Rights on what is left

Laurent-Lucchetti, Jérémy

University of Bern and Oeschger center for climate change

September 2009

Rights on what is left

Jérémy Laurent-Lucchetti[*,†]

December 8, 2010

[*]Bern University, department of economics and Oeschger Center for Climate Change.

[†]I am very grateful for stimulating conversations with Christian Almer, Stefan Ambec, Olivier Bochet, Sidartha Gordon, Pierre Lasserre, Hervé Moulin, Bernard Sinclair-Desgagné and William Thomson as well as for feedback from participants of the Montreal Natural Resources and Environmental Economics Workshops and the Social Choice and Welfare Conference. I specifically want to thank Justin Leroux, his valuable advice greatly improve this work. I gratefully acknowledge the support of the Swiss National Center of Competence in Research on Climate (NCCR Climate), funded by the Swiss National Science Foundation (SNF).

Allocating property rights on an open access resource which has been freely exploited in the past is often very problematic. Involved agents typically rely on one of two competing principles to determine future allocation. The first priority principle, *first in time, first in rights* favors the status quo while the other one, *historical accountability*, is a corrective justice argument. We construct a simple model inspired by the claims problem literature to show that these two positions are in fact compatible: they define bounds to the set of possible allocations. We detail a family of methods which meets these bounds and characterize the two extreme points of this family: the equal sharing and the uniform gains methods.

Keywords: Claims problems, axiomatic solutions, distributive justice, environmental economics, property rights, tragedy of the commons.

JEL Classification Numbers: D30, D63, D70, H23, Q2, Q3, Q56.

1 Introduction

The so-called "tragedy of the commons"[1] concerns the free access regime to a common resource (fisheries, water, stock of clean air...), where every agent exploits independently the common. It is known that this regime leads to over-exploitation of the resource. The lack of efficiency of this free access regime is due to the absence of clear property rights: each agent can choose freely whether to exploit or not the resource and at what level. Efficiency requires reducing the exploitation level by regulating the access to the common. In practice, an approach employed increasingly for coping with the problem of rationing access to a common is the "Cap and Trade" programme (see Tienteberg, 2003, for a survey of concrete applications), which involves permits trading.

In a Cap and Trade approach, a total resource access limit (the cap) is defined and allocated among users. A permits market is then created in order to facilitate the trading of property rights among users. Compliance is established by simply comparing actual use with the assigned user-specific cap as adjusted by any acquired or sold permits. In practice, regulatory tools are implemented by defining specific property rights over the resource. Several methods for allocating initial entitlements have been used (lotteries, auctions...) but the most common method for the applications discussed here is allocating property rights free of charge, based upon *historic use*.

However, fair allocation in this context is a controversial issue because involved agents typically rely on two different principles to determine the allocation of property rights: agents which have already highly exploited the resource claim that they "acquired" a larger share of the resource by investing more in it, while agents which have exploited less claim that they should obtain a larger share of what is remaining because they have benefited less from it. The first argument, known as "*first in time, first in rights*" in the literature (see for example Ott and Sachs, 2002), considers agent's current level of exploitation as a firm property right over the resource. This

[1] See Ostrom (1999) or Moulin (2004) for comprehensive reviews of the "tragedy".

argument can be tracked as far as Locke (Second Treatise of Government) who argues that an agent could claim to ownership of a common resource by working on this resource (by "mixing" his labor with the resource). This argument is further developed in the "entitlement theory of justice" of Robert Nozick (1974). In the pollution case, it translates as follows: because the high-emitting countries were unaware that they were overusing a commons—emissions is the by-product of a wealth creation process—it would be unfair to ask them to restrict their use to a lower total share than other agents. The competing argument, traditionally named "*historical accountability*", considers that agents whom have benefited less from a common in the past should obtain a higher amount of what remains. This argument of *corrective justice* considers the resource has a common property. In the pollution case, this principle is often associated with a general "right to development" linked to the future needs of agents whom have emitted less in the past and, as such, are deemed to be less responsible for the accumulated stock of past emissions.

These principles are seldom explicitly advocated but underlie many of the arguments on the subject. In fact, they are often associated with specific methods for allocating the rights: respectively *proportional* sharing (with respect to past exploitation) and *equal sharing* (of what is left of the resource). For several environmental problems, the first principle is often selected and property rights are typically allocated proportionally to past emissions (e.g, the Acid Rain Program of the 1990 Clean Air Act in the USA). This obviously contradicts the *historical accountability* principle and, in practice, contestations are overcome by the presence of a regulatory agency which enforces the allocation. However, the problem remains intact for situations where no regulatory agency exists. The allocation of property rights over CO_2 emissions is one of the prominent examples of such a situation: a voluntary agreement must be reached because no international authority can enforce a given allocation. Our goal is to design a method for allocating property rights which may overcome objections based on these fairness arguments.

The literature in environmental economics has long recognized the importance of fairness issues. For the climate change example, a first strand of literature (see for example, Rose et al., 1998, and Tol, 2001) focuses on determining the impact of different allocations methods on welfare and emission paths using different setups calibrated to real world data (i.e. integrated assessment model for Rose et al.). A second strand is interested in identifying "fairness bias" (see for example Johansson-Stenman and Konow, 2010) or the strategic use of fairness argument to improve one's payoff (see Lange et al., 2010, for a recent reference). Our focus here is quite different: we formally assess the link between the main fairness arguments—considered as axioms—and the allocation methods that they logically imply. The main contribution of the paper lies in the characterization of allocation methods meeting *both* principles.

Following a well established opinion in the climate change literature, we feel that defining a family of methods for allocating property rights which meets both bounds will help reach the voluntary agreements of all agents. This argument is exposed by Müller (1999) as follows: "*the chances of overcoming a doomsday negotiating scenario are enhanced—indeed, maximized as far as selection bases are concerned—by adopting what I shall refer to as an "equitable" selection base including all (and only) the proposals put forward by the parties concerned, provided (i) they can be justified by an equity principle, (ii) they do not give rise to double counting, and (iii) the principles involved are "morally independent".*"

We thus start by defining formally both equity principles and show that they are independent. In our setup, the first property *first in time, first in rights* translates as follows: if an agent has exploited a resource more than an other, then he should not obtain a lower amount of the *whole* resource. Note that a stronger version of this principle has been proposed in the literature[2]: if an agent has exploited a resource more than an other, then he should not obtain a lower share of the *remaining* resource.

[2]See, for an example in a different framework, Rose et al. (1998).

We view this version as overly strong because it extends the rights on what has not yet been exploited. In fact, even the most fervent proponents of the argument do not support this interpretation, because it excludes the "Lockean Proviso" (see Nozick, 1974): though individuals have a right to acquire private property from nature, they must leave "enough and as good in common...to others." The proviso says that though every appropriation of property is a diminution of another's rights to it, it is acceptable as long as it does not make anyone worse off than they would have been without any private property. Extending the property rights from past exploitation to the non-exploited resource clearly violates the proviso. We thus choose to retain a definition closer to the original ethical justification[3]. The definition of the second property, *historical accountability*, is more consensual in the literature. It translates as follows: if an agent has exploited a resource less than an other, then he should not obtain a lower share of the *remaining* resource.

We use a model inspired by the rationing and surplus sharing literature[4] to address the problem. The remaining resource to be shared is equal to the initial stock of the resource minus the sum of the quantities already exploited by each agent. The problem consists in finding an allocation over what is left of the resource, using past exploitation as the relevant information for allocating shares. This problem differs from the traditional rationing—and surplus sharing—one because we cannot consider past exploitation as legitimate "claims": there is no *a priori* reasons to consider the past exploitation of an agent as a minimum or maximum value for his future share.

We first discuss traditional surplus-sharing methods in this context. We introduce two properties, *sustainability* and *autarky*, which contribute to the characterization of the most progressive and the most regressive methods among the family of methods

[3]However, if one chooses to retain this version of the principle, excluding the Proviso, then it will be clear from the proof of our results that only one method meets both principles: the *equal sharing* method.

[4]See Young (1987) and Young (1990) for applications of these models to the taxation problem. See Moulin (2002) and Thomson (2003) for comprehensive surveys.

we propose. By a "progressive" method, we mean one which gives a higher quantity of what is remaining to agent i, relatively to agent j, if agent i has exploited less the resource than j before, and conversely for a "regressive" method (as in Young, 1990). The goal of these properties is to give priority in the distribution to those agents which have already exploited less or, respectively, exploited more, the resource. *Sustainability* considers that an agent should not obtain more than the quantity he would have obtained if all agents had exploited the resource similarly as him. This property, along with *first in time, first in rights*, *historical accountability*, and *consistency* (a well-known property on claims problem literature) characterizes the uniform gain solution, the most progressive method meeting *first in time, first in rights*. *Autarky* says that the share of an agent should only depend upon her own past exploitation and the amount to be shared, and not upon the distribution of past exploitation. This property, along with *first in time, first in rights* and *historical accountability*, characterizes the equal sharing method. Many observers advocated this method as being the most progressive one (i.e, Posner and Sunstein, 2002). Surprisingly, this is the most regressive method meeting *historical accountability.*

Then, we propose a family of methods which meets both bounds. It turns out that a simple representation of this family of methods possesses a "progressivity" parameter which allows to select among a convex set of the two methods equal sharing and uniform gains. Finally, we propose a dynamic interpretation of the model in order to address the problem of regulating successive flows of pollution or the renewable resource case.

2 The model and the main properties

Let $N = \{1, ..., n\}$ be the set of agents (firms, individuals or countries). Each agent $i \in N$ has already exploited a quantity $x_i \geq 0$ of a resource. Let $x = (x_i)_{i \in N}$ be the profile of these quantities. The initial stock of this resource is denoted by t, where

$t \in \mathbb{R}_+$. This amount may be the existing stock of the resource (e.g. oil or gas reservoir, water tank) or the available quantity above a threshold (e.g. clean air) exogenously set. The problem is to share what remains of the resource, the amount r, which is equal to $t - \sum_N x_i \geq 0$. The triple (N, r, x) denotes a "*rights sharing problem*", or simply a "problem".

A *method* associates with any problem (N, r, x) a profile of individual quantities y denoted $y = s(N, r, x)$ such that $y_i \geq 0$ for all $i \in N$ and $\sum_N y_i = r$. In contrast with the traditional rationing problem (and surplus sharing) the quantities y_i are not constrained *a priori* by the already exploited shares x_i. Any agent i can obtain a quantity y_i higher or lower than x_i. We denote by z_i the *total quantity* of an agent i: $z_i = y_i + x_i$.

The link between past exploitation and future quantities is done through two properties. They assign basics responsibilities to all agents given their *relative* past exploitation[5]:

First in time, first in rights: For all N, all i, $j \in N$, $i \neq j$ and all x,

$$x_i \geq x_j \Rightarrow z_i \geq z_j$$

Historical accountability: For all N, all i, $j \in N$, $i \neq j$ and all x,

$$x_i \geq x_j \Rightarrow y_i \leq y_j$$

These principles are compatible and define bounds upon the set of possible allocations. These bounds are binding, some well-known methods do not meet one or the other of the properties: for example, the proportional method fails *historical ac-*

[5]These properties are related to the *ranking* property introduced in the claims problem literature (see Moulin, 2002).

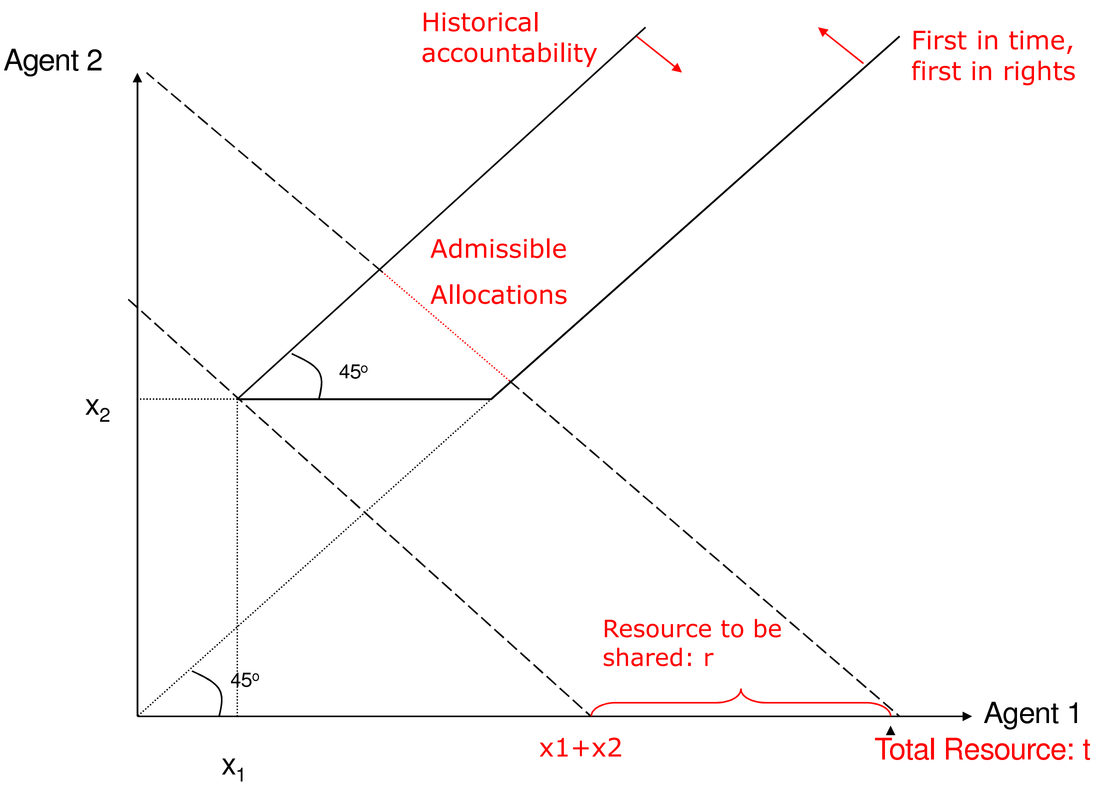

Figure 1: The two agents case

countability. Figure 1 represents the boundaries imposed by these properties on the set of possible allocations for the two agents case.

3 Progressive and regressive methods

We introduce two properties, *Sustainability* and *Autarky*, which assign stronger responsibilities to agents given their past exploitation. The first property asks for some prior notations: define the profile x^i as the profile in which all agents have exploited the same quantity as agent i: $x^i_1 = x^i_2 = ... = x^i_n = x_i$ and r^i as the remaining amount in such a profile: $r^i = \max(t - nx_i, 0)$. Then, *Sustainability* defines a bound on each agents' share: no agent should obtain more than the quantity he would have obtained if all agents had exploited the resource similarly as himself.

Sustainability: For all N, all $i \in N$, all r and all x,

$$y_i = s_i(N, r, x) \leq \bar{y}_i = s_i(N, r^i, x^i)$$

Sustainability is especially binding for agents which have already benefited more from the resource. As such, it can be viewed as a protective criterion for agents which have exploited relatively less the resource in the past[6].

The second property says that no agent should be responsible for the distribution of past exploitation: the allocation of an agent should only depend on the level of her past exploitation and the remaining resource to be shared.

Autarky: For all N, all $i \in N$, all r, all x and all x',

$$\left. \begin{array}{c} x'_i = x_i \\ r' = r \end{array} \right\} \Longrightarrow y'_i = y_i.$$

Autarky implies that the *distribution* of past exploitation should be irrelevant for sharing what is left of the resource. As such, it is a protective criterion for those agents which have relatively exploited more the resource in the past: they should not be penalized for having already exploited more than other agents.

The last property is a well-known property on claims problem literature: *con-*

[6] For a discussion on such protective criteria, see Herrero and Villar (2001) and Yeh (2004). These papers discuss of properties which insure a preferential treatments to small claims in the rationing problem.

sistency[7]. Interpreted as a general principle of distributive justice, it says that an allocation is fair when restricted to any subgroup of the population: by removing one agent, or a subgroup of agents, from the society N and the resources allocated to this agent, or these agents, the allocation of shares within the reduced society remains the same.

Consistency: For all N, all $i \in N$, all r and all x,

$$s(N,r,x)_{N\setminus i} = s(N\setminus i, r - s_i(N,r,x), x_{N\setminus i})$$

We are now ready to characterize the uniform gain method and the equal sharing method. The proofs of both theorems are in the appendix. Formally, the uniform gains method is defined as follows:

Uniform gains method: For all N, all $i \in N$ and all x,

$ug_i(N,r,x) = (\lambda - x_i)_+$, where λ is s.t $\sum_N (\lambda - x_i) = r$

Theorem 1. *The uniform gains method is the only rights sharing method which meets first in time, first in rights, historical accountability, sustainability and consistency.*

The *uniform gains* method aims at equalizing the total shares $((z_i)_{i\in N})$ among all agents. It will be clear from the proof of our results that the *uniform gains* method is the method which erases as much as possible the original inequality in the distribution given *first in time, first in rights.*

We define the equal sharing method as follows:

[7]See Thomson, 1996, for a comprehensive survey on this property.

Equal sharing method: For all N, all $i \in N$, all r and all x,

$es_i(N, r, x) = \frac{r}{n}$

Theorem 2. *The equal sharing method is the only rights sharing method which meets first in time, first in rights, historical accountability and autarky.*

It is noteworthy that the *equal sharing* method is the method which preserves the most the original inequality in the distribution of past exploitation given *historical accountability.*

4 A family of methods

We now propose a family of methods meeting both bounds, *first in time, first in rights* and *historical accountability.* It turns out that a simple representation of this family of methods possesses a "progressivity" parameter a which allows to select among a convex set of the two methods *equal sharing* and *uniform gains*:

Fair methods: For all N, all $i \in N$, all t and all x,

$fm_i(N, r, x) = a_i + (\frac{t}{N} - x_i)$

where $a_i = \alpha(\max(x_i - \frac{t}{N}, \lambda)) + (1-\alpha)(x_i - \frac{\sum_N x_j}{N})$, λ such that $\sum_N \max(x_i - \frac{t}{N}, \lambda) = 0$ and $0 \leq \alpha \leq 1$.

This family of method simply assigns to each agent i an equal quantity of the total resource $(\frac{t}{N})$ minus his own past exploitation (x_i) and adds a positive or negative term a_i which allows to select a more or less progressive method. When $\alpha = 0$, we obtain precisely the *equal sharing* method :$a_i = x_i - \frac{\sum_N x_j}{N}$ for all i, j, and when

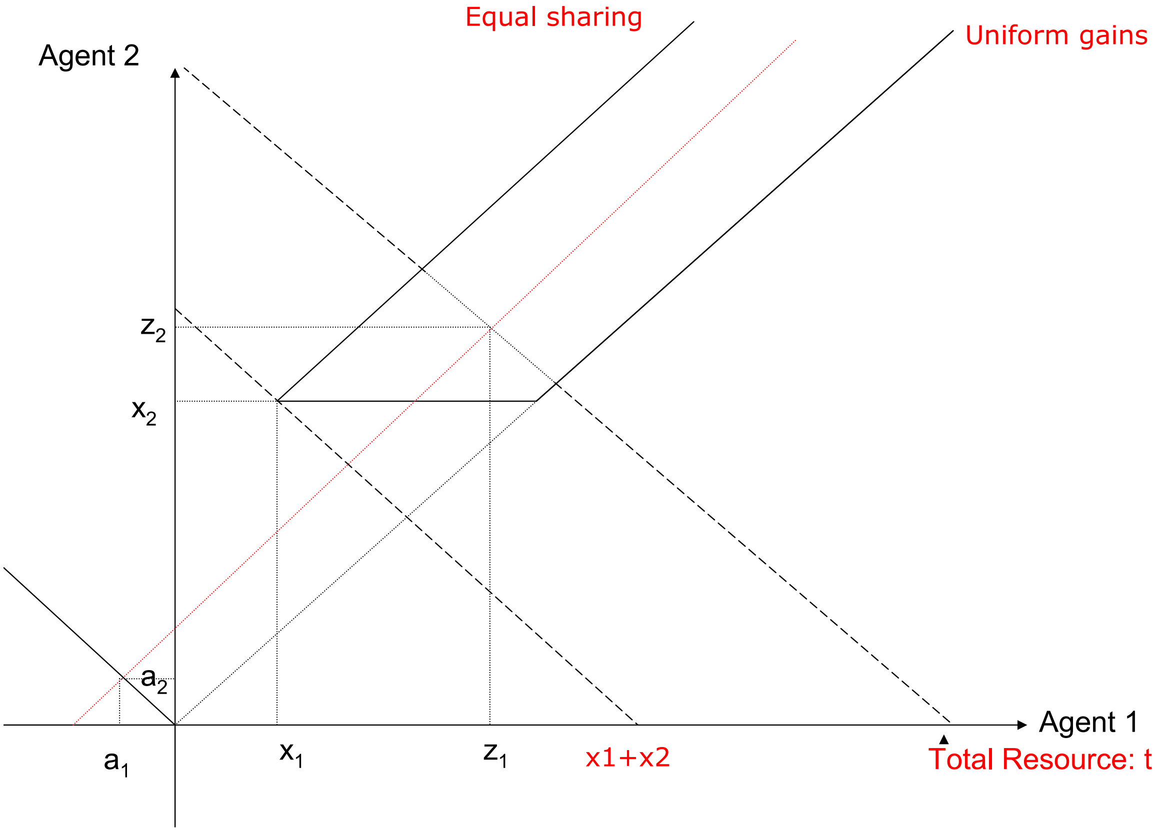

Figure 2: A method of the family, two agents case

$\alpha = 1$ the profile of quantities y coincides exactly with the *uniform gains* allocation: $a_i = \max(x_i - \frac{t}{n}, \lambda)$ with λ such that $\sum_N a_i = 0$. Thus, one can view α as a "progressivity" parameter, and accommodates a predetermined goal of preservation of the inequality in the original distribution of past exploitation/emissions. Figure 2 represents this family of methods for the two agents case.

5 Dynamic interpretation of the model and successive sharing

For numerous environmental problems, one may want to regulate the successive flows of pollution instead of the total stock of the resource (e.g. for the regulation of CO_2 emissions). The treatment of a renewable resource also asks for a more "dynamic" interpretation of the model due to the occurrence of successive sharing. We provide such an interpretation along with a crucial property tied to the problem we name Time Consistency (also known as Path Independence or Composition Down[8]).

In order to understand the problem of successive sharing, let consider two steps (two "periods"): the first one in which the total amount of the resource to be shared is equal to t and the second one in which it is equal to t', with $t' \geq t$. The problem is to share t in the first stage, and the increment $t' - t$ in the second step. We can interpret the quantity $t' - t$ as either being the new amount of resources "created" during the first period (for a renewable resource) or as the current amount of resource that the regulator wishes to allocate at this period (in which case the profile of consumption should be interpreted as flows of consumption for a given period). We can either consider to compute the solution in the second step based on terms of t' and x (the initial profile of consumption) or we can have a more local approach and just share the increment ($r' \equiv t' - t$) among the agents based on the profile of shares obtained in the first step. Then, in the new situation, the agents have what they had previously plus their share of the increment. The property *Time Consistency* allows the regulator to indifferently use one or the other methods: when *Time Consistency* holds, the regulator can use the profile x to share t' or y to share $t' - t$, the final allocation of the second step y' remains the same:

Time Consistency: For all N, all $i \in N$, all t, $t' \geq t$ and all x,

[8] For a recent example see Moulin (2000).

$$s(N, r', x) = s(N, r' - r, s(N, r, x)) + s(N, r, x)$$

Our family of methods obviously meets this dynamic consistency property. An important consequence is that a method in this family stays consistent between successive sharing *even* when the regulator does not know at each step the amount to be shared at the next step. Note that the intuition remains true for every successive amount to be shared $t \leq t' \leq t''$...

6 Conclusion

Allocating property rights on an open access resource which has been freely exploited in the past is often very problematic. The most common method for the applications discussed here is allocating property rights free of charge, based upon *historic use.* Fair allocation in this context is a very controversial issue because involved agents typically rely on two different principles to determine the allocation: *first in time, first in rights* and *historical accountability.* We construct a simple model inspired by the claims problem literature to show that these two positions are in fact compatible: they define bounds to the set of possible allocations. Our main contribution is to provide a family of methods meeting both bounds and characterize the extreme points of this family: the *equal sharing* and the *uniform gains* methods.

The equal sharing method is traditionally associated with the *historical accountability* principle but, surprisingly, it is the most regressive method among the family we propose. The uniform gains method is the one which aims to equalize as much as possible the *total shares* among all agents within the bounds imposed by *first in time, first in rights.* Thus, this method should be the one associated with the egalitarian goal if one considers the family of methods we propose.

For the important problem of allocating CO_2 emissions, many observers have urged that in an international agreement, emissions rights should be allocated by reference to population (see for example, Posner and Sustein, 2008). It should be noted that our model encompasses this situation: we can simply define the society N as being composed of individuals and the profile of emissions x as being the *per capita* profile of emissions (i.e. the ratio of emissions of a country over its population).

References

[1] Baer, P. 2002. *Climate Change Policy, A Survey.* Equity, Greenhouse Gas Emissions, and Global Common Resources, pp. 393-408.

[2] Gerlagh, R., and M. A. Keyzerb. 2001."Sustainability and the intergenerational distribution of natural resource entitlements." *Journal of Public Economics*, 79, 315-341.

[3] Herrero, C., and A. Villar. 2001."The three musketeers: four classical solutions to bankruptcy problems." *Mathematical Social Sciences*, 42, 307-328.

[4] Johansson-Stenman, O., and J. Konow. 2010. "Fairness Concerns in Environmental Economics-Do They Really Matter and If So How?." Mimeo. School of Business, Economics and Law at University of Gothenburg.

[5] Moulin, H. 1992."Welfare bounds in the cooperative production problem." *Games and Economics Behavior*, 4(3), 373-401.

[6] Moulin, H. 2000."Priority rules and other asymmetric rationing methods." *Econometrica*, 68(3), 643-684.

[7] Moulin, H., 2004. *Fair division and collective welfare.* MIT Press.

[8] Moulin, H. 2002. *Axiomatic Cost and Surplus Sharing.* Handbook of Social Choice and Welfare, Elsevier Science.

[9] Müller, B. 1999. *Justice in Global Warming Negotiations: How to Obtain a Procedurally Fair Compromise.* Oxford: Oxford Institute for Energy Studies.

[10] Nozick, R. 1974. *Anarchy, State, and Utopia*, Basic Books, Inc.

[11] Ostrom, Elinor. 1999. Coping with tragedies of the commons, *Annual Review of Political Science* 2, 493-535.

[12] Ott, H.E. and W. Sachs. 2002. *The Ethics of International Emissions Trading.* In: Pinguelli-Rosa L. & M. Munasinghe (eds.). Ethics, Equity and International Negotiations on Climate Change. Cheltenham, UK & Northampton, MA, USA: Edward Elgar: 159-179.

[13] Posner, E. A., and C. R. Sunstein. 2008. "Justice and Climate Change," Mimeo, The Harvard Project on International Climate Agreements.

[14] Rose, A., Stevens, B., Edmonds, J.A. and M.A. Wise. 1998. "International equity and differentiation in global warming policy—an application to tradeable emission permits." *Environmental and Resource Economics*, 12, 25-51.

[15] Thomson, W. 1996." Consistent allocation rules." mimeo.

[16] Thomson, W. 2003."Axiomatic and Game-Theoretic Analysis of Bankruptcy and Taxation Problems: a Survey." *Mathematical Social Sciences*, 45, 249-297.

[17] Tol, R. 2001."Equitable cost-benefit analysis of climate change policies." *Ecological Economics*, 36, 71-85.

[18] Tietenberg, T. 2003. "The tradable-permits approach to protecting the commons: lessons for climate change," *The Oxford Review of Economic Policy* 19(3), 400-419.

[19] Yeh, C.H. 2004."Sustainability, exemption, and the constrained equal awards rule: a note." *Mathematical Social Sciences*, 47, 13-110.

[20] Young, H.P. 1987."On dividing an amount according to individual claims or liabilities," *Mathematics of Operations Research*, 12, 298-414.

[21] Young, H.P. 1990."Progressive taxation and equal sacrifice," *American Economic Review*, 80(1), 253-266.

A Proof: Preliminaries

In both proofs we use a property that is not described in the main body of the paper, *equal treatment of equal.* This property is a basic fairness requirement, broadly used in the distributive justice literature (see for example Moulin, 2004). It states that agents with the same characteristics should be treated equally:

Equal treatment of equal: For all N, all i, $j \in N$, $i \neq j$ and all x,

$x_i = x_j \Rightarrow y_i = y_j$

The reader can easily check that in our set up *first in time, first in rights* and *historical accountability* imply together *equal treatment of equal.*

B proof of theorem 1

Proof. Note that, formally, we use *sustainability*, *first in time, first in rights*, *consistency* and *equal treatment of equal* to characterize the *uniform gains method.* Without loss of generality we rank agents from lowest to highest past exploitation: $x_1 \leq ... \leq x_n$.

Step 1

First, consider any profile x and a total resource t such that $nx_1 \leq ... \leq nx_n \leq t$.

- by *sustainability* and *equal treatment of equal*: $y_n \leq 1/n(t - nx_n) = t/n - x_n$.
- however, by *first in time, first in rights*: $y_n + x_n \geq t/n$, otherwise an agent $i \neq n$ must obtain a total share $y_i + x_i \geq t/n \geq y_n + x_n$ which contradicts *first in time, first in rights.*
- thus, $y_n = t/n - x_n$.

- by *first in time, first in rights*, $y_{n-1}+x_{n-1} \leq y_n+x_n = t/n$. Also, $y_{n-1}+x_{n-1} \geq 1/(n-1)(t-(x_n+y_n)) = t/n$, otherwise an agent $i \neq n, n-1$ must obtain a total share $y_i + x_i \geq y_{n-1} + x_{n-1}$ which contradicts *first in time, first in rights.*

- thus $y_{n-1} = t/n - x_{n-1}$.

- we could repeat the argument for all $i < n-1$. Thus, $y_i = t/n - x_i$ for all i which coincides with the uniform gains method.

Step 2

Now, consider t such that $nx_1 \leq ... \leq nx_{n-1} \leq t \leq nx_n$.

- by *sustainability*, $y_n = 0$.

- by *first in time, first in rights*, $y_{n-1} \geq 1/(n-1)(t-(x_n)) - x_{n-1}$.

- by *sustainability*, $y_{n-1} \leq s_{n-1}(N, r^{n-1}, x^{n-1})$.

- by adding *consistency*, $s_{n-1}(N, r^{n-1}, x^{n-1}) = s_{n-1}(N\backslash n, r^{n-1} - s_n(N, r, x), x^{n-1}_{N\backslash n})$.

- thus, considering the combination of the two above properties and *equal treatment of equal*, $y_{n-1} \leq 1/(n-1)(t-(x_n)) - x_{n-1}$ where $1/(n-1)(t-(x_n)) - x_{n-1} = s_{n-1}(N, r^{n-1}, x^{n-1})$.

- Thus, $y_{n-1} = 1/(n-1)(t-(x_n)) - x_{n-1}$.

- as in step 1, we could repeat the argument for all $i \neq n, n-1$. Thus, $y_i = 1/(n-1)(t-(x_n)) - x_i$ for all $i \neq n$ and $y_n = 0$, which coincides with the uniform gains method.

Step 3

Now, consider r such that $nx_1 \leq ... \leq t \leq x_{n-1} \leq nx_n$.

- by *sustainability*, $y_n = y_{n-1} = 0$.

- as in step 2, by *first in time, first in rights*, *sustainability*, *consistency* and *equal treatment of equal*, $y_{n-2} = 1/(n-2)(T - (x_n + x_{n-1})) - x_{n-2}$.

- as in step 1, we could repeat the argument for all $i \neq n, n-1, n-2$. Thus, $y_i = 1/(n-1)(t - (x_n + x_{n-1})) - x_i$ for all $i \neq n, n-1$ and $y_n = y_{n-1} = 0$ which coincides with the uniform gains method.

- repeat for all possible values of t (recall that because of our notation $nx_1 \leq t$).

□

C Proof of theorem 2

Proof. Note that, formally, we use *autarky*, *historical accountability* and *equal treatment of equal* to characterize the *equal sharing* method. Without loss of generality we rank agents from lowest to highest past exploitation: $x_1 \leq ... \leq x_n$. Pick x' such that $x'_i = x_1$ for all i, and t' such that $r' = r$.

- by *equal treatment of equal*, $y'_i = \frac{r}{n}$ for all i

- by *autarky*, $y'_1 = y_1 = \frac{r}{n}$

- if $n = 2$, by budget balance ($\sum_N y_i = r$), $y_2 = y_1 = \frac{r}{2}$ which coincides with the *equal sharing* method.

- otherwise, by *historical accountability*, $y_2 \leq y_1 = r/n$ and $y_2 \geq (r-(r/n))/n-1$ (otherwise, an agent $i \neq 1, 2$ obtains an $y_i \geq y_2$ which contradicts *historical accountability*). Given that $(r-(r/n))/n-1 = \frac{r}{n}$, $y_2 = \frac{r}{n}$.

- we could repeat the argument for all $i > 2$. Thus, $y_i = \frac{r}{n}$ for all i which is precisely the *equal sharing* method. Given that x was picked arbitrarily, the proof is complete.

$\square$