + \beta p^k(1 - \delta))$ between the two factors. The relative returns, that are the factor returns net of cost, are greater the further away factors are from optimal levels. The second first order condition is the binding cash constraint (Equation (9)).

The model's solution is unique as it provides the first order conditions for optimal *constrained* behavior of the firm. The solution is simplified as the return from production for constrained firms is strictly greater than the return from saving the money; so that by maximizing current profits, the firm is also maximizing future profits. (Constrained firms produce below optimal where marginal returns are higher than marginal cost). Not only is the return high for both inputs, but capital can also be sold and transformed into cash the next period.

2.2 Theoretical Predictions

One feature of the working capital model is that finance is needed for more than one factor of production. Under constraints, the firm is forced to choose between factors in its allocation of scarce cash, leading to countercyclical capital behavior. The substitution between factors is driven by binding constraints and changes in output price and not by changes in relative factor prices. These dynamics are unique to the working capital model and are not accounted for by standard investment models with financial constraints.¹⁷ The working capital model also ties output demand to the demand for finances by the firm. As such, whether financial constraints are binding depends on the level of output demand. For firms near the margin of being constrained, firms have sufficient resources to finance a limited range of price realizations but not for realizations beyond their resources. Thus, constraints are more likely to bind when output price increases. Output response to shocks is therefore differentiated between increasing and decreasing price shocks as firms move into and out of constrained states.

The firm's choice of labor and capital is entirely described by the set of Equations (6), (7), (8) and (9) given initial state variables. The solution implies that the growth of the firm is characterized by three phases: 1) *Always Constrained Phase*: at very low levels of cash and capital stock, the firm will always be constrained regardless of the price; firm behavior is defined by the Constrained FOCS; 2) *Sometimes Constrained Phase*: at medium levels of cash and capital stock, the firm is unconstrained when price is low but may become constrained when price is high; firm behavior is governed by the Unconstrained FOCs for a low range of prices and then switches to the Constrained FOCS when credit constraints become binding; and 3) *Never Constrained Phase*: at high levels of cash and capital stock the firm is never constrained regardless of the price shock; firm behavior always follows the Unconstrained FOCS. I derive two testable theoretical predictions from the working capital model that distinguishes it from other models.

Capital Countercyclicality Under Always Constrained Phase

The first proposition is that capital responds to shocks countercyclically when constraints are binding. That is, positive price shocks are associated with a decrease in capital. However, the opposite is true when financial constraints binds (from inspection of the optimal first order conditions). The formal proof of the result is as follows:

Proposition: Given constraints are binding, $x < wL^* + p^k(K^* - (1 - \delta)K_0) - bc$, where L^* and K^* is the solution to Equations 6 and 7: the change in capital due to a change in price will be negative $\frac{dK}{dP} < 0$.

Proof: Fully differentiating Equations (8) and (9) with respect to the two choice variables, L and K , and the parameter of interest P and X yields:

$$(10) \quad \frac{P}{w} [F_{LL}dL + F_{LK}dK] + \frac{F_L}{w}dP = \frac{P}{p^k} [F_{KL}dL + F_{KK}dK] + \frac{F_K}{p^k}dP$$

$$(11) \quad dX = w dL + p^k dK$$

Combine the two Equations (10) and (11) by substituting out dL yields the following:

¹⁷Appendix 2 elaborates on the contrast between the working capital model and the standard model.

$$(12) \quad \left(-F_{LL} \frac{p^k}{w} + F_{LK} + F_{KL} - F_{KK} \frac{w}{p^k} \right) dK = \left(\frac{F_{KL}}{p^k} - \frac{F_{LL}}{w} \right) dX + \left(\frac{w}{p^k} F_K - F_L \right) \frac{dP}{P}$$

$$(13) \quad \frac{dK}{dP} = \frac{\left(\frac{w}{p^k} F_K - F_L \right)}{P \left(-F_{LL} \frac{p^k}{w} + F_{LK} + F_{KL} - F_{KK} \frac{w}{p^k} \right)}$$

Equation (12) states that the total change in capital is decomposed into the change in cash, X , and the change in price, P . The change in capital due to the change in price can be expressed by Equation (13) as cash is predetermined and does not change due to price, $\frac{dX}{dP} = 0$. The numerator is negative as Equation 8 rearranged is: $\frac{w}{p^k} F_K - F_L = -\frac{w(1-\delta)}{P}$. The denominator is positive as $F_{LL} < 0$, $F_{KK} < 0$ and $F_{KL} > 0$. Thus $\frac{dK}{dP} < 0$.

Capital behaves countercyclically under binding constraints due to two mechanisms. First, the difference between capital and labor as durable and non durable factors of production implies that the factors contribute differently to the next period's assets given by $PF(K, L) + (1 - \delta)K$. Labor only contributes to the value of production whereas capital contributes to production *and* retains value after production for future use. A change in price alters the value of production but not the accumulated value. As a result, a current period price shock will affect the marginal rate of value substitution between labor and capital.¹⁸ Secondly, a binding cash constraint forces the firm to choose between the two factors and thus consider the marginal rate of value substitution.¹⁹

Figure 3 illustrates the changes to factor demand due to an increase in the price in LK space. Isovalue curves, like isoquants, depict the labor and capital combinations for the value that the factors generate, where value is defined as the sum of revenue and the depreciated value of capital: $PF(K, L) + (1 - \delta)K$. The budget line is the cash constraint (Equation 9). The firm begins initially at point A where isovalue curve V1 is tangent to the budget line. An increase in the price enables the firm to produce the same output with relatively less labor than capital - the isovalue curve V2 becomes flatter. The new tangency point occurs at B and to the left of A where isovalue curve V3 lies tangent to the budget line.²⁰

The reverse happens in response to a negative price shock. A decrease in demand decreases the marginal value of labor relative to the marginal value of capital. Firms do not adjust capital at the same rate as labor because the value of depreciated capital has not changed. The total change in capital, Equation (12), can be decomposed into an income effect, from the change in X , and a substitution effect from the stochastic changes in P . If the borrowing constraint was modeled as a function of net worth or the price, this will show up in Equation (12) as an additional term, i.e $dK = \Lambda dX + \Phi dP + \Omega dbc$. It follows that changes to the borrowing constraint act like an income

¹⁸Equation 8 can be rewritten as: $\frac{F_L}{F_K + \frac{(1-\delta)}{P} p^k} = \frac{w}{p^k}$. From inspection, a change in P changes the marginal rate of value substitution.

¹⁹The ratio of the marginal rate of value substitution holds also for firms at the optimal. Unconstrained firms do not exhibit countercyclical investment behavior because they are able to increase both labor and capital in response to positive shocks due to non binding constraints. There, capital increases less relative to labor in accordance with the marginal rate of value substitution. To show that unconstrained firms also experience the same marginal rate of value substitution, the unconstrained optimal FOCs can be re- expressed as Equation (8). The capital FOC $\beta PF_K(K, L) = p^k \cdot \frac{(r+\delta)}{(1+r)}$ is the simplified version of $\beta PF_K(K, L) + \beta p^k(1 - \delta) = p^k$ where $p^k(r + \delta) = p^k(1 + r) - p^k(1 - \delta)$. Dividing labor FOC and the non simplified capital FOC will yield the result as Equation (8).

²⁰This is holding cash in hand constant. Price changes are over time, and cash is invariant to price but varies with time. A change in cash would push the budget line out and the tangency point will expand likewise.

effect that can accentuate or dampen the fundamental response to demand.²¹

Asymmetric Output Response to Shocks

The response to price shocks described earlier is illustrated by Figure 2, which is calculated assuming a Cobb-Douglas production function, a borrowing allowance of zero and a specific set of parameters.²² The first panel shows the behavior of a firm that has very low cash and capital and is always constrained. The last panel shows unconstrained optimal behavior. The middle panel shows the combination of the two when the firm switches from optimal to constrained behavior with increasing magnitudes of the price shock. In this example, the *Sometimes Constrained* firm has enough cash to afford optimal inputs at the mean price level equal to one. The firm is not constrained for shocks below the mean and becomes constrained for shocks above the mean. First, note in Figure 2, that as prove above, capital reacts countercyclically when firms are always constrained, but procyclically when never constrained.

The second proposition is that constraints bind with increasing price and as a result, output response to positive will be different from negative changes in the price. The asymmetry captured by the middle panel of Figure 2 is driven by constraints binding when the firm wants to expand production and not binding when the firm contracts. The left and right panels of Figure 2 clearly show that when firms are credit constrained, output is much less responsive to prices. Figure 4 illustrates the output response to a positive shock using iso-value curves and budget lines. Take two firms that are both producing optimally at point A in Figure 4. One firm is never financially constrained - it has ample internal finance or access to external credit. The firm uses the optimal amount of capital and labor to determine output. The other firm is on the margin of being financially constrained.²³ A positive shock shifts the iso-value curve outward and beyond the budget set of the credit constrained firm. While the firm without constraints can increase output to point NFC, the constrained firm can only increase as far as point FC. Thus, under positive shocks, the output of the credit constrained firm responds less than that of the unconstrained firm.

3 Loss in Producer Surplus, Inefficiency and Firm Growth

Financial constraints cause suboptimal input levels and distort the efficient relative factor ratio in response to shocks. Both of these effects contribute to lower output levels, leading to losses in producer surplus. As future production is dependent on revenue, the loss in profits in turn affects firm growth over time. The effects of financial constraints are magnified as working capital constraints become binding exactly when good production opportunities arise. I simulate the working capital model and the standard investment model using Matlab to illustrate the effects of financial constraints on output levels, efficiency, profits and long term firm value and growth.

²¹For example, if the borrowing allowance increases with price, this is equivalent to an increase in cash - which is a change in income.

²²Here, the figure is intended to motivate the theoretical results. The choice of parameters is discussed in more detail in Section 3 when the model is simulated to examine output inefficiencies and the growth of the firm over time.

²³The firm is limited in the sense that it can only just afford to produce optimally at point A where the shock level is equal to one. For any shock greater than one, the firm will be constrained, just as in the middle panel of Figure 2.

The policy function is given by the first order conditions, Equations (6), (7), (8) and (9). A Cobb-Douglas production function $F(K_t, L_t) = K_t^\alpha L_t^\gamma$ is used where $\alpha + \gamma < 1$ to ensure a stationary solution. Parameters are set as follows: return on capital $\alpha = 0.30$, return on labor $\gamma = 0.60$, time discount factor $\beta = 0.9$ ²⁴, rate of capital depreciation $\delta = 0.10$, standard deviation of the log price $\sigma = 0.1$, real interest rate $r = \frac{1}{\beta} - 1$, gross interest rate $R = \frac{1}{\beta}$, price of capital $p^k = 1$ and wage $w = 0.25$. The transitory shock is assumed to be lognormally distributed with a mean of one, that is $\ln P_{t+1} \sim N(-\frac{1}{2}\sigma^2, \sigma^2)$ where $E(\ln P_{t+1}) = -\frac{1}{2}\sigma^2$. This implies $E[P_{t+1}] = 1$. To generate average statistics, I simulate the model over 40 time periods with 1000 different simulated paths and 21 different values of the borrowing constraint bc (from 0 to 2000). This generates a total of 840000 observations for each model.

A first order effect of financial constraints is that output levels are restricted, $Q_c < Q^*$. I calculate the ratio between constrained outcomes and the optimal level (in accordance with the simulated path of shocks) to illustrate the average loss due to financial constraints. Table 3 shows that under the working capital model, when firms are in the *Always Constrained* phase they produce on average²⁵ only 38% of optimal output. During the *Sometimes Constrained* phase, they produce only 87%. In contrast, the standard model with constraints predicts that output will be 60% of the optimal level. Suboptimal output levels lead to lower profit levels. Under the working capital model with binding constraints, profits are only 8% of the optimal level. This is about half of the 15 percent predicted by the standard model. These results suggests that the opportunity cost of producing suboptimally due to financial constraints is much greater when working capital is taken into account.

The loss in producer surplus²⁶ due to suboptimal production is illustrated in Figure 5. The output of the 3 phases from Figure 2 is re-plotted with price on the y-axis and quantity on the x-axis to show the difference in supply between phases. Given price, P , the loss in producer surplus is defined by the triangular shaped area bounded by constrained supply, unconstrained supply and price. This loss is attributed to the firm producing at output level which are not profit maximizing, or analogously, not cost minimizing. Financial constraints restrict firms from producing at minimum cost and inefficiencies arise due to producing below scale and additionally, due to factor composition. Table 4 summarizes the cost per dollar of revenue and the labor to capital ratios. Compared to the unconstrained case, the cost of producing one dollar of revenue is around 15 percent higher under binding constraints, and 10 percent under sometimes constrained for the working capital model. Also, the labor to capital ratio is 20 percent higher than the unconstrained optimal ratio. Note that these ratios are not directly comparable to the standard capital adjustment model as the firm's optimization problem differs in capital choice.²⁷ These results illustrate that financial constraints causes inefficiencies due to non cost minimizing input levels and distorted factor ratios.

²⁴Cooper Prescott and Miles assume $\beta \approx 0.96$

²⁵Averages are taken over all values of the borrowing parameter.

²⁶There is also a loss in consumer surplus due to suboptimal supply. I emphasize producer surplus because I model the behavior of firms.

²⁷Under the standard model, financial constraints effect only capital accumulation (and labor is always at optimal relative to capital stock and shocks). Constrained firms invest more relative to unconstrained firms as capital is below steady state. This causes the cost per revenue dollar under constraints for the standard model to be higher than that of the working capital model. Under the working capital model, both capital and labor are below the unconstrained optimal and therefore the firm cannot devote resources to accumulate capital. The differences in the optimization problem between the two models is also reflect in the labor to capital ratios. For the standard model firms under constraints have a smaller labor to capital ratio than unconstrained firms, consistent with capital accumulation below steady state. For the working capital model, labor to capital ratios are greater under constraints than unconstrained, consistent with firms turning to labor and stalling investment to increase production under constraints.

The losses in output and profits informs us about the static losses caused by financial constraints. Dynamically, the loss in profits persists over time as output determines revenue, which in turn affects production possibilities the next period. As a result, the growth of the firm is hampered and the time to maturity becomes extended. Here, maturity is defined as a state where the firm is able to produce at optimal scale in response to shocks. The longer the firm remains constrained, the greater the losses in the long term value of the firm.

The growth path of the firm is simulated starting at the same initial conditions with no credit available and is illustrated in Figure 6. The standard capital adjustment model growth path, the far left line, is much steeper than that of the working capital model, the dotted line. As expected, the working capital model predicts a much slower long run growth path than the standard model. In the same figure, the effect of a positive and a negative shock of two standard deviations introduced at period ten on the growth path of the working capital model is shown. A positive shock puts the firm on a higher growth path and a negative shock puts the firm on a lower path resulting in a longer time to maturity.²⁸ This suggests that under financial constraints, the growth path of the firm will be much more variable in a stochastic environment.

Another indicator for the long run growth rate of the firm is the time to maturity. The longer the time taken, the slower the rate of growth. Maturity for the standard model can be easily identified as the steady state capital level. After reaching this level, the firm can fully respond to any transitory shocks.²⁹ With the working capital model, the firm’s ability to produce at optimal scale depends on the magnitude of the demand shock. I define maturity as reaching a threshold cash level that allows production at optimal scale 90% of the time. That is, to be considered ‘matured’ the firm does not need to have enough cash to meet high demand shocks with only have a 10% probability of occurring.³⁰ This measure recognizes the fact that financial constraints affect the response to stochastic shocks and not just the static level of output or capital stock.

The average time to maturity from the simulated data is shown in Table 5. The standard model predicts that on average, the firm matures in 2.35 periods (which includes the 1 period required for capital to install). Under the same conditions, the working capital model predicts that on average maturity takes 8.38 periods (which includes the 1 period delay in the receipt of revenue). The variation for the time to maturity is much greater under the working capital model - the standard deviation is 6.43 versus 2.80 periods for the standard model. This may be attributable to the sensitivity of the firm’s growth path to shocks, as noted earlier and as seen in Figure 6. These numbers suggest that the standard model may seriously understate the effects of financial constraints on firm growth. For example, if the periods were defined as quarters, the standard model predict maturity at 6 months while the working capital model predicts maturity in 2 years.

The delay to maturity is largely attributed to the time spent in the *Sometimes Constrained* phase. Figure 7 illustrates the time to maturity as function of the amount the firm can borrow (the borrowing allowance). The difference between total time to maturity and the time to leave the *Always Constrained* phase equals the time the firm is in the *Sometimes Constrained* phase. For the standard capital adjustment model, the firm is constrained every period right up to reaching steady

²⁸The change in the entire growth path due to a shock also suggests that working capital may also act a propagation mechanism for shocks. Shocks are carried for at least one period past the time the shock occurs as revenue is received one period later.

²⁹This is as investment is invariant to transitory shocks and labor is self financing.

³⁰Cash threshold calculated as $X_{threshold} = wL(A, K) |_{A=1.1274, K=\bar{K}} + p^k(K(A, K) |_{A=1.1274, K=\bar{K}} - (1 - \delta)\bar{K})$ noting that unconstrained labor and capital is not a function of cash.

state.³¹ For the working capital model, the continuously bounded stage is shorter (the average time is 2.22 periods) but firms are still vulnerable to restricting constraints on average for another 6 periods. This contributes to the prolonged effects of financial constraints on firm growth.

Long term value is the sum of profits across time and Figure 8 shows the median bands of the ratio between constrained long term value and the non constrained long term value for both the working capital model and the standard model, across different borrowing allowances. At a borrowing allowance of zero, constrained firms have only 63% of the value of non constrained firms. Even though constrained firms eventually catch up in terms of capital stock and ability to respond to positive shocks, they can never catch up to the long term value of unconstrained firms of the same age. Dynamically, financial constraints have a permanent effect on firm value in the long run.

To summarize, financial constraints restrict optimal output which results in loss of producer surplus. Inefficiencies arise not just from producing below scale (shown by cost per dollar of revenue) but also from distortions to factor composition (shown by labor to capital ratios). Financial constraints have a persisting effect over time. The value of the firm is inevitably lower as the constraints inhibit it from making the most out of profitable production opportunities. Furthermore, the predicted effects of financial constraints are much more severe in the model with working capital than in the standard model.

4 Empirical Analysis

The predicted behavior of firms under financial constraints is different in the working capital model than in the standard model. The effects of financial constraints on output levels, efficiency and firm growth are much more severe in the working capital model. I test the predictions unique to the working capital model using firms level panel data from Bangladesh. First, the need to finance working capital implies that firms are more likely to become financially constrained when demand shocks increase. Second, under binding constraints, investment responds countercyclically to demand shocks. Third, the timing of when constraints bind imply that output response to positive demand shocks will differ from negative demand shocks for firms near the margin of being constrained. If these predictions are consistent with what we see in the data, this lends support for the working capital model of financial constraints and offers suggestive evidence for the simulated results of the model.

4.1 The Bangladesh Panel Survey

Firms in Bangladesh have very little access to external finance. The country's investment climate is considered one of the worst amongst all the developing countries.³² Foreign inflows are minimal due to investor's concerns about political instability and high levels of corruption. Domestic private investment is also low, partly due to the dominance of state owned enterprises³³ but largely due to the underdeveloped banking sector in Bangladesh. There are very few private banks.³⁴ The private

³¹In the absence of permanent shocks

³²Bangladesh has consistently been classified as an under performer in attracting foreign direct investment by the UN Conference of Trade and Development. The Inward FDI Performance Index 2002-04 ranks Bangladesh 122 out of 140 countries. (Country Report, EIU 2005)

³³The public sector owns approximately 40% of Bangladesh manufacturing and utility assets.

³⁴There are approximately 30 private commercial banks, 10 foreign banks and 5 development financial institutions.

banks offer more competitive interest rates than the four state owned banks that dominate the financial sector but they suffer from capital inadequacy and insider trading. Foreign bank activities are usually restricted to offshore and foreign trade business. Stock markets are still in a stage of infancy - the Dhaka Stock Exchange and the Chittagong Stock Exchange opened only in 1995. The market capitalization of the stock exchanges relative to GDP in 2004 was only 4.2%. It is clear that financial constraints are particularly salient for Bangladeshi firms.

The Bangladesh Panel Survey, part of the group of Enterprise Surveys, is conducted by the World Bank and is unique in that it is a panel data set taken semi-annually over the years 2003 to 2005.³⁵ There are 259 privately owned firms in the panel representing six different manufacturing sectors. Surprisingly few firms drop out of the survey. There are 241 firms that are present in all 6 periods. Firms were sampled from the two major cities, Dhaka and Chittagong, and are representative of the industrial composition of the Bangladesh economy. More than half the sample is in either the Garment or Textile industry, 28% and 26% respectively. The rest of the sample is distributed in Food (15%), Leather (12%), Electronics (9%) and Chemicals (10%). There is substantial variation in firm size. The interquartile range is 264 employees with the median at 150. About 37% of firms have fewer than 100 employees, and 85% have fewer than 500 employees. There is a tendency for firm surveys in developing countries to over sample larger firms, which may not be representative of the microenterprises that often characterize developing economies.

The panel contains several indicators for financial constraints. My first measure of financial constraint is the manager's subjective assessment of whether access to financing is a problem. They report on a scale from 0 to 4 with 1 corresponding to a minor problem and 4 corresponding to a severe problem.³⁶ I define the dummy $acc = 1$ if the firm described access to finance as moderate to severe problem.³⁷ The second measure uses the composition of the sources of finance. As shown in Figure 1, firms report the share of finance from each of 14 different sources. I define a second financial constraint dummy, $internalF = 1$ if 100% of financing comes from internal funds and the firm reported at least some problem with access to financing.³⁸ A firm that finances operations entirely from internal funds is one that does not utilize external credit. Under the model's framework, this implies either the firm is matured and does not need external finance (and these firms are not considered financially constrained by $internalF$), or the firm is financially constrained. I also include indicators typically used in the literature such as age and size of the firm (Cooley and Quadrini (2001); Cabral and Mata (2003)). The dummy $age5 = 1$ if the firm is 5 years old or less and $size100 = 1$ if the number of employees is 100 or fewer. The measures that utilize financial composition and subjective assessment indicate that a substantial proportion of the firms are financially constrained, consistent with the poor investment climate of Bangladesh; 67% are constrained according to the internal finance indicator and 44% according to the access to finance indicator. The proportion of firms constrained according to indicators $age5$ and $size100$ are considerably lower, 15% and 38% respectively. This suggests that age and size may not adequately identify financially constrained firms, or perhaps these measures are less appropriate for developing countries. There is considerable

³⁵Enterprise Surveys were previously called the Investment Climate Surveys (ICS). The Bangladesh Panel Survey was carried out by the World Bank in conjunction with the South Asia Enterprise Development Facility and the Bangladesh Enterprise Institute.

³⁶The access to finance question is asked only in years 2004-2005. Missing values were supplemented with predicted values from age, size and bank loan variables. The prediction matched 76% of the actual values.

³⁷Moderate to severe problem corresponds to responses of 2, 3 or 4.

³⁸Internal finance is available for all years but the response rate dropped to approximately 50% in 2004. Missing values were supplemented with predicted values from access. The predicted values matched 60% of the actual values.

overlap between the indicators, particularly between the first two indicators. (See Table 6.) Internal finance and access to finance indicators exhibit movement by firms into and out of being financially constrained whereas the age and size indicators do not. (Transitional probabilities are shown in Table 7.)

Firms report percentage price changes for output price and raw material prices. The price index is constructed by setting base year price equal to 10 and adding subsequent percentage price changes.³⁹ Considering that more than half of the firms in the sample trade (either through exporting, importing or both), I assume that firms are price takers. Under this assumption, price changes reflect demand shocks.

Summary statistics of key outcome variables are given in Table 8. Constrained firms have lower output, capital stock, labor, investment and output growth than unconstrained firms and the difference is statistically significant. While there are differences between outcome variables, there are no significant difference in price variables. Table 9 shows there is no significant difference between the two groups in changes in output price, raw material price or wage but there is a small difference in the change in interest rate. Looking at firm characteristics (see Table 10), constrained firms have fewer workers which is consistent with expectations. Contrary to expectation, there is no significant difference in age. This may reflect the weaker correlation between age and freedom from financial constraint in developing countries. The distribution over sectors is similar between constrained and unconstrained firms.

4.2 When Do Constraints Bind?

The working capital model predicts that constraints are more likely to bind when price shocks increase than when shocks decrease, as shown by the middle panel of Figure 2. This prediction is driven by the need for more working capital during high demand and by the assumption that credit available to the firm is invariant to price shocks. If credit availability changes with price, as is assumed in models where borrowing is dependent on net worth, constraints will be less likely to bind when output price increases and more likely to bind when price decreases. Empirically testing when constraints bind is important not only to test a key implication of the working capital model but also to test the borrowing assumption, a point about which there is no consensus in the literature. Furthermore, understanding when constraints bind would help policy makers identify when alleviating financial constraints is most crucial.

According to the model, the state of being constrained should depend on wealth, the state variables capital and cash, the price shock and other parameters. The empirical specification is as follows:

$$CnstrIN_{it} = \lambda_0 + \lambda_1 \Delta shock_{it} + \beta_1 \ln K_{i,t-1} + \beta_2 \ln R_{i,t-1} + \beta_3 \Delta X_{it} + \varphi time + \varepsilon_{it}$$

The dependent variable, $CnstrIN = 1$ if the firm was not constrained in the previous period and is constrained in the current period. Internal finance and access to finance were used in the analysis as the constraint indicators, because age and size never switch from unconstrained to constrained. Controls for firm characteristics are: sectors, log age, change in log wage and change in interest

³⁹Although the survey asks for the firm to report the top 3 output (or raw material) price changes, very few reported the second or third top output price. I use the top output price to proxy for firm level price changes.

rates averaged over sector and time. Change from firm specific mean of log output price is used as the change in shock. The expected sign for the coefficient λ_1 for the response to price changes is positive.

Empirical results are shown in Table 11. Across all specifications, the estimates for λ_1 are positive and significant at the 5% level. As predicted by the working capital model, and contrary to conventional wisdom, positive price shocks are associated with movements into a constrained state. In other words, constraints bind precisely when good opportunities arise.

4.3 Investment Response to Shocks

The model's prediction is that when financial constraints are binding, investment reacts countercyclically to demand shocks. Motivated by the solution to the firm's dynamic problem that states that the firm's choice of inputs depends only on the state variables, the stochastic factor, and the exogenous parameters, I estimate the following specification:

$$\begin{aligned} \Delta \ln K_{it} = & \alpha_0 + \alpha_1 cnstr + \alpha_2 \Delta shock_{it} + \alpha_3 cnstr * \Delta shock_{it} \\ & + \beta_1 \ln K_{it-d} + \beta_2 cnstr * \ln R_{it-d} + \beta_3 \ln R_{it-d} + \beta_4 \Delta X_{it} + \varphi time + \varepsilon_{it} \end{aligned}$$

The dependent variable is the change in log capital stock. Initial capital stock, reported present value of machinery, equipment, land, buildings and leasehold improvement, comes from the 2002 Investment Climate Assessment (ICA) survey.⁴⁰ Capital investment in each period is the net spending⁴¹ on additional machinery, equipment, vehicles, land and buildings. The lagged log value of capital and revenue, R , are used for the state variables, capital stock and cash in hand respectively. Revenue is interacted with the constraint variable, $cnstr$, as cash should only matter when the firm is constrained. The stochastic variable is the change in log output price, $shock$. I also control for industrial sector, lagged firm size, change in log wages, sector- and time-specific interest rates, and a linear time trend.

The model predicts that investment is countercyclical when the firm is in the always financially constrained phase. I use two different methods to measure binding constraints: First, I define the firm as consecutively constrained if the firm is financially constrained in periods t and $t - 1$, and second, I define the firm as consistently constrained if it is constrained for all periods the indicator is observed. The consecutively constrained indicator is firm- and time- variant whereas the consistently constrained indicator varies across firms only.

The empirical test is to see if the investment of constrained firms responds negatively to a price shock while the investment of unconstrained firms responds positively. That is, the coefficient on the interaction between the financial constraint indicator and the price shock, α_3 , should be negative and greater than the coefficient for the price shock, α_2 . In addition, α_2 should be greater than zero. Results are shown in Table 12 for both semi-annual and annual changes in capital stock. I find that

⁴⁰The 2003-2005 panel is linked to the 2002 Investment Climate Assessment (ICA) survey. The ICA surveyed 1000 firms and has a more comprehensive and detailed survey questionnaire. The panel survey follows up firms interviewed in the 2002 ICA and firms are matched according to identification numbers. Comparing establishment years reported 2002 ICA and 2003 wave of the panel suggests that there may be matching errors. Approximately 40% of the firms have discrepancies in the establishment year. There are 52 firms with a discrepancy of over 5 years. These firms were left out of the sample in empirical analysis.

⁴¹Net investment is the additional spending minus sales of additional machinery, equipment, vehicles, land and buildings.

investment of unconstrained firms responds positively to an increase in output price. A 10 percent increase in the output price leads to between 15 to 30 percent increase in capital semi-annually or 20 to 35 percent annually. This is significant at the 5 percent level across all specifications using different indicators for financial constraints. The coefficient α_3 is negative across all specification, consistent with the model.

Of the 6 different indicators of financially constrained, only two indicators consistently show significant difference in response to output shocks between constrained and unconstrained firms both in semi-annual changes and annual changes: the Internal Finance ‘Consistently constrained for all periods’ and *Age5*, the indicator for when the age of the firm is less than 5 years old . These are shown in estimates (2), (6), (8) and (12). The model’s predictions for countercyclicality are during the firm’s ‘Always Financially Constrained’ phase. As such, it makes sense that this phase maybe better captured when a) the firm is consistently only financing through internal funds and b) the firm is very young. Interestingly, this also suggests that Access to Finance, as a qualitative assessment by the manager, and the size of the firm are not very good indicators for firms that are extremely financially constrained.

Of the estimates (2), (6), (8) and (12), in all estimates except for estimate (8), investment is significantly countercyclical when the firm is constrained. Estimate (2) and (6) suggest that a 10 percent increase in the output price leads to a decrease in semi-annual investment of around 0.3 and 0.4 percent respectively. Estimate (8) shows that on an annual level, firms that are internally financed would only increase investment by 5 percent, compared to unconstrained firms that respond by an increase of 35 percent. However, the strongest result is in estimate (12) where the estimated coefficient suggests that a 10 percent increase in the output price leads to a decrease in annual investment of around 11 percent. The difference in the result between estimate (8) and (12) may be due to the internal finance indicator capturing firms that may be less financially constrained than those captured by age. This may also explain the difference in the size of the estimated coefficients in the semi-annual regressions (2) and (6) where the effect is larger using age.

There are at least two reasons why evidence of investment countercyclicality is not found in all of the regressions. First, disinvestment may not occur if secondary markets for machinery, equipment and vehicles are thin. If capital cannot easily be liquidated, the firm faces disincentives to invest and may be unresponsive to shocks. Secondly, identifying financial constrained firms using proxies may inadequately capture the firm’s true financial position.

Investment cashflow sensitivity is estimated by the coefficients for lagged revenue and interacted lagged revenue. For unconstrained firms, the coefficient β_3 is close to zero and not significant for the majority of the specifications, as expected. This indicates that, consistent with the first order condition, cash on hand does not enter the unconstrained firm’s optimal decision. Contrary to the predictions of the working capital model, there is very little evidence that the investment of constrained firms is sensitive to cashflow. One potential explanation is that the part of the total change in capital due to the change in cash (the first term on the right hand side of Equation 12) is dominated by the part due to the change in price. In other words, the income effect is small relative to the substitution effect.⁴² A second possible explanation is that the cash positions of firms are not accurately measured by last period’s revenue.

⁴²This is consistent with the need to capture investment opportunities in investment cashflow sensitivity regressions as emphasized in the literature.

My empirical strategy assumes that price shocks measure demand shocks. It is possible that instead, price shocks reflect supply shocks, either changes in technology or in factor prices. If price shocks represent technology shocks, the model's predictions and the interpretation of the results are unchanged. If price shock reflect changes in factor prices, particularly a change in the price of capital, investment would appear to behave countercyclically. However, there is no theoretical basis to suggest that the direction of the response to factor price changes will differ between constrained and unconstrained firms, unless the shocks are different between the two groups. From summary statistics of price changes in Table 9, we see that there are no significant differences in the changes of output price, wages nor raw material prices between constrained and unconstrained firms. Furthermore, these are controlled for in the regression. In summary, even though it is possible that prices may reflect supply shocks, supply shocks cannot explain the significant difference in investment response between constrained and unconstrained firms.

4.4 Output Response to Shocks

Section 4.2 has shown that constraints are more likely to bind when positive production opportunities arise. This implies that financial constraints limit the firm's output response to positive shocks but do not affect the firm's response to negative shocks. The empirical specification to test the output response of financially constrained firms is as follows:

$$\begin{aligned} \Delta \ln Q_{it} = & \alpha_0 + \alpha_1 \Delta shock_{it}^+ + \alpha_2 \Delta shock_{it}^- + \alpha_3 cnstr_{it} * \Delta shock_{it}^+ + \alpha_4 cnstr_{it} * \Delta shock_{it}^- \\ & + \beta_1 \ln K_{it-d} + \beta_2 cnstr * \ln R_{it-d} + \beta_3 \ln R_{it-d} + \beta_4 \Delta X_{it} + \varphi time + \varepsilon_{it} \end{aligned}$$

The change in log output is percentage growth and is measured as the difference from the firm specific mean. I categorize firms into; *Always Constrained* and *Sometimes Constrained* using internal finance and access to finance as the financial indicators.⁴³ All other explanatory variables are the same as those in the investment specification. The specification was estimated using OLS with robust standard errors clustering on firms. Positive shocks are defined as shocks above firm mean and negative shock defined as below mean.

The empirical test is to see whether sometimes constrained firms respond differently to positive shocks than unconstrained firms. Unconstrained firms increase output in response to positive shocks, so α_1 is expected to be positive. *Sometimes Constrained* firms are limited in their ability to expand, so α_3 is expected to be negative. Unconstrained and *Sometimes Constrained* firms are predicted to respond to negative shocks in the same way, so α_4 is not expected to be significant. According to the model, *Always Constrained* firms may have a different response to negative shocks as these firms are producing at a much steeper part of the production function.

Results are shown in Table 13. When the financial constraint dummy is defined based on access, virtually all of the estimated coefficients are insignificant. When the internal finance measure is used, the results are broadly consistent with the working capital model. First, the response to positive shocks is significantly different for unconstrained and *Sometimes Constrained* firms. Unconstrained firm increase output but *Sometimes Constrained* firms do not. Second, unconstrained and *Sometimes*

⁴³Recall that age and size variables do not indicate the firms transitioning into being financial constrained. *Always Constrained* is defined as a dummy equal to one when the firm is constrained for all periods and is time invariant. Likewise, *Sometimes Constrained* is defined as a dummy equal to one when firms are constrained at least once but less than for all periods.

Constrained firms do not differ in their response to negative shocks. This is indicated by the insignificant coefficient on the *SometimesCnstr * NegativeShock*. Third, consistent with theory, results indicate that output growth of constrained firms is sensitive to cashflow whereas the output growth of unconstrained firms is not.

5 Conclusion

Finance is scarce in developing countries where poverty hinges precariously on economic growth. How firms develop within an environment of limited access to external credit, and how financial constraints affect the behavior of the firm are of crucial importance to understanding investment and growth. Existing models have mainly been developed for and tested using data from developed economies, yet, despite the vast differences in context, these models are continually applied to developing countries. As a result, economic factors that are not necessarily relevant to developed economies but are crucial for developing countries are overlooked. A clear example of this is the largely neglected role of working capital. In countries such as the US, working capital is mostly irrelevant due to the abundance of short term credit availability. However, in developing countries where external credit is virtually non-existent, entrepreneurs have to resolve the time delay between incurring the cost of production and the receipt of revenue themselves. The need for working capital becomes very relevant when access to credit is scarce.

This paper develops a basic dynamic working capital model of the firm with financial constraints. By taking into account the need to finance both working capital and investment, and the possibility of financing from internally generated funds, the model provides insights into the effects of financial constraints on the firm's operations that are not captured by existing models. First, the working capital model shows that in addition to scale inefficiencies caused by constrained suboptimal output levels, financial constraints also distort optimal factor ratios in response to demand shocks. Investment becomes countercyclical to shocks under binding constraints. Not only are profits lower but the cost of generating a dollar of revenue are higher for the constrained firm than for an unconstrained firm. Secondly, not only is the constrained firm earning less at each period, but the suboptimal level of revenue it generates negatively affect production and growth over time. Thirdly, financial constraints prevent the firm from taking advantage of production opportunities. The working capital model relates the demand for financing to the demand for inputs to production. Firms are bound by constraints precisely when they wish to expand and are not bound during times of contraction.

The model offers a flexible theoretical framework of the firm under financial constraints. The assumptions of the model are very general - the only modifications to standard assumptions are the timing of revenue receipt and the absence of adjustment costs to investment. The theoretical predictions for investment response to price shocks do not rest on any assumptions regarding the functional form of the production function. The solution is analytically tractable and captures the essential predictions of existing models but offers additional insights into the effects of financial constraints. While this paper has focused on presenting a parsimonious model to illustrate the inclusion of the concept of working capital, the model can be easily modified with investment adjustment costs⁴⁴, borrowing constraints as functions of other variables, or incorporated into a general equilibrium

⁴⁴In another paper, I develop an extension of the working capital model to include time to build capital adjustments to look at precautionary savings by the firm.

framework. Although the interest of this paper has been on the firm's dynamic response to demand shocks, the model may yield interesting insights into other areas such as firm behavior in response to interest rate changes⁴⁵, output volatility over the business cycle⁴⁶, sector development with limited access to external credit⁴⁷ and the effects of trade liberalization under financial constraints⁴⁸.

In this paper, I empirically test three predictions in particular: constraints bind when prices increase, the countercyclical behavior of capital under binding constraints and the asymmetric output response. These predictions differ from the commonly used Jorgenson model of investment with financial constraints and allow comparisons of the working capital model to the standard model. I find that the working capital model's predictions are consistent with empirical evidence. These findings suggest that studying the dynamic behavior of firms may be a promising strategy for identifying which firms are credit constrained. Furthermore, these two predictions provide a means of identification of constrained firms that does not rely on the occurrence of a natural experiment or endogenous firm characteristics.

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⁴⁵The model allows the firm to choose between internal finance (vulnerable to real shocks) and external credit (vulnerable to interest rate shocks).

⁴⁶The working capital model exhibits propagation of temporary shocks past the period when the shock occurred, even though the shock is i.i.d.

⁴⁷Differences in factor intensities between industries could generate different rates of growth but also different sector responses to shocks under financial constraints.

⁴⁸Financial constraints restrict expansion to markets, therefore under trade liberalization where market opportunities %%% I develop this in another paper.

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Appendix

Contrast With the Standard Model of Financial Constraints

Standard model of financial constraints

The standard model utilizes the Jorgenson model of investment with financial constraints (see Bond and Meghir (1994) for review). Capital takes one period to install and the firm chooses current labor inputs and the next period’s capital. The implicit assumption is that there is no delay in the receipt of revenue and the model is given as:

$$\begin{aligned}
 & \max_{L_t, K_{t+1}} E_t \left[\sum_{t=0}^{\infty} \beta^t (P_t F(K_t, L_t) - wL_t - p^k I_t) \right] \\
 (14) \quad & s.t \quad wL_t + p^k (K_{t+1} - (1 - \delta)K_t) + b_t = P_t F(K_t, L_t) + (1 + r)b_{t-1} \\
 (15) \quad & s.t \quad K_{t+1} = (1 - \delta)K_t + I_t \\
 & s.t \quad b_t > bc \\
 & \lim_{t \rightarrow \infty} b_t = 0 \\
 & K_0 \text{ given} \\
 & b_0 \text{ given}
 \end{aligned}$$

There are four characteristics of the standard model that contrasts with a model with working capital:

1) Labor is always at optimal regardless of financial constraints. This is an equilibrium condition as the marginal product of labor is immediately received to not only cover the cost of labor but also to fund capital.⁴⁹ Take for instance that a firm does not have enough cash to purchase steady state capital (where $E[MPK] = p^k(r + \delta)$ the rental cost of capital). The firm at optimal labor where $MPL = w$, chooses to decrease a dollar’s worth of labor and put it towards capital. Although

⁴⁹This implies that there must be a slight timing difference between when labor decisions are made and when capital decisions are made.

capital has increased by a dollar, capital is funded by revenue which has declined by more than a dollar. Overall, the decrease in labor will lead to a decrease in capital.

2) Firms under this model do not switch between constrained and not constrained states due to transitory shocks. With the Jorgenson model, firms are constrained until they reached steady state capital stock and then are never constrained (unless there are permanent shocks). The firm cannot become constrained again after reaching steady state as the firm only need to investment the depreciated amount of steady state capital at each period. Even under an extreme negative shock, the revenue from production at steady state capital stock would strictly cover the cost of the depreciated value of capital, that is $P_t F(K^*, L_t) \gg \delta K^*$.

3) Investment is strictly procyclical to shocks. The budget constraint, Equation 14, dictates the level of investment and as such, a change in revenue will lead to a change in investment in the same direction.

4) Output response is symmetrical under the capital adjustment model with financial constraints. Labor is always optimally chosen and as such, output responds fully to demand shocks.⁵⁰ The capital adjustment model does not allow for firms to move in and out of financially constrained states and as such firms under this framework will never find themselves suddenly limited under a (transitory) positive shock.

Overall, the standard model of financial constraints will understate the effects of financial constraints on firm growth and responses to shocks because of these four characteristics compared to the working capital model of financial constraints.

⁵⁰The curvature of the production function however will cause increases to be less than decreases in output.

Table 1: Working Capital Requirements

US				Bangladesh				
<i>Industry</i>	<i>N. firms</i>	<i>NonCashWC</i> <i>Sales</i>	<i>WC</i> <i>Sales</i>	<i>Sector</i>	<i>N. firms</i>	<i>NonCashWC</i> <i>Sales</i>	<i>WC</i> <i>Sales</i>	<i>I</i> <i>Sales</i>
Chemical [†]	152	12.82%	17.35%	Chemical	85	26.19%	31.91%	3.15%
Food	114	7.46%	11.20%	Food	147	41.87%	48.12%	4.34%
Electronics	199	16.72%	30.31%	Electronic	91	24.02%	28.24%	1.24%
Shoe	28	20.82%	27.79%	Leather	99	35.65%	39.18%	4.60%
Apparel	58	18.09%	23.55%	Garment	306	18.54%	21.46%	1.88%
-	-	-	-	Textiles	252	37.88%	42.69%	4.61%
Weighted Average			21.94%	Weighted Average			34.24%	3.28%

Note: *Non – CashWC*: inventory + accounts receivable -accounts payable, *WC*: inventory+cash+accounts receivable - accounts payable $\frac{I}{Sales}$:Investment to sales. Short term credit available to the firm is not observable on accounting sheets whereas short term debt is accounted for by accounts payable.

Source:US Firm Data: Jan 2002.Aswath Damodaran, 'Working Capital Ratios by Sector', calculated using Value Line database of 7091 firms. Available from <http://pages.stern.nyu.edu/~adamodar> . [†]Chemicals is the weighted average of Basic Chemicals, Diversified Chemicals and Specialty Chemicals. Bangladesh Firm Data: Investment Climate Assessment Bangladesh 2002, Enterprise Surveys, World Bank.

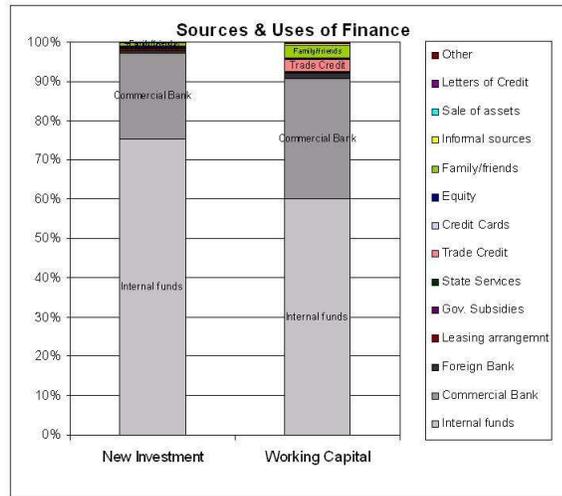


Figure 1: Sources and Uses of Finance - Bangladesh

Source: Bangladesh Panel Survey 2003-2005, Enterprise Surveys, World Bank.

Note: Survey instrument for sources of financing was introduced for the 2004-2005 rounds only. Average percentage calculated across all rounds.

Table 2: Time line of Production and Receipts

	Period 1	Period 2
State	Initial Capital Stock: K_0 Initial Cash Stock: X_1 Realized Price: $P_1 = \bar{P} + \varepsilon_1$	New: $K_1 = I_1 + (1 - \delta)K_0$ New: $X_2 = P_1 F(K_1, L_1) + (1 + r)(X_1 - wL_1 - p^k(K_1 - (1 - \delta)K_0))$ Realized Price: $P_2 = \bar{P} + \varepsilon_2$
Choice	Make input choices: L_1 and K_1 Pay input costs: $wL_1 + p^k(K_1 - (1 - \delta)K_0)$ Produce: $F(K_1, L_1)$	Make input choices: L_2 and K_2 Pay input costs: $wL_2 + p^k(K_2 - (1 - \delta)K_1)$ Produce: $F(K_2, L_2)$

Note: At the start of the period, the firm has capital and cash. The price that the firm will receive for its product is realized and then it decides on input choices of labor and capital. The next period, the firm receives the revenue which, along with savings, constitutes cash for the next period's production.

Table 3: Comparing Constrained Outcomes to Unconstrained

	Model	$\frac{Q}{Q^*}$	$\frac{L}{L^*}$	$\frac{K}{K^*}$	$\frac{Profit}{Profit^*}$	N.Obs
Always Constrained	Working Capital	38%	37%	33%	8%	18159
	Standard	60%	56%	58%	15%	23276
Sometimes Constrained	Working Capital	87%	86%	85%	64%	115833
	Standard	na	na	na	na	
Never Constrained	Working Capital	99%	99%	99%	95%	685008
	Standard	100%	100%	100%	100%	795724

Note: *Always Constrained:* firms are consistently under binding constraints. *Sometimes Constrained:* firms shift between nonbinding and binding constraints depending on demand shock. *Never Constrained:* firms that never run into binding constraints 90% of the time. This is also why even under the never constrained phase, the working capital model does not predict 100% of optimal. **Measurement:** Percentages are calculated as the constrained outcome variable divided by the corresponding unconstrained outcome for the simulated prices series over time. The average is taken over all borrowing constraints and all observations within constrained phases.

Table 4: Production Inefficiency - Cost per Dollar of Revenue and Labor to Capital Ratio

	Working Capital Model			Standard Capital Adjustment Model		
	Cost per Dollar of Revenue	Labor to Capital Ratios	N.obs	Cost per Dollar of Revenue	Labor to Capital Ratios	N.obs
Always Constrained	0.825	1.910	25655	0.973	1.63	28276
Sometimes Constrained	0.787	1.581	129337	Not Applicable		
Never Constrained	0.712	1.525	685008	0.730	1.88	811724

Note: Medians reported. Cost per Revenue Dollar calculated as $\frac{wL + p^k I}{PQ}$.

Table 5: Time to Maturity

model	mean	sd	min	max	N
Standard	2.35	2.80	1	14	840000
Working Capital	8.38	6.43	2	27	840000

Note: The standard investment model is the Jorgenson model of investment with financial constraints.

Simulation: 40 time periods, 1000 different simulated paths and 21 different borrowing parameters (from 0 to 2000). Time to maturity: Standard model is steady state capital stock, for Working Capital model is the cash required to respond to 90% of the shock.

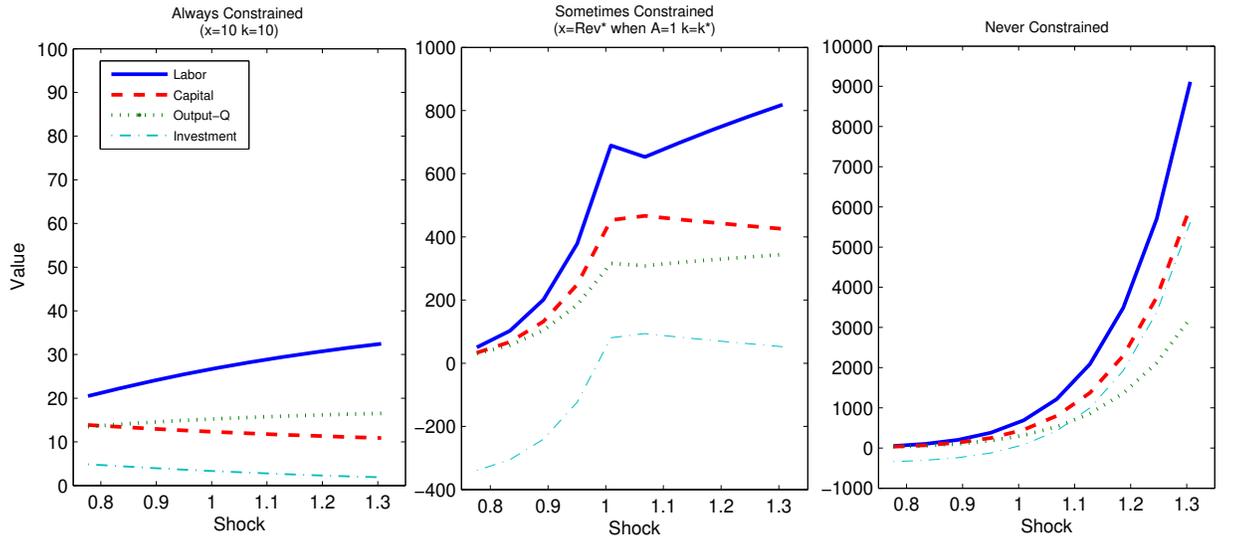


Figure 2: Response of the Firm to Shocks- Three Phases

Parameters: $P_t F(K_t, L_t) = P_t K_t^\alpha L_t^\gamma$, $\alpha = 0.30$, $\gamma = 0.60$, $\beta = 0.9$, $\delta = 0.10$, $\ln P_{t+1} \sim N\left(-\frac{1}{2}\sigma^2, \sigma^2\right)$, $\sigma = 0.1$,

$E(P_{t+1}) = 1$, $p^k = 1$, $w = 0.25$.

Sometimes Constrained: initial cash is set such that firm can afford optimal solution when the shock is equal to one.

Note: Panel A illustrates responses in output, labor, capital and investment to price shocks when firm behavior is dictated by constrained first order conditions. Panel B illustrates these same variables when the firm switches from unconstrained to constrained states. Panel C is the behavior of the unconstrained firm. Note that the scale of the y-axis differs across the three panels.

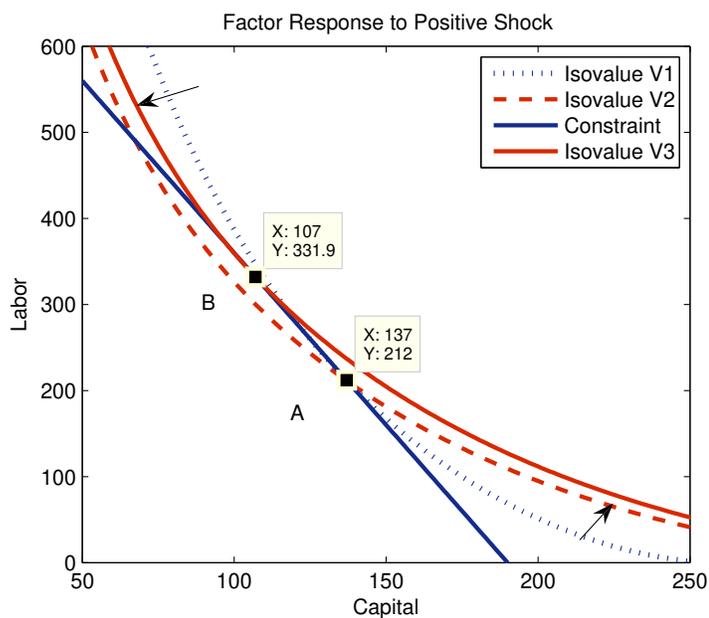


Figure 3: Factor Response to Positive Shock

Note: $V = AK^\alpha L^\gamma + (1 - \delta)p^k K$. Isoquants: $L = \left(\frac{V - (1 - \delta)p^k K}{AK^\alpha} \right)^{\frac{1}{\gamma}}$ Budget lines: $L = \frac{1}{w} (X + (1 - \delta)p^k K - p^k K - bc)$
 $\alpha = 0.30 \quad \gamma = 0.60 \quad p^k = 1 \quad w = .25 \quad X = 100 \quad K = 100 \quad \delta = .10$ Shock is 1.7

A price increase causes the isoquant curve to pivot from V1 to V2 and the new tangency point B is to the left of the initial starting point A.

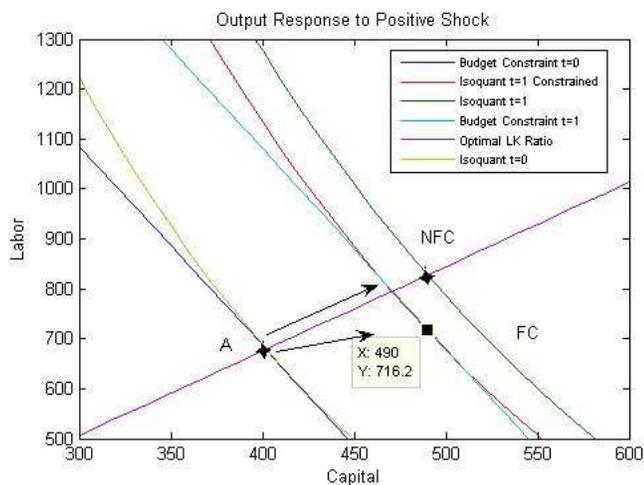


Figure 4: Output Response to Shocks

Parameters: same as Figure 2 Initial Conditions: Shock = 1 at $t = 0$, Shock = 1.02, at $t = 1$. Cash: cost of optimal inputs when Shock = 1

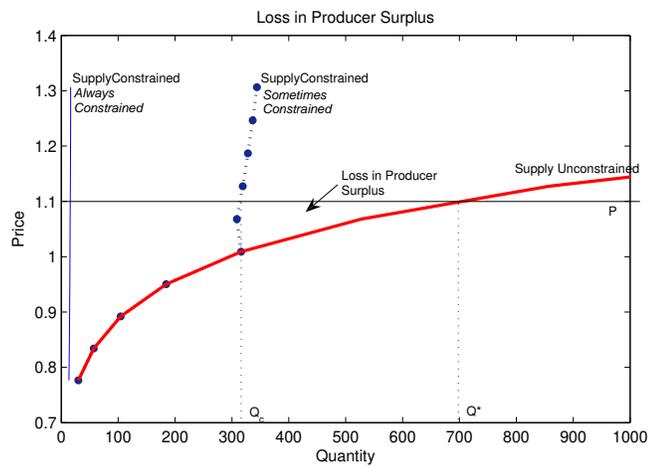


Figure 5: Loss in Producer Surplus

Note: Loss in Producer Surplus: Same 3 cases as those in Figure 2, with Shock as the y-axis and Output on the x-axis. Average Loss in Producer Surplus calculated as $Loss_{i,t,bc} = \left(\frac{(P^* - P)(Q^* - Q)}{2} \right)$ Sum across time and averaged over simulated paths.

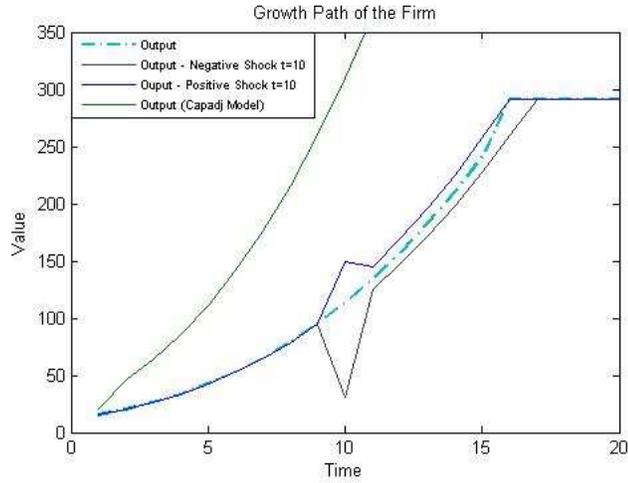


Figure 6: Growth Path of the Firm

*Note:*Initial conditions: Cash=10 and Capital=10. The growth path predicted by the working capital model is much flatter than the path predicted by the standard investment model. A positive shock (of 2 standard deviations) shifts the path upward, whereas a negative shock shifts the path downward.

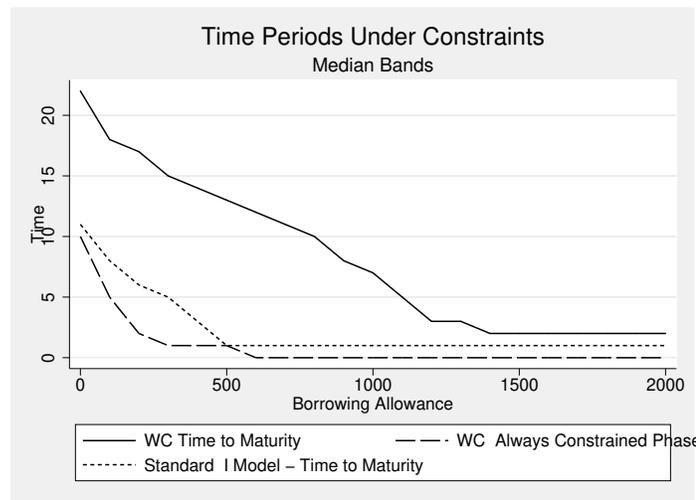


Figure 7: Period of Time Under Constraints

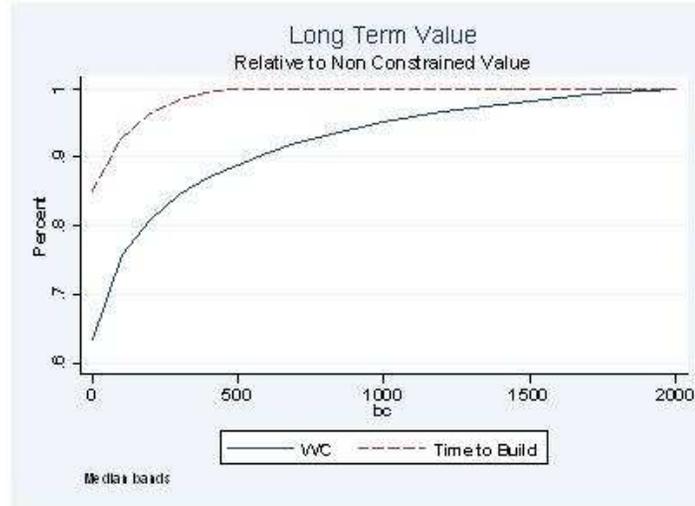


Figure 8: Long Term Value of the Firm and the Effects of Financial Constraints

Note: Long Term Value calculated as sum of profits over 40 periods. All firms reach maturity by the end of the 40 periods, as such the value after 40 would be the same across all firms regardless of borrowing constraints.

Table 6: Overlap Between Financially Constrained Indicators

Indicators Jointly Equal to 1			
	Internal Finance	Access to Finance	Age is less than 5yr
Access to Finance	76.69%		
Age is less than 5yr	45.55%	48.97%	
Size is less than 100 employees	55.77%	60.19%	55.24%

Measurement: $internalF = 1$ if 100 percent of financing comes from internal funds and the firm reported access to financing as some problem; $acc = 1$ if the firm reported access to finance as a moderate to severe problem ; $age5 = 1$ if the firm is 5 years old or less and $size100 = 1$ if the number of employees is 100 or less. *Note:* To illustrate the relationship between the internal finance and access to finance indicator, this table is shown with the original values of these indicators before supplementing predicted values for missing observations.

Table 7: Transition Probability of Financial Indicators

$t \setminus t+1$	Internal Finance			$t \setminus t+1$	Access to Finance		
	0	1	Total		0	1	Total
0	71.08	28.92	100.00	0	71.10	28.90	100.00
1	24.20	75.80	100.00	1	26.48	73.52	100.00
Total	38.12	61.88	100.00	Total	51.77	48.23	100.00

Table 8: Summary Statistics

	Not Constrained	Constrained	Total	Difference	t-statistic
Value of Output	10.582	9.603	9.937	0.979	9.626
Capital Stock	10.223	9.301	9.614	0.922	8.694
Revenue	10.649	9.664	9.999	0.985	9.650
Labor Costs	8.422	7.487	7.804	0.935	11.060
Change in Capital Stock (semi-annual)	0.065	0.032	0.045	0.034	2.658
Change in Capital Stock (annual)	0.110	0.061	0.084	0.049	2.483
Output Growth (semi-annual)	0.099	0.040	0.063	0.058	1.128

Note: Financial constraint indicator is internal finance, variables defined in logs. The null hypothesis that the difference in means between constrained and unconstrained firms is equal to zero is rejected for all variables at the 5% significance level.

Table 9: Summary Statistics- Prices

Semi-annual change	Not Constrained	Constrained	Total	Difference	t-statistic
Output Price	-0.009	-0.019	-0.015	0.010	1.522
Raw Material Price	0.068	0.079	0.074	-0.010	-1.644
Log Wages	0.046	0.079	0.066	-0.033	-0.810
Interest rate*	-0.003	0.001	0.000	-0.004	-7.978

Note: Financial constraint indicator is internal finance, variables defined in logs. * Due to low response rate, the interest rate is calculated as the average over sector and time. The null hypothesis that the difference in means between constrained and unconstrained firms is equal to zero cannot be rejected for all variables at the 5% significance level except for interest rates.

Table 10: Summary Statistics - Firms Characteristics

	Not Constrained	Constrained	Total	Difference	t-statistic
N. Workers	369.93	221.53	271.92	148.41	8.54
Age	16.68	15.15	15.67	1.53	1.91
Garment	0.24	0.31	0.28	-0.07	-2.47
Textile	0.33	0.25	0.27	0.08	2.96
Food	0.16	0.14	0.15	0.02	0.98
Leather	0.09	0.14	0.12	-0.05	-2.44
Electronics	0.04	0.09	0.07	-0.05	-3.36
Chemical	0.14	0.08	0.10	0.07	3.68

Note: Sector variables are indicator variables. The null hypothesis that the difference in means between constrained and unconstrained firms is equal to zero is rejected for all variables at the 5% significance level except for age.

Table 11: Constraints and Shocks

DPROBIT- Robust standard errors				
Cluster Standard Errors on Firm id				
Reported Marginal Effects				
Become Constrained	Access		Internal Finance	
shock	0.389 (2.17)**	0.387 (2.22)**	0.322 (2.30)**	0.312 (2.27)**
Initial R	0.018 (2.98)***	0.016 (2.52)**	0.013 (2.33)**	0.011 (2.13)**
Initial K	-0.007 (1.48)	-0.006 (1.15)	-0.012 (2.57)**	-0.008 (1.91)*
time trend	-0.027 (4.35)***	-0.057 (3.32)***	0.018 (1.41)	0.016 (3.01)***
Control for Firm Characteristics	no	yes	no	yes
Observations	1217	1216	1216	1217
N.firms	250.00	250.00	250.00	250.00
Pseudo R2	0.02	0.03	0.03	0.02
Log likelihood	-533.08	-527.38	-348.78	-351.99

Robust z statistics in parentheses * significant at 10%; ** significant at 5%; *** significant at 1%

Table 12: Investment Response to Shocks

OLS												
Dependent Variable: Change in Log Capital Stock												
Shock: Change in Log Output Price												
	Semi Annual Changes						Annual Changes					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Constant	0.076	0.103	0.106	0.080	0.051	0.114	0.135	0.196	0.235	0.184	0.154	0.240
	(0.78)	(1.20)	(1.60)	(1.02)	(0.59)	(1.69)*	(0.74)	(1.34)	(1.80)*	(1.28)	(0.89)	(1.96)*
Constraint	0.098	0.119	0.020	0.077	0.095	-0.048	0.249	0.261	0.057	0.173	0.197	-0.042
	(1.05)	(1.58)	(0.25)	(1.22)	(1.17)	(0.74)	(1.41)	(1.90)*	(0.39)	(1.38)	(1.18)	(0.28)
Shock	0.200	0.301	0.179	0.160	0.188	0.156	0.263	0.350	0.253	0.272	0.323	0.251
	(2.71)***	(2.83)***	(2.99)***	(2.10)**	(2.30)**	(2.53)**	(2.30)**	(2.44)**	(2.35)**	(2.24)**	(2.43)**	(2.85)***
Cnstr*shock	-0.185	-0.304	-0.154	-0.104	-0.143	-0.160	-0.205	-0.300	-0.155	-0.203	-0.290	-0.363
	(1.84)*	(2.48)**	(1.30)	(1.04)	(1.44)	(1.77)*	(1.58)	(1.94)*	(1.10)	(1.60)	(1.99)**	(2.26)**
Lagged Capital	-0.012	-0.014	-0.012	-0.012	-0.011	-0.011	-0.018	-0.021	-0.019	-0.018	-0.017	-0.016
	(1.98)**	(2.11)**	(1.93)*	(1.97)*	(1.89)*	(1.82)*	(1.80)*	(1.99)**	(1.85)*	(1.82)*	(1.71)*	(1.62)
Cnstr*Lagged Rev	-0.014	-0.017	-0.004	-0.009	-0.009	0.004	-0.032	-0.032	-0.011	-0.020	-0.020	0.001
	(1.55)	(2.30)**	(0.45)	(1.40)	(1.17)	(0.52)	(1.81)*	(2.22)**	(0.70)	(1.61)	(1.25)	(0.04)
Lagged Revenue	0.014	0.015	0.008	0.010	0.011	0.006	0.025	0.022	0.012	0.016	0.020	0.009
	(2.17)**	(2.63)***	(1.53)	(1.78)*	(1.82)*	(1.28)	(1.85)*	(1.72)*	(1.10)	(1.34)	(1.39)	(0.87)
Time trend	-0.019	-0.018	-0.019	-0.019	-0.025	-0.019	-0.040	-0.039	-0.041	-0.040	-0.047	-0.041
	(3.71)***	(3.37)***	(3.74)***	(3.80)***	(3.93)***	(3.79)***	(3.73)***	(3.50)***	(3.78)***	(3.78)***	(3.84)***	(3.89)***
Firm characteristics	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Observations	979	979	979	979	979	979	777	777	777	777	777	777
R-squared	0.05	0.06	0.04	0.04	0.04	0.04	0.09	0.08	0.07	0.07	0.07	0.07
N.Firms	202.00	202.00	202.00	202.00	202.00	202.00	198.00	198.00	198.00	198.00	198.00	198.00

Robust t statistics in parentheses* significant at 10%; ** significant at 5%; *** significant at 1%

(1), (7) Internal Finance: Consecutively binding. $cnstr_{t-1}$ & $cnstr_t = 1$ (varies with time and across firms)

(2), (8) Internal Finance: Consistently constrained for all periods indicator is observed. (firm specific)

(3), (9) Access to finance reported as problem(moderate to extreme): Consecutively binding (varies with time and across firms)

(4), (10) Access to finance reported as problem(moderate to extreme): Consistently constrained for all periods indicator is observed. (firm specific)

(5), (11) Number of employees less than 100: Consecutively binding (varies with time and across firms)

(6), (12) Age of firm is less than 5 years old: Consecutively binding (varies with time and across firms)

Table 13: Output Response to Shocks

OLS - Robust Standard Errors				
Changes from Mean				
Dependent Variable: Log Output (Quantity)				
Shocks: Log Output Price				
	Internal Finance		Access	
	(1)	(2)	(3)	(4)
Constant	0.010	0.400	-0.408	-0.535
	(0.03)	(1.67)*	(0.76)	(1.00)
Always Cnstr	-0.854	-1.155	-0.137	-0.150
	(1.81)*	(3.01)***	(0.17)	(0.20)
Sometimes Cnstr	-0.946	-1.279	-0.620	-0.403
	(2.58)**	(4.29)***	(1.10)	(0.74)
Positive Shock	2.109	1.476	-1.215	-0.542
	(1.59)	(4.20)***	(0.94)	(0.38)
Always Cnstr*Positive Shock	-2.576	-1.920	3.309	2.856
	(1.28)	(1.30)	(1.03)	(0.89)
Sometimes Cnstr*Positive Shock	-2.923	-1.762	0.505	0.272
	(1.95)*	(2.46)**	(0.35)	(0.18)
Negative Shock	-5.988	-3.407	0.391	-0.804
	(2.24)**	(4.92)***	(0.17)	(0.38)
Always Cnstr*Negative Shock	4.705	1.695	2.826	3.685
	(1.62)	(1.33)	(0.65)	(0.94)
Sometimes Cnstr*Negative Shock	4.424	1.323	-2.277	-1.595
	(1.61)	(1.37)	(0.98)	(0.72)
Lagged Log Capital Stock	-0.092	-0.113	-0.094	-0.115
	(4.11)***	(4.60)***	(4.59)***	(5.06)***
Lagged Log Revenue	0.076	-0.001	0.110	0.078
	(3.93)***	(0.03)	(2.26)**	(1.76)*
Always Cnstr*Lagged Log Revenue	0.053	0.088	-0.014	0.007
	(1.19)	(2.22)**	(0.16)	(0.08)
Sometimes Cnstr*Lagged Log Revenue	0.072	0.102	0.044	0.026
	(2.51)**	(3.87)***	(0.86)	(0.53)
Time trend	0.041	0.042	0.043	0.047
	(3.13)***	(1.90)*	(3.29)***	(2.20)**
Control for Firm Characteristics	no	yes	no	yes
Observations	954	953	954	953
R-squared	0.16	0.21	0.17	0.22
N.firms	200.00	200.00	200.00	200.00

Robust t statistics in parentheses
* significant at 10%; ** significant at 5%; *** significant at 1%