Improving Portfolio Optimization by DCC And DECO GARCH: Evidence from Istanbul Stock Exchange

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In this paper, the performance of global minimum variance (GMV) portfolios constructed by DCC and DECO-GARCH are compared to that of GMV portfolios constructed by sample covariance and constant correlation methods in terms of reduced volatility. Also, the performance of GMV portfolios are tested against that of equally weighted and cap weighted portfolios. Portfolios are constructed from the stocks listed in Istanbul Stock Exchange 30 index (hereafter, ISE-30). The results show that GMV portfolios constructed by DCC-GARCH outperformed the other portfolios. In addition, the performance of GMV portfolios estimated by DCC and DECO-GARCH methods are improved by extending calibration period from three years to four years and lowering rolling window term from one week to one day, while the performances of other GMV portfolios decrease. It shows the effect of time varying variance and dynamic correlations on portfolio optimization at Turkish stock market.

Key words: DCC-GARCH, DECO-GARCH, GMV portfolio

Jel Classification: C32, C51, C61, G11

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1. Introduction

Investors and even portfolio managers often monitors the benchmark indices which are generally constructed by exchanges relying on cap weighted average. These benchmarks only give the investors the general idea about the general market movement. Since they lack the requirements for robust benchmark or portfolio construction, investments decisions depend on these benchmarks lead the investors and portfolio managers to underperform. Therefore, robust portfolio optimization is one of the major issues for portfolio managers and other market participants.

Since Markowitz (1952) developed the mean-variance framework, there have been profound developments on the portfolio optimization. Sharpe (1963) proposes the CAPM as the single factor model to estimate covariance matrices. Elton and Gruber (1973) introduce the constant correlation methodology to reduce the burden of large scale parameter estimation. Instead of simple model of Sharpe (1963), multi-factor model estimation is applied by Chan, Karceski and Lakonishok (1999).

Standard deviations and the pair-wise correlations are the elements for covariance construction. Employing unconditional standard deviations and constant correlations to estimate covariance matrix are always debated by finance literature and market as well. The presence of time varying variances and correlations are shown by Engle (1982). GARCH (1,1), introduced by Bollerslev(1986), capture the ARCH effect and model the time varying variance with less constraints relative to ARCH.

The first adaptation of univariate GARCH process is carried out by Bollerslev, Engle and Wooldridge (1988). They employ the univariate GARCH process to do multivariate parameterization. It is known as vech form of multivariate GARCH. They also propose diagonal vech form of that model by which the numbers of parameters, which are to be estimated, were reduced. However, it creates a computational burden when the sample size increases. Since the number of parameters to be estimated is too many, it is hard to achieve a feasible estimation.

Engle and Kroner (1995) find the way of producing positive definite covariance matrix through BEKK model. However, it is a great problem that estimating the conditional covariances as the sample size increased. In order to solve this problem, Bollerslev (1990) introduces constant conditional correlation GARCH (CCC-GARCH) model. By this model, standard deviations of each asset are produced by univariate GARCH process and it is assumed that the pair-wise correlations are constant. Therefore, CCC approach does not model the time varying conditional correlation. The standard deviations within the covariance matrix are calculated relying on the GARCH constraints such as non-negativity. So, under the conditions of the guaranteed non-negative conditional variances and the invertible conditional matrix generating positive definite covariance matrix is certainly obtained.

However, Tse and Yu (1999) prove that the constant correlation is not valid when the estimation process is multivariate. The pair-wise correlations are also time-varying and they need to be modeled to produce consistent errors.

The challenging problem of constant correlation is solved by the dynamic conditional correlation GARCH (DCC-GARCH), proposed by Engle (2001). Mathematical framework of this model, developed by Engle and Sheppard (2001), has main two steps algorithm to have time varying covariance matrix. First step is to find conditional standard deviations through the univariate GARCH and second step is to model the time varying correlations relying on lagged values of residuals and covariance matrices. After that, conditional covariance matrix could be found by using conditional standard deviations and dynamic correlations.

Tse and Tsui (2002) support estimation accuracy of dynamic correlation model. They assume that the pair-wise correlations follow moving average process and they find the conditional variances by univariate GARCH. They show that errors of maximum likelihood estimator are reduced by the multivariate GARCH estimation with dynamic correlation.

Considering that DCC-GARCH captures the time varying correlations and variances, it is very well structured model to estimate time varying covariance matrix. However, the estimation of conditional correlation matrix for a portfolio with large number of assets causes the difficulty of estimation. The way of reducing the scale of estimation was proposed by Engle and Kelly (2009). Averaging of pair dynamic correlations, they reduce the burden of large scale parameterization. This process is called as Dynamic Equicorrelation GARCH (DECO-GARCH). It reduces the sample risk caused by large scale covariance matrix. However, there is always estimation risk because of assigning one value to each pair-wise correlation instead of their real values.

These relatively new techniques, DCC and DECO-GARCH, are employed to construct GMV portfolio in this paper. Considering the high volatile structure of emerging stock markets, one of the volatile emerging market indexes, Istanbul Stock Exchange 30 Index (ISE-30) and its constituents are used to test the performance of GMV.

In order to test the effect of time varying variance and dynamic correlations on portfolio optimization, the estimation accuracy of these methods is compared to that of the sample covariance and constant correlation methodology in terms of reduced volatility of the GMV portfolios. The performances of non-optimized portfolios such as equally weighted and cap weighted portfolios are also compared to that of those GMV portfolios.

The rest of the paper is organized as following: Part 2 gives brief information about theoretical framework of mean-variance optimization and covariance estimation techniques employed in this paper. While Part 3 describes the data and evaluates the empirical results, concluding remarks are finally given at Part 4.

2. Theoretical Framework

Estimating Covariance Matrix by constant volatilities and correlations:

As modern portfolio theory (MPT) proposes, main objective of diversification is to minimize risk in a given level of return. While all efficient portfolios nest on the efficient frontier, GMV is the one at the beginning of that frontier and it has lowest volatility amongst other efficient portfolios.

In brief, the mathematical construction of the GMV portfolio is as follows: Having n number of assets, weight w vectors along with the covariance matrix Σ, the objective and subjective functions of the optimization process of GMV portfolio is

$$\min_{w} \frac{1}{2} w'\Sigma w \qquad \text{subject to } w'1 = 1; 1' = [1,1,1.....,1]$$

Then the weights of GMV portfolio that minimizes the portfolio variance are calculated as $w_{gmv} = \frac{\Sigma^{-1}1}{1'\Sigma^{-1}1}$. Since there is no argument about mean in GMV portfolio construction, so there is no constraint on mean. The key part is in that equation is finding the covariance matrix denoted as Σ. When constant standard deviations and pair-wise correlations are used to estimate covariance matrix, this optimization procedure is called *sample covariance estimation*. Considering sample covariance estimation, $\Sigma = S'S \circ R$ where R is n by n square matrix of constant correlations and S refers to the vector of constant standard deviations such as $S = [s_1, s_2, ..., s_n]$ where s_i = ith asset in the portfolio.

In constant correlation estimation, R is constructed by using average of pair-wise correlations such that

$$\overline{R} = \overline{\rho} \times 1_{n \times n} + (1 - \overline{\rho}) \times I_{n \times n} \Rightarrow \overline{R} = \begin{bmatrix} 1 & \overline{\rho} & \cdots & \overline{\rho} \\ \overline{\rho} & \ddots & & \vdots \\ \vdots & & \ddots & \overline{\rho} \\ \overline{\rho} & \cdots & \overline{\rho} & 1 \end{bmatrix}$$

where $\overline{\rho}$ is average constant correlation, $1_{n \times n}$ is $n \times n$ matrix of ones and $I_{n \times n}$ is $n \times n$ identity matrix. Then, new covariance matrix is estimated by Hadamard product of constant average correlation matrix and the matrix constructed by multiplication of vectors of constant standard deviations such that $\Sigma_{cc} = S'S \circ \overline{R}$. This technique is named as *constant correlation estimation* and developed by Elton and Gruber (1973). Aftermath, finding optimal weights for GMV portfolio is the same with sample covariance such that $w_{gmv} = \frac{\Sigma_{cc}^{-1} 1}{1' \Sigma_{cc}^{-1} 1}$.

The Dynamic Conditional Correlation Model (DCC-GARCH)

Being a multivariate GARCH model, DCC-GARCH assumes that returns of the assets (r_t) distribute normally with zero mean and they have covariance matrix such as C_t.

$$r_t \sim N(0, C_t)$$

Conditional covariance matrix is found by using conditional standard deviations and dynamic correlation matrix. Let S_t is $1 \times n$ vector of conditional standard deviations modeled by univariate GARCH process such that

$$S_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i e_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \qquad (1)$$

where $e_{t-i} = \sigma_{t-i} \zeta_{t-i}$ and $\zeta_{t-i} \sim N(0,1)$. In order to find time varying correlation matrix, Engle (2002) proposes a model for the time varying covariance such that

$$K_t = (1 - \sum_{i=1}^{p} \alpha_i - \sum_{j=1}^{q} \beta_j)\overline{K} + \sum_{i=1}^{p} \alpha_i \left(e_{t-i} e'_{t-i}\right) + \sum_{j=1}^{q} \beta_j K_{t-j} \tag{2}$$

subject to
$$\alpha_i \geq 0$$
$$\beta_j \geq 0$$
$$\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$$

As it can be seen from that equation above, the GARCH process is adopted to model time varying covariance matrices. $\overline{K}$ is the unconditional covariance and it is initially obtained by sample covariance estimation. K_t is forecasted by lagged residuals (e_{t-i}), which are standardized by conditional standard deviations, and lagged covariances (K_{t-i}). Therefore, in estimating conditional covariance matrix first, conditional standard deviations of each asset in the portfolio are modeled by univariate GARCH. It is important to note that the constraints of GARCH process are still considered to construct positive definite covariance matrix. In order to find estimators of that model, the log likelihood function can be written as

$$\begin{aligned} L &= -\frac{1}{2} \sum_{t=1}^{T} \left(k \log(2\pi) + \log(|S_t M_t S_t|) + r_t' S_t^{-1} M_t^{-1} S_t^{-1} r_t \right) \\ &= -\frac{1}{2} \sum_{t=1}^{T} \left(k \log(2\pi) + 2\log(|S_t|) + \log(|R_t|) + e_t' M_t^{-1} e_t \right) \end{aligned} \tag{3}$$

After finding optimum estimators that maximize the log likelihood function above, it is easy to produce covariance series. But, it is necessary to note that each covariance matrix is not constructed by conditional standard deviations yet. This covariance matrix series is generated by relying on initial unconditional covariance matrix. Then new covariance matrix for next time point is generated by previous one and standardized residuals as in simple univariate GARCH process. So, univariate GARCH process is employed to extract time varying positive definite correlation matrices from that covariance matrix series such that

$$M_t = \left(\sqrt{K_t^d} \right)^{-1} K_t \left(\sqrt{K_t^d} \right)^{-1}$$

In that equation, K_t^d refers to the diagonal matrix of variances which are obtained from the diagonal items of K_t as following

$$K_t^d = \begin{bmatrix} S_{11}^2 & 0 & \cdots & 0 \\ 0 & S_{22}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & S_{nn}^2 \end{bmatrix}.$$

The matrix notation for time varying correlation indicates that the way of calculating correlations such as dividing covariances by standard deviations extracted from the covariance matrix. The matrix notation can be interpreted by algebraically such that $\rho_{i,j,t} = \frac{S_{i,j,t}}{\sqrt{S_{ii}^2 S_{jj}^2}}$.

Finally, conditional covariance matrix can be found by Hadamard product of the matrix of conditional standard deviations and time varying correlation matrix such that

$$C_t = S_t' S_t \circ M_t$$

This methodology gives the conditional covariance matrix for each data point of the calibration period. To find the conditional covariance matrix that is used to optimize weights of assets in the portfolio, one day conditional standard deviations of assets and their dynamic correlation matrix are forecasted.

The Dynamic Equicorrelation Model (DECO-GARCH)

Engle and Kelly (2009) propose a different version of DCC-GARCH model, named DECO-GARCH. They set the average of conditional correlation equal to all pair correlations in order to reduce burden of the computation of large scale correlation matrices. They use the same structure to construct covariance matrix as in the DCC-GARCH model such that $C_t = S_t' S_t \circ \overline{M}_t$. However, the conditional correlation matrix would be different because of taking average of conditional correlations as the below equation shows

$$\overline{\rho}_t = \frac{1}{n(n-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^{n} \hat{\rho}_{ij,t}$$

where, $\overline{\rho}_t$ is defined as conditional equicorrelation as $\hat{\rho}_{ij,t}$ refers to the pair-wise correlation. After finding the average correlation, the new conditional correlation matrix is constructed such that,

$$\overline{M}_t = \overline{\rho}_t 1_{n\times n} + (1 - \overline{\rho}_t) I_{n\times n}$$

That equation is almost same with the equation that calculates the constant correlation matrix. The only difference is that correlation matrix is extracted from covariance matrix series modeled by univariate GARCH process and therefore, average correlation matrix in that equation is conditional and time varying.

Engle and Kelly (2009) defined the log- likelihood function such that,

$$L = -\frac{1}{T}\sum_t \left[\log\left([1-\rho_t]^{n-1}[1+(n-1)\rho_t] \right) + \frac{1}{1-\rho_t}\left(\sum_i \left(e_{i,t}^2\right) - \frac{\rho_t}{1+(n-1)\rho_t}\left(\sum_i e_{i,t}\right)^2 \right) \right] \quad (4)$$

Considering DCC, it is necessary to estimate $\frac{n\times(n-1)}{2}$ pair-wise correlations. Instead of estimating each pair-wise correlation, only one parameter for conditional correlation is estimated by DECO. When the sample size is quite large, the DCC model incurs sample size risk due to the fact that the number of parameters which are to be estimated would be many. It creates a burden on programming and also noise on the forecasted data. Considering DECO approach, the computational burden may be decreased by assigning same correlation to the each element of conditional correlation matrix but, there would be model risk for the sample in which each asset returns has quite different dynamic pair-wise correlations. So, this paper also answers that question whether the pair-wise correlations of Turkish stocks are not quite different from each other to be negligible or not. If the results indicate that performance of GMV portfolio estimated by DECO is better than that of GMV portfolio constructed by DCC, it can be concluded that pair-wise correlations move very close to mean. Otherwise, this is not the case.

3. Data and Empirical Results

3.1. Data

In the paper, the data, composed of the daily prices of stocks within the ISE-30 Index from January 2004 to the end of the September 2010, is collected from FOREX FX2000 6.1.233. Then, daily prices are transformed to daily logarithmic returns. A rolling-window estimate of the variance-covariance matrix with a calibration period of three years is performed by ruling out the short sale and then, 1 week out-of sample forecasting of optimal portfolio returns are generated. Calibration period of three years is extended to four years to test the effect of changing calibration period on the performance of the GMV portfolio. Then, in order to make the portfolio allocation more dynamic, the out-of sample forecasting period is lowered to 1 day.

Since the list of constituents of the index changes over time, a fixed list of 23 stocks within the ISE-30 Index dating from January 2004 is used to simulate a cap-weighted portfolio for fair comparison. The descriptive statistics of the ISE-30 Index consist of three years of data, and constituents ISE-30 Index are given by the Table 1. Since there are some short sale restrictions at Istanbul Stock Exchange, the short sale constraint is added to constraint set of optimizer.

Table 1: Descriptive Statistics (Ise-30 Index and Its Constituents)

	Mean	Std. Dev.	Max.	Min.	Skewness	Excess Kurtosis	JB Test (Prob.)
AKBNK	0.11%	2.60%	11.28%	-10.30%	0.1584	1.0186	0.001
AKENR	0.05%	2.46%	13.35%	-11.00%	0.1913	3.2670	0.001
ARCLK	0.02%	2.36%	12.76%	-9.35%	0.3229	2.2598	0.001
DOHOL	0.08%	2.60%	10.01%	-9.80%	-0.1853	0.7324	0.001
DYHOL	0.05%	2.76%	11.90%	-10.23%	0.0575	1.0645	0.001
ECILC	0.14%	2.60%	14.20%	-12.03%	0.3352	3.7143	0.001
ENKAI	0.17%	2.13%	12.84%	-10.19%	0.4792	2.9533	0.001
EREGL	0.17%	2.34%	10.09%	-11.15%	0.0262	1.5035	0.001
GARAN	0.19%	2.67%	11.88%	-16.21%	-0.2449	1.9679	0.001
ISCTR	0.10%	2.70%	9.48%	-11.25%	0.0711	0.6291	0.001
KCHOL	0.06%	2.39%	9.20%	-10.43%	0.1073	0.6980	0.001
KOZAA	0.30%	3.98%	20.59%	-13.98%	0.9269	2.9904	0.001
KRDMD	0.22%	3.51%	21.13%	-12.14%	0.6236	4.0294	0.001
PETKM	0.05%	2.41%	17.23%	-13.70%	0.5545	5.2626	0.001
SAHOL	0.06%	2.44%	9.58%	-9.20%	0.0304	0.5920	0.003
SISE	0.11%	2.37%	9.53%	-8.16%	0.0996	0.7390	0.001
SKBNK	0.28%	3.68%	17.44%	-22.91%	0.5564	4.5639	0.001
TCELL	0.15%	2.58%	8.82%	-15.00%	-0.0161	1.7240	0.001
TEBNK	0.22%	3.18%	19.19%	-15.07%	0.3721	2.7968	0.001
THYAO	0.02%	2.43%	13.27%	-20.48%	-0.4480	10.0071	0.001
TUPRS	0.15%	2.29%	11.65%	-10.03%	0.3943	2.5442	0.001
VESTL	-0.07%	2.07%	12.29%	-8.38%	0.1531	2.2729	0.001
YKBNK	0.13%	2.71%	14.87%	-10.88%	0.3004	2.1896	0.001
ISE-30	0.10%	1.85%	7.18%	-8.53%	-0.1477	0.8782	0.001

According to the Table 1, for four years period of data daily logarithmic returns of the stocks and ISE-30 Index is lower than 1% while the daily volatility of those is between 1.85% and 3.98%. Also spread between maximum and minimum values of daily returns are quite high and moreover, the minimum values of daily logarithmic returns are negative. Since the return series of all stocks and the index are right skewed (positive skewness), the distribution of data is asymmetric. Relying on the kurtosis values, the logarithmic return series of stocks and index have positive excess kurtosis. So, the distribution the data series can be indicated as leptokurtic (the presence of fat-tails). Since skewness is different from zero and there is high excess kurtosis, the data distribution shows non-normality. This claim is supported by the results of the JB test. Since the probability values of JB test is lower than 0.01 (99%, confidence level) for all stocks and ISE-30 Index, ISE-30 Index and its constituents show non-normality. Those results present the high volatile and non-normality structure of daily logarithmic returns of ISE-30 Index and its constituents.

3.2. Empirical Results

Reduced volatility is the major criteria to test the performances of GMV portfolios. First, the out of sample returns within 1 week rolling window period is forecasted by using calibration period of three years. Then, the forecasting period is lowered from one week to one day. So, Out-of sample returns are forecasted from the beginning of January 2008 till the end of September 2010 for 3 years calibration. The Table 2 and 3 show the risk and return measures of the GMV portfolios, generated by different covariance estimation methods with three year calibration period, for one week and one day rolling windows. Also the performance of the each GMV is compared to that of equally weighted and cap weighted portfolios. On the tables, SC, EQW, CAPW, CC refer to sample covariance estimation, equally weighted portfolio, cap-weighted portfolio and constant correlation covariance estimation respectively.

Table 2: GMV Portfolio- Risk and Return Measures (Calibration Period: 3 Years; Rolling Window: 1 Week)

3 Year-Weekly Roll	*SC*	*EQW*	*CAPW*	*CC*	*DCC*	*DECO*
Mean Return	0.058%	0.055%	0.073%	0.061%	0.031%	0.030%
Annualized Mean Return	15.650%	15.000%	20.286%	16.703%	8.063%	7.987%
Volatility	**1.399%**	**1.493%**	**1.659%**	**1.598%**	**1.411%**	**1.629%**
Annualized Volatility	22.215%	23.697%	26.340%	25.374%	22.393%	25.856%
Skewness	-0.320	-0.227	-0.083	-0.208	-0.399	-0.489
Excess Kurtosis	1.788	1.681	1.426	2.870	5.041	6.004
Historical VAR (99%)	4.760%	4.863%	5.226%	5.476%	4.670%	6.413%

Table 3: GMV Portfolio- Risk and Return Measures (Calibration Period: 3 Years; Rolling Window: 1 Day)

3 Years-Daily Roll	*SC*	*EQW*	*CAPW*	*CC*	*DCC*	*DECO*
Mean Return	0.060%	0.057%	0.075%	0.063%	0.050%	0.046%
Annualized Mean Return	16.208%	15.537%	20.914%	17.268%	13.401%	12.320%
Volatility	**1.393%**	**1.491%**	**1.657%**	**1.595%**	**1.374%**	**1.606%**
Annualized Volatility	22.121%	23.668%	26.310%	25.317%	21.816%	25.502%
Skewness	-0.323	-0.230	-0.086	-0.226	-0.346	-0.519
Excess Kurtosis	1.747	1.691	1.433	2.795	4.712	5.715
Historical VAR (99%)	4.672%	4.863%	5.226%	5.488%	4.745%	6.303%

According to the results, volatility of the portfolios decreases as the forecasting period (rolling window) is lowered. GMV portfolio constructed by DCC-GARCH produce the lowest volatility while portfolio optimization is carried out by more dynamically (when rolling window is one day).

In addition, all portfolio returns are negative skewed and the distribution of the portfolio returns shows fat-tail since all portfolios have positive excess kurtosis. Value at Risk Analysis is also employed. DCC-GARCH and sample covariance give lowest historical VaR for three year calibration period.

To test the effect of using larger scale of data, the calibration period is extended. Table 4 and 5 indicate the risk and return measures of the GMV and other portfolios for four year calibration period with one week and one day rolling windows.

Table 4: GMV Portfolio- Risk and Return Measures (Calibration Period: 4 Years; Rolling Window: 1 Week)

4 Year-Weekly Roll	***SC***	***EQW***	***CAPW***	***CC***	***DCC***	***DECO***
Mean Return	0.009%	0.020%	0.040%	0.037%	-0.035%	-0.011%
Annualized Mean Return	2.383%	5.227%	10.727%	9.892%	-8.508%	-2.831%
Volatility	**1.490%**	**1.584%**	**1.725%**	**1.668%**	**1.491%**	**1.723%**
Annualized Volatility	23.647%	25.147%	27.379%	26.486%	23.671%	27.357%
Skewness	-0.258	-0.176	-0.119	-0.196	-0.481	-0.706
Excess Kurtosis	1.410	1.442	1.425	2.317	4.382	5.865
Historical VAR (99%)	4.739%	4.872%	5.987%	5.616%	5.298%	7.256%

Table 5: GMV Portfolio- Risk and Return Measures (Calibration Period: 4 Years; Rolling Window: 1 Day)

4 Year-Daily Roll	***SC***	***EQW***	***CAPW***	***CC***	***DCC***	***DECO***
Mean Return	0.011%	0.021%	0.041%	0.068%	0.031%	0.066%
Annualized Mean Return	2.732%	5.506%	10.941%	18.610%	8.011%	18.040%
Volatility	**1.487%**	**1.583%**	**1.724%**	**1.605%**	**1.153%**	**1.340%**
Annualized Volatility	23.608%	25.133%	27.361%	25.473%	18.297%	21.276%
Skewness	-0.253	-0.178	-0.120	-0.620	-1.062	-1.617
Excess Kurtosis	1.414	1.446	1.431	3.209	9.657	14.559
Historical VAR (99%)	4.695%	4.872%	5.987%	5.948%	4.131%	4.995%

According to the tables above, volatility of all portfolios except ones optimized DCC and DECO-GARCH, is raised by extending the calibration period from three years to four years. The performances of the portfolios estimated by DCC and DECO-GARCH increases substantially in terms of reduced volatility when four year calibration period accompanied with one day rolling window is employed. DCC-GARCH covariance estimation gives lowest volatility amongst all portfolios. Historical VaR analysis also supports this result, because the lowest historical VaR is achieved by DCC process having four year calibration and one day rolling window periods.

Visualizing changes on conditional volatilities over forecasting time horizon is another way to compare the performances of GMV portfolios in terms of volatility. Figure 1 and 2 present the conditional volatilities of each portfolio for each calibration period and the period of rolling window. The conditional volatility is modeled by univariate EGARCH (1, 1).

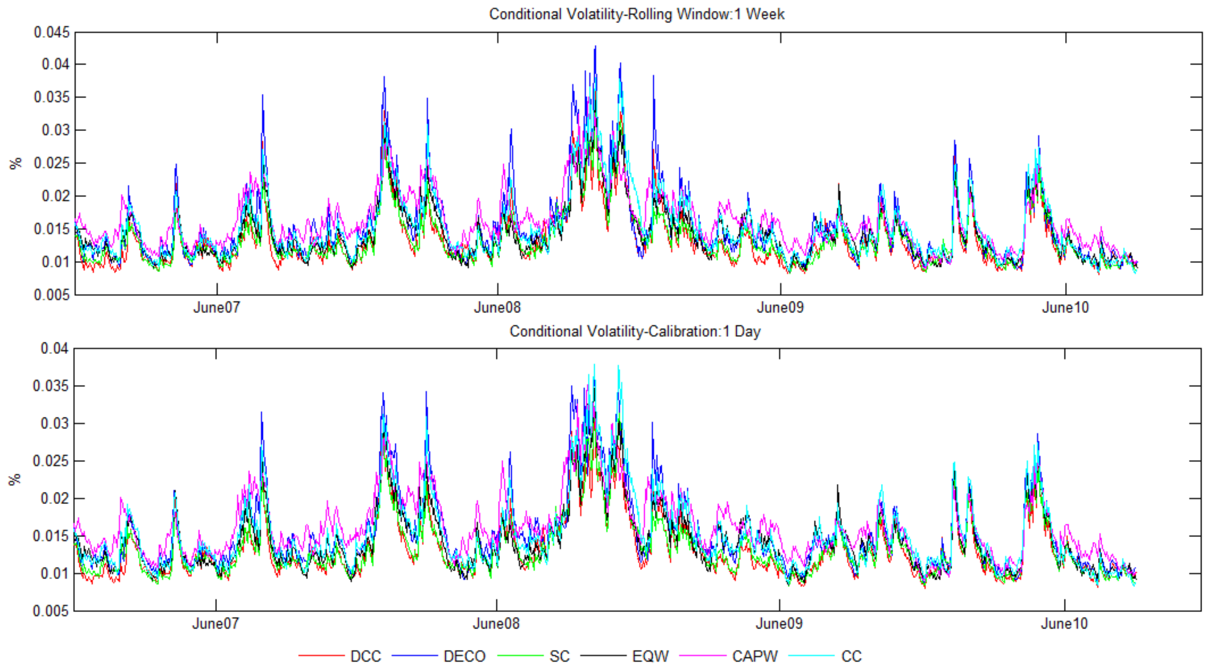

Figure 1: Conditional Volatility- Calibration Period: 3 Years

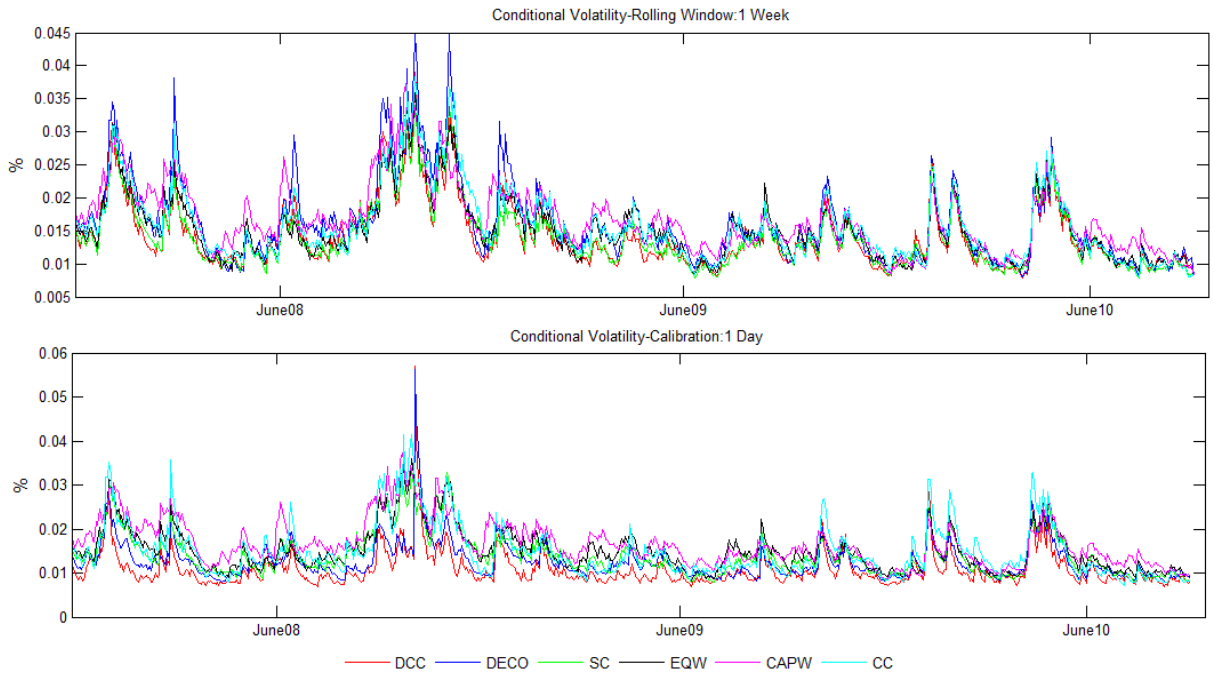

Figure 2: Conditional Volatility- Calibration Period: 4 Years

Considering one day rolling window period, the conditional volatility series of portfolios constructed by DCC is below the volatilities of other portfolios for each calibration period. Although conditional volatility of sample covariance moves at lower level than DCC as one week rolling window period is employed, the lowest conditional volatility level is achieved by DCC with four year calibration and one day rolling window.

In order to analyze the estimation quality of methods, we compared the performance of forecasted GMV portfolios to the performance of the true GMV portfolios. Since the forecasted GMV portfolio with lowest volatility is reached by four year of calibration period, in-sample estimation for GMV portfolios is carried out by using realized returns of forecasting period starts from January 2008 to September 2009. Four different efficient frontiers are drawn by using four different covariance estimation methods, which are sample covariance, constant correlation, DCC-GARCH and DECO-GARCH. Those efficient frontiers are given by figure below:

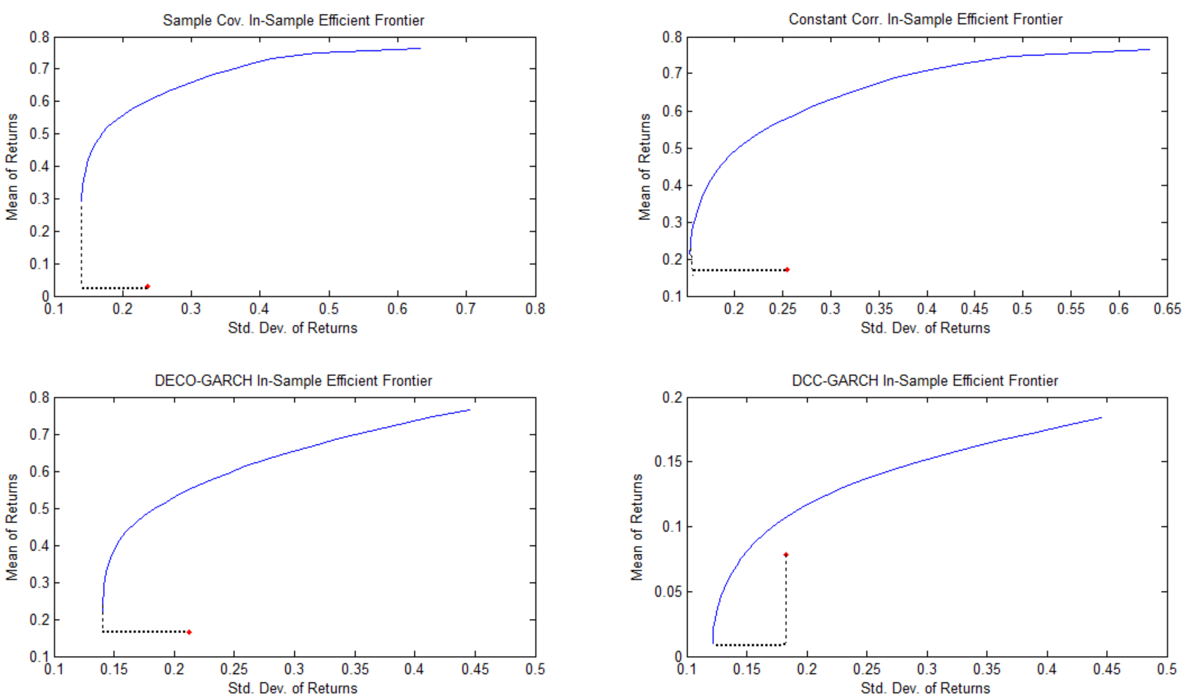

Figure 3: In -Sample Estimation-Mean Variance Efficient Frontier

Relying on the in-sample estimation, lowest volatility is achieved by DCC-GARCH and DECO-GARCH is the second one. Forecasting accuracy can be measured by spread between the volatility of forecasted GMV and that of true GMV. To quantify this graphical analysis, volatilities of forecasted GMV and true GMV portfolios are given table below:

Table 6: In-Sample Estimation: Forecasted GMV Volatilities and True GMV Volatilities

Covariance Matrix Estimation Methods	*Forecasted GMV Vol.*	*True GMV Vol.*	*Spread* *
Sample Covariance	1.49%	0.89%	0.60%
Constant Correlation	1.60%	0.97%	0.63%
DCC-GARCH	**1.15%**	**0.77%**	**0.38%**
DECO-GARCH	1.34%	0.88%	0.46%

*Spread=|Forecasted GMV Volatility-True GMV Volatility|

According to Table 6, the portfolio that is closest to true GMV portfolio is the one constructed by DCC-GARCH estimation. While DECO estimation follows DCC, other estimation methods create substantial deviations from true volatilities. The spreads generated by dynamic correlations models are almost 30%-40% lower than the spreads produced by other estimation methods. Therefore, it can be inferred that GMV portfolios with dynamic correlations outperformed the other portfolios in terms of reduced risk.

4. Conclusion

Employing GARCH process in mean-variance optimization framework has been the subject of the finance literature for a long time. Considering the co-movements of assets in a portfolio, the multivariate form of GARCH process is employed to have efficient covariance estimators.

In this paper, the estimation accuracy of two different types of multivariate GARCH models, DCC and DECO-GARCH, are compared to classical sample covariance and constant correlation estimation. Also, performances of the forecasted GMV portfolios are compared to equally weighted portfolio and cap-weighted portfolio in terms of reduced volatility. Volatility series of each portfolio is extracted by univariate EGARCH process due to the fact that the presence of time varying variance. Also, estimation accuracy of each GMV portfolio is tested by employing in-sample estimation.

According to the results of out of sample forecast, when more dynamic portfolio allocation (one day allocation) is used, extending calibration period from three years to four years reduce the volatility of GMV portfolios constructed by DCC and DECO. However, this is not case for other covariance estimation methods, equally weighted and cap weighted portfolios.

It is important to note that performance of GMV optimized DECO is still poor for three years calibration period, even if its performance is improved by lowering rolling window term from one week to one day. Also, it performs quite poor with four year calibration and weekly roll. DECO is second best only under the conditions of four year calibration and daily roll. However, DCC estimation is the best one in terms of reduced volatility when allocation period is reduced from one week to one day for each calibration term. Also, the lowest volatility is achieved by DCC when calibration period is four years and rolling window term is one day.

Relying on the results of in-sample estimation for four year period, the GMV portfolios constructed by DCC and DECO-GARCH are closer to the true GMV than the other portfolios. The spread between volatility of forecasted GMV portfolio and that of true GMV portfolio is detected and it is found that lowest spread belongs to DCC. DECO is the second one. Moreover, volatility spreads of DCC and DECO are almost 30% and 40% lower than those of other covariance estimation methods. In-sample estimation results support out-of sample estimation findings related to better performance of DCC relative to other estimation methods under the conditions of more dynamic allocation and extended calibration period.

Finally, there is a necessary question is why DECO performs worse than DCC and sample covariance, except in the case of four year calibration period with one day rolling window? Answer for that question is actually one of the major results related to the structure of Turkish stock market. Before answer that question, digging up empirical results would provide logical path to answer that question precisely. If the performance of GMV optimized by constant correlation method is detected, it is easy to find it is the worst GMV portfolio for both out-of and in-sample estimation.

Bottom line, DECO and constant correlation methods perform worse than the others especially when three year calibration period is used. This means the estimation error incurred by these two methods are higher than that of the other methods. Reason causing high estimation error is variance of pair-wise correlations. Therefore, it can be concluded that pair-wise correlations among Turkish stocks do not cluster closely around mean. Differences among those correlations are not negligible.

However, when longer calibration term such as four years and more dynamic allocation period such as one day is considered, DECO is the second best GMV. This is the clue for another important inference for Turkish stock market. Pair-wise correlations also change over time. They are not constant. If more dynamic portfolio allocation is preferred, techniques that model dynamic correlations such as DCC and DECO-GARCH should be used.

Covariance estimation techniques that employ each pair-wise correlations, incur sampling error because of large number of estimated parameters. However, the benchmark index, which is investigated in this paper, consists of only twenty three stocks for the investigation period. Therefore, the scale of covariance matrices estimated by each method are not too large and so do not create too much noise causing sampling error. Consequently, DCC gives quite low volatility compared to other estimation techniques, when longer calibration term and shorter rolling window period is considered.

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