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2009

Online at https://mpra.ub.uni-muenchen.de/27334/
MPRA Paper No. 27334, posted 09 Dec 2010 13:41 UTC

Working Paper Series: 2008/09 – 023

Demand and Supply of Currencies of Small Denominations: A Theoretical Framework

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Demand and Supply of Currencies of Small Denominations: A Theoretical Framework*

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February 2009

Abstract

The paper presents theoretical framework of demand and supply of currencies of small denominations. In our framework both demand and supply equations emerge out of an optimization framework. Demand functions for small denominations are obtained from a linear expenditure system. Our main contention is that economic agents would like to hold a fixed number of small changes, independent of their respective total cash holdings. However, in our model the fixed quantity is influenced by the probability that in a currency transaction, the counterparty would be able to provide the small change if needed. The supply function is derived from an optimization problem where the central bank balances its operational cost with the probability that an individual would be able to carry out "small" transactions independently, without the help of counterparty. In this demand-supply framework, the probability that a randomly chosen individual in an economy would hold certain currency combinations is interpreted as "price". We attempt to show that in a dynamic environment, such interaction could be understood by specifying a cob-web type model where expectations are formed based on previous period's experience. As an operational rule, it is proposed that the central bank should increase the supply of small denominations at a rate marginally above the growth rate of economically active population and stop minting as soon as some of the small denominations start return in the currency chest. We also suggest how demand for "small change" could be estimated from the "lifetime" of the "smallest" denomination.

Keywords: Small Change**,** Denomination, Currency Management, Poisson Distribution
Journal of Economic Literature Classification: C0, D0, E40, E5
Running Title: Demand and Supply of Currencies

* The views expressed in this study are personal and not necessarily of the views of Indian Institute of Management Lucknow. The author bears full responsibility for any error.

Demand and Supply of Currencies of Small Denominations: A Theoretical Framework

1. INTRODUCTION

For a central bank, a rigorous study of the short run variation in currency demand is important. A systematic study of demand for currency leads to better currency management by the central bank. Net injection or absorption of liquidity in an economy is crucially dependent on public's demand for currency. Given its importance, the behavior of currency in the aggregate has been studied exhaustively, either in isolation or as a part of a bigger macroeconomic model (Jadhav, 1994; Cassino *et al*, 1997; Palanivel and Klein, 1999; Bhattacharya and Joshi, 2001, 2002). These studies point out that aggregate currency variation in an economy can be explained and predicted with reasonable accuracy.

Currencies in an economy are, however, available in different denominations. Not only the aggregate currency, accurate estimates of denomination-wise demand of currencies are also important because they enable the central bank to manage the payment needs in an economy correctly. If demand for a particular denomination exceeds supply, the economy pays in terms of more transaction cost. On the other hand, the cost of printing "excess" currency notes than is required could be substantial, especially for a developing country. Denomination-wise demand is, therefore, more closely related to operational aspects of currency management. Economic historians have documented several episodes in which economic agents complained about there being shortages of specific denominations.[1] These episodes are not just of historical interest because they have occurred in many countries during recent period as well (Wallace, 2003). Despite high practical importance, this aspect of currency demand has till now received limited attention in the literature.

In a pioneering attempt, initially Ghosh *et al* (1991) and then Sarkar *et al* (1993) studied requirements of currencies of different denominations in the Indian context. Borrowing from the already existing literature on aggregate demand for currency, these studies proposed a

[1] Sargent and Velde (2002) document some of these episodes, for example that in England circa 1400 (p. 134) and that in France circa 1337 (p. 135).

regression model to forecast the total value of currency of all denominations taken together. The major challenge was to break up the aggregate demand into components. Unavailability of actual currency transaction data constrained these studies to assume that currency transactions follow log-normal distribution. Currency transaction distributions were then simulated for different values of the parameters and for each transaction value in the support range of the distribution, minimal requirements of currencies of different denominations were obtained. Finally, denomination-wise requirements were obtained as weighted sums of all these requirements, where the weights were computed from the transaction distribution and assumed values of transaction velocities.

It may be noted that the results in both Ghosh *et al* (1991) and Sarkar *et al* (1993) are based on certain assumptions on demand for currency (e.g., the assumption of log-normal distribution in case of currency transactions) that are unlikely to be met in reality. Empirically, it is observed that small currency transactions in an economy tend towards round numbers and denominations that are actually available (Durand, 1961).[2] An implication of this observation is that currency transaction distributions in reality will have certain spikes (especially in the left tail), which will not be covered well in continuous distributions like lognormal distribution. Further, it is difficult to identify the spikes because with the rate of inflation experienced by the economy, they may shift abruptly.[3] Another major limitation in these studies is that the transaction "velocities", which play a major role in forecasting actual denominational requirements, are not estimated from real-life data. Rather, several plausible values of them are imposed in the model.

On a more general note, earlier studies like Ghosh *et al* (1991) and Sarkar *et al* (1993), though rich in terms of statistical theory, are devoid of any economic content. Currency holdings of different denominations are results of optimizing behavior of economic agents. From the demand side, household and business sector's problem is to balance the inventory

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[2] Sometimes producers fix prices just below a round figure so that it does not cross a psychological barrier (e.g., prices like 99/- instead of 100. This type of pricing is treated as an exception.

[3] Because of menu cost, price changes in many items may be infrequent, but when they increase, they tend to increase sharply (Ball and Mankiw, 1995). For example, suppose a cup of coffee in a restaurant initially costs Rs. 10. In the face of an average annual rate of inflation of 5.0 per cent, restaurant owners may not increase the price annually in exact proportion to inflation, but wait for one more year and then raise the price to Rs. 11. Note that in the first case, the currency transaction could be carried out by a single Rs. 10 note. In the second case, however, any cash transaction requires at least two notes / coins (e.g., Rs. 10 and Rs. 1).

maintenance cost and the transaction cost. From the supply side, the central bank's problem is to balance the operational cost and the transaction cost. Ideally, understanding optimal requirements of denominations require a rigorous understanding of the interactions of such forces behind demand and supply.

Recent literature has attempted to address some of these theoretical issues (Sargent and Velde, 2002; Redish and Weber 2007).[4] However, despite important contributions, the scope of the literature is still limited because of three reasons. First, it has focused its attention on commodity money that has intrinsic value. This specification of commodity money is useful in providing a strong theoretical foundation in understanding some of the historical episodes, but it also limits the applicability of the proposed models in the current context. Second and more importantly, the focus in the literature has primarily been on the interaction of buyers and sellers. Theoretical specifications typically pose a paired matching problem in which equilibrium emerges as a result of optimization of transaction costs by buyers and sellers. A major limitation of this approach is that the literature has not focused adequate attention on central bank behavior. In particular, it has not discussed how central bank behavior affects expectations of economic agents and how such expectations, in turn, affect equilibrium behavior of holding small denominations. Third, as we intend to show, to understand both demand and supply of small denominations, it is necessary to focus on specific combinations of denominations that facilitates transactions up to a specific range. As a consequence, the literature, so far, has only a limited success in suggesting operational rules for the central bank in managing currency requirements.

In this paper, we attempt to build a theoretical model of demand and supply of currencies of "small" denominations.[5] The conscious choice of the narrower focus on small denominations is due to four reasons. First, the framework proposed by us in this study can be extended to include all denominations. However, as we attempt to show, both the demand and the supply equations for larger denominations would be more complex. To estimate such demand and supply equations, one may need detailed data that are generally not available in the public

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[4] For example, Sargent and Velde (2002) (SV for short) specified a "cash in advance" model with two goods and two monies. One of the two moneys is interpreted as being a large coin ("dollar"), and the other a small coin ("penny"). In the SV model, one of the goods can be purchased with either of the two coins. The other good can only be purchased with pennies.

[5] Section 3 specifies the exact definition of "small" denominations.

domain. Second, a significant portion of the operational cost borne by the central bank is due to small denominations. Therefore, though narrower focus in the study makes the model simple, from the central bank perspective, the practical aspect of the problem is not unimportant. Third, historical episodes reviewed by earlier studies tend to point out recurrent incidents of lack of small change in different economies at different points of time. Although accurate estimates of transaction costs are not available in these studies, historical documentations seem to indicate that the cost – from the perspective of a single economic agent – should not be ignored. In fact, due to its practical importance, first Cippola (1956) and later Sargent and Velde (2002) coined the term "the big problem of small change". Fourth, common sense suggests that the specification of log-normal distribution for transactions, as in Ghosh *et al* (1991) and Sarkar *et al* (1993) would work reasonably well once the left tail in the currency transaction distribution is excluded. Thus, for larger denominations, the central bank has the option to follow the methods prescribed by Ghosh *et al* (1991) and Sarkar *et al* (1993).

In this paper, our focus is to understand the forces that form demand and supply of currencies of small denominations. Our contention is that such demand and supply need to be studied together to improve the efficiency of the operational aspect of currency demand management. From the supply side, i.e., from the central bank perspective, the efficiency has two angles: First, currency transactions in the economy should be as hassle-free as possible. Second, the central bank should be able to carry out its operations with the least cost possible. Mathematically, the operational problem could be specified as an optimization problem in which, *with a given set of denominations*, the central bank fixes the probability that an economic agent would be able to carry out transactions up to a certain range *independently (e.g., without the help of the counterparty)* and minimize the printing and other cost of notes required for that range.[6]

Clearly, the proportion of hassle free transactions is positively associated with per capita availability of number of notes or coins of different denominations over a broad range. An unconstrained increase, however, is not feasible for two reasons. First, it will lead to an increase in the printing and other operational costs of the central bank. Second and more importantly, if the economy is flooded with currencies of small denominations, economic agents may assume that in any currency transaction, the required change would be available from the counterparty.

[6] The cut-off limit for that probability would be a policy parameter.

This type of reasoning would act as a disincentive to the economic agent in holding smaller denominations. In our model such expectations from the demand side play a key role. In fact, the probability that a randomly chosen individual in the economy would hold certain currency combinations (so that he can either pay the exact amount as the "party" or give exact changes as the "counterparty") is strongly related to transaction cost and plays the role of "price" in our model. We attempt to show that in a dynamic environment, such interaction could be understood by specifying a cob-web type model where expectations on central bank behavior are formed based on previous period's experience.

The plan of the paper is as follows: Section 2 identifies the minimal composite basket of "small" denominations by an individual in an environment where the monetary authority follows a combination of power-of-two with the decimal principle in choosing denominations. Our focus is on identifying the minimum composite currency requirements which would enable an individual to pay all possible transactions up to a certain range *independently (e.g., without the help of a counterparty)*. Due to symmetry, the same currency combinations enable the individual to give exact "small change" as a counterparty when at the "receiving end". As we shall see, our results on minimal currency baskets are different from Summer (1993), Telser (1995) and probably closer to reality. In Section 3, we specify a simple model of demand and supply for currencies of small denominations based on optimizing behavior of economic agents. In our framework, the agents' currency holdings would depend on the probability that the counterparty would have the minimum composite currency basket to give the exact change. On the supply side, we attempt to identify the proportions in which the central bank should focus on printing these denominations so that probability of hassle in a currency transaction in the economy does not increase beyond a certain point. Section 4 specifies a cob-web type model that elucidates the dynamic interactions of demand and supply of small denominations and attempts to discuss a few operational strategies from the perspective of a central bank Finally Section 5 summarizes the major findings with some observations and discusses the scope of further generalizations.

2. MINIMUM REQUIREMENT OF COMBINATION OF CURRENCY DENOMINATIONS FOR SMOOTH TRANSACTIONS

The first step in operational management of currency demand is to decide the currency denominations that an economy should have. In this context, Sumner (1993) and Telser (1995) have argued that the optimal system of denominations of currency (coins and notes) would minimize the number of denominations while concurrently it should increase the probability of proffering exact change. In that proposed optimal system, the denominations differ from each other by a factor of three (that is, denominations of 1, 3, 9 and so forth). One can then describe the whole system in terms of power-of-three (that is 3^0, 3^1, 3^2 and so forth) principle. Telser (1995) shows that the problem is analogous to Bachet's problem of finding the smallest number of weights capable of weighing any unknown integer quantity between one and an infinite upper bound, using a bilaterally symmetric balance. When the person doing the weighing puts weights in both pans, in the optimal set of weights each weight is three times as heavy as the next smaller weight.

Despite being an academically useful result, the power-of-three principle is not of any practical use because in reality, the two most common organizing principles that are followed are (1) power-of-two principle, and (2) a combination of power-of-two with the decimal principle (Tschoegl, 1997). The powers commonly range from -3 (that is, from one-eighth of the standard unit) to two (that is, to four times the standard unit).[7] Today, all currency systems take the decimal principle as their starting point. Still, a survey of cross-country practices by Tschoegl (1997) reveals that denominations of coins minted since 1990 show many examples of the mixing of decimal system with the power-of-two principle. The same study also recognizes that a decimal system with half and double decimals could provide too many denominations. Many countries, therefore, provide only one additional denomination beyond that given by the decimal system and half decimal system. Worldwide, 20 appears to be more popular than 25 as a way to split the interval between 10 and 50.

In this paper, we assume that an economy follows the power of 2 with the decimal principle. The powers of 2 we consider are -1 and 1 only. The decimal principle implies that notes are printed in denominations of 1, 10, 100, 1000 etc. The power-of-two principle in combination with decimal system in the economy is specified in the following way:

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[7] This principle also shows up in financial markets. Contracts in futures markets tend to use decimal ticks for a minimum price change in the price of the futures being traded (Tschoegl, 1997).

- The power of 2^1 leads to denominations like 2, 20, 200 etc.
- The power of 2^{-1} leads to denominations like 5, 50, 500 etc.

For simplicity, we exclude the possibility of coins.

As mentioned in Section 1, the specification is very close to actual reality. For example, in the Indian context, it exhaustively includes all denominations that are currently in use. India, however, uses the power-of-two principle in a slightly limited way. Among the denominations mentioned here, only the denomination of 200 is currently not in use in the Indian economy. It may be noted that for all practical purpose Rs. 1000 is the highest denomination that is currently in use in India.

While determining the optimal denominations, our focus is on identifying combinations of denominations that would enable an individual to carry out currency transaction of any value up to a limited range, *independently, without any contribution of the counterparty.* To motivate towards the general result, first we consider the least number of notes needed for transactions from 1 to 9. Table 1 reports the minimum currency requirements up to that range.

Table 1: Minimum Denominations Required for Transactions from 1 to 9

Transaction Value	Minimum Currency Denominations										
	Individual Paying Independently			Individual Transacting with the Help of a Counterparty							
				Individual Pays				Individual Receives			
	1	2	5	1	2	5	10	1	2	5	10
1	1	0	0	1	0	0	0	0	0	0	0
2	0	1	0	0	1	0	0	0	0	0	0
3	1	1	0	1	1	0	0	0	0	0	0
4	0	2	0	0	2	0	0	0	0	0	0
5	0	0	1	0	0	1	0	0	0	0	0
6	1	0	1	1	0	1	0	0	0	0	0
7	0	1	1	0	1	1	0	0	0	0	0
8	1	1	1	0	0	0	1	0	1	0	0
9	0	2	1	0	0	0	1	1	0	0	0

The minimum possible combination of various denominations required to be able to carry out all possible transactions, is actually the super set of the denominations required for all individual transactions possible. **Thus for transactions up to Rs. 9, the super-set is {1, 2, 2, 5}, e.g., the combination of one Rs.1, two Rs.2 and one Rs.5 denomination.**

Note that some of these transactions could be carried out in a more efficient manner (e.g., involving less number of denominations) *with the help of a counterparty*. For example, if the counterparty has a Rs. 10 note, then an efficient way of carrying out a transaction of Rs. 9 would be through one Rs. 10 and one Rs. 1 note. However, note that while involvement of counterparty ensures smooth transaction for certain transaction amounts, supersets that would ensure transactions at all points from Rs. 1 to Rs. 9 would be larger in size. The minimum number of notes required in this case is 5 instead of 4, the paying individual must have currency combinations {1, 2, 5, 10} and the receiving counterparty only a one rupee note, e.g. {1}. If in addition, we impose efficiency of transaction at every amount between 1 and 9, the requirements become even more stringent. While the currency holdings of the paying individual would remain the same, e.g., {1, 2, 5, 10}, the receiving individual must now have an additional Rs. 2 note, implying a holding of the combination {1, 2}. Thus for efficient exchange at every level from 1 to 9, we need six notes appropriately distributed among two randomly selected individuals in contrast to four notes with which a transaction up to this range could be made. As the first option is likely to be more expensive for the central bank, we assume that the central bank's priority would be to make an individual as far independent in currency transactions as feasible.

To generalize the result further, note that for any transaction value that is a multiple of Rs.10 and is between Rs. 10 and Rs.90 (both inclusive), the elements of the same super set would be multiplied by a factor of 10, e.g., the super set would be {10, 20, 20, 50}. Thus, for any transaction up to Rs. 99, the super set would be: {1, 2, 2, 5, 10, 20, 20, 50}.

Similarly, for any transaction that is a multiple of Rs. 100 and is between Rs.100 and Rs. 900 (both inclusive), the super set would be {100, 200, 200, 500}. Therefore, for any transaction up to Rs. 999, the super set would be: {1, 2, 2, 5, 10, 20, 20, 50, 100, 200, 200, 500}.

In general, for transactions up to $(10^n - 1)$, the super set would be:

$\{1, 2, 2, 5, 10, 20, 20, 50, 100, 200, 200, 500, \ldots, 10^{n-1}, 2\times10^{n-1}, 2\times10^{n-1}, 5\times10^{n-1}\}$

Thus an individual needs to have 4n currencies in the specific combination mentioned above so that she can carry out any transaction up to (10^n -1) *without any additional help from the counterparty or a third person*.

It may be noted that if any individual, without depending on the counterparty, can carry out transactions up to (10^n -1), reduction of transaction cost is more than it apparently seems. First, there is the argument of symmetry. For example, suppose the transaction value is Rs. X where X is in between 1 to 100. If an economic agent can pay the counterparty directly, the counterparty can also pay that individual the balance if only one Rs. 100 is given. Second, if the value of the transaction exceeds Rs.100, then also our results partly apply. Only, in that case, one needs to ensure that agents engaged in transaction have adequate currencies in multiples of Rs. 100 along with denominations that can carry out transactions up to Rs. 99.

3. DEMAND AND SUPPLY OF CURRENCIES OF SMALL DENOMINATIONS

In this section, we analyze the demand for and the supply of "small" notes. The "small" notes defined in this section are: 1, 2 and 5, e.g. denominations that are necessary for transaction values that have a non-zero number in the unit place. Subsection 3.1 specifies the optimization framework for demand and Subsection 3.2 for that of supply.

3.1 Demand for "Small" Denominations

Consider an economy with *K* currency denominations in increasing order with values P_1, P_2, …, P_K. Given these denominations, consider the problem of a single economic agent. Let the utility function of holding these denominations be given by $U(Q_1, Q_2, \ldots, Q_K)$, where Q_i is the number of the *i*-th denomination. The agent's problem is to allocate M, the total amount of cash holding, across different denominations. Naturally, a general specification of the optimization problem of the agent is:

$$(1) \qquad \text{Max } U(Q_1, Q_2, \ldots, Q_K) \qquad \text{subject to} \quad \sum_{i=1}^{K} P_i Q_i = M$$

The equation of demand will depend on the functional form of the utility function. In this study, we assume that the preferences are of the following form:

$$(2) \qquad U(Q_1, Q_2, \ldots, Q_K) = \sum_{i=1}^{K} \gamma_i \log(Q_i - \beta_i) \quad \text{where} \quad Q_i > \beta_i\,,\ \gamma_i > 0, \sum_{i=1}^{K} \gamma_i = 1 \qquad i = 1, 2, \ldots, K$$

The above specification is standard and leads to a linear expenditure system of demand equations, e.g.,

$$(3) \qquad P_i Q_i = P_i \beta_i + \gamma_i \left(M - \sum_{i=1}^{K} P_i \beta_i \right) \qquad \text{where} \quad \gamma_i > 0, \sum_{i=1}^{K} \gamma_i = 1 \qquad i = 1, 2, \ldots, K$$

The basic interpretation of equation (3) is as follows: whatever be the actual cash holdings, agents' have some minimal requirements for all denominations. Once the minimal requirement is met, agents' would like to allocate the remaining cash holdings proportionately for all denominations.

We now impose the following assumption that for small denominations γ_i is an infinitesimal, e.g.,

Assumption 1: For any "small'' denomination, $\gamma_i \approx 0$, implying $Q_i \approx \beta_i$

The motivation behind this assumption is that agents in our model do not want to pile up "small" denominations as their wealth level or cash holdings increases. Rather, they would want a fixed number of such denominations purely for transaction purpose. An implication of this assumption is: demand for "small" denominations would be independent of the total amount of cash holdings of a person.

Common sense suggests Assumption 1 is not bad as a starting point for "small" denominations like 1, 2 and 5. Further, Assumption 1 may hold for denominations of higher values (e.g.,10, 20 and 50) as well. However, as we move to larger denominations, the role of aggregate cash balance and γ_i's, the allocation parameters, become more and more important. For the complete estimation of the linear expenditure system, detailed data on denominational break ups of cash holdings of economic agents are required. However, note that for "small" denominations like 1, 2 and 5, the functional form of demand becomes simpler due to Assumption 1. As Section 4 will reveal, the simplicity in functional form makes it amenable to different type of treatment, in so far as estimation is concerned.

For empirical standpoint, it is now important to examine the aggregation implications and identify the factors that would affect the parameter for aggregate or average minimal requirement. Note that the minimal requirement of small denominations could be different for different economic agents. However, when aggregated, the total demand becomes a constant proportion of the population actively engaged in currency transactions. Let this proportion for the *i*-th "small" denomination be denoted by B_i.

One of the major factors that would affect the aggregate demand for the *i*-th "small" denomination would be the probability that an individual would be able to receive the requisite change when needed. For small denominations like 1, 2, and 5 this depends upon the situation that when an agent pays Rs. 10 (or any number that is a multiple of 10) in a currency transaction, the counterparty would be able to pay him the requisite "small" change whatever be the value of the transaction. Note that if this probability is very high, it may be rational for the agent not to carry small denominations. If on the other hand, the probability is perceived as low, there could be tendencies on the part of the agent to stockpile small denominations, aggravating the "real" need in the economy.

In this paper, the aggregate demand for B_i is specified as a function of the probability that a randomly chosen individual carries a combination of the superset {1, 2, 2, 5} denominations in his possession. If this probability is α, then the aggregate demand for a small denomination in an economy becomes a function of α, e.g.

(4) $$Q_i^{AGG} = B_i(\alpha)$$

Note that here α, which is strongly related to the probability of a hassle free transaction, behaves like a "price" in the demand equation. If the "price" is more, e.g., the probability that the counterparty is able to give the necessary "small" change is high, the demand for "small" denominations decreases.

3.2 Supply of "Small" Denominations

In order to specify an optimization framework for operational management of currency denominations by the central bank, a few simplifying assumptions are proposed.[8] The assumptions are as follows:

Assumption 2: For all "small" denominations, the total per-unit cost (e.g., printing cost, supply chain and logistics management cost, destruction cost etc.) is same.

In practice, different small denominations have different costs of printing, destruction etc. per unit. This is due to the fact that the different denominations have varying sizes, paper or metal quality, security features along with other printing costs and life-time. For the sake of simplicity, we have assumed that all "small" denominations have the same cost of printing. This assumption saves a lot of algebraic computations and can easily be relaxed.

Assumption 3: Distribution of small denominations is homogeneous across income groups

This is a simplifying assumption and it implies that the sole motive of holding small denominations is transaction. In reality, the transactions that economic agents are indulging may vary as per their income distributions or revenue streams. For example, very affluent people may not carry out transactions of very less value (e.g., lesser than Rs. 10 etc.) frequently. Similarly, poor people may not carry out transactions more than Rs.1000 as frequently as rich people. However, this assumption is rooted in the derivation of the demand equation in Subsection 3.1, where it was assumed that holding of "small" denominations does not depend on the total cash balance a person would like to hold.

Assumption 4: Holdings of different "small" denominations are independent of one another

Actually, if a person has more than two Rs. 5 notes, then she may like to have a single Rs. 10 note to carry as less number of notes as possible. But here we are assuming that for a given person, the holding of different denominations is independent of one another. Once again, this assumption is also rooted in the demand for "small" denominations. As the demand for any such

[8] Some of these assumptions can be easily generalized; however, any such generalization would lead to unwarranted model complexity.

denomination did not depend on other denominations, the actual holdings are likely to be independent.

With Assumptions 2-4, we specify an optimization framework for the central bank. As an illustration, initially we concentrate on the supply of "small" denominations, e.g., 1, 2 and 5. Let S = {1, 2, 5}. Suppose that the central bank supplies N_i number of notes of denomination i ($i \in S$). In this study, we assume that the numbers of a specific denomination i ($i \in S$) that a randomly chosen economic agent has in her possession is a random variable X_i.

For demand for currencies of small denominations, the income distribution or aggregate cash holding is likely to play a limited role, as agents would hold these currencies solely for consumption. Thus, as per Assumption 3, the probability distribution of X_i is identical for all agents in the economy. Now, invoking Assumption 4, the probability that a randomly chosen economic agent will have at least one unit of denominations of 1 and 5 and 2 units of denominations 2 becomes a multiple of the individual probabilities, i.e.,

(5) $P[X_1>0, X_2>1, X_5>0] = P[X_1>0]\, P[X_2>1]\, P[X_5>0]$

A major problem here is to specify a particular probability distribution. In this paper, our actual specification is constrained by data availability. Typically, published data only reveal the aggregate numbers of different denominations circulating in the economy. Let the total number of agents engaged in currency transaction in an economy be N. Then, let the mean number of availability of denomination i ($i \in S$) in the population be z_i, where $z_i = (N_i/N)$. Note that z_i may be interpreted as per-agent availability of denomination i in the economy.

In our specification: $X_i \sim$ Poisson (Z_i), e.g.,

(6) $$P[X_i = x_i] = e^{-z_i} \frac{(z_i)^{x_i}}{x_i!}$$

Despite its limitations, Poisson distribution is often considered as a starting point for the specification of a count variable. With the specification of Poisson distribution, the expressions in the right hand side of equation (5) could then be written in the form:

(7) $$f(z_1, z_2, z_5) = (1 - e^{-z_1})(1 - e^{-z_2} - z_2 e^{-z_2})(1 - e^{-z_5})$$

The central bank would like to keep this probability at a "desirable" level [say, α=(1-δ) where δ is small]. Among various combinations of z_1, z_2 and z_5 it would choose the one that would minimize the cost. For simplicity, we assume that the fixed cost borne by the central bank is zero. Let the cost per note – same for each small denomination (Assumption 2) - be ν. [9] Here, ν is a technology parameter whose value is known to the central bank. The total cost of the central bank is then: ν $(N_1+N_2+N_5)$, or equivalently, the total per-agent cost is: $\nu(z_1+z_2+z_5)$. The central bank would like to supply the quantities of these denominations in such a manner that the probability that a randomly chosen economic agent has a currency combination {1, 2, 2, 5} is α and the total per-agent cost is minimum. Thus the optimization problem of the central bank in case of "small" denominations is as follows:

(8) Min $(z_1+z_2+z_5)$ subject to $f(z_1, z_2, z_5) = \alpha$

Note that a general framework of optimization involving all denominations is theoretically possible. However, in order it to be practically useful, more information on actual distributions of currencies is needed. The major difficulty is that Assumption 3 will not hold in case of larger denominations. This is because for larger denominations, cash holdings across the population are likely to be segmented. For example, in the Indian context, rich people may like to hold a few Rs. 1000 notes while the poor may not hold them at all.

To model such segmentation effectively, one needs to specify a more general probability distribution than Poisson distribution. One possibility is to use weighted Poisson distributions. If the weights are continuous and follow Γ distribution, then the resulting distribution would be a negative binomial distribution. The negative binomial distribution takes care of the typical over-dispersion problem observed in the empirical applications involving Poisson distribution. However, to estimate negative binomial distribution, one would require more detailed data on denomination-wise cash holdings in the population. As mentioned in Section 1, this is one of the major reasons behind our narrower focus on "small" denominations.

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[9] An implication of this specification is that we are ruling out the fixed cost in currency management operations. The underlying implication is that we are assuming that the central bank utilizes an existing infrastructure. While in the long-run, the assumption will not hold, for the short-run case, it will be a reasonable assumption.

To identify the supply function for "small" denominations, in this study we solve for z_i numerically for different values of α. While the actual algorithm consists of searching over specific grids for each z_i, the search in our case had been narrowed considerably. This is because each term in the right hand side of equation (7) is a probability and therefore, must be between 0 and 1. However, because the value of their product is α, each term and also, each pair of terms, in the right hand side of equation (7) must be greater than or equal to α. Thus, the feasible range of variation for each term in equation (7) is narrowed to $[\alpha,1]$. Grid values considered for the z_i's are also restricted accordingly. Our algorithm based on grid search calculates the minima of the objective function by looking at the restricted grids.

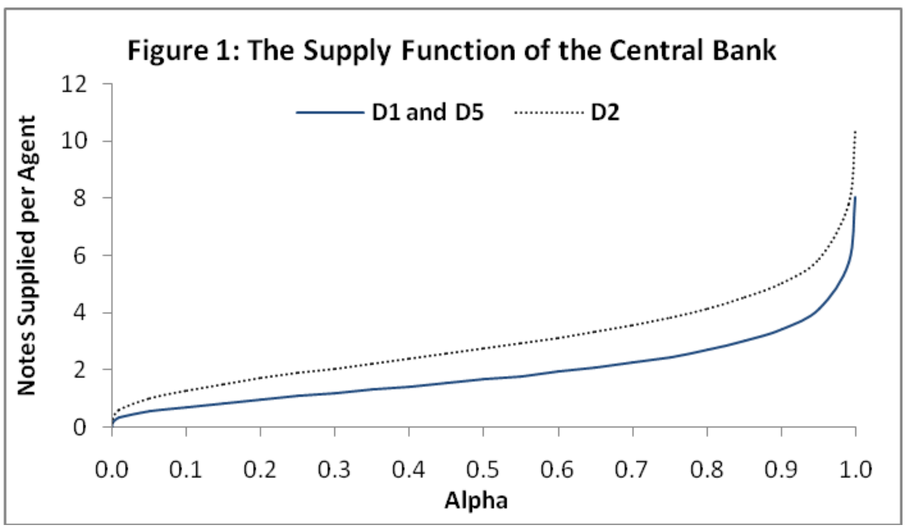

Figure 1 plots the supply function of the central bank for different values of α. Here, the graph labeled "D1 and D5" denotes the required number of notes of denominations 1 and 5 per agent and D2 denotes the same for denomination 2. For any particular value of α, if the central bank supplies currency notes as per the graphs "D1 and D5" and "D2", then probability that a randomly chosen individual in the economy will have the {1, 2 ,2, 5} superset as a subset of his currency holding would be α. Clearly, as α increases, more notes per agent are needed for all denominations.

Note that near α=0, the slope of the supply function for both the graphs is high. The supply functions are approximately linear for $\alpha \in$ [0.05, 0.80]. For α>0.80, the slope of the functions once again starts to increase sharply. For α>0.9, the slope increases at a very fast pace. As $\alpha \to 1$, the number of notes required for each denomination tends towards infinity.

Note that the supply functions for denominations 1 and 5 are identical. Further, near α=0, the proportion of notes for 1, 2 and 5 denominations supplied by the central bank is 1:2:1. However, as α increases, the ratio is not in 1:2:1, and comparatively less number of notes of denomination 2 is required.

In the context of the nature of the supply function, a major remark on the role of the homogeneity assumption is necessary. If population is heterogeneous, weighted Poisson distributions or negative binomial distributions would be better options. In such cases, however, the supply function would tend to shift upward. To illustrate this, consider the situation where per-agent supply of notes of any particular denomination by the central bank is 4.0. Note that in this case a randomly selected agent has that denomination (e.g., P[X>0] in case of Poisson (4) distribution) is 0.9817. However, suppose that the agents are heterogeneous. Although per-agent supply of the denomination is still 4.0, 90.0 per cent of agents in the economy hold only 3 units of that denomination whereas the remaining 10.0 per cent hold 13 units. In this situation, the probability that a randomly selected agent would hold that denomination decreases to 0.9552. Thus, in such situations, for a fixed value of α, more supply of coins would be necessary.

4. A COB-WEB TYPE MODEL OF INTERACTION OF DEMAND AND SUPPLY OF SPECIFIC DENOMINATIONS

Analysis in Section 3 reveals that at any given point of time, the demand for a “small” denomination would be a multiple of the economically active population, the multiplication factor being related to the transaction cost that agents in the economy are comfortable with. The transaction cost in the economy is summarized in our model by agents’ perception about the availability of “small” denominations. In this section, we examine the implications of equilibrium behavior, both in the static as well as in a dynamic environment.

In the static model in Section 3, central bank can maintain equilibrium for a particular denomination if it knows the transaction cost that the agents in the economy will be comfortable with. In case supply exceeds demand, the currency holdings of the central bank in its inventory will increase, as unwanted denominations return to the central bank through the banking sector. In the case where demand exceeds the supply, the "gap" in small change would be filled up by informal arrangements like bulk purchase or by the use of commodity or private money.

In reality, however, demand and supply constantly interact with each other as the perceptions of agents may depend upon the actual experience in the previous period. To illustrate the dynamic behavior, we specify a cob-web type model of demand and supply of "small" denominations.

Suppose that demand is formed by previous period's experience. Also, for illustration, we assume a simple linear demand equation, e.g.,

(9) $Q_{i,t}^{AGG} = a + b\,\alpha_{t-1}$ $\quad\quad b \le 0$

The specification implies that with as the probability that an individual will have "small" change increases (decreases) in the previous period, agents tend to hold less (more) small change assuming that small change would be made available by the counterparty.

Note that in our specification, the supply function of the central bank depends on the current value of α. Figure 1 suggests that if $\alpha \in [0.1, 0.8]$, then the supply function is nearly linear. Suppose, we approximate the supply function in this range by the line:

(10) $Q_{i,t}^{S} = c + d\,\alpha_t$ $\quad\quad d \approx 0.045$ for "D1 and D5" and $d \approx 0.035$ for "D2"

Then, as in a standard cobweb model, if $|b/d| < 1$, transaction costs in the form of α_t would converge to a stable equilibrium. An implication is: a very small stable value of the parameter b relative to d ensures convergence of the public's and the central bank's transaction cost through an iterative process over time. Such interaction of demand and supply also helps to understand the situations in which there could be a "big problem of small change". Such problem could occur in situations where the per capita supply of small denominations is limited. Further, the equilibrium could be an unstable one for a high value of |b|. When the equilibrium for a particular denomination is unstable, commodities or individual money fills up the gap. Transaction needs sometime also tune to the actual availability of denominations. Bulk purchase is one option that is exercised by economic agents to reduce transaction cost.

Experience of monetary history for different countries suggests that |b| would be smaller relative to |d| in most real-life situation. However, note that even if |b| is high, central bank has the option of increasing α. As α increases, the slope of the supply equation in Section 3 increases at a fast rate. For example, if the central bank operates in a range where $\alpha > 0.9$, convergence to a stable equilibrium is more likely than in cases where $\alpha < 0.9$, even with a high value of |b|.

The rest of the section now focuses on operational strategies of the central bank. One strategy is to have a detailed survey on cash holdings by different economic agents. Such survey data, however, are often not readily available and meets with practical difficulties. Without such data, a possible beginning would be to estimate the current level of transaction cost in the economy. Suppose, the central bank starts with per-agent supply of y_1, y_2 and y_5 notes or coins respectively for denominations 1, 2 and 5. The central bank then can compute the probability that a randomly selected agent would be able to carry out a transaction in the range [1,9] without any help of the counterparty. If this probability is found to be in the range of [0.1, 0.8], then the central bank may start increasing the supply of the necessary denominations at a rate above the growth rate of the economically active population. The crucial signal that the optimal level is reached would be the point beyond which small denominations of good quality supplied by the central bank start coming back to the central bank through the banking sector. The central bank can, of course, improve the handlings of currency if it regularly collects indents for different denominations from financial intermediaries and obtains feedback about service quality from the economic agents.

We now attempt to discuss another strategy where the old historical data can be used to estimate the economy's need for small change under some simplifying assumption. To elucidate the strategy, note that in the long-run, depending upon the average rate of inflation, the transaction distribution in the economy will shift. We propose a method of estimation of the approximate requirement of the lowest denomination and its upper limit from the lifetime of "minimum" denominations in an economy.

Without loss of generality, suppose that in a given year 0, denomination 1 has become the smallest denomination in an economy. Now, suppose due to inflation and the concomitant shift in the transaction distribution, the denomination 1 is withdrawn in year *t* because agents do not want to hold that denomination. Suppose, the rate of inflation in year *j* (*j*=1, 2, …, t) is π_j. and the per-agent circulation of denomination 1 in year *j* is y_j. We now propose two intuitively appealing assumptions:

Assumption 5: The minimum currency transaction size in year 0 in the economy is 1.

Assumption 6: The minimum transaction in the long-run shifts proportionally to the compounded rate of inflation in the economy between years [0,t].

Let us denote y^* as the following:

(11) $y^* = \text{Max}\ (y_0, y_1, y_2, \ldots, y_t)$

Then the per-agent demand for denomination 1 in the economy may be estimated as y^*. Also, Assumption 6 implies that G_t, the minimum transaction size in year t, will be:

(12) $$G_t = \prod_{j=1}^{t} (1 + \pi_j)$$

No agent in the economy want to hold denomination 1 to carry out a transaction of size G_t, implying that G_t could be an upper bound on the demand for denomination 1.

Note that central banks traditionally maintain detailed records on minting. Hence, the exact point of time when specific denominations are withdrawn from an economy completely will be known to the central bank. Hence, in the absence of detailed data on cash holdings, the lifetimes of "minimum" denominations provide valuable clues about the maximum requirements of "small change" in an economy.

In this context, note that the information on demand obtained from the lifetime of a particular denomination also becomes useful at a later stage. This happens when the need for denominations 1, 2 and 5 vanishes completely and they are substituted by 10, 20 and 50. This is the situation when transaction size shifts about ten fold compared to the base period, the denominations 10, 20 and 50 now take the new role of 1, 2 and 5. Our conjecture is: in such a situation, the earlier learning about the transaction cost that the people would like to have in case of denominations 1, 2, and 5 is likely to be similar in case of 10, 20 and 50. Thus maintaining a low rate of inflation and the knowledge that eventually the demand would shift to the next decimal level gives the central bank considerable flexibility to estimate the transaction cost the agents in an economy would be comfortable with.

5. CONCLUSION

The paper attempted to present a theoretical framework of demand and supply of currencies of small denominations. Unlike previous studies, in our framework both demand and supply equations emerged out of an optimization framework for paper currencies that have no intrinsic value. Further, the framework was tuned to derive specific roles or strategies for the central bank or the monetary authority.

Demand functions for small denominations were obtained as a special case of a linear expenditure system. Our main contention was that economic agents would like to hold a fixed number of small changes, independent of their respective total cash holdings. However, the fixed number is influenced by the probability that in a currency transaction, the counterparty would be able to provide the small change if needed. The supply function in our framework was derived from an optimization problem where the central bank balanced its operational cost with the probability that an individual would be able to carry out "small" transactions independently, without the help of counterparty. We showed that the probability that a randomly chosen individual in the economy would hold certain currency combinations (so that he can either pay the exact amount as a "party" or give exact changes as the "counterparty") is strongly related to transaction cost. In our model, the probability played the role of "price". We attempted to show that in a dynamic environment, such interaction could be understood by specifying a cob-web type model where expectations are formed based on previous period's experience. As an operational rule, it was proposed that the central bank should increase the supply of small denominations at a rate marginally above the growth rate of economically active population and stop minting as soon as some of the small denominations return in the currency chest.

More research is required to remove some of the lacunae in this study. For example, some of the assumptions in our model need validation or refutation from empirical observations. In particular, the specification of Poisson distribution in the model is guided by data availability. The probabilities, and hence the supply function in the model can change substantially if holdings of "small change" in the population is not homogeneous. In short, a major conclusion appears to be that to convert the "big problem of small change" into a "small problem", a major requirement would be a much more concerted effort on empirical behavior of holdings of different denominations by agents in an economy.

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