Predicting Sustainable Retirement Withdrawal Rates Using Valuation and Yield Measures

Wade Donald Pfau

National Graduate Institute for Policy Studies (GRIPS)

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by

Wade D. Pfau
Associate Professor
National Graduate Institute for Policy Studies (GRIPS)
7-22-1 Roppongi, Minato-ku, Tokyo 106-8677 Japan
Email: wpfau@grips.ac.jp
phone: 81-3-6439-6225

Abstract

This study attempts to quantify whether a 4 percent withdrawal rate can still be considered as safe for U.S. retirees in recent years when earnings valuations have been at historical highs and the dividend yield has been at historical lows. We find that the traditional 4 percent withdrawal rule is likely to fail for recent retirees. The maximum sustainable withdrawal rate (MWR) for retirees may continue declining even after the peak in earnings valuations in 2000. Our lowest point estimate for an MWR with a 60/40 allocation between stocks and bonds is 1.46 percent for new retirees in 2008. We also discuss confidence intervals for these predictions. The regression framework with variables to predict long-term stock returns, bond returns, and inflation (the components driving the retiree's remaining portfolio balance) produces estimates that fit the historical data quite well, and we use backtesting for a further robustness check. Nevertheless, there are important qualifications for these predictions. In particular, they depend on out-of-sample estimates as the circumstances of the past 15 years have not been witnessed before, and there is always potential for structural changes which could leave recent retirees in better shape than suggested by the model. Looking forward, this methodology can guide new retirees toward a reasonable range for their MWR so that the 4 percent rule need not be blindly followed.

JEL Codes: C15, C20, D14, G11, G17, N22

Keywords: safe withdrawal rates, retirement planning, market valuation, price-earnings ratio, dividend yield, stock returns, bond returns

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Introduction

Campbell and Shiller (1998) find that earnings valuation ratios provide predictive power for long-term stock market returns. Most notably, the dividend-price ratio (DY) and the ratio of current stock price to average real earnings over the previous 10 years (PE10) are both statistically significant predictors of the subsequent 10-year real returns on stocks. Figure 1 updates a portion of Campbell and Shiller’s findings, showing that PE10 in logged form and DY respectively explain 30.3 and 20.3 percent of the variation in the subsequent 10-year real stock returns. Extending this approach to real bond returns, Figure 1 also shows that nominal bond yields explain 16.7 percent of the variation in real bond returns over the subsequent 10 years.

We aim to investigate how well valuation and yield measures predict the maximum sustainable withdrawal rate (MWR) that a person can successfully use with their retirement savings to obtain inflation-adjusted income over a 30-year period. MWRs represent a sustainable spending rate from savings, and Arnott (2004) argues that sustainable spending rates are not fixed numbers. Rather, they change with changing yields. He reminds that the key components of returns are income, growth, and changing valuation multiples. For stocks, returns depend on their dividend yield, earnings growth, and any changes in the valuation placed on earnings. If the current dividend yield is below its historical average, then future stock returns will also tend to be lower. In contrast, when the PE10 is low, markets tend to exhibit mean reversion and relatively higher future returns can be expected. Returns on bonds, meanwhile, depend on the initial bond yield and on subsequent yield changes. Low bond yields will tend to translate into lower returns due to less income and heightened interest rate risk.

While MWRs are closely connected to long-term asset returns over the 30-year horizon, MWRs also exhibit important differences from straightforward geometric real returns. MWRs are based on a portfolio of assets split between stocks and bonds. MWRs also experience sequence of returns risk, as withdrawals are made periodically from savings so that the retiree's geometric returns will not be the same as those for the underlying assets. As such, withdrawals made early in the retirement period have a bigger impact on the final outcomes than withdrawals late in the retirement period. It can be difficult for retirees to recover from early losses. Fullmer (2008) and Pfau (2010b) provide further discussion of this issue. MWRs are also sensitive to when the lower bound of zero wealth is reached, which make MWRs particularly sensitive to asset volatility and could cause the MWR to be lower than suggested by the geometric returns of the underlying assets.
assets. Inflation is also specifically important to MWRs in a manner beyond inflation’s impact on real asset returns, since withdrawal amounts are adjusted annually to reflect the cumulative inflation since retirement. High inflation will compound the difficulty of sustaining a real withdrawal amount.

For a retirement portfolio split between stocks and bonds, the MWR over a 30-year horizon is the initial withdrawal amount as a percentage of savings at retirement that can then be adjusted for inflation in subsequent years and will provide income for precisely 30 years. Bengen (1994) originated a methodology for finding a safe real withdrawal rate using historical data. His “SAFEMAX” is the minimum of all the MWRs in the historical period. Using data on the U.S. S&P 500 and intermediate-term U.S. government bonds (ITGB) since 1926, he suggested that a withdrawal rate of 4 percent of the portfolio value at retirement, which can then be adjusted for inflation in subsequent years, will safely provide income for at least 30 years when the stock portion of the portfolio is between 50 and 75 percent.¹

Figure 2 provides an illustration of the 30-year MWRs for each retirement year between January 1883 and January 1980 using the data on large-capitalization stocks and 10-year U.S. government bonds from Robert Shiller’s webpage (http://www.econ.yale.edu/~shiller/data.htm). With this data, the SAFEMAX is below 4 percent. The MWR fell under 4 percent for retirements beginning in January of 1965, 1966, 1968, and 1969, even under the extreme assumption (called “perfect foresight”) that allows new retirees in each year to pick the fixed asset allocation over their retirement period that provides the largest MWR. MWRs are shown for allocations of 100 percent bonds, a 60/40 split between stocks and bonds, and the "perfect foresight" MWR just described. The bottom part of Figure 2 shows the corresponding optimal asset allocation for the perfect foresight case, indicating that 100 percent stocks provided the highest MWRs in most retirement years. Looking specifically at the 60/40 portfolio of stocks and bonds, MWRs ranged

¹ MWRs vary from study to study because they are quite sensitive to underlying assumptions including asset allocation, whether any fees are deducted, whether withdrawals occur at the beginning or end of the period, and how frequently the portfolio is rebalanced. Also, in particular, MWRs are sensitive to the dataset used, and they tend to be highest when the bond component is the above-mentioned ITGB from Ibbotson Associates' Stocks, Bonds, Bills, and Inflation (SBBI) monthly data on total returns for U.S. financial markets since 1926. When the stock component is the S&P 500, ITGB keeps the SAFEMAX above 4 percent, but the SAFEMAX is less than 4 percent when long-term government or corporate bonds are used. On the other hand, Bengen (2006) finds that including small-capitalization stocks with the S&P 500 and ITGB can increase the SAFEMAX to about 4.5 percent.
from a high of over 10 percent in 1921 and 1922, to a low of under 4 percent between 1963 and 1969. With such volatility in past MWRs, new retirees might certainly welcome guidance about the realistic range of MWRs for their own retirements.

Debates about whether such guidance can realistically be provided have raged on Internet message boards related to personal finance since as early as 2002. Rob Bennett and John Walter Russell are perhaps the earliest and strongest champions of applying earnings valuations to withdrawal rate studies. Bennett (2010) explains his use of a regression model with PE10 as the explanatory variable to predict that the safe withdrawal rate for a new retiree in 2000 with an 80/20 asset allocation is 2.02 percent. This estimate seemingly comes from a lower confidence limit derived using the methodology described in Russell (undated). Bennett's Retirement Risk Evaluator spreadsheet lets users choose various parameters to develop personalized MWR estimates. As well, Otar (2009) uses the price-earnings ratio as an indicator of how much longer a retiree's portfolio will sustain withdrawals. Bob's Financial Website (2008) provides a compendium of resources about earnings valuations and MWRs. He plots MWRs for a 50/50 asset allocation with data between 1926 and 1971 against EY10 and finds that EY10 explains 76 percent of variation in MWRs. He is agnostic about the implications of this finding. Greaney (2002) considers similar evidence and is not persuaded that market valuations imply a need for recent retirees to reduce their withdrawal rates below 4 percent. We cannot find any instances in the existing literature of regression analysis that considers more than just EY10 or PE10 as an explanatory variable.

Kitces (2008a) provides a more formal study of the relationship between MWRs and PE10, though he does not use regression analysis. He divides the historical PE10 values into quintiles and then shows the lowest and highest MWRs within each quintile. He concludes that retirees should be extra cautious when retiring at times with high PE10 ratios, but he still expects that a 4.5 percent withdrawal rate will be safe. His focus was more in the other direction, i.e. that retirees who observe a low PE10 value at retirement (below 12) could safely increase their withdrawal rate to 5.5 percent. In a subsequent blog post, though, Kitces (2008b) notes that he was persuaded by Bennett that the previously unseen high valuation levels of the late 1990s and early 2000s could potentially lead to new lows for MWRs in those years.

Related to Kitces (2008b) admission, Figure 3 illustrates why recent retirees may be extra concerned about the impacts of earnings valuations on their retirements. It is natural to think that withdrawal rate studies based on historical data will have accounted for the full range of observed
valuation levels, and that the 4 percent rule has withstood the test of time. This is correct only to an extent. If one is using asset returns data through the end of 2009 and wishes to consider the MWR for a retirement lasting 30 years, then the SAFEMAX will be based on retirements beginning only up to 1980. Figure 3 provides scatterplots of the 60/40 MWR against EY10 and PE10. EY10 can explain 54.8 percent of the variation in MWRs, while the logarithm of PE10 explains 56.4 percent of this variation. Other sources cited above generally were based on the SBBI dataset since 1926. For the Shiller data, the $R^2$ for the period since 1926 is about 70 percent for both EY10 and PE10. This explanatory power is surprisingly high compared to what had first drawn Campbell and Shiller’s attention to 10-year stock predictions. Also equally intriguing, the figure shows just above the x-axis all the values for EY10 and PE10 for the years since 1980 when the corresponding MWRs are not yet known. For EY10, 1929 was the only year before 1980 that witnessed a value under 4 percent, while in 10 years since 1980 (since 1997, more specifically) EY10 values have been under 4 percent. And with PE10, the highest PE10 ratio in a January before 1980 was 27.08, which happened in 1929. Since 1980, though, the PE10 ratio has been above 27.08 in 8 years, including 1997-2002, 2004, and 2007. A highpoint of 43.77 occurred in January 2000. Figure 3 also shows the out-of-sample predictions from the simple regressions and finds that the EY10 regression is much more forgiving of low EY10 values than the PE10 regression is of high PE10 values. PE10 predicts worse MWR outcomes but is more vulnerable to out-of-sample estimation errors. For this reason, predictions for recent MWRs should rely more on EY10 than PE10.

// Figure 4 About Here //

Figure 4 provides greater detail by showing the time series plot for the 60/40 MWR, the 10-year average of real earnings divided by the current stock price (EY10), the 10-year moving average of dividend yields (DY10), and the nominal bond yield (I) at the start of the year. The figure shows that EY10 occasionally rises far higher than the MWR. For instance, in 1921 both EY10 and the MWR were at their highest levels in history, and EY10 was almost double the MWR at that time. But when EY10 is low, the MWR has generally fallen as well to stay slightly below. The bottom part of the figure shows the exceptions for this pattern, which tend to occur before 1900 and in the years around the start of the Great Depression. The most EY10 ever fell below the MWR was by 1.26 percentage points in 1929. Since 1931, EY10 has always been larger than the MWR. This tendency will need to be broken, though, if the MWRs for recent retirees can possibly be 4 percent. EY10 has been under 4 percent in 1997-2002, and 2004-2007. Its lowest value was 2.28 percent in 2000.
But Figure 4 also shows that it will be increasingly difficult for the MWR to rise above the EY10 in recent years, because the income portion of returns is also low in the 2000s compared to historical averages. DY10 has been under 3 percent since 1997 and under 2 percent since 2003. But before 1997, it had never fallen below 3 percent. The possibility exists that companies replaced dividend payments with stock buybacks that increase earnings per share, but Campbell and Shiller (1998) and Arnott (qtd. in Arends, 2010) do not find this explanation to be sufficient to eliminate worries about lower dividend yields. Meanwhile, though nominal bond yields have not reached their lows of the 1940s, their yields in recent years will not help much to raise MWRs. Altogether, this figure suggests that recent retirees may be in danger of finding that the 4 percent rule will not be sustainable.

This study attempts to quantify whether a 4 percent withdrawal rate can still be considered as safe for U.S. retirees in recent years when stock market valuations have been at historical highs and the dividend yield has been at historical lows. We find that the 4 percent rule is likely to fail for recent retirees. Unlike the previous analyses that only investigated PE10 or EY10 and predicted that the lowest MWR will be for 2000 retirees, we find that MWRs may continue to decline after 2000 as the lowest MWR of the 1940s, their yields in recent years will not help much to raise MWRs. Altogether, this figure suggests that recent retirees may be in danger of finding that the 4 percent rule will not be sustainable.

Methodology and Data

The maximum sustainable withdrawal rate (MWR) is the variable we seek to explain and predict. For each retirement year, it is the highest withdrawal rate that would have provided a sustained real income over a fixed 30-year retirement duration. At the beginning of the first year
of retirement, an initial withdrawal is made equal to the MWR times accumulated wealth. Remaining assets then grow or shrink according to the asset returns for the year. At the end of the year, the remaining portfolio wealth is rebalanced to the targeted asset allocation. In subsequent years, the withdrawal amount adjusts by the previous year’s inflation rate and the order of portfolio transactions is repeated. Withdrawals are made at the start of each year and are not affected by asset returns, so the current withdrawal rate (the withdrawal amount divided by remaining wealth) differs from the MWR in subsequent years. If the withdrawal pushes the account balance to zero, the withdrawal rate was too high and the portfolio failed. No attempt is made to consider taxes, which makes these findings applicable to Roth IRAs when considered on an after-tax basis. Also, we assume that retirees do not need to pay any portfolio management or advisor fees. For each retirement year, we calculate the MWR for portfolios with various combinations of stocks and bonds in five percentage point increments for a total of 21 asset allocation choices.

We develop a regression model with the intention of predicting the 30-year MWR for a retiree based on market information freely available at the time of retirement. For each regression specification, 21 regressions are run in order to provide distinct prediction models for each asset allocation. We will choose one regression specification to cover the various asset allocations, which suggests that the specification must include variables to predict real stock and bond returns and inflation. Campbell and Shiller established a link between real stock returns, PE10 (or EY10), and DY, suggesting that these variables belong in the regression specification. For bonds, the current bond yield (I) may provide reasonable predictive power as a high yield implies both that current income from the bond will be high and that any subsequent mean reversion to lower yields will raise bond prices and boost returns. Through the Fisher effect, the nominal bond yield consists of a real yield and expected inflation, and it may provide insight into future inflation rates since both real rates and inflation tend to show persistence. This suggests a parsimonious model to explain and predict MWRs:

\[
MWR_t = \beta_0 + \beta_1 EY10_t + \beta_2 DY10_t + \beta_3 I_t + \varepsilon_t
\]

for \( t \) ranging from 1883 to 1980. We also consider models with fewer variables, and models in which the logarithm of PE10 replaces EY10 as the earnings valuation indicator.

Our data is from Robert Shiller’s website (http://www.econ.yale.edu/~shiller/data.htm). The data includes stock index values, stock returns, bond yields, bond returns, the price index, aggregate dividends, and aggregate corporate earnings since 1871. After constructing the explanatory variables, we estimate a regression model using data since 1883. The PE10 measure
is the stock price in January divided by the average real earnings on a monthly basis over the previous 10 years. Campbell and Shiller justify this measure as a way to remove cyclical factors from earnings, though there is no particular theoretical reason to pick precisely 10 years. The EY10 measure is 100 divided by PE10. The dividend yield (DY) is aggregate dividends divided by the stock price. We find that a 10-year moving average for the dividend yield (DY10) provides a better model fit. This can be justified as a way to obtain the underlying trend in dividend payments after removing the cyclical trend in stock prices. Unlike EY10, the 10-year moving average for DY is not the average of previous dividends over current price, but rather the average dividend yield. Since EY10 already includes the current price, another variable is not needed for that. Bond yields (I) are for 10-year government bonds.

Obtaining confidence intervals for the predictions is important. But estimating correct standard errors is confounded by the problem of overlapping observations and serial correlation. With the Shiller data, there are 98 observations between 1883 and 1980, but because the MWRs are based on 30-year rolling intervals, and because EY10 and DY10 are constructed from 10 years of data, the number of nonoverlapping observations is less than 3. The standard errors obtained from the regression estimates are too small, indicating a degree of preciseness for the estimates that does not exist. The smallness of the standard errors is seen because the 95 percent confidence intervals for the predicted values from the historical data cover few actual historical data points. The Newey-West estimator is available as an alternative to correct the standard errors for overlapping observations, but we find that these estimates hardly make any improvement over ordinary least squares (OLS). This finding is confirmed by Britten-Jones, Neuberger, and Nolte (2010), who show that Newey-West and Hansen-Hodrick estimators provide standard errors that are still severely biased downward. These authors develop an alternative estimator. But their technique cannot be applied to MWRs, since MWRs cannot be separated into distinct period returns. In a given year, up to 30 retirees will each be withdrawing different amounts from their portfolios. As far as we can determine, econometric techniques have yet to be developed to obtain corrected standard errors for this situation.

As an expedient alternative, we estimate pseudo-confidence intervals in order to provide some idea about the uncertainty associated with the forecasts. We keep the standard errors from OLS, but when we determine the critical-value of the t-distribution to make 95 percent confidence intervals, we use a small value for the degrees of freedom in order to widen the confidence intervals so that they cover a larger portion of the historical data. Our unorthodox
confidence intervals illustrate a realistic range around the fitted values that is consistent with the variability found in the historical data.

**Model Fit and Predictions**

Figure 5 plots the adjusted-$R^2$ values of the explanatory power for MWRs provided by various variable combinations across the range of stock allocations. The differences in explanatory power between using EY10 and log(PE10) are negligible, and our explanations focus on the EY10 case. As we seek one model to cover all asset allocations, the parsimonious 3-variable model (EY10, DY10, I) explains on an adjusted-$R^2$ basis between 53.4 percent (for the zero stock allocation case) and 74.5 percent (for the 55 percent stock allocation case) of the variation in MWRs across the asset allocation range. If I is excluded, the combination of EY10 and DY10 provides just as much explanatory power for high stock allocations, but the combination performs miserably for low stock allocations. The figure does also suggest that there is value in considering more than just EY10 or PE10 alone when seeking to explain past MWRs, as including DY10 and I strongly increase the explanatory power across the range of allocations, and these variables have theoretical justification. Finally, the figure also shows that DY10 or I alone do not explain the MWRs as well as either EY10 or PE10.

Figure 6 shows the model fit and predictions for the 60/40 MWR. The top part of the figure shows the results for EY10 with DY10 and I, while the bottom uses PE10 instead. Both plots also show the pseudo-confidence intervals for the in-sample and out-of-sample model predictions. The figure illustrates the remarkable closeness of fit for the model predictions and the historical data. Visually, it is difficult to distinguish between the model predictions for historical data when comparing these two specifications. Focusing on the EY10 case, especially since the early 1960s, the fitted values of the regression are extremely close to the actual MWRs. Earlier, though, the fit is not always so precise. The biggest model failure occurred between 1909 and 1920, when the model predicts higher MWRs than actually observed. In 4 of those years, the difference was over 1 percentage point. The worst prediction across the historical period occurred in 1918, when the predicted MWR was 8.14 percent and the actual MWR was 6.11 percent. Figure 4 made clear why this mistake happened, as earnings valuation levels were very low and MWRs did rise to the same degree as EY10. Since 1920, the difference between actual and predicted MWRs was over 1 percentage point in only 6 years. In the late 1950s, the model did not
give sufficient warning for how quickly MWRs fell from their relative peak in 1949, and the model also overestimated MWRs by a small amount in 1929 and in the late 1930s. On the other hand, the model failed to predict the full rise in MWRs in the late 1940s and early 1950s. The confidence intervals with 1 degree of freedom provides coverage for 96 percent of the historical MWRs between 1883 and 1980.

In addition to showing the historical model fit, the other main purpose of the figure is to provide the model predictions for MWRs in the years since 1980. The model with EY10, DY10, and I predicts that MWRs continue rising in the early 1980s to a peak of 8.84 percent in 1982. Then they begin a long process of decline over the next 20 years. Predicted MWRs for the 60/40 asset allocation fall under 5 percent in 1992, under 4 percent in 1996, under 3 percent in 1999, and under 2 percent in 2003. EY10 experienced its lowest value in 2000, so studies using only that variable predict that the lowest MWR will be for the 2000 retiree. But when we add the income components of returns, we find that the worst was yet to come. The estimate for the 2000 retiree is 2.7 percent, but the decline continues until 2008 when the MWR reaches 1.46 percent. For 2010 retirees, it rises only to 1.82 percent. As seen in the figure, the predictions are even lower when using PE10, with MWRs under 2 percent since 1999 and the low reaching 1.25 percent in 2008. But for reasons discussed above, the estimates using EY10 should be more reliable. The confidence intervals for these estimates widen dramatically in the early 1980s when bond yields were at extreme highs. They narrow in the 1990s, but widen again in recent years as the values of the explanatory variables move out of the range of past observance. The upper limit for the confidence intervals fluctuate around 6 percent since the 1990s, though it is probably wishful thinking to hope that a 6 percent withdrawal rate can be sustainable.

To investigate the robustness of the findings, Figure 7 provides model predictions assuming this study was conducted using MWR data available through 1950. This year was selected because 1949 provided a relatively high MWR of about 8 percent for the 60/40 asset allocation. It will be interesting to see whether the regression model could have predicted the almost continuous decline in MWRs over the subsequent 15 years. The model predictions missed the high 1950 value, but they do track the subsequent decline. However, the model did overshoot the trough for MWRs in the 1960s by around 0.5 percentage points. As well, while the model did track the continuous rise in withdrawal rates throughout the 1970s, it did consistently underestimate MWRs by 1 to 2 percentage points for these years as well. This finding may make readers feel more comfortable that MWRs will not be as low in the 2000s as Figure 6 suggests.
On the other hand, the ability to track the actual trends in MWRs in the years since 1950 does indicate that an actual relationship exists between MWRs and market valuations.

**Asset Allocation**

Figure 8 shows both the asset allocation providing the highest MWR in each retirement year (which was also shown in the bottom of Figure 2) and the predicted optimal asset allocation from the regression with EY10, DY10, and I. The predicted optimal asset allocation is chosen as the allocation providing the highest predicted MWR across the 21 possible allocations. These asset allocations are the fixed values that the retiree uses during the entire 30-year period. The bottom part of the figure also shows the costs associated with using the predicted asset allocation instead of the optimal asset allocation, in terms of the decrease in the MWR experienced by that retiree. This figure shows that the model does a suitable job of providing asset allocation recommendations in the historical period. There were just two notable periods of mistakes. At the start of the Great Depression, the actual optimal stock allocation reduced to as low as 35 percent, but the model maintained a high stock allocation, resulting in an almost one percentage point lower MWR for several years. In 1979 and 1980, the model predicted a reduced stock allocation, while 100 percent stocks continued to support the highest MWRs. This mistake would have cost retirees almost 1.5 percentage points from the MWR. Those years witnessed the highest nominal bond yields in history, which may explain the model's preference for bonds.

Since 1980, the actual optimal asset allocations are not yet known, but the model continues predicting lower bond allocations throughout the 1980s and early 1990s. Interestingly, the predicted optimal asset allocation in 2000 is 100 percent bonds, but it returns to 100 percent stocks for all subsequent years. Retirees will generally find a 100 percent stock allocation to be too risky, and Dimson, Marsh, and Staunton (2004) and Pfau (2010a) explain how stock returns in the twentieth-century U.S. were unusually high from an international perspective. It is reasonable to expect them to be lower in the future even for reasons unrelated to earnings valuations and dividend yields. The findings of Figure 8 should not be treated as anything more than a speculative first look at how the regression can guide decisions about asset allocation.

**Conclusions**

Given the volatility of MWRs over the years, an important question is whether retirees could have any inclination for whether they were retiring at a time which would allow for a
relatively high or low MWR. Market valuation and yield measures at retirement may provide a
tool to help predict how much retirees can safely expect to withdraw from their portfolios. Our
findings do suggest that the issue should be taken seriously. The connection to MWRs is stronger
than Campbell and Shiller’s findings for 10-year stock returns. The predictive power is far from
perfect, but the fitted model closely predicts actual historical MWRs. The use of backtesting
further shows the ability of the model to make predictions extending beyond the sample data.

Nevertheless, it would be a great pity if retirees scale down their retirement expenditures
and live a more frugal lifestyle only to find at the end that a higher withdrawal rate could have
been sustainable. For this reason, the predictions must not be made lightly. A number of caveats
about these findings must be clear. First of all, though the fitted model does closely predict actual
outcomes, the predictions are not perfect, as the adjusted-R^2 never rises above about 75 percent.
For instance, the fitted model underestimated the 1949 MWR by 1.71 percentage points, and the
backtested model using data through 1949 would have guided retirees in the 1960s and 1970s to
withdraw 0.5 to 1 percentage points less than possible. As well, attempting to estimate the
MWRs of 2000-era retirees requires extreme out-of-sample predictions, as EY10 and DY10 were
reaching all-time lows beyond anything previously experienced. If the relationships outside the
sample range for EY10, PE10, and DY10 do not display the same linear relationship, then the
predicted MWRs may overstate (or understate) what will actually happen. The risk of data mining
must also be taken seriously, as though we have tried to be careful about using robust
assumptions, there remain a number of arbitrary choices. These include whether to use PE10 or
EY10, the choice of data set, the choice of nominal bond yields rather than real yields and
inflation, and the decision to use a 10-year moving average of the dividend yield. Other problems
include that the overlapping observations make it difficult to estimate confidence intervals, and
that changing economic circumstances may alter the relationship between MWRs and the
explanatory variables. Furthermore, the previous worst outcomes in modern U.S. history were for
retirees in the 1960s, and a fundamental difference from the 2000s is that inflation was
substantially higher before. If the nominal bond yield does not provide a proper specification for
inflation, then the lower inflation of today may prove material in supporting higher MWRs than
the model predicts. Finally, retirees must maintain flexibility and the fixed real withdrawal
strategy tested here is only a starting point to define baseline parameters about what may be
sustainable. Other asset classes such as a Treasury-Inflation Protected Bonds, small capitalization
stocks, international assets, real estate, commodities and other alternative investments, as well as
annuity products could all provide a way to diversify away from the risks of overvalued assets.
References
Figure 1
Earnings Valuations and Yields Against Subsequent 10-Year Real Returns

Return = 25.58 - 7.15 \times \log(PE10)
R^2 = 0.30321

Return = -0.1 - 1.46 \times DY
R^2 = 0.20297

Return = -0.52 - 0.62 \times I
R^2 = 0.16724

Figure 2
Maximum Sustainable Withdrawal Rates (MWR) by Asset Allocation

Note: See "Methodology and Data" section for full explanation of assumptions and data sources.
Figure 3
Maximum Sustainable Withdrawal Rates (MWR),
Earnings Yields (EY10) and Price-Earnings Ratios (PE10)
Asset Allocation: 60% Stocks, 40% Bonds

Note: See "Methodology and Data" section for full explanation of assumptions and data sources.
Figure 4
Maximum Sustainable Withdrawal Rates (MWR)
Earnings Yield (EY10), Dividend Yield 10-Year Moving Average (DY10), and Nominal Interest Rate (I)
Asset Allocation: 60% Stocks, 40% Bonds

Note: "Methodology and Data" section for full explanation of assumptions and data sources.
Figure 5
Fit of Regression Model Across Stock Allocations
Estimated for Retirement Years 1883-1980

Note: "Methodology and Data" section for full explanation of assumptions and data sources.
Figure 6
Fitted and Predicted MWRs
Asset Allocation: 60% Stocks, 40% Bonds
Estimated for Retirement Years 1883-1980

Explanatory Variables: EY10, DY10, I

Actual MWR
Predicted MWR
"95% Confidence Interval", Assumed degrees of freedom: 1

Explanatory Variables: PE10, DY10, I

Actual MWR
Predicted MWR
"95% Confidence Interval", Assumed degrees of freedom: 1

Note: See "Methodology and Data" section for full explanation of assumptions and data sources.
Figure 7
Fitted and Predicted MWRs - Robustness Check
Backtesting: Model Estimates Based on Data Through 1949
Asset Allocation: 60% Stocks, 40% Bonds

Note: See "Methodology and Data" section for full explanation of assumptions and data sources.
Figure 8
Optimal Asset Allocation – Actual and Predicted

Note: See "Methodology and Data" section for full explanation of assumptions and data sources.