Partial privatization and unidirectional transboundary pollution

Kazuhiko Kato

Faculty of Economics, Asia University

2. December 2010

Partial privatization and unidirectional transboundary pollution[†]

Kazuhiko Kato[‡]

December 2, 2010

Abstract

We determine whether or not a local regional government should privatize its local public firm in a mixed duopoly when it faces the problem of unidirectional transboundary pollution. We consider two regions in an economy, one located upstream and the other, downstream. Where both the local public firm owned by the local government upstream and the private firm are located and compete upstream, we analyze two cases: (h) the private firm is owned by private investors upstream and (f) it is owned by private investors downstream. A comparison of the two cases presents the following results. Partial privatization is desirable for local welfare upstream in (h), but it is not always desirable in (f). In both (h) and (f), it is desirable for local welfare downstream and for the overall welfare of the economy when the degree of environmental damage and the fraction of transboundary pollution upstream are low. However, when they are high, the results change for (h) and (f).

JEL classification: L13; L33; Q53; R38

Keywords: Mixed Duopoly, Privatization, Transboundary pollution

[†]We thank the participants for their helpful comments in the non-linear economic theory workshop at Chuo University (November 2010).

[‡]Faculty of Economics, Asia University, 5-24-10 Sakai, Musashino shi, Tokyo 180-8629, Japan; E-mail: kkato@asia-u.ac.jp

1 Introduction

Transboundary pollution such as that caused by acid rain and water and air pollution has been attracting attention since the middle-nineteenth century. For example, acid rain, which has long been recognized as a serious environmental problem in Europe, has become a serious issue in East Asia over the past few decades.[1] For another example, for the past several years, waste that is believed to be generated in Russia, China, and Korea has been regularly found on the shores of northern Japan, where it is carried by the sea. To solve this problem, Japan and Korea held working-level talks in February 2009.

Meanwhile, global warming continues to worsen the environment worldwide. It can affect the fraction of transboundary pollution to a large extent because global warming may cause the Westerlies to meander and increase the frequency of natural calamities.[2] Therefore, while analyzing the transboundary pollution problem, we pay attention to not only the total amount of pollution but also its transboundary fraction from one region to the other.

In some of the countries and regions mentioned above, there still exist mixed markets where public and private firms compete. In these mixed markets, the privatization of public firms is a major issue because it changes their objectives and thus, their behavior. Since the welfare-maximizing public firm takes into consideration environmental damage, the effect of privatization on social welfare depends on the affairs associated with

[1]Nagase and Silva (2007) provide, in detail, the extent of damage caused by acid rain in China and Japan. Ichikawa and Fujita (1995) estimate the contribution of China to be about one-half of the total with respect to the wet deposition of sulfate in Japan. For other transboundary pollution issues, Ohara et al. (2001) indicate the threat of an increase in ozone, sulfate, and nitrate, causative factors of the existing urban ozone in China, which may greatly impact the air quality in Japan in the future.

[2]The meandering Westerlies will also affect the present amount of air-borne pollutants and toxic chemicals (which cause acid rain) that are carried between the countries. Heavy rains transport city waste that may be lying on the riverbed or in waste collection sites located along the river into the river. Floods then transfer this waste to downstream areas. An increase in the atmospheric temperature and surface level of the sea and a decrease in the salinity of the seas due to the melting glaciers may alter the flow of the oceans and thus affect the amount of waste that is carried from one country to another.

transboundary pollution. In particular, when we consider the partial privatization of public firms, the welfare-maximizing degree of this partial privatization may be affected by the fraction of transboundary pollution. Of course, in many cases, partial privatization in one country or region affects not only its own welfare but also welfare of other countries or regions through changes in the equilibrium outcome, including the influence of transboundary pollution. We therefore pose the following two questions: (1) How does the fraction of transboundary pollution affect the welfare-maximizing degree of partial privatization? (2) Does partial privatization in one country or region enhance welfare of other countries or regions and that of the whole world?

In order to answer these questions, we have developed a model. The model considers two regions, with one located downstream of the other, and two firms, a public firm and a private firm. In our model, the market is opened only upstream, and both firms exist there. We consider two cases: (h) the private firm is owned by private investors upstream and (f) it is owned by private investors downstream. In the literature of the mixed oligopoly theory, we often observe different results between (h) and (f).[3] This is because the behavior of the public firm changes from (h) to (f) and the profit of the private firm is included in the objective of the public firm in (h) but not in (f).

In recent years, some studies have addressed the environmental issue in a mixed oligopoly. Bárcena-Ruiz and Garzón (2006), Beladi and Chao (2006), Wang and Wang (2009), and Wang, Wang, and Zhao (2009) consider emission tax in the domestic market, whereas Ohori (2006a, 2006b) considers the same in the international market. Kato (2006, 2010) and Naito and Ogawa (2009) compare some environmental policies. They examine environmental regulation in a mixed oligopoly and analyze the effects of privatization. Cato (2008) investigates the relationship between the degree of environmental damage and privatization. However, because these works deal with the environmental problem in one region, they do not take into consideration transboundary pollution.

From among the earlier work conducted on transboundary pollution, Nagase and

[3]For example, Fjell and Pal (1996), Fjell and Heywood (2004), Matsumura (2003), and Lu (2006) demonstrate the different results in the case wherein the competitors of the public firm are domestic private firms and the case wherein they are foreign private firms.

Silva (2007) is closely related to our motivation. Nagase and Silva (2007) consider the situation where one region (China) is located upstream of another region (Japan) under unidirectional transboundary pollution.[4] However, their main interest is to examine an environmental policy-making game between the two, and therefore, it differs from ours with regard to focusing on the effect of privatization. In China, a large number of public firms have been privatized since the 1990s.[5] However, mixed oligopoly is still prevalent in several industries that depend on energy from fossil fuels, especially coal. Thus, the analysis of transboundary pollution in the framework of the mixed oligopoly theory may lead to a new approach in the research on the transboundary pollution problem.

The remainder of the paper is organized as follows. Section 2 describes our model. Section 3 derives the equilibrium outcome under different cases of private firm ownership and conducts a welfare comparison. Section 4 compares the results obtained in the previous section. Section 5 concludes the main text. Detailed calculations for the equilibrium outcome in each case and proofs of the propositions are given in the Appendices.

2 Model

Consider an economy of two regions: regions A and B. Region A is located upstream of region B. In this economy, there is one local public firm (firm 0) owned by the local regional government of A and one private firm (firm 1) owned by private investors from either region A or region B. Both the firms are located in region A and produce a homogeneous product that harms the environment. We call this product a "dirty good."

Firms 0 and 1 compete in quantity. The output of firm i is denoted by q_i $(i = 0, 1)$. Total output is denoted by $Q = q_0 + q_1$. We assume that the cost function of firm i is given by $c_i(q_i) = cq_i^2/2$. Given the inverse demand function of the dirty good, $p = p(Q)$,

[4] Nagase and Silva (2007) consider a competitive market and allow an abatement effort and an emission tax policy.

[5] For an overview of the reform of state-owned enterprises in China, see Fernández and Fernández-Stembridge (2007).

and then, the profit of firm i is

$$\pi_i(q_0, q_1) = p(Q)q_i - \frac{c_i q_i}{2}.$$

A representative consumer exists in each region. The representative consumer in region A consumes the dirty good and a clean numeraire good. The representative consumer in region B only exists.

The representative consumer in region A maximizes $U(Q)+y$ subject to $pQ+y=m$, where p denotes the price of the dirty good, y denotes the amount of the numeraire good, whose price is normalized to 1, and m denotes the income of the representative consumer. We assume that $U(Q)$ is

$$U(Q) = aQ - \frac{Q^2}{2}. \tag{1}$$

Therefore, we obtain the following inverse demand function, $p(Q) = a - Q$ by solving the utility maximization problem of the representative consumer in region A.

In our model, pollution is generated by either production or consumption and is harmful to the environment. Producing or consuming one unit of a dirty good generates one unit of pollution. The pollution is converted into environmental damage which reduces the consumer surplus via a lump-sum transfer. We do not consider the case where pollution is generated by both production and consumption. In our setting, pollution is generated only in region A and there is no difference between pollution through production and that through consumption. Therefore, in the subsequent instructions and analyses, we consider the case that pollution is generated by production. The total pollution in region l is denoted by E_l ($l = A, B$); the total environmental damage in region l is denoted by $D_l(E_l) = d(E_l)^2/2$.

We assume that pollution is transboundary and can affect the environment in region B. We now explain how transboundary pollution is considered in the model. Pollution is generated only in region A because both firms produce in region A; the amount of pollution generated is Q. We assume that region A is located upstream of region B (along a river or in the path of a periodic wind), and therefore, some of the pollution is transported to region B. The fraction of pollution that remains in region A is $\theta \in [0, 1]$;

therefore, the fraction of pollution transported to region B is $(1-\theta)$. Thus, the pollution levels in regions A and B are θQ and $(1-\theta)Q$, respectively.

This paper examines two cases of ownership of the private firm: case (h), where firm 1 is owned by private investors from region A, and case (f), where it is owned by private investors from region B. Figure 1 shows the two cases and the amount of pollution of two regions by the unidirectional transboundary pollution.

In the model, welfare is defined as the sum of consumer surplus, producer surplus, and environmental damage.

First, we consider case (h), wherein firm 1 is owned by private investors from region A. Welfare in region A is given by

$$w_A = \int_0^Q p(s)ds - \frac{cq_0^2}{2} - \frac{cq_1^2}{2} - \frac{d(\theta Q)^2}{2} + m. \tag{2}$$

Welfare in region B is given by

$$w_B = -\frac{d\{(1-\theta)Q\}^2}{2}. \tag{3}$$

Welfare in the economy is defined as the sum of the welfare in regions A and B. Thus,

$$W = \int_0^Q p(s)ds - \frac{cq_0^2}{2} - \frac{cq_1^2}{2} - \frac{d(\theta Q)^2}{2} - \frac{d\{(1-\theta)Q\}^2}{2} + m. \tag{4}$$

Second, we consider case (f). In this case, firm 1 is owned by private investors from region B. Welfare in region A, welfare in region B, and the total welfare are respectively given by

$$w_A = \int_0^Q p(s)ds - p(Q)q_1 - \frac{cq_0^2}{2} - \frac{d(\theta Q)^2}{2} + m, \tag{5}$$

$$w_B = p(Q)q_1 - \frac{cq_1^2}{2} - \frac{d\{(1-\theta)Q\}^2}{2}, \tag{6}$$

$$W = \int_0^Q p(s)ds - \frac{cq_0^2}{2} - \frac{cq_1^2}{2} - \frac{d(\theta Q)^2}{2} - \frac{d\{(1-\theta)Q\}^2}{2} + m. \tag{7}$$

We denote the welfare of region l as "local welfare l" and the welfare in the entire economy as "total welfare." Further, we define the local regional government of l as "local government l."

Here, we define the objective function of each firm. The objective functions of public firm U_0 and private firm U_1 are respectively given by

$$U_0 = \alpha W + (1-\alpha)\pi_0, \quad \alpha \in [0,1], \tag{8}$$

$$U_1 = \pi_1. \tag{9}$$

When $\alpha = 0$, firm 0 is a pure profit-maximizer, and when $\alpha = 1$, it is a pure local welfare-maximizer. Here, α is understood as the share holding of the public sector and $1-\alpha$ is that of the private sector.[6] The objective of firm 1 is to maximize its own profits.

Finally, we consider the following timing of the game. Before the game begins, the public firm is perfectly owned by local government A, that is, $\alpha = 1$. When the game starts, local government A chooses the level of α, and then, the two firms choose their quantity simultaneously.

3 Equilibrium outcomes and welfare comparison

In this section, we derive the equilibrium outcome and compare three types of welfare before and after privatization in cases (h) and (f). First, we examine case (h).

3.1 Case (h)

We first consider the case where firm 1 is owned by private investors from region A.

Local welfare A, local welfare B, and total welfare are respectively defined as (2), (3), and (4).

In the second stage, each firm maximizes its objective by choosing its quantity. The first-order condition of the maximization problem of firms 0 and 1 are respectively given by

$$\frac{\partial U_0}{\partial q_0} = a - (2 - \alpha + c + d\alpha\theta^2)q_0 - (1 + d\alpha\theta^2)q_1 = 0, \tag{10}$$

$$\frac{\partial U_1}{\partial q_1} = a - q_0 - (2+c)q_1 = 0. \tag{11}$$

[6] For a rationalization of this objective function, see Bös (1991) and Matsumura (1998).

Solving the above first-order conditions, we obtain

$$q_0^h = \frac{a(1+c-d\alpha\theta^2)}{(1+c)(3+c)-(2+c)\alpha+(1+c)d\alpha\theta^2}, \tag{12}$$

$$q_1^h = \frac{a(1+c-\alpha+d\alpha\theta^2)}{(1+c)(3+c)-(2+c)\alpha+(1+c)d\alpha\theta^2}, \tag{13}$$

$$w_A^h = \frac{2a^2(1+c)\{(1+c)(4+c-2d\theta^2)-(5+2c-4d\theta^2-2cd\theta^2)\alpha\}}{2\{(1+c)(3+c)-(2+c)\alpha+(1+c)d\alpha\theta^2\}^2}, \\ + \frac{a^2\alpha^2\{3+c-d\theta^2(3+2cd\theta^2)\}}{2\{(1+c)(3+c)-(2+c)\alpha+(1+c)d\alpha\theta^2\}^2}, \tag{14}$$

$$w_B^h = -\frac{a^2d(2+2c-\alpha)^2(1-\theta)^2}{2\{(1+c)(3+c)-(2+c)\alpha+(1+c)d\alpha\theta^2\}^2}, \tag{15}$$

$$W^h = \frac{a^2\{2(1+c)^2(4+c-2d)-2(1+c)(5+2c-2d)\alpha-2d\alpha^2\theta^2(2+cd\theta^2)\}}{2\{(1+c)(3+c)-(2+c)\alpha+(1+c)d\alpha\theta^2\}^2} \\ + \frac{a^2\{(3+c-d)\alpha^2+2d(2+2c-\alpha)^2\theta+4d(1+c)(-2-2c+3\alpha+c\alpha)\theta^2\}}{2\{(1+c)(3+c)-(2+c)\alpha+(1+c)d\alpha\theta^2\}^2}+m, \tag{16}$$

where the superscript h denotes the equilibrium outcome in the second stage in case (h) except for α. With regard to α, the superscript denotes the equilibrium outcome in the full game. In the subsequent section, this superscript is also used to represent the equilibrium outcome in the second stage. To restrict our attention to the case of the interior solution, we assume that $1+c \geq d$. We also assume that $c \geq 1$ in order to simplify the subsequent analyses.

Here, we examine the comparative statics for the equilibrium output of each firm with respect to α. We find that

$$\frac{\partial q_0^h}{\partial \alpha} < 0, \ \frac{\partial q_1^h}{\partial \alpha} > 0, \ \text{and} \ \ \frac{\partial Q^h}{\partial \alpha} < 0, \ \ \text{if and only if} \ \ d\theta^2 > \frac{1}{2}. \tag{17}$$

In terms of local welfare A, there are two distortions in the region. One is caused by underproduction with regard to the duopolistic market and the other is caused by excess production with regard to environmental damage. A high level of d and θ imply that a large fraction of pollution remains in region A and environmental damage is large. In this case, the latter distortion dominates the former one, and therefore, the local public firm decreases its output when it gives greater weight to local welfare A.

In the first stage, local government A chooses α in order to maximize local welfare

A.[7] Solving for α, we obtain

$$\alpha^h = \frac{(1+c)^2}{1+3c+c^2}. \tag{18}$$

The result shows that partial privatization is desirable for local welfare A. We also find that α^h does not depend on the fraction of transboundary pollution and the degree of environmental damage. Rather, these results depend on the functional forms of demand, cost, and environmental damage.[8]

Does partial privatization of the local public firm enhance local welfare in the other region and the total welfare? (welfare comparison)

We examine whether the optimal privatization for local welfare A enhances local welfare B and the total welfare. Comparing local welfare B and total welfare at $\alpha = 1$ and $\alpha = \alpha^h$, we obtain the following proposition.

Proposition 1. *When $\theta = 1$ or $d\theta^2 = 1/2$, $w_B|_{\alpha=1} = w_B|_{\alpha=\alpha^h}$. Consider the case where $\theta \neq 1$ and $d\theta^2 \neq 1/2$. Then,*

$$w_B|_{\alpha=1} - w_B|_{\alpha=\alpha^h} > 0 \qquad \text{if} \quad d > \tfrac{1}{2} \text{ and } \theta \in \left(\sqrt{\tfrac{1}{2d}}, 1\right),$$

$$w_B|_{\alpha=1} - w_B|_{\alpha=\alpha^h} < 0 \qquad \text{if} \quad \begin{cases} d > \frac{1}{2} \text{ and } \theta \in \left[0, \sqrt{\frac{1}{2d}}\right), \\ d < \frac{1}{2} \text{ and } \theta \in [0, 1). \end{cases}$$

Proof. See Appendix B. □

Figure 2 illustrates Proposition 1 for each value of the fraction of transboundary pollution and the degree of environmental damage.

The intuition behind Proposition 1 is as follows. When $\theta = 1$, no fraction of the pollution caused in region A is transported to region B, and therefore, α does not affect

[7] The second-order condition of the maximization problem is satisfied. See Appendix A.

[8] The amount of total output and output level of each firm affect the decision with respect to α^h. Specifically, the total output level affects both the marginal utility of the representative consumer and the marginal environmental damage. The larger is the total output, the larger are the marginal utility and marginal environmental damage. On the other hand, the output level of each firm affects its marginal production cost: the difference between the marginal production costs of firms is maximized at $\alpha = 1$ and minimized at $\alpha = 0$. As local government chooses α taking into account both total output level and production inefficiency, partial privatization would not always be chosen given other functional forms.

local welfare B. When $\theta \neq 1$, some portion of the pollution generated in region A is transported to region B. Local welfare B is based on environmental damage. We know that the environmental damage function is a function of the total output and that the total output decreases (increases) with an increase in α when $\theta > (\leq)\ 1/\sqrt{2d}$. Suppose the case where d and θ are small (large). When the local public firm is not privatized, that is, $\alpha = 1$, it produces more (less) and the total output is larger (smaller) than when $\alpha = \alpha^h$. The larger (smaller) the total output is, the larger (smaller) the total emission is. Therefore, $\alpha = \alpha^h$ ($\alpha = 1$) is more desirable than $\alpha = 1$ ($\alpha = \alpha^h$) for local welfare B.

Next, we investigate the total welfare. We compare total welfare at $\alpha = 1$ and $\alpha = \alpha^h$. Calculating $W|_{\alpha=1} - W_{\alpha=\alpha^h}$, we obtain the following proposition.

Proposition 2.

$$W|_{\alpha=1} - W|_{\alpha=\alpha^h} > 0 \qquad \textit{if} \qquad d > \tfrac{1}{2} \textit{ and } \theta \in \left(\sqrt{\tfrac{1}{2d}}, \bar{\theta}\right],$$

$$W|_{\alpha=1} - W|_{\alpha=\alpha^h} < 0 \qquad \textit{if} \quad \begin{cases} d > \frac{1}{2} \textit{ and } \theta \in \left[0, \sqrt{\frac{1}{2d}}\right), \\ d > \frac{1}{2} \textit{ and } \theta \in [\bar{\theta}, 1], \\ d < \frac{1}{2} \textit{ and } \theta \in [0, 1], \end{cases}$$

where $\bar{\theta}$ is the solution of $W|_{\alpha=1} - W_{\alpha=\alpha^h} = 0$.

Proof. See Appendix C. □

The intuition behind Proposition 2 is as follows. When θ and d are small, we find that partial privatization enhances welfare of both regions A and B from (18) and Proposition 1. Therefore, the total welfare is larger after partial privatization. When θ and d are large, partial privatization increases local welfare A but decreases local welfare B. Remember that local welfare B is composed of $-D_B(E_B)$ and $E_B = (1-\theta)Q^h$. When θ and d are sufficiently large, E_B becomes sufficiently small. A decrease of local welfare B is overcome by an increase of local welfare A. Therefore, the total welfare is larger under partial privatization when θ and d are either small or large. Figure 3 shows these results.

3.2 Case (f)

We consider the case where firm 1 is owned by private investors from region B.

Local welfare A, local welfare B, and total welfare are respectively defined as (5), (6), and (7).

In the second stage, each firm maximizes its objective by choosing its quantity. The first-order condition of the maximization problem of firms 0 and 1 are respectively given by

$$\frac{\partial U_0}{\partial q_0} = a - (2 - \alpha + c + d\alpha\theta^2)q_0 - (1 - \alpha + d\alpha\theta^2)q_1 = 0, \tag{19}$$

$$\frac{\partial U_1}{\partial q_1} = a - q_0 - (2 + c)q_1 = 0. \tag{20}$$

Solving the above first-order conditions, we obtain

$$q_0^f = \frac{a(1 + c + \alpha - d\alpha\theta^2)}{(1 + c)(3 - \alpha + c + d\alpha\theta^2)}, \tag{21}$$

$$q_1^f = \frac{a(1 + c - \alpha + d\alpha\theta^2)}{(1 + c)(3 - \alpha + c + d\alpha\theta^2)}, \tag{22}$$

$$w_A^f = \frac{a^2\{(1 + c)^2(6 + c - 4d\theta^2) - 2(1 + c)c\alpha(1 - d\theta^2) - (2 + 3c)(1 - d\theta^2)^2\alpha^2\}}{2(1 + c)^2(3 - \alpha + c + d\alpha\theta^2)^2}, \tag{23}$$

$$\begin{aligned} w_B^f = {} & \frac{a^2\{(2 + c)(1 + c - \alpha)^2 - 4(1 + c)^2(1 - 2\theta)d + (2 + c)\alpha^2 d\theta^4\}}{2(1 + c)^2(3 - \alpha + c + d\alpha\theta^2)^2} \\ & + \frac{2a^2(-2 - 4c - 2c^2 + 2\alpha + 3c\alpha + c^2\alpha - 2\alpha^2 - c\alpha^2)d\theta^2}{2(1 + c)^2(3 - \alpha + c + d\alpha\theta^2)^2}, \end{aligned} \tag{24}$$

$$W^f = \frac{a^2\{(1 + c)^2(4 + c - 2d - 2\alpha + 4d\theta - 4d\theta^2 + 2d\alpha\theta^2) - c\alpha^2(1 - d\theta^2)^2\}}{(1 + c)^2(3 - \alpha + c + d\alpha\theta^2)^2} + m. \tag{25}$$

Here, we analyze the comparative statics for the equilibrium output of each firm with respect to α. We find that

$$\frac{\partial q_0^f}{\partial \alpha} < 0, \ \frac{\partial q_1^f}{\partial \alpha} > 0, \ \text{and} \ \frac{\partial Q^f}{\partial \alpha} < 0, \ \text{ if and only if } d\theta^2 > 1. \tag{26}$$

In the first stage, local government A chooses α in order to maximize local welfare A. Solving for α, we obtain[9]

[9]In Appendix D, we show that the second-order condition of the maximization problem is satisfied. For the calculation of α^f, see Appendix E.

$$
\alpha^f = \begin{cases} \bar{\alpha} & if \begin{cases} 0 < d < \frac{c}{1+2c} \ and \ \theta \in [0, 1], \\ \frac{c}{1+2c} < d \ and \ \theta \in [0, \sqrt{\frac{c}{d(1+2c)}}], \\ \frac{3+2c}{2(1+c)} < d \ and \ \theta \in [\sqrt{\frac{3+2c}{2d(1+c)}}, 1], \end{cases} \\ 1 & if \begin{cases} \frac{c}{1+2c} < d < 1 \ and \ \theta \in [\sqrt{\frac{c}{d(1+2c)}}, 1], \\ 1 < d \ and \ \theta \in [\sqrt{\frac{c}{d(1+2c)}}, \sqrt{\frac{1}{d}}], \end{cases} \\ 0 & if \begin{cases} 1 < d < \frac{3+2c}{2(1+c)} \ and \ \theta \in [\sqrt{\frac{1}{d}}, 1], \\ \frac{3+2c}{2(1+c)} < d \ and \ \theta \in [\sqrt{\frac{1}{d}}, \sqrt{\frac{3+2c}{2d(1+c)}}], \end{cases} \end{cases}
$$

where

$$
\bar{\alpha} = \frac{(1+c)\{3+2c-2d\theta^2(1+c)\}}{(3+6c+2c^2)(1-d\theta^2)}. \tag{27}
$$

From the result, we find that α^f depends on the fraction of transboundary pollution and the degree of environmental damage. Figure 4 illustrates α^f for each d and θ. When both d and θ are small or large (region *I* or *IV*), partial privatization ($\alpha^f = \bar{\alpha}$) is chosen. When they take a middle value, local public firm A is fully privatized (region *III*) or is not privatized at all (region *II*).

The intuition behind the result is as follows. First, we consider the case where d and θ are sufficiently large. In this case, environmental damage is severe in region A, and thus, the local public firm produces less when $\alpha = 1$ than when $\alpha = \bar{\alpha}$. Suppose a marginal decrease of α at $\alpha = 1$. A marginal increase of output of the public firm does not affect welfare because the public firm is a local welfare maximizer when $\alpha = 1$. However, the marginal decrease of output of the private firm improves welfare because it reduces the environmental damage. Therefore, partial privatization enhances welfare.

Second, we consider the case where d and θ are sufficiently small. In this case, the degree of environmental damage is low, and thus, we regard this case as no environmental problem in a mixed duopoly to some extent. Suppose a marginal decrease of α at $\alpha = 1$. As is the same reason mentioned in the previous paragraph, a marginal decrease of output of the public firm does not affect welfare. However, a marginal increase of output of the private firm increases consumer surplus. Therefore, partial privatization enhances welfare.

Finally, we consider the case where d and θ take a middle value. In this case, the equilibrium output in a mixed duopoly is nearly the same as that in a pure duopoly. For example, consider the case where $d\theta^2 = 1$. In this case, the reaction function of the local public firm does not depend on α: the reaction function of each firm is symmetric. And thus, either full privatization or no privatization can be chosen.

Does privatization of the local public firm enhance local welfare in the other region and the total welfare? (welfare comparison)

We examine whether the optimal privatization for local welfare A enhances local welfare B and the total welfare. We compare local welfare B and total welfare at $\alpha = 1$ and $\alpha = \alpha^f$.

Here, we compare local welfare B before and after privatization. Because the level of α^f depends on the values of parameters, we separate the cases for each α^f. Figure 4 shows the level of α^f for the values of parameters: $\bar{\alpha}$ is chosen by the local government A in regions I and IV, 0 in region III, and 1 in region II. In each region, the results of the welfare comparison before and after privatization are as follows.

Proposition 3.

$$
\begin{aligned}
& w_B|_{\alpha=1} - w_B|_{\alpha=\bar{\alpha}} > 0 && \text{if } d > \tfrac{3+2c}{2(1+c)} \text{ and } \theta \in \left(\sqrt{\tfrac{3+2c}{2d(1+c)}}, 1\right], \\
& w_B|_{\alpha=1} - w_B|_{\alpha=\bar{\alpha}} < 0 && \text{if } \begin{cases} d > \tfrac{c}{1+2c} \text{ and } \theta \in \left[0, \sqrt{\tfrac{c}{d(1+2c)}}\right), \\ d < \tfrac{c}{1+2c} \text{ and } \theta \in [0,1] \end{cases} \\
& w_B|_{\alpha=1} - w_B|_{\alpha=0} > 0 && \text{if } d > 1 \text{ and } \theta \in \left(\sqrt{\tfrac{1}{d}}, \sqrt{\tfrac{3+2c}{2d(1+c)}}\right].
\end{aligned}
$$

Proof. See Appendix F. □

According to Proposition 3, when the degree of environmental damage and the fraction of transboundary pollution remaining in region A are low, privatization of the local public firm in region A enhances local welfare B, but when they are high, privatization worsens the welfare. Figure 5 shows these results.

The intuition behind Proposition 3 is as follows. Local welfare B is based on the sum of firm 1's profit and environmental damage. We know that the environmental damage function is a function of the total output and that the total output decreases with an increase in α when $\theta > 1/\sqrt{d}$. We also see that firm 1's profit increases with an increase in α when $\theta > 1/\sqrt{d}$ because the price of the dirty good increases with an decrease in the total output, and the output of firm 1 increases as a result of the strategic substitution effect. Thus, we find that local welfare B increases with an increase in α when $\theta > 1/\sqrt{d}$. When $\theta < 1/\sqrt{d}$, the results are opposite, that is, local welfare B decreases with an increase in α.

Lastly, we compare total welfare between $\alpha = 1$ and $\alpha = \alpha^f$. As in the case of the welfare comparison for region B, we separate the cases for each α^f. The results of the welfare comparison in terms of before and after privatization are as follows for each case.

Proposition 4.

$$\begin{aligned} &W|_{\alpha=1} - W|_{\alpha=\bar{\alpha}} > 0 \quad && if\ d > \tfrac{3+2c}{2(1+c)}\ and\ \theta \in \left(\sqrt{\tfrac{3+2c}{2d(1+c)}}, 1\right], \\ &W|_{\alpha=1} - W|_{\alpha=\bar{\alpha}} < 0 \quad && if\ \begin{cases} d > \tfrac{c}{1+2c}\ and\ \theta \in \left[0, \sqrt{\tfrac{c}{d(1+2c)}}\right), \\ d < \tfrac{c}{1+2c}\ and\ \theta \in [0,1] \end{cases} \\ &W|_{\alpha=1} - W|_{\alpha=0} > 0 \quad && if\ d > 1\ and\ \theta \in \left(\sqrt{\tfrac{1}{d}}, \sqrt{\tfrac{3+2c}{2d(1+c)}}\right]. \end{aligned}$$

Proof. See Appendix G. □

Figure 6 shows Proposition 4. According to Proposition 4, when the degree of environmental damage and the fraction of transboundary pollution remaining in region A are low, privatization of the local public firm in region A enhances the total welfare because local welfare A and B both increase. However, when the same are high, local welfare B is worsened considerably, and the total welfare decreases. Thus, in terms of total welfare, privatization is not desirable.

4 Comparison between cases (h) and (f)

We compare the results obtained in cases (h) and (f). There are three major points.

1. Partial privatization is chosen in case (h), but partial privatization, full privatization, or no privatization can be chosen in case (f).

2. Partial privatization enhances w_A, w_B, and W in both cases when the degree of environmental damage and the fraction of transboundary pollution remaining in region A are low.

3. Partial privatization enhances W and reduces w_B in case (h), but it reduces both w_B and W in case (f) when the degree of environmental damage and the fraction of transboundary pollution remaining in region A are high.

5 Concluding remarks

This paper examines the effect that the privatization of a local public firm has on local welfare in two regions and on the overall welfare of the two regions when the fraction of unidirectional transboundary pollution varies. We analyze this problem by considering two separate cases of ownership of a private firm.

We discuss the possible implication of our results. Consider the example of the relationship between China upstream and Japan downstream. Since the twenty-first century, several Japanese firms have entered the Chinese market. From China's point of view, it is more complex to calculate the optimal degree of privatization in terms of welfare of China in this situation than in the situation where the competitor of the public firm is a domestic private firm; the optimal degree of privatization varies for each value of the degree of environmental damage and the fraction of transboundary pollution. When the pollutant has a moderate degree of environmental damage, the Chinese government should pay particular attention to the trend of the fraction of transboundary pollution, since there is a possibility that its fraction is affected by recent extreme weather conditions.

This paper uses a simple framework to consider the privatization problem in the context of the unidirectional transboundary pollution problem. Therefore, several extensions of this analysis are possible. For example, we can consider the case wherein firms can abate the possible pollution. If firms can reduce their pollution by investing some abatement effort, the public firm produces more when it can invest abatement effort than when it cannot, because in the latter case, it has to reduce its output in order to reduce pollution. As a result, the welfare-maximizing degree of partial privatization changes and the effect of partial privatization of the upstream public firm on welfare of each region might change to a large extent. We can also extend our model to examine the case wherein a market for dirty goods exists in both countries, wherein a generation of pollution occurs in the country located downstream, and wherein the government can impose the environmental regulations such as emission taxes and quotas on firms. We leave these analyses for future research.

Appendix A

The first-order condition of the maximization problem of local government *A* Partially differentiating w_A^h with respect to α, we obtain

$$\frac{\partial w_A^h}{\partial \alpha} = \frac{a^2(1+c)\{(1+c)^2 - (1+3c+c^2)\alpha\}(1-2d\theta^2)^2}{\{(1+c)(3+c+d\alpha\theta^2) - (2+c)\alpha\}^2} = 0. \tag{28}$$

We can easily find that the denominator is positive. We focus on the numerator. When $d\theta^2 = 1/2$, w_A^h does not depend on α. When $d\theta^2 \neq 1/2$, we can derive the optimal degree of partial privatization level for local government A, that is, α^h.

The second-order condition of the maximization problem of local government *A*

To determine whether α^h is the maximizing value for w_A^h, we calculate the second-order condition of the maximization problem for local government A. Then, we obtain

$$\frac{\partial^2 w_A^h}{\partial \alpha^2} = -\frac{a^2(1+c)(1-2d\theta^2)^2 X_0(c,d,\theta,\alpha)}{\{(1+c)(3+c+d\alpha\theta^2) - (2+c)\alpha\}^4} \leq 0, \tag{29}$$

where

$$\begin{aligned} X_0(c,d,\theta,\alpha) \;=\;& (1+c)(-3+c+3c^2+c^3) + 2(2+c)(1+3c+c^2)\alpha \\ & +(3-2\alpha)d\theta^2 + (9-8\alpha)cd\theta^2 + (9-8\alpha)c^2d\theta^2 \\ & +(3-2\alpha)c^3d\theta^2 > 0. \end{aligned} \tag{30}$$

Note that a strict inequality holds when $d\theta^2 \neq 1/2$. Therefore, the second-order condition is satisfied when $d\theta^2 \neq 1/2$.

Appendix B

Proof of Proposition 1. Calculating local welfare B when $\alpha = 1$ and $\alpha = \alpha^h$, we respectively obtain

$$w_B^h|_{\alpha=1} = -\frac{a^2(1+2c)^2(1-\theta)^2 d}{2\{1+3c+c^2+(1+c)d\theta^2\}^2}, \quad (31)$$

$$w_B^h|_{\alpha=\alpha^h} = -\frac{a^2(1+5c+2c^2)^2(1-\theta)^2 d}{2\{1+7c+5c^2+c^3+(1+c)^2 d\theta^2\}^2}. \quad (32)$$

Comparing the above, we obtain the following equation:

$$w_B^h|_{\alpha=1} - w_B^h|_{\alpha=\alpha^h} = -\frac{a^2cd(1+c)(1-\theta)^2(1-2d\theta^2)\{2+17c+37c^2+22c^3+4c^4+2(1+c)(1+4c+2c^2)d\theta^2\}}{2\{1+3c+c^2+(1+c)d\theta^2\}^2\{1+7c+5c^2+c^3+(1+c)^2 d\theta^2\}^2}.$$

From the above equation, we find that $w_B|_{\alpha=1} = w_B|_{\alpha=\alpha^h}$ when $\theta = 1$. Consider the case where $\theta \neq 1$. Whether or not $w_B|_{\alpha=1} - w_B|_{\alpha=\alpha^h}$ is positive depends on the sign of $1 - 2d\theta^2$. Thus, we can derive Proposition 1. □

Appendix C

Proof of Proposition 2. Calculating the total welfare when $\alpha = 1$ and $\alpha = \alpha^h$, we respectively obtain

$$W^h|_{\alpha=1} = \frac{a^2\{1+5c+8c^2+2c^3+(1+2c)^2(2\theta-1)d-4c^2d\theta^2-2cd^2\theta^4\}}{2(1+3c+c^2+d\theta^2+cd\theta^2)^2} + m, \quad (33)$$

$$W^h|_{\alpha=\alpha^h} = \frac{a^2\{(1+6c+2c^2)(1+7c+5c^2+c^3)+(1+5c+2c^2)^2(2\theta-1)d\}}{2(1+7c+5c^2+c^3+d\theta^2+2cd\theta^2+c^2d\theta^2)^2} - \frac{a^2\{4c(1+7c+5c^2+c^3)d\theta^2+2c(1+c)^2d^2\theta^4\}}{2(1+7c+5c^2+c^3+d\theta^2+2cd\theta^2+c^2d\theta^2)^2} + m. \quad (34)$$

The difference between them is

$$W^h|_{\alpha=1} - W^h|_{\alpha=\alpha^h} = \frac{a^2c(-1+2d\theta^2)X_1(c,d,\theta)}{2\{1+3c+c^2+(1+c)d\theta^2\}^2\{1+7c+5c^2+c^3+(1+c)^2d\theta^2\}^2},$$

where

$$\begin{aligned}X_1(c,d,\theta) =& c + 7c^2 + 5c^3 + c^4 + 2d + 19cd + 54c^2d + 59c^3d + 26c^4d + 4c^5d \\ &- 2(1+c)(2 + 17c + 37c^2 + 22c^3 + 4c^4)d\theta \\ &+ 2d(1 + 9c + 21c^2 + 25c^3 + 12c^4 + 2c^5 + d + 6cd + 11c^2d + 8c^3d + 2c^4d)\theta^2 \\ &- 4(1+c)^2(1 + 4c + 2c^2)d^2\theta^3 + 2(1+c)^3(1+2c)d^2\theta^4. \qquad (35)\end{aligned}$$

When $d\theta^2 = 1/2$, there is no difference between them. We consider the case where $d\theta^2 \neq 1/2$. Whether or not the difference is positive depends on both the sign of $-1+2d\theta^2$ and that of $X_1(c,d,\theta)$. At first glance, it is not clear whether or not $X_1(c,d,\theta)$ is positive. In the subsequent analyses, we examine the property of $X_1(c,d,\theta)$.

First, we check the monotonicity of $X_1(c,d,\theta)$ in $\theta \in [0,1]$. Partially differentiating $X_1(c,d,\theta)$ with respect to θ, we find that

$$\begin{aligned}\frac{\partial X_1(c,d,\theta)}{\partial\theta} =& 2d\{-(1+c)(2 + 17c + 37c^2 + 22c^3 + 4c^4) \\ &+ 2(1 + 9c + 21c^2 + 25c^3 + 12c^4 + 2c^5 + d + 6cd + 11c^2d + 8c^3d + 2c^4d)\theta \\ &- 6(1+c)^2(1 + 4c + 2c^2)d\theta^2 + 4(1+c)^3(1+2c)d\theta^3\}. \qquad (36)\end{aligned}$$

Summing up the above terms, we find that $\partial X_1(c,d,\theta)/\partial\theta = 2dX_2(c,d,\theta)$, where

$$\begin{aligned}X_2(c,d,\theta) =& -2(1-\theta)\{1 + 9c + 21c^2 + 25c^3 + 12c^4 + 2c^5 + 2(1+c)^3(1+2c)d\theta^2\} \\ &+ 2(1-\theta)(1+c)^2(1 + 4c + 2c^2)d\theta - c(1 + 12c + 9c^2 + 2c^3) \\ &- 4c(1+c)^2d\theta^2. \qquad (37)\end{aligned}$$

The above calculation shows that the second term is positive whereas the other terms are negative. As the upper bound of d is assumed to be $1+c$, we substitute $1+c$ into d only in the second term of the above equation. Summing up the calculation, we obtain

$$\begin{aligned}X_2(c,d,\theta) =& -2(1-\theta)\{1 - \theta + (9-7\theta)c + (21-17\theta)c^2 + (25-19\theta)c^3 \\ &+ 2(6-5\theta)c^4 + 2(1-\theta)c^5 + 2(1+c)^3(1+2c)d\theta^2\} \\ &- c(1 + 12c + 9c^2 + 2c^3) - 4c(1+c)^2d\theta^2 < 0. \qquad (38)\end{aligned}$$

Therefore, we find that $\partial X_1(c,d,\theta)/\partial\theta < 0$.

Second, we check the sign of $X_1(c,d,0)$ and $X_1(c,d,1)$. When $\theta = 0$, we find

$$X_1(c,d,0) = c + 7c^2 + 5c^3 + c^4 + 2d + 19cd + 54c^2d + 59c^3d + 26c^4d + 4c^5d > 0. \quad (39)$$

When $\theta = 1$, we find

$$X_1(c,d,1) = -c(-1+2d)(1+7c+5c^2+c^3+d+2cd+c^2d). \quad (40)$$

When $d > 1/2$, this term is negative. As mentioned previously, $X_1(c,d,\theta)$ decreases with respect to θ, and therefore, there exists a unique $\bar{\theta} \in [0,1]$ at which $X_1(c,d,\bar{\theta})$ is equal to 0. When $d < 1/2$, this term is positive. Then, $X_1(c,d,\theta)$ is always positive in $\theta \in [0,1]$.

Finally, we examine the magnitude of the relation between $\sqrt{1/(2d)}$ and $\bar{\theta}$. Substituting $\sqrt{1/(2d)}$ into θ in $X_1(c,d,\theta)$, we find

$$X_1(c,d,\sqrt{\frac{1}{2d}}) = d(1+c)(3+c)(1+2c)(1+5c+2c^2)\left(1-\sqrt{\frac{1}{2d}}\right)^2 \geq 0, \quad (41)$$

where a strict inequality holds when $d \neq 1/2$. As $X_1(c,d,\theta)$ is a decreasing function with respect to θ and $X_1(c,d,\bar{\theta}) = 0$, we find that $\sqrt{1/(2d)} \leq \bar{\theta}$, where a strict inequality holds when $d \neq 1/2$.

On the basis of the above analyses, we can draw Figure 3 and derive Proposition 2. □

Appendix D

The first-order condition of the maximization problem of local government *A* Partially differentiating w_A^f with respect to α, we obtain

$$\begin{aligned}\frac{\partial w_A^f}{\partial \alpha} &= \frac{2a^2(1-d\theta^2)\{(1+c)(3+2c-2(1+c)d\theta^2)-(3+6c+2c^2)(1-d\theta^2)\alpha\}}{(1+c)^2(3+c-\alpha+d\alpha\theta^2)^3} \\ &= 0. \quad (42)\end{aligned}$$

When $d\theta^2 = 1$, w_A^f does not depend on α. When $d\theta^2 \neq 1$, we derive $\bar{\alpha}$ by solving the above equation with respect to α. Note that because both the sign and value of $\bar{\alpha}$ vary with the value of the parameters of c, d, and θ, it is necessary to examine $\bar{\alpha}$ in detail. For further details regarding α^f, see Appendix E.

The second-order condition of the maximization problem of local government A

To determine whether $\bar{\alpha}$ is the maximization value for w_A^f, we calculate the second-order condition of the maximization problem of local government A. Then, we obtain

$$\frac{\partial^2 w_A^f}{\partial \alpha^2} = -\frac{4a^2(1+c)(1-d\theta^2)^2 Y_0(c,d,\theta,\alpha)}{(1+c)^2(3+c-\alpha+d\alpha\theta^2)^4} \leq 0, \tag{43}$$

where

$$\begin{aligned} Y_0(c,d,\theta,\alpha) &= c(3+3c+c^2)+(3+6c+2c^2)\alpha \\ &\quad +3(1-\alpha)d\theta^2+6(1-\alpha)cd\theta^2+(3-2\alpha)c^2d\theta^2 > 0. \end{aligned} \tag{44}$$

Note that a strict inequality holds when $d\theta^2 \neq 1$. Therefore, the second-order condition is satisfied when $d\theta^2 \neq 1$.

Appendix E

Derivation of α^f

Consider the case where $d\theta^2 \neq 1$. There is a possibility that $\bar{\alpha}$ is negative or that $\bar{\alpha}$ is greater than 1. In the subsequent analyses, we ascertain the sign and value of $\bar{\alpha}$ for each value of parameter.

First, we derive the condition where $\bar{\alpha}$ is positive. In order to obtain a positive $\bar{\alpha}$, the following conditions have to be satisfied:

$$d\theta^2 < (>)\ \frac{3+2c}{2(1+c)} \text{ and } d\theta^2 < (>)\ 1. \tag{45}$$

As $\sqrt{(3+2c)/\{2d(1+c)\}} > \sqrt{1/d}$, we obtain

$$\bar{\alpha} > 0 \ \text{ if } \begin{cases} d\theta^2 < 1, \\ d\theta^2 > \frac{3+2c}{2(1+c)}. \end{cases} \tag{46}$$

Next, we examine whether or not α^f is less than 1. Calculating $1-\bar{\alpha}$, we obtain

$$1-\bar{\alpha} = \frac{c-(1+2c)d\theta^2}{(3+6c+2c^2)(1-d\theta^2)}. \tag{47}$$

When the above equation is positive, $\bar{\alpha}$ is less than 1. Thus, the conditions where $\bar{\alpha}$ is less than 1 are given by

$$d\theta^2 < (>)\ 1 \text{ and } d\theta^2 < (>)\ \frac{c}{(1+2c)}. \tag{48}$$

As $1 > c/(1+2c)$, we obtain

$$\bar{\alpha} < 1 \quad \text{if} \quad \begin{cases} d\theta^2 < \frac{c}{1+2c}, \\ d\theta^2 > 1. \end{cases} \tag{49}$$

Summing up the above conditions while taking into account the fact that θ must be in $[0, 1]$, we can draw Figure 4 and derive α^f.

Appendix F

Proof of Proposition 3. Calculating local welfare B for each value of α^f, we obtain

$$w_B^f|_{\alpha=0} = \frac{a^2(2+c-4d+8d\theta-4d\theta^2)}{2(3+c)^2}, \tag{50}$$

$$w_B^f|_{\alpha=1} = \frac{a^2\{c^2(2+c)-4(1+c)^2(1-2\theta)d-2(2+2c+c^2)d\theta^2+(2+c)d^2\theta^4\}}{2(1+c)^2(2+c+d\theta^2)^2}, \tag{51}$$

$$w_B^f|_{\alpha=\bar{\alpha}} = \frac{a^2\{c^2(2+c)^3-(3+6c+2c^2)^2(1-2\theta)d\}}{2\{3+8c+5c^2+c^3+(1+c)^2d\theta^2\}^2} + \frac{a^2\{-(9+28c+32c^2+14c^3+2c^4)d\theta^2+(1+c)^2(2+c)d^2\theta^4\}}{2\{3+8c+5c^2+c^3+(1+c)^2d\theta^2\}^2}. \tag{52}$$

According to Figure 4, local government A does not privatize firm 0 in region *II*. In this case, local welfare B is unchanged. In region *III*, it is necessary to compare $w_B|_{\alpha=1}$ with $w_B|_{\alpha=0}$. The result is shown by

$$w_B|_{\alpha=1} - w_B|_{\alpha=0} = -\frac{2a^2(1-d\theta^2)Y_1(c,d,\theta)}{(1+c)^2(3+c)^2(2+c+d\theta^2)^2}, \tag{53}$$

where

$$Y_1(c,d,\theta) = (2+c)(1+3c+c^2)+(2+c)^2\theta^2+(1+c)^2(1-\theta)^2(5+2c+d^2\theta^2) > 0. \tag{54}$$

Therefore, whether or not $w_B|_{\alpha=1}$ is larger than $w_B|_{\alpha=0}$ depends on the sign of $1 - d\theta^2$.

In regions I and IV, it is necessary to compare $w_B|_{\alpha=1}$ with $w_B|_{\alpha=\bar{\alpha}}$. The result is shown by

$$w_B|_{\alpha=1} - w_B|_{\alpha=\bar{\alpha}} = -\frac{a^2\{c-(1+2c)d\theta^2\}Y_2(c,d,\theta)}{2(1+c)^2(2+c+d\theta^2)^2\{3+8c+5c^2+c^3+(1+c)^2d\theta^2\}^2}, \quad (55)$$

where

$$\begin{aligned} Y_2(c,d,\theta) \;=\;& c(2+c)(7+16c+10c^2+2c^3)+d(2+c)(1+2c)(5+6c+2c^2)\theta^2 \\ &+d(1+c)^2(12+31c+20c^2+4c^3)(1-\theta)^2 \\ &+(1+c)^2d^2\theta^2\{(5+10c+4c^2)(1-\theta)^2+2(2+c)\theta^2\} > 0. \qquad (56)\end{aligned}$$

Therefore, whether or not $w_B|_{\alpha=1}$ is larger than $w_B|_{\alpha=\bar{\alpha}}$ depends on the sign of $c-(1+2c)d\theta^2$.

Figure 5 and Proposition 3 sum up the above analyses. □

Appendix G

Proof of Proposition 4. Calculating total welfare for each value of α^f, we obtain

$$W_{\alpha=0} = \frac{a^2\{4+c-2d+4d(1-\theta)\theta\}}{(3+c)^2}+m, \quad (57)$$

$$W_{\alpha=1} = \frac{a^2\{2+4c+4c^2+c^3-2d(1+c)^2(1-2\theta)-2(1+c+c^2)d\theta^2-cd^2\theta^4\}}{(1+c)^2(2+c+d\theta^2)^2}+m, \quad (58)$$

$$\begin{aligned} W_{\alpha=\bar{\alpha}} =\;& \frac{a^2\{(3+c)(3+12c+18c^2+10c^3+2c^4)-(1-2\theta)(3+6c+2c^2)^2d\}}{2(3+8c+5c^2+c^3+d\theta^2+2cd\theta^2+c^2d\theta^2)^2} \\ &-\frac{2a^2\{6+21c+26c^2+12c^3+2c^4+c(1+c)^2d^2\theta^4\}}{2(3+8c+5c^2+c^3+d\theta^2+2cd\theta^2+c^2d\theta^2)^2}+m. \qquad (59)\end{aligned}$$

According to Figure 2.6, total welfare as well as local welfare B is unchanged in region II. In region III, it is necessary to compare $W|_{\alpha=1}$ with $W|_{\alpha=0}$. The result is given by

$$W|_{\alpha=1} - W|_{\alpha=0} = -\frac{2a^2(1-d\theta^2)Y_3(c,d,\theta)}{(1+c)^2(3+c)^2(2+c+d\theta^2)^2}, \quad (60)$$

where

$$\begin{aligned} Y_3(c,d,\theta) \;=\;& -1+2c+c^2+d(1+c)^2(5+2c)(1-\theta)^2+d(3+3c+3c^2+c^3)\theta^2 \\ &+(1+c)^2\{(1-\theta)^2+\theta^2\}d^2\theta^2 > 0. \qquad (61)\end{aligned}$$

Therefore, whether $W|_{\alpha=1}$ or not is larger than $W|_{\alpha=0}$ depends on the sign of $1 - d\theta^2$.

In regions I and IV, it is necessary to compare $W|_{\alpha=1}$ with $W|_{\alpha=\bar{\alpha}}$. The result is shown by

$$W|_{\alpha=1} - W|_{\alpha=\bar{\alpha}} = -\frac{a^2\{c-(1+2c)d\theta^2\}Y_4(c,d,\theta)}{2(1+c)^2(2+c+d\theta^2)^2(3+8c+5c^2+c^3+d\theta^2+2cd\theta^2+c^2d\theta^2)^2}, \tag{62}$$

where

$$\begin{aligned} Y_4(c,d,\theta) = & 17c + 47c^2 + 41c^3 + 15c^4 + 2c^5 + 3(1+c)^2d^2\theta^4 \\ & + (1+c)^2d(1-\theta)^2\{12+31c+20c^2+4c^3+(5+10c+4c^2)d\theta^2\} \\ & + (7+24c+25c^2+12c^3+2c^4)d\theta^2 > 0. \end{aligned} \tag{63}$$

Therefore, whether or not $w_B|_{\alpha=1}$ is larger than $w_B|_{\alpha=\bar{\alpha}}$ depends on the sign of $c - (1 + 2c)d\theta^2$.

Figure 6 and Proposition 4 sum up the above analyses. □

References

Bárcena-Ruiz, J. C. and Garzón M. B., (2006), "Mixed oligopoly and environmental policy", *Spanish Economic Review*, **8**: 139-160.

Beladi, H. and Chao, C.-C., (2006), "Does privatization improve the environment?", *Economics Letters*, **93**: 343-347.

Bös, D., (1991), *Privatization: a theoretical treatment*, Clarendon Press, Oxford.

Cato, S., (2008), "Privatization and the environment", *Economics Bulletin*, **12**: 1-10.

Fernández, J. A. and Fernández-Stembridge, L., (2007), *China's state-owned enterprise reforms – an industrial and CEO approarch*, Routledge.

Fjell, K. and Heywood, S., (2004), "Mixed oligopoly, subsidization and the order of firm's moves: the relevance of privatization", *Economics Letters*, **83**: 411-416.

Fjell, K. and Pal, D., (1996), "A mixed oligopoly in the presence of foreign private firms", *Canadian Journal of Economics*, **29**: 737-743.

Ichikawa, Y. and Fujita, S., (1995), "An analysis of wet deposition of sulfate using a trajectory model for East Asia", *Water, Air, and Soil Pollution*, **85**: 1927-1932.

Kato, K., (2006), "Can allowing to trade permits enhance welfare in mixed oligopoly?", *Journal of Economics*, **88**: 263-283.

Kato, K., (2010), "Emission Quota versus Emission Tax in a Mixed Duopoly", forthcoming in *Environmental Economics and Policy Studies.*

Lu, Y., (2006), "Endogenous timing in a mixed oligopoly with foreign competitors: The linear demand case", *Journal of Economics*, **88**: 49-68.

Matsumura, T. (1998), "Partial privatization in mixed duopoly", *Journal of Public Economics*, **70**: 473-483.

Matsumura, T., (2003), "Stackelberg mixed duopoly with a foreign competitor", *Bulletin of Economic Research*, **55**: 275-287.

Nagase, Y. and Silva, E. C. D., (2007), "Acid rain in China and Japan: a game-theoretic analysis", *Regional Science and Urban Economics*, **37**: 100-120.

Naito, T. and Ogawa, H., (2009), "Direct versus indirect environmental regulation in a partially privatized mixed duopoly", *Environmental Economics and Policy Studies*, **10**: 87-100.

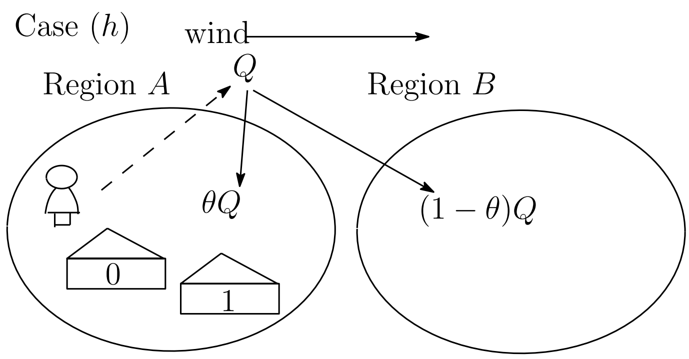
Ohara, T., Uno, I., Wakamatsu, S., and Murano, K., (2001), "Numerical simulation of the springtime transboundary air pollution in East Asia", *Water, Air, and Soil Pollution*, **130**: 295-300.

Ohori, S., (2006a), "Optimal environmental tax and level of privatization in an international duopoly", *Journal of Regulatory Economics*, **29**: 225-233.

Ohori, S., (2006b), "Trade liberalization, consumption externalities and the environment: a mixed duopoly approach", *Economics Bulletin*, **17**: 1-9.

Wang, L. F.S. and Wang, J., (2009), "Environmental taxes in a differentiated mixed duopoly", *Economic Systems*, **33**: 389 – 396.

Wang, L. F.S., Wang, Y., and Zhao, L., (2009), "Privatization and the Environment in a Mixed Duopoly with Pollution Abatement", *Economics Bulletin*, **29**: 3112 – 3119.



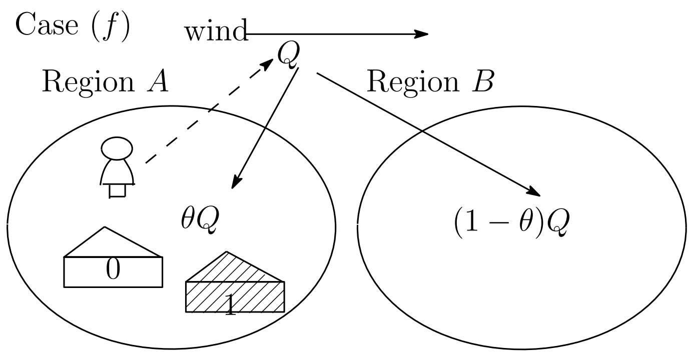


Figure 1: Two cases of ownership are considered in this paper. Case (h): firm 1 is owned by private investors from region A; Case (f): firm 1 is owned by private investors from region B (shaded object).

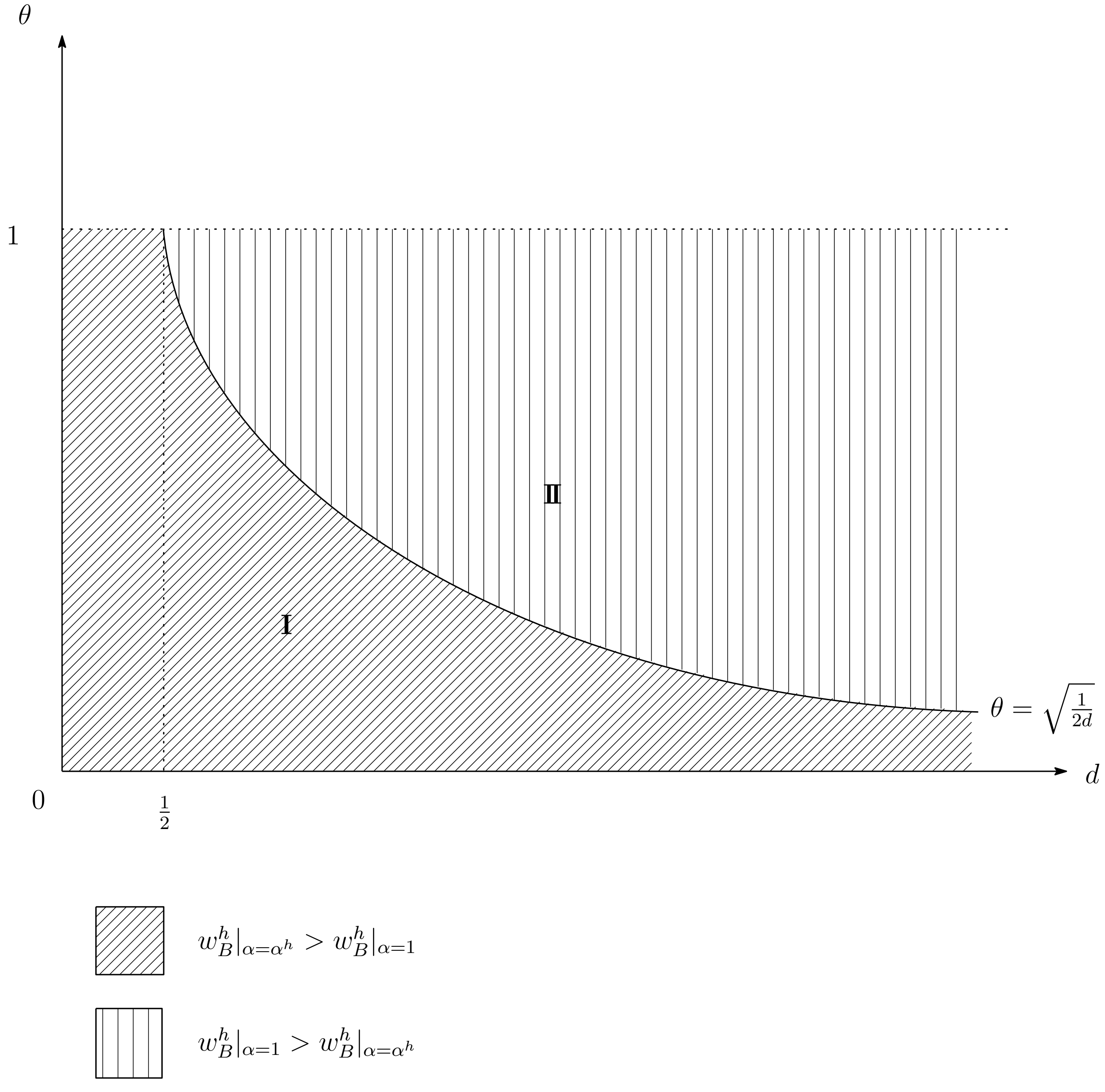


Figure 2: Comparisons between $w_B^h|_{\alpha=1}$ and $w_B^h|_{\alpha=\alpha^h}$ for each value of parameter of d and θ.

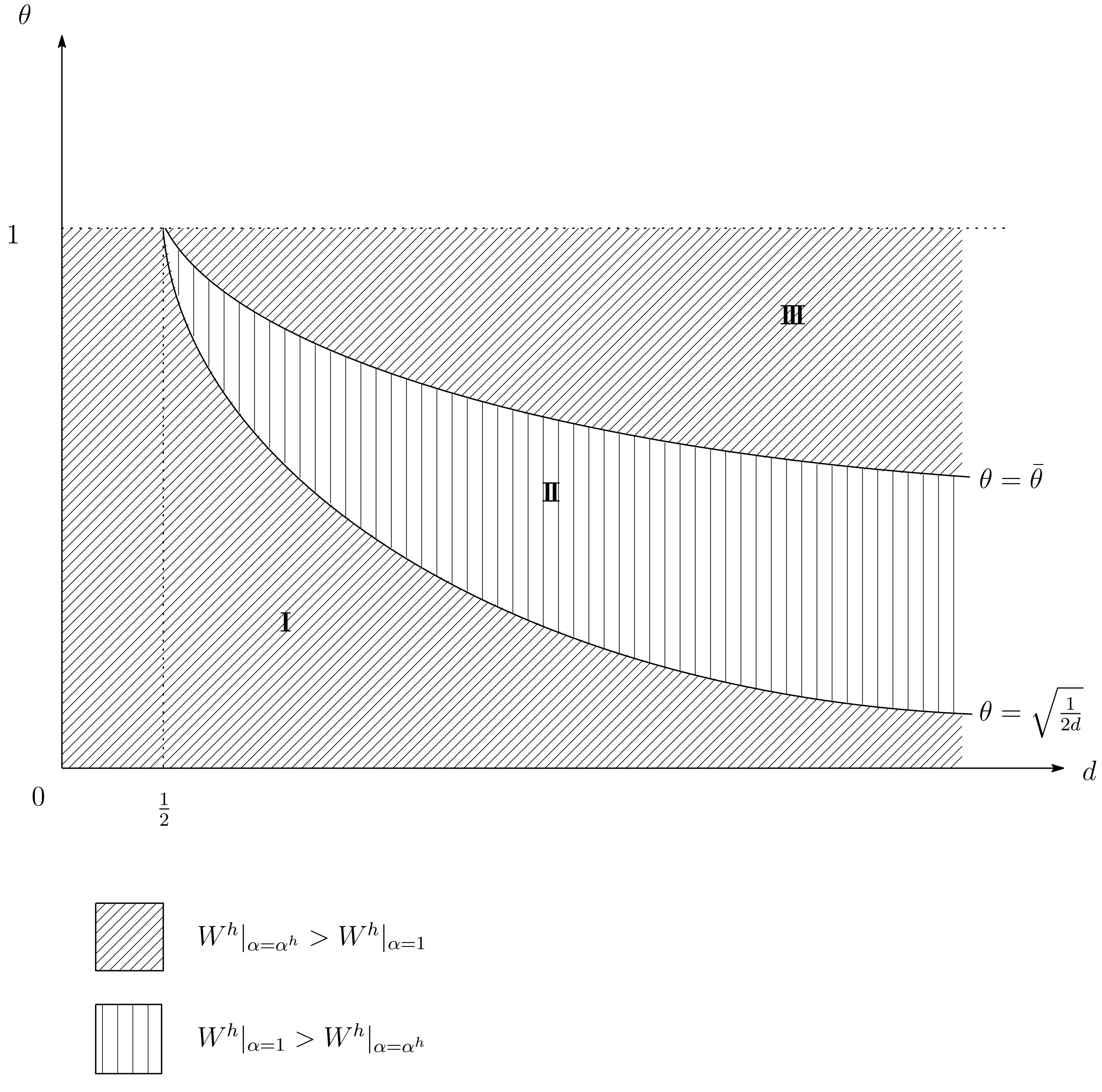


Figure 3: Comparison between $W^h|_{\alpha=1}$ and $W^h|_{\alpha=\alpha^h}$ for each value of parameter of d and θ.

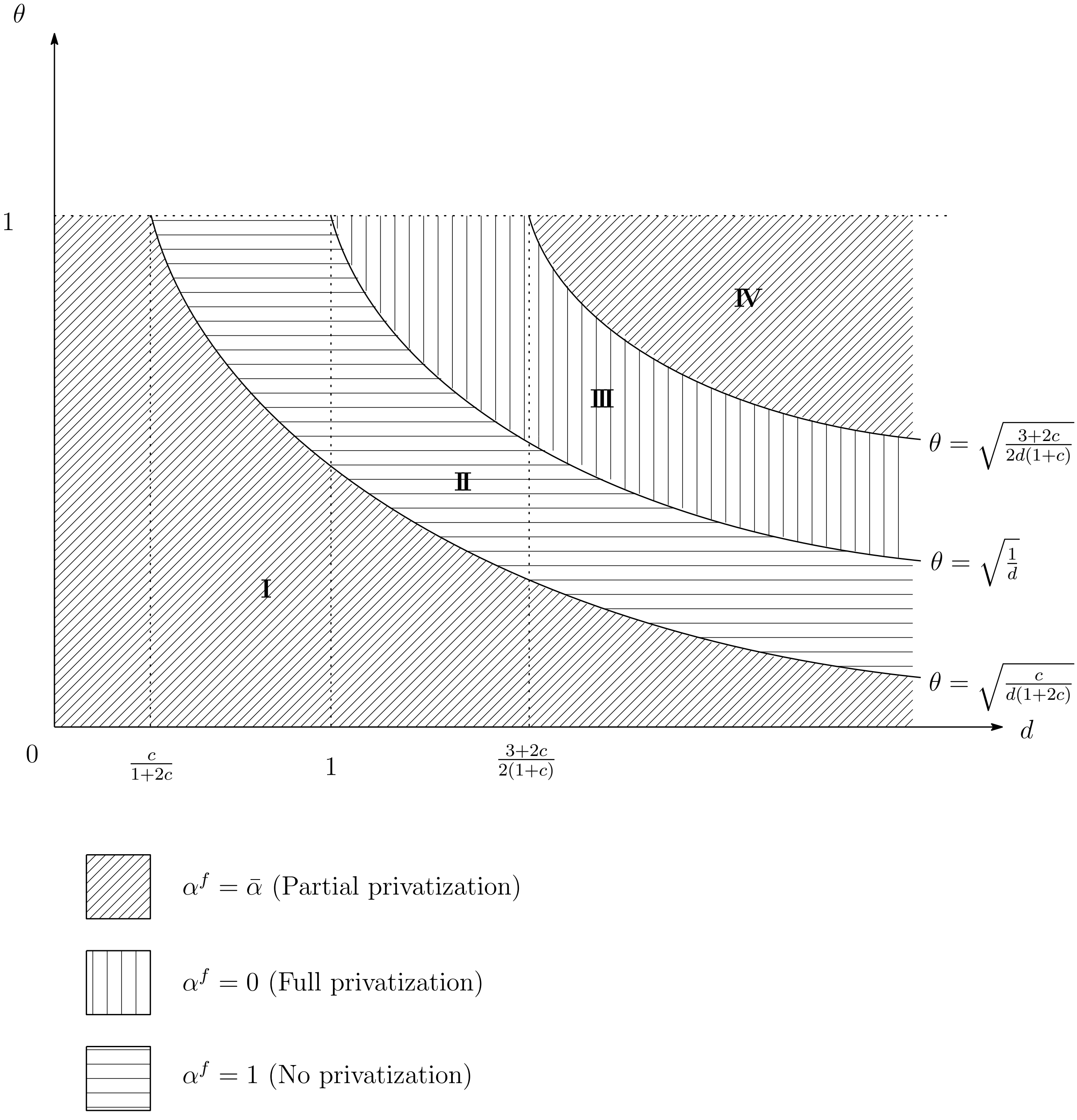


Figure 4: α^f for each value of parameter of d and θ.

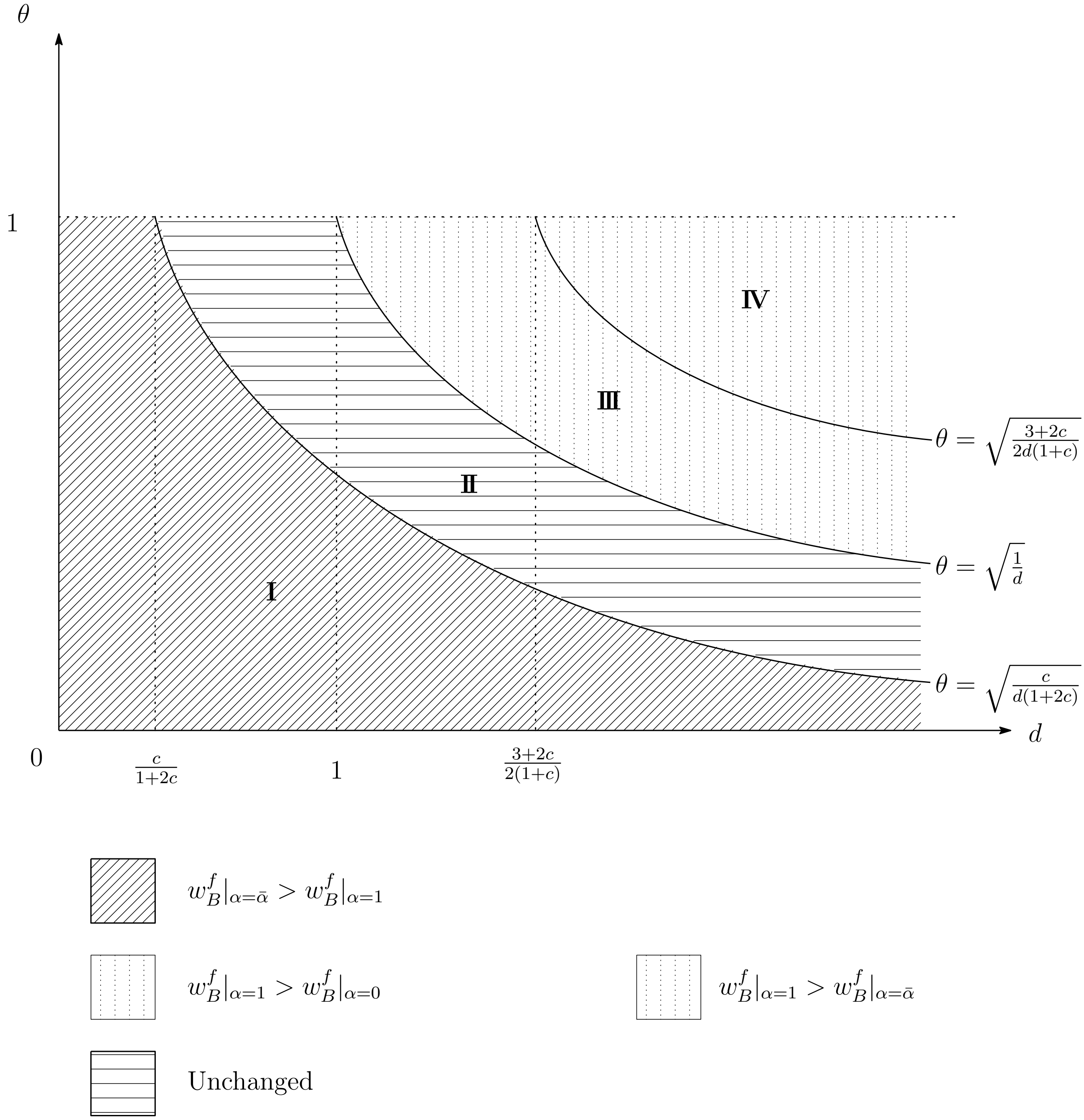


Figure 5: Comparisons between $w_B^f|_{\alpha=1}$, $w_B^f|_{\alpha=\alpha^h}$, and $w_B^f|_{\alpha=0}$ for each value of parameter of d and θ.

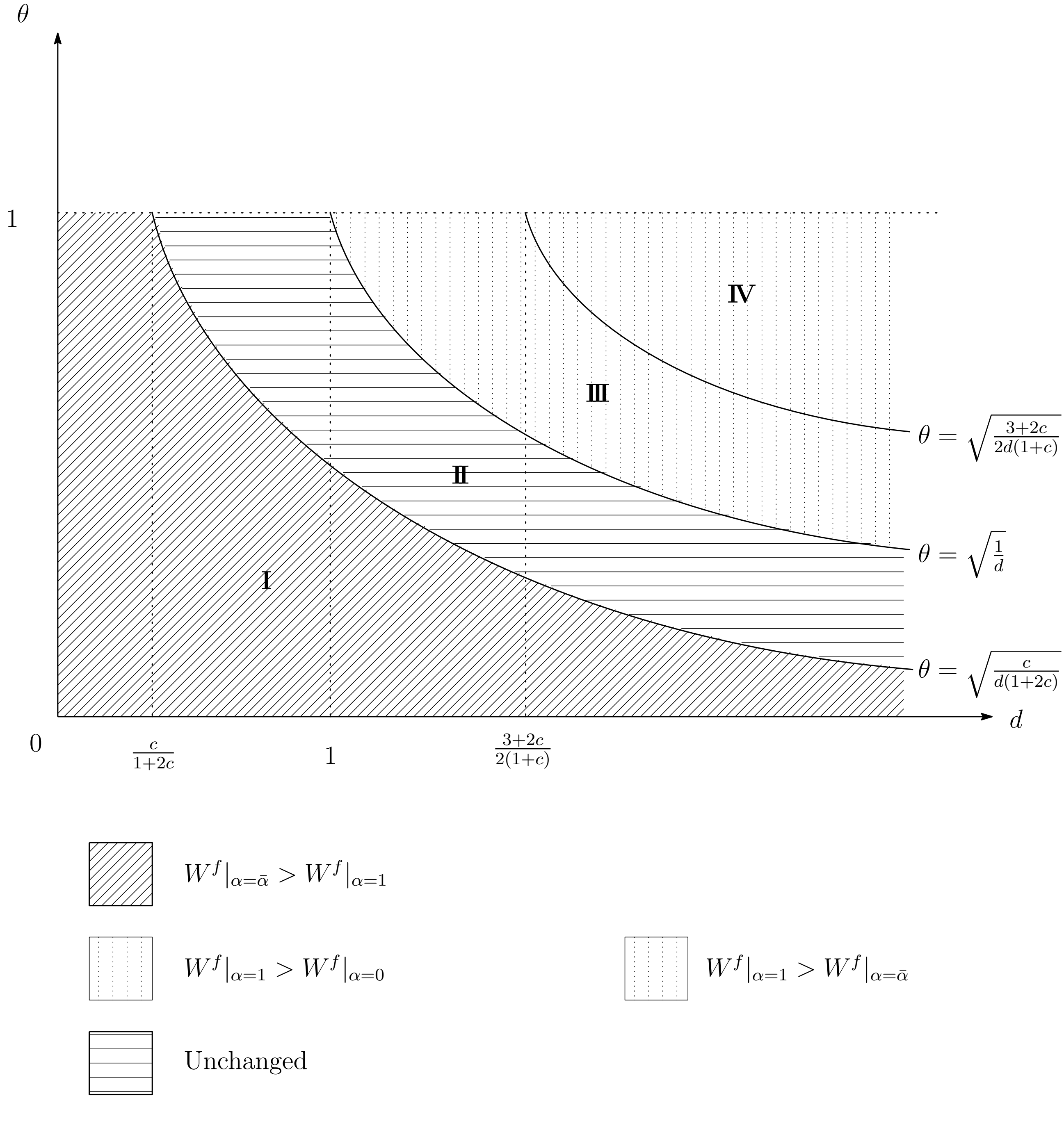


Figure 6: Comparisons between $W^f|_{\alpha=1}$, $W^f|_{\alpha=\alpha^h}$, and $W^f|_{\alpha=0}$ for each value of parameter of d and θ.