Forecasting Australian Macroeconomic variables, evaluating innovations state space approaches

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Innovations state space time series models that encapsulate the exponential smoothing methodology have been shown to be an accurate forecasting tool. These models for the first time are applied to Australian macroeconomic data. In addition new multivariate specifications are outlined and demonstrated to be accurate.

**Keywords**: exponential smoothing, state space models, multivariate time series

1 Introduction

Accurately forecasting economic conditions is an integral part of government and business. In this paper, innovation state space models that capture various forms of the exponential smoothing methodology are assessed for the first time using Australian macroeconomic data. In addition, new formulations are tested and shown to produce relatively accurate forecasts.

Only a few studies have considered the approach investigated in this paper. The first was Hyndman, Koehler, Snyder & Grose (2002) which developed and applied this approach to data from the M3\(^1\) forecasting competition. Their findings clearly demonstrated that the model produced relatively accurate forecasts. A multivariate extension of Hyndman et al. (2002) was first proposed by de Silva, Hyndman & Snyder (2010) which was assessed using US macroeconomic data. Again, the findings demonstrated that the approach produced relatively accurate forecasts.

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\(^{1}\)For more information on the M3 data, refer to Makridakis & Hibon (2000)
Many studies have demonstrated that state space models are useful economic time series modeling techniques. For example, Aoki & Havenner (1991) modeled various macroeconomic variables including gross national product and money supply. Recently Panher (2007) showed how state space models can be used to capture economic relationships relating to the money market. In addition Leu & Sheen (2007) outlined a small state space model of the Australian economy. None of these papers however, assessed forecasting performance.

Only a handful of Australian macroeconomic forecasting experiments exist. These include Adams, Dixon, McDonald, Meagher & Parmenter (1994) which assessed the forecasts of the Computer General Equilibrium model known as the Monash model. Summers (1999) which outlines the Bayesian Vector Autoregression (BVAR) model used at the Melbourne Institute. Subsequent work (Summers 2001) showed the effectiveness of forecasts generated using the BVAR specification during the Asian financial crisis. A relatively recent study is Tsiaplias & Chua (2010) which examines the use of large data sets using the same approach as Stock & Watson (2002a,b).

The structure of this paper is as follows, section 2 outlines the innovations state space model including the new multivariate specification. Section 3 outlines the design of the forecasting experiment as well as the data used. The results are presented in Section 4. A summary together with some general concluding remarks are presented in Section 5.

2 The Innovation State Space Specification

The innovation state space specification was first proposed by Anderson & Moore (1979). The form of the model is very general, hence it is able to adapt to many different data generating processes. This investigation considers those relating to the exponential smoothing methodology. Specifically the framework outlined in Hyndman, Koehler, Ord & Snyder (2008).

2.1 Univariate Specification

Let an observation at time $t$, where $t = 1, 2, \ldots, T$, be denoted as $y_t$. The general form of the innovations state space model specification can then be represented as:

$$y_t = hx_{t-1} + e_t,$$

$$x_t = Fx_{t-1} + \alpha e_t, \quad e_t \sim iidN(0, \sigma^2),$$

where the term $x_t$ denotes a $k$-vector of latent states. The $k$-vector $h$ is often referred to as the structure vector. The form of this vector is determined prior to fitting the model and typically comprises zeros and ones. Like $h$, $F$ is also determined prior to fitting the model. Although, in some instances selected
components of the $k \times k$ $F$ matrix may be determined by the data. The vector $\alpha$ is also of length $k$ and represents a set of smoothing coefficients.

It is important to note the difference of the error specification between the innovations approach and the state space formulation often presented (see for example Harvey 1989). By specifying a common source of error for the observation and state transition equations (equations (2.1) and (2.2) respectively), the model is estimated using conventional likelihood approaches (as discussed in section 2.4). This is different from the Harvey specification as the likelihood can only be estimated by using the Kalman Filter. Further, it has been shown that the when the multiple errors are constrained to be independent, the innovations specification is more general (Anderson & Moore 1979, Hyndman et al. 2008).

The innovations state space formulation encapsulates 30 exponential smoothing specifications (Taylor 2003). Only a subset of these models are considered here. There are two reasons for only considering a subset, one the variables modeled here are seasonally adjusted reducing the available number to models to ten. Secondly, only additive trend models were considered as some of the multiplicative trend models can be unstable (Akram et al. 2009). The total number of models considered therefore is six. A list of these univariate models is presented in Appendix A.1.

For the sake of brevity, it is shown how only two of these models are encapsulated by the general form depicted in equations (2.1) and (2.2). The first being the Holt linear trend (or local trend model) additive error model. This model is obtained by setting:

$$h = [1, 1], \quad x_t = [\ell_t, b_t], \quad F = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad \alpha = [\alpha, \beta],$$

The terms $\ell_t$ and $b_t$ denote the level and trend states (sometimes $b_t$ is interpreted as the growth rate) at time $t$, whilst $\alpha$ & $\beta$ denote the smoothing coefficients of the level and trend components respectively.

The multiplicative error counterpart is obtained by replacing $e_t$ with the relative error term $\frac{y_t - y_{t-1}}{y_{t-1}}$, where $y_t = \ell_{t-1} + b_{t-1}$. Then using the same equalities as before the following specification is revealed:

$$y_t = (\ell_{t-1} + b_{t-1})(1 + e_t), \quad (2.3)$$
$$\ell_t = (\ell_{t-1} + b_{t-1})(1 + \alpha e_t), \quad (2.4)$$
$$b_t = b_{t-1} + \beta(\ell_{t-1} + b_{t-1})e_t. \quad (2.5)$$

The other models considered can be regarded as variants of the Holts linear trend specification. The first is when no trend is present ($b_t = 0, \forall t$). The second is when a damped term is included in the trend equation such that (for the additive error case):

$$b_t = \phi b_{t-1} + \beta e_t.$$
2.2 Multivariate Specification

In addition to the univariate specification a multivariate version of the additive error models is also consid-
ered. This specification is known as the Vector Innovations Structural Time Series (VISTS) framework. This framework was shown to produce accurate forecasts when contrasted with traditional alternatives such as the Vector Autoregression specification (de Silva et al. 2010).

Let \( y_t \) denote a set of series at time \( t \), the multivariate general form can then be expressed as:

\[
\begin{align*}
y_t &= H x_{t-1} + e_t, \\
x_t &= F x_{t-1} + Ae_t,
\end{align*}
\]

where \( y_t \) denotes an \( N \)-vector of observations at time \( t \). The terms \( H, F \) and \( A \) denote various coefficient matrices, \( x_t \) a \( k\times N \)-vector of states and \( e_t \) denotes an \( N \)-vector of innovations. As before, the innovations are assumed to follow a Gaussian distribution. It is also assumed that \( \Sigma \), the covariance matrix of \( e_t \), is diagonal.

The multivariate Holt linear trend (also sometimes referred to as the vector local trend model) additive error specification has the form:

\[
\begin{align*}
y_t &= \ell_{t-1} + b_{t-1} + e_t, \\
\ell_t &= \ell_{t-1} + b_{t-1} + A_\ell e_t, \\
b_t &= b_{t-1} + A_b e_t
\end{align*}
\]

where \( \ell_t \) and \( b_t \) denote \( N \)-vectors of the level and trend at time \( t \). The terms \( A_\ell \) and \( A_b \) denote \( N \times N \) matrices of persistence coefficients. By setting

\[
H = [I_N, I_N], \quad x_t = [\ell_t, b_t], \quad F = \begin{bmatrix} I_N & I_N \\ 0 & I_N \end{bmatrix}, \quad A = [A_\ell, A_b],
\]

of equations (2.6) and (2.7), the multivariate Holt linear trend additive error specification is revealed.

Only the additive error models are evaluated here as multivariate error models are yet to be de-
veloped. Two variations of the Holt's linear trend model are considered, one when there is no trend \( (b_t = 0, \forall t) \) and two, when a damped term is included in the trend equation. Specifically,

\[
b_t = \Phi b_{t-1} + A_\beta e_t.
\]

In this instance \( \Phi \) and \( A_\beta \) denote \( k \times k \) coefficient matrices. To avoid identification issues (de Silva et al. 2010), either \( \Phi \) or \( A_\beta \) must be constrained to be diagonal. In the following analysis \( A_\beta \) is constrained to
be zero. A full specification of the three VISTS models is presented in Appendix A.2.

2.3 New Multivariate Specification

The motivation of the new specification is similar to the Beveridge-Nelson class of VISTS models (de Silva et al. 2009). It takes the form:

\[ y_t = \ell_{t-1} + c_{t-1} + e_t \]  
\[ \ell_t = \ell_{t-1} + A_t e_t \]  
\[ c_t = \Phi_1 c_{t-1} + \Phi_2 c_{t-2} + \Phi_3 c_{t-3} + A_c e_t \]

The level (equation (2.12)) can now be considered to capture the permanent component and the trend (equation (2.13)), the transitory component of each series. The terms \( A_\ell, A_c \) and \( \Phi \) denote \( k \times k \) coefficient matrices. As in the case of vector damped local trend model \( A_c \) is constrained to be diagonal to avoid identification issues.

The new specification maps into the general form by equating:

\[ H = [I_N, I_N, 0_N, 0_N], \quad x_t = [\ell_t, c_t, c_{t-1}, c_{t-2}], \quad F = \begin{bmatrix} I_N & 0_N & 0_N & 0_N \\ 0_N & \Phi_1 & \Phi_2 & \Phi_3 \\ 0_N & I_N & 0_N & 0_N \\ 0_N & 0_N & I_N & 0_N \end{bmatrix}, \quad A = [A_\ell, A_c, 0_N, 0_N]. \]

Three variations of this model are considered. The distinguishing feature between these models is the lag length of the transitory component. The three lag lengths considered are one, two and three. Considering up to three lags is consistent with Pagan & Dungey (2000) who specify a lag length of three for a small macroeconomic model of the Australian economy in the form of a structural vector autoregression. Appendix A.3 presents each model considered.

2.4 Implementation

As indicated previously the innovation state space specification can be estimated using conventional likelihood techniques. The likelihood function is numerically optimised and takes the form:

\[ L(\psi, x_0, \Sigma) = \left(\frac{2\pi}{N} \prod_{i=1}^{N} \sigma_i^2\right)^{-1/2} \exp \left( -\frac{1}{2} \sum_{t=1}^{T} \sum_{i=1}^{N} e_{it}^2 \sigma_i^{-2} \right), \]  

where \( \psi \) denotes the unknown coefficients, \( x_0 \) denotes the state seed values and \( \Sigma \) denotes a diagonal covariance matrix of the innovations. The non-zero (diagonal) elements of \( \Sigma \) are denoted as \( \sigma_i, i = \)
The initial state values, $x_0$, are determined by running a regression on the first ten observations. The regression consists of a constant and a linear trend only. The constant is used for the initial level value, $\ell_0$. The coefficient belonging to the linear time trend is reserved to be the starting value for $b_0$.

For the new set of models (section 2.3) the starting value for $\ell_0$ is the first observation ($y_t$) and the starting values for various $c$’s are the difference between the first and second observations.

The parameter matrices that feed into (2.14) to initiate the estimation procedure are constrained to be diagonal. Experience indicates that the final results are insensitive to the starting values. Therefore, arbitrary values are selected corresponding to the conditions stated below.

The likelihood is maximised subject to two conditions: one being invertibility

$$\text{mod } |\lambda(D)| < 1,$$

(2.15)

where $D = F - AH$, and the other being stationarity

$$\text{mod } |\lambda(F)_{N+1}| < 1.$$  

(2.16)

The invertibility condition denoted by equation (2.15) refers to the modulus of the largest eigenvalue of $D$. In relation to equation (2.16), it should be noted that the first $N$ eigenvalues are one by construction, the next largest eigenvalue (hence the subscript $N + 1$), is therefore constrained to be of modulus less than one.

3 Experiment design

In total, six forecasting alternatives are considered. All specifications are implemented using the freeware software package R (R Development Core Team 2009). The methods considered are:

1. Naïve Prediction.
3. Vector Auto-Regression (VAR).
4. Univariate Innovation structural times series models (UISTS).
5. Vector Innovation structural times series models (VISTS).

The first approach is used as a benchmark. Using the Naïve method as a benchmark in economic forecasting experiments is consistent with many studies including Meese & Rognoff (1983). In addition it has the extra benefit of being consistent with the economic concept of the efficient markets hypothesis.
The next two approaches considered arguably represent the most popular time series approaches. ARIMA processes have been used extensively throughout all areas of forecasting. The VAR process is commonly accepted to be the multivariate form of option two despite being only a partial extension. A maximum lag length of three is set for the VAR processes, whereas the ARIMA processes are unconstrained.

Alternatives four and five are the innovation models outlined in the previous sections. Note, the new models outlined in section 2.3 are considered part of the VISTS class of models defined in section 2.2.

All time series approaches listed above can be implemented using automatic routines. Specifically, a routine that automatically selects the lag order using an information criterion is easily designed (options two and three). Similarly, the choice between exponential specifications (options four and five) can also be automatically determined using information criterion (Hyndman & Khandakar 2008). In the applications that follow the Akaike information criterion is employed.

3.1 Data

Four macroeconomic variables that are considered to represent the main economic indicators of the Australian economy are used to measure forecasting performance. These are real gross domestic product per capita (RGDPpc), interest rates (90-day bill Treasury Note), the inflation rate and the effective full time unemployment rate.

RGDPpc essentially measures the size of the economy in terms of output adjusting for prices and population growth. It is important to adjust by these factors as they give a truer picture of the state of the economy. Since 2008, for example, the growth rate in real GDP per capita is consistently lower by up to 0.6%. This is significant given the near zero growth rates of real GDP recorded during the same period coupled with the significant increases in population.

Interest rates influence every agent in an economy. This investigation will consider the 90-day Treasury note for two reasons. One, this represents a short-term interest rate that reflects closely the Reserve Bank of Australia’s position in regards to the credit market, and two it is one of few interest rate variables readily available dating back to January 1986 (the beginning of sample).

The inflation rate used in these forecasts correspond to the annual inflation rate measured on a quarterly basis. The inflation rate for ‘all groups’ was selected as arguably this version of the inflation rate has more influence on the market as a whole rather than the trimmed mean or median versions.

The effective rate of unemployment is derived from aggregate hours worked and labour market participation (see Appendix B). Being a function of aggregate hours worked it is not subject to the limitations of the definition of being (un)employed. Further it is likely to reflect under-employment conditions better. As such, the measure is believed to be more reflective of labour market conditions than the unemployment rate (Davidson & de Silva 2009). This is especially important given the significant labour market
de-regulation in recent years.

The multivariate alternatives (options three and five) are based on an eight variable system. The additional variables are US real gross domestic product per capita (US-RGDPpc), US treasury bill, the terms of trade (TOT), and the trade weighted index (TWI). These variables capture different aspects of Australia's international exposure. This is particularly important in the Australian context as international economies significantly influence Australia's overall economic conditions.

A rolling forecast experiment is conducted. To begin with each model is fitted to observations spanning 1986 Q1 to 2005 Q4. For each model, eight-step-ahead forecasts are generated. These forecasts are stored and the process is repeated again, this time fitting the models to observations ranging 1986 Q1 to 2006 Q1. The process is repeated until the sample is exhausted. In total nine eight-step-ahead forecasts for each variable is generated. There are ten seven-step-ahead forecasts and 11 six-step-ahead forecasts and so on. As the most recent observation is 2009 Q4, 16 one-step-ahead forecasts are generated. At each stage the set of forecasts is stored. The next section summarises the relative accuracy of each model using the stored forecasts.

4 Results

The performance of the innovations structural time series models will be assessed in three ways:

1. New multivariate innovations structural state space models verses the Naïve approach.
2. Innovation models verses the Naïve approach.
3. Innovations verses autoregressive alternatives.

In each case the average absolute forecast error by series as well as the relative performance of the alternatives will be considered.

4.1 New VISTS compared to Naïve

Figure 4.1 presents the average absolute forecast across horizon for each macroeconomic variable. In addition, Table 4.1 shows the percentage of times each model generates the 'best' forecast as well as the average rank for each model (note: the lower the average rank the more accurate the model).

In each case VISTS-AR refers to the new specification outlined in section 2.3. The number following indicates the number of the lags in the transitory component.

In most cases Figure 4.1 demonstrates that the VISTS-AR2 model outperforms all alternatives. The major exception being interest rate predictions. The few exceptions among the other
variables are the forecasts relating to short horizons of the RGDP per capita and the effective unemployment rate.

<table>
<thead>
<tr>
<th>Horizon</th>
<th>% of Firsts</th>
<th>Average Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>VISTS-AR1</td>
<td>VISTS-AR2</td>
</tr>
<tr>
<td>1</td>
<td>21</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>3-4</td>
<td>33</td>
<td>37</td>
</tr>
<tr>
<td>5-6</td>
<td>25</td>
<td>40</td>
</tr>
<tr>
<td>7-8</td>
<td>27</td>
<td>39</td>
</tr>
</tbody>
</table>

Table 4.1: Ranks by horizon and model

The findings of the Figure 4.1 are supported by the measures reported in Table 4.1. The measures in the left panel indicates that the VISTS-AR2 specification outperforms all models from the third horizon onwards. Interestingly, the most accurate model for horizons one and two is the VISTS-AR1 specification.

These findings are confirmed by the right panel where all VISTS models have lower average rank across horizons two to eight.
4.2 Innovations vs Naïve

Four alternatives to the Naïve approach are considered in this section, they are the univariate innovation state space alternative (option 2) denoted as ETS\(^2\), the VISTS models, and two selected models, the vector damped local trend (VDLT) and the VISTS-AR(2) models. Both the ETS and VISTS are implemented using automatic selection produces using the AIC (for more details refer to Hyndman & Khandakar 2008, de Silva et al. 2010). The reason for considering the VDLT separately is based on the conclusion reached by Fildes (2001) and Gardner (2006) that it is difficult to beat the damped trend models. Similarly, given the performance of the VISTS-AR(2) in the previous section, it would seem the same could be said of it also.

Figure 4.2 illustrates the forecasts accuracy for each model. The forecast accuracy of the ETS and Naïve approaches are indistinguishable for inflation and unemployment. It seems the ETS approach is defaulting to a naïve prediction model which is a special case of the local level model. For the other two variables the performance of the ETS and Naïve models are distinct, the ETS clearly outperforms the Naïve approach for RGDPpc whereas the reverse is true for the chosen interest rate variable.

\(^2\)This notation is based on Hyndman et al. (2008) where E-error type, T-trend type and S-season type
Consistent with the findings in the previous section the VISTS-AR2 model seems to perform relatively well. This is becomes more apparent as the horizon increases. Table 4.2 presents the percentage of times the alternatives generated the most accurate forecasts. As before the right panel of the table presents the average rank.

The VISTS model using the automatic implementation seems to be inferior to the VDLT supporting the findings of Fildes (2001), Gardner (2006). Overall the VISTS-AR2 and VDLT models perform relatively well.

The right panel of Table 4.2 indicates that both the univariate and VISTS-AR2 models perform relatively well from the second horizon onwards. This is an interesting observation given that the univariate approach does not seem to produce the most accurate forecast on a regular basis. This suggests the univariate option seems to perform consistently well albeit only generating the most accurate forecast a few times.

### 4.3 Innovations vs ARIMA and VAR

In this section the performance of the innovations approach is gauged against the ARIMA and VAR alternatives. In all cases (except the VISTS-AR2) the most appropriate model is selected using the AIC. Figure 4.3 presents the average absolute error across horizon.

According to Figure 4.3 the performance varies according to horizon and variable. Table 4.3 provides a clearer indication of which model performs best. In particular, the left panel of Table 4.3 the alternative that performs best is the VISTS at horizons one to four and the VISTS-AR2 at horizons five to eight (note the VAR also performs equally as well at horizons 5-6).

The findings presented in the right hand side of the panel of Table 4.3 support the conclusions that the VISTS-AR2 performs better at longer horizons. At horizons one and two the ARIMA and ETS models perform relatively well.
Figure 4.3: Mean Absolute Forecast Error by horizon and series (Black=VISTS-AR2, Red=ETS, Orange = ARIMA and GREEN=VAR, Purple=VISTS)

<table>
<thead>
<tr>
<th>% of Firsts</th>
<th>Average Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizon</td>
<td>VISTS</td>
</tr>
<tr>
<td>1</td>
<td>21</td>
</tr>
<tr>
<td>2</td>
<td>23</td>
</tr>
<tr>
<td>3-4</td>
<td>30</td>
</tr>
<tr>
<td>5-6</td>
<td>20</td>
</tr>
<tr>
<td>7-8</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 4.3: Ranks by horizon and model

5 Conclusion

An innovation state space approach that captures various forms of the exponential smoothing methodology has been evaluated in this paper. The methodology has been established in the forecasting literature for some time, however up until now had never been tested on Australian macroeconomic data.

In addition to reviewing established formulations a new formulation that decomposed a series into permanent and transitory components was also presented.

The final section of the paper presented the results of three pieces of analysis. In general, the VISTS-AR(2) model is a particularly good model as it consistently produced relatively accurate
forecasts, especially as the horizon increased.

Further, the findings indicate that the innovations approach (both univariate and multivariate forms) is a robust forecasting tool. Therefore, it is recommended, based on the evidence presented that these models should be considered as viable options when forecasting Australian macroeconomic variables.

References


URL: http://www.R-project.org


A Innovation models

A.1 Univariate Models

Local Level Model

Additive error

\[ y_t = \ell_{t-1} + e_t \] (A.1)
\[ \ell_t = \ell_{t-1} + \alpha e_t \] (A.2)

Multiplicative Error

\[ y_t = \ell_{t-1}(1 + e_t) \] (A.3)
\[ \ell_t = \ell_{t-1}(1 + \alpha e_t) \] (A.4)

Local Trend Model (Holt Linear Model)

Additive error

\[ y_t = \ell_{t-1} + b_t + e_t \] (A.5)
\[ \ell_t = \ell_{t-1} + b_{t-1} + \alpha e_t \] (A.6)
\[ b_t = b_{t-1} + \beta e_t \] (A.7)

Multiplicative error

\[ y_t = (\ell_{t-1} + b_t)(1 + e_t) \] (A.8)
\[ \ell_t = (\ell_{t-1} + b_{t-1})(1 + \alpha e_t) \] (A.9)
\[ b_t = b_{t-1} + \beta(\ell_{t-1} + b_{t-1}) e_t \] (A.10)
Damped Local Trend Model

Additive Error

\[ y_t = \ell_{t-1} + b_t e_t \]  \hspace{1cm} (A.11)
\[ \ell_t = \ell_{t-1} + b_t + \alpha e_t \]  \hspace{1cm} (A.12)
\[ b_t = \phi b_{t-1} + \beta e_t \]  \hspace{1cm} (A.13)

Multiplicative Error

\[ y_t = (\ell_{t-1} + b_t)(1 + e_t) \]  \hspace{1cm} (A.14)
\[ \ell_t = (\ell_{t-1} + \phi b_{t-1})(1 + \alpha e_t) \]  \hspace{1cm} (A.15)
\[ b_t = b_{t-1} + \beta (\ell_{t-1} + \phi b_{t-1}) e_t \]  \hspace{1cm} (A.16)

A.2 Multivariate Models

Vector Local Level Model

\[ y_t = \ell_{t-1} + e_t \]  \hspace{1cm} (A.17)
\[ \ell_t = \ell_{t-1} + A_t e_t \]  \hspace{1cm} (A.18)

Vector Local Trend Model

\[ y_t = \ell_{t-1} + b_{t-1} + e_t \]  \hspace{1cm} (A.19)
\[ \ell_t = \ell_{t-1} + b_{t-1} + A_t e_t \]  \hspace{1cm} (A.20)
\[ b_t = b_{t-1} + A_t e_t \]  \hspace{1cm} (A.21)
Vector Damped Local Trend Model

\[ y_t = \ell_{t-1} + b_{t-1} + e_t \]  \quad (A.22)
\[ \ell_t = \ell_{t-1} + b_{t-1} + A_t e_t \]  \quad (A.23)
\[ b_t = \Phi b_{t-1} + A_b e_t \]  \quad (A.24)

A.3 New Multivariate Models

VISTS-AR(1)

\[ y_t = \ell_{t-1} + c_{t-1} + e_t \]  \quad (A.25)
\[ \ell_t = \ell_{t-1} + A_t e_t \]  \quad (A.26)
\[ c_t = \Psi_1 c_{t-1} + A_c e_t \]  \quad (A.27)

VISTS-AR(2)

\[ y_t = \ell_{t-1} + c_{t-1} + e_t \]  \quad (A.28)
\[ \ell_t = \ell_{t-1} + A_t e_t \]  \quad (A.29)
\[ c_t = \Psi_1 c_{t-1} + \Psi_2 c_{t-2} + A_c e_t \]  \quad (A.30)

VISTS-AR(3)

\[ y_t = \ell_{t-1} + c_{t-1} + e_t \]  \quad (A.31)
\[ \ell_t = \ell_{t-1} + A_t e_t \]  \quad (A.32)
\[ c_t = \Psi_1 c_{t-1} + \Psi_2 c_{t-2} + \Psi_3 c_{t-3} + A_c e_t \]  \quad (A.33)
B Effective Full Time unemployment rate calculation

The Effective Full time Unemployment Rate addresses this by calculating the effective number of full time jobs. This is achieved in two steps:

1. Aggregate Hours divided by 150 (this figure represents the number of hours per month a full timer works). This gives the effective number of full time positions in the work force.

2. The number calculated in 1 is then divided by the number of persons in the labour force and then multiplied by 100.

The measure is captures the relative (as opposed to the absolute) health of the labour market.